Department of Economics

WORKING PAPER

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Incentive contracts when agents distort probabilities

Working Paper No. 2101

May 2021

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April 27, 2021

Abstract

I show that stochastic contracts are powerful motivational devices when agents distort probabilities. Stochastic contracts allow the principal to target probabilities that, when distorted by the agent, enhance the agent's motivation to exert effort on the delegated task. This novel source of incentives is absent in traditional contracts. A theoretical framework and an experiment demonstrate that stochastic contracts targeting small probabilities, and thus exposing the agent to a large degree of risk, generate higher performance levels than traditional contracting modalities. A result that contradicts the standard rationale that optimal contracts should feature a tradeoff between insurance and efficiency. This unintuitive finding is attributed to probability distortions caused by likelihood insensitivity—cognitive limitations that restrict the accurate evaluation of probabilities.

JEL Classification : C91, C92, J16, J24.
Keywords: Contracts, Risk Attitude, Incentives, Probability Weighting, Experiments.

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*I thank Charles Noussair, Patricio Dalton, Wieland Müller, Aurelien Baillon, Peter Wakker, Robert Dur, Eeva Mauring, Cole Williams, Geoffrey Castillo, and Stephanie Koornneef for valuable comments. I also thank participants in seminars at Erasmus University Rotterdam, Maastricht University, Tilburg University, University of Vienna, Vienna University of Economics and Business, and the Foundations of Utility and Risk conference at York. The experiments reported in this paper were funded by the Center for Economic Research (CentER) at Tilburg University.

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1. Introduction

A fundamental result from incentive theory is that a principal, who cannot observe the agent's action to exert effort, motivates the agent with a contract that features a tradeoff between insurance and efficiency (Holmstrom, 1979). On the one hand, the agent is not fully insured against the risk emerging from performance on the delegated task not being fully determined by his effort. Instead, the agent receives a contract with a schedule of payments that depend positively on performance. On the other hand, the risk exposure implied by the offered incentive contract should be moderate not to disincentivize the agent to accept it.

Recent literature exploring the influence of behavioral biases on contract design shows that simple and costless extensions to this traditional solution can enhance motivation. For instance, contracts in which wage-irrelevant production goals are specified by the principal generate higher effort levels when agents exhibit reference-dependence preferences (Corgnet et al., 2018).[1] This paper shows that stochastic contracts introducing risk in the agent's environment motivate agents who suffer from probability distortion to a greater extent than traditional contracting modalities.

Abundant empirical evidence from the literature of decision theory shows that individuals, when making decisions under risk, tend to overweight small probabilities and underweight medium-sized to large probabilities (Abdellaoui, 2000, Gonzalez and Wu, 1999, Wu and Gonzalez, 1996, Tversky and Fox, 1995, Tversky and Kahneman, 1992). With a contract that exposes the agent to some desirable degree of risk, the principal not only activates these probability distortions, but is also able to target those probabilities that, when distorted by the agent, enhance his motivation to exert effort on the delegated task. Since these incentives are absent in traditional contracts, because they seek to expose the agent to as little risk as possible, stochastic contracts have the potential to generate greater output at no extra cost for the principal.

I consider a simple version of stochastic contracts in which the agent obtains one of two possible outcomes: a monetary compensation that depends on the agent's performance on the delegated task, and a payment that does not depend on performance. When offered this contract, the agent faces the risk that his effort might not count toward his compensation. Moreover, the principal can introduce the desired amount of additional risk in the agent's environment by adjusting the probability that the performance-contingent compensation is paid. Therefore, under full commitment, the agents' decision about how much effort to

[1] Other examples of these simple extensions are contracts that include contests for status in the organization when individuals exhibit a preference for status (Besley and Ghatak, 2008, Auriol and Renault, 2008, Moldovanu et al., 2007) and contracts that allow dynamically inconsistent agents to set personal production targets (Kaur et al., 2015).

exert not only depends on the monetary incentives offered by the contract, but also, and more importantly, on the perceived probability that effort influences his compensation. The principal's problem in the studied framework amounts to choosing the risk exposure that best motivates the agent.

To understand how stochastic contracts can outperform traditional contracts, consider a setting in which both contracts are cost-equivalent for the principal. That is, the *expected* monetary compensation associated to supplying a level of output when an agent works under the probability contract is equal to the monetary compensation given when the same level of output is supplied under a traditional contract. An expected value maximizer will be equally motivated under both contracts and as a consequence will exert the same level of effort. However, when the assumption that the agent perceives probabilities accurately is relaxed, and instead it is assumed that he overweights the probability that the performance-contingent outcome realizes, the stochastic contract motivates this agent to exert higher effort. The underlying reason for such boost in motivation is that this distortion of probabilities inflates the agent's perceived benefits of supplying higher levels of effort under the stochastic contract.

A simple theoretical framework serves two purposes. First, it pins down the conditions guaranteeing the main result of the paper, which is that stochastic contracts exposing the agent to large amounts of risk generate more motivation than more traditional contracts. When the agent's probability weighting function attains a lower-bound, representing the necessary probability overweighting to make him risk seeking, the principal is better off offering the stochastic contract with a very small probability. Second, the theory provides a set of predictions that are empirically tested with a laboratory experiment.

A controlled laboratory experiment demonstrates that stochastic contracts, when implemented with a small probability, i.e. $p = 0.10$, yield higher performance in an effort intensive task as compared to a cost-equivalent piece-rate contract. In contrast, I find that stochastic contracts implemented with larger probabilities, namely $p = 0.30$ or $p = 0.50$, yield no differences in performance as compared to a cost-equivalent piece-rate. The experiment also features an elicitation of the utility and probability weighting functions of subjects. These data show that subjects display linear utilities and an average weighting function with inverse-S shape. I demonstrate that this pattern of probability distortion explains the treatment effects found in the effort task. In addition, analyses of the data show that probability distortions due to *likelihood insensitivity*, which refers to the cognitive inability of individuals to accurately evaluate probabilities (Tversky and Wakker, 1995), explain the difference in performance between the stochastic contract implemented with $p = 0.10$ and the piece-rate contract.

While stochastic contracts are typically treated as a theoretical idea in the literature and their application might be prohibited in countries where gambling is forbidden, I show that their incentives can be brought to practice using standard tools of personnel economics. For instance, in a setting in which output is stochastic, bonus and commission contracts offering monetary rewards after some production target is achieved expose the worker to different degrees of risk. For instance, when the production target is set high the associated probability of achievement is small and the agent is exposed to more risk than if the target was set more modest. This paper shows that if the probability of obtaining the bonus or commission is overweighted by the worker, the principal can induce more motivation from this contract than if she were to motivate the agent using a fixed wage or a piece-rate contract. I provide a detailed explanation of this application of stochastic contracts and provide some more in the last section of the paper.

This paper contributes to at least two strands of literature. Its theoretical and empirical results add in several ways to the literature of behavioral contract theory (See Koszegi (2014) for a review). The main contribution to this literature is the result that, when agents distort probabilities, stochastic contracts introducing large amounts of risk motivate the agent to a greater extent than traditional contracts. This is at odds with the standard result from incentive theory stating that the principal faces a trade-off between incentives and insurance. While the optimality of stochastic contracts has been put forward in other settings, such as multitasking environments (Ederer et al., 2018), when agents exhibit aspiration levels (Haller, 1985), or when agents are loss averse (Herweg et al., 2010), I am the first to show theoretically and empirically that they are desirable when agents exhibit probability weighting.

To the best of my knowledge only Spalt (2013) has studied optimal contract design under probability weighting. The most relevant distinction with respect to that paper is that I focus on the agent's incentive compatibility constraint. That is, I study the incentives that result from offering stochastic contracts with different degrees of risk to establish the degrees of risk that best motivate the agent. Spalt's (2013) analysis does not consider these incentives and, due to the studied setting in his paper, ignores that constraint. Another relevant difference with that paper is that I study the incentives of a general class of contracts introducing risk in the agent's compensation. These incentives can be brought to practice in multiple ways, not only by means of compensation plans with stock options.[2]

Finally, the results of this study also contribute to the literature of decision theory. To the best of my knowledge I am the first to provide applications of probability weighting

[2]Other crucial difference with respect to Spalt (2013) is that he analyzes the profitability of compensation plans with stock options using a calibration exercise that employs parameters estimated in classical experiments. Instead, I demonstrate analytically that stochastic contracts are more effective in motivating agents than more traditional contracting modalities. Also, my experiment is designed to directly link the subjects' performance under probability contracts to their risk preferences. This experimental design feature allows me to cleanly establish whether the subjects' probability weighting function, and not other factors, drives the result that the proposed contracts can generate greater performance.

elicitation techniques to the context of incentives. Furthermore, the experimental results illustrate the importance of using parametrized probability weighting functions that separate the components of likelihood insensitivity and optimism/pessimism. I use the different methods proposed by Wakker (2010) and applied in Abdellaoui et al. (2011) to isolate these two components of probability weighting, and show that they contribute unequally to the effectiveness of stochastic contracts.

2. The model

Consider a principal (she) who delegates a task to an agent (he). The agent's decision consists of exerting an effort level $e \in [0, \bar{e}]$ on the task. This decision depends on the disutility associated to exerting effort, as well as on the monetary incentives included in a take-it or leave-it contract that is offered by the principal *before* the agent makes the decision to exert effort.

The agent experiences marginally increasing disutility from higher effort levels. Specifically, the disutility from effort is represented by the cost function $c(e)$, a continuously differentiable, strictly increasing, and convex function.

Assumption 1. *$c(e)$ is a $\mathcal{C}^2$ function with $c'(e) > 0$, $c''(e) > 0$, and $c(0) = 0$.*

Moreover, it is assumed that effort translates into output in a deterministic way.

Assumption 2. *$y = f(\theta, e) = \theta e$ for all $\theta \in [0, 1]$.*

The parameter $\theta \in [0, 1]$, included in the production function, captures the agent's ability on the task. Hence, Assumption 2 states that agents with higher ability can deliver higher output levels without having to exert as much effort.

Settings in which the link between effort, output, and ability is deterministic have been labeled as "false moral hazard" in the literature (See Laffont and Martimort (2002) Ch. 7.2). The rationale for investigating the effectiveness of stochastic contracts using this framework rather than a standard moral hazard model, is that this setting allows me to establish whether introducing risk in the agent's environment enhances motivation in a situation in which he would not have to face any risk. In an extension of the model, presented in Appendix B, I consider a framework in which output is stochastic. The results therein confirm the main result presented in this section: the principal can derive more motivation by introducing large amounts of risk in the agent's environment with the stochastic contract.

To incentivize the agent to exert high effort, the principal offers the agent a contract with a transfer $t(y)$. It is assumed that the transfer enters the agent's utility through the function $b\left(t(y)\right)$ about which I make the following assumption:

Assumption 3. *$b(t)$ is a $\mathcal{C}^2$ function with $b(0) = 0$ and $b'(t) > 0$.*

Note that Assumption 3 does not impose restrictions on the sign of the second derivative of $b\left(t(y)\right)$. That is because the results of the model will be evaluated under the two signs that this derivative attains.

Two types of contracts will be considered: traditional contracts and stochastic contracts. As a benchmark of traditional contracts, I use linear contracts. Formally, these contracts consists of a transfer $t_d(y) := ay$, where $a > 0$ represents a monetary quantity.[3] While linear contracts may be optimal under rather specific conditions, e.g. Holmstrom and Milgrom (1987), this type of incentive scheme is broadly used by organizations in practice and is prevalent in the study of incentives.

All in all, the agent's utility when offered the contract t_d is:

$$U(t_d) = b(ay) - c(e). \tag{1}$$

Alternatively, the principal can incentivize the agent to work on the task using a stochastic contract. Such a contract also offers a monetary compensation that depends on the agent's level of output, but, unlike the piece-rate, this compensation is not given with certainty. Instead, the agent receives it with a probability $p \in (0, 1]$, chosen ex-ante by the principal. As a consequence, the principal has two channels to motivate the agent: i) via the monetary rewards given in exchange of the level of output that is supplied and ii) via changes in the likelihood that such rewards are indeed paid.

Formally, stochastic contracts are lottery-like compensation schedules of the type $t_s(y) := (Ay, p; 0, 1-p)$, where $A > 0$ represents a monetary quantity.[4] The timing of the contract is as follows. The principal moves first choosing $p \in (0, 1]$ and A. After this choice is made, p and A are communicated to the agent before he makes a decision about the level of e to be exerted. Next, the agent chooses e. Finally, when the contracted work-span concludes, a random device to which the principal credibly commits determines whether or not the agent's compensation depends on the supplied level of y.

[3] A more general representation of these contracts is $t_s = F + Ay$ where $F \geq 0$ is a fixed pay that does not depend on the agent's performance. Since F does not generate any incentives for the agent to exert effort, the normalization $F = 0$ is considered throughout the paper.

[4] A more general representation of the stochastic contract is $t_s = (F + Ay, p; F, 1-p)$ for some fixed-payment $F \geq 0$. This representation is more realistic inasmuch as it leaves some non-zero base-pay to the agent. To make the linear contract comparable to this contract, I also use the normalization $F = 0$.

I assume that the agent's risk preferences are characterized by rank-dependent utility (RDU, henceforth) (Quiggin, 1982). These risk preferences model probability distortions using probability weighting functions $w(p)$ about which I make the following assumption.

Assumption 4. *A probability weighting function is* $w(p) : [0, 1] \to [0, 1]$ *such that:*

- $w(p)$ *is* $\mathcal{C}^2$*;*
- $w'(p) > 0$ *for all* $p \in [0, 1]$*;*
- $w(0) = 0$ *and* $w(1) = 1$*;*
- *There exists* $\tilde{p} \in [0, 1]$ *such that* $w''(p) < 0$ *if* $p \in [0, \tilde{p})$ *and* $w''(p) > 0$ *if* $p \in (\tilde{p}, 1]$*;*
- $\lim_{p\to 0^+} w'(p) > 1$ *if* $\tilde{p} > 0$*;*
- $\lim_{p\to 1^-} w'(p) > 1$ *if* $\tilde{p} < 1$*;*
- *If* $\tilde{p} \in (0, 1)$*, there exists a* $\hat{p} \in (0, 1)$ *such that* $w(\hat{p}) = \hat{p}$*.*

According to Assumption 4, $w(p)$ is an increasing and two-times continuously differentiable function that maps the unit interval onto. The probability weighting function contains *at least* two fixed-points: one at $p = 0$ and another one at $p = 1$. Furthermore, $w(p)$ can exhibit three possible shapes: a concave shape if $\tilde{p} = 1$, a convex shape if $\tilde{p} = 0$, and an inverse-S shape if $\tilde{p} \in (0, 1)$. The latter shape generates an additional interior fixed-point, $\hat{p} \in (0, 1)$.[5]

All in all, the rank-dependent expected utility of the agent when offered t_s is:

$$RDU(t_s) = w(p)b(Ay) - c(e). \tag{2}$$

Notice that when $w(p) = p$, RDU collapses to expected utility theory (EUT, from here onward). An agent with risk preferences characterized by EUT exhibits the following expected utility when working under t_s:

$$\mathbb{E}\left(U(t_s)\right) = pb(Ay) - c(e). \tag{3}$$

A key difference between EUT and RDU is that under RDU the agent's risk attitudes are cojointly determined by the curvature of b and the curvature of $w(p)$, which is informally called *probabilistic risk attitudes*. Hence, whether the agent is risk averse or risk seeking depends on the interaction between these two factors.

Another theory of risk that incorporates probability distortions through probability weighting functions is Cumulative Prospect Theory (CPT, henceforth) (Tversky and Kahneman, 1992). CPT is a more descriptive version of RDU. An agent with CPT preferences also

[5] As noted by Wakker (2010), a weighting function with *cavexity*, that is first concave and then convex, does not necessarily ensure the existence of an interior point fixed-point. However, the assumptions $\lim_{p\to 0^+} w'(p) > 1$ if $\tilde{p} > 0$ and $\lim_{p\to 1^-} w'(p) > 1$ if $\tilde{p} < 1$ along with *cavexity* guarantee the existence of an interior fixed point $\hat{p} \in (0, 1)$.

exhibits probabilistic risk attitudes, but in addition also displays relativistic perception of outcomes with respect to a *reference point*. In the interest of space, I relegate the formal description of CPT preferences and the analysis of the incentives produced by the proposed contracts on agents with CPT preferences to Appendix C.

2.1. Probabilistic risk attitudes and their decomposition

This subsection can be omitted if the reader is acquainted with the concepts of likelihood insensitivity (Tversky and Wakker, 1995), and pessimism and optimism toward risk (Yaari, 1987, Abdellaoui, 2002). As previously mentioned, characterizing the agent's risk preferences with RDU introduces probabilistic risk attitudes (Wakker, 1994), which are the influence of the agent's sensitivity to the probabilities included in the contract on his total risk attitude.

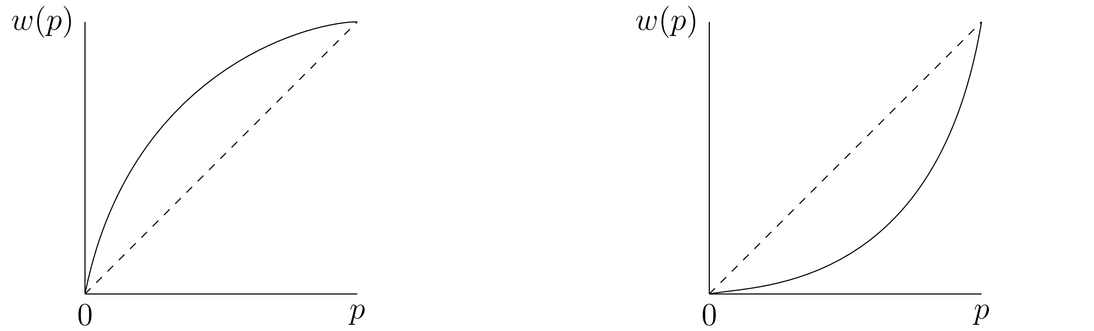
To precisely investigate the way in which probabilistic risk attitude affects the effectiveness of stochastic contracts, I distinguish between two components of probability weighting. This decomposition is based in Wakker (2010). The first component captures *motivational* deviations from EUT stemming from pessimist or optimist attitudes toward risk. These factors affect probability evaluations because of the agent's irrational belief that unfavorable outcomes, in the case of pessimism, or favorable outcomes, in the case of optimism, realize more often. Pessimism is represented with a convex weighting function and optimism is represented with a concave weighting function. Figure 1 presents graphical examples of optimism and pessimism.

Definition 1. *Pessimism (optimism) is characterized by a probability weighting function $w(p)$ with the properties of Assumption 4 and $\tilde{p} = 0$ ($\tilde{p} = 1$).*

It is useful to determine when an agent suffers from stronger degrees of optimism or pessimism. The following definition, due to Yaari (1987), formalizes these comparisons.

