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Private Labels in Marketplaces

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Abstract

This paper investigates the implications of vertical integration with private labels in the marketplace model opposed to the classic wholesale model. Differently from classic retailers, on a marketplace firms set end-consumer prices and the intermediary collects fees. When introducing a lower-quality version of a product, a marketplace owner does not have an incentive to increase the cost of the outside seller and foreclose him. In order to protect revenues from the seller channel, a marketplace owner overprices his product, compared to a retailer or stand-alone monopolist, and decreases the fee. I demonstrate that offering a lower quality is indeed optimal for both marketplace owner and classic retailer, with the former differentiating more from the seller's offering. This harms the seller less, but improves the consumer surplus less compared to a retailer.

JEL Classification: D21, D40; L12, L22, L42, L81;

Keywords: Marketplace, Private Labels, Online Platforms, Vertical Integration, Vertical Differentiation, Retailer, Vertical Contracts

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1 Introduction

The marketplace model, where sellers set end-consumer prices while marketplace owners collect fees, has become more prominent in the growing e-commerce space.¹ The combination of network effects and digital technologies has resulted in large marketplaces with considerable market power (e.g. Amazon in US, Flipkart in India, JD.com in China). Like classic retailers, marketplaces develop their own versions of products and position them in direct competition with hosted sellers. Regulation authorities in the EU and U.S. see such vertical integration as problematic as it creates a “dual” role for the marketplace: on one side it is a “gatekeeper” who sets the terms of access to the consumers side (e.g. fees, commissions), and on the other side it is a direct competitor. Potentially, a marketplace owner may leverage his market power in order to benefit his own product by raising the cost for rivals to compete on the marketplace or even foreclose outside sellers.

The strategy of developing an own version of a product to compete with an outside seller’s offering has been long used by retailers. Both theoretical and empirical research² have investigated the effects of a private label introduction in the classic wholesale structure. Results show that, by offering a lower quality aimed at consumers with lower willingness to pay, a retailer is able to increase his bargaining power vis-a-vis the seller (especially for niche categories), driving his wholesale price down. Consequently, such entry reduces prices, increases consumer surplus and the profits of the retailer, but cannibalizes the profits of the outside seller.

Given that a marketplace has a different vertical organization from a retailer, the impact of an introduction of a private label is not per se clear. While a retailer retains control over the pricing of all products, a marketplace owner determines the price of his own product by competing with the outside sellers. In addition, a marketplace owner imposes fees on his competitors, while sellers set a wholesale price for their products in the classic retail structure. Therefore, the effect of a private label in a marketplace on prices, outside seller’s profits, and consumer welfare are not straightforward.

The paper contributes to the ongoing regulatory debate by investigating how a private label strategy differs in the marketplace setting compared to a classic retailer structure. First, the introduction of a lower-quality private label in a marketplace forces the seller to reduce his price and forgo profits. Interestingly, the marketplace owner overprices the in-house product relative to a retailer or stand-alone monop-

¹Worldwide the share of revenues in the digital purchase sector attributable to marketplaces was roughly 50% in 2019 (Sabanoglu (2020)). In Europe marketplaces sales in 2020 accounted for 60% of e-commerce sales (RetailDetail (2020)).

²See Berges-Sennou et al. (2004) for overview and Draganska et al. (2010), Meza and Sudhir (2010) for empirical studies.

olist and has an incentive to decrease the fee he charges to the seller post entry. When it comes to the optimal quality of the in-house product, lower-quality entry is indeed preferred by both marketplace owner and retailer. The former, however, differentiates his private label more from the seller's product. This result is consistent with empirical observations showing Amazon's private labels having, on average, lower ratings (Marketplace Pulse (2019)) and their reviews revealing low quality (Marketplace Pulse (2019), Ellis and Hicken (2020)).

To understand the logic behind these results, consider a natural setting in which consumers differ in their valuation for products' quality. There is a seller, who is a monopolist in offering a quality product on the market, using the services of an intermediary (a marketplace or a retailer). If the intermediary is acting as a marketplace, it first sets a per-unit fee and then the seller sets the end-consumer price, taking the fee as given. In this sense the fee acts as a cost factor for selling on the marketplace. If the intermediary is acting as a retailer, the seller first determines the wholesale price of his product, then the retailer resells the product to the consumers. The intermediary (marketplace or retailer) can choose to introduce a private label. In such a case, this decision, as well as the choice of the private label's quality, takes place before the marketplace owner determines the fee or the seller sets the wholesale price. Following the empirical observations, the main focus in the paper falls on the introduction of a lower-quality private label.

To put the results into perspective, think briefly about the market outcome before a private label gets introduced. As already discussed by Johnson (2017) the marketplace and the wholesale vertical structures are a priori equivalent from the point of view of consumers. End-consumer prices are the same and so are demand and industry profits. However, the firm setting the fixed fee/wholesale price in the upstream contract is the one that extracts the larger share of the profits.

Suppose that a trade intermediary introduces a private label. Consider the pricing stage. In a marketplace the seller and the marketplace owner compete in prices. When there is a lower-quality private label, the seller has to best respond by lowering his price and reducing his markup to a competitive level. This makes his optimal pricing more responsive to changes in the fee, which he regards as cost, and passes more of the fee to the consumer side. The marketplace owner also prices competitively, but faces an opportunity cost in the form of the fee. Each unit his private label cannibalizes from the seller is a unit he does not collect fees from. If the fee is high enough, the price of the private label is going to exceed the one set by an independent monopolist. In contrast, a retailer sets both end-consumer prices as a multi-product monopolist and internalizes cannibalization effects. As a result, both products' optimal margins are independent from each other, and the retailer prices the private label as if he was an independent monopolist in the market.

Turning to the upstream contract stage, there are two main channels influencing the optimal fee decision in the marketplace. First, because of the lower-quality alternative offered to consumers, the demand for the seller's product decreases. This directly reduces revenues for the marketplace owner from the fee channel. Additionally, because the seller reacts by pricing more competitively, the share of the fee he passes on to the consumer side increases, which further decreases revenues for the marketplace owner. These two effects together push the marketplace owner to greatly reduce his margin on the seller's price. Second, the marketplace owner has an interest to collect at least as much margin as he does from his own product. The second channel only partially offsets the reduction in the fee from the first channel, resulting in overall lower fee post entry. The fee is however at a level, which results in overpricing of the private label above a stand-alone monopolist or retailer. In the classic retail structure the optimal wholesale price decreases as well. However, because the end-consumer prices are independent, the sole drive in this reduction is the demand-stealing effect.

There are a few general similarities in the two vertical structures. The introduction of a private label always drives the price of the seller down - the better the private label, the stronger the discount. Still, due to the overpricing of the private label, in a marketplace the prices are higher. Further, the trade intermediary has more demand than the seller, meaning that in the case of Amazon, a private label is bound to have a higher ranking and even become best seller. However, because the marketplace owner protects his fee revenue channel, the seller has relatively more sales on a marketplace than in a classic retail structure. Inherent from the pre-entry case, for a given quality of the private label, the seller generates higher revenues in the classic retail structure. These, nonetheless, get squeezed out in both structures after the introduction of the private label.

When it comes to the optimal choice of the private label quality, the marketplace owner further disadvantages the in-house product by choosing relatively lower quality than the retailer. This offsets the fact that for the same quality of the private label seller's profits are lower in marketplace mode than in retailer setting, and at the end, the seller is actually less harmed by marketplace-owner entry than by retailer entry. Lastly, because a marketplace offers a lower, but overpriced quality, consumer surplus is higher in the retailer setting.

The literature on the implications of vertical integration in marketplaces and platforms is rapidly growing with papers focusing on different aspects of the problem. As far as my knowledge, however, there is no other paper, which addresses the question of how vertical integration through private labels differs, depending on the business model implemented. Additionally, none of these papers allows for endogenous choice of quality (or level of differentiation) of the private label. Hagiu et

al. (2021) investigate the welfare implications of platform’s choice of operating mode (either pure seller or pure marketplace) following a ban of “dual” mode. Anderson and Bedre-Defolie (2021) consider the effects of vertical integration by a platform on product variety. Etro (2021) explores the incentives for a platform owner to promote its own product for all sales in a category and whether this decision is aligned with consumers’ interest. Hervas-Drane and Shelegia (2020) address the question on retailer opening up a marketplace in order to learn about new product categories and then, due to capacity constraint, decides which he will fully overtake as reseller, which he will leave to third-party sellers, and in which he will co-exist with the outside seller, while enjoying a fixed share of the demand of inattentive consumers for the same homogeneous good. Madsen and Vellodi (2021) and Lam and Liu (2020) investigate the role of data when demand is uncertain, with the former addressing the implications of vertical integration by the platform with respect to innovation incentives. Padilla et al. (2020) address the question on the use of consumer and seller data by a vertically integrated platform for the purpose of improving the overall quality of the intermediation service. Zenny (2020) focuses on the possibility for a platform to bias the search algorithm in its own favor, when consumers have to search for prices. Finally, Gautier et al. (2021) and Pouyet and Trégouët (2021) consider the importance of network effects.

This paper borrows from and contributes to the literature on private labels in classic retailer setting (Mills (1995), Bontems et al. (1999), Raju et al. (1995), Scott Morton and Zettelmeyer (2004), etc.³). The closest to the analysis provided here are the papers by Bontems et al. (1999) and Heese (2010). Both papers analyze the incentives for a retailer to introduce a private label and the optimal choice of its quality under the assumption that the retailer has a marketing disadvantage which results in relatively higher cost for quality. This drives the retailer to choose a lower quality and carry both products in equilibrium. In the limit, when the cost-disadvantage disappears, the retailer chooses a lower quality as long as the heterogeneity of consumer preferences is not too high (in which case seller gets foreclosed). I show that a marketplace owner has even less incentive than a retailer to copy the outside firm’s product, as this means more loss of rents from the higher-quality good.

The research is also related to the literature on vertical relations and different pricing structures in vertical markets. The move from wholesale (seller sets wholesale price, retailer sets consumer price) to agency model (retailer determining revenue share, while seller sets end price) is theoretically founded in Johnson (2017). He shows that this raises overall surplus as well as increases the downstream firm’s

³For a summary see Berges-Sennou et al. (2004)

profit at the expense of the upstream firms. Gautier et al. (2019) show how in search markets a hybrid of the two can arise. Hagiu and Wright (2015) discuss whether an intermediary wants to act as a reseller or marketplace depending on whether marketing activities can create spillovers between products. Complementing these studies the model in this paper accounts for vertical integration and draws a line of comparison between traditional retailer and marketplace setting, but does not consider the choice of optimal vertical contract.

The paper is structured as follows. Section 2 presents the model and discusses the main assumptions made with respect to the demand side of the market. The analysis starts with quick review of the vertical structures without the presence of an in-house product in Section 3, then proceeds with the implications of vertical integration in Section 4. Section 5 concludes. Proofs of propositions, as well as additional material can be found in the Appendix.

2 The Model

The supply side of the market is represented by an intermediary, either a marketplace (M) or a retailer (R), and a single seller (S) who offers product 1 of (exogenous) quality s_1 on the market through the intermediary. The intermediary can offer its own competing version of the seller's product denoted by 2 with quality s_2 . Providing quality is costly and the marginal cost of producing one unit of a good with quality s is given by an increasing convex function $c(s) = \alpha s^2$.

Depending on the vertical structure, the choice variables of the seller and the intermediary vary. In the marketplace setting, the marketplace owner collects a per unit fee f^4 on the sales of the seller, who takes it as given when setting the end-consumer price p_1 . If the marketplace owner offers a private label, he sets the end-consumer price p_2 . As prices are set simultaneously, this results effectively in price competition. In the classic wholesale setting, the seller offers his product to the retailer at a wholesale price w and the retailer sets the end-consumer price p_1 . If the retailer offers a private label, he sets both end-consumer prices p_1 and p_2 , acting as a multi-product monopolist.

The demand side is represented by a unit mass of consumers who have a heterogeneous preference for quality denoted by θ , which is distributed over the interval $[0, \bar{\theta}]$ according to some monotone density function $g(\theta)$. In the spirit of Mussa and

⁴Online marketplaces use various fee structures. For example Amazon sets a fixed per unit fee of \$/€0.99 for small sellers and an ad-valorem fee on end-consumer price for bigger sellers. Here I assume a fixed fee as it best represents a direct cost factor for the seller and it allows for more tractable analysis. In Section 4.6.1 I briefly discuss the case of an ad-valorem fee and show that the main results remain robust.

Rosen (1978) the utility of consumer i with preference parameter θ_i of purchasing a product of quality s_j for price p_j is given by $u(\theta_i, s_j, p_j) = \theta_i s_j - p_j$. If the price is too high, consumers have the option of not purchasing anything which gives them zero utility ($u(0) = 0$). This implies that the demand for the seller product of quality s_1 offered at price p_1 is given by $1 - G(\tilde{\theta}_1)$, where $\tilde{\theta}_1$ is the marginal consumer, who is indifferent between buying the seller's good or not, i.e. $\tilde{\theta}_1 s_1 - p_1 = 0 \Rightarrow \tilde{\theta}_1 = \frac{p_1}{s_1}$. In the presence of a, for example, lower-priced $p_2 < p_1$ lower-quality $s_2 < s_1$ private label there is an indifferent consumer $\hat{\theta} = \frac{p_1 - p_2}{s_1 - s_2}$, who is indifferent between buying the two products. In this case the demand for the high quality product is given by $1 - G(\hat{\theta})$ and the demand for the lower-quality product is given by $G(\hat{\theta}) - G(\tilde{\theta}_2)$.

Depending on the vertical structure implemented, the order of decisions varies. The game in the marketplace setting evolves in the following way. In the first stage the marketplace owner decides whether he wants to introduce a private label and chooses its quality. Then, depending on that decision, he sets the fixed fee for selling on the marketplace. In the final stage, the seller sets the end-consumer price taking the fee as given, while the marketplace owner sets the end-consumer price for the private label. The game in the retailer setting unfolds as follows. In the first stage the retailer decides whether he wants to offer a private label and chooses its quality. Observing this decision in the second stage, the seller sets the wholesale price. Finally, the retailer sets both end-consumer prices, taking the wholesale price as given. As these are dynamic games of complete information the solution method applied is Subgame Perfect Equilibrium.

