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Sudden Stops and Reserve Accumulation in the Presence of Liquidity Risk

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Sudden Stops and Reserve Accumulation in the Presence of International Liquidity Risk*

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Abstract

We propose a small open economy model where agents borrow internationally and invest in liquid foreign assets to insure against liquidity shocks, which temporarily shut out the economy of short-term credit markets. Due to the presence of a pecuniary externality individual agents borrow too much and hold too little liquid assets relative to a social planner. This inefficiency rationalizes macroprudential policy interventions in the form of reserve accumulation at the central bank coupled with a tax on foreign borrowing. Unless combined with other measures, a tax on foreign borrowing is detrimental to welfare; it reduces agents' incentives to invest in liquid assets and thereby increases financial instability. Our model can quantitatively match the simultaneous depreciation of the exchange rate and contractions in output, gross trade flows, foreign liabilities and liquid reserves during Sudden Stop episodes.

JEL **codes: D62, E44, F32, F34, F41**

Key words: international reserves, sudden stops, liquidity, macroprudential policy, pecuniary externalities

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1 Introduction

Over the last three decades many emerging market economies have experienced surging gross capital inflows. At the same time, these economies have accumulated large amounts of liquid foreign assets mainly in the form of foreign exchange reserves. The fact that emerging markets pay high interest rates to foreign investors while earning ultra-low returns on their reserves has sparked an intensive debate about the economic rationale behind this pattern. A prominent empirical literature[1] argues that emerging market economies use liquid foreign assets to insure against sharp reversals in capital flows (Sudden Stops). This literature is successful at explaining observed levels of reserve accumulation, but fails to address the normative question whether these levels indeed reflect an optimal intervention by policymakers. Theoretical quantitative models that study optimal macroeconomic policies, on the other hand, usually abstract from foreign reserve accumulation; accordingly these models, too, are silent about the optimal holdings of foreign reserves by the central bank.[2]

In the present paper we contribute to filling this gap in the literature. We study optimal macroeconomic policy in a small open economy model with emerging market crises, where the central bank's policy toolkit includes foreign reserve accumulation. The central feature of our model is the occurrence of liquidity crises. During these episodes domestic firms lose access to international markets for unsecured short-term debt to finance imports of intermediate inputs. Holdings of foreign reserves are useful for firms, since they can serve as collateral to facilitate transactions with international suppliers. The overall borrowing capacity of the economy is tied to the real exchange rate which causes an inefficiency in the unregulated equilibrium:[3] A *pecuniary externality* arises because individual agents do not internalize the effect of their actions on the exchange rate and thereby borrowing capacity. As a consequence, private agents engage in inefficiently low precautionary behavior and expose the economy to excessive risk of Sudden Stops relative to a (constrained) social planner.

Our theoretical and quantitative analysis yields three main findings: i) Reserve accumulation at the central bank is part of an optimal policy mix, ii) reserve accumulation interacts non-trivially with other macroprudential policies, such as capital controls and iii) the model can jointly account for the level of reserves, foreign borrowing and gross goods flows as well as their dynamics during Sudden Stop episodes.

We show that inefficiencies in private behavior rationalize the observed accumulation

[1]See e.g.Obstfeld, Shambaugh, and Taylor (2010) and Calvo, Izquierdo, and Loo-Kung (2013).

[2]Exceptions include Bianchi, Hatchondo, and Martinez (2018), Arce, Bengui, and Bianchi (2019) and Shousha (2017). We discuss our relation to these studies below.

[3]This type of borrowing constraint has been widely studied since Mendoza (2002). Our model is most similar to Bianchi (2011).

of reserves at central banks. During normal times the social planner accumulates more liquid foreign assets than private agents, but chooses a very similar level of gross borrowing. Private agents therefore *underinvest* in liquid assets and *overborrow* in net debt. By accumulating more reserves the planner reduces the fall in consumption from 8% to 4% and the depreciation in the exchange rate from 30% to 15% in a typical Sudden Stop. An optimal policy mix to decentralize the planner allocation requires an intervention both in private borrowing and liquidity holdings, for example a tax on debt combined with reserve accumulation at the central bank. In a severe crisis, however, the inefficiency in liquid asset choices can change sign. Since agents do not internalize that they further destabilize the exchange rate by purchasing liquid foreign assets, private holdings of these assets can be excessive. In this situation, a policymaker can prevent inefficient liquidity hoarding by limiting convertibility of the domestic currency.

A key insight of our analysis is that the presence of liquid assets fundamentally changes the welfare properties of simple regulatory interventions. A common result in the previous literature is that simple capital controls in the form of a constant tax on foreign debt can be used to reduce overborrowing and improve welfare.[4] Our findings are in sharp contrast: While a tax on debt can be used to reduce both gross and net foreign borrowing, the tax also reduces private holdings of liquid assets. This occurs because the tax on debt makes agents value current consumption more relative to future consumption, which reduces private incentives to accumulate liquid assets. As a result, the tax fails to improve financial stability and causes a welfare loss. The intuition behind this result is general and highly relevant for the design of regulatory interventions: Policies that just aim at reducing gross borrowing, distort incentives for accumulating liquid assets and can therefore lead to financial instability. To avoid unintended consequences such policies have to be combined with liquidity regulation.

To study the quantitative success of our model we present a set of stylized facts on gross goods and capital flows around sudden stop episodes. In particular Sudden Stops are associated with sharp contractions in imports, exports, gross foreign liabilities and international reserves. At the same time the real exchange rate strongly depreciates. We calibrate our model to match selected moments of a set of emerging market economies and find that the model quantitatively replicates the stylized facts on Sudden Stops. Further, the model generates observed levels of international borrowing and liquid asset holdings. We show that the presence of liquidity crises is crucial for the model to match the patterns in the data.[5] In the absence of liquidity crises, agents hold no liquid assets in equilibrium, so neither the level nor the dynamics of international reserves are replicated.

[4] See, for example, Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2018).

[5] Our model nests the model of Bianchi (2011) as a special case, where no liquidity shocks occur.

In addition, the model without liquidity shocks fails to match the contractions in gross goods flows.

Our focus on the role of foreign reserves in facilitating trade in intermediate inputs is motivated by a number of empirical facts.[6] First, intermediate inputs to production dominate overall international trade. At least 60% of international trade is in intermediate inputs. Using OECD data we find that the relevant number in our context is close to 85%. Second, most of trade is not paid for in advance but financed with short-term debt with maturity below one year.[7] Third, there is recent evidence that in times of crisis emerging market firms' ability to finance inputs through borrowing is severely restricted.[8] Sustaining trade in intermediate inputs during crises is hence an important reason for emerging markets to hold liquid reserves, even though clearly not the only one. We nevertheless restrict attention to this single mechanism, which allows us to obtain clear analytical results in a model that is able to quantitatively match empirical regularities. Importantly, however, our normative results are compatible with other explanations for the accumulation of liquid assets, as long as a pecuniary externality is present and liquid assets have some positive effect on domestic output during crises. If this is the case the various inefficiencies in private behavior we identify are present and the result that regulatory interventions in borrowing interfere with incentives to accumulate liquidity applies.

Related literature Our framework builds on the workhorse small open economy model developed by Mendoza (2002) and its normative analysis in Bianchi (2011).[9] Specifically, our model shares the debt-deflation mechanism and the pecuniary externality arising from the fact that the borrowing capacity is tied to the value of domestic output. In the setup of Bianchi (2011), this externality gives rise to overborrowing, which can be addressed by the social planner through a variety of macroprudential measures, for example a tax on foreign borrowing. Importantly, however, there is no explicit role for liquid reserves in these contributions. We show that the introduction of liquidity crises introduces such a role, which allows the model to match not only level and dynamics of reserve holdings, but

[6] The precautionary demand for foreign reserves to hedge against shocks to the trade balance has been studied in the economic literature since Heller (1966). We build on this literature but also stress the role of reserves as insurance against collapse in short-term financing, which is related to modern theories of reserve demand.

[7] IMF (2009) provides evidence that around 40% of trade is intermediated through short-term bank loans and another 40% is direct credit from suppliers to importers.

[8] Swanson (2019) shows that trade credit contracts even more than imports in sudden stop episodes. Interestingly contractions in trade credit supply seem to have played little role in the turmoil at the onset of the Great Recession.

[9] Note that there is a series of related paper that study financial amplification mechanisms and optimal policy in small open-economy models without an explicit role for liquid foreign reserves(see e.g. Benigno, Chen, Otrok, Rebucci, and Young, 2013; Jeanne and Korinek, 2018; Bianchi and Mendoza, 2018).

also gross goods flows during Sudden Stops. Moreover, our model generates novel policy implications which stress the importance of coordinating capital controls with reserve accumulation.