Definition 2. *An agent i with weighting function $w(p)_i$ is more optimistic (pessimistic) than an agent j with weighting function $w(p)_j$ if $w(p)_i = \psi(w(p)_j)$, with $\psi : [0,1] \to [0,1]$ a strictly increasing, continuous, and concave (convex) function.*

The second component influencing probabilistic risk attitude is likelihood insensitivity (Tversky and Wakker, 1995, Wakker, 2001). This component captures the notion that individuals distort probabilities because they are not sufficiently sensitive towards changes in intermediate probabilities and are overly sensitive to changes in extreme probabilities. This deviation from EUT is due to cognitive and perceptual limitations. An extreme characterization of likelihood sensitivity is a step-shaped probability weighting function assigning $w(p) \approx 0.5$ to all interior probabilities $p \in (0,1)$, i.e. $w(p)$ with the properties



(a) Example of optimism

(b) Example of pessimism

Figure 1: Motivational sources of probability distortion

of Assumption 4 and $\hat{p} = 0.5$, $\lim_{p \to 0^+} w(p) = 0.5$, and $\lim_{p \to 1^-} w(p) = 0.5$. An opposing characterization to likelihood insensitivity is that of an EUT agent who is fully sensitive to probabilities, $w(p) = p$. Figure 2 presents graphical examples of different degrees of likelihood insensitivity.[6]

Definition 3. *Likelihood insensitivity is characterized with a probability weighting function $w(p)$ with the properties of Assumption 4 and $\tilde{p} = 0.5$.*

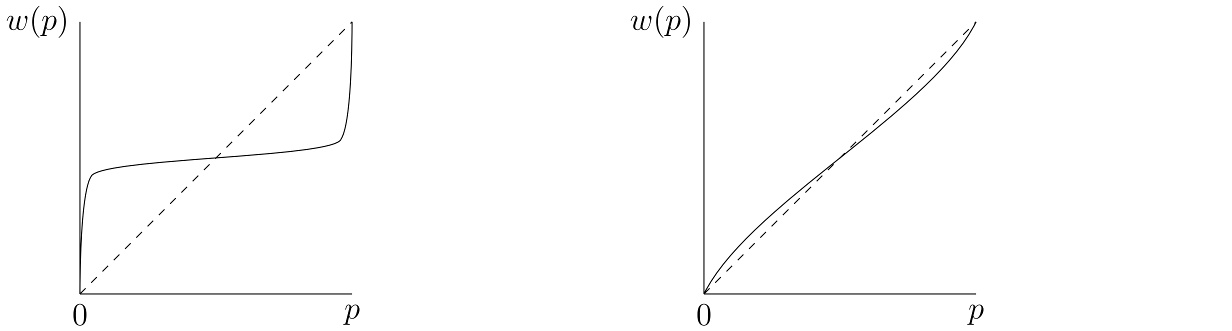
It will become useful to understand the conditions for which an agent suffers from stronger likelihood insensitivity. The following definition based on Tversky and Wakker (1995)'s subadditivity provides us with such a comparative.

Definition 4. *An agent i with weighting function $w(p)_i$ is more likelihood insensitive than an agent j with weighting function $w(p)_j$ if $w(p)_i = \phi(w(p)_j)$, where $\phi : [0, 1] \to [0, 1]$ a strictly increasing, continuous, and subadditive function in the sense of Tversky and Wakker (1995).*

The co-existence of optimism or pessimism, and likelihood insensitivity generates probabilistic risk attitudes that can be represented with a probability weighting function with an inverse-S shape. The location of the interior fixed-point, $\hat{p}$ of such weighting function depends on whether the agent displays pessimism or optimism. For instance, a pessimist agent who is also likelihood insensitive, exhibits a $w(p)$ with an interior fixed-point located in the interval $\hat{p} \in (0, 0.5)$.

When comparing $t_s(y)$ and $t_d(y)$ special focus is given to the roles of likelihood insensitivity and optimism. These two components yield different requirements with regard to the implementation of the contracts. In particular, if likelihood insensitivity leads to higher effort when $t_s(y)$ is offered, then the higher performance obtained with this type of contracts is due

[6] These two phenomena have been addressed in the psychological literature as *curvature* and *elevation* (Gonzalez and Wu, 1999). I instead use the jargon used in economics.



(a) Example of extreme likelihood insensitivity (b) Example of moderate likelihood insensitivity

Figure 2: Cognitive sources of probability distortion

to cognitive limitations that can be inherent to the agent's perception of probability, and that can be readily available to the principal. Instead, if optimism yields that $t_s(y)$ generates higher output, then the principal needs to contract with agents that are optimistic when facing risk. This is a stringent requirement, given the abundant evidence that individuals are generally averse to risk, and thus pessimistic.

2.2. Contract comparisons

In this subsection the considered contracts are compared with respect to the output levels that they generate. To facilitate these comparisons, I make a final assumption about the monetary incentives offered by both contracts. In particular, I assume that stochastic contracts offer, on expectation, the same monetary reward as the linear contract. Formally, let $A \equiv \frac{a}{p}$, so that $\mathbb{E}(t_s) = ay = t_d$. This equivalence allows me to solely focus on the incentives produced by probability contracts implemented with different probability p while keeping the monetary payment constant. Importantly, note that a consequence of this assumption is that stochastic contracts nest linear piece-rates, i.e. as $p \to 1$, then $A \to a$. Hence, when introducing risk is undesirable the principal can simply set $p = 1$.

Proposition 1 shows that the effectiveness of the probability contract relative to the piece-rate contract depends on the agent's risk attitudes. The proofs of the main results of the paper are relegated to Appendix A.

Proposition 1. *Under Assumptions 1-4, the stochastic contract:*

(i) generates lower output than the linear contract if the agent is EUT and $b''\left(t(y)\right) < 0$*;*

(ii) generates higher output than the linear contract if implemented with infinitesimal probabilities and the RDU agent exhibits $\frac{w(p)}{p} \geq \exp\left(\int_p^1 \frac{\rho\left(\frac{ay}{\mu}\right)}{\mu} d\mu\right)$ *at the p implemented*

by the principal, where $\rho\left(\frac{ay}{p}\right) := -\frac{\frac{ay}{p}b''\left(\frac{ay}{p}\right)}{b'\left(\frac{ay}{p}\right)}$.

Proposition 1 part (i) formalizes the conventional wisdom that introducing risk in the agent's environment is counterproductive when utility exhibits diminishing returns to transfers, i.e. $b''(t) < 0$. In that case, the principal would be better off incentivizing the agent with the linear contract. That EUT is an adequate characterization of the agent's risk preference is critical to this result.

Part (ii) of Proposition 1 presents a more interesting result. It states that, under RDU preferences, the stochastic contract implemented with very small probabilities elicits higher performance than the linear contract if the agent's probability weighting function sufficiently overweights the probability specified by the principal. The lower-bound $\exp\left(\int_p^1 \frac{\rho\left(\frac{ay}{\mu}\right)}{\mu} \mathrm{d}\mu\right)$ specifies the necessary degree of probability overweighting to be attained. Intuitively, stochastic contracts introducing large amounts of risk are more motivating if the agent's probabilistic risk seeking attitude, implied by the degree at which the probability specified by the principal is overweighted, outweighs the potentially risk averse attitude stemming from the agent's utility curvature. If this is indeed the case, the agent is risk seeking and has a taste for risky contracts.

I provide further intuition of Proposition 1 part (ii) with the following examples.

Example 1: Linear utility. Let $b''(\cdot) = 0$. Then $\rho\left(\frac{ay}{\mu}\right) = 0$ and the lower-bound from Proposition 1 (ii) becomes $w(p) \geq p$. In words, the agent needs to overweight probabilities in some non-empty probability interval in order to be more motivated under the stochastic contract. To make things more concise consider Prelec (1998)'s weighting function $w(p) = \exp\left(-\beta(-ln(p))^{\alpha}\right)$. Suppose first that $\alpha = 1$, then $w(p) = p^{\beta}$, a power function. The requirement $p^{\beta} \geq p$ holds if $\beta < 1$; the weighting function needs to be concave, which according to Definition 1 is equivalent to the agent exhibiting optimism. Suppose instead that $\beta = 1$. Then $w(p) = \exp\left(-(-ln(p))^{\alpha}\right)$ and $w(p) \geq p$ holds as long as $\alpha < 1$; the weighting function exhibits an inverse-S shape, which according to Definition 3 is equivalent to the agent having likelihood insensitivity.

Example 2: CRRA utility. Let now $\rho\left(\frac{ay}{\mu}\right) = 1 - k$ where $k \in \mathbb{R}$. The lower-bound from Proposition 1 (ii) becomes $w(p) \geq p\exp\left((1-k)\int_1^p \frac{1}{\mu}\mathrm{d}\mu\right) \Leftrightarrow w(p) \geq p^k$. In words, the agent's probabilistic risk seeking attitudes should outweigh the eventual risk aversion emerging from his utility curvature. Consider again Prelec (1998)'s weighting function. Suppose first that $\alpha = 1$, then $w(p) \geq p^k \Leftrightarrow \beta < k$ which entails that the weighting function must attain a level of concavity. In light of Definition 2 this is equivalent implies attaining some level of optimism. Consider next $\beta = 1$. The stochastic contract generates higher performance if $\exp\left(-(-ln(p))^{\alpha}\right) > p^k$. If $\alpha < 1$, then the concavity of $w(p)$ in the interval $p \in (0, \frac{1}{\mathrm{e}})$ should be stronger than that implied by p^k. According to Definition 4, this is equivalent to attaining some degree of likelihood insensitivity.

A common property in the previous examples is that the agent needs to attain a degree of probability overweighting to become risk seeking at small probabilities. As a general note, the most prominent proposals of probability weighting functions, e.g. Prelec (1998), Goldstein and Einhorn (1987), and Tversky and Kahneman (1992), exhibit extreme sensitivity to small probabilities. That is, they exhibit $\lim_{p\to 0} \frac{w(p)}{p} = \infty$ implying strong probabilistic risk seeking attitudes for small probability events. [7] Thus, as long as the potential risk averse attitudes from utility curvature are bounded, i.e. $\lim_{p\to 0} \exp\left(\int_p^1 \frac{\rho\left(\frac{ay}{\mu}\right)}{\mu} \mathrm{d}\mu\right) < B$ for some $B < \infty$, the agent with probabilistic risk attitudes characterized by those probability weighting functions will be more motivated under stochastic contracts that expose him to large degrees of risk.[8]

In the remainder of the section I investigate how the motivational and cognitive components of probability weighting enhance or inhibit the implementation of the stochastic contract. The following corollaries show that either optimism or likelihood insensitivity, on their own, can, when sufficiently strong, guarantee that the stochastic contract is more motivating than the linear contract.

Corollary 1. *Let* $\lim_{p\to 0} \exp\left(\int_p^1 \frac{r\left(\frac{ay}{\mu}\right)}{\mu} d\mu\right) < B$ *for some* $B < \infty$. *There exists a level of optimism that guarantees Proposition 1 part (ii).*

To understand Corollary 1 recall that optimism is equivalent to the probability weighting function being concave with all interior probabilities being overweighted. Also note that concavity of the weighting function implies that overweighting of small probabilities is more pronounced than overweighting of large and intermediate probabilities. Hence, stronger optimism, in the sense of Definition 2, implies that small probabilities are more overweighted at a larger extent, making it more likely that the agent becomes risk seeking at small probabilities. The corollary states that if the potential risk averse attitudes emerging from the curvature of the utility function are bounded, then a sufficiently strong degree of optimism

[7]Well-known non-continuous probability weighting functions such as those proposed by Chateauneuf et al. (2007) and Kahneman and Tversky (1979) also exhibit extreme sensitivity near impossibility and near certainty. In fact, the non-continuity of these weighting functions stems from the observation that subjects are overly sensitive to extreme probabilities.

[8]A prominent example in which this Proposition 1 (ii) does not hold emerges when b adopts the properties of the CARA family. Under that assumption, then the condition to guarantee Proposition 1 (ii) becomes $w(p) \geq p \exp\left(Day\left(\frac{1}{p} - 1\right)\right)$, where $D \in \mathbb{R}$. It is evident that the right-hand side of that inequality is larger than one unless $p = 1$.

will make the agent risk seeking and thus more motivated when exposed to large degrees of risk.

Corollary 2. *Let* $\lim_{p\to 0} \exp\left(\int_p^1 \frac{r\left(\frac{ay}{\mu}\right)}{\mu} d\mu\right) < B$ *for some* $B < \infty$. *There exists a level of likelihood insensitivity that guarantees Proposition 1 part (ii).*

Likelihood insensitivity, on its own, implies small probabilities being overweighted. As the agent becomes more likelihood insensitive, that is as his weighting function becomes more subadditive, small probabilities are overweighted to a larger extent, making more likely that the agent is risk seeking at small probabilities. The corollary states that if the potential risk averse attitudes emerging from the curvature of the utility function are bounded, a sufficiently strong degree of likelihood insensitivity will make the agent risk seeking and more motivated when exposed to large degrees of risk.

To summarize, the two parts in Proposition 1 yield opposing results regarding the effectiveness of the proposed contract. Part (i) formalizes the notion that introducing risk in the agent's compensation is detrimental to performance. Part (ii) presents the conditions under which, an RDU agent is more motivated under a contract that exposes him to a large amount of risk. Corollary 1 and Corollary 2 show that sufficiently strong probability weighting either from optimism or likelihood insensitivity guarantee this result.

An alternative analysis that not only incorporates the agent's incentive compatibility, as is done throughout this section, but that also considers the participation constraint and the principal's objective function is presented in Appendix D. Such analysis confirms the main conclusion achieved with this simple model: when contracting with an agent with RDU preferences, the principal is better off implementing stochastic contracts as long as the agent exhibits a strong sensitivity to probabilities that makes him risk seeking. Appendix B shows that this result emerges under less stringent conditions in a setting in which y is stochastic. Finally, Appendix C shows that under CPT preferences this result is achieved under similar conditions emerge for the domain of gains, while less stringent conditions are required when the contract locates the agent in the domain of losses.

3. Experimental Method

3.1. The general setup

The experiment was conducted at Tilburg University's CentERLab. The participants were all students at that university and were recruited using an electronic system. The data consist of 15 sessions with a total of 172 subjects. On average, a session lasted approximately

80 minutes. Between eight and eighteen subjects took part in a session. The currency used in the experiment was Euros. I used Z-Tree (Fischbacher, 2007) to implement and run the experiment. Subjects earned on average 15.83 Euros. The instructions of the experiment are presented in Appendix H.

The experiment consisted of two parts. Upon arrival, participants were informed that their earnings from either part one, or those from part two would become their final earnings and that this would be decided by chance at the end of the experiment.[9] In the first part of the experiment subjects performed a task that demanded their effort and attention. The task consisted of summing five two-digit numbers multiple times. Each summation featured randomly drawn numbers by the computer, ensuring similar levels of difficulty between subjects. When a participant knew the answer to the numbers that appeared in his screen, he could submit it using the computer interface. Immediately after submission, a new summation appeared on the computer screen and the participant was invited to solve the new summation. In total, subjects had 10 rounds of four minutes each to complete as many summations as they could.

In the second part of the experiment, the subjects' task was to choose between two binary lotteries multiple times. This part of the experiment was designed to elicit their utility and probability weighting functions. To elicit these two functions, I used the two-step method developed by Abdellaoui (2000). This method has the advantage of not making assumptions about the way in which subjects evaluate probabilities nor the way in which they evaluate monetary outcomes.[10] Subjects were informed that only one lottery, chosen at random at the end of the experiment, was played and counted toward their earnings for the second part of the experiment.

After the second part of the experiment was over, subjects were given feedback about their performance and were informed about their earnings in the first part of the experiment. They were also informed about the lottery that was chosen for compensation for the second part of the experiment and its realization. In addition, subjects learned whether part one or part two counted toward their final earnings.

[9] This randomization of payments could be a source of concern if subjects distort probabilities. However, as it will be shown in §6, subjects on average exhibit $w(p) \approx 0.5$, so the probability underlying this randomization of payments was approximately evaluated accurately. Moreover, as it will explained later on, isolation guarantees this randomization to generate proper measurement of effort and risk attitude.

[10] A drawback of this method is that it can be violate incentive compatibility when subjects are aware of the chained nature of the questions they face. I overcome this disadvantage by adding questions that are not used in the analysis of the data at random, and by randomizing the appearance of the lotteries of decision sets 7 to 11 which will be described later on.

3.2. Treatments

There were four different treatments differing in the type of incentives given to subjects to perform the effort task. Subjects were randomly assigned to one of these treatments. The baseline treatment is *Piecerate*. Subjects assigned to that treatment were paid 0.25 Euros for every correctly solved summation. The other three treatments also offered monetary rewards that depended on individual performance on the task. However, they also introduced the risk that performance in a round did not count towards their earnings. The magnitude of that risk was varied across the treatments. These treatments seek to represent stochastic contracts implemented with different probabilities.

The Treatments *LowPr*, *MePr* and *HiPr* featured a low, medium, and high probability, respectively, that performance in a given round counted toward earnings. In LowPr subjects faced a 10% chance that performance in a given round counted toward earnings. This was implemented by telling subjects assigned to this treatment that only one round (out of ten) was to be chosen at random at the end of the experiment and that only performance in that round was going to be paid. Similarly, in MePr and HiPr, three and five rounds, respectively, were randomly chosen at the end of the experiment and performance in those rounds was paid. This representation of stochastic contracts assumes isolation, which implies that the subjects' decision to exert effort in a round does not take into account decisions made or to be made in other rounds.[11] [12] A failure of isolation would yield similar average performance between the treatments because, as it will be explained next, they face similar monetary incentives across treatments.

As in the theoretical framework, the monetary incentives offered in the treatments Piecerate, LowPr, MePr and HiPr were calibrated such that subjects faced, on expectation, similar monetary incentives across the treatments. For instance, a subject assigned to LowPr received 2.50 Euros for each correct summation in the round that was chosen for compensation, which was tenfold of what a subject assigned to Piecerate earned for each correctly solved summation. This difference in monetary payments exactly accounts for the probability difference that performance in a round is paid between these treatments. By the same token, subjects assigned to the MePr and HiPr treatments received a compensation of 0.85 and 0.50

[11] Isolation is strongly supported by the literature of experimental economics in designs where the *random incentive system*, i.e. paying one round or one exercise at random at the end of the experiment, is implemented (See for instance Baltussen et al. (2012), Hey and Lee (2005) and Cubitt et al. (1998)).

[12] A common misunderstanding regarding the random incentive system is assuming that the independence axiom is a necessary condition to guarantee appropriate experimental measurement, which in the present setup implies that subjects making effort choices as if each decision was paid and in the absence of income effects. While the independence axiom, along with some dynamic principles, *suffices* to guarantee proper measurement, isolation, on its own, ensures proper experimental measurement under the random incentive system even when the independence axiom does not hold (Baltussen et al., 2012).

Euros, respectively, for each correctly solved summation in the rounds that were randomly chosen for compensation.

The probabilities governing the treatments LowPr, MePr and HiPr were chosen according to the common finding in the literature of decision-making: subjects distort probabilities according to an inverse-S shape probability weighting function with an interior fixed point at approximately $p = 0.33$ (See Wakker (2010) pp. 204 for an extensive list of references finding this pattern). If subjects in the experiment follow this regularity, they should on average overweight the probability that a round is chosen with 10% chance, underweight the probability that a round is chosen with 50% chance, and approximately evaluate accurately the probability that a round is chosen with 30% chance.

3.3. Elicitation of risk preference

The second part of the experiment consisted of 11 decision sets. Decision sets 1 to 6 constitute the first step of Abdellaoui (2000)'s methodology, which is based on Wakker and Deneffe (1996). These decision sets elicit a sequence of outcomes $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ that made the subject indifferent between lottery $L = (x_{j-1}, 2/3; 0.5, 1/3)$ and lottery $R = (x_j, 2/3; 0, 1/3)$ for $j = \{1, ..., 6\}$. Indifference was found through bisection.[13] These two lotteries were designed so that the elicited sequence of outcomes yielded equally spaced utility levels for each subject. Formally $u(x_j) - u(x_{j-1}) = u(x_{j-1}) - u(x_{j-2})$ for all for $j = \{1, ..., 6\}$.

The starting point of the program, x_0, was set at $\frac{2}{5}$th of what a subject earned in the first part of the experiment. This is done to more accurately relate the subjects' risk preference, more specifically their utility curvature which can change with the magnitude of monetary incentives, to their behavior on the first part of the experiment. Subjects were not informed about this calibration. The left panel of Table 1 presents an example illustrating the bisection procedure used for Decision sets 1 to 6.

Decision sets 7 to 11 constitute the second step of Abdellaoui (2000)'s methodology. These decision sets were designed to elicit a sequence of probabilities,

$$\{w^{-1}(p_1), w^{-1}(p_2), w^{-1}(p_3), w^{-1}(p_4), w^{-1}(p_5)\},$$

where $p_{j-1} = \frac{j-1}{6}$ and $j = \{2, ..., 6\}$. These probabilities made subjects indifferent between

[13] The bisection procedure works as follows: a subject was required to express his preference between two initial versions of lotteries L and R. After having made a choice, the outcome x_j of lottery R changed as a function of the subject's choice, such that either the outcome of the chosen lottery was replaced by a less attractive alternative, or the outcome of the not chosen one was replaced by a more attractive alternative, while the other lottery remained the same. When facing the new situation, the subject was invited to make a choice again between the modified lotteries L and R. This process was repeated four times for each decision set.

the lottery $L = (x_6, w^{-1}(p_{j-1}); x_0, 1 - w^{-1}(p_{j-1}))$ and the degenerate lottery x_{j-1}. These two lotteries were designed so that the elicited probabilities yield equally spaced probability weights, i.e. $w(p_j) - w(p_{j-1}) = w(p_{j-1}) - w(p_{j-2})$ for $p_{j-1} = \frac{j-1}{6}$ and $j = \{2, ..., 6\}$. Again, indifference between these lotteries was found through bisection. The right panel of Table 1 presents an example illustrating the bisection procedure for these decision sets.