In order to guarantee unique solutions, as well as introduce necessary simplifications at certain stages of the analysis, the following assumption over the demand generating distribution of preferences $g(\theta)$ is made:

Assumption 1. *The demand generating probability distribution function of preferences $g_{(\mu, \sigma, \xi)}(\theta)$ belongs to the family of Generalized Pareto Distributions*

$$g_{(\mu, \sigma, \xi)}(\theta) = \begin{cases} \frac{1}{\sigma} \left(1 + \frac{\xi(\theta - \mu)}{\sigma}\right)^{-\frac{1}{\xi} - 1} & \text{for } \xi \neq 0 \\ \frac{1}{\sigma} \exp\left(-\frac{\theta - \mu}{\sigma}\right) & \text{for } \xi = 0 \end{cases}$$

with location parameter $\mu = 0$, scale parameter $\sigma > 0$, and shape parameter $\xi \leq 0$.

An important property of the family of Generalized Pareto Distributions is that the Mills' ratio is a linear function given by $\frac{1 - G(\theta)}{g(\theta)} = \gamma(\theta) = \sigma + \xi\theta$, hence $\gamma'(\theta) = \xi$ is constant.⁵ In order to guarantee uniqueness of equilibria in the pricing stage, it is assumed that $\xi \leq 0$, or that $g(\theta)$ is log-concave. This includes the families of

⁵See most recently used by Rhodes et al. (2021). A nice overview of this property is also discussed in Johnson and Myatt (2015), which expands Itoh (1983).

uniform ($\xi = -1$) and exponential distributions ($\xi = 0$). The highest valuation parameter is then $\bar{\theta} = -\frac{\sigma}{\xi}$.

Throughout the analysis of this paper I assume that the qualities offered in the market by either the seller or the intermediary are viable ($\bar{\theta}s_i - c_i > 0$ and $\bar{\theta} - c'_i > 0$ for $i = 1, 2$) and efficient ($\bar{\theta}s_i - c_i > \bar{\theta}s_j - c_j$ for $i \neq j$). As the main focus falls on the private label strategy of the intermediary, I do not model a quality choice by the outside seller. In Section 4.4 I demonstrate that entry with a lower-quality private label is optimal conditional on the seller offering the monopolist quality. This is equivalent to assuming the seller was offering the (socially) optimal quality pre entry (*i.e.*, maximizing $\bar{\theta}s_1 - c_1$).⁶

3 The Case Without Private Label

To put the differences in vertical structures arising due to vertical integration into perspective, this section reviews the case without private label. The results here are not new and have been addressed in Johnson (2017). In general, one of the firms sets the terms of trade (either fixed fee or wholesale price), and the other firm sets the end-consumer price. Given that there is only one product offered, this results in the same level of double marginalization in both vertical structures leading to the same end-consumer price, demand and industry profits. The only difference is in the allocation of industry profits - under the assumption of log-concave demand the firm moving on the upstream level (fee/wholesale price) extracts a larger share of industry profits. Hence, an intermediary is better off, when the marketplace model is implemented, which explains why Amazon has been pushing sellers from its retail to its marketplace business (see Berthene (2019)).

To see this in more detail, consider the marketplace model and the classic wholesale model with the seller offering product 1 with quality s_1 . The game has two stages - in the first stage (1S) the per-unit fixed fee f or wholesale price w , respectively, are set. In the second stage (2S) the end-consumer price p_1 for the seller's product is chosen. The demand for a single product with quality s_1 is $(1 - G(\tilde{\theta}_1))$, where $\tilde{\theta}_1 = \frac{p_1}{s_1}$ is the marginal consumer buying the last unit. The profit functions are given by:

$$\begin{array}{ll} 1S & \max_f \pi_M = f(1 - G(\tilde{\theta}_1)) & \max_w \pi_S = (w - c_1)(1 - G(\tilde{\theta}_1)) \\ 2S & \max_{p_1} \pi_S = (p_1 - f - c_1)(1 - G(\tilde{\theta}_1)) & \max_{p_1} \pi_R = (p_1 - w)(1 - G(\tilde{\theta}_1)) \end{array}$$

⁶The optimal quality a seller offers is not affected by the type of vertical structure in the market, and is the same as if he was an integrated monopolist in an independent market. This holds as long as the vertical contract specifies a per-unit price/fee.

The problems are equivalent for $f = w - c_1$. The optimization problem of the firm setting the end-consumer price yields:

$$p_1 - f - c_1 = p_1 - w = \frac{1 - G(\tilde{\theta}_1)}{g(\tilde{\theta}_1)} s_1 = \gamma(\tilde{\theta}_1) s_1 \quad (1)$$

Under the assumption of log-concave demand the RHS of the expression above is a monotone non-increasing function and there exists a unique optimal price p_1^* .

Now consider the first stage of the game (1S). As pointed out by Johnson (2017), one can alternatively assume that the players in the first stage choose end-consumer price p_1 , while w and f equilibrate respectively according to $w = p_1 - \gamma(\tilde{\theta}_1) s_1$ and $f = p_1 - \gamma(\tilde{\theta}_1) s_1 - c_1$ from optimality condition (1). This yields the same optimization problem:

$$\max_{p_1} \pi = (p_1 - \gamma(\tilde{\theta}_1) s_1 - c_1)(1 - G(\tilde{\theta}_1))$$

with solution to the F.O.C.:

$$p_1 - c_1 = \gamma(\tilde{\theta}_1) s_1 + [1 - \gamma'(\tilde{\theta}_1)] \gamma(\tilde{\theta}_1) s_1$$

The equation above represents the industry margin on each unit sold. From (1) the first term is the margin of the firm who sets the end-consumer price. Hence, the margin a seller in a wholesale model and a marketplace owner in the marketplace model collect is $[1 - \gamma'(\tilde{\theta}_1)] \gamma(\tilde{\theta}_1) s_1$. Thus, the firm moving first obtains larger share of the profits when $\gamma'(\tilde{\theta}_1) < 0$, or when demand is log-concave. Under Assumption (1) one can show that the margin $f = w - c_1$ is equal to the stand-alone monopolist margin (see Appendix).

4 The Implications of Vertical Integration

I now solve the game in the presence of a private label. Throughout the pricing and commissions stages, consistent with the empirical observations, I assume that both intermediaries offer a quality of the private label that is at most the quality of the seller, *i.e.*, $s_2 \leq s_1$. Later on I demonstrate that this is, in fact, optimal.⁷ Thus, the demand for the high quality product is given by $(1 - G(\hat{\theta}))$, where $\hat{\theta} = \frac{p_1 - p_2}{s_1 - s_2}$ is the indifferent between the two products consumer. The demand for the lower quality is given by $(G(\hat{\theta}) - G(\tilde{\theta}_2))$ with $\tilde{\theta}_2 = \frac{p_2}{s_2}$ being the marginal consumer buying the last unit in the market. I first start with a short analysis of the classic retail structure.

⁷Offering a higher-quality private label is briefly discussed in the Appendix.

4.1 Classic Retail Setting

Retailer's multi-product monopoly pricing By ways of backwards induction the analysis starts with the pricing stage, where the retailer sets both end-consumer prices. He takes the wholesale price w as given and it enters his optimization problem as a marginal cost:

$$\max_{p_1, p_2} \pi_R = (p_1 - w)(1 - G(\hat{\theta})) + (p_2 - c_2)(G(\hat{\theta}) - G(\tilde{\theta}_2))$$

If $\frac{s_1}{s_2}c_2 \leq w \leq \bar{\theta}\Delta s + c_2$ there is an interior solution to the optimization problem. The retailer always internalizes the cannibalization effect for the lower-quality product and prices it as stand-alone monopolist, *i.e.*, $p_2^R - c_2 = \gamma(\tilde{\theta}_2)s_2$. The markup set for the seller's product is at least the margin collected on the private label plus a premium for the higher quality it offers $p_1^R - w = (p_2 - c_2) + \gamma(\hat{\theta})\Delta s$, where $\Delta s = s_1 - s_2$. In case of constant-curvature demand, as demonstrated by Johnson and Myatt (2015), the premium adds up to the monopoly markup set by a stand-alone monopolist, or $p_1^R - w = \gamma(\tilde{\theta}_1)s_1$. Hence, if the seller does not adjust the wholesale price post entry, the end-consumer price remains the same. However, demand is going to shrink, as $\hat{\theta} > \tilde{\theta}_1$. This is the main drive in the reduction of the wholesale price, which is discussed next.

Optimal wholesale price After the introduction of the private label the seller adjusts the wholesale price according to the changed demand and competition conditions:

$$\max_w \pi_S = (w - c_1)(1 - G(\hat{\theta}))$$

This means that the optimal margin he sets on top of his cost is given by:

$$w^n - c_1 = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1^n}{\partial w} \right)^{-1}$$

where $\frac{\partial p_1^n}{\partial w}$ is the end-consumer price's pass-through with respect to the wholesale price after the introduction of the private label. Because the end-consumer prices of both products downstream are independent, the pass-through remains the same as before, *i.e.*, $\frac{\partial p_1^n}{\partial w} = \frac{\partial p_1^o}{\partial w} = \frac{1}{1-\gamma'}$. Therefore the only drive for change in the wholesale price is the demand-stealing effect of the lower quality in the market. As a result, the seller reduces the wholesale price with the decrease being larger the better the quality of the private label. This directly drives the end-consumer price p_1^R down.

4.2 Marketplace Setting

Marketplace's price competition outcome In a vertically integrated marketplace the seller and the marketplace owner set prices simultaneously maximizing profits:

$$\begin{aligned} \max_{p_1} \pi_S &= (p_1 - f - c_1)(1 - G(\hat{\theta})) \\ \max_{p_2} \pi_M &= f(1 - G(\hat{\theta})) + (p_2 - c_2)(G(\hat{\theta}) - G(\tilde{\theta}_2)) \end{aligned}$$

If the fee is not too high and the seller is not foreclosed,⁸ he sets a competitive margin factoring in the fixed fee as additional cost:

$$p_1^M - f - c_1 = \frac{1 - G(\hat{\theta})}{g(\hat{\theta})} \Delta s = \gamma(\hat{\theta}) \Delta s \quad (2)$$

Similarly, the marketplace owner sets a competitive markup for the private label, but also faces an opportunity cost in the form of the fee:

$$p_2^M - c_2 = \underbrace{(1 - \omega)\gamma(\tilde{\theta}_2)s_2 - \omega\gamma(\hat{\theta})\Delta s}_{\text{competitive margin}} + \underbrace{\omega f}_{\text{opportunity cost}} \quad (3)$$

where $\omega = \frac{g(\hat{\theta})s_2}{g(\tilde{\theta}_2)\Delta s + g(\hat{\theta})s_2} \in [0, 1]$ represents the ratio of the marginal change in demands for the seller product to private label product after a small change in price p_2 . As the private label demand depends on both the indifferent consumer $\hat{\theta}$ and the marginal consumer $\tilde{\theta}_2$, the fee is weighted down in the optimal price decision.

There exists a cutoff value for the fee, for which the price of the private label is going to be higher than the stand-alone monopoly price. For $f > \frac{\sigma s_1 + \xi c_1}{1 - \xi} + \frac{\xi}{1 - \xi} \gamma(\hat{\theta}) \Delta s$ the marketplace owner overprices the private label compared to an independent monopolist or a retailer. The first term $\frac{\sigma s_1 + \xi c_1}{1 - \xi} = f^o$ is equal to the initial pre-entry fee, which, in turn, is equal to the monopoly markup of an integrated monopolist for the higher-quality product.⁹ As long as the qualities offered in the market are efficient and viable, the higher quality always generates higher revenues and profits, as consumers have higher willingness to pay. Therefore, the marketplace owner receives higher revenues from the seller's channel than from the private label before adjusting the fee. How the fee changes after the introduction of the private label is subject to the next subsection.

⁸See Appendix for discussion on foreclosure level of fee \bar{f} .

⁹This means that even if the marketplace owner is limited in his abilities to extract the maximum fee pre entry, it still can be the case that the private label gets overpriced.

Optimal fee. Given the optimal pricing decisions of the two players, the marketplace owner chooses the optimal fee by maximizing the value function

$$\rho(p_1^*(f), p_2^*(f), f) = \underset{p_2}{argmax} \pi(p_1^*(f), p_2, f)$$

Taking the first derivative and applying the Envelope theorem, one obtains the expression of the optimal fee:

$$f^n = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1^n}{\partial f} \right)^{-1} + (p_2^M - c_2) \quad (4)$$

Basically, the marketplace owner sets a margin on top of the seller's markup while extracting at least the margin he collects from the private label. The first part $\gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1^n}{\partial f} \right)^{-1}$ is similar to the wholesale price in the retailer setting. Hence, the demand-stealing effect pushes the margin down. However, there is an additional effect. Because the seller needs to compete face to face with the marketplace owner on the downstream level, his price becomes more sensitive to changes in the fee and as a result passes more of the fee to the consumer side. In other words, the pass-through $\frac{\partial p_1^n}{\partial f} = \frac{1}{1-\gamma'} - \frac{\gamma'}{1-\gamma'} \frac{\partial p_2}{\partial f}$ increases post entry compared to pre entry $\frac{\partial p_1^o}{\partial f} = \frac{1}{1-\gamma'}$ and drives the fee further down. The revenue $(p_2^M - c_2)$ from the private label can only partially compensate for this decrease, resulting in lower fee post entry.

Despite the decrease in the fee, it is still above the threshold for which the private label gets overpriced (as discussed above). The intuition follows from the fact that the fee is collected on a higher margins product and is at least the margin of the private label. The latter in turn has the higher revenue channel margin as an opportunity cost. The results for the marketplace pricing and fee stages are summarized in the following proposition:

Proposition 1. *A marketplace owner reduces the fee he charges to the seller after the introduction of a private label and overprices the private label compared to a stand-alone monopolist or classic retailer.*

Proof. The technical proof can be found in the Appendix.