Most closely related to this paper are recent contributions which jointly study reserve accumulation and emerging market crises in quantitative frameworks, in particular Arce, Bengui, and Bianchi (2019), Bianchi, Hatchondo, and Martinez (2018) and Shousha (2017). Arce, Bengui, and Bianchi (2019) show that foreign reserve accumulation can be used to implement the constrained efficient allocation in the model of Bianchi (2011), if reserves carry the same interest as foreign borrowing. In their framework, however, reserves are not useful per se, but are only held by the regulator to offset excessive private borrowing. If the return on reserves is below the international interest rate, reserve accumulation is strictly dominated by other macroprudential policies in their model. In contrast, we provide a framework where reserve accumulation is part of an optimal policy mix, even if reserves pay no interest. Bianchi, Hatchondo, and Martinez (2018) study the optimal accumulation of international reserves in a framework of sovereign default. Sovereigns issue long-term debt and invest in international reserves simultaneously because reserves provide liquid resources in states where borrowing opportunities deteriorate. Our focus on the other hand lies on studying private borrowing and public reserve accumulation, which dominate gross capital flows in the data.[10] Finally Shousha (2017) provides a framework where reserves are held by the central bank for their value as collateral in a liquidity crisis. However, in his framework private decisions are not inefficient per se, but individual agents have no access to liquid assets, so reserves are held at the central bank.[11] Our model on the other hand rationalizes reserve accumulation by the monetary authority as a macroprudential measure, even though private agents could in principle insure themselves through the accumulation of liquid assets.

Our study is further related to a wide empirical literature that studies the idea that reserves are held to provide insurance against Sudden Stops and more generally the dynamics of gross capital flows during emerging market crises.[12] We use the findings from this literature to motivate our modeling approach and to test the quantitative success of the model. Our equilibrium analysis allows us to provide insights for the design of macroprudential policies. We further connect to recent theoretical contributions stressing

[10]See section 2.

[11]A similar argument holds in the models of Benigno and Fornaro (2012), Caballero and Panageas (2008), Jeanne and Rancière (2011) and Jeanne and Sandri (2016) which are more loosely related to our study.

[12]Obstfeld, Shambaugh, and Taylor (2010) Calvo, Izquierdo, and Loo-Kung (2013), Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012), Broner, Didier, Erce, and Schmukler (2013), Avdjiev, Hardy, Kalemli-Ozcan, and Servén (2017)

the precautionary accumulation of foreign reserves in static setups.[13] A common finding in this literature, which relates to our paper, is that agents who may lose access to external credit in future periods are willing to invest in liquid assets with low returns. By providing a dynamic model we can evaluate our theory quantitatively and provide insights into the cyclical nature of policy interventions.

Several other explanations for the accumulation of international reserves, including the presence of growth externalities and facilitation of FDI, have been proposed.[14] We see our study as complementary to this literature. As we argue above, our normative findings are compatible with other explanations for the accumulation of liquid assets, if reserves provide some positive effects on output during a crisis.

Note further that the use of the cash-in-advance constraint on imports follows a well established literature to explain liquidity holdings.[15] Recently, such constraints have been used to explain large liquidity holdings of corporates (Bacchetta, Benhima, and Poilly, 2019) and households (Telyukova, 2013).

Finally, inefficiencies associated with private liquidity choices are also studied in a literature that focuses on systemic vulnerabilities arising in the banking sector, in static models without a focus on emerging markets.[16] Similar to our paper, a key result in this literature is that private agents hold an inefficiently low amount of liquid assets, emphasizing the need for liquidity regulation.

The rest of the article proceeds as follows. Section 2 presents the empirical evidence motivating our analysis and section 3 lays out the modeling framework and characterizes the competitive equilibrium. In section 4 we characterize the constrained-efficient allocation and discuss the inefficiencies present in the competitive equilibrium. Section 5 illustrates the main insights of our model in a quantitative analysis and section 6 concludes.

[13] Important contributions include but are not limited to Corneli and Tarantino (2016), Hur and Kondo (2016), Aizenman and Lee (2008), Jeanne (2016).

[14] See e.g. Aguiar and Amador (2011), Song, Storesletten, and Zilibotti (2011), Korinek and Servén (2016), Coeurdacier, Guibaud, and Jin (2015), Bacchetta and Benhima (2012), Jung and Pyun (2016).

[15] The use of such constraints goes back to Lucas (1982), Lucas and Stokey (1987) and Svensson (1985).

[16] Recent contributions include Malherbe (2014), Calomiris, Heider, and Hoerova (2015), Kara and Ozsoy (2016), Walther (2016), Lutz and Pichler (2017).

2 Sudden Stops and Capital Flows in the Data

Our work is motivated by a recent empirical literature emphasizing the importance of gross capital flows.[17] These studies document (i) that gross capital flows are large and volatile compared to net flows, (ii) gross as well as net capital flows are highly pro-cyclical and (iii) gross capital outflows of emerging markets mainly take the form of international reserves.

Avdjiev, Hardy, Kalemli-Ozcan, and Servén (2017) further decompose capital flows by sectors. They find that the cyclical behavior of capital flows is mainly driven by private sector borrowing and international reserves, which are both pro-cyclical. Furthermore, sovereign borrowing is negatively correlated with domestic output. These results provide support for our focus on private sector borrowing and a regulator intervening in the accumulation of liquid assets.

Following the empirical literature, we consider patterns of gross capital and trade flows around sudden stop episodes in the data.[18] The data we use is taken from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2006) and the Word Developement Indicators.

The patterns discussed above can clearly be seen in Figure 1. We show all non-stationary variables as log-deviations from their trend.[19] The ratios of the trade balance and the change in NFA to GDP are already stationary and are therefore shown in levels. By definition output strongly contracts and the trade balance sharply increases during the sudden stop. Both gross flows fall strongly, but imports contract more than exports. Moreover, there are is a decline in gross capital outflows, driven by a reduction in foreign reserves. Still the net foreign asset position increases, since gross liabilities contract even more strongly. Since gross liabilities are much larger than reserve holdings, even though they contract less in relative terms, the net foreign asset position strongly increases. Note that for consistency all variables are measured in dollars, which explains the large contractions in output. This contraction partly arises from the depreciation in the exchange rate, while in purchasing power parity it is significantly smaller.

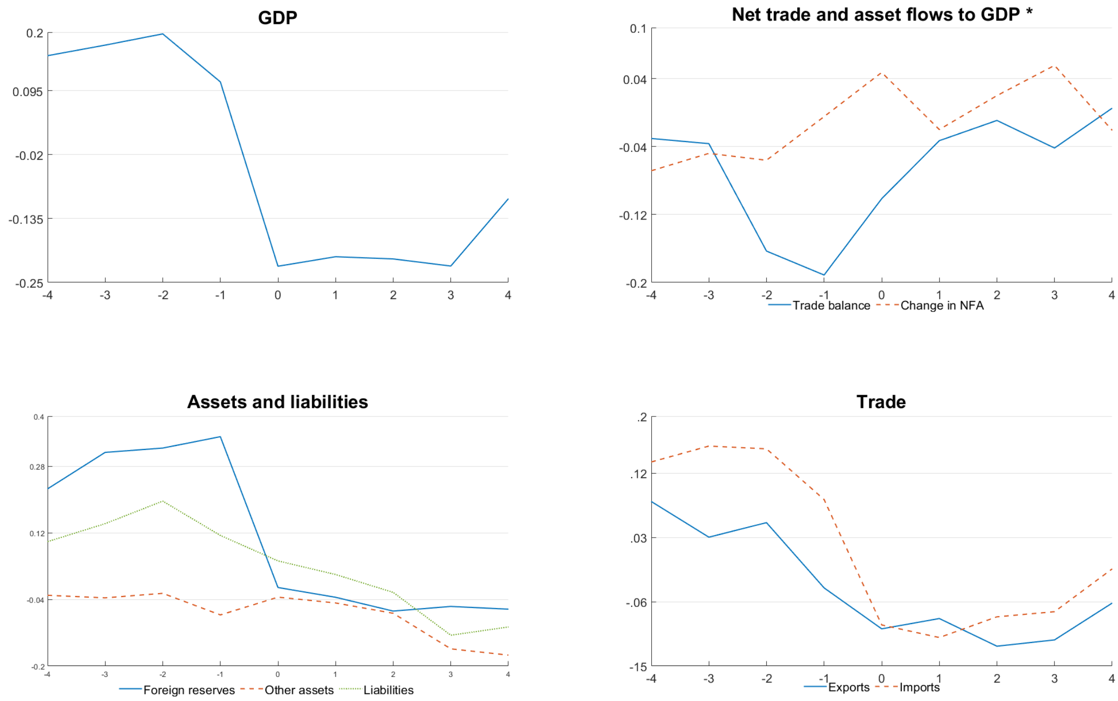
Foreign assets other than reserves do not seem to be correlated with the sudden stop event in an obvious way and fluctuate much less compared to foreign reserve holdings. Reserves and other assets are in similar magnitudes, so overall fluctuations in foreign

[17]Contributions include Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012), Broner, Didier, Erce, and Schmukler (2013), Avdjiev, Hardy, Kalemli-Ozcan, and Servén (2017).