Table 1: Example of the Abdellaoui's (2000) algorithm

	Left Panel			**Right Panel**		
iteration #	Lotteries	Interval	Choice	Lotteries	Probability	Choice
1	L=$(1, 0.66; 0.50, 0.33)$ R=$(3.7, 0.66; 0, 0.33)$	[1, 6.4]	L	L=$(x_1, 1)$ R=$(x_6, 0.50; 1, 0.5)$	$[0, 1]$	L
2	L=$(1, 0.66; 0.50, 0.33)$ R=$(5.05, 0.66; 0, 0.33)$	[3.7,6.4]	R	L=$(x_1, 1)$ R=$(x_6, 0.75; 1, 0.25)$	$[.5, 1]$	L
3	L=$(1, 0.66; .050, 0.33)$ R=$(4.38, 0.66; 0, 0.33)$	[3.7,5.05]	R	L=$(x_1, 1)$ R=$(x_6, 0.87; 1, 0.13)$	$[.75, 1]$	R
4	L=$(1, 0.66; 0.50, 0.33)$ R=$(4.04, 0.66; 0, 0.33)$	[3.7,4.38]	L	L=$(x_1, 1)$ R=$(x_6, 0.81; 1, 0.19)$	$[.75, .87]$	L
5	L=$(1, 0.66; 0.50, 0.33)$ R=$(4.21, 0.66; 0, 0.33)$	[4.04,4.38]	L	L=$(x_1, 1)$ R=$(x_6, 0.85; 1, 0.15)$	$[.81, .87]$	L
		$x_1 \in [4.21, 4.38]$			$p_1 \in [0.85, 0.87]$	

Note: This table illustrates the bisection method used to elicit utility and probability functions. The lotteries in this table are expressed in the form $(m, p; n, 1-p)$ where m and n are prizes, and p is a probability. The left panel presents the bisection method to elicit utility and the right panel presents the bisection method to elicit probability functions.

4. Hypotheses

The theoretical model generates a set of hypotheses that are tested with the experiment. When formulating these hypotheses two additional assumptions about subjects' preference are made. First, subjects exhibit linear utility. This assumption is consistent with numerous experimental findings (Abdellaoui et al., 2008, 2007, Abdellaoui, 2000, Wakker and Deneffe, 1996) and with the fact that payments in laboratory experiments are modest. Second, subjects are RDU decision makers with a probability weighting function that conforms to the common finding of inverse S-shape with a fixed-point located at $\hat{p} \approx 0.33$.

Hence, risk attitude is expected to be fully determined by the probability weighting function. Specifically, subjects are expected to be risk seeking for probabilities $p \in (0, 0.33)$ and risk averse in $p \in [0, 33, 1]$. Example 1 from Section §2.4 becomes essential to understand the following hypotheses.

The first hypothesis is based on Proposition 1 part (ii). It states that stochastic contracts implemented with probability $p = 0.10$ motivate subjects to a greater extent than linear piece-rates. That is because subjects are risk seeking at those probabilities. Instead, stochastic contracts implemented with probabilities $p = 0.30$ and $p = 0.50$ yield the same and lower motivation, respectively, as compared to Piecerate. Subjects are expected to be risk averse at those probabilities.

Hypothesis 1. *Subjects exhibit an average performance levels across treatments that conform to the ranking:*

$$\textit{LowPr} > \textit{MePr} = \textit{Piecerate} > \textit{HiPr}.$$

Empirical support in favor of Hypothesis 1 does not conclusively validate the model. Factors other than probability weighting might generate these performance differences. If the model is accurate, the expected performance differences across treatments should be explained by the subjects' tendency to distort probabilities. The following hypothesis captures this conjecture about the underlying mechanism behind the treatment effects.

Hypothesis 2. *(i) Subjects assigned to LowPr who overweight small probabilities exhibit higher performance as compared to subjects assigned to Piecerate.*

(ii) Subjects assigned to HiPr who underweight intermediate probabilities exhibit lower performance as compared to subjects assigned to Piecerate.

Finally, I formulate hypotheses about the influence of motivational or cognitive factors of probability weighting on the expected performance differences. Since utility is expected to be linear, Corollary 1 can be interpreted as optimism toward risk, regardless of its level, guaranteeing on its own the greater performance of stochastic contracts implemented with small probabilities against linear contracts.

Hypothesis 3. *Optimism explains the greater average performance of LowPr against Piecerate.*

Furthermore, under linear utility, Corollary 2 can be interpreted as stating that likelihood insensitivity, regardless of its level guaranteeing on its own the greater performance of stochastic contracts implemented with small probabilities against linear contracts.

Hypothesis 4. *Likelihood insensitivity explains the greater average performance of LowPr against Piecerate.*

5. Results

5.1. Treatment effects

In this subsection I compare performance in the effort task across the treatments. Performance is the total number of correctly solved summations by a subject. Table 2 presents the descriptive statistics of performance by treatment. This table shows that, as predicted by Hypothesis 1, the stochastic contract implemented with $p = 0.10$ generates higher performance than the piece-rate contract. Specifically, subjects assigned to the LowPr treatment solved on average 20.56 % more summations than subjects assigned to Piecerate ($t(84.454) = 2.360, p = 0.010$).[14] The effect size of this treatment difference is of 0.50 standard deviations which is significant at the 5% confidence level.[15] This is the main result of the paper: subjects are more motivated when the contract exposes them to large additional degrees of risk as compared to a cost-equivalent contract without such risk exposure.

Table 2: Descriptive statistics of performance by treatments

Treatment	LowPr	MePr	HiPr	Piecerate	Total
Mean	98.116	87.900	83.750	81.377	87.686
Median	91	87	82.500	77	85
St.dev.	34.659	28.134	24.358	31.684	30.412
N	43	40	44	45	172

In contrast, stochastic contracts implemented with higher probabilities generate similar average performance as the linear contract. Subjects assigned the MePr treatment solved 87.9 correct summations on average and subjects assigned the HIPR treatment solved 83.7 correct summations on average, neither of which are statistically different from the average number of correct summations solved by subjects assigned to Piecerate.[16] These findings partially support Hypothesis 1, which accurately predicts that MePr induces similar performance as Piecerate, but incorrectly predicts that HiPr generates lower performance than Piecerate. Conjectures about this partial confirmation of Hypothesis 1 are provided at the end of this subsection.

[14] A Wilcoxon-Mann-Whitney test also rejects the null hypothesis of no average difference between Piecerate and LowPr ($z = 2.634, p = 0.008$).

[15] The significance of the effect size was evaluated with a bootstrapped 95% confidence interval with 10000 replications.

[16] The t-tests of these comparisons are ($t(83) = 1.005, p = 0.159$) and ($t(82.44) = 0.386, p = 0.693$), respectively. Wilcoxon-Mann-Whitney tests of these comparisons yield ($z = 1.321, p = 0.186$) and ($z = 0.895, p = 0.3710$, respectively.

Among the treatments representing stochastic contracts, the LowPr generates greater average performance. This treatment generates 17% higher average performance than HiPr ($t(75.215) = 2.232, p = 0.014$), and 11% higher average performance than MePr ($t(79.575) = 1.479, p = 0.071$).[17] Therefore, statistical inference using pairwise testing suggests that LowPr generates highest average performance as compared to the other treatments.

I estimate regressions of individual performance on treatment dummies, dummies that capture different possible shapes of the utility function, and dummies that capture different possible shapes of the weighting function. These regressions establish how robust are the aforementioned treatment effects when average risk attitude is controlled for. If the treatment effects found above are robust to the inclusion of these controls, the performance differences between the treatments are not an artifact of more risk seeking or less risk averse subjects assigned to some of the treatments.

The dummy variables included in the regressions were constructed using data from the second part of the experiment. Subjects' utility functions were classified as having a linear, concave, convex, or mixed shape. Details of this classification are provided in Appendix E.[18] Furthermore, probability weighting functions of subjects were classified as displaying lower subadditivity (LS, from here onward) and/or upper subadditivity (US, from here onward). A weighting function with LS assigns larger decision weights to low probabilities than to intermediate probabilities. A weighting function with US assigns larger decision weights to large probabilities than to intermediate probabilities.[19] In some specifications a different classification for weighting functions is included, this alternative classification was based on the strength of the possibility effect relative to the certainty effect. The variable "Possibility" takes a value of one if the possibility effect is stronger than the certainty effect and zero otherwise.[20] Details of the two classifications of probability weighting functions are provided in Appendix F.

Table 3 presents regression estimates. For all specifications, the coefficient associated to assignment to LowPr is significant and positive at the 5% significance level, which corroborates the aforementioned result that subjects assigned to that treatment display higher average performance than subjects assigned to Piecerate, the benchmark treatment of the regression.

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[17] Wilcoxon-Mann-Whitney tests of these differences yield ($z = 1.966, p = 0.049$) and ($z = 1.035, p = 0.15$), respectively. The effect sizes of these differences are of 0.480 standard deviations and 0.322 standard deviations, respectively.

[18] In short, a variable $\Delta_j^{''} := (x_j - x_{j-1}) - (x_{j-1} - x_{j-2})$ for $j = 2, 3, 4, 5, 6$, is constructed for each subject. A subject is classified as having linear utility if most values $\Delta_j^{''}$ are close to zero, concave utility if most values $\Delta_j^{''}$ are positive, convex utility if most values $\Delta_j^{''}$ are negative, and mixed utility otherwise.

[19] Specifically, a subject in the experiment exhibited LS when $w^{-1}\left(\frac{1}{6}\right) < w^{-1}\left(\frac{2}{6}\right) - w^{-1}\left(\frac{1}{6}\right)$. Also, a subject exhibited US when $1 - w^{-1}\left(\frac{5}{6}\right) < w^{-1}\left(\frac{5}{6}\right) - w^{-1}\left(\frac{4}{6}\right)$.

[20] A subject had a stronger possibility effect when $1 - w^{-1}\left(\frac{5}{6}\right) < w^{-1}\left(\frac{1}{6}\right)$.

Table 3: Regression of performance on treatments

	(1) Performance	(2) Performance	(3) Performance	(4) Performance
LowPr	16.739**	16.558**	16.001**	16.526**
	(7.090)	(7.508)	(7.532)	(7.589)
MePr	6.522	6.714	6.335	6.585
	(6.487)	(6.610)	(6.677)	(6.724)
HiPr	2.372	1.684	1.616	0.758
	(5.985)	(5.888)	(6.308)	(6.016)
Concave		14.359	15.067	15.090
		(9.401)	(9.529)	(9.681)
Convex		7.623	8.527	7.185
		(10.109)	(10.469)	(10.513)
Mixed		3.864	3.698	4.259
		(6.625)	(6.699)	(6.785)
US			0.904	
			(5.183)	
LS			2.924	
			(5.053)	
Possibility				4.901
				(7.637)
Certainty				7.062
				(7.791)
Constant	81.378***	79.819***	78.497***	74.667***
	(4.726)	(5.025)	(5.242)	(7.371)
R^2	0.045	0.062	0.065	0.064
Observations	172	172	172	172

Note: This table presents the estimates of the Ordinary Least Squares regression of the model $Performance_i = \gamma_0 + \gamma_1 LowPr + \gamma_2 MePr + \gamma_3 HiPr + Controls'\Lambda + \epsilon_i$, with $E(\epsilon | MepR, LowPr, HiPr, Controls) = 0$. "Performance" is the number of correctly solved sums solved by a subject in the first part of the experiment, "LowPr", "MePr" and "HiPr" are binary variables that indicate if a subject was assigned to the treatment offering stochastic contracts implemented with low, medium or high probability, respectively. "Piecerate" is the benchmark of the regression. Robust standard errors in parenthesis. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Similarly, the coefficient of LowPr is significantly higher than the estimate associated with HiPr ($F(1, 163) = 5.75, p = 0.017$) and significantly higher than that associated to MePr ($F(1, 163) = 2.16, p = 0.071$). Thus, among the studied contracts, the LowPr produces the highest performance.

The first result is that the stochastic contract with small probability, exposing subjects to more additional risk, yields higher performance than the other three contracts.

Result 1. *Average performance across treatments conforms to the ranking:*

$$\textit{LowPr} > \textit{MePr} = \textit{Piecerate} = \textit{HiPr}.$$

A possible explanation to Result 1 is that LowPr generates higher performance because it circumvents income effects (See Azrieli et al. (2018) and Lee (2008)). In contrast, these effects are present in Piecerate and can be a source of demotivation for subjects toward the last rounds of the experiment. This explanation also accommodates the result that LowPr generates higher average performance than MePr and HiPr because, by paying less rounds, that treatment is more effective in minimizing income effects. If income effects are the main reason behind the treatment effects, then we should not observe performance differences across treatments in the first round, when income effects are absent. A regression of performance in a given round on treatment dummies, round dummies, and relevant controls is estimated with standard errors clustered at the individual level. Regression estimates show that in the first round subjects assigned to LowPr achieve 1.67 higher average summations as compared to subjects in Piecerate ($p = 0.013$). Subjects in LowPr also exhibit higher average performance in the first round as compared to subjects in HiPr ($p = 0.015$) and subjects in HiPr ($p = 0.083$). Hence, the aforementioned performance differences emerge in the absence of income effects.

To conclude, the data on performance in the effort task *partly* supports Hypothesis 1. However, Hypothesis 1 was structured around the common findings that individuals exhibit linear utility, overweight probabilities smaller than $p = 0.33$, and underweight all probabilities thereafter. Instead, the analyses presented in this subsection suggest that subjects in the experiment overweighted on average the probability $p = 0.10$ and evaluated approximately accurately the probabilities $p = 0.3$ and $p = 0.5$. In the next subsection, it is shown that subjects indeed display an average weighting function with that shape.

5.2. Utility and probability weighting functions

In this subsection I analyze the data obtained in the second part of the experiment. As described in §3, decision sets 1 to 6 elicited the sequence of outcomes $\{x_1, x_2, x_3, x_4, x_5, x_6\}$

for each subject. This sequence captures a subject's preference over the monetary outcomes in the lotteries. Analyses of these data show that the majority of subjects, i.e. 75% of subjects, exhibit linear utility functions and that the average utility function is linear, which is in line with the findings of Wakker and Deneffe (1996), Abdellaoui (2000), Abdellaoui et al. (2008), and Abdellaoui et al. (2011) and is consistent with the critique put forward by Rabin (2000). Given this result, and since the main focus of the paper is on probability weighting functions and probabilistic risk attitude, I relegate the complete analysis of the shape of utility functions to Appendix E.

Decision sets 7 to 11 elicited the sequence of probabilities

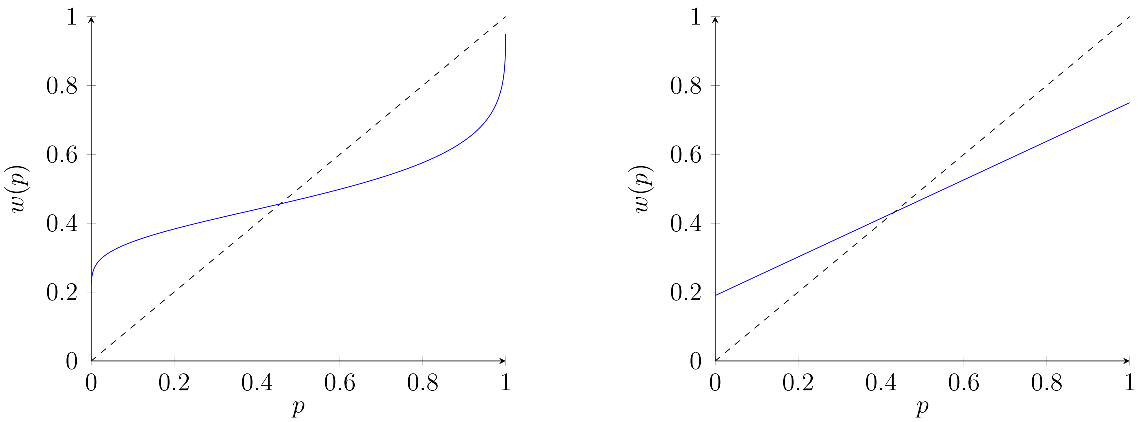
$$\{w^{-1}(p_1), w^{-1}(p_2), w^{-1}(p_3), w^{-1}(p_4), w^{-1}(p_5)\},$$

for each subject. These data are analyzed to examine how subjects evaluated probabilities. This is done by means of regressions that relate elicited probabilities to the probability weights that they map. The rationale for using regressions as the main analysis of these data is that i) they provide a good indication of the average degree of probability weighting, ii) the resulting estimates can be used to compare the degree of probability weighting in the experiment to those reported in previous studies, and iii) with the resulting estimates one can construct indexes of likelihood insensitivity and optimism, which according to Corollary 1 and Corollary 2 are relevant factors behind the documented efficiency of stochastic contracts.[21] Alternative analyses of these data, including individual analyses and non-parametric analyses, are presented in Appendix F.

Various and well-known proposals of probability weighting are assumed to estimate the regressions. Specifically, I use the neo-additive probability weighting function (Chateauneuf et al., 2007), Tversky and Kahneman (1992)'s probability weighting function, Prelec (1998)'s two-parameter probability weighting function, and Goldstein and Einhorn (1987)'s log-odds probability weighting function. That the regressions are estimated using various functionals of $w(p)$ ensures robustness, i.e. that the estimated results do not stem from the underlying assumptions of a particular functional form.

The resulting estimates are presented in Table 4 and Figure 3. Under all specifications it is found that subjects display an average probability weighting function with a strong inverse-S shape and, for weighting functions having two parameters, less pessimism than previously documented. Detailed comparisons of these estimates with respect to those found in previous studies are provided in Appendix G. This shape of the average probability weighting function

[21] Comparisons across studies (see argument ii), must be taken with a grain of salt inasmuch as resulting differences cannot only be attributed to differences in preferences, but also to the different stakes and methods used to elicit risk preferences.



(a) using Prelec (1998) (b) using Chateauneuf et al. (2007)

Figure 3: Average probability weighting functions

along with the linearity of the average utility function constitutes the second result.

Result 2. *Subjects exhibit on average linear utility and a probability weighting function with strong inverse S-shape and moderate pessimism.*

The conjunction of strong inverse-S curvature and moderate pessimism produces a probability weighting function that strongly overweights small probabilities and moderately distorts medium-sized probabilities. For example, using the estimates of Panel 1 in Table 4 it can be established that for the subjects in this experiment the probability $p = 0.10$ is on average perceived to be $w(0.10) = 0.25$, while the probabilities $p = 0.30$ and $p = 0.5$ are on average perceived to be $w(0.30) = 0.363$ and $w(0.50) = 0.477$, respectively. These patterns of probability distortion accommodate the findings of the first part of the experiment, namely that LowPr generates higher output than Piecerate, and that HiPr, MePr, and Piecerate produce similar performance.

5.3. Overweighting of probabilities, likelihood insensitivity, and the treatment effect

This subsection reconciles the data from the first and second part of the experiment. I first present empirical evidence supporting Hypothesis 2 by demonstrating that the higher average performance of subjects assigned to LowPr is caused by their tendency to overweight small probabilities. Second, I show that likelihood insensitivity, alone, explains the treatment effects documented in §5.1, validating Hypothesis 4.