This result shows that, despite its “dual” role, a marketplace owner has no incentive to increase the cost of the outside seller and does not leverage its market power to benefit the own product. On the contrary, a marketplace owner disadvantages the own product by “overpricing” it, reduces the fee (a direct cost for the seller) and in this way protects the seller's revenue channel.

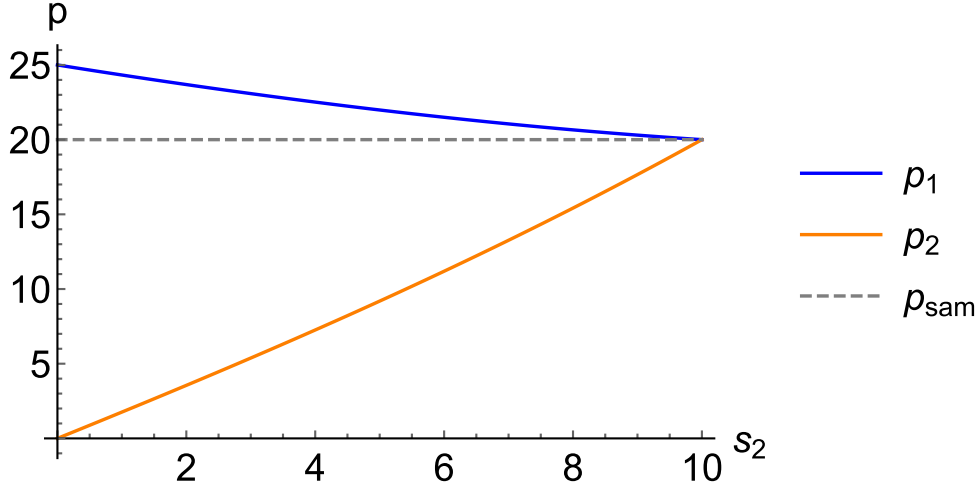


Figure 1: Price comparison in a marketplace for different $s_2 < s_1$, where p_{sam} is the stand-alone monopoly price for quality s_1 . Calibrated for uniformly distributed preferences, $\bar{\theta} = 3$, $\alpha = 0.1$, $s_1 = \frac{\bar{\theta}}{3\alpha} = 10$.

4.3 Intermediate Discussion

Before proceeding with the optimal quality stage, let us look at the implications for the market outcome in the marketplace setting and draw a comparison to the outcome in the classic retail structure for any quality $s_2 \leq s_1$.

Remark 1. *The better the quality of the private label s_2 on the marketplace, the higher its price and the lower the end-consumer price p_1 of the seller's product. In the limit, when $s_2 = s_1$, the prices of both products equal the stand-alone monopoly price.*

The result is well illustrated by Figure 1, which plots the end-consumer prices for $s_2 < s_1$. The margin the seller sets for his product is equal to $\gamma(\hat{\theta})\Delta s$ and as $s_2 \rightarrow s_1$ this one converges to zero. The better the quality of the private label the closer its price to the stand-alone monopoly price of the higher quality s_1 . In the limit, when $s_2 = s_1$, the marketplace owner can effectively foreclose the seller from the market, as he can undercut him and gain the whole demand.

In the retailer case the end-consumer prices behave in a similar way. As the quality of the private label improves, its price is going to increase as higher quality means higher willingness to pay by consumers and thus, higher prices. The decrease in the wholesale price drives the end-consumer price of the seller product down, and in a similar way $w = 0$ when $s_2 = s_1$, leaving the seller with zero profits. In this limit the retailer, as well, can slightly increase p_1^R and gain the whole demand.

The first difference between the two vertical structures is that the private label gets overpriced in a marketplace. Figure 2 shows the difference in marketplace and retail end-consumer prices. The lower the quality of the private label, the higher

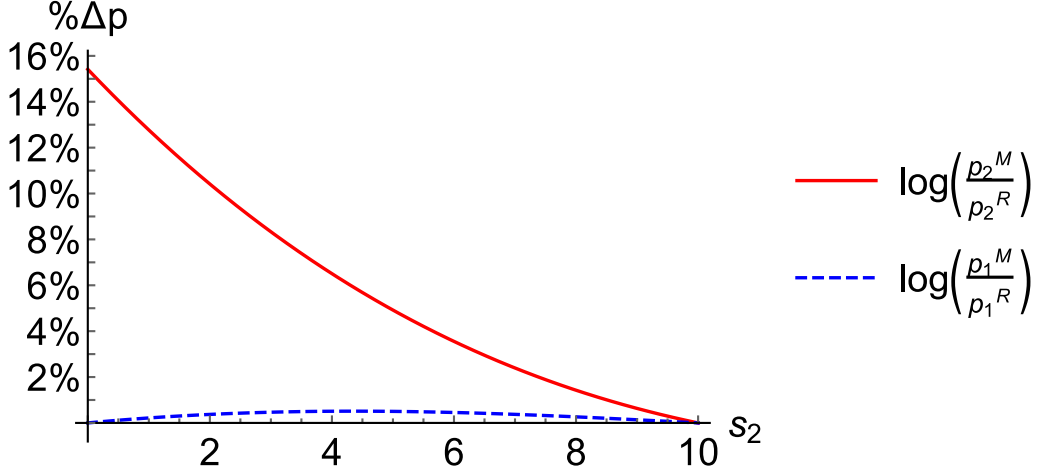


Figure 2: Differences in marketplace and retailer end-consumer prices for quality of the private label s_2 . Calibrated for uniformly distributed preferences, $\bar{\theta} = 3$, $\alpha = 0.1$, $s_1 = \frac{\bar{\theta}}{3\alpha}$.

its relative overpricing. This has a second-order effect on the end-consumer price set by the seller, which is also slightly higher in the marketplace than in the classic retail structure.

An auxiliary result following from the fact that a private label gets overpriced is that a seller ends up having higher sales with a vertically integrated marketplace than with a vertically integrated retailer. To see this consider the indifferent consumer $\hat{\theta} = \frac{p_1 - p_2}{s_1 - s_2}$. In the marketplace and the retailer setting this one is, respectively, given by:

$$\hat{\theta}^M = \frac{\gamma(\hat{\theta})\Delta s \left(1 + \left(\frac{\partial p_1^M}{\partial f}\right)^{-1}\right) + c_1 - c_2}{s_1 - s_2} < \hat{\theta}^R = \frac{\gamma(\hat{\theta})\Delta s \left(1 + \left(\frac{\partial p_1^R}{\partial w}\right)^{-1}\right) + c_1 - c_2}{s_1 - s_2}$$

The former is clearly smaller, as pre entry the end-consumer prices' pass-through is the same, but while it remains the same for the retailer case post entry, the pass-through in the marketplace increases, hence $\left(\frac{\partial p_1^M}{\partial f}\right)^{-1}$ is lower. In a similar way, the retailer has higher sales of the private label, as $\hat{\theta}^R$ is higher and $\tilde{\theta}_2^R = \frac{p_2^R}{s_2} < \frac{p_2^M}{s_2} = \tilde{\theta}_2^M$ is lower due to the overpriced private label in the marketplace setting. In total, more units of both products are sold in a retail structure than in a marketplace.

Because the seller's price suffers from double marginalization, his sales are lower than the sales of the private label in both vertical structures. This means that whenever a marketplace owner or a retailer introduce a cheaper version of a product, they will obtain higher sales than the seller. In the case of Amazon, this implies that its product will have a higher ranking compared to the seller and is bound to become best seller.

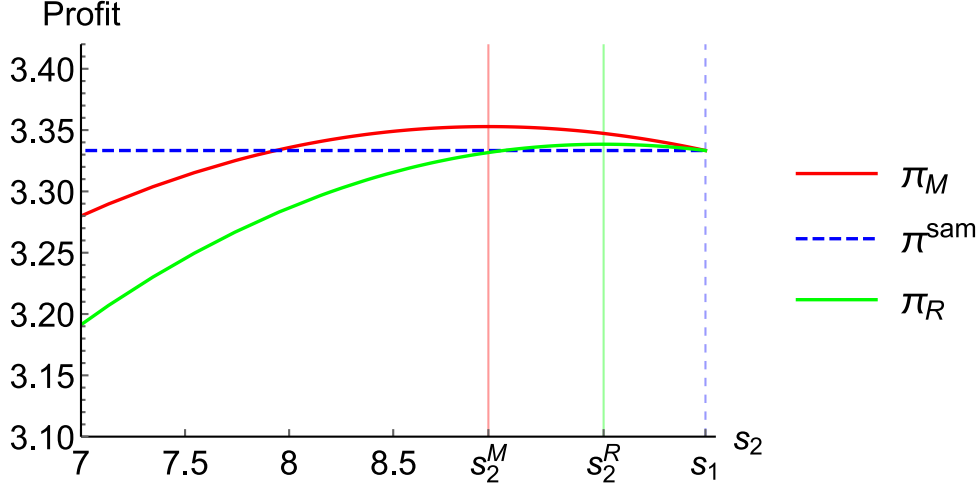


Figure 3: Profits of marketplace owner (red), retailer (green) and stand-alone monopolist with quality s_1 (blue, dashed) as a function of quality s_2 with respective maximum. Calibrated for uniformly distributed preferences, $\bar{\theta} = 3 s_1 = \frac{\bar{\theta}}{3\alpha}$, $\alpha = 0.1$.

Finally, as in the case without private label, a marketplace owner extracts a higher share of the revenues generated through the seller's product compared to a retailer. For a given quality of the private label s_2 , the seller is always left with more profits in the classic retail structure than in the marketplace setting. However, as we will see in the following section, this result gets reversed, once the quality of the private label is endogenous.

4.4 Optimal Quality of the Private Label

So far we have seen that the marketplace owner overprices the private label in order to protect revenues from the higher-quality product. In addition, the marketplace owner has no incentive to increase the cost of the outside seller, but on the contrary, decreases the fee post entry. In this section I derive the optimal quality a retailer and marketplace owner choose for the private label. To keep the analysis tractable, I assume that preferences are uniformly distributed and the cost function takes the form of $c(s) = \alpha s^2$. This implies that the optimal quality in a monopoly market offered by the seller is given by $s_1 = \frac{\bar{\theta}}{3\alpha}$.¹⁰

Proposition 2. *The marketplace owner optimally chooses lower quality than both the retailer's and the seller's quality.*

Proof. Technical proof is available in the Appendix.

The result is best illustrated by Figure 3, which plots the profits of the marketplace owner, the retailer, and a stand-alone monopolist, carrying the seller's product,

¹⁰See Appendix for proof that this is the optimal quality chosen by a stand-alone monopolist and is the same in both vertical structures.

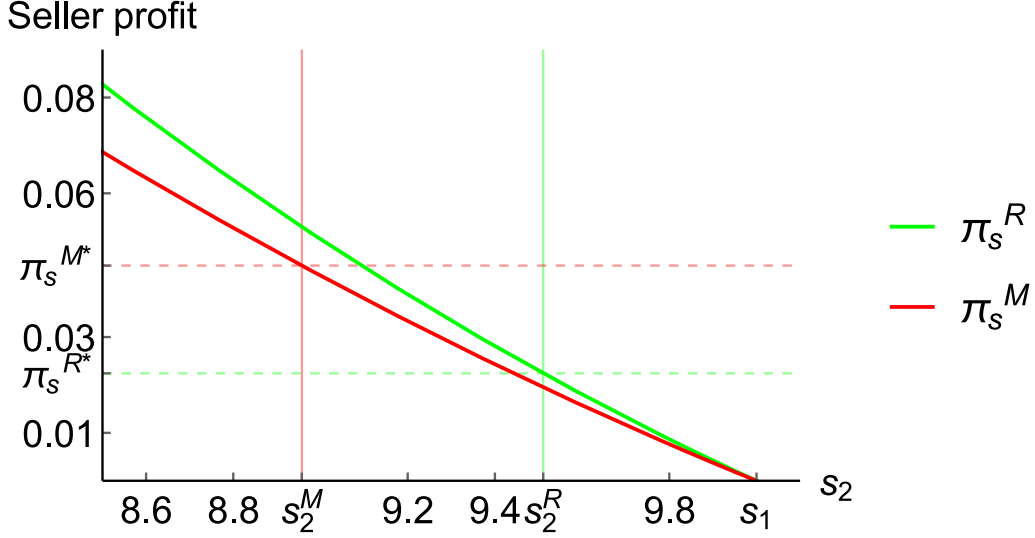


Figure 4: Profits of seller under marketplace (red) and retailer (green) mode with optimal quality of the private label for each mode. Calibrated for uniformly distributed preferences, $\bar{\theta} = 3$, $s_1 = \frac{\bar{\theta}}{3\alpha}$, $\alpha = 0.1$.

for a given level of the private label's quality. First, for all $s_2 < s_1$ the profit of the marketplace owner is always higher than the one of the retailer. This is inherent from the fact that the marketplace owner extracts more profits from the seller as a first-mover. This implies that a seller generates higher profits in the retailer structure compared with the marketplace for a given s_2 . Therefore, following the overpricing of the private label and decreased fee post entry, the marketplace owner will further protect the seller revenue channel by optimally choosing a lower than the seller's product quality for his private label, and will differentiate more than the retailer would do. The latter, as well, does not find it profitable to copy the quality of the seller, because he enjoys a stronger discount on the wholesale price through the competitive pressure of the private label, which attracts more demand for the higher-quality product.

4.5 Empirical and Welfare Implications

As discussed in section 4.3, the seller generates lower profits in a marketplace than in a classic retail setting for a given s_2 . However, as the marketplace owner differentiates more than the retailer when choosing s_2 in order to protect the seller's channel, this outcome gets reversed. As illustrated in Figure 4 the seller actually ends up with more profits after the introduction of the private label in a marketplace than in a retailer setting. The difference in profits $\pi_S^M(s_2^{M*}) - \pi_S^R(s_2^{R*})$ increases in the heterogeneity of consumers (as $\bar{\theta}$ increases) and decreases in the cost factor α . In other words, in markets in which consumers have stronger heterogeneity in preferences for

quality, this difference is more prominent.