[18]The dates for sudden stop episodes are taken from Calvo, Izquierdo, and Mejía (2004) who identify sudden stops as periods where year-on-year fall in capital flows (net of changes in reserves) fall two standard deviations below their mean and output simultaneously contracts. They use data between 1990 and 2004.

[19]We use a standard hp filter with parameter 100, which is widely used for yearly data.

Figure 1: Sudden Stops in the Data



Note: Variables are shown in relative deviations from trend; * is shown in absolute values.

assets during sudden stops are mostly driven by reserves. We therefore use international reserves as a proxy for total liquid assets held by domestic agents in our quantitative analysis and use the terms 'foreign liquid assets' and 'reserves' interchangeably in the rest of this paper. To the extent that these other assets are also used to provide liquidity to domestic agents, we underestimate the importance of the channels we study. Note that even in the data it is difficult to disentangle which agent can actually use the liquidity associated with foreign reserves, as monetary authorities in emerging markets deposit significant shares of their reserve holdings in their banking systems.[20]

In the following section we develop a model that can quantitatively match the facts presented here.

[20]Emerging market banks hold large amounts of international interbank deposits. Moreover, central banks engage in trading forward contracts on foreign currency with their domestic banking system. If, for example, a central bank buys dollars forward from a domestic bank, this bank will then hold dollar liquidity until the contract matures (see, for example, Wooldridge, 2006 and McCauley and Zukunft, 2008).

3 The Model

We develop a two-sector small-open economy model based on Bianchi (2011) and introduce liquidity risk. The economy is populated by an infinitely lived representative bank-firm-household who consumes tradable and nontradable goods. Agents can borrow long-term on international debt markets subject to a standard collateral constraint.[21] Furthermore they can invest in a non-interest bearing liquid foreign asset. Production of tradable output requires imported goods as an essential input. In normal times imports can be financed through short-term international loans, such as trade credit or international bank loans. There are times, however, when these short-term debt markets shut down and only safe liquid assets can serve as collateral for these transactions.[22] These are episodes when global financial markets are in turmoil and liquid funding dries up for reasons exogenous to the domestic economy. In what follows, we often refer to such an event as a *liquidity shock*. As we show below, the shut down in short-term debt markets can turn into a full Sudden Stop, with an associated contraction of domestic production and a sharp reversal in the current account, if agents hold too little liquid assets.

Note that we don't model a separate balance sheet for the central bank, but assume that liquid assets are held directly by the domestic agent. Due to the simplicity of our model, central bank reserves are perfect substitutes to privately held liquid foreign assets, if it is commonly understood by domestic agents and international lenders that the central bank will use its reserves to act as a lender-of-last-resort in the event of a liquidity crisis.[23] We maintain this assumption throughout this analysis and therefore don't take a strong stand on which agent actually holds these assets on their balance sheet. What matters in this section is that on the margin the amount of liquid assets held in the domestic economy is determined by private decisions. In section 4 we study an economy where liquid assets are chosen by a central authority.

3.1 Bank-Firm-Household Optimization Problem

The representative agent maximizes life-time utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

[21] Long-term refers to an inter-temporal loan, which lasts for one model period, as opposed to a short-term loan, which has to be repaid in the same period.

[22] Alternatively the assets could directly be used for the transaction

[23] This equivalence is trivial if one assumes, that the central bank accepts domestic non-tradable output as collateral and sets a haircut exactly, such that all its reserves are alloted to domestic agents. This means each agent will have access to exactly the same amount of reserves. Based on Uribe (2006) we conjecture that the equivalence still holds, if a market clearing interest rate is set.

where $\mathbb{E}_0$ is the expectation operator conditional on information available at date 0 and β is the discount factor. The consumption basket c_t is a CES aggregator with elasticity of substitution $1/(\eta+1)$ between tradable goods c_t^T and nontradable goods c_t^N given by

$$c_t = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta}. \tag{2}$$

In each period agents receive a fixed endowment of nontradable goods y^N. We extend the model of Bianchi (2011) by assuming that tradable output y_t^T is produced using intermediate input goods ν_t. Tradable output is given by $y_t^T = z_t F(\nu_t)$, where z_t denotes total factor productivity in the tradable sector, which follows a finite-state Markov process and $F(.)$ denotes a twice continuously differentiable, concave production function. Net output is thus given by $[z_t F(\nu_t) - p_\nu \nu_t]$, where p_ν denotes the exogenous price of intermediate inputs. In principle the framework can be extended to include endogenous production using capital and labor inputs in both sectors.[24] To keep both theoretical and quantitative analysis concise, we exclude those extensions.

All prices and contracts are written in terms of the tradable good, which acts as a numeraire with a price of one. Agents have access to a one-period non-state contingent foreign bond b_{t+1} that pays a fixed interest rate $R > 1$ and to a non-remunerated liquid foreign asset l_{t+1}, i.e. essentially a storage technology. In equilibrium agents will choose negative amounts of b, which corresponds to foreign borrowing.[25] We denote the price of nontradables by p_t^N. Following the literature we refer to p_t^N as the real exchange rate in the remainder of the text. The budget constraint of the representative agent reads

$$c_t^N p_t^N + c_t^T + l_{t+1} + \frac{b_{t+1}}{R} = [z_t F(\nu_t) - p_\nu \nu_t] + p_t^N y_t^N + l_t + b_t, \tag{3}$$

where b_{t+1} are foreign bond holdings chosen at time t and negative values of b indicate foreign debt of domestic agents.

Inter-period debt is subject to a collateral constraint limiting total debt to not exceed a fraction κ of the market value of eligible collateral:

$$b_{t+1} \geq -\kappa^N p_t^N y_t^N - \kappa^T [z_t F(\nu_t) - \nu_t p_\nu] - \theta l_{t+1}, \tag{4}$$

where κ^N and κ^T denote the shares of nontradable and tradable net output, respectively, which can be pledged to foreign investors.[26] The crucial feature of this constraint is that

[24] Extensions to endogenous labor and capital accumulation are numerous in the literature.

[25] Note that we implicitly assume that private agents directly borrow on international markets in foreign currency. See Mendoza and Rojas (2019) for a model where domestic agents borrow from domestic banks in dollar denominated debt.

[26] Constraints of this type have widely been used in the international macroeconomic literature since,

nontradable goods are part of the collateral and are valued at their endogenous price p_t^N. Since individual agents do not anticipate their incremental effect on equilibrium prices, this may lead to an inefficiently low precautionary behavior (see e.g. Lorenzoni, 2008; Korinek, 2010; Jeanne and Korinek, 2010; Bianchi, 2011).

Constraint 4 is a natural extension of the constraint in Bianchi (2011) to account for endogenous production of tradable goods and the presence of liquid assets, which can also serve as collateral. Their use as collateral here is not necessary for liquid assets to be held in equilibrium and our normative results apply for any $\theta \in [0, 1]$. The share of liquid assets that can serve as collateral determines how much of an extra unit of liquid assets can be financed through debt issuance and how much has to be financed through reduced consumption. We provide a microfoundation of this constraint in Appendix A.

We now turn to the central, new feature of our model. At the beginning of the period intermediate inputs ν_t have to be imported before production takes place. Agents can only finance these imports through unsecured short-term credit.[27] In normal times end-of-period production provides sufficient collateral to finance desired levels of imports, but during liquidity crises lenders are only willing to accept liquid assets. This gives rise to the following stochastic constraint:

$$S_t \nu_t p_\nu \leq l_t. \tag{5}$$

Note that effectively this constraint is a cash-in-advance constraint on imported inputs, where S_t is a stochastic variable that governs the state of the short-term debt market and follows a first-order two-state Markov process. In normal times ($S_t = 0$) agents have access to unlimited short-term credit and the constraint is never binding. However, in case of a liquidity shock ($S_t = 1$) only liquid assets can serve as collateral for short-term credit. If the stock of liquid assets is low, imported inputs have to be reduced, so output, tradable consumption and the price of nontradable goods fall. This price decline reduces the capacity to borrow inter-temporally, which further lowers the price of nontradables and triggers a Fisherian debt-deflation mechanism. We provide a microfoundation of this constraint along with the inter-temporal borrowing constraint in Appendix A. We

as far as we know, Mendoza (2002) and are generally understood to capture financial frictions associated with borrower default in a reduced form manner. As pointed out by Schmitt-Grohé and Uribe (2018) it is possible that, while this constraint binds on the individual level, the effect of trade-able consumption on the exchange rate is so strong, that on aggregate higher consumption and borrowing is possible. Policy implications in such economies are very different, as generally there is underborrowing in net debt. We assume that this is not the case throughout our normative analysis. In our calibrated economy, the constraint binds on aggregate, whenever it binds individually.