To empirically verify the validity of Hypothesis 2, I extend the statistical models presented in Table 3 by including interactions between the dummy variable indicating assignment to LowPr and variables that capture overweighting of small probabilities. Specifically, I use the

Table 4: Parametric estimates of average probability weighting function

Panel 1 $w(p) = c + sp$		
	$\hat{c}$	$\hat{s}$
	0.194***	0.566***
	(0.021)	(0.036)
Log-Likelihood		220.288
N		860
Panel 2 $w(p) = \frac{p^{\psi}}{\left(p^{\psi}+(1-p)^{\psi}\right)^{\frac{1}{\psi}}}$		
		$\hat{\psi}$
		0.598***
		(0.016)
Adj. R^2		0.838
N		860
Panel 3 $w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$		
	$\hat{\gamma}$	$\hat{\delta}$
	0.281***	0.921***
	(0.025)	(0.020)
Adj. R^2		0.863
N		860
Panel 4 $w(p) = \exp\left(-\beta(-\ln(p))^{\alpha}\right)$		
	$\hat{\alpha}$	$\hat{\beta}$
	0.284***	0.841***
	(0.025)	(0.015)
Adj. R^2		0.864
N		860

Note: This table presents estimates of the average probability weighting function of subjects when different parametric forms are assumed. Panel 1 presents the maximum likelihood estimates of the equation $w(p) = c + sp$ with truncation at $w(p) = 0$ and at $w(p) = 1$. Panel 2 presents the non-linear least squares estimation of the function $w(p) = \frac{p^{\psi}}{(p^{\psi}+(1-p)^{\psi})^{\frac{1}{\psi}}}$. The third panel presents the non-linear least squares estimates of the parametric form $\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$. The last panel presents the non-linear least squares estimates of the function $w(p) = \exp\left(-\beta(-\ln(p))^{\alpha}\right)$. Robust standard errors in parenthesis. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

binary variables LS, Possibility, and Overweight$_{p=\frac{1}{6}}$ to capture small probability overweighting. The first two variables were already explained defined in §5.1, while the last variable takes a value of one if a subject overweights the probability $p = \frac{1}{6}$ and zero otherwise.[22]

Column (1) in Table 5 presents the OLS estimates of the regression when LS is used to capture overweighting of small probabilities. I find that subjects assigned to LowPr and who have weighting functions with lower subadditivity display an average performance level that is significantly higher than that of subjects in Piecerate. In contrast, subjects assigned to LowPr with weighting functions without lower subadditivity display an average performance level that is statistically indistinguishable to that of subjects in Piecerate. Hence, only subjects with a weighting function assigning larger decision weights to small probabilities relative to the weights assigned to medium-ranged probabilities display higher performance levels under LowPr as compared to subjects assigned to Piecerate. Columns (2) and (3) in Table 5 show that similar conclusions can be reached using the other two variables that capture overweighting of small probabilities.[23]

That overweighting of probabilities explains the treatment effect constitutes the third result.

Result 3. *Subjects assigned LowPr and who overweight small probabilities exhibit higher performance with respect to subjects in Piecerate.*

Finally, I investigate whether the higher performance of subjects under LowPr is due to optimism or likelihood insensitivity, as both factors can explain why subjects overweight small probabilities and exhibit risk seeking attitude for small probabilities. To that end, subjects are first classified as likelihood insensitive and/or optimistic. Following Wakker (2010) and Abdellaoui et al. (2011), I estimate for each subject, i, the neo-additive probability weighting function:

$$w(p_{ij}) = c_i + s_i p_{ij} + e_i, \tag{4}$$

where j indicates a probability from the elicited sequence in decision sets 7 to 11 and e_i

[22] These variables relate in the following way: a subject for whom LS takes a value of one surely overweights the probability $p = \frac{1}{6}$ and might exhibit a possibility effect that is stronger than the certainty effect. Similarly, a subject for whom Possibility takes a value of one surely overweights $p = \frac{1}{6}$ and exhibits LS.

[23] Unlike the analyses in which LS and Possibility were used to capture probability overweighting, the coefficient associated to LowPr remains significant when Overweight$_{p=\frac{1}{6}}$ is used in column (2). This significance suggests that the treatment effect is not entirely captured by the mere tendency of subjects to overweight the probability $p = \frac{1}{6}$. Instead, the treatment effect is explained by the subjects' tendency to overweight the probability $p = \frac{1}{6}$ *relative* to other probabilities. For example, assigning higher weights to $p = \frac{1}{6}$ as compared to the weights given to $p = \frac{1}{2}$ or $p = \frac{5}{6}$. This is already suggestive that overweighting of probabilities due to likelihood insensitivity, which entails a relative overweighting of small probabilities with respect to medium-sized probabilities, better explains the treatment effect.

Table 5: The influence of probability overweighting on the treatment effects

	(1) Performance	(2) Performance	(3) Performance
LowPr*Mechanism	17.127***	17.418**	16.634**
	(8.378)	(8.312)	(7.873)
Mechanism	-2.141	3.014	-5.307
	(5.557)	(6.158)	(5.528)
LowPr	7.083	17.306**	2.679
	(8.199)	(8.252)	(10.792)
MePr	6.971	6.517	7.039
	(6.559)	(6.602)	(6.535)
HiPr	1.410	1.725	1.019
	(6.485)	(6.584)	(6.391)
Concave	15.766*	14.934*	14.719
	(8.851)	(8.928)	(8.742)
Convex	10.768	8.257	8.865
	(18.129)	(18.333)	(17.968)
Mixed	4.073	3.968	6.837
	(6.802)	(6.900)	(6.851)
US	0.332	1.377	
	(4.958)	(4.941)	
Constant	80.358***	78.495***	83.151***
	(5.183)	(5.471)	(5.783)
Mechanism variable	LS	Overweight$_{p=\frac{1}{6}}$	Possibility
R^2	0.083	0.063	0.08
Observations	172	172	172

Note: This table presents the estimates of the Ordinary Least Squares regression of the model $Performance_i = \gamma_0 + \gamma_1 LowPr + \gamma_2 LowPr * Mechanism + \gamma_3 Mechanism + \gamma_4 MePr + \gamma_5 MePr + \gamma_6 HiPr + Controls'\Gamma + \epsilon_i$, with $E(\epsilon_i | MePr, LowPr, HiPr, Controls, Mechanism) = 0$. "Performance" is the number of correctly solved sums solved by a subject in the first part of the experiment. "LowPr", "MePr" and "HiPr" are binary variables that indicate if a subject was assigned to a treatment offering stochastic contract implemented with low, medium or high probability, respectively. "Piecerate" is the benchmark of the regression. In column (1) Mechanism is equal to "LS" a binary variable that takes a value of one if a a subject has a weighting function with lower subadditivity and zero otherwise. In column (2) Mechanism is equal to "Overweight$_{p=\frac{1}{6}}$" a binary variable that takes a value of one if a subject overweights the probability $p = \frac{1}{6}$. In column (3) Mechanism is equal to "Possibility " a binary variable takes a value of one if a subject has a weighting function with the possibility effect being stronger than the certainty effect and zero otherwise. Robust standard errors in parenthesis. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

is an error term. To allow for S-shape estimates, this weighting function was estimated with truncation at $w(0)$ and $w(1)$.

The magnitude of the estimate $\hat{s}_i$ indicates subject's i sensitivity to probabilities. If $\hat{s}_i < 1$, the subject is not sufficiently responsive to changes in probabilities and is classified as likelihood insensitive. Instead, if $\hat{s}_i \geq 1$, the subject is sufficiently or too sensitive to changes in probabilities and is classified as likelihood sensitive. I find that 96 subjects are classified as likelihood insensitive and 61 subjects are classified as likelihood sensitive.[24] Importantly, the degree of likelihood insensitivity is balanced across treatments. For instance, there is no empirical evidence to reject the null hypothesis of no difference in likelihood insensitivity between LowPr and Piecerate ($t(77.248) = 0.657$).

In addition, the magnitude of $\hat{c}_i$ determines subject's i optimism. Whenever $\hat{c}_i > 0$ the subject assigns large weights to best-ranked outcomes and, as a consequence, exhibits optimism. Alternatively, if $\hat{c}_i < 0$ the subject exhibits pessimism. I find that 97 subjects display optimism while 75 subjects display pessimism. Degrees of optimism are also balanced across treatments. For instance, there is no empirical evidence to reject the null hypothesis of no difference in optimism between LowPr and Piecerate ($t(77.248) = -0.6570.304, p = 0.512$).

Binary variables capturing the above classifications are added to the regressions presented in Table 3. These variables are labeled "Optimism" and "Likelihood Insensitive". Also, interactions between these variables and the binary variable capturing assignment to LowPr are included in some specifications. The coefficient associated to these interactions evaluate the strength of the treatment effect for subjects who exhibit likelihood insensitivity and/or optimism.

The resulting estimates are presented in Table 6. I find empirical support for Hypothesis 4 in columns (2) and (4). Specifically, I find that likelihood insensitive subjects assigned to LowPr display higher average performance as compared to subjects assigned to Piecerate. In contrast, subjects assigned to LowPr and who were not classified as likelihood insensitive did not exhibit performance differences with respect to the baseline treatment. These findings support the result that likelihood insensitivity, on its own, ensures the efficiency of stochastic contracts when they are implemented with a small probability. In addition, columns (3) and (4) show that subjects displaying optimism and who were assigned to LowPr exhibit average performance levels that are statistically indistinguishable from those of subjects in Piecerate. Therefore, optimism, on its own, is unable to explain the treatment effects documented in §5.1. This result invalidates Hypothesis 3.

To gain robustness, a similar exercise is performed using the probability weighting

[24] Fifteen subjects had a negative estimated parameter $\hat{s}_i < 0$ which has no clear interpretation and are thus left unclassified.

Table 6: The influence of likelihood insensitivity and optimism on treatment effects

	(1) Performance	(2) Performance	(3) Performance	(4) Performance
LowPr*Likelihood ins.		23.692***		25.501**
		(10.625)		(11.133)
LowPr*Optimist			5.668	-11.884
			(8.730)	(10.689)
LowPr	15.907**	8.986	15.173	10.157
	(6.562)	(10.387)	(10.058)	(11.850)
MePr	7.140	7.111	4.699	7.158
	(6.497)	(6.502)	(6.616)	(6.525)
HiPR	2.879	2.953	1.061	3.050
	(6.407)	(6.413)	(6.410)	(6.445)
Likelihood ins.	6.811	4.487	6.892	4.025
	(5.492)	(6.125)	(5.572)	(6.451)
Optimist	-10.343*	-9.756*	-10.612*	-8.991
	(5.571)	(5.617)	(6.245)	(6.513)
Concave	14.689	15.076*	14.610*	15.323*
	(8.686)	(8.700)	(8.750)	(8.794)
Convex	7.006	4.838	6.962	4.719
	(17.929)	(18.120)	(17.990)	(18.180)
Mixed	3.936	3.879	3.858	4.076
	(6.792)	(6.798)	(6.860)	(6.869)
Certainty	-9.285*	-9.195*	-9.192*	-9.429*
	(4.978)	(6.789)	(5.085)	(5.0961)
Constant	87.361***	88.1047***	87.436***	87.987***
	(6.302)	(6.366)	(6.369)	(6.404)
R^2	0.093	0.097	0.093	0.098
Observations	172	172	172	172

Note: This table presents the estimates of the Ordinary Least Squares regression of the model $Performance_i = \gamma_0 + \gamma_1 LowPr{*}Likelihoodins. + \gamma_2 LowPr{*}Optimism + \gamma_3 LowPr + \gamma_4 MePr + \gamma_5 HiPr + \gamma_6 Likelihoodins. + \gamma_7 Optimism + Controls'\Gamma + \epsilon_i$, with $E(\epsilon_i|MePr, LowPr, HiPr, Piecerate, Optimism, Likelihoodins., Controls, Mechanism) = 0$. "Performance" is the number of correctly solved sums solved by a subject in the first part of the experiment, "LowPr", "MePr" and "HiPr" are binary variables that indicate if a subject was assigned to a treatment offering a stochastic contract implemented with low, medium or high probability, respectively. "Piecerate" is the benchmark of the regression. "Likelihood ins." is a binary variable that takes a value of one if the subject is likelihood insensitive and zero otherwise. "Optimism' is a binary variable that takes a value of one if the subject displays optimism and zero otherwise. Robust standard errors in parenthesis. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

functions proposed by Prelec (1998) and Goldstein and Einhorn (1987). These functions contain, each, two parameters. One of the parameters mainly influences likelihood insensitivity while the other parameter mainly influences optimism. On the basis of the magnitude of these parameters, I classify subjects according to whether they are likelihood insensitive and/or optimistic. Table 14 in Appendix G presents the results of regressions when these alternative classifications are used. Altogether, the regression estimates therein corroborate the aforementioned results and, thus, the empirical validity of Hypothesis 4.

Result 4. *Likelihood insensitivity and not optimism explains the treatment effects.*

6. Applications and Discussion

This paper demonstrated that stochastic contracts exposing individuals to large degrees of risk yield more motivation than traditional linear contracts. This result is explained by the individuals' tendency to overweight small probabilities, which, when considerable, makes them risk seeking and generates a preference for risky compensation schemes. I show that likelihood insensitivity, the cognitive component of probability weighting, explains this result.

While stochastic contracts are abstract constructs and might be forbidden in countries where gambling is illegal, their incentives can be brought to practice using well-known tools of personnel economics. In the following, I discuss some ways in which these incentives can be implemented.

- **Bonuses.** Consider a setting in which the agent's effort on the task and output relate stochastically. The principal can take advantage of this stochastic relationship by offering a contract that pays a bonus in the contingency that an output target is attained. The findings of this paper show that the principal should set a high target, yielding a small probability of achievement and exposing the agent to a large amount of risk. Workers with RDU preferences suffering from likelihood insensitivity, will be more motivated under this contract as compared to a cost-equivalent linear piece-rate contract. I formalize this application in Appendix B.

- **Excessive entrepreneurship and autonomy in the firm.** The risk neutral principal can sell the agent a highly risky project within the firm, making him residual claimant. The RDU agent suffering from likelihood insensitivity will buy the project as he overweights the probability that the project will be profitable. Moreover, the results of his paper suggest that the agent will be more motivated when he is made the residual claimant of the project, and is thus fully exposed to risk, than when the principal protects him from the risk associated to that project with a standard work contract.

- **Stock options.** A volatile firm can offer CEOs compensation plans that include stock options. Naturally at the moment in which the contract is signed the future stock price is unknown. First, as shown by Spalt (2013) the agent with RDU preferences will accept these contracts despite the firm being risky. These risk seeking attitudes emerge because the agent overweights the probability associated to obtain large gains from calling the option. Second, this paper suggests that when the agent's higher effort shifts the distribution of future stock prices, contracts with stock options generate higher motivation than other less risky standard performance-pay contracts. That is because the perceived contribution of the agent's effort to the probability of high future stock prices is overweighted.

A common property among the aforementioned applications is that the incentives of stochastic contracts are implemented using natural sources of uncertainty: output realizations given effort, future stock prizes, and project success. Indexing the outcomes of the contract to natural uncertain events allows the principal to circumvent the problem of lack credibility that might arise if she were to generate the contract's uncertainty using an artificial device, e.g. a roulette or dice. Assuring that the principal has no influence over the realization of uncertainty allows her to more credibly commit to the contract. Moreover, recent research suggests that individuals display more insensitivity toward ambiguity than toward risk (Baillon et al., 2018, Abdellaoui et al., 2011). Since likelihood insensitivity was found to be the main explanation for effectiveness of the contract, implementing the contract using natural sources of ambiguity can potentially enhance the gains from its usage.

The present study has several limitations that might open avenues for future research. First, it is assumed through the paper that the principal is fully informed about the agent's risk attitudes. Future research could relax this assumption. Specifically, the model presented in §2 can be extended to incorporate a stage of adverse selection. The principal's task in such framework is to design a menu of contracts that not only to motivates agents, but also that screens agents according to their risk preference.

Second, this paper considered a static setting. A more comprehensive investigation of the probability contract could examine its incentives in a setting of repeated interaction between principal and agent. On the theoretical side, it has been shown that optimal contracts in dynamic settings exhibit properties that depend on the agent's expectation over the contracting span, e.g. martingale property. When expectations are distorted, due to probability weighting, it is unclear whether these properties are enhanced or fade away, and whether these conditions are more favorable for stochastic contracts. On the empirical side, an extension to a dynamic setup allows for a more robust analysis of incentives. That is because such setting could shed light on whether and how probability weights are adjusted with experience, and how the incentives of stochastic contracts change as a function of the number of interactions.

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Appendix A. Proofs

Proposition 1

Proof. Assumption 2 and the equivalence $A = \frac{a}{p}$ are used to rewrite equation (2) as follows,

$$RDU(t_s) = w(p)b\left(\frac{ay}{p}\right) - c\left(\frac{y}{\theta}\right). \tag{5}$$

The optimal output level chosen by the agent with RDU preferences when working under t_s, y^{**}, satisfies the following first order condition:

$$b'\left(\frac{ay^{**}}{p}\right)\frac{w(p)}{p}a - c'\left(\frac{y_R^{**}}{\theta}\right)\frac{1}{\theta} = 0. \tag{6}$$

That y^{**} is a maximum requires the second order condition to be negative. Formally,

$$b''\left(\frac{ay^{**}}{p}\right)\frac{w(p)}{p^2}a^2 - c''\left(\frac{y^{**}}{\theta}\right)\frac{1}{\theta^2} < 0. \tag{7}$$

If the principal chooses not to introduce risk in the agent's environment, $p = 1$, equation (5) becomes

$$U(t_s) = b\left(ay\right) - c\left(\frac{y}{\theta}\right), \tag{8}$$

and the corresponding optimal output level, y^*, satisfies the following first-order condition:

$$b'\left(ay^*\right)\frac{w(p)}{p}a - c'\left(\frac{y^*}{\theta}\right)\frac{1}{\theta} = 0. \tag{9}$$

To investigate how y^{**} and y^* relate, implicitly differentiate (6) with respect to y^{**} and p to obtain:

$$\frac{\mathrm{d}y^{**}}{\mathrm{d}p} = \frac{\left(\frac{w'(p)p - w(p)}{p^2}\right)ab'\left(\frac{ay^{**}}{p}\right) - \frac{w(p)a^2y}{p^3}b''\left(\frac{ay^{**}}{p}\right)}{c''\left(\frac{y^{**}}{\theta}\right)\frac{1}{\theta^2} - b''\left(\frac{ay^{**}}{p}\right)\frac{w(p)a^2}{p^2}}. \tag{10}$$

To prove part (i) of the Proposition, the sign of (10) is analyzed under $w(p) = p$. In that case, equation (10) collapses to:

$$\frac{\mathrm{d}y^{**}}{\mathrm{d}p} = \frac{-\frac{a^2y^{**}}{p^2}b''\left(\frac{ay^{**}}{p^2}\right)}{c''\left(\frac{y^{**}}{\theta}\right)\frac{1}{\theta^2} - b''\left(\frac{ay^{**}}{p}\right)\frac{a^2}{p}}. \tag{11}$$

If y^{**} is a maximum, the denominator of (11) must be positive due to (7) evaluated at $w(p) = p$. Hence, the restrictions $a > 0$, $y^{**} > 0$, and $p \in [0, 1]$ imply that, at the optimum, $\frac{\mathrm{d}y^{**}}{\mathrm{d}p} > 0$ if $b''\left(\frac{a^2y^{**}}{p^2}\right) < 0$. In that case the principal is better off offering t_s with $p = 1$, which is equivalent to offering the linear contract t_d. That is because under $p=1$ then $y^{**}=y^*$, but for any $p<1$ then $y^{**}<y^*$. Instead, if $b''\left(\frac{a^2y^{**}}{p^2}\right)>0$, then $\frac{\mathrm{d}y^{**}}{\mathrm{d}p}<0$. In that case the principal offers t_s with p as small as possible, exposing the agent to large degrees of additional risk.