The introduction of a new product in an uncovered market with vertically differentiated products always improves consumer surplus and overall welfare. However, because in the marketplace there is a relatively lower quality, which is overpriced, the consumer surplus gets less improved than in a classic retailer (see Figure 5). The same holds for total welfare. The differences increase in $\bar{\theta}$ and decrease in α .

Finally, the result that a marketplace owner chooses optimally a lower quality for his product is consistent with the observations made by Marketplace Pulse (2019) about Amazon's products having on average lower ratings and reviews revealing bad quality of the products.

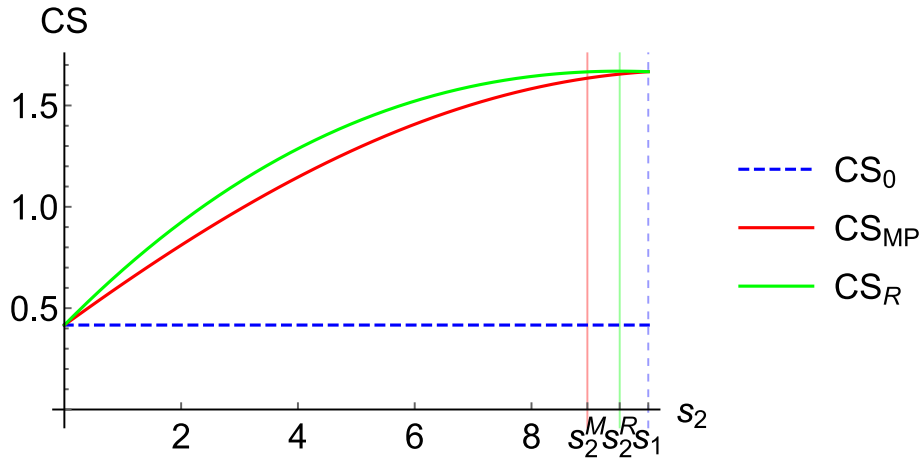


Figure 5: Consumer surplus pre (blue, dashed) and post entry under marketplace (red) and retailer (green) mode with optimal quality of the private label. Calibrated for uniformly distributed preferences, $\bar{\theta} = 3$, $\alpha = 0.1$, $s_1 = \frac{\bar{\theta}}{3\alpha} = 10$.

4.6 Extensions

Here I discuss briefly the implications of an ad-valorem fee opposed to per-unit fixed fee. Additionally, I provide a short discussion on non-linear contracts.

4.6.1 Ad-valorem Fee Contract

In reality, marketplaces like Amazon set a fixed fee for small-scale sellers, but implement an ad-valorem fee contract for large-scale sellers. The difference in the two vertical contracts in the absence of a private label is discussed by Johnson (2017). In general, if there is no private label introduced and there is only the intermediary and the seller offering his product, the end-consumer price is lower with ad-valorem fee than with fixed fee $p_1^*(r) < p_1^*(f)$. This leads to more demand and higher industry profits. The seller receives a lower share of these profits, which results in less

profits under ad-valorem than under fixed fee contract. The marketplace owner and consumers, however, are better off with ad-valorem fee contract.

Before proceeding with the results of this section, it must be noted that under the case with no private label it is already hard to express the optimal ad-valorem commission r in a closed form. As the introduction of a private label adds an additional layer of complexity in the analysis, the results presented here are (partially) an outcome of a numerical simulation. I assume uniform distribution of preferences and a cost function given by $c(s) = \alpha s^2$.

Let us first briefly go over the case without private label. The profit of the seller is given by

$$\pi_S = ((1 - r)p_1 - c_1)(1 - G(\tilde{\theta}_1))$$

and yields an optimal monopolist margin of $p_1 - \frac{c_1}{1-r} = \gamma(\tilde{\theta}_1)s_1$, where the ad-valorem fee r collected on the revenues enters the pricing decision as a cost-augmenting factor. The marketplace owner then sets r so that it maximizes

$$\pi_M = rp_1(1 - G(\tilde{\theta}_1))$$

The optimal ad-valorem fee r^o is then the solution of the following equation:

$$\frac{r^o}{1 - r^o} = \frac{p_1^o \gamma(\tilde{\theta}_1) s_1}{c_1} \left(\frac{\partial p_1^o}{\partial r} \right)^{-1}$$

As $\frac{r^o}{1 - r^o}$ is increasing in r^o , it allows us to track the changes on the right hand side and how they affect the level of the ad-valorem fee. In general, like the fixed fee, the ad-valorem fee decreases in the cost c_1 .

Consistent with the result, derived by Johnson (2017), the marketplace owner has higher revenues with ad-valorem fee than with fixed fee. One interesting result is that the revenue per unit with ad-valorem fee $r^o p_1^o > f^o$ is higher than the per-unit fixed fee. Recall that under Assumption (1) the pre-entry fixed fee f^o was equal to the stand-alone monopolist margin. This implies that with ad-valorem fee, the marketplace owner extracts an even higher revenue per unit than a stand-alone monopolist.

Then, let us again, following the evidence, focus on the case when a marketplace owner introduces a lower-quality private label. All of the results from the fixed fee contract carry on to the ad-valorem fee scenario. The profits of the seller and the marketplace owner change to:

$$\begin{aligned} \pi_S &= ((1 - r)p_1 - c_1)(1 - G(\hat{\theta})) \\ \pi_M &= r p_1(1 - G(\hat{\theta})) + (p_2 - c_2)(G(\hat{\theta}) - G(\tilde{\theta}_2)) \end{aligned}$$

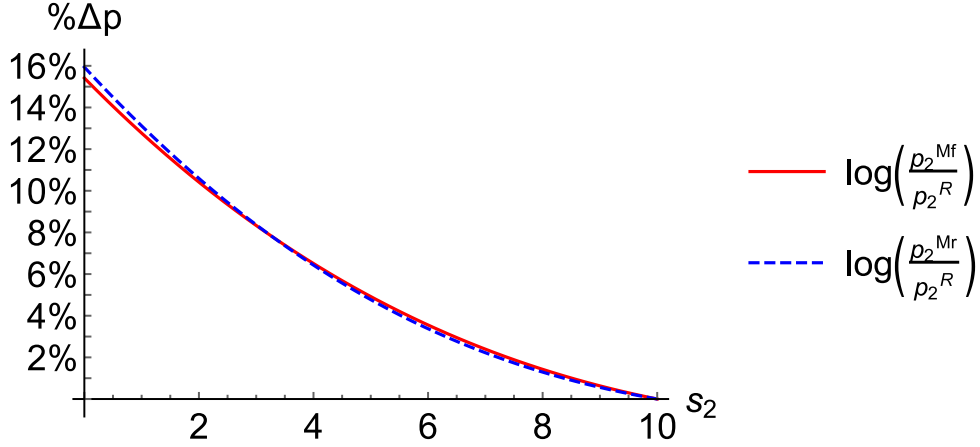


Figure 6: Differences in marketplace private label end-consumer prices under fixed (red) and ad-valorem (blue, dashed) fee to retailer's private label price for quality of the private label s_2 . Calibrated for uniformly distributed preferences, $\hat{\theta} = 3$, $\alpha = 0.1$, $s_1 = \frac{\hat{\theta}}{3\alpha} = 10$.

The seller switches to setting a competitive margin $p_1 - \frac{c_1}{1-r} = \gamma(\hat{\theta})\Delta s$.

The result that the marketplace owner overprices the private label compared to a retailer or independent monopolist carries on with ad-valorem fee contract. The share of the seller's price collected by the marketplace owner $r^n p_1^n$ serves the same purpose in the optimal expression for the private label price

$$p_2 - c_2 = (1 - \omega)\gamma(\hat{\theta}_1)s_2 - \omega\gamma(\hat{\theta})\Delta s + \omega r^n p_1^n$$

and represents an opportunity cost. The overall overpricing after the adjustment of the ad-valorem fee is similar to the case with a fixed fee (see Figure 6).

Further, the price of the seller decreases with the improvement of the private label quality due to the competitive pressure. Because it is lower than under fixed fee contract, the seller has higher sales with ad-valorem fee than with fixed fee, or with wholesale price.

After the introduction of the private label the optimal ad-valorem fee r increases. This, however, cannot be interpreted as an incentive for the marketplace owner to increase the cost of the seller, even though it has this indirect effect in his optimality condition. As pointed out by Bishop (1968) (who compares fixed and proportionate taxes imposed by the government on a monopolist), comparing effects of changes in a fixed fee and ad-valorem fee cannot be done in terms of $\frac{\partial p_1}{\partial f}$ and $\frac{\partial p_1}{\partial r}$, as f has the same dimensionality as p_1 , but r is a dimensionless ratio. Therefore, one has to consider and compare f with rp_1 .

The per-unit collected revenue rp_1 , like the fixed fee f , decreases after the introduction of the private label. The logic behind this result is, more or less, the same

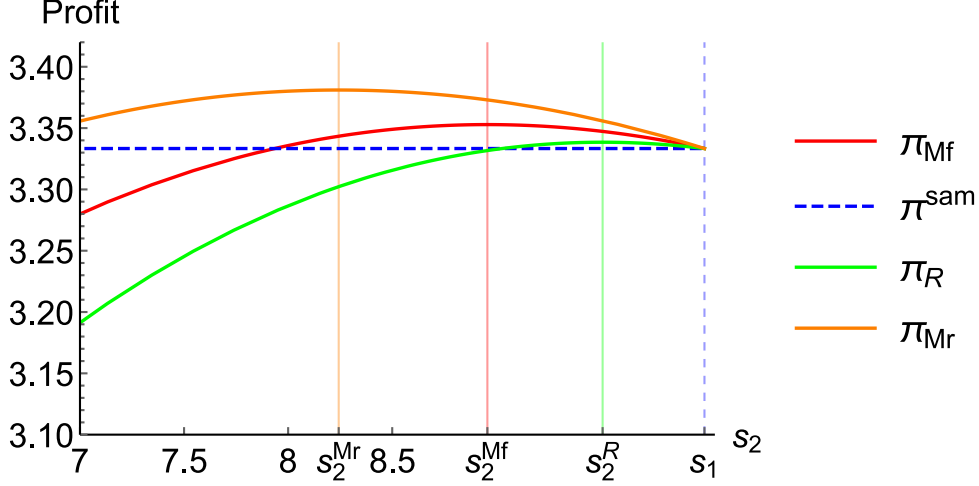


Figure 7: Profits of MPO with fixed (red) and ad-valorem (orange) fee, retailer (green), and stand-alone monopolist with quality s_1 (blue, dashed) as a function of quality s_2 with respective optimum. Calibrated for uniformly distributed preferences, $\bar{\theta} = 3$, $\alpha = 0.1$, $s_1 = \frac{\bar{\theta}}{3\alpha} = 10$.

as in the scenario with fixed fee. Solving for optimal rp_1 from $\frac{\partial \pi_M}{\partial rp_1} \stackrel{!}{=} 0$ pre and post entry yields:

$$r^o p_1^o = \gamma(\tilde{\theta}_1) s_1 \left[\left(\frac{\partial p_1^o}{\partial rp_1} \right)^{-1} + r^o \right]$$

$$r^n p_1^n = \gamma(\hat{\theta}) \Delta s \left[\left(\frac{\partial p_1^n}{\partial rp_1} \right)^{-1} + r^n \right] + (p_2 - c_2)$$

where $\frac{\partial p_1^o}{\partial rp_1} = 1 + \frac{\gamma'}{p_1^o} (p_1^o - 1 + r \frac{\partial p_1^o}{\partial rp_1}) \frac{\partial p_1^o}{\partial rp_1}$ increases to $\frac{\partial p_1^n}{\partial rp_1} = 1 + \frac{\gamma'}{p_1^n} (p_1^n - 1 + r \frac{\partial p_1^n}{\partial rp_1}) \left(\frac{\partial p_1^n}{\partial rp_1} - \frac{\partial p_2}{\partial rp_1} \right)$. Given that the price of the private label is relatively the same and there is also a demand-stealing effect, it is no surprise that post entry the margin $r^n p_1^n$ decreases. Thus, the increase in r cannot be interpreted as an attempt by the marketplace owner to retain the same revenue level as pre entry. On the contrary, this result shows that, independent from the type of the fee - ad-valorem or fixed - the marketplace owner overprices his product and reduces the rents he extracts from the seller channel, effectively acting against, and not in favor of his product.¹¹

¹¹Technically speaking, the ad-valorem fee increases because for the revenues the marketplace owner loses on the higher quality due to the imposed competition, he compensates through the margin $(p_2 - c_2)$:

$$\frac{r^n}{1 - r^n} = \frac{p_1^n \gamma(\hat{\theta}) \Delta s}{c_1 - (1 - r^n)(p_2 - c_2)} \left(\frac{\partial p_1^n}{\partial r} \right)^{-1}$$

This effectively reduces the cost factor in the denominator $c_1 - (1 - r)(p_2 - c_2)$ and offsets the post-entry reduction in the rest of the factors $(p_1^n \gamma(\hat{\theta}) \Delta s \left(\frac{\partial p_1^n}{\partial r} \right)^{-1})$.

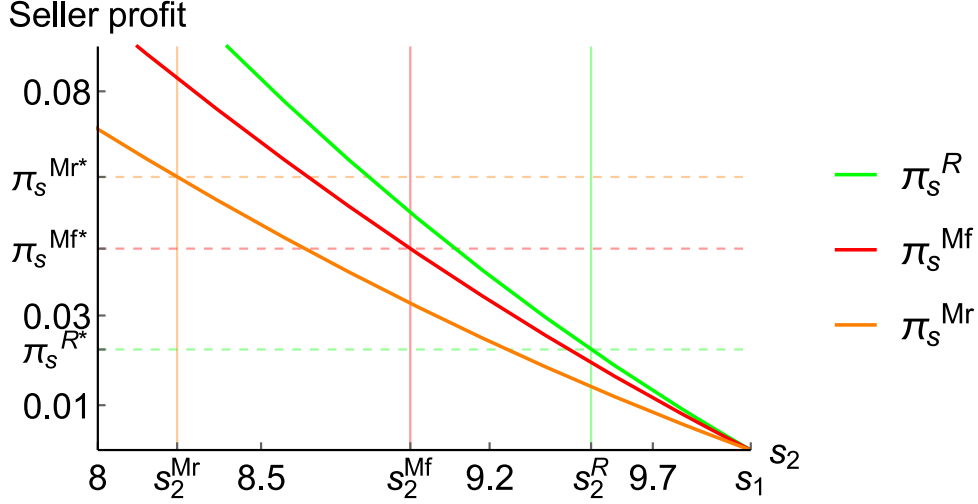


Figure 8: Profits of seller under marketplace fixed (red) and ad-valorem (blue) fee, and retailer (green) mode with optimal quality of the private label and ex-post profits in each mode. Calibrated for uniformly distributed preferences, $s_1 = \frac{\bar{\theta}}{3\alpha} = 10$, $\alpha = 0.1$.