[27] This assumption is not as stark as it might seem. IMF (2009) presents evidence that worldwide only 20% of imported goods are paid in advance, while 80% are credit financed. Half of these 80% is direct trade credit from exporters to importers while the other half is intermediated.

discuss the role of the liquidity constraint in more detail in section 3.2, after defining a competitive equilibrium in our model.

3.2 Competitive Equilibrium

The representative household maximizes life-time utility (1) subject to the budget constraint (3), the collateral constraint (4), the liquidity constraint (5) and the non-negativity constraint on liquid assets $l_{t+1} \geq 0$ by choosing $\{b_{t+1}, l_{t+1}, c_t^T, c_t^N, \nu_t\}$, taking prices as given. This maximization problem yields the following optimality conditions

$$\lambda_t = u_T(t), \tag{6}$$

$$p_t^N = (\frac{1-\omega}{\omega})(\frac{c_t^T}{c_t^N})^{\eta+1}, \tag{7}$$

$$\xi_t S_t p_\nu = (\lambda_t + \mu_t \kappa^T)[z_t F_{\nu_t}(.) - p_\nu], \tag{8}$$

$$\frac{\lambda_t}{R} - \mu_t = \beta \mathbb{E}_t\{\lambda_{t+1}\}, \tag{9}$$

$$\lambda_t - \theta\mu_t - \psi_t = \beta \mathbb{E}_t\{\lambda_{t+1} + \xi_{t+1}\}, \tag{10}$$

where λ is the nonnegative multiplier associated with the budget constraint, μ is the nonnegative multiplier of the collateral constraint, ξ is the nonnegative multiplier of the liquidity constraint and ψ is the nonnegative multiplier of the non-negativity constraint on liquid assets.[28] Further, $u_T(t)$ denotes the derivative of the utility function with respect to tradable consumption.

Equation (6) is the optimality condition for tradable consumption equating the marginal utility to the shadow value of current wealth. Condition (7) equates the relative price of tradable and nontradable goods with their marginal rate of substitution. Note that an increase in c^T generates an increase in the price of nontradable goods thereby increasing the collateral value of nontradables. Equation (8) is the intra-temporal optimality condition for intermediate inputs. If the liquidity constraint is nonbinding in period t, the optimality condition simplifies to $z_t F_{\nu_t}(.) = p_\nu$. Otherwise, intermediate inputs are determined by last periods liquidity holdings and $z_t F_{\nu_t}(.) > p_\nu$. Equation (9) is the intertemporal optimality condition for foreign bonds, which is completely standard. When the collateral constraint binds, the marginal benefit of borrowing exceeds the expected marginal costs of repayment by the shadow price of the constraint. Finally, the optimality condition for liquid assets is given by equation (10), which captures the central

[28] This constraint is not really necessary. As long as there is a positive probability of a liquidity shock, household endogenously hold positive liquid assets in equilibrium. Otherwise they would run the risk of producing 0 output, as intermediate inputs are an essential input to production. We nevertheless state it here to make clear that agents cannot borrow in the liquid asset.

mechanisms in our model. While liquid assets do not carry any interest, they are still useful to private agents because they anticipate that liquid assets can be used to buy intermediate inputs in case of a liquidity shock as captured by the term ξ_{t+1}. Since all agents are identical, market clearing conditions are given by

$$c_t^N = y^N, \tag{11}$$

$$c_t^T = z_t F(\nu_t) - p_\nu \nu_t + l_t + b_t - l_{t+1} - \frac{b_{t+1}}{R}. \tag{12}$$

Finally, the following complementary slackness conditions have to hold

$$b_{t+1} + \kappa^N p_t^N y_t^N + \kappa^T [z_t F(\nu_t) - p_\nu \nu_t] + \theta l_{t+1} \geq 0, \quad \mu \geq 0, \tag{13}$$

$$l_t - S_t p_\nu \nu_t \geq 0, \quad \xi \geq 0, \tag{14}$$

$$l_{t+1} \geq 0, \quad \psi \geq 0, \tag{15}$$

as well as the laws of motion for the exogenous variables $\{S_t, z_t\}$. The unregulated competitive equilibrium is defined as follows

Definition 1. *A unregulated recursive competitive equilibrium is a set of*

i a pricing function $\hat{p}^N(b, l, S, z)$

ii decision functions $\{\hat{b}(b, l, S, z), \hat{l}(b, l, S, z), \hat{c}^T(b, l, S, z), \hat{c}^N(b, l, S, z), \hat{\nu}(b, l, S, z)\}$

iii and multipliers $\{\hat{\lambda}(b, l, S, z), \hat{\mu}(b, l, S, z), \hat{\xi}(b, l, S, z), \hat{\psi}(b, l, S, z)\}$

that satisfies the equations (6) - (15) given the laws of motion for the exogenous states S and z.

3.3 Discussion of the Liquidity Constraint

The liquidity constraint (5), is central to the mechanism studied in this paper, as introduces a separate role for liquid assets. In comparison in the model of Bianchi (2011) the equilibrium is not affected, if beginning of period debt and liquid asset holdings are increased by the same amount. In that model the extra liquid asset can only be used to repay the extra debt.[29] The same is true in our model if $S_t = 0$. However, if $S_t = 1$ an extra unit of liquid assets allows agents to produce more than one unit of output on the margin, while they have to repay just one extra unit of debt[30]. Agents anticipate this and

[29]Since liquid assets earn zero return, while agents have to pay interest on debt, liquid assets are not held in equilibrium.

[30]It is straightforward to see that the marginal product in case of a liquidity shock will always be strictly larger than 1 in equilibrium

build up precautionary liquidity holdings. Note that in the absence of liquidity shocks, intermediate inputs are always at their unconstrained level and tradable production only depends on exogenous productivity. As a result, no liquid assets are held in equilibrium and the model reduces to Bianchi (2011), with a stochastic endowment of tradable goods. This shows that the model with liquidity shocks nests the model by Bianchi (2011) as a special case.

Importantly, the liquidity constraint does not introduce a new pecuniary externality to the model, as there is no price involved.[31] Liquidity choices are still potentially inefficient because of the interaction of the two constraints: If a liquidity shock hits the economy, the level of liquid assets affects the amount of tradeable output and consumption. Tradeable consumption in turn affects the real exchange rate, which is the source of the pecuniary externality. This effect is not internalized by individual agents when making their decisions, which, as we show in the following analysis, generally leads to underinvestment in liquid assets during normal times.

A potential concern is that trade flows over short periods of time are not large enough to warrant the large observed amounts of liquid asset holdings in emerging economies. Obstfeld, Shambaugh, and Taylor (2010) point out that trade flows alone could at most account for reserve outflows of 1/2% of GDP per week, which would give both policy makers and private agents time to adjust their liquid asset positions. They argue that only rapid withdrawals of domestic bank deposits in order to purchase foreign currency, coupled with unwillingness of foreign investors to buy domestic currency can explain the large precauntionary liquid asset holdings. We would like to point out that our model can be understood this way as well. In the event of a liquidity shock, private agents expect a depreciation of the exchange rate and a shortage of foreign liquid assets. In this situation even firms, which have no immediate use for them, will hoard foreign liquid assets. This leads to an instant outflow of liquid assets from the domestic banking system that by far exceeds the actual flow of imports. As a result, firms which have actual liquidity needs will have access to insufficient amounts of reserves.

Finally, note that access to short-term debt markets is determined by a purely exogenous process which is unrelated to economic fundamentals. One can think of this as the risk aversion of international investors. Empirical studies show that global risk aversion is among the most robust predictors for sudden capital flow reversals in emerging markets (see e.g. Eichengreen, Gupta, and Mody, 2016; Calvo, 2013). Furthermore, the assumption that domestic output completely loses its value as collateral for international short-term credit might seem very stark. In principle, the model can easily be extended by introducing a time-varying share of domestic output entering in the liquidity constraint.

[31] We show this point formally in the section 4.1.

In such a model, access to short-term debt markets would indeed be related to domestic output. While this might add to the quantitative realism of the model it would come at the cost of analytical clarity. We prefer the simplest possible version of the model, which can match the key features in the data we are interested in.