To prove part (ii), consider the general case in which $w(p)$ acquires the properties of Assumption 4. A necessary condition for equation (10) to be negative, at the optimum, is:

$$w'(p)\leq\frac{w(p)}{p}\left(1-\rho\left(\frac{ay^{**}}{p}\right)\right), \tag{12}$$

where $\rho(x):=-\frac{xb''(x)}{b'(x)}$, the coefficient of relative risk aversion. The class of probability weighting functions $w(p)$ that satisfy (12) are found using Grönwall's lemma. First, find the solution to:

$$v'(p)=\frac{v(p)}{p}\left(1-\rho\left(\frac{ay^{**}}{p}\right)\right), \tag{13}$$

where $v(p)$ is a weighting function that exhibits the same properties as $w(p)$. The solution to the above ordinary differential equation is given by:

$$\int\frac{v'(p)}{v(p)}\mathrm{d}p=\int\frac{1}{p}\left(1-\rho\left(\frac{ay^{**}}{p}\right)\right)\mathrm{d}p\Leftrightarrow v(p)=\exp\left(-\int_p^1\frac{1-\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right). \tag{14}$$

Second, the way in which $w(p)$ and $v(p)$ relate is investigated computing the derivative of the ratio $\frac{w(p)}{v(p)}$ as follows,

$$\frac{\mathrm{d}\left(\frac{w(p)}{v(p)}\right)}{\mathrm{d}p}=\frac{(v(p)w'(p)-w(p)v'(p))}{v(p)^2}=\frac{v(p)\left(w'(p)-\frac{w(p)}{p}\left(1-\rho\left(\frac{ay^{**}}{p}\right)\right)\right)}{v(p)^2}, \tag{15}$$

where the second equality results from replacing $v'(p)$ with the derivative of (14). Using the last equality of the above equation together with (12) it can be established that $\frac{\mathrm{d}\left(\frac{w(p)}{v(p)}\right)}{\mathrm{d}p}\leq 0$. Thus, the minimum of $\frac{w(p)}{v(p)}$ is attained at $p=1$ and it must be that for any $p\in(0,1]$:

$$\frac{w(p)}{v(p)}\geq\frac{w(1)}{v(1)}=1. \tag{16}$$

Therefore, the solution to (12) is bounded by (14) in the following way:

$$\frac{w(p)}{p}\geq\exp\left(\int_p^1\frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right). \tag{17}$$

If (17) holds for some non-empty interval in $p\in(0,1)$ then $\frac{\mathrm{d}y^{**}}{\mathrm{d}p}\leq 0$ and the principal is better off offering t_s with the smallest probability in that interval. Alternatively, when (17) cannot hold for any p, $\frac{\mathrm{d}y^{**}}{\mathrm{d}p} > 0$ and the principal must offer t_s with $p = 1$, which is equivalent to the linear contract.

The remainder of the proof investigates the values of p under which (17) can hold. To that end, differentiate the right-hand side of (17) with respect to p to obtain:

$$\frac{\mathrm{d}\exp\left(\int_p^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right)}{\mathrm{d}p} = -\exp\left(\int_p^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right)\frac{\rho\left(\frac{ay^{**}}{p}\right)}{p}. \tag{18}$$

If $\rho\left(\frac{ay^{**}}{p}\right) > 0$, the right-hand side of (17) decreases with p. In that case the lowest value that the right-hand side of (17) attains is at $p = 1$ and is equal to one. Instead, if $\rho\left(\frac{ay^{**}}{p}\right) < 0$ the right-hand side of (17) increases with p. For that case, the highest value attained by the right-hand side of (17) is one at $p = 1$.

Next, differentiate the left-hand side of (17) with respect to p to obtain:

$$\frac{\mathrm{d}\left(\frac{w(p)}{p}\right)}{\mathrm{d}p} = \frac{pw'(p) - w(p)}{p^2}. \tag{19}$$

Using Grönwall's lemma it can be established that $pw'(p) - w(p) \leq 0 \Leftrightarrow \frac{w(p)}{p} \geq 1$, implying that $\frac{\mathrm{d}\left(\frac{w(p)}{p}\right)}{\mathrm{d}p} \leq 0$ under probability overweighting.

Under $\rho\left(\frac{ay^{**}}{p}\right) \leq 0$, probability overweighting, $\frac{w(p)}{p} > 1$ suffices to guarantee (17). That is because, the highest value attained by the right-hand side of that equation is at $p = 1$ and is equal to one, so $\frac{w(p)}{p} > 1$ for any $p < 1$ ensures that (17) holds with strict inequality. Since $\frac{w(p)}{p} > 1$ implies $\frac{\mathrm{d}\left(\frac{w(p)}{p}\right)}{\mathrm{d}p} \leq 0$, then any $p < p^*$ guarantees (17), where $p^* = \hat{p}$ is the interior fixed-point below which all probabilities are overweighted as defined in Assumption 4.

Instead, under $\rho\left(\frac{ay^{**}}{p}\right) > 0$ a necessary condition for (17) to hold for $p < 1$ is:

$$\lim_{p\to 0^+} \frac{w(p)}{p} > \lim_{p\to 0^+} \exp\left(\int_p^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right). \tag{20}$$

Note that since $\lim_{p\to 1^-} \exp\left(\int_p^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right) = 1$, overweighting of probabilities $\frac{w(p)}{p} > 1$ is a necessary condition, but not a sufficient one, for equation (20) to hold.

If (20) holds, there exists a $p^* \in (0, 1]$ such that (17) holds with equality. That is because $\frac{w(p)}{p}$ and $\exp\left(\int_0^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right)$ intersect at least once at $p = 1$, the point where these two expressions attain the minimum value. Also, if these two functions also intersect anywhere in

$p \in (0,1)$, the smallest point at which that intersection takes place is p^*. Since $\frac{\mathrm{d}\left(\frac{w(p)}{p}\right)}{\mathrm{d}p} \leq 0$ under probability overweighting, then any $p < p^*$ guarantees equation (17) with strict inequality.

To finalize the proof, note that both $\rho\left(\frac{ay^{**}}{p}\right) \leq 0$ and $\rho\left(\frac{ay^{**}}{p}\right) > 0$ yield a set of probabilities, $p \in (0, p^*)$ for which t_s implemented with those probabilities elicits higher output. Given that (17) holds for any $p \in (0, p^*)$, then at those probabilities $\frac{\mathrm{d}y^{**}}{\mathrm{d}p} \leq 0$ implying that the principal is better off setting $p \to 0^+$, infinitesimal probability.

■

Corollary 1

Proof. Denote by $o(p) : [0,1] \to [0,1]$ the class of probability weighting functions adopting the properties of Assumption 4 plus $\hat{p} = 1$. That is, all probability weighting functions with optimism in the sense of Definition 1. For optimism, on its own, to guarantee Proposition 1 part (ii) it is required that $o(p)$ attains the following lower-bound,

$$\frac{o(p)}{p} \geq \exp\left(\int_p^1 \frac{r\left(\frac{ay_R^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right). \tag{21}$$

While under optimism $\frac{o(p)}{p} > 1$ holds for all $p \in (0,1)$, the above equation yields a stronger requirement since $\lim_{p\to 1^-} \exp\left(\int_p^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right) = 1$.

By Definition 2, stronger optimism implies that for any $p \in (0,1)$ the expression $\left|\frac{o''(p)}{o'(p)}\right|$ becomes larger. The largest possible level of optimism, i.e. the most concave $o(p)$ function, exhibits $\lim_{p\to 0^+} \left|\frac{o''(p)}{o'(p)}\right| = \infty$. Instead, the smallest possible degree of optimism is given by a weighting function that exhibits $\lim_{p\to 0^+} \left|\frac{o''(p)}{o'(p)}\right| = \epsilon$ for arbitrarily small $\epsilon > 0$. These two extreme characterizations of optimism, together with the assumption $o(p)$ is $\mathcal{C}^2$ and the assumption $\lim_{p\to 0^+} \exp\left(\int_p^1 \frac{r\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right) < B$ for some $B < \infty$ imply that there exists a weighting function with a degree of concavity such that (21) holds with equality as $p \to 0^+$. Denote that threshold degree of concavity of a probability weighting function with optimism by $\hat{o}(p)$.

Any probability weighting function with the properties of Assumption 4 and $\hat{p} = 1$ that exhibits more optimism than $\hat{o}(p)$, i.e. that is such that $\left|\frac{o''(p)}{o'(p)}\right| \geq \left|\frac{\hat{o}''(p)}{\hat{o}'(p)}\right|$, ensures (21). If the agent's weighting function has such degree of optimism, the principal is better off choosing $p \to 0^+$ because in that case (21) holds, which in turn implies $\frac{\mathrm{d}y^{**}}{\mathrm{d}p} < 0$.

■

Corollary 2

Proof. Denote by $l(p):[0,1]\rightarrow[0,1]$ the class of probability weighting functions adopting the properties of Assumption 4 plus $\hat{p}=0.5$. That is, all probability weighting functions generating likelihood insensitivity in the sense of Definition 3. For likelihood insensitivity, on its own, to guarantee Proposition 1 part (ii) it is required that $l(p)$ attains the following lower-bound,

$$l(p) \geq \exp\left(\int_p^1 \frac{r\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right). \tag{22}$$

While under likelihood insensitivity $\frac{l(p)}{p}>1$ holds for all $p\in(0,0.5)$, the above equation yields a stronger requirement since $\lim_{p\to 1^-}\exp\left(\int_p^1 \frac{\rho\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right)=1$.

By Definition 4, stronger likelihood insensitivity implies $\lim_{p\to 1^-} l'(p)$ and $\lim_{p\to 0^+} l'(p)$ becoming larger; more weight assigned to extreme probability events. Extreme likelihood insensitivity implies that for small probabilities $\lim_{p\to 0^+} l'(p)=\infty$. Instead, the mildest form of likelihood insensitivity is $\lim_{p\to 0^+} l'(p)=\epsilon$ for ϵ arbitrarily close to one. These two extreme characterizations of insensitivity, together with the assumptions $l(p)$ is $\mathcal{C}^2$ and $\lim_{p\to 0^+}\exp\left(\int_p^1 \frac{1-r\left(\frac{ay^{**}}{\mu}\right)}{\mu}\mathrm{d}\mu\right)<B$ for some $B<\infty$ imply that there exists a probability weighting function with a degree of likelihood insensitivity such that (22) holds with equality as $p\to 0^+$. Denote the probability weighting function attaining such threshold degree of likelihood insensitivity by $\hat{l}(p)$.

Any probability weighting function with the properties of Assumption 4 and $\hat{p}=0.5$ that is more subadditive than $\hat{l}(p)$, i.e. that exhibits $\lim_{p\to 0^+}\hat{l}(p)>\lim_{p\to 0^+} l(p)$, ensures (22). If the agent's weighting function exhibits such degree of likelihood insensitivity, the principal is better off choosing $p\to 0^+$ because in that case (22) holds, which in turn implies $\frac{\mathrm{d}y^{**}}{\mathrm{d}p}<0$.

■

Appendix B. Stochastic output and applications

In this Appendix I show that the main result of the theoretical framework applies when the relationship between effort, e, and output y, is stochastic. Taking advantage of this framework I formalize one of the applications of the contract discussed in the Discussion section, the last section of the paper.

Let $y \in [0, \bar{y}]$ be a stochastic variable. This can be thought as θ, the ability variable, not being known to the agent himself, or as y being not only influenced by effort, e, and ability but also by exogenous shocks to performance. I assume that y is distributed according to the probability density function $g(y|e)$ which admits a cumulative density function $G(y|e)$.

To make things simple, assume that the agent's action consists on exerting a high effort level or a low effort level $e = \{e_L, e_H\}$. In this setting, only high effort is costly, that is $c(e) = c$ if $e = e_H$ and $c(e) = 0$ if $e = e_L$. Finally, as it is standard in the literature, I assume that output and effort relate according to the monotone likelihood ratio property.

Assumption 5. *Effort and output relate according to* $\frac{\partial}{\partial y}\left(\frac{g(y|e_H)}{g(y|e_L)}\right) > 0$.

The following proposition shows that the RDU agent is more motivated under the contract $t_s(y)$ when it is implemented with small probabilities. Hence, as in Proposition 1 (ii), the agent is more motivated when exposed to additional degrees of risk.

Proposition 2. *Under Assumptions 2- 5, an agent with RDU preferences exerts high effort more often under the stochastic contract implemented with infinitesimal probabilities if* $\frac{w(p)}{p} > 1$ *at the* p *specified by the principal.*

Proof. The RDU agent is motivated under $t_d(y)$ if the following incentive compatibility condition holds:

$$\int_0^{\bar{y}} b(ay)\mathrm{d}w\left(1 - G(y|e_H)\right) - \int_0^{\bar{y}} b(ay)\mathrm{d}w\left(1 - G(y|e_L)\right) \geq c. \tag{23}$$

Instead, the RDU agent is motivated under $t_s(y)$ implemented with some $p \in (0,1)$ when the following inequality holds:

$$w(p)\left(\int_0^{\bar{y}} b\left(\frac{ay}{p}\right)\mathrm{d}w\left(1 - G(y|e_H)\right) - \int_0^{\bar{y}} b\left(\frac{ay}{p}\right)\mathrm{d}w\left(1 - G(y|e_L)\right)\right) \geq c. \tag{24}$$

Denote the non-additive expectation by $\tilde{\mathbb{E}}\left(y|e\right) := \int_0^{\bar{y}} y\mathrm{d}w\left(1 - G(y|e)\right)$. The stochastic contract elicits e_H more often as long as:

$$w(p)\left(\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_H\right)-\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_L\right)\right)>\tilde{\mathbb{E}}\left(b\left(ay\right)\middle| e_H\right)-\tilde{\mathbb{E}}\left(b\left(ay\right)\middle| e_L\right). \tag{25}$$

Rearranging (25) and multiplying it by $\frac{1}{p}$ gives:

$$\frac{w(p)}{p}>\frac{1}{p}\left(\frac{\tilde{\mathbb{E}}\left(b\left(ay\right)\middle| e_H\right)-\tilde{\mathbb{E}}\left(b\left(ay\right)\middle| e_L\right)}{\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_H\right)-\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_L\right)}\right). \tag{26}$$

The derivative of left-hand side of (26) with respect to p is:

$$\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{w(p)}{p}\right)=\frac{w'(p)p-w(p)}{p^2}, \tag{27}$$

From the proof of Proposition 1 and from Example 1, it can be established that $\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{w(p)}{p}\right)>0$ if $w(p)<p$, i.e. under probability underweighting. However, at $p=1$, the largest probability that can be set by the principal, equation (26) cannot hold since $w(1)>1$, an impossibility. Hence, under probability underweighting the stochastic contract does not elicit high effort.

Let $\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{w(p)}{p}\right)<0$ which, according to Proposition 1, holds under $w(p)>p$. In such case p should be set as small as possible to make $\frac{w(p)}{p}$ as large as possible. For equation (26) to hold, it suffices to show that the right-hand side of (26) shrinks as p decreases. That is because at $p=1$ the right-hand side and left-hand side of that equation are both equal to one and the left-hand side of that equation increases as p decreases.

Use the integration by parts to rewrite the denominator of (26) as:

$$\left(\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_H\right)\right)-\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_L\right)=\frac{a}{p}\int_0^{\bar{y}}b'\left(\frac{ay}{p}\right)\left(w\Big(1-F(y|e_H)\Big)-w\Big(1-F(y|e_L)\Big)\right). \tag{28}$$

The derivative of $p\left(\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_H\right)-\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_L\right)\right)$ with respect to p is:

$$-\frac{a^2}{p^2}\int_0^{\bar{y}}yb''\left(\frac{ay}{p}\right)\left(w\Big(1-F(y|e_H)\Big)-w\Big(1-F(y|e_L)\Big)\right), \tag{29}$$

which is positive under the assumption that $b''(\cdot)<0$ and since $w\Big(1-F(y|e_H)\Big)-w\Big(1-F(y|e_L)\Big)>0$ due to Assumption 5. Hence, since as p decreases (26) not only $\frac{w(p)}{p}$ increases, provided that the agent overweights probabilities, but also the right-hand side of (26) decreases. Hence, under probability overweighting $t_s(y)$ elicits high effort more often than $t_d(y)$ if $\frac{w(p)}{(p)}>1$ at the p specified by the principal.

Finally, to relate this result to that given in Proposition 1 part (ii) notice that the derivative of $\left(\tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_H\right) - \tilde{\mathbb{E}}\left(b\left(\frac{ay}{p}\right)\middle| e_L\right)\right)$ with respect to p is:

$$\begin{aligned} -\frac{a}{p^2}\int_0^{\bar{y}} b'\left(\frac{ay}{p}\right)\Big(w\big(1-F(y|e_H)\big) - w\big(1-F(y|e_L)\big)\Big) - \\ \frac{a^2}{p^3}\int_0^{\bar{y}} yb''\left(\frac{ay}{p}\right)\Big(w\big(1-F(y|e_H)\big) - w\big(1-F(y|e_L)\big)\Big). \end{aligned} \tag{30}$$

A sufficient condition for (30) to be strictly positive is:

$$1 < -\frac{\int_0^{\bar{y}} \frac{ay}{p} b''\left(\frac{ay}{p}\right)\Big(w\big(1-F(y|e_H)\big) - w\big(1-F(y|e_L)\big)\Big)}{\int_0^{\bar{y}} b'\left(\frac{ay}{p}\right)\Big(w\big(1-F(y|e_H)\big) - w\big(1-F(y|e_L)\big)\Big)}. \tag{31}$$

Thus, a necessary, but not sufficient, condition for equation (26) to hold when multiplied on both sides by p is given by (31), a condition that resembles the lower bound from the condition of Proposition 1 part (ii). ■

Proposition 2 generalizes the result from Proposition 1 part (ii) that the agent is more productive when additional risk is introduced in his environment with the stochastic contract. Hence, the result from Proposition 1 part (ii) is not an artifact of the assumed deterministic relationship between output and effort but emerges from the agent's risk preferences. Under probability overweighting at the probability chosen by the principal, the agent is more motivated under the stochastic contract that under the linear piece-rate.

Lump-sum bonus vs. linear contract

As an example of how the incentives of the stochastic contract can be brought to the workplace, consider a principal who is deciding to switch from a linear piece-rate to another cost-equivalent compensation scheme. Specifically, consider a lump-sum bonus contract, which introduces larger degrees of risk in the agent's environment.

Formally, the bonus contract pays a monetary quantity $B > 0$ in the contingency that the level of output supplied by the agent y surpasses some threshold $\hat{y}$. Instead, the piece-rate contract pays ay for any $y \in [0, \bar{y}]$. That these two payment modalities are cost-equivalent to the principal implies that:

$$B\left(1 - G(\hat{y}|e_H)\right) = a\mathbb{E}\left(y|e_H\right). \tag{32}$$

Let $b(t(y))$ be strictly concave and assume that at the threshold output level $\hat{y}$, $w((1 - G(\hat{y}|e_H) >$

$(1 - G(\hat{y}|e_H))$, i.e. the agent overweights the probability that the bonus contract will be attained. Using (32) along with Jensen's inequality,

$$b(B) = b\left(\frac{\mathbb{E}\,(ay|e_H)}{(1 - G(\hat{y}|e_H))}\right) > \frac{\mathbb{E}\,(b(ay)|e_H)}{w\,(1 - G(\hat{y}|e_H))}. \tag{33}$$

Denote the non-additive expectation by $\tilde{\mathbb{E}}\,(y|e) := \int_0^{\bar{y}} y \mathrm{d}w\,(1 - G(y|e))$. Assume that, despite $w((1 - G(\hat{y}|e_H) > (1 - G(\hat{y}|e_H))$, the agent exhibits $\tilde{\mathbb{E}}\,(b(ay)|e_H) < \mathbb{E}\,(b(ay)|e_H)$. Thus, the agent exhibits pessimism about risk for most of the probability interval except at the $\hat{y}$ set by the principal. This is a reasonable assumption as long as $w(p)$ exhibits an inverse-S shape with pessimism and $\hat{y}$ is set high enough, so that the small probability that it implies is overweighted.