Finally, with ad-valorem fee the result of the marketplace owner choosing optimally a lower quality than the seller continues to hold. Given that the vertical structure generates larger industry revenues and a larger share goes to the marketplace owner, now there is even more incentive to differentiate the product, compared to the fixed fee case (see Figure 7). This, again, reverses the result that a seller generates lower profits with ad-valorem fee and leaves the seller with even more profits ex-post compared to a fixed fee contract or retailer setting (see Figure 8). Interestingly, numerical simulations also show that, even though for a given quality of the private label consumer surplus is higher under ad-valorem fee, this result gets reversed by the quality choice of the marketplace owner, and the consumer surplus is the lowest ex-post among the three vertical contracts.

4.6.2 Two-part Tariff Contract

In the classic wholesale structure two-part tariffs have been used to avoid the double-marginalization problem. In a scenario, where the seller sets a per-unit wholesale price w and a fixed tariff T_R , it is optimal to set $w = c_1$ and $T_R = \pi_R = (p_1 - c_1)Q_1$. Thus, the end-consumer price is equal to the stand-alone monopoly price and the whole surplus goes to the seller, the retailer obtains zero profits. Similarly, in the marketplace setting, when there is no private label present, the marketplace owner can set the per-unit fee $f = 0$ and extract the whole surplus through a fixed tariff $T_M = (p_1 - c_1)Q_1$.

Once the retailer introduces a private label of lower quality, the seller changes

the optimal tariff to the competitive surplus of the higher quality, while $w = c_1$. In such a case, the only channel, through which the retailer obtains positive profits, is the one of the private label. Therefore, the best strategy for him is to perfectly copy the seller's product and foreclose him.

If a marketplace owner introduces a lower-quality private label, he has three instruments, through which he can maximize his profits: the price of the private label p_2 , the fixed per-unit fee f , and the tariff T_M . As he has interest in maximizing the joint producer surplus, his objective function changes, both in the pricing and the fee stages:

$$\begin{aligned} \max_{p_2} \pi_M &= (p_1 - c_1)(1 - G(\hat{\theta})) + (p_2 - c_2)(G(\hat{\theta}) - G(\tilde{\theta}_2)) \\ \max_f \pi_M &= (p_1(f) - c_1)(1 - G(\hat{\theta}(f))) + (p_2(f) - c_2)(G(\hat{\theta}(f)) - G(\tilde{\theta}_2(f))) \end{aligned}$$

This implies that the optimal price of the private label is now given by $p_2 - c_2 = \gamma(\tilde{\theta}_2)s_2 - \omega\gamma(\tilde{\theta}_1)s_1 + \omega(p_1 - c_1)$. As the seller sets a competitive margin $p_1 - f - c_1 = \gamma(\hat{\theta})\Delta s$, this means that the optimal price of the private label, as a function of the fee is given by $p_2 - c_2 = (1 - \omega)\gamma(\tilde{\theta}_2)s_2 + \omega f$. Solving for the optimal fee, we get $f = \gamma(\tilde{\theta}_2)s_2$. Hence, the private label will be priced at the stand-alone monopoly price, and so will the seller's product. In this sense, the prices in both vertical structures will be the same. However, because the marketplace owner obtains the revenues of a multi-product monopolist, he will choose a socially sub-optimal lower quality of the private label as demonstrated by Mussa and Rosen (1978), Itoh (1983), Maskin and Riley (1984), Besanko et al. (1987) and Tirole (1988). The seller won't be foreclosed and consumer surplus will be higher.

The non-linear contract avoids double marginalization, therefore the first instance of disadvantaging the own product by overpricing it in a marketplace setting is not present. Comparing changes in the fees for linear and non-linear contracts does not make up for substantial discussion either. However, the results in this section outline the drastic difference in roles adopted by the intermediary in the different business models. Even though a retailer prices as a multi-product monopolist, he, de facto, acts as direct competitor to the outside seller and fully forecloses him. Despite his "dual" role as a "gatekeeper" and price competitor, a marketplace owner acts as a multi-product monopolist, accommodating both products, and protecting more the outside product by differentiating and setting a sub-optimal quality for the private label.

5 Conclusion

This paper contributes to the discussion on implications of vertical integration in marketplaces. It is the first paper to consider heterogeneous preferences for quality and endogenous quality choice of the private label in a marketplace structure and draw a comparison with the classic wholesale structure. In a marketplace a seller sets a price directly to consumers, while the marketplace owner collects fees on his sales. In the classic wholesale model a retailer buys the seller product at a wholesale price determined by the seller and then chooses end-consumer prices. Before the introduction of a private label both structures are identical from consumer's perspective, as marginalization on each level is the same across business models. However, the one setting the upstream contract (fee or wholesale price) collects the larger share of the profits. After the introduction of the private label the two vertical structures diverge. While a retailer internalizes competition effects and sets both end-consumer prices as independent stand-alone monopolists, a marketplace owner directly competes in prices with the seller on the platform. As a result, the marketplace owner overprices the private label product compared to a retailer or stand-alone monopolist in order to protect revenues from the higher-quality higher-margin product. Because the price competition makes the seller's price more sensitive to changes in the fee and this way passes more from it to the demand side of the market, the marketplace owner decreases the fee post entry. Finally, the marketplace owner optimally chooses to enter with a lower-quality product and by doing so differentiates more and harms the seller less than a classic retailer. Due to the relatively lower quality but higher price, the consumer surplus gets less improved in a marketplace than in a classic wholesale structure after the introduction of a private label.

The current analysis points at multiple possibilities for future research. First, the model assumes a monopoly at each level of the vertical structure. Future research might want to extend the analysis to competition on either level. Depending on what level there is more competition on, the upstream or the downstream, private labels may play more or less important roles. Competition for consumers would typically drive prices down and push intermediaries to align their objective functions with the objectives of consumers, as demonstrated by Etro (2021) in a different setting. Another interesting direction for future research is what are the implications for asymmetric information regarding demand. Amazon collects data from hosting sellers on its platform. It is claimed that Amazon may have better view on the actual demand distribution, as it sees when people consider, but do not buy a product. An interesting extension would be to assume that the seller does not know the exact distribution of demand for quality. In this case a marketplace owner might sometimes find it profitable to enter with a higher quality than the seller's

product. Interesting would be also if a marketplace owner has an incentive to share the knowledge on demand with the seller, as in seminal works by Vives (1984, 1990) sharing information on uncertain demand in Bertrand setting with substitutes goods is a dominant strategy.

As the main focus of the paper falls on the private label strategy of the intermediary depending on the vertical contract implemented, I do not allow for the outside seller to react to the introduction of the private label and adjust his quality accordingly. If the seller would be able to adjust his quality in anticipation, his main drive would be to differentiate upwards in a similar way to what happens with independent sellers in markets with vertical differentiation. Endogenizing the quality of the seller in the current setting of the model leads to an additional layer of calculations, without bringing additional insights into the private label strategies of the marketplace owner and the retailer. Therefore, I leave the quality competition game between a vertically integrated marketplace and outside seller to future research.

A Omitted discussions and technical proofs

Notes on double marginalization. The vertical structure of an intermediary and seller imposes double marginalization on the product of the seller. This results in higher end-consumer price and lower demand compared to the case of a vertically integrated unit. In particular, let us compare the marginal consumers under vertical separation $\tilde{\theta}_1^D$ and a single monopolist $\tilde{\theta}_1^M$ for a product of quality s_1 . We know that:

$$\begin{aligned}\tilde{\theta}_1^D &= \frac{p_1^D}{s_1} = \frac{\gamma(\tilde{\theta}_1^D)s_1 + f + c_1}{s_1} = \frac{[2 - \gamma'(\tilde{\theta}_1^D)]\gamma(\tilde{\theta}_1^D)s_1 + c_1}{s_1} \\ \tilde{\theta}_1^M &= \frac{p_1^M}{s_1} = \frac{\gamma(\tilde{\theta}_1^M)s_1 + c_1}{s_1}\end{aligned}$$

This implies that the loss of demand due to double marginalization can be expressed as:

$$\tilde{\theta}_1^D - \tilde{\theta}_1^M = [2 - \gamma'(\tilde{\theta}_1^D)]\gamma(\tilde{\theta}_1^D) - \gamma(\tilde{\theta}_1^M)$$

Under Assumption (1) the Mills ratio $\gamma(\cdot)$ is a monotone non-increasing function with $\gamma'(\cdot) = \text{const} = \xi$. Then:

$$\tilde{\theta}_1^D - \tilde{\theta}_1^M = [2 - \xi]\gamma(\tilde{\theta}_1^D) - [\gamma(\tilde{\theta}_1^D) - \xi(\tilde{\theta}_1^D - \tilde{\theta}_1^M)] \Leftrightarrow (1 - \xi)(\tilde{\theta}_1^D - \tilde{\theta}_1^M) = (1 - \xi)\gamma(\tilde{\theta}_1^D)$$

This helps us compare the upstream margin $(1 - \xi)\gamma(\tilde{\theta}_1^D)s_1$ (either f or $w - c$) and the markup of a vertically integrated monopolist $\gamma(\tilde{\theta}_1^M)s_1$:

$$(1 - \xi)\gamma(\tilde{\theta}_1^D)s_1 \square \gamma(\tilde{\theta}_1^M)s_1 = s_1(\gamma(\tilde{\theta}_1^D) - \xi(\tilde{\theta}_1^D - \tilde{\theta}_1^M)) = (1 - \xi)\gamma(\tilde{\theta}_1^D)s_1$$

and see that the upstream markup equals the integrated monopoly markup on top of the downstream markup for $\gamma(\cdot)$ linear.

Multi-product firm/Retailer higher-quality pricing. The price of the higher quality $p_1^R - w = (p_2^R - c_2) + \gamma(\hat{\theta})\Delta s$ depends on the curvature of the Mills ratio $\gamma(\cdot)$. The markup is bigger, equal, or smaller than the markup $\gamma(\tilde{\theta}_1)s_1$ a stand-alone monopolists sets for a product of the same quality s_1 and marginal cost w , if $\gamma(\cdot)$ is respectively (globally) convex, linear, or concave. To see that compare

$$p_1^R - w = \gamma(\tilde{\theta}_2)s_2 + \gamma(\hat{\theta})\Delta s \square p_1^m - w = \gamma(\tilde{\theta}_1)s_1$$

We know that $\tilde{\theta}_2 < \tilde{\theta}_1 < \hat{\theta} \Rightarrow G(\tilde{\theta}_2) \leq G(\tilde{\theta}_1) \leq G(\hat{\theta}) \Rightarrow 1 - G(\tilde{\theta}_2) \geq 1 - G(\tilde{\theta}_1) \geq 1 - G(\hat{\theta})$. For $\gamma(\theta)$ monotone function when comparing both prices one can apply

the Mean Value Theorem, i.e. $\exists \hat{\theta}_1 \in (\tilde{\theta}_1, \hat{\theta})$, $\exists \hat{\theta}_2 \in (\tilde{\theta}_2, \hat{\theta})$:

$$\begin{aligned} \gamma(\tilde{\theta}_2)s_2 + \gamma(\hat{\theta})\Delta s &\square \gamma(\tilde{\theta}_1)s_1 \Leftrightarrow \\ s_2 \left[\gamma(\tilde{\theta}_2) - \gamma(\hat{\theta}) \right] &\square s_1 \left[\gamma(\tilde{\theta}_1) - \gamma(\hat{\theta}) \right] \Leftrightarrow \\ s_2 \left[(\tilde{\theta}_2 - \hat{\theta})\gamma'(\theta) \Big|_{\hat{\theta}_2} \right] &\square s_1 \left[(\tilde{\theta}_1 - \hat{\theta})\gamma'(\theta) \Big|_{\hat{\theta}_1} \right] \Leftrightarrow \\ s_2 \frac{s_1 p_2 - s_2 p_1}{s_2(s_1 - s_2)} \left[\gamma'(\theta) \Big|_{\hat{\theta}_2} \right] &\square s_1 \frac{s_1 p_2 - s_2 p_1}{s_1(s_1 - s_2)} \left[\gamma'(\theta) \Big|_{\hat{\theta}_1} \right] \Leftrightarrow \\ \gamma'(\theta) \Big|_{\hat{\theta}_2} &\square \gamma'(\theta) \Big|_{\hat{\theta}_1} \end{aligned}$$

Depending on of $\gamma(\theta)$ being convex, linear, or concave, the price of the higher quality will be higher, equal, or lower than the price an independent monopolists with cost w chooses for the high quality product.

Decreasing wholesale price. The optimal margin is $w^n - c_1 = [1 - \gamma']\gamma(\hat{\theta})\Delta s$. As $s_2 \rightarrow s_1$ it is easy to see that this margin is going to converge to zero. On the other end of the specter as $s_2 \rightarrow 0$, it can be verified that $p_2 \rightarrow 0$, hence $\hat{\theta} \rightarrow \tilde{\theta}_1$ and the margin of the seller converges to the pre-entry case $w^o - c_1 = [1 - \gamma'(\tilde{\theta}_1)]\gamma(\tilde{\theta}_1)s_1$.