4 Constrained Efficient Equilibrium

We now consider a constrained social planner that faces the same collateral constraint and liquidity constraint but internalizes how prices are determined in equilibrium. In particular, the planner chooses b_{t+1}, l_{t+1}, c_t^T, c_t^N and ν_t to maximize aggregate welfare (1) subject to the resource constraints (11) and (12), the collateral constraint (4), the working capital constraint (5) and the pricing rule of the competitive equilibrium allocation (7).[32]

As opposed to private agents, the planner internalizes the effect of debt and liquid assets on the price of nontradable goods. Critically, the planner anticipates that higher outstanding debt reduces tradable consumption, which lowers the exchange rate and borrowing capacity in states where the collateral constraint binds. Conversely, more liquid assets increase tradable consumption in adverse states of the world, which raises the exchange rate and prevents strong declines in the borrowing ability of the economy. In recursive form the social planner's problem can be expressed as

$$
\begin{aligned}
V(b,l,S,z) &= \max_{\{b',l',c^T,\nu_t\}} U(c(c^T,y^N)) + \beta \mathbb{E} V(b',l',s'), \qquad (16)\\
\text{s.t.} & \\
c^T & + \frac{b'}{R} + l' = zF(\nu) - p_\nu \nu + b + l,\\
b' & \geq -\kappa^N \frac{1-\omega}{\omega} \left(\frac{c^T}{y^N}\right)^{(\eta+1)} y^N - \kappa^T [zF(\nu) - p_\nu \nu] - \theta l', \quad \mu \geq 0,\\
l & \geq S p_\nu \nu, \quad \xi \geq 0,\\
l' & \geq 0,
\end{aligned}
$$

where we have used the pricing rule of the competitive equilibrium to replace the price p_t^N in the collateral constraint of the social planner. The first order conditions in sequential

[32]By constraining the social planner problem to the pricing rule of the competitive equilibrium we follow the constrained-efficiency concept of Kehoe and Levine (1993), which has been widely used for example in Bianchi (2011). Note that not allowing the social planner to directly intervene in the exchange rate is a non-trivial constraint. As shown by Benigno, Chen, Otrok, Rebucci, and Young (2016) the planner can actually completely undo the effects of the borrowing constraint by using consumption taxes to manage the real exchange rate in a similar model.

form are given by

$$\begin{aligned}\lambda_t^{SP} &= u_T(t) + \mu_t^{SP}\Psi_t, &(17)\\ \frac{\lambda_t^{SP}}{R} - \mu_t^{SP} &= \beta\mathbb{E}_t\{\lambda_{t+1}^{SP}\}, &(18)\\ \lambda_t^{SP} - \theta\mu_t^{SP} - \psi_t^{SP} &= \beta\mathbb{E}_t\{\lambda_{t+1}^{SP} + \xi_{t+1}^{SP}\}, &(19)\\ \xi_t^{SP} S_t p_\nu &= (\lambda_t^{SP} + \kappa^T\mu_t^{SP})[z_t F_\nu(.) - p_\nu], &(20)\end{aligned}$$

where we use superscript *sp* to distinguish the multipliers of the competitive equilibrium from the multipliers of the constrained-efficient allocation and $\Psi_t = \kappa(p_t^N c_t^N / c_t^T)(1+\eta) > 0$. This term summarizes the equilibrium effect on the collateral value of nontradable goods for a marginal change in tradable consumption.[33]

It is important to note that the optimality condition for ν implies that the planner makes the same choice for imported inputs as agents in the competitive economy given the states. In particular the equation implies that ν is pinned down by the condition $z_t F_\nu(.) - p_\nu = 0$ if possible and by the constraint $S_t p_\nu \nu_t \leq l_t$ otherwise.

4.1 Comparison of Private and Planner Optimality conditions

The key difference between the unregulated competitive equilibrium and the constrained-efficient allocation becomes evident by comparing equation (6) with equation (17). In particular, the social planner's marginal utility gain from tradable consumption is composed of a direct effect on utility $u_T(t)$ and an indirect effect as the planner understands that an increase in tradable consumption relaxes the collateral constraint. This effect on the price of tradable goods and thereby on the aggregate borrowing ability of the economy is not internalized by individual agents. Individual agents thus value wealth less than the social planner in states where the collateral constraint binds. Importantly, in our framework the difference in the valuation of wealth distorts both individual borrowing and liquidity choices, which we now discuss in more detail.

Macroprudential interventions Following Bianchi (2011) we first consider states where the collateral constraint is currently not binding i.e. $\mu_t^{SP} = 0$.[34] This simplifies equations and allows us to develop important insights into the nature of macroprudential interventions, which the planner uses in states where there is currently no debt crisis. We turn to the case where the constraint currently binds below. As noted before, the

[33] This term is equivalent to Bianchi (2011).

[34] For consistency we assume in the remainder of this section that both optimality conditions are evaluated at the constrained-efficient allocation. Agents form their expectations knowing this. We discuss how to consistently interpret private LMs in this setting in Appendix C.

planner's choice for purchases of imports coincides with the competitive one. We can therefore focus on the consumption-borrowing-liquidity choices.

The overborrowing result becomes evident by using (6) and (17) with $\mu_t = 0$ to rewrite the competitive and constrained-efficient Euler equations for inter-temporal debt holdings respectively

$$
\begin{aligned}
u_T(t) &= \beta R \mathbb{E}_t\{u_T(t+1)\}, \\
u_T(t) &= \beta R \mathbb{E}_t\{u_T(t+1) + \mu^{SP}_{t+1}\Psi_{t+1}\}.
\end{aligned}
$$

Clearly the planner faces higher future costs of current debt holdings given by $\mu^{SP}_{t+1}\Psi_{t+1}$ as she anticipates the negative effect on prices. This implies that the private first order condition cannot be satisfied, and the agents have an incentive to marginally increase borrowing.[35] Private agents therefore overborrow (taking liquid asset holdings as given) compared to constrained-efficiency leaving the economy excessively prone to financial distress. The farther away the economy is from the borrowing constraint ($\mathbb{E}_t\mu^{SP}_{t+1}$ close to 0), the smaller the difference between the two allocations becomes.

Critically, in our model the wedge in the valuation of wealth also distorts optimal liquidity choices. Using the fact that $\lambda_t = u_T(t)$ and $\xi_t = \frac{(\lambda_t+\mu_t\kappa^T)[zF_\nu(.)-p_\nu]}{p_\nu}$ in the competitive equilibrium and t $\lambda^{sp}_t = u_T(t) + \mu^{sp}_t\Psi_t$ and $\xi^{sp}_t = \frac{(\lambda^{sp}_t+\mu^{sp}_t\kappa^T)[zF_\nu(.)-p_\nu]}{p_\nu}$ in the constrained-efficient allocation, the Euler conditions for liquid assets can be rewritten to

$$
\begin{aligned}
u_T(t) &= \beta \mathbb{E}_t\Big\{u_T(t+1)\frac{z_{t+1}F_\nu(t+1)}{p_\nu} + \mu_{t+1}\kappa^T\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu}\Big\}, \\
u_T(t) &= \beta \mathbb{E}_t\Big\{u_T(t+1)\frac{z_{t+1}F_\nu(t+1)}{p_\nu} + \mu^{SP}_{t+1}\kappa^T\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu} \\
&\quad + \mu^{SP}_{t+1}\Psi_{t+1} + \mu^{SP}_{t+1}\Psi_{t+1}\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu}\Big\}.
\end{aligned}
$$

Here, the competitive and planner optimality conditions differ in three ways. Firstly, it is clear that the gain of holding liquid assets for the social planner exceeds the gain of private agents by $\mu^{SP}_{t+1}\Psi_{t+1}$. This difference arises for exactly the same reason as in the optimality conditions for borrowing, since the planner values savings more than individual agents. Note that the valuation of wealth is unrelated to the actual use of the liquid asset in liquidity crises and therefore not the central part of our analysis. Moreover, in the absence of liquidity shocks, this term on its own would not cause the planner to hold positive amounts of liquid assets because the benefit from holding liquid assets is still strictly smaller than the interest rate on borrowing in a model without liquidity shocks.

[35] Formally we could introduce the planners' choice as a constraint in the decentralized optimization problem. This constraint would then have a positive LM, so that the Euler equation is satisfied

The other differences are more interesting, as they are new in our model and more subtle. The last term in the planner's Euler equation $\mu_{t+1}^{SP}\Psi_{t+1}\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu}$ captures the benefit the planner attaches to the positive effect of the additional net output on the real exchange rate.[36] In a liquidity crisis, a higher stock of liquid assets increases output, which allows for higher tradable consumption. This in turn raises the real exchange rate, which makes the borrowing constraint less tight. Since private agents do not internalize this effect, the term unambiguously raises the planner's incentives to hold liquid assets relative to the competitive equilibrium.