Under the aforementioned assumptions, equation (33) can be rewritten as:

$$b(B) > \frac{\tilde{\mathbb{E}}\,(b(ay)|e_H)}{w\,(1 - G(\hat{y}|e_H))} \tag{34}$$

Finally, notice that Assumption 5 guarantees both $\tilde{\mathbb{E}}\,(b(ay)|e_H) - \tilde{\mathbb{E}}\,(b(ay)|e_L) > 0$ and $w\,(1 - G(\hat{y}|e_H)) - w\,(1 - G(\hat{y}|e_L)) > 0$. However, sufficiently pronounced small probability overweighting together with pessimism can guarantee:

$$\frac{w\,(1 - G(\hat{y}|e_L))}{w\,(1 - G(\hat{y}|e_H))} < \frac{\tilde{\mathbb{E}}\,(b(ay)|e_L)}{\tilde{\mathbb{E}}\,(b(ay)|e_H)}. \tag{35}$$

The condition in (35) together with (34) lead to:

$$b(B) > \frac{\tilde{\mathbb{E}}\,(b(ay)|e_H) - \tilde{\mathbb{E}}\,(b(ay)|e_L)}{w\,(1 - G(\hat{y}|e_H)) - w\,(1 - G(\hat{y}|e_L))}. \tag{36}$$

The above equation can be rewritten as:

$$b(B)\,(w\,(1 - G(\hat{y}|e_H)) - w\,(1 - G(\hat{y}|e_L))) > \tilde{\mathbb{E}}\,(b(ay)|e_H) - \tilde{\mathbb{E}}\,(b(ay)|e_L)\,. \tag{37}$$

Therefore, the lump-sum bonus elicits high effort more often than the linear piece-rate under sufficiently strong likelihood insensitivity (inverse-S) together with pessimism. However, to achieve such result, the principal needs to set the threshold output $\hat{y}$ high enough to induce strong overweighting of probabilities.

Appendix C: Agents with CPT preferences

In this Appendix, I analyze the incentives generated by stochastic contracts when agents have risk preferences characterized by CPT (Tversky and Kahneman, 1992). I find that under mild additional conditions, the result stated in Proposition 1 part (ii) holds: stochastic contracts that expose the agent to large amounts of risk can generate higher output than linear piece-rate contracts. This finding is not surprising since CPT incorporates probability distortions in the same way as RDU.

Agents with CPT preferences evaluate possible outcomes in the stochastic contract relative to a reference point $r \geq 0$. Outcomes below the reference point are coined *losses* and outcomes above it are *gains*. In the original formulation of CPT, r represents the status quo, or the monetary amount that the agent owns and is thus exogenous to the principal's choice. In the following, I adopt the assumption that the reference point is exogenous to the principal's offer.

The main difference of CPT with respect to RDU is that the agent can exhibit different risk preferences in the domain of gains and the domain of losses. This is partly because outcomes are evaluated with a value function that exhibits the following properties:

Assumption 6. *$V(t_s(y), r)$ is the piecewise function,*

$$V(t_s, r) = \begin{cases} b(\frac{ay}{p} - r) & , \text{ if } \frac{ay}{p} \geq r, \\ -\lambda b(r - \frac{ay}{p}) & , \text{ if } \frac{ay}{p} < r. \end{cases}$$

With $r \geq 0$, $\lambda > 1$, $b'(\cdot)$ for all $y \in [0, \bar{y}]$, $b''(\cdot) < 0$ if $\frac{ay}{p} > r$, and $b''(\cdot) > 0$ if $\frac{ay}{p} < r$.

In words, the value function, V, is an increasing function that is concave in the domain of gains and convex in the domain of losses, generating thus risk averse and risk seeking attitudes, respectively. Additionally, the worker is loss averse, i.e. for him losses loom larger than equally-sized gains. This property is captured by the parameter $\lambda > 1$ which only enters the value function for the domain of losses.

The CPT agent also transforms the probabilities included in t_s using a probability weighting function. However, transformations of probability can be different for gains and losses. Let $w(p)$ be the probability weighting function used to transform probabilities in the domain of gains. This weighting function exhibits the properties from Assumption 4.

Moreover, let $z(p)$ be the probability weighting function used to transform probabilities in the domain of losses. To simplify matters, it is assumed that $w(p)$ and $z(p)$ relate through the duality $z(p) = 1 - w(1-p)$. Hence, the weighting function for losses adopts the same properties as that for gains, and only differs in that probability transformations are applied to loss ranks, or a ranking of outcomes from least-desirable to most-desirable, rather than to gain ranks. [25]

All in all, the utility of the agent with CPT preferences when offered $t_s(y)$ is equal to:

$$CPT(t_s(y), t) = \begin{cases} w(p)v\left(\frac{ay}{p} - r\right) - c(e) & \text{, if } \frac{ay}{p} \geq r \geq 0, \\ -z(p)\lambda v\left(r - \frac{ay}{p}\right) - c(e) & \text{, if } r > \frac{ay}{p} > 0. \end{cases} \tag{38}$$

Consider the case in which the agent with CPT preferences works under t_d. While this contract does not contain risk, which disregards probability weighting functions, the assumption that the agent makes decisions relative to a reference point is kept. Maintaining this assumption is consistent with abundant evidence showing that in settings of deterministic choice, individuals exhibit reference-dependent preferences (Kahneman et al., 1991).

When offered $t_d(y)$, the agent with riskless prospect theory preferences (Kahneman et al., 1991) exhibits the following utility:

$$CPT(t_d(y)) = \begin{cases} b\left(ay - r\right) - c(e) & \text{, if } ay \geq r \geq 0, \\ -\lambda b\left(r - ay\right) - c(e) & \text{, if } r > ay > 0. \end{cases} \tag{39}$$

We are in a position to compare the two contracts with respect to the output that they deliver. Proposition 3 provides the conditions under which the principal is better off exposing the agent to large amounts of risk with the stochastic contract.

Proposition 3. *Under Assumptions 1, 2, 4, and 6, an agent with CPT preferences delivers higher output with a stochastic contract implemented with infinitesimal probabilities as compared to a linear piece-rate if either:*

(i) $w(p) \geq p \exp\left(ay \int_p^1 \frac{\mathcal{A}\left(\frac{ay}{\mu} - r\right)}{\mu^2} d\mu\right)$ *at the probability specified by the principal and* $\frac{ay_C^{**}}{p} \geq r$, *or*

(ii) $z(p) \geq p \exp\left(ay \int_p^1 \frac{\mathcal{A}^l\left(\frac{ay}{\mu} - r\right)}{\mu^2} d\mu\right)$ *at the probability specified by the principal and* $\frac{ay_C^{**}}{p} < r$.

Where $\mathcal{A}\left(\frac{ay}{\mu} - r\right) := \frac{-b''\left(\frac{ay}{\mu} - r\right)}{b'\left(\frac{ay}{\mu} - r\right)}$ *and* $\mathcal{A}^l := \frac{-b''\left(r - \frac{ay}{\mu}\right)}{b'\left(r - \frac{ay}{\mu}\right)}$.

Proof. When working under $t_s(y)$, the agent with CPT preferences supplies a level of output

[25] Formally, an agent with CPT preferences facing a lottery $(x_1, p_1; x_2, p_2; ...; x_n, p_n)$ ranks the outcomes using an increasing arrangement $x_1 < x_2 < ...x_{r-1} < r < x_{r+1}... < x_n$ and evaluates the outcomes of the lottery relative to r through the function $v(y, r)$. The lottery outcomes $x_{r+1}, ...x_n$ are gains and the outcomes $x_1, ...x_{r-1}$ are losses. The individual assigns decision weights to gains in the following way $\pi_n = w(p_n), \pi_{n-1} = w(p_{n-1} + p_n) - w(p_n)), ..., \pi_{r+1} = 1 - \sum_{j=r+1}^{n} w(p_j)$ and assigns decision weights to losses in the following way $\pi_1 = z(p_1), \pi_2 = z(p_1 + p_2) - z(p_1)), ..., \pi_{r-1} = 1 - \sum_{j=r-1}^{n} z(p_j)$.

y_C^{**} satisfying the following system of equations:

$$\frac{a}{p}w(p)b'\Big(\frac{ay_C^{**}}{p}-r\Big)-c'\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta}=0,\text{ if }\frac{ay_C^{**}}{p}\geq r, \tag{40}$$

$$\frac{a}{p}\left(1-w(1-p)\right)\lambda b'\Big(r-\frac{ay_C^{**}}{p}\Big)-c'\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta}=0,\text{ if }\frac{ay_C^{**}}{p}<r. \tag{41}$$

That y_C^{**} is a maximum requires that the second order condition is negative. Formally,

$$\frac{a^2}{p^2}w(p)b''\Big(\frac{ay_C^{**}}{p}-r\Big)-c''\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta^2}<0,\text{ if }\frac{ay_C^{**}}{p}\geq r, \tag{42}$$

and

$$\frac{a^2}{p^2}\left(1-w(1-p)\right)b''\Big(r-\frac{ay_C^{**}}{p}\Big)-c''\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta^2},\text{ if }\frac{ay_C^{**}}{p}<r. \tag{43}$$

I first consider the case in which the agent is in the domain of gains. To investigate whether the principal must set $p<1$ differentiate implicitly equation (40) with respect to y_C^{**} and p to obtain:

$$\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p}=\frac{\left(\frac{w'(p)p-w(p)}{p^2}\right)ab'\left(\frac{ay_C^{**}}{p}-r\right)-\frac{w(p)a^2y}{p^3}b''\left(\frac{ay_C^{**}}{p}-r\right)}{c''\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta^2}-b''\left(\frac{ay_C^{**}}{p}-r\right)\frac{w(p)a^2}{p^2}}. \tag{44}$$

If $\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p}>0$, the principal is better off setting $p=1$ and obtaining y_C^* satisfying

$$ab'\Big(ay_C^*-r\Big)-c'\left(\frac{y_C^*}{\theta}\right)\frac{1}{\theta}=0. \tag{45}$$

Instead, $\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p}\leq 0$ implies that the principal must set the smallest possible p. From (44) it can be established that a necessary condition to obtain $\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p}<0$, and thus that the principal has incentives to choose $p<1$, is:

$$w'(p)\leq\frac{w(p)}{p}\left(-\frac{ay}{p}\mathcal{A}\left(\frac{ay}{p}-r\right)+1\right), \tag{46}$$

where $\mathcal{A}\left(\frac{ay}{p}-r\right)=-\frac{b''\left(\frac{ay}{\mu}-r\right)}{b'\left(\frac{ay}{\mu}-r\right)}$. I proceed as in the proof of Proposition 1 using Grönwall's lemma. Let $v(p)$ be a weighting function with the properties of $w(p)$ and consider the following ordinary differential equation,

$$v'(p)=\frac{v(p)}{p}\left(-\frac{ay}{p}\mathcal{A}\left(\frac{ay}{p}-r\right)+1\right), \tag{47}$$

which is solved by

$$v(p) = \exp\left(-ay\int_p^1 \frac{1-\mathcal{A}\left(\frac{ay}{\mu}-r\right)}{\mu^2}\mathrm{d}\mu\right). \tag{48}$$

To study how $w(p)$ and $v(p)$ relate compute the following derivative of their ratio:

$$\frac{\mathrm{d}\left(\frac{w(p)}{v(p)}\right)}{\mathrm{d}p} = \frac{(v(p)w'(p)-w(p)v'(p))}{v(p)^2} = \frac{v(p)\left(w'(p)-\frac{w(p)}{p}\left(-\frac{ay}{p}\mathcal{A}\left(\frac{ay}{p}-r\right)+1\right)\right)}{v(p)^2}, \tag{49}$$

where the second equality results from replacing $v'(p)$ using equation (48). The above equation together with (46) yield that $\frac{\mathrm{d}\left(\frac{w(p)}{v(p)}\right)}{\mathrm{d}p} \leq 0$. Hence, the minimum of $\frac{w(p)}{v(p)}$ must be attained at $p=1$ and it must be that $\frac{w(p)}{v(p)} \geq \frac{w(1)}{v(1)} = 1$ for any $p \in (0,1]$.

Therefore, the solution to (46) is bounded by (48), in the following way:

$$\frac{w(p)}{p} \geq \exp\left(ay\int_p^1 \frac{\mathcal{A}\left(\frac{ay}{\mu}-r\right)}{\mu^2}\mathrm{d}\mu\right). \tag{50}$$

As long as (50) holds in the domain of gains the principal is better setting smallest possible probability $p \to 0^+$.

Consider now the domain of losses. To analyze the influence between y_C^{**} and p, implicitly differentiate (41) with respect to those variables to obtain:

$$\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p} = \frac{\left(\frac{\frac{w'(1-p)}{p}-(1-w(1-p))}{p^2}\right)\frac{a}{p}b'\left(r-\frac{ay_C^{**}}{p}\right) - \frac{(1-w(1-p))a^2y}{p^3}\lambda b''\left(r-\frac{ay_C^{**}}{p}\right)}{c''\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta^2}+\lambda b''\left(r-\frac{ay_C^{**}}{p}\right)\frac{(1-w(1-p))a^2}{p^2}}. \tag{51}$$

From (51) it can be established that a sufficient condition for $\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p} \leq 0$ is:

$$w'(1-p) \leq \frac{(1-w(1-p))}{p}\left(1-\frac{ay_C^{**}}{p}\mathcal{A}^l\left(r-\frac{ay_C^{**}}{p}\right)\right), \tag{52}$$

where $\mathcal{A}^l\left(r-\frac{ay_C^{**}}{p}\right) := -\frac{b''\left(r-\frac{ay_C^{**}}{p}\right)}{b'\left(r-\frac{ay_C^{**}}{p}\right)}$. Following a similar procedure as that presented above for the domain of gains and recognizing the duality $z(p) = 1-w(1-p)$, we can establish that the solution to the above differential inequality is:

$$\frac{z(p)}{p} \geq \exp\left(ay\int_p^1 \frac{\mathcal{A}^l\left(\frac{ay}{\mu}-r\right)}{\mu^2}\mathrm{d}\mu\right). \tag{53}$$

If equation (53) holds in the domain of losses then $\frac{\mathrm{d}y_C^{**}}{\mathrm{d}p} \leq 0$ and the principal is better setting smallest possible probability $p \to 0^+$.

■

As with RDU preferences, the principal derives greater motivation using the stochastic contract implemented with infinitesimal probabilities. However, this result emerges when the agent sufficiently overweights the probability specified by the principal, so that the probabilistic risk seeking attitude of the agent outweighs the potential risk averse attitudes stemming from his value function. Specifically, when the favorable outcome of t_s is evaluated as a gain, $\frac{ay}{p} \geq r$ the lower-bound of probability overweighting to be attained by the probability weighting function to make the agent risk seeking is $\exp\left(ay \int_p^1 \frac{\mathcal{A}\left(\frac{ay}{\mu}-r\right)}{\mu^2}\mathrm{d}\mu\right)$. Instead, when $\frac{ay}{p} < r$ such lower-bound is $\exp\left(ay \int_p^1 \frac{\mathcal{A}^l\left(\frac{ay}{\mu}-r\right)}{\mu^2}\mathrm{d}\mu\right)$. As it will explained below, the requirement under gains is more stringent.

To conclude this appendix, I comment on the role of loss aversion and diminishing sensitivity, two factors that determine risk attitude under CPT preference and that are absent under RDU. Notice that the coefficient of loss aversion, λ, does not enter in the lower-bound of Proposition 3 part (ii), when the agent is located in the domain of losses. Thus, that the agent exhibits more or less loss aversion is immaterial to the effectiveness of this stochastic contract with respect to the linear contract.

The second component is diminishing sensitivity, i.e. that the value function is concave for gains and convex for losses. This property implies that the requirement on probability overweighting for the effectiveness of stochastic contracts is more stringent in the domain of gains. To see how note that $\mathcal{A}^l\left(\frac{ay}{\mu} - r\right)$, the expression included in the lower-bound for the domain of losses, is negative, while $\mathcal{A}\left(\frac{ay}{\mu} - r\right)$, the expression included in the lower-bound for the domain of gains, is positive. So in the domain of losses, the condition for Proposition 3 can be attained with probability underweighting, while in the domain of gains probability overweighting is necessary. Intuitively, the convex curvature of the value function in the domain of losses generates risk seeking, and facilitates that the agent becomes more motivated with a contract that introduces risk. Instead, in the domain of gains, the value function is concave which generates risk aversion, making more difficult that a contract that introduces risk is attractive to the agent unless sufficient probabilistic risk seeking is induced.

Appendix D. The principal's problem

The purpose of this Appendix is to complement the theoretical model presented in §2 by providing a solution to the princpal's problem. Hence, not only is the revenue of the principal maximized using the incentive compatibility constraint, but also taking into account the participation constraint. The results from this Appendix show that Proposition 1 (ii) holds when the participation constraint is taken into account.

Assume that the agent's risk preferences are characterized by RDU. That is, the agent distorts cumulative probabilities using the weighting function $w(p)$ described by Assumption 4. To simplify matters, also assume that the agent's utility belong to the CRRA family:

Assumption 7. *Let $b(t) = t^{\rho}$ where $\rho \in \mathbb{R}$.*

This assumption must be taken with a grain of salt. It states that Proposition 1 (ii) can hold, as shown in Example 2. Therefore, the present exercise is an investigation of whether by including the participation constraint, the principal is further restricted in her actions.

Furthermore, I assume that the principal is risk-neutral and her decision consists on choosing $p \in (0, 1]$ included in $t_s(y)$ such that the agent accepts the contract and is motivated to exert as much effort as possible. Due to the equivalence $A = \frac{a}{p}$, that the principal chooses $p = 1$ is tantamount to choosing the piece-rate contract $t_d(y)$. In other words, the principal problem amounts to choose among contracts that are cost-equivalent but that differ on the amount of risk that will be faced by the agent.

All in all, the principal's program is:

$$
\begin{aligned}
\min_{p \in (0,1]} \quad & Apy \\
\text{s.t.} \quad & w(p)\,(Ay)^{\rho} - c\left(\frac{y}{\theta}\right) \geq \bar{U}, \\
& A\rho w'(p)\,(Ay)^{\rho-1} - c'\left(\frac{y}{\theta}\right)\frac{1}{\theta}.
\end{aligned}
\tag{54}
$$

The solution to the principal's problem is presented in Proposition 4. The solution to the principal's problem is identical to that presented in Proposition 1 (ii). That is, the principal is better off offering $t_s(y)$ with infinitesimal probabilities as long as the agent sufficiently overweights those very small probabilities.

Proposition 4. *Under Assumptions 1, 2, 4, and 7, the solution to (54) consists on implementing the stochastic contract with infinitesimal probabilities if $w(p) \geq p^{\rho}$ at the*

p implemented by the principal and $t_d(y)$ otherwise.

Proof. Recognizing the equivalence $A = \frac{a}{p}$, and denoting by ν_1 and ν_2 the Lagrangian multipliers of the participation and incentive compatibility constraints, respectively, the Lagrangian of the principal program can be written as:

$$\mathcal{L} = ay - \nu_1 \left(w(p) \left(\frac{ay}{p} \right)^{\rho} - c\left(\frac{y}{\theta} \right) - \bar{U} \right) - \nu_2 \left(\frac{arw(p)}{p} \left(\frac{ay}{p} \right)^{\rho - 1} - c'\left(\frac{y}{\theta} \right) \frac{1}{\theta} \right). \tag{55}$$

Taking into account that changes in p might induce changes in y, the first-order condition of the Lagrangian in (55) with respect to p is:

$$\begin{aligned} -a\frac{\mathrm{d}y}{\mathrm{d}p} - \nu_1 \left(w'(p) \left(\frac{ay}{p} \right)^{\rho} - \frac{a\rho w(p)}{p} \left(\frac{\mathrm{d}y}{\mathrm{d}p} - \frac{y}{p} \right) \left(\frac{ay}{p} \right)^{\rho - 1} \right) \\ - \nu_2 \left(\frac{a\rho w'(p)}{p} \left(\frac{ay}{p} \right)^{\rho - 1} - \frac{a\rho w(p)}{p^2} \left(\frac{ay}{p} \right)^{\rho - 1} - \frac{a^2\rho(\rho - 1)w(p)}{p^2} \left(\frac{\mathrm{d}y}{\mathrm{d}p} - \frac{y}{p} \right) \left(\frac{ay}{p} \right)^{\rho - 2} \right) = 0. \end{aligned} \tag{56}$$

After some manipulations, equation (56) can be rewritten as:

$$(\nu_1 y + \nu_2 \rho) \left(w'(p)y + \rho w(p) \left(\frac{\mathrm{d}y}{\mathrm{d}p} - \frac{y}{p} \right) \right) = \nu_2 \rho \left(w(p) \frac{\mathrm{d}y}{\mathrm{d}p} \right) + \frac{a\frac{\mathrm{d}y}{\mathrm{d}p}}{\frac{a^2}{p^2} \left(\frac{ay}{p} \right)^{\rho - 2}}. \tag{57}$$

To analyze the optimal value of p determined by (57) assume first that $\frac{\mathrm{d}y}{\mathrm{d}p} = 0$. In that case, equation (57) gives:

$$(\nu_1 y + \nu_2 \rho) \left(w'(p) - \frac{w(p)\rho}{p} \right) = 0. \tag{58}$$

Hence, under $\nu_1 > 0$ and $\nu_2 > 0$ equation (58) holds if

$$w'(p) = \frac{\rho w(p)}{p}. \tag{59}$$

The solution to the ordinary differential equation in (59) is

$$w(p) = p^{\rho}. \tag{60}$$

Hence, $\frac{\mathrm{d}y}{\mathrm{d}p} = 0$ is achieved for agents with weighting functions complying with $w(p) = p^{\rho}$ and in such case the principal is indifferent between offering t_s with infinitesimal p or the linear contract t_d.