To understand how the optimal wholesale price behaves in the interval $s_2 \in (0, s_1)$ look at the optimal solution

$$w^n - c_1 = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1^n}{\partial w} \right)^{-1}$$

where $\frac{\partial p_1^n}{\partial w}$ is the pass-through of the end-consumer price with respect to the wholesale price. The original wholesale price can be expressed in the same way

$$w^o - c_1 = \gamma(\tilde{\theta}_1)s_1 \left(\frac{\partial p_1^o}{\partial w} \right)^{-1}$$

The pass-through of both end-consumer prices - the original and the new one - is given by $\frac{1}{1-\gamma'(\tilde{\theta}_1)}$ and $\frac{1}{1-\gamma'(\hat{\theta})}$, respectively, and are constant and equal to $\frac{1}{1-\xi}$ (Assumption (1)). As discussed in the main section 4.1 the only drive in reduction of the wholesale price is the demand-stealing effect. Thus, for a given p_1^\diamond it holds $\hat{\theta}^\diamond > \tilde{\theta}_1^\diamond$ and therefore

$$w^n - c_1 = \gamma(\hat{\theta}^\diamond)\Delta s[1 - \xi] < w^o - c_1 = \gamma(\tilde{\theta}_1^\diamond)s_1[1 - \xi]$$

To prove that the wholesale price decreases as s_2 improves, we can use that

$\gamma(\theta) = \sigma + \xi\theta$, simplify the expressions and show that the wholesale price decreases as the quality of the private label improves. The end-consumer prices are given by $p_1 = \frac{\sigma s_1 + w}{1 - \xi}$ and $p_2 = \frac{\sigma s_2 + c_2}{1 - \xi}$. Then the expression for the wholesale price reduces to

$$w^n = \frac{\sigma \Delta s - \xi c_2 + c_1}{1 - \xi}$$

which is decreasing in s_2 as long-as the private label quality is viable, *i.e.*, $\bar{\theta} s_2 > c_2$.

Foreclosure fee level. The seller gets foreclosed, when there is effectively no demand for his product, *i.e.*:

$$\bar{\theta} - \hat{\theta} = -\frac{\sigma}{\xi} - \frac{p_1^*(f) - p_2^*(f)}{\Delta s} = 0$$

The optimal prices as a function of the fee are given by

$$p_1^*(f) = \frac{\sigma s_1(1 - \xi + \xi\omega) + c_1(1 - \xi) - (\sigma s_2 + \xi c_2) + f^n(1 - \xi - \xi\omega)}{1 - 2\xi + \xi^2(1 - \omega)}$$

$$p_2^*(f) = \frac{\sigma s_2(1 - \xi + \xi\omega) + c_2(1 - \xi) - \omega(\sigma s_1 + \xi c_1) + \omega f^n(1 - 2\xi)}{1 - 2\xi + \xi^2(1 - \omega)}$$

The fee level \bar{f} , for which there is no demand for the seller, is then given by:

$$\bar{f} = \frac{(\sigma s_1 + \xi c_1)(1 - \xi(1 - \omega)) - (\sigma s_2 + \xi c_2)}{-\xi(1 - \xi)(1 - \omega)}$$

For the case without private label (effectively $s_2 = 0$), the foreclosure level is well above the optimal fee $\frac{\sigma s_1 + \xi c_1}{-\xi} > f^o = \frac{\sigma s_1 + \xi c_1}{1 - \xi}$.

For the case with private label, we have

$$\bar{f} - f^n = \frac{(\sigma \Delta s + \xi(c_1 - c_2))(1 - 2\xi + \xi^2(1 - \omega))}{-\xi(1 - \xi)(1 - \omega)(1 - \xi - \left(\frac{\partial p_1}{\partial f}\right)^{-1})}$$

which is positive as long as the qualities of the goods are efficient and have positive demand when priced at marginal cost.

Decreasing post-entry fee (Proof of Proposition 1 Part 1). Let us explore the limits of the new fee $f^n = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1}{\partial f}\right)^{-1} + (p_2^M - c_2)$ given the quality of the private label s_2 . As $s_2 \rightarrow 0$ it converges to the original fee, *i.e.* $f^n \rightarrow f^o$.¹² As $s_2 \rightarrow s_1$, the first term in the optimal fee expression converges to zero, making

¹²The original fee can be expressed as $f^o = \gamma(\tilde{\theta}_1)s_1 \left(\frac{\partial p_1}{\partial f}\right)^{-1}$.

the margin of the seller equal to zero. This implies $f^n = p_1 - c_1 = p_2 - c_2$ and the fee is equal to the total margin generated by the private label of quality s_1 , i.e. $\gamma(\tilde{\theta}_1)s_1$, which is equal to the original fee f^o (See Appendix A Notes on double marginalization).

The proof that the fee actually decreases for any s_2 between zero and s_1 is more technical. By Assumption (1) the Mills ratio is given by the linear function $\gamma(\theta) = \sigma + \xi\theta$. Plugging it in the expression for the optimal fee and end-consumer price p_1 before entry it yields:

$$f^o = (1 - \xi)(\sigma s_1 + \xi p_1) = \frac{\sigma s_1 + \xi c_1}{1 - \xi}$$

The end-consumer prices in the marketplace post entry can be expressed as:

$$p_1^*(f) = \frac{\sigma s_1(1 - \xi + \xi\omega) + c_1(1 - \xi) - (\sigma s_2 + \xi c_2) + f^n(1 - \xi - \xi\omega)}{1 - 2\xi + \xi^2(1 - \omega)}$$

$$p_2^*(f) = \frac{\sigma s_2(1 - \xi + \xi\omega) + c_2(1 - \xi) - \omega(\sigma s_1 + \xi c_1) + \omega f^n(1 - 2\xi)}{1 - 2\xi + \xi^2(1 - \omega)}$$

and the new fee is given by:

$$f^n = \frac{(\sigma s_1 + \xi c_1)\left(\left(\frac{\partial p_1}{\partial f}\right)^{-1} (1 - \xi(1 - \omega)) - \omega\right) + (\sigma s_2 + \xi c_2)(1 - \xi(1 - \omega) - \left(\frac{\partial p_1}{\partial f}\right)^{-1})}{(1 - \xi)(1 - \xi - \xi\left(\frac{\partial p_1}{\partial f}\right)^{-1})(1 - \omega)}$$

Subtracting the old fee yields:

$$f^n - f^o = -\frac{(\sigma\Delta s + \xi(c_1 - c_2))\left(1 - \xi(1 - \omega) - \left(\frac{\partial p_1^n}{\partial f}\right)^{-1}\right)}{(1 - \xi)\left(1 - \xi - \xi\left(\frac{\partial p_1^n}{\partial f}\right)^{-1}\right)(1 - \omega)}$$

The denominator is strictly positive. The first expression in the nominator being positive $\sigma\Delta s + \xi(c_1 - c_2) > 0$ is equivalent to a basic assumption that the higher-quality product is efficient and has positive demand when products are priced at marginal cost, i.e., $\bar{\theta}s_1 - c_1 > \bar{\theta}s_2 - c_2$, where $\bar{\theta} = -\frac{\sigma}{\xi}$. Therefore, given $\frac{\partial p_1^n}{\partial f} = \frac{1 - \xi}{1 - \xi}$ the fee decreases post entry if:

$$1 - \xi(1 - \omega) - \left(\frac{\partial p_1^n}{\partial f}\right)^{-1} > 0 \Leftrightarrow \frac{\partial p_2}{\partial f} \geq \frac{\omega}{1 - \xi + \xi\omega}$$

In order to demonstrate this indeed holds, let us first take a step back and show that $\frac{\partial p_1}{\partial f} < 1 \Leftrightarrow \frac{\partial p_2}{\partial f} < 1$ and that $\frac{\partial p_1}{\partial f} > 0 \Leftrightarrow \frac{\partial p_2}{\partial f} > 0 > \frac{1}{\xi}$. Thus, it has to be

shown that $\frac{\partial p_2}{\partial f} \in [0, 1]$ for $f^n = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1}{\partial f}\right)^{-1} + (p_2 - c_2)$. We show this holds for $\xi \in [0, 1]$.

Let us first explore the limits of the interval. For $\xi \rightarrow 0$ the distribution of preferences $g(\theta)$ converges to the exponential distribution. This family of distributions has the property that the Mills ratio is a constant, *i.e.*, $\gamma(\theta) = \sigma$. The expression of the optimal price from equation 3 can be reduced to:

$$p_2 - c_2 = \sigma s_2 - \omega \sigma s_1 + \omega f$$

Taking the first derivative w.r.t. the fee we obtain:

$$\frac{\partial p_2}{\partial f} = -\frac{\partial \omega}{\partial f} \sigma s_1 + \frac{\partial \omega}{\partial f} f + \omega$$

where $\omega = \frac{g(\hat{\theta})s_2}{g(\hat{\theta}_2)\Delta s + g(\hat{\theta})s_2} \in [0, 1]$. The price that the outside seller sets for the high quality product is given by $p_1 - f - c_1 = \gamma(\hat{\theta})\Delta s = \sigma \Delta s$ implying that the sensitivity to changes in the fee is exactly equal to 1. The optimal fee is then given by $f = \sigma \Delta s + p_2 - c_2 = \sigma s_1$. Hence, we obtain that $\frac{\partial p_2}{\partial f} = \omega \in [0, 1]$. This also satisfies $\frac{\partial p_2}{\partial f} \geq \frac{\omega}{1 - \xi + \xi \omega} = \omega$.

For the limit $\xi = -1$ the distribution of preferences $g(\theta)$ is given by the uniform distribution, where $\omega = \frac{s_2}{s_1}$ and $0 < \frac{\omega}{1 - \xi + \xi \omega} = \frac{s_2}{3s_1 - s_2} < \frac{\partial p_2}{\partial f} = \frac{3s_2}{4s_1 - s_2} < 1$ for $s_2 < s_1$. To see this, consider the optimization problem of the seller and the marketplace owner in the uniform case:

$$\begin{aligned} \max_{p_1} \pi_s(p_1, p_2) &= (p_1 - f - c_1) \left(1 - \frac{p_1 - p_2}{\bar{\theta}(s_1 - s_2)}\right) \\ \max_{p_2} \pi_p(p_1, p_2) &= f \left(1 - \frac{p_1 - p_2}{\bar{\theta}(s_1 - s_2)}\right) + (p_2 - c_2) \left(\frac{p_1 - p_2}{\bar{\theta}(s_1 - s_2)} - \frac{p_2}{\bar{\theta}s_2}\right) \end{aligned}$$

Deriving the FOCs gives the best response functions:

$$p_1 = BR_s(p_2) = \frac{\bar{\theta}(s_1 - s_2) + f + p_2}{2} + \frac{c_1}{2} \quad \wedge \quad p_2 = BR_p(p_1) = \frac{s_2(p_1 + f)}{2s_1} + \frac{c_2}{2}$$

Solving for the optimal prices of the higher-quality product and the private label as function of the fee f yields:

$$\begin{aligned} p_1^*(f) &= \frac{1}{4s_1 - s_2} (2s_1\bar{\theta}(s_1 - s_2) + 2s_1c_1 + s_1c_2 + (2s_1 + s_2)f) \\ p_2^*(f) &= \frac{1}{4s_1 - s_2} (s_2\bar{\theta}(s_1 - s_2) + s_2c_1 + 2s_1c_2 + 3s_2f) \end{aligned}$$

For $\xi \in (0, 1)$ the proof proceeds like this. Start with the first order condition of the marketplace owner w.r.t. the price of the private label and take its derivative w.r.t. the fee:

$$\begin{aligned} \frac{\partial \pi_p}{\partial p_2} &= f \left(-g(\hat{\theta}) \right) \left(-\frac{1}{\Delta s} \right) + G(\hat{\theta}) - G(\tilde{\theta}_2) + (p_2 - c_2) \left[-\frac{g(\hat{\theta})}{\Delta s} - \frac{g(\tilde{\theta}_2)}{s_2} \right] \stackrel{!}{=} 0 \Leftrightarrow \\ & f \frac{g(\hat{\theta})}{\Delta s} + G(\hat{\theta}) - G(\tilde{\theta}_2) = (p_2 - c_2) \left[\frac{g(\hat{\theta})}{\Delta s} + \frac{g(\tilde{\theta}_2)}{s_2} \right] \quad / \partial f \Rightarrow \\ & \frac{g(\hat{\theta})}{\Delta s} + f \frac{g(\hat{\theta})}{\Delta s} \frac{-\xi - 1}{\gamma(\hat{\theta})\Delta s} \left(\frac{\partial p_1}{\partial f} - \frac{\partial p_2}{\partial f} \right) + \left[\frac{g(\hat{\theta})}{\Delta s} \left(\frac{\partial p_1}{\partial f} - \frac{\partial p_2}{\partial f} \right) - \frac{g(\tilde{\theta}_2)}{s_2} \frac{\partial p_2}{\partial f} \right] = \\ & = \frac{\partial p_2}{\partial f} \left[\frac{g(\hat{\theta})}{\Delta s} + \frac{g(\tilde{\theta}_2)}{s_2} \right] + (p_2 - c_2) \left[\frac{g(\hat{\theta})}{\Delta s} \frac{-\xi - 1}{\gamma(\hat{\theta})\Delta s} \left(\frac{\partial p_1}{\partial f} - \frac{\partial p_2}{\partial f} \right) + \frac{g(\tilde{\theta}_2)}{s_2} \frac{-\xi - 1}{\gamma(\tilde{\theta}_2)s_2} \frac{\partial p_2}{\partial f} \right] \end{aligned}$$

Step 1: one can substitute $\frac{\partial p_1}{\partial f} - \frac{\partial p_2}{\partial f} = \frac{1 - \frac{\partial p_2}{\partial f}}{1 - \xi}$ and multiply by $(1 - \xi)$:

$$\begin{aligned} (1 - \xi) \frac{g(\hat{\theta})}{\Delta s} + f \frac{g(\hat{\theta})}{\Delta s} \frac{-\xi - 1}{\gamma(\hat{\theta})\Delta s} \left(1 - \frac{\partial p_2}{\partial f} \right) + \frac{g(\hat{\theta})}{\Delta s} \left(1 - \frac{\partial p_2}{\partial f} \right) - (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} \frac{\partial p_2}{\partial f} &= \\ &= (1 - \xi) \frac{\partial p_2}{\partial f} \left[\frac{g(\hat{\theta})}{\Delta s} + \frac{g(\tilde{\theta}_2)}{s_2} \right] + \\ &+ (p_2 - c_2) \left[\frac{g(\hat{\theta})}{\Delta s} \frac{-\xi - 1}{\gamma(\hat{\theta})\Delta s} \left(1 - \frac{\partial p_2}{\partial f} \right) + (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} \frac{-\xi - 1}{\gamma(\tilde{\theta}_2)s_2} \frac{\partial p_2}{\partial f} \right] \end{aligned}$$