Finally, in both optimality conditions the term $\mu_{t+1}^{i}\kappa^T\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu}$ enters. This term captures the shadow value of liquid assets in relaxing the borrowing constraint through raising net tradable output. This effect is separate from the one above, as it is unrelated to the exchange rate, but reflects the fact that the additional output can be collateralized which directly relaxes the constraint. It is therefore understood by both planner and individual agents. Importantly, however, the values that planner and individuals attach to relaxing the constraint do not coincide in general. Understanding the relationship between the two, however, requires us to consider situations where the borrowing constraint is currently binding, i.e. $\mu_t^{SP} > 0$.[37] Rearranging the planner and competitive optimality conditions we get the following relationship between the two multipliers

$$\mu_t^{SP} = \mu_t + \frac{\mu_t^{SP}\Psi_t}{R} - \beta\mathbb{E}_t\{\mu_{t+1}^{SP}\Psi_{t+1}\}. \tag{21}$$

Since these are endogenous, general equilibrium objects we cannot show whether one or the other multiplier is larger in general. Nevertheless, this equation is very useful to develop intuition about the difference in how the planner and individual households respond to the presence of the constraint. In particular, both understand the benefit from relaxing the constraint and increasing consumption, captured by their respective μ_t^i, however, the planner makes two more considerations. First, she understands that relaxing the constraint and increasing consumption raises the real exchange rate, which in turn further relaxes the constraint. This effect is captured by the term $\frac{\mu_t^{SP}\Psi_t}{R}$. Moreover, the planner also understands that relaxing the constraint today increases the borrowing, which tightens the constraint tomorrow, as given by the term $\beta\mathbb{E}_t\{\mu_{t+1}^{SP}\Psi_{t+1}\}$. This precautionary motive lowers the planner's shadow value relative to the individual agent. While in general it is unclear which effect dominates, note that the negative term is discounted with β which is smaller than $\frac{1}{R}$, but is otherwise the same expression shifted by one period. On average, the planner thus values relaxing the constraint more than

[36] Note that the last two terms in the equation can be simplified to $\mu_{t+1}^{SP}\Psi_{t+1}\frac{z_{t+1}F_\nu(t+1)}{p_\nu}$. We keep them separate to conceptually distinguish the incentives at play.

[37] It is obvious from the two optimality conditions that $\mu_t > 0$ if $\mu_t^{SP} > 0$. See Appendix C for details.

the individual agent.

Furthermore μ_t^{SP} is much larger than μ_t in states where μ_t^{SP} is currently large because a binding collateral constraint forces the economy to deleverage which implies a slack borrowing constraint in the following period, i.e. $\mathbb{E}_t\{\mu_{t+1}^{SP}\}$ is close to 0. Especially in a crisis, the planner hence values relaxing the borrowing constraint clearly more than individual agents.

Returning to the optimality conditions for liquid asset holdings, the presence of the shadow values is ambiguous in general. However, in our numerical solution we find that the planner always chooses to hold (weakly) more liquid assets than agents in the competitive equilibrium in situations where the constraint does currently not bind. This indicates that the negative term is strictly dominated by the positive term discussed above. Finally, it is important to note here, that all three differences in the optimality conditions for liquid assets between private agents and the social planner are related to the value of μ_{t+1}^{SP}. In particular, if the economy is very far from the constraint so $\mu^i = 0$ and $\mathbb{E}_t\{\mu_{t+1}^i\} = 0$, the optimality conditions coincide. This shows that the motive for liquidity regulation in our model only arises because of the interaction of the liquidity and collateral constraints and not because of a new externality.

Ex-post interventions We close this section by analyzing the optimal choice of liquid assets in a situation where the collateral constraint currently binds,[38] i.e. $\mu_t^{SP} > 0$. This is interesting in our case, because the planner can still meaningfully affect the equilibrium, by choosing liquid assets and borrowing subject to the constraint. In Bianchi (2011) on the other hand, the equilibrium under a binding constraint is already fully determined and the competitive and planner choices coincide. In our model there is effectively only one inter-temporal choice when the constraint binds, since the choice of liquid assets also pins down borrowing. We therefore do not discuss borrowing separately.

In contrast to the first part of the section differences in incentives here do not reflect macroprudential interventions by the planner, but differences in how she responds to a crisis. The Euler equations for the liquidity choice at the constraint are given by

$$\begin{aligned} u_T(t) - \theta\mu_t &= \beta\mathbb{E}_t\big\{\lambda_{t+1} + \xi_{t+1}\big\}, \\ u_T(t) + \mu_t^{SP}\Psi_t - \theta\mu_t^{SP} &= \beta\mathbb{E}_t\big\{\lambda_{t+1}^{SP} + \xi_{t+1}^{SP}\big\}. \end{aligned}$$

For readability we have not plugged in the terms on the right hand side, but in principle all insights from above still apply here. However, in cases where the constraint strongly binds (μ_t^{SP} is large), the economy is forced to deleverage. This means that borrowing is

[38]Note that we have already discussed the differences in optimality conditions for borrowing in this case above, by relating the two Lagrange multipliers. We use this relation again below.

low, so the probability of a binding constraint tomorrow and therefore $\mathbb{E}_t\mu^i_{t+1}$ is small. As we just discussed, for small $\mathbb{E}_t\mu^i_{t+1}$ the right hand sides of the two equations become very close, so we can focus on differences on the left hand side.

We have pointed out above that in a crisis μ^{SP}_t is likely much larger than μ_t. This has two effects. Firstly, since a share θ of liquid assets can be collateralized, a higher multiplier on the borrowing constraint makes holding liquid assets more attractive to the planner. This effect pushes her towards borrowing more and accumulating more liquid assets. At the same time, however, the planner understands, that by selling liquid assets, she can increase current consumption of tradable goods which boosts the real exchange rate and thereby also increases borrowing capacity. We find this effect highly interesting, as it captures a form of inefficient liquidity hoarding: Individual agents do not understand that they can increase the aggregate borrowing capacity by reducing their accumulation of liquid assets. Note that this is more likely to occur in economies where the share of liquid assets that can be collateralized is low, as the opposing effect is smaller. Intuitively this occurs because the planner has to give up more consumption today for holding an extra marginal unit of liquidity, if its collateral value is low.

While our analytical results do not allow us to characterize one dominating effect, our numerical results show that both are possible. In our calibrated economy we find that in parts of the state space, where the constraint binds very strongly, the planner prefers to reduce liquid assets more drastically than individual agents. We now turn to a discussion of which instruments the planner can use to implement the constrained-efficient allocation.

4.2 Decentralizing the planner allocation

We first characterize optimal price based policy and show that, in order to restore constrained-efficiency, a policy maker needs to impose both, a state dependent tax on foreign debt and a state dependent subsidy on liquid assets.[39] We then continue by showing that in the states where the subsidy is positive, the planner can implement the constraint efficient allocation without paying out subsidies, by accumulating reserves at the central bank and committing to act as a lender of last resort in a liquidity crisis.

4.2.1 Price based policy tools

Assume that the policy maker imposes a state contingent tax on debt, τ^b_t, and a state contingent subsidy on liquid assets, τ^l_t, on debt and liquidity holdings chosen in period

[39] For simplicity we refer to the two instruments as tax and subsidy, even though the subsidy on liquid assets turns negative in some states, which effectively makes it a tax.

t. Details on the implementation of these taxes and a derivation of the equations below can be found in Appendix C.

With the tax and subsidy the price of bonds becomes $\frac{1}{R(1+\tau_t^b)}$ and the price of liquid assets becomes $\frac{1}{(1+\tau_t^l)}$ in the budget constraint of private agents. Note that τ_t^b is always and τ_t^l is (usually) positive. The reason why we consider τ_t^b a tax and τ_t^l a subsidy is that bond holdings are always negative in equilibrium, while liquid assets holdings are by definition positive. Net tax revenues are rebated lump-sum to the household. The Euler equation for debt holdings and liquid assets of private agents are given by

$$\begin{aligned} \frac{\lambda_t}{R(1+\tau_t^b)} - \mu_t &= \beta \mathbb{E}_t\{\lambda_{t+1}\}, && (22) \\ \frac{\lambda_t}{(1+\tau_t^l)} - \theta\mu_t - \psi_t &= \beta \mathbb{E}_t\{\lambda_{t+1} + \xi_{t+1}\}. && (23) \end{aligned}$$

The planner allocation can be implemented by setting the tax and subsidy such that the Euler equations of the private agent coincide with the planner's optimality conditions. This results is summarized in the following proposition.

Proposition 1. *The constrained-efficient allocation can be decentralized using a state contingent tax on debt* τ_t^b *and a state contingent subsidy on liquid assets* τ_t^l *set to satisfy*

$$\tau_t^b = \begin{cases} \frac{\mathbb{E}_t\{\mu_{t+1}^{SP}\Psi_{t+1}\}}{\mathbb{E}_t\{u_T(t+1)\}}, \quad > 0, & \text{if } \beta\mathbb{E}_t\{u_T(t+1) + \mu_{t+1}^{SP}\Psi_{t+1}\} > \frac{u_T(t)}{R}, \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

$$\tau_t^l = \frac{\beta\mathbb{E}_t\{[\kappa^T\mu_{t+1}^{SP} - \kappa^T\mu_{t+1}]\frac{F_\nu(t+1)-p_\nu}{p_\nu}\} + \mu_{t+1}^{SP}\Psi_{t+1}\frac{F_\nu(t+1)}{p_\nu} + \theta(\mu_t^{SP} - \mu_t) - \Psi_t\mu_t^{SP}}{\beta\mathbb{E}_t\{u_T(t+1) + \mu_{t+1}\kappa^T\frac{F_\nu(t+1)}{p_\nu}\} + \theta\mu_t}, \quad (25)$$

and tax revenues rebated as lump-sum transfers.