Consider now the more interesting case in which $\frac{\mathrm{d}y}{\mathrm{d}p} \leq 0$. In such case, the right-hand side of (57) is negative, and for the equality to be maintained under $\nu_1 > 0$ and $\nu_2 > 0$ it is necessary that:

$$w'(p) \leq \frac{\rho w(p)}{p}. \tag{61}$$

.

From Grönwall's lemma, applied in the proof of Proposition 1, it can be established that (61) holds for any weighting function such that $w(p) \geq p^\rho$. Hence, the principal must choose $t_s(y)$ with $p \to 0^+$ if $w(p) \geq p^\rho$ because in that case $\frac{\mathrm{d}y}{\mathrm{d}p} \leq 0$. Notice that this condition is identical to the one presented in Example 2. Instead, when $w(p) < p^\rho$ the principal is better off offering $t_d(y)$ by setting $p = 1$.

Finally it is analyzed whether the aforementioned solution to the principal's program is valid. To do so, I investigate the shape of the Lagrangian in Equation (55). The second-order condition with respect to p is:

$$\frac{a\rho y}{p^2}\left(\frac{ay}{p}\right)^{\rho-1}\left(\nu_1 + \frac{\nu_2\rho}{y}\right)\left(w'(p) - \frac{\rho w(p)}{p}\right) - \left(\frac{ay}{p}\right)^{\rho}\left(\nu_1 + \frac{\nu_2\rho}{y}\right)\left(w''(p) - \frac{p\rho w'(p) - rw(p)}{p^2}\right) \tag{62}$$

Equation (62) becomes positive if $w''(p) < 0$. Hence the candidate solution, $p \to 0^+$ if $w(p) \geq p^\rho$, is valid as long as $p \in (0, \tilde{p})$, where $\tilde{p}$ is the inflection point below which $w(p)$ is concave. However, if $w''(p) > 0$, the second order condition in (62) becomes negative, implying that the objective function attains a minimum value at extremes, either $p = \tilde{p}$ or $p = 1$.

Note that according to Assumption 4, $w(p)$ might display $\hat{p} \neq \tilde{p}$, i.e. the inflection point is different than the fixed point. Consider first the case $\hat{p} > \tilde{p} > 0$. In such case, at $p = \tilde{p}$ the weighting function exhibits $\frac{w(\tilde{p})}{\tilde{p}} > 1$. This implies that he incentive compatibility and participation constraints of the program presented become larger than at $p = 1$, which implies that at $p = \tilde{p}$ the Lagrangian attains a lower value. Thus, the principal chooses $p = \tilde{p}$ if $p \in (\tilde{p}, 1)$ and $\hat{p} > \tilde{p}$. However, if $w(\tilde{p}) \geq \tilde{p}^\rho$, then $\frac{\mathrm{d}y}{\mathrm{d}p} \leq 0$, the principal is better off choosing sufficiently small p. So the candidate solution is corroborated for this case.

Let now $0 < \hat{p} \leq \tilde{p}$. For the interval $p \in [\tilde{p}, 1]$, the solution is $p = 1$ since yields $\frac{w(\tilde{p})}{\tilde{p}} < 1$ which leads to lower values of the incentive compatibility and participation constraints than those implied by $p = 1$. This solution is valid since $w(1) \leq 1^\rho$ holds implying that $\frac{\mathrm{d}y}{\mathrm{d}p} \geq 0$; the principal is better off setting as large as possible probabilities.

■

Proposition 4 shows that considering the full principal problem does not affect the optimal implementation of incentives presented in Proposition 1 part (ii). That is mainly because by offering the stochastic contract with small probabilities the principal is not affecting the expected compensation of the agent. This is due to the equivalence $A = \frac{a}{p}$. Hence, that the expected value maximizer accepts t_d necessarily implies that contract t_s will be accepted. Furthermore, an agent who, due to probability overweighting, is more motivated to exert higher effort levels under t_s with small probabilities than with t_d, must be also more willing to accept working under t_s than under t_d. That is because his distortion of probabilities enhances the utility that he expects to derive from t_s vis-a-vis the utility he gets from t_d. This rationale makes the participation constraint superfluous to the problem.

Appendix E: Utility functions

This appendix investigates the properties of the elicited utility functions. Decision sets 1 to 6 of the second part of the experiment are designed to elicit the sequence of outcomes $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ for each subject. This elicited sequence has the relevant property that it ensures equally-spaced utility values, i.e. $u(x_j) - u(x_{j-1}) = u(x_{j-1}) - u(x_{j-2})$, allowing me to characterize a subject's preference over monetary outcomes by mapping each utility value, $u(x_j)$to the subject's stated preference x_j.

I focus on two properties of the utility function: the sign of the slope and the curvature. To that end, I construct two variables, the first variable is $\Delta'_i := x_j - x_{j-1}$, for $j = 1, ..., 6$ and the second is $\Delta''_j := \Delta'_j - \Delta'_{j-1}$ for $i = 2, ..., 6$. The sign of Δ'_j as j increases determines the sign of the slope, i.e. whether a subject prefers larger monetary outcomes to smaller monetary outcomes. Similarly, the sign of Δ''_j as j increases determines the utility curvature. For example, a subject with $\Delta'_j > 0$ and $\Delta''_j > 0$ for all j exhibits a preference for larger monetary outcomes and experiences smaller utility increments with larger monetary outcomes, this is equivalent to say that this subject has an increasing and concave utility function.

The first analysis focuses on classifications at the individual level. I classify subjects according to the curvature of their utility function. Since I have multiple observations for each subject and it was possible that subjects made mistakes, this classification is based on the sign of Δ''_j with the most occurrence. Specifically, a subject with at least three negative Δ''_js was classified as having a convex utility, a subject with at least three positive Δ''_js had a concave utility and subject with three or more Δ''_is had a linear utility. A subject with a utility function that cannot be classified as concave, convex, or linear, had a mixed utility. Furthermore, to statistically asses the sign of a Δ''_j, I construct confidence intervals around zero. In particular, I multiply the standard deviation of each Δ''_j by the factors 0.64 and -0.64. Thus, if Δ''_j follows a normal distribution, 50% of the data should lie within the confidence interval.[26]

The data suggest that all subjects in the experiment exhibit an increasing sequence $\{x_1, ..., x_6\}$ which denotes, not surprisingly, a generalized preference for larger amounts of money. Table 7 presents the classification of subjects according to the curvature of their utility function. The data suggest that the majority of subjects exhibit linear utility functions.

[26] More stringent confidence intervals were also used for the analysis. These confidence intervals were also constructed using the standard deviation of a Δ''_j which was multiplied by different factors, such as 1 and -1, 1.64 and -1.64, and 2 and -2. The qualitative results of these analyzes are not different from the main result presented here that the majority of subjects exhibit a linear utility function. This is not surprising inasmuch as these confidence intervals are more stringent and yield less subjects classified as having a mixed utility function and more subjects exhibiting a linear utility function.

Specifically, 77% of the subjects have linear utility, while the rest of the subjects have mixed utility (13% of the subjects), and concave utility (7% of the subjects). A proportions test suggest that the proportion of subjects with linear utility is significantly larger than 50% ($p<0.001$). Moreover, this test also yields that the proportion of subjects having linear functions is significantly larger than the proportion of subjects with mixed ($p<0.001$) and concave utility ($p<0.001$).

The result that more than two-thirds of the subjects exhibit linear utility is at odds with the principle of diminishing sensitivity, a key property of cumulative prospect theory (CPT). However, disregarding CPT as a possible representation for the subjects' preferences for money on the basis of this classification may be incorrect. As pointed out by Wakker and Deneffe (1996), their trade-off method, used to elicit $\{x_1, x_2, x_3, x_4, x_5\}$, requires lotteries with large monetary outcomes in order to obtain utility functions with curvature. Therefore, one of the advantages of the experimental design, that it elicits the utility function and the probability weighting function of subjects using monetary stakes that reflect the monetary incentives in the first part of the experiment, is also the reason that diminishing sensitivity is not be observed.

Table 7 also presents the results of the aforementioned analysis when it is assumed that subjects have CPT preferences with a reference point equal to the monetary equivalent of a subject's beliefs about his performance in the first part of the experiment. Monetary outcomes above this reference point are considered gains and outcomes below the reference point are considered losses. This alternative analysis also leads to the conclusion that the majority of the subjects exhibit a linear utility function. Specifically, I find that 65 % of the subjects have linear utilities in the domain of gains and 98% of the subjects exhibit linear utilities in the domain of losses.

To understand how the aforementioned results aggregate, I analyze the sequence $\{x_1, ..., x_6\}$ when each outcome x_j is averaged for all subject. Table 8 presents the descriptive statistics of the resulting outcomes. I find that the average outcome x_j is increasing with j, implying that on average subjects exhibit a taste for larger monetary outcomes. Moreover, the column displaying the average values of the variable Δ'_j shows that as j increases, increments of x_j become larger. Thus, while on average subjects exhibit linear utility, this tendency ceases as monetary outcomes in the lotteries become larger. In fact, for large values of x_j the average utility function displays concavity. This result is also found by Abdellaoui (2000).

The last analysis of the data consists on fitting well-known parametric families of utility functions. Specifically, I assume a power utility, belonging to the CRRA family of utility functions, and an exponential function, belonging to the CARA family of utility functions. Table 9 the regression estimates when non-linear least squares is used to fit the data to the

Table 7: Classification of subjects according to utility curvature

Reference Point	Domain	Convex	Concave	Linear	Mixed	Total
No/Zero	No/Gains	3	13	133	23	172
Belief	Gains	3	12	43	21	79
Belief	Losses	0	1	90	2	93

Note: This table presents the subjects classification according to the shape of their utility function. Subjects are classified as having a convex, concave, linear or mixed utility function based on the sign of Δ_j with more occurrence. The first row presents the classification when the analysis is performed with all the data. The second and third columns feature the analysis taking into account the Beliefs non-zero reference point, which is the monetary equivalent of a subject's beliefs about her performance in the first part of the experiment. The second row presents the analysis when the monetary outcomes of the lotteries are above the reference point, whereas the third row presents the analysis when the monetary outcomes of the lotteries are below the reference point

assumed utility function. For the two parametric specifications I find that the average utility function of the subjects is approximately linear. For instance, when the power utility function $u(x) = x^k$ is assumed, the parameter attains a value of 0.995. This finding is consistent with the large proportion of subjects that were classified as having a linear utility function in the individual analysis and the modest increments that the averaged outcomes x_j exhibit as j increases presented in Table 8.

These analyses are also performed under the assumption that subjects have CPT preferences with a reference point equal to the monetary equivalent of the subject's belief in the first part of the experiment. According to Table 8, subjects exhibit an average preference for larger monetary amounts in both domains. Also, the descriptive statistics suggest a decreasing tendency of the utility function to be linear as the outcome becomes larger in the domain of gains and lower in the domain of losses. The latter finding implies that in the domain of gains the average utility function tends to concavity, while in the domain of losses the function it tends to convexity. Furthermore, the data suggest that diminishing sensitivity manifests at different degrees across the domains, with subjects exhibiting more in the domain of gains. This difference is explained by fact that only positive outcomes were used to elicit the sequence $\{x_1, x_2, x_3, x_4, x_5, x_6\}$. This leaves little room for subjects to exhibit as much sensitivity in the domain of losses as in the domain of gains. Note that I chose to elicit preferences using only positive outcomes since the second part of the experiment was designed to understand the subjects' risk preferences over the monetary incentives at stake in the first part of the experiment. A more complete analysis of diminishing sensitivity across domains, and of risk preferences in general, requires lotteries featuring negative outcomes.

Table 8: Aggregate results x_1,x_2,x_3,x_4,x_5, and x_6

j	x_j	Δ'_j	x_j	Δ'_k	x_j	Δ'_j
1	2.579 (1.990)	1.579	3.761(4.037)	3.037	1.576 (0.548)	0.576
2	4.573 (4.445)	1.993	8.167 (5.226)	4.129	2.167(0.931)	0.590
3	6.684 (6.792)	2.110	12.545(7.564)	4.378	2.761(1.280)	0.593
4	9.179 (9.420)	2.495	17.812 (9.826)	5.266	3.515 (1.800)	0.754
5	11.773 (11.880)	2.594	23.156(11.598)	5.344	4.353 (2.589)	0.837
6	14.379 (14.418)	2.605	28.400 (13.608)	5.243	5.287 (3.727)	0.934
Ref.Point	No/Zero		Beliefs		Beliefs	
Domain	No/Gains		Gains		Losses	

Note: This table presents the average, standard deviations of the sequence $x_1, x_2, x_3, x_4, x_5, x_6$ along with the difference $\Delta'_j = x_j - x_{j-1}$. Standard deviations are presented in parenthesis. Columns 2 and 3 present these statistics when all the data is taken into account. Columns 4, 5, 6, and 7, present these statistics when it is assumed that subjects make decisions around Beliefs as a reference point. Columns 4 and 5 present the mean and median of $x_1, x_2, x_3, x_4, x_5, x_6$ along with $\Delta'_j = x_j - x_{j-1}$ for values above Beliefs for each subject. Columns 6 and 7 present the mean and median of $x_1, x_2, x_3, x_4, x_5, x_6$ along with $\Delta'_j = x_j - x_{j-1}$ for values below Beliefs for each subject.

I also estimate the parameters of the utility function for each domain assuming a power or an exponential utility function. For the domain of losses, the estimated coefficients suggest approximate linearity, with an estimated coefficient $k = 0.992$ when a power utility function is assumed. A similar result is found for the domain of losses, where the estimation yields $k = 1.035$.

All in all, the data suggest that subjects have linear utility functions. This finding is robust to the assumption that subjects have CPT preferences and the reference point is assumed to be their belief. This is not a surprising finding given the magnitude of the stakes used to elicit the subject's risk preferences. Furthermore, the conclusion that the utility function is linear implies that probability risk attitudes fully determine the risk attitudes. Implying that performance differences across treatments must be explained by probability distortions.

Table 9: Parametric estimates of average utility function

Exponential (CARA) $1 - \exp(-\gamma(x_{j-1} + \frac{\epsilon}{2}))$			
$\hat{\gamma}$	0.977 (0.001)	0.946 (0.001)	1.337 (0.001)
Adj. R^2	0.922	0.887	0.303
N	1032	412	619
Power Utility (CRRA) $(x_{j-1} + \frac{\epsilon}{2})^k$			
$\hat{k}$	0.995 (0.001)	0.992 (0.001)	1.035 (0.007)
Adj. R^2	0.925	0.971	0.756
N	1032	412	619
Ref.Point	No/Zero	Beliefs	Beliefs
Domain	No/Gains	Gains	Losses

Note: This table presents the estimates of the non linear least squares regression. The upper panel assumes that the parametric form $1 - \exp(-\gamma(x_{j-1} + \frac{\epsilon}{2}))$ and the lower panel assumes the parametric form $(x_{j-1} + \frac{\epsilon}{2})^k$. The first column presents uses all the data. The second and third column present the data for the domain of gains and the domain of losses, respectively, when the reference point is Beliefs. Standard errors in parenthesis.

Appendix F: Individual analysis of probability weighting functions

This appendix presents alternative analyses of the probability weighting functions. Decision sets 7 to 11 included in the second part of the experiment were designed to elicit the subjects' weighting functions. In the main body of the paper, I present parametric analyses of the average data. In this appendix, I present non-parametric analyses of these data performed at the individual level.

The first analysis classifies each subject according to the shape of the elicited probability weighting function and is based on Bleichrodt and Pinto (2000). There were five possible shapes of the probability weighting function. A subject could display a weighting function with either lower subadditivity (LS), upper subadditivity (US) or with both properties. These three properties result from comparing the behavior of the probability weighting function at extreme probabilities to the behavior of the same function at intermediate probabilities. Moreover, a subject could display a concave or a convex probability weighting function.

To classify a subject into one of these five categories, I created the variable $\partial_{j-1}^{j} := \frac{w(p_j)-w(p_{j-1})}{w^{-1}(p_j)-w^{-1}(p_{j-1})}$, which captures the average slope of the probability weighting function between probabilities j and $j-1$. I also created the variable $\bigtriangledown_{j-1}^{j} \equiv \partial_{j-1}^{j} - \partial_{j-2}^{j-1}$, which represents the change of the average slope of the weighting function between successive probabilities.

To understand the subjects' behavior at extreme and intermediate probabilities I focus on the sign of the variables $\bigtriangledown_{0.16}^{0.33}$ and $\bigtriangledown_{0.83}^{1}$. If a subject exhibits $\bigtriangledown_{0.16}^{0.33} < 0$, his probability weighting exhibits LS. In other words, his probability weighting function assigns larger weights to small probabilities than to medium-ranged probabilities. Moreover, if a subject has $\bigtriangledown_{0.83}^{1} > 0$, then his probability weighting function exhibits the property of US. That is, his weighting function assigns larger weights to large probabilities than to medium-ranged probabilities. The resulting dummy variables LS and US or Both were used in the main body of the paper to investigate the effect of these properties of the weighting function on the treatment effects.

In addition, I examine the sign of $\bigtriangledown_{j-1}^{j}$ as j increases to determine the shape of the weighting function of each subject over the whole probability interval. A subject was classified as having a concave weighting function if at least three (out of five) $\bigtriangledown_{j-1}^{j}$ had a negative sign and he did not exhibit US. Alternatively, a subject had a convex probability weighting function if at least three (out of five) $\bigtriangledown_{j-1}^{j}$ were positive and he did not exhibit LS. Note that these classifications allow for the possibility of response error.

Table 10 presents the results of the individual classification. I find that 57 % of subjects exhibit LS, 75% of subjects exhibit US and 44% of subjects display probability weighting functions with both LS and US. Therefore, most subjects in the experiment had weighting functions that yield overweighting of small probabilities or underweighting of large probabilities. Also, almost half of subjects exhibit probability weighting functions that assign large weights to small and large probabilities. These proportions are however considerably lower than those reported by Bleichrodt and Pinto (2000). Moreover, I find that 39% of the subjects exhibit convex weighting functions and only 13% of the subjects exhibit concave weighting functions. Thus, more subjects in the experiment exhibit pessimism. Furthermore, the proportion of subjects in the experiment with either concave or convex probability weighting functions is higher than that reported by Bleichrodt and Pinto (2000), who finds that only 15% of the subjects have probability weighting functions with either of these shapes.

Table 10: Classification of subjects according to the shape of their weighting function

Reference Point	Domain	Convex	Concave	LS	US	LS & US
No/Zero	No/Gains	68	23	98	129	76
Beliefs	Gains	29	9	49	63	38
Beliefs	Losses	39	14	49	66	38

Note: This table presents the classification of subjects according to the shape of their probability weighting function. Subjects are classified as having a probability weighting function with US, LS or both. Also, subjects are classified as having a convex or concave probability weighting function if they do not exhibit LS and US, respectively. This classification depends on the sign of $\bigtriangledown_{j-1}^{j}$. The first row presents the classification with all the data. The second and third columns feature the analysis assuming that the monetary equivalent of a subject belief in the real-effort task is the reference point. The second row presents the analysis when the monetary outcomes of the lotteries are above the reference point. The third row presents the analysis when the monetary outcomes of the lotteries are below the reference point.