Step 2: Collect all expressions multiplied by $\left(1 - \frac{\partial p_2}{\partial f} \right)$ and bring them on the LHS and collect the rest multiplied by $\frac{\partial p_2}{\partial f}$ on the RHS:

$$\begin{aligned} & \left[(2 - \xi) + (f - (p_2 - c_2)) \frac{-\xi - 1}{\gamma(\hat{\theta})\Delta s} \right] \frac{g(\hat{\theta})}{\Delta s} \left(1 - \frac{\partial p_2}{\partial f} \right) \\ &= \frac{\partial p_2}{\partial f} (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} \left[2 + (p_2 - c_2) \frac{-\xi - 1}{\gamma(\tilde{\theta}_2)s_2} \right] \end{aligned}$$

Step 3: We only need to show that $\frac{\partial p_2}{\partial f} \in [0, 1]$ for $f^n = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1}{\partial f} \right)^{-1} + (p_2 - c_2)$ and $p_2 - c_2 = \frac{\omega}{1 - \omega} \gamma(\hat{\theta})\Delta s \left[\left(\frac{\partial p_1}{\partial f} \right)^{-1} - 1 \right] + \gamma(\tilde{\theta}_2)s_2$, meaning that we can plug in the optimal values into the above expression:

$$\left[(2 - \xi) + \frac{(-\xi - 1)(1 - \xi)}{1 - \xi \frac{\partial p_2}{\partial f}} \right] \frac{g(\hat{\theta})}{\Delta s} \left(1 - \frac{\partial p_2}{\partial f} \right) =$$

$$= \frac{\partial p_2}{\partial f} (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} \left[2 + \frac{-\xi - 1}{\gamma(\tilde{\theta}_2)s_2} \left(\frac{\omega}{1 - \omega} \gamma(\hat{\theta}) \Delta s (-\xi) \left[\frac{1 - \frac{\partial p_2}{\partial f}}{1 - \xi \frac{\partial p_2}{\partial f}} \right] + \gamma(\tilde{\theta}_2)s_2 \right) \right]$$

Step 4: Multiply by $1 - \xi \frac{\partial p_2}{\partial f}$ and write the equation in quadratic form:

$$\begin{aligned} & (2 - \xi) \frac{g(\hat{\theta})}{\Delta s} \left[1 - (1 + \xi) \frac{\partial p_2}{\partial f} + \xi \left(\frac{\partial p_2}{\partial f} \right)^2 \right] + (-\xi - 1)(1 - \xi) \frac{g(\hat{\theta})}{\Delta s} \left(1 - \frac{\partial p_2}{\partial f} \right) \stackrel{!}{=} \\ & (1 - \xi)^2 \frac{g(\tilde{\theta}_2)}{s_2} \left[\frac{\partial p_2}{\partial f} - \xi \left(\frac{\partial p_2}{\partial f} \right)^2 \right] + (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} (1 + \xi) \xi \frac{\omega}{1 - \omega} \frac{\gamma(\hat{\theta}) \Delta s}{\gamma(\tilde{\theta}_2)s_2} \left[\frac{\partial p_2}{\partial f} - \left(\frac{\partial p_2}{\partial f} \right)^2 \right] \\ & \Leftrightarrow \left(\frac{\partial p_2}{\partial f} \right)^2 \xi \left[(2 - \xi) \frac{g(\hat{\theta})}{\Delta s} + (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} \left((1 - \xi) + (1 + \xi) \frac{\gamma(\hat{\theta}) \Delta s}{\gamma(\tilde{\theta}_2)s_2} \frac{\omega}{1 - \omega} \right) \right] + \\ & + \frac{\partial p_2}{\partial f} \left[\frac{g(\hat{\theta})}{\Delta s} (-1 - \xi) - (1 - \xi) \frac{g(\tilde{\theta}_2)}{s_2} \left((1 - \xi) + \xi(1 + \xi) \frac{\gamma(\hat{\theta}) \Delta s}{\gamma(\tilde{\theta}_2)s_2} \frac{\omega}{1 - \omega} \right) \right] + \\ & + (1 - \xi + \xi^2) \frac{g(\hat{\theta})}{\Delta s} \stackrel{!}{=} 0 \end{aligned}$$

Step 5: Divide by $\frac{g(\hat{\theta})}{\Delta s}$ and notice that $\frac{\frac{g(\tilde{\theta}_2)}{s_2}}{\frac{g(\hat{\theta})}{\Delta s}} = \frac{1 - \omega}{\omega}$:

$$\begin{aligned} & \left(\frac{\partial p_2}{\partial f} \right)^2 \xi \underbrace{\left[(2 - \xi) + (1 - \xi) \left((1 - \xi) \frac{1 - \omega}{\omega} + (1 + \xi) \frac{\gamma(\hat{\theta}) \Delta s}{\gamma(\tilde{\theta}_2)s_2} \right) \right]}_a + \\ & + \frac{\partial p_2}{\partial f} \underbrace{\left[(-1 - \xi) - (1 - \xi) \left((1 - \xi) \frac{1 - \omega}{\omega} + \xi(1 + \xi) \frac{\gamma(\hat{\theta}) \Delta s}{\gamma(\tilde{\theta}_2)s_2} \right) \right]}_b + \\ & + \underbrace{(1 - \xi + \xi^2)}_c \stackrel{!}{=} 0 \end{aligned}$$

The solution to the quadratic equation is given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The denominator $2a$ is always negative for $\xi \in (-1, 0)$. $-b$ is always positive, although not apparent straight from the expression. While $(1 - \xi) \geq 0$ for $\xi \leq 0$ and $(-1 - \xi) \leq 0$ for $\xi \geq -1$, the expression $\left((1 - \xi) \frac{1 - \omega}{\omega} + \xi(1 + \xi) \frac{\gamma(\hat{\theta}) \Delta s}{\gamma(\tilde{\theta}_2)s_2} \right)$ is not that obvious. Whether the latter is greater than zero is equivalent to:

$$(1 - \xi)(1 - \omega)\gamma(\tilde{\theta}_2)s_2 > -\xi(1 + \xi)\omega\gamma(\hat{\theta})\Delta s$$

$$\Leftrightarrow (1 - \xi)(1 - G(\tilde{\theta}_2)) > -\xi(1 + \xi)(1 - G(\hat{\theta}))$$

$(1 - \xi) > -\xi(1 + \xi)$ for all ξ and the total demand $(1 - G(\tilde{\theta}_2))$ is greater than the demand for the higher-quality product $(1 - G(\hat{\theta}))$. Thus, $-b$ is always positive.

As the root function is always positive, then the only solution satisfying the conditions we are looking for is such that $-b - \sqrt{b^2 - 4ac} < 0$ and $-b - \sqrt{b^2 - 4ac} > 2a$. The former is equivalent to $-4ac > 0$, which holds as $a < 0$ and $c > 0$, therefore $\frac{\partial p_2}{\partial f} > 0$. The latter is equivalent to $a + b < -c \Leftrightarrow -(1 - \xi)^3 \frac{1 - \omega}{\omega} < 0$, which is satisfied for $\xi \in (-1, 0)$, therefore $\frac{\partial p_2}{\partial f} < 1$.

Check if the formula holds for the limit $\xi \rightarrow -1$. The final quadratic equation reduces to:

$$\begin{aligned} \frac{\partial p_2^2}{\partial f} (-1) \underbrace{\left[3 + 2 \left(2 \frac{\Delta s}{s_2} + 0 \right) \right]}_a + \frac{\partial p_2}{\partial f} \underbrace{\left[0 - 2 \left(2 \frac{\Delta s}{s_2} + 0 \right) \right]}_b + \underbrace{3}_c \stackrel{!}{=} 0 \Leftrightarrow \\ \frac{\partial p_2^2}{\partial f} (-4s_1 + s_2) - \frac{\partial p_2}{\partial f} 4(s_1 - s_2) + 3s_2 \stackrel{!}{=} 0 \end{aligned}$$

Then the root is given by $\frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{3s_2}{4s_1 - s_2}$.

For the other limit $\xi \rightarrow 0$ the quadratic equation reduces to a linear equation:¹³

$$\frac{\partial p_2^2}{\partial f} \underbrace{0}_a + \frac{\partial p_2}{\partial f} \underbrace{\left[-1 - 1 \left(1 \frac{1 - \omega}{\omega} + 0 \right) \right]}_b + \underbrace{1}_c \stackrel{!}{=} 0 \Leftrightarrow \frac{\partial p_2}{\partial f} = \omega$$

Finally, to show that $\frac{\partial p_2}{\partial f} \geq \frac{\omega}{1 - \xi + \xi\omega}$, we can use the general formula above by plugging $\left(\frac{\partial p_2}{\partial f} + \frac{\omega}{1 - \xi + \xi\omega} \right)$ instead of $\frac{\partial p_2}{\partial f}$ and showing that the function still has a positive root. Denote $x = \frac{\omega}{1 - \xi + \xi\omega}$ for simplicity. Then we obtain a new quadratic equation:

$$\left(\frac{\partial p_2}{\partial f} \right)^2 a + (b + 2ax) \frac{\partial p_2}{\partial f} + c + ax^2 + bx = \left(\frac{\partial p_2}{\partial f} \right)^2 a_1 + b_1 \frac{\partial p_2}{\partial f} + c_1$$

We are interested in the root $\frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1} > 0$. As $a_1 = a < 0$ and $-b_1 = -b - 2ax > 0$, we need $-b_1 - \sqrt{b_1^2 - 4a_1c_1} < 0 \Leftrightarrow 0 < -4a_1c_1$. This one is positive

¹³As Step 4 really does not change the expression in this case, Step 3 can also be taken as sufficient for the proof

if $c_1 > 0$, or:

$$-\frac{\xi(1-\xi)^2(1-\omega) \left[\frac{\gamma(\hat{\theta})\Delta s}{\gamma(\tilde{\theta}_2)s_2}(1+\xi)\omega + \omega - \xi(1-\omega) \right]}{(1-\xi(1-\omega))^2} > 0$$

For $\xi \in [-1, 0]$ the above expression is for sure positive, thus $\frac{\partial p_2}{\partial f} > \frac{\omega}{1-\xi+\xi\omega}$ and the fee decreases post entry.

Overpricing of private label (Proof of Proposition 1 Part 2). The optimal price of the private labels is given by the F.O.C.:

$$\frac{\partial \pi_p}{\partial p_2} = f \left(-g(\hat{\theta}) \right) \left(-\frac{1}{\Delta s} \right) + G(\hat{\theta}) - G(\tilde{\theta}_2) + (p_2 - c_2) \left[-\frac{g(\hat{\theta})}{\Delta s} - \frac{g(\tilde{\theta}_2)}{s_2} \right] \stackrel{!}{=} 0$$

Solving for $p_2 - c_2$ and dividing by $\frac{g(\hat{\theta})}{\Delta s} + \frac{g(\tilde{\theta}_2)}{s_2}$:

$$\begin{aligned} p_2^M - c_2 &= \frac{g(\hat{\theta})s_2}{g(\tilde{\theta}_2)\Delta s + g(\hat{\theta})s_2} \left[f - \frac{1 - G(\hat{\theta})}{g(\hat{\theta})}\Delta s + \frac{1 - G(\tilde{\theta}_2)}{g(\hat{\theta})}\Delta s \right] = \\ &= \frac{g(\hat{\theta})s_2}{g(\tilde{\theta}_2)\Delta s + g(\hat{\theta})s_2} \left[f - \gamma(\hat{\theta})\Delta s \right] + \frac{g(\tilde{\theta}_2)\Delta s}{g(\tilde{\theta}_2)\Delta s + g(\hat{\theta})s_2} \left[\frac{1 - G(\tilde{\theta}_2)}{g(\tilde{\theta}_2)}s_2 \right] \Rightarrow \\ &= (1-\omega)\gamma(\tilde{\theta}_2)s_2 - \omega\gamma(\hat{\theta})\Delta s + \omega f \end{aligned}$$

where $\omega = \frac{g(\hat{\theta})s_2}{g(\tilde{\theta}_2)\Delta s + g(\hat{\theta})s_2}$ with the part $(1-\omega)\gamma(\tilde{\theta}_2)s_2 - \omega\gamma(\hat{\theta})\Delta s$ representing the competitive margin a firm with a lower quality would set in an independent market and ωf - the weighted opportunity cost for the marketplace owner.

Following the result from the retailer pricing case that $\gamma(\hat{\theta})\Delta s + \gamma(\tilde{\theta}_2)s_2 = \gamma(\tilde{\theta}_1)s_1$, one can express the optimal price for the private label as $p_2 - c_2 = \gamma(\tilde{\theta}_2)s_2 + \omega(f - \gamma(\tilde{\theta}_1)s_1)$. Using that the Mills ratio is $\gamma(\theta) = \sigma + \xi\theta$ together with the expression for the optimal price from above yields that the marketplace owner sets a higher margin than a pure monopolist for $f > \frac{\sigma s_1 + \xi c_1}{1-\xi} + \frac{\xi}{1-\xi}\gamma(\hat{\theta})\Delta s$.