Note that the private Lagrange multipliers in the expressions above are different from the competitive ones as they arise from optimality conditions in an economy where the optimal policies are already implemented. Details can be found in Appendix C. To economize on notation we omit this distinction.

The optimal tax rate for foreign debt is identical to the expression in Bianchi (2011). This tax is weakly positive representing the uninternalized marginal cost of borrowing of private agents. Further, the tax rate increases as the probability of a binding collateral constraint increases and is zero if the probability of a binding collateral constraint is zero. Note that we follow a convention in the literature and set the tax rate to zero in cases where individual agents would like to borrow, but are prevented from doing so by the constraint. This is slightly arbitrary, but necessary, because in this case infinitely many combinations of τ^l and τ^b could implement the constrained-efficient allocation. Note that a positive tax in such a situation, would lower the multiplier μ_t, which would raise the optimal subsidy on liquidity. This is the case, because effectively there is only one inter-temporal choice, if borrowing and liquidity holdings are linked by the constraint, so the planner needs only one instrument.[40] We omit a discussion of the subsidy on liquid assets, as we have investigated the underlying incentives in the previous section. Note that exactly the same terms as discussed above enter the numerator of the expression.

4.2.2 Quantity based policy tools

We now show that as an alternative, the planner can implement the constraint efficient allocation without paying out subsidies, by accumulating reserves at the central bank and committing to act as a lender of last resort in a liquidity crisis. This exercise is interesting, not only because reserve accumulation is widely used in practice, but also because as the sovereign might not have the fiscal capacity to pay out subsidies during a crisis.

Under the the optimal policy private agents will not hold any liquid assets and the non-negativity constraint (15) binds during normal times . In states of severe crisis where the planner chooses less liquid assets than the competitive equilibrium, she needs to impose a tax on liquid assets to prevent liquidity hoarding. The optimal tax on borrowing is unaffected by these liquidity policies. We summarize these findings in the following proposition:

Proposition 2. *Alternatively, the constrained-efficient allocation can be decentralized using a state contingent tax on debt $\hat{\tau}_t^b$, a tax on liquid assets $\hat{\tau}_t^l$ and by accumulating liquid reserves $\hat{l}_{t+1}$ at the central bank:*

$$\hat{l}_{t+1} = l^*_{t+1} \tag{26}$$

where l^*_{t+1} denotes the socially optimal level of liquid assets and

$$\hat{\tau}_t^l = \begin{cases} 0, & if \psi_t > 0 \\ \tau_t^l & otherwise \end{cases} \tag{27}$$

And the optimal tax on debt is given by equation (24), i.e. $\hat{\tau}_t^b = \tau_t^b$ Tax revenues and proceeds from trading liquid assets are rebated as lump-sum transfers.

[40] In models where there is only one inter-temporal choice, like Bianchi (2011), the tax rate at the constraint doesn't affect the equilibrium allocation at all.

In all states where $\hat{\tau}_t^l$ is different form zero, it is strictly negative, since it is used to discourage liquidity hoarding. This means that no subsidies are paid out by the planner. However, private liquidity holdings are zero everywhere, so no tax is actually collected in equilibrium either. In fact $\hat{\tau}_t^l$ is not unique, as any larger tax would also achieve zero private liquidity holdings. The planner could also impose an infinite tax on liquid assets, which effectively corresponds to prohibiting purchases of foreign reserves. This could be achieved by suspending convertibility of the domestic currency.

While no subsidies are paid out, the planner still needs resources to accumulate liquid assets. We show in the quantitative analysis, that she strongly reduces liquid asset holdings during crises and only accumulates liquid assets during expansions. Therefore fiscal constraints should not impede the implementation of this policy. Finally, note that a wide range of alternative instruments can be used to implement the constrained-efficient allocation. For example a reserve requirement can replace the subsidy on liquid assets in states where it is positive. For brevity we omit the explicit formula here, as it adds little economic insight.

5 Quantitative Analysis

In this section, we describe the calibration of the model and evaluate implications regarding Sudden Stops, optimal policy and welfare. We solve the model globally using fixed point iteration on the conditional expectations.[41]

5.1 Calibration and functional forms

We chose the following standard functional forms:

$$u(c_t) = \frac{c_t^{1-\varepsilon} - 1}{1 - \varepsilon}, \tag{28}$$

$$c_t = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta}, \tag{29}$$

$$z_t F(\nu_t) = z_t \nu_t^{\gamma_\nu}. \tag{30}$$

Our calibration uses standard values from the literature and data for 16 economies classified as emerging markets and developing economies by the IMF's World Economic Outlook[42]. The data is taken from the updated version of the dataset provided by Lane and Milesi-Ferretti (2006) and from the Word Developement Indicators (WDI) One period

[41] We use a variation of the algorithm in Wolf (2019).

[42] We use data for the following countries: Argentina, Brazil, Chile, Croatia, Columbia, India, Indonesia, Mexico, Malaysia, Poland, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela.

in the model represents a year and the values assigned to the model parameters are summarized in Table 1. The risk aversion is set to 2 and the world interest rate to 4% which are standard values in the DSGE-SOE literature. The domestic discount factor is set to .92 to target the average net foreign asset position of -28% of GDP between 1990-2011.

Table 1: Calibration

Parameter	Value	Source/Target
Risk aversion	σ =2	Standard value DSGE-SOE
World interest rate	$R = 1.04$	Standard value DSGE-SOE
Discount factor	$\beta = .92$	NFA=-30%
Elasticity of substitution	$1/(1+\eta)$ =.83	Bianchi (2011)
Weight on tradables in CES	ω=.3	Share of tradable output = 30%
Share of pledgeable liquidity	$\theta = .95$	Reserve Outflows during Sudden Stops
Intermediate inputs share	$\gamma_\nu = .45$	Bianchi and Mendoza (2018) & imported inputs/GDP
Share of pledgeable output, tradable	$\kappa^T = .3$	Probability of Sudden Stop=5.4%
Share of pledgeable output, nontradable	$\kappa^N/\kappa^T = 1$	Bianchi (2011)
Probability of liquidity shock	$P_{gb} = 10\%$	International reserves/GDP = 14%
TFP Process	$\rho_z = .537, \sigma_s = .0134$	Mendoza (2010)

The rate of substitution between tradable and nontradable goods determines the sensitivity of price of nontradables, with respect to relative changes in consumption of the two goods. Empirical estimates for this parameter range from 0.4 to 0.83. Following Bianchi (2011) we choose a conservative value and set $1/(\eta+1) = .83$. The parameters ω determines the tradable share in the CES aggregator and is calibrated to match a 30% share of tradable output which is the average value for the set of countries from 1990-2011. In line with previous studies we define tradable output as the sum of primary and manufacturing goods.

The share of pledgeable liquid assets is calibrated to match the decline in foreign reserves during a Sudden Stop episode. As pointed out in section 3.1 the share of liquid assets that can serve as collateral determines how much of an extra unit of liquid assets has to be financed through reduced consumption. Effectively, θ therefore determines how strongly reserve holdings respond to changes in the marginal utility of tradable consumption. In a crisis tradable consumption is low and its marginal utility is high, which triggers a fall in reserves. To match the observed magnitude of decline in reserves during a Sudden Stop we set $\delta = .95$. A value close to one seems reasonable since liquid assets can be easily pledged as collateral. The production share of intermediate inputs γ_ν is set to .45 as in Bianchi and Mendoza (2018), who target the average share of total intermediate inputs to gross output for all OECD members. While it is not clear whether this target is relevant for emerging markets, setting $\gamma_\nu = .45$ generates an average ratio of imported intermediate inputs to GDP of 23.5%. This value is consistent with the data for our set of countries.[43]

The coefficient of the collateral constraint is set to $\kappa^T = .3$ to match the yearly observed frequency of Sudden Stops reported by Eichengreen, Gupta, and Mody (2008). In line with Calvo, Izquierdo, and Mejía (2004), Sudden Stops are defined as periods where capital inflows (net of changes in reserves) fall two standard deviations below their mean and output contracts. Following Bianchi (2011) we set $\kappa^T = \kappa^N$.

The probability of a liquidity shock is set to 10% which yields an international reserves to GDP ratio of 14% consistent with the cross-country average observed between 1990 and 2011. The transition probability matrix is symmetric which yields an average duration of a Sudden Stop slightly above one year.[44]

Finally, the stochastic process for tradable sector total factor productivity is specified as

$$log(z_t) = \rho log(z_{t-1}) + \epsilon_t, \tag{31}$$

where ϵ_t is an iid N(0,σ_z^2) shock. The parameters of this process are set to $\rho_z = 0.537$ and $\sigma_s = 0.0134$ which are the first autocorrelation and the standard deviation of aggregate total factor productivity reported by Mendoza (2010).