For the sake of robustness, I perform an alternative classification of LS and US also proposed by Bleichrodt and Pinto (2000). In comparison to the above classification, weights given to extreme probabilities are contrasted to the corresponding objective probability. In particular a subject has a weighting function with LS if $w^{-1}\left(\frac{1}{6}\right) < 0.16$. Similarly, a subject has a weighting function with US if $1 - w^{-1}\left(\frac{5}{6}\right) < 0.16$. This alternative classification of LS and US is admittedly less accurate. The reason is that assigning large weights to extreme probabilities does does not guarantee that the weights assigned to medium-ranged probabilities are small.

The results of the alternative classification are presented in Table 11. I find that a similar proportion of subjects exhibit US and LS. Specifically, 40.12% of subjects exhibit LS and 38.37% subjects exhibit US. Also, only 20% of subjects exhibit both LS and US. These proportions are considerably lower than those obtained with the initial classification and are also smaller to those reported by Bleichrodt and Pinto (2000).

Table 11: Classification of subjects according to LS, US, or both

Reference Point	Domain	LS	US	Both
No/Zero	No/Gains	55	89	25
Beliefs	Gains	18	49	8
Beliefs	Losses	37	40	17

Note: This table presents the classification of subjects according to the shape of their weighting functions. Subjects are classified as having weighting functions with LS if $w^{-1}\left(\frac{1}{6}\right) < w^{-1}\left(\frac{2}{6}\right) - w^{-1}\left(\frac{1}{6}\right)$. Subjects have weighting functions with US if $1 - w^{-1}\left(\frac{5}{6}\right) < w^{-1}\left(\frac{5}{6}\right) - w^{-1}\left(\frac{4}{6}\right)$. When these two properties hold, subjects are classified in Both.

The last considered classification, evaluates the strength of the possibility effect relative to the certainty effect. A subject exhibits a weighting function with a possibility effect that is stronger than the certainty effect when $1 - w^{-1}\left(\frac{5}{6}\right) > w^{-1}\left(\frac{1}{6}\right)$. Table 12 shows that the majority of subjects in the experiment have probability weighting functions with the certainty effect exceeding the possibility effect. This result is in line with the findings of Tversky and Fox (1995). Nevertheless, the proportion of subjects for which Certainty exceeds Possibility is not negligible as it constitutes close to 32 % of subjects.

Table 12: Classification of subjects according to strength of possibility effect

Reference Point	Domain	Certainty	Possibility	Equal
No/Zero	No/Gains	107	55	10
Beliefs	Gains	57	18	4
Beliefs	Losses	50	37	6

Note: This table presents the classification of subjects according to the strength of the possibility effect with respect to the certainty effect. Subjects are classified Possibility, that is having probability weighting function where the possibility effect exceeds the certainty effect if $1 - w^{-1}\left(\frac{5}{6}\right) > w^{-1}\left(\frac{1}{6}\right)$. Instead, if $1 - w^{-1}\left(\frac{5}{6}\right) < w^{-1}\left(\frac{1}{6}\right)$ subjects were classified certainty. Finally, subjects with $1 - w^{-1}\left(\frac{5}{6}\right) = w^{-1}\left(\frac{1}{6}\right)$were classified Equal.

As in the main body of the paper, I consider the possibility that subjects have CPT preferences with a reference point equal to the monetary equivalent of their beliefs about performance in the first part of the experiment. All previous analyses are also performed under the assumption that the monetary equivalent of a subject's belief in the real-effort task is the reference point.[27] The results of these analyses are also presented in Table 10, Table 11, and Table 12. All in all, I find that the aforementioned results are robust to subjects having CPT preferences. In particular for both domains there is a large proportion of subjects with US and/or LS. Also, regardless of the domain, more subjects exhibit weighting functions with the certainty effect being stronger than the possibility effect.

In conclusion, the analyses of the data at the individual level suggest that the majority of subjects have weighting functions with US or LS. Moreover, I find that less than half of subjects exhibit both properties at the same time, which is a remarkable difference with respect to Bleichrodt and Pinto (2000). Finally, as in Abdellaoui (2000) and Tversky and Fox (1995), I find that the certainty effect is stronger than the possibility effect for a larger share of individuals.

[27] It is important to emphasize that the nature and intuition of the classification under the assumption that Beliefs is the reference point differs from the original classification. The reason for this difference is that the data does not admit enough $\bigtriangledown_{j-1}^{j}$s to analyze the shape of the probability weighting function of a subject for the domain of gains as well as for the domain of losses. Instead, I analyze the shape of a subject's probability weighting function for the domain wherein the majority of his $\bigtriangledown_{j-1}^{j}$s lie. Thus, this analysis could shed light on whether subjects who have most of their choices in the domain of losses exhibit weighting functions of different shape than subjects who have most of their choices in the domain of gains.

Appendix G: Additional analyses

Descriptions and comparisons with previous studies

Panel 1 in Table 4 presents the estimates of a truncated regression of the neo-additive functional, $w(p) = c + sp$.[28] The resulting estimates display $\hat{c} > 0$ and $\hat{c} + \hat{s} < 1$, which imply that subjects on average overweighted small probabilities and underweighted large probabilities. Furthermore, $\hat{c}$ and $\hat{s}$ are larger and smaller, respectively, than the estimates reported in Abdellaoui et al. (2011), suggesting that subjects in my experiment exhibit higher degrees of optimism toward risk as well as higher degrees of likelihood insensitivity.

A more traditional parametric representation of the probability weighting function was proposed by Tversky and Kahneman (1992). Their proposal relates probabilities and their associated weights according to the following non-linear function: $w(p) = \frac{p^{\psi}}{\left(p^{\psi}+(1-p)^{\psi}\right)^{\frac{1}{\psi}}}$. The second panel of Table 4 shows that the non-linear least squares method generates an estimate $\hat{\psi} = 0.59$, which is lower than those reported in previous studies. Specifically, classical experiments report estimates in the range 0.60-0.75 (Bleichrodt and Pinto, 2000, Abdellaoui, 2000, Wu and Gonzalez, 1996, Tversky and Kahneman, 1992). Therefore, subjects in my experiment display a weighting function with more severe probability distortion.

A crucial disadvantage of Tversky and Kahneman's (1992) weighting function is that likelihood insensitivity and optimism/pessimism influence ψ, so their effect on probabilistic risk attitudes cannot be identified. To overcome such disadvantage, I also use the log-odds weighting function proposed by Goldstein and Einhorn (1987), $w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$, which can, up to some extent, separate these two components. The estimates of a non-linear least squares regression are presented in Panel 3. The magnitude of $\hat{g}$ indicates that the average weighting function has a strong inverse-S shape and the magnitude of $\hat{\delta}$ an strikingly small degree of pessimism. These coefficients are lower and higher, respectively, than those found in previous studies (Bruhin et al., 2010, Bleichrodt and Pinto, 2000, Abdellaoui, 2000, Gonzalez and Wu, 1999, Wu and Gonzalez, 1996, Tversky and Fox, 1995). Thus, subjects in the experiment had an average weighting function with more likelihood insensitivity and optimism than previously documented.

Lastly, I also estimate a regression assuming Prelec (1998)'s probability weighting function with two parameters, $w(p) = \exp\left(-\beta(-\ln(p))^{\alpha}\right)$. This parametric functional also separates, up to some extent, optimism from likelihood insensitivity. Panel 4 presents the estimates of a non-linear least squares regression. The estimate $\hat{\alpha}$, which is statistically lower than one,

[28] The assumed truncation at the extremes, $w(0)$ and $w(1)$, provides the estimation with the flexibility to admit weighting functions with S-shape.

entails that the average probability function has a strong inverse-S shape. Moreover, the estimate $\hat{\beta}$, which is also statistically lower than one, entails that subjects on average display optimism. Previous estimations of this probability weighting function report larger values of α and β (Murphy and Ten Brincke, 2018, L'Haridon et al., 2018, Fehr-duda and Epper, 2012, Abdellaoui et al., 2011, Bleichrodt and Pinto, 2000). Hence, these subjects display an average probability weighting function with a stronger inverse-S shape and more optimism as compared to previous studies.

Including an expectations-based reference point

For the sake of robustness, I perform the aforementioned estimations accounting for the possibility that subjects have CPT preferences. In such analysis I assumed the subject's reference point to be the monetary equivalent of each subject's belief in the first part of the experiment. Lottery outcomes above this reference point belong to the domain of gains, while lottery outcomes below this reference point belong to the domain of losses. I perform separate regressions for each domain. The results are presented in Table 13. I find that for all considered functional forms of probability weighting and for both domains, subjects display weighting functions with inverse-S shapes and more optimism than previously found. As a consequence, the results presented in this section are robust to the assumption that subjects' preferences can be represented by CPT preferences.

Table 13: Parametric estimates of the weighting function with reference point

	(1)	(2)	(3)
Panel 1: Neo-additive (truncated)			
$w(p) = c + sp$			
$\hat{c}$	0.194 ***(0.021)	0.228*** (0.024)	0.155*** (0.024)
$\hat{s}$	0.566*** (0.035)	0.463*** (0.037)	0.686*** (0.044)
Log-Likelihood	220.288	75.200	166.842
Panel 2: Tversky & Kahneman (1992)			
$w(p) = \frac{p^{\psi}}{\left(p^{\psi}+(1-p)^{\psi}\right)^{\frac{1}{\psi}}}$			
$\hat{\psi}$	0.598*** (0.016)	0.597*** (0.012)	0.785*** (0.037)
Adj. R^2	0.838	0.827	0.866
Panel 3: Goldstein and Einhorn (1987)			
$w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$			
$\hat{g}$	0.281***(0.025)	0.196***(0.027)	0.426*** (0.042)
$\hat{\delta}$	0.921***(0.020)	0.892*** (0.029)	0.982***(0.032)
Adj. R^2	0.863	0.845	0.888
Panel 4: Prelec (1998)			
$w(p) = \exp\left(-\beta(-\ln(p))^{\alpha}\right)$			
$\hat{\alpha}$	0.284***(0.025)	0.143*** (0.025)	0.357*** (0.033)
$\hat{\beta}$	0.841***(0.015)	0.596***(0.024)	0.944*** (0.019)
Adj. R^2	0.864	0.907	0.851
N	860	304	550
Ref.Point	No/Zero	Beliefs	Beliefs
Domain	No/Gains	Gains	Losses

Note: This table presents the estimates of the probability weighting function when parametric estimates are assumed. Panel 1 presents the maximum likelihood estimates of the equation $w(p) = c + s(p)$ when truncation at $w(p) = 0$ and at $w(p) = 1$ is assumed. Panel 2 presents the non-linear least squares estimation of the function $w(p) = \frac{p^{\psi}}{(p^{\psi}+(1-p)^{\psi})^{\frac{1}{\psi}}}$. The third panel presents the non-linear least squares estimates of the parametric form $\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$. The last panel presents the non-linear least squares estimates of the function $w(p) = \exp\left(-\beta(-\ln(p))^{\alpha}\right)$. The first column in all the panels presents the estimates when all the data are used. The second and third columns present the estimations when it is assumed that Beliefs is the reference point and only data for the domain of gains and the domain of losses, respectively, is used for the estimations. Robust standard errors in parenthesis. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Robustness on the influence of likelihood insensitivity on treatment effects

Table 14: The influence of likelihood insensitivity and optimism on treatment effects

	(1) Performance	(2) Performance	(3) Performance	(4) Performance	(5) Performance	(6) Performance
LowPr		21.976***	16.778*		20.642**	15.264*
*Likelihood ins.		(10.085)	(10.085)		(8.510)	(9.112)
LowPr*			2.450			5.135
Optimist			(12.974)			(12.227)
LowPr	20.492***	15.635	8.753	21.177***	11.846	6.711
	(7.696)	(13.864)	(18.827)	(7.735)	(12.342)	(13.774)
MePr	12.827*	12.850*	12.768*	12.708*	12.907*	12.698*
	(6.679)	(6.704)	(6.727)	(6.692)	(6.715)	(6.746)
HiPR	6.711	6.727	6.411	7.753	7.829	7.709
	(5.665)	(5.688)	(5.692)	(5.697)	(5.709)	(5.699)
Likelihood ins.	1.194	-0.075	0.410	-1.399	-4.034	-3.821
	(4.850)	(4.934)	(4.972)	(4.759)	(4.815)	(4.807)
Optimist	-5.818	-5.520	-7.887	-4.103	-4.207	-7.013
	(5.119)	(5.296)	. (5.452)	(4.924)	(4.938)	(4.940)
Concave	16.859*	17.095*	16.321*	15.825*	15.218*	14.631*
	(9.257)	(9.388)	(9.641)	(8.760)	(8.655)	(8.685)
Convex	6.206	5.386	9.296	8.038	5.723	9.774
	(11.581)	(11.848)	(12.770)	(11.387)	(11.889)	(12.279)
Mixed	6.808	7.122	6.324	6.499	7.075	6.133
	(7.348)	(7.365)	(7.221)	(7.377)	(7.522)	(7.187)
Certainty	-9.639*	-9.734*	-9.094	-9.437*	-9.714*	-8.630
	(5.793)	(5.844)	(5.657)	(5.494)	(5.527)	(5.423)
Constant	82.533***	83.128***	83.989***	82.474***	84.110***	84.716***
	(7.305)	(7.190)	(7.233)	(6.733)	(6.667)	(6.614)
Parametric family	Prelec (1998)	Prelec (1998)	Prelec (1998)	Goldstein and Einhorn (1987)	Goldstein and Einhorn (1987)	Goldstein and Einhorn (1987)
Likelihood ins.	$\hat{\alpha} < 1$	$\hat{\alpha} < 1$	$\hat{\alpha} < 1$	$\hat{g} < 1$	$\hat{g} < 1$	$\hat{g} < 1$
Optimist	$\hat{\beta} < 1$	$\hat{\beta} < 1$	$\hat{\beta} < 1$	$\hat{\delta} > 1$	$\hat{\delta} > 1$	$\hat{\delta} > 1$
R^2	0.118	0.119	0.124	0.114	0.120	0.127
Observations	156	156	156	157	157	157

Note: This table presents the estimates of the Ordinary Least Squares regression of the model $Performance_i = \gamma_0 + \gamma_1 LowPr * Likelihoodins. + \gamma_2 LowPr * Optimism + \gamma_3 LowPr + \gamma_4 MePr + \gamma_5 HiPr + \gamma_6 Likelihoodins. + \gamma_7 Optimism + Controls'\Gamma + \epsilon_i$, with $E(\epsilon_i | MePr, LowPr, HiPr, Piecerate, Optimism, Likelihoodins., Controls, Mechanism) = 0$. "Performance" is the number of correctly solved sums solved by a subject in the first part of the experiment, "LowPr", "MePr" and "HiPr" are binary variables that indicate if a subject was assigned to a treatment offering a stochastic contract implemented with low, medium or high probability, respectively. "Piecerate" is the benchmark of the regression. "Likelihood ins." is a binary variable that takes a value of one if the subject is likelihood insensitive and zero otherwise. "Optimist' is a binary variable that takes a value of one if the subject displays optimism and zero otherwise. Robust standard errors in parenthesis. *** denotes significance at the 0.01 level, ** denotes significance at the 0.05 level, * denotes significance at the 0.1 level.

Appendix H: Instructions

This is an experiment in the economics of decision-making. The instructions are simple and if you follow them carefully and make certain decisions, you might earn a considerable amount of money, which will be paid to you via bank transfer at the end of the experiment. The amount of money that you earn will depend on your decisions and effort, and partly on chance. Once the experiment has started, no one is allowed to talk to anybody other than the experimenter. Anyone who violates this rule will lose his or her right to participate in this experiment. If you have further questions when reading these instructions please do not hesitate to raise your hand and formulate the question to the experimenter.

The experiment consists of two parts. Your earnings in part one or part two of the experiment will be chosen at the end of the experiment and become your final earnings. Whether the earnings of part one or the earnings of part two will be your final earnings will be established by roll of a die.

Part one

In this part of the experiment your task is to complete summations. Your earnings in this part of the experiment depend only on the number of correct summations that you deliver. You need to complete as many summations as you can in 10 rounds, each round lasts four minutes. In other words you will have a total of 40 minutes to complete as many summations as you can.

Each summation consists of five-two digit numbers. For example 11+22+33+44+55=? Once you know the answer to the sum of these five two digit numbers, input the answer in the interface, Click OK, and a new set of numbers will appear on your screen.

For your better understanding you will face with two examples next.

[Examples displayed]

The previous examples show what you have to do in this part of the experiment. The only thing left to be explained is to specify how you are going to earn money by completing the summations.

Piecerate Treatment Payment rule: In this part of the experiment each correct summation will add 25 Euro cents to your experimental earnings.

Remember: you have 40 minutes to complete summations, and only correct summations will count towards your earnings at a rate of 25 euro cents each. If you understood these instructions Press ”OK”. When everyone is ready we will start with this part of the experiment.

LowPr Treatment **Payment rule:** In this part of the experiment 1 out of all the 10 rounds will be randomly chosen. The specific round is chosen at random by the computer at the end of this part of the experiment. This is, once you completed summations in all the 10 rounds, only the correct summations in a randomly chosen round will count towards your earnings at a rate of 250 euro cents per correct summation.

Remember: you have 40 minutes to complete summations, and only correct summations in 1 specific round, chosen randomly by the computer at the end of the experiment, will count towards your earnings at a rate of 250 euro cents each. If you understood these instructions Press ”OK”. When everyone is ready we will start with this part of the experiment.

MePr Treatment **Payment rule:** In this part of the experiment 3 out of all the 10 rounds will be randomly chosen. The specific rounds are chosen at randomby the computer at the end of this part of the experiment. This is, once you completed summations in all the 10 rounds, only the correct summations in that randomly chosen round will count towards your earnings at a rate of 85 euro cents per correct summation.

Remember: you have 40 minutes to complete summations, and only correct summations in 3 specific rounds, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 85 euro cents each. If you understood these instructions Press ”OK”. When everyone is ready we will start with this part of the experiment.

HiPr Treatment **Payment rule:** In this part of the experiment 5 out of all the 10 rounds will be randomly chosen. The specific rounds are chosen at random by the computer at the end of this part of the experiment. This is once you completed summations in all the 10 rounds, only the correct summations in that randomly chosen round will count towards your earnings at a rate 50 euro cents per correct summation.

Remember: you have 40 minutes to complete summations, and only correct summations in 5 specific rounds, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 50 euro cents each. If you understood these instructions Press ”OK”. When everyone is ready we will start with this part of the experiment.

Part two

In this part of the experiment your task is to choose among two possible alternatives. Your earnings on this part of the experiment depend on how good your choices are.Particularly, you will face with 11 decision sets. In each of these sets you need to choose between the option L, that delivers a fixed amount of money, and the option R that is a lottery between two monetary amounts. Each decision set contains six choices.

Be Careful! Every time you make a choice between L and R, the monetary prizes of the options are going to change and you ought to make a choice again. One of your choices will be randomly picked by the computer, will be played and its realization will count towards your earnings for this part of the experiment. You will be faced with one example next. [Example displayed]

If it is clear what you have to do in this part of the experiment. Press ”OK” to start, once everyone is ready this part of the experiment will begin.

Survey

- Gender:
- Age:
- What is your education level? (Bachelor, Exchange, Pre-Master, Master, PhD):
- What is the name of your program of studies?
- How difficult did you find the task? (where 1 stands for easy and 5 for Very difficult)
- Rate how confident you are that you can do the task good enough so you can be in the top half of performers in this group as of now. (1-Not confident, 10- Very confident)
- Are you any good at adding numbers? (1-Not good at all, 10-Very good)
- Are you generally a person who is fully prepared to take risks or do you try to avoid taking risks?
- Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks".

- People can behave differently in different situations. How would you rate your willingness to take risks while driving? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks in financial matters? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks with your health? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks in your occupation? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks in your faith in other people? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"