After deriving expression of the new optimal fee $f^n = \gamma(\hat{\theta})\Delta s \left(\frac{\partial p_1}{\partial f} \right)^{-1} + (p_2 - c_2)$ and plugging it into the optimal private label price from above, the private label continues to be overpriced if $\frac{\partial p_1}{\partial f} < 1 \Leftrightarrow \frac{\partial p_2}{\partial f} < 1$. This has been shown above to hold for $\xi \in [-1, 0]$:

$$p_2 - c_2 = \underbrace{\frac{\omega}{1-\omega}\gamma(\hat{\theta})\Delta s \left[\left(\frac{\partial p_1}{\partial f} \right)^{-1} - 1 \right]}_{>0} + \gamma(\tilde{\theta}_2)s_2$$

Discussion on Remark 1. To show that statements $p_1 - c_1 = \gamma(\hat{\theta})\Delta s + f \rightarrow \gamma(\tilde{\theta}_1)s_1$ and $p_2 - c_2 = \gamma(\tilde{\theta}_2)s_2 - \omega\gamma(\tilde{\theta}_1)s_1 + \omega f \rightarrow \gamma(\tilde{\theta}_1)s_1$ hold, recall the proof of Proposition 1. There we saw that in the limit $s_2 = s_1$ the fee equals $\gamma(\tilde{\theta}_1)s_1$. This proves statement that the private label price converges to the stand-alone monopolist price. Additionally, $\gamma(\hat{\theta})\Delta s = 0$ for $s_2 = s_1$, therefore the end-consumer price of the seller's product also converges to the stand-alone monopoly price.

The optimal price of the private label can be expressed as the stand-alone monopoly price plus a positive term consistent with the proof of Proposition 1:

$$p_2^*(s_2) = \frac{\sigma s_2 + c_2}{1 - \xi} + \frac{\omega(\sigma\Delta s + \xi(c_1 - c_2)) \left(\left(\frac{\partial p_1}{\partial f} \right)^{-1} - 1 \right)}{(1 - \xi)(1 - \xi - \xi \left(\frac{\partial p_1}{\partial f} \right)^{-1})(1 - \omega)}$$

In the limits of $s_2 \in [0, 1]$ p_2 is equal to the stand-alone monopoly price. Hence, the second term is zero in these limits. As it is positive for intermediate values, then this implies that the second term of the pricing expression above must be concave in s_2 . Thus, the price of the private label increases as s_2 improves.

The optimal price of the seller's product is given by:

$$p_1^*(s_2) = \frac{\sigma s_1 + c_1}{1 - \xi} + \frac{(\sigma\Delta s + \xi(c_1 - c_2)) \left(\left(\frac{\partial p_1}{\partial f} \right)^{-1} - \omega \right)}{(1 - \xi)(1 - \xi - \xi \left(\frac{\partial p_1}{\partial f} \right)^{-1})(1 - \omega)}$$

For the extreme case where $\xi = 0$ the price of the seller above is given by $p_1^*(s_2) = \sigma s_1 + c_1 + \sigma\Delta s$ and $\frac{\partial p_1}{\partial s_2} = -\sigma < 0$. For the other extreme, where $\xi = -1$, we have $\omega = \frac{s_2}{s_1}$ and $\left(\frac{\partial p_1}{\partial f} \right)^{-1} = \frac{4s_1 - s_2}{2s_1 + s_2}$ (see Proof of Proposition 1). Thus, the optimal price of the seller is given by:

$$p_1^*(s_2) \Big|_{\xi=-1} = \frac{\sigma s_1 + c_1}{2} + \frac{(\sigma\Delta s - (c_1 - c_2))(4s_1^2 - 3s_1s_2 - s_2^2)}{2(8s_1 + s_2)}$$

Then $\frac{\partial p_1}{\partial s_2} \Big|_{\xi=-1} \stackrel{!}{<} 0$ is equivalent to

$$\begin{aligned} [(-\sigma + c_2')(4s_1^2 - 3s_1s_2 - s_2^2) + (\sigma\Delta s - (c_1 - c_2))(-3s_2 - 2s_2)]2(8s_1 + s_2) \\ - 2(\sigma\Delta s - (c_1 - c_2))(4s_1^2 - 3s_1s_2 - s_2^2) < 0 \end{aligned}$$

which is true for all efficient and viable quality levels s_2 and s_1 with $s_2 < s_1$.

For the intermediate case, where $\xi \in (-1, 0)$, the analysis becomes intractable. In general, one can expect that the price of the seller behaves similarly as in the special cases of $\xi \in \{-1, 0\}$.

Higher sales of private label. In this section I derive the conditions under which a private label becomes a best-seller, or in other words, has higher sales than that of the original product. Thus, compare:

$$Q_1 = 1 - G(\hat{\theta}) \stackrel{?}{<} Q_2 = G(\hat{\theta}) - G(\tilde{\theta}_2) \Leftrightarrow \gamma(\hat{\theta})\Delta s \frac{\omega}{1-\omega} \stackrel{?}{<} \frac{1}{2}\gamma(\tilde{\theta}_2)s_2$$

For the retailer case we have that the optimal prices are given by:

$$p_1^* = \frac{\sigma s_1(2 - \xi) + c_1 - (\sigma s_2 + \xi c_2)}{(1 - \xi)^2} \quad \text{and} \quad p_2^* = \frac{\sigma s_2 + c_2}{1 - \xi}$$

Therefore, the above inequality can be expressed as:

$$\begin{aligned} 2\omega(\sigma\Delta s + \xi(p_1 - p_2)) \stackrel{?}{<} (1 - \omega)(\sigma s_2 + \xi p_2) &\Leftrightarrow 2\omega(\sigma s_1 + \xi p_1) \stackrel{?}{<} (1 + \omega)(\sigma s_2 + \xi p_2) \\ &\Leftrightarrow 2\omega(\sigma s_1 + \xi c_1) \stackrel{?}{<} [(1 - \omega)(1 - \xi) + 2\omega](\sigma s_2 + \xi c_2) \end{aligned}$$

For the limit where $\xi = -1$ we have that $\omega = \frac{s_2}{s_1}$ and the inequality reduces to $\frac{c_2}{c_1} \stackrel{?}{<} \frac{s_2}{s_1}$ which is only satisfied for a cost function $c(s)$, which is convex and increasing in the quality parameter s . For linear $c(s)$ both products will sell the same amount. For the limit where $\xi = 0$ we have $\omega = \frac{\exp(-\frac{\hat{\theta}}{\sigma})s_2}{\exp(-\frac{\hat{\theta}}{\sigma})s_2 + \exp(-\frac{\tilde{\theta}_2}{\sigma})\Delta s}$. Then the inequality reduces to

$$\frac{2\exp(-\frac{\hat{\theta}}{\sigma})s_2}{2\exp(-\frac{\hat{\theta}}{\sigma})s_2 + \exp(-\frac{\tilde{\theta}_2}{\sigma})\Delta s} \stackrel{?}{<} \frac{s_2}{s_1} \Leftrightarrow 2 \stackrel{?}{<} \exp\left(1 + \frac{c_1 s_2 - c_2 s_1}{\sigma s_2 \Delta s}\right)$$

which is fulfilled for increasing convex, linear, and some weakly concave in quality cost function $c(s)$. As for $\xi = -1$ linear cost function leads to the same sales of both product, one may deduce that for $\xi \in (-1, 0)$ the private label outsells the original product for convex, linear and some weakly concave functions.

For the marketplace setting, we have that the seller has higher sales than in the classic wholesale structure, while the private label has lower. This implies that the private label outsells the original product for strictly convex cost function for $\xi = -1$. For $\xi = 0$ the same end-consumer prices arise in both pricing structures, leading to the same demands. Thus, also in the marketplace setting for $\xi = 0$ the private label outsells the original product for increasing convex, linear, and some weakly concave in quality cost function $c(s)$. One may deduce that as ξ increases over $[-1, 0]$, the set of functions for which the private label has higher sales increases as well.

Optimal stand-alone monopolist quality s_1 . For uniformly distributed preferences, the stand-alone monopoly price and quantity are given by $p_1^{sam} = \frac{\bar{\theta}s_1+c_1}{2}$ and $q_1^{sam} = \frac{\bar{\theta}s_1-c_1}{2\theta s_1}$, and the monopolist profit is $\pi_S^{sam} = \frac{(\bar{\theta}s_1-c_1)^2}{4\theta s_1}$. In both vertical structures, when there is no private label (as discussed in Section 3) both the end-consumer price and the sold quantities are the same: $p_1^R = p_1^M = \frac{3\bar{\theta}s_1+c_1}{4}$ and $q_1^R = q_1^M = \frac{\bar{\theta}s_1-c_1}{4\theta}$. The margins $f = w - c_1 = \frac{\bar{\theta}s_1-c_1}{2}$ then imply that the seller's profits are given by $\pi_S^R = \frac{(\bar{\theta}s_1-c_1)^2}{8\theta s_1}$ and $\pi_S^M = \frac{(\bar{\theta}s_1-c_1)^2}{16\theta s_1}$ in the classic wholesale and the marketplace structure, respectively. Hence,

$$\max_{s_1} \pi_S^{sam} = \frac{(\bar{\theta}s_1 - c_1)^2}{4\theta s_1} \equiv \max_{s_1} \pi_S^R = \frac{(\bar{\theta}s_1 - c_1)^2}{8\theta s_1} \equiv \max_{s_1} \pi_S^M = \frac{(\bar{\theta}s_1 - c_1)^2}{16\theta s_1}$$

Then for all the F.O.C. fulfills:

$$\underbrace{(\bar{\theta}s_1 - c_1(s_1))}_{\stackrel{!}{=}0 \Rightarrow \pi_s=0} [2s_1(\bar{\theta} - c_1'(s_1)) - (\bar{\theta}s_1 - c_1(s_1))] \stackrel{!}{=} 0 \Rightarrow$$

$$2s_1(\bar{\theta} - c_1'(s_1)) - (\bar{\theta}s_1 - c_1(s_1)) \stackrel{!}{=} 0 \Leftrightarrow c_1'(s_1^*)s_1^* = \frac{\bar{\theta}s_1^* + c_1(s_1^*)}{2}$$

If we substitute in with $c(s) = \alpha s^2$ we get that the optimal quality is given by $s_1^* = \frac{\bar{\theta}}{3\alpha}$.

Proof of Proposition 2. First, for all $s_2 < s_1$ the profit of the marketplace owner is always higher than the one of the retailer. This is inherent from the fact that the marketplace owner extracts more profits from the seller as a first-mover. Then, for $s_2 = s_1$ both intermediaries earn the same profit of an integrated monopolist. Third, the first-derivative of the profits of the marketplace owner and the retailer with respect to the quality of the private label s_2 evaluated at $s_1 = \frac{\bar{\theta}}{3\alpha}$ are given, respectively, by:

$$\left. \frac{\partial \pi_M}{\partial s_2} \right|_{s_2=s_1=\frac{\bar{\theta}}{3\alpha}} = -\frac{\bar{\theta}}{81} < \left. \frac{\partial \pi_R}{\partial s_2} \right|_{s_2=s_1=\frac{\bar{\theta}}{3\alpha}} = -\frac{\bar{\theta}}{144}$$

Both the marketplace owner and the retailer have negative slopes at the point of the perfect copied product ($s_2 = s_1$), meaning that both would optimally choose a lower quality for the private label. Because, the retailer internalizes the price competition effects between the two products, it is easier to derive s_2^{R*} , which is given by:

$$s_2(s_1) = \frac{6\bar{\theta} + \alpha s_1 - \sqrt{9\bar{\theta}^2 + 12\bar{\theta}\alpha s_1 - 8\alpha^2 s_1^2}}{9\alpha} = \frac{\bar{\theta}(19 - \sqrt{109})}{27\alpha}$$

Substituting into the first derivative of the marketplace owner profit, one obtains:

$$\left. \frac{\partial \pi_M}{\partial s_2} \right|_{s_2=s_2^{R*}} = -\frac{\bar{\theta}(56303 - 3383\sqrt{109})}{486(91 - \sqrt{109})^2} \approx -\frac{\bar{\theta}}{150.3} < 0$$

Finally, we need to show that both profit functions are concave in s_2 . For the profit function of the retailer, the second derivative with respect to the quality s_2 reduces to

$$\frac{\partial^2 \pi_R}{\partial s_2^2} = \frac{\alpha(27\alpha s_2 - 10\bar{\theta})}{24\bar{\theta}} \leq 0 \Leftrightarrow s_2 < \frac{19\bar{\theta}}{27\alpha}$$

which is true for all $s_2 < s_1 = \frac{\bar{\theta}}{3\alpha}$.

For the marketplace owner's profit function, the second derivative is given by:

$$\frac{\partial^2 \pi_M}{\partial s_2^2} = \frac{\alpha(243\alpha^4 s_2^4 + 1674\alpha^3 s_2^3 \bar{\theta} + 3024\alpha^2 s_2^2 \bar{\theta}^2 - 1152\alpha s_2 \bar{\theta}^3 - 1520\bar{\theta}^4)}{6\bar{\theta}(8\bar{\theta} + 3\alpha s_2)^3}$$

Let $k = \frac{\alpha s_2}{\bar{\theta}}$. Then the expression above is negative if:

$$243k^4 + 1674k^3 + 3024k^2 - 1152k - 1520 < 0$$

For $s_2 \leq \frac{\bar{\theta}}{3\alpha}$ it means that $k \leq \frac{1}{3}$ and the expression above can be at most equal to $-1503 < 0$.

The fact that the profit curve of the marketplace owner is higher, concave, downward sloping and steeper at $s_2 = s_1$, and has negative first derivative at $s_2 = s_2^{R*}$, implies that it achieves maximum for $s_2^{M*} < s_2^{R*} < s_1$.

Higher-quality private label In this section I shortly discuss the outcome if a marketplace owner or retailer introduce a higher quality than the seller. The case is to some extent a mirror image to the lower-quality case, with the difference that now the seller has the relatively less costly product to offer. As his quality is on the highest margin in a stand-alone monopolist market, his product is still the more profitable one. Therefore, in this case as well, the private label gets overpriced (compared to the retailer case, where monopoly prices are set) and the seller's price is slightly higher. Similarly the fee decreases to further protect revenues from the seller product. Because the marketplace owner extracts the larger share of channel revenues stemming from the seller's product, there is more incentive to differentiate when choosing $s_2^* > s_1$. However, because higher qualities are suboptimal and have lower margins due to the increasing convex costs, both types of intermediaries prefer to enter with lower quality than with higher quality.

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