5.2 Sudden Stops

In this section, we compare a typical Sudden Stop generated by our model with the characteristics of actual Sudden Stops observed in the data. To this end, we simulate the model economy for 100.000 periods. We then use the same procedure to identify Sudden Stop events as Calvo, Izquierdo, and Mejía (2004) on the simulated paths. In particular Sudden Stops are defined as periods where the trade balance increases two standard deviation above its mean and output falls one standard deviation below trend. We then average across all identified Sudden Stop episodes.

Figure 2 contains the central quantitative findings of this paper. In particular, it compares a typical Sudden Stop in the model with the data counterpart. We show all non-stationary variables as log deviations from trend, so that magnitudes can be understood

[43] We compute this number based on data from the OECD Structural Analysis Databases. For details see Appendix B. If this number seems high, note that we include imported capital goods in imports to production, given that capital goods are not part of our model.

[44] This is in line with the estimated average duration of a Sudden Stop reported by Eichengreen, Gupta, and Mody (2008).

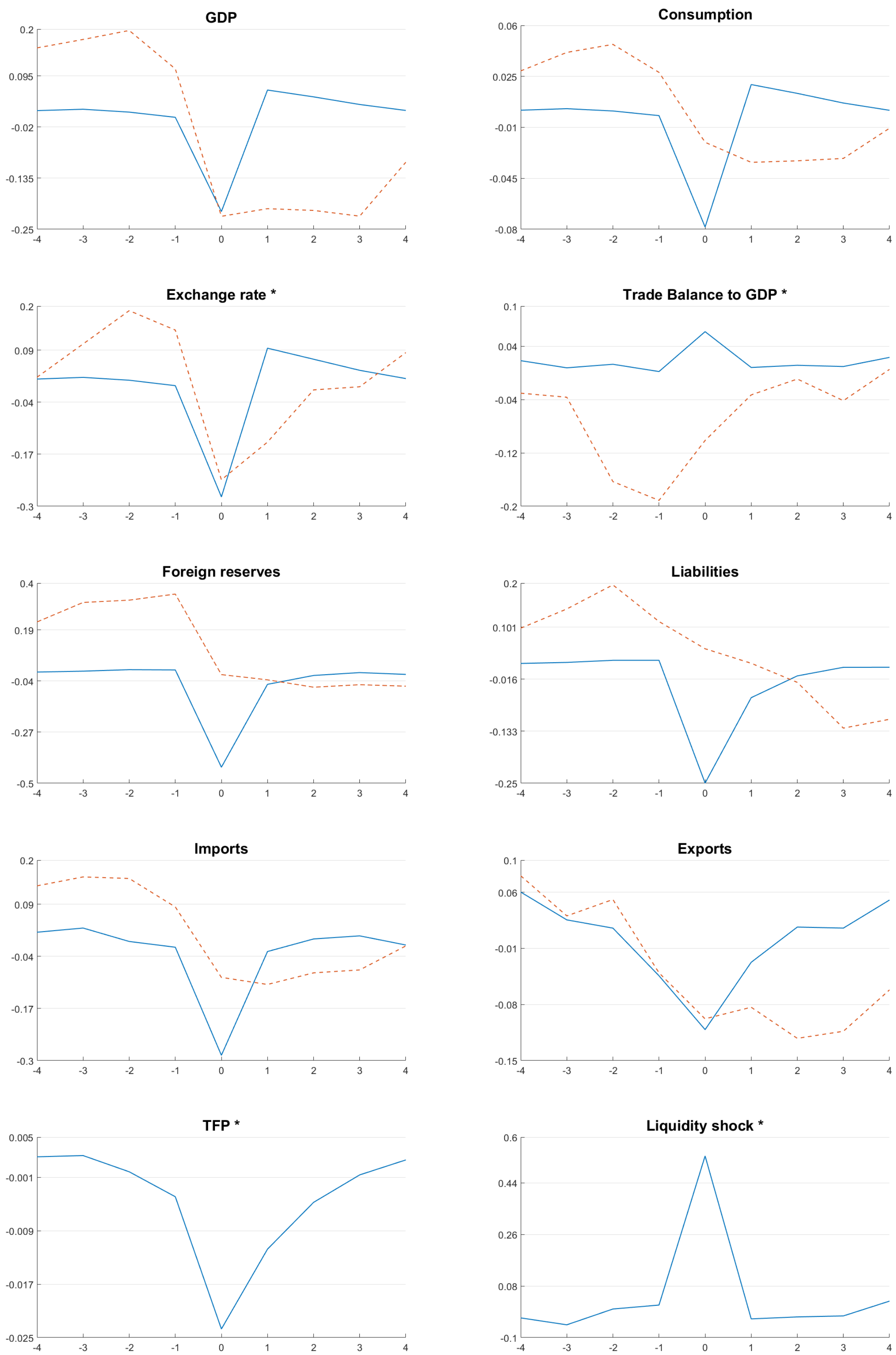
as relative deviations. The trade balance relative to GDP and the real exchange rate are already stationary and their magnitudes have natural interpretations. We therefore show them in terms of deviations from their long-run means.

Given the simplicity of our model, we find that it replicates Sudden Stop episodes surprisingly well across many dimensions. Firstly, the decline in foreign reserves was targeted and is matched well. The responses in consumption, GDP, imports and exports are all in similar magnitudes to the data. Finally, the model somewhat underpredicts the reversal in the trade balance and overpredicts the fall in gross liabilities. The lowest left panel shows that typically Sudden stops occur in the model when tradable sector productivity is around 2% below trend. Importantly, note that the model qualitatively accounts for all Sudden stop characteristics observed in the data. However, the model fails to account for the trends observed before Sudden Stops. In particular, Sudden stops occur in the data, when GDP, consumption, the exchange rate, foreign reserves and gross liabilities are above trend and the trade balance below its mean.

Note further that the model fails to match the persistent response of most variables following Sudden Stops observed in the data. This is a common feature of models that don't feature investment, since there is no endogenous mechanism that could generate persistence. Moreover, the liquidity shock in our model only takes two extreme values and has a symmetric transition matrix, so exogenous persistence could only arise through productivity. Further, the strong deleveraging process induced by the binding collateral constraint leads to an increase in the net foreign asset position. As a result, the exchange rate increases above its mean already in the period after the Sudden Stop which raises consumption and GDP in the model. In reality, other mechanisms, such as externalities in the production sector, default of domestic firms and declines in foreign direct investment are likely to contribute to the observed persistence of real variables. A model where access to international short-term markets follows a somewhat persistent AR process might also be able to get closer to the data in this dimension.

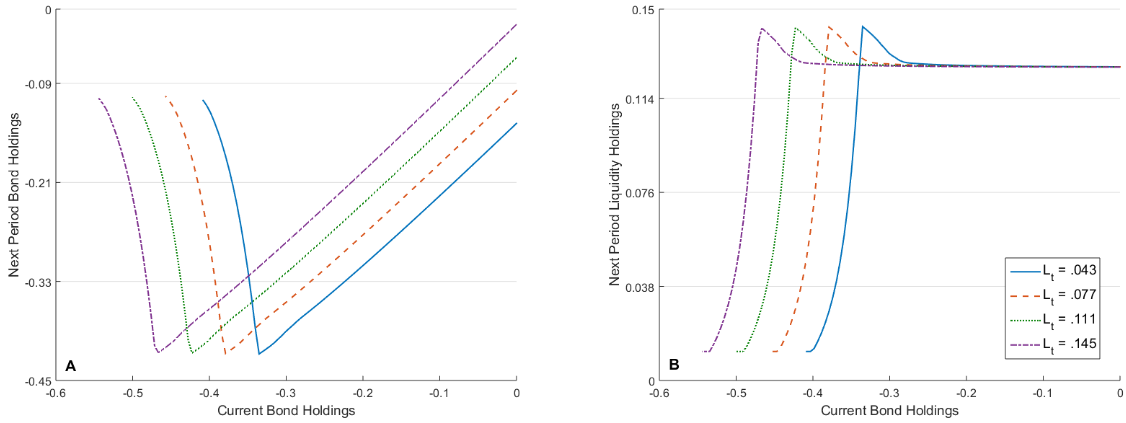
Finally, note that around half of the Sudden Stops in our model economy are caused by a liquidity shock. In section E of the Appendix we show a comparison between Sudden Stops caused by liquidity shocks and those purely caused by declining productivity. The contractions in output, the exchange rate and asset positions are very similar. Sudden stops that are purely due to declines in productivity, however, lead to negligible falls in gross goods flows. This shows that the inclusion of liquidity shocks is necessary for the model to match the dynamics in goods flows observed in the data.

Figure 2: Sudden Stops in the Model and the Data



Note: The blue solid line corresponds to the model simulations, the red dashed line to the data. Dates for Sudden stops are taken from Calvo, Izquierdo, and Mejía (2004). Accordingly in the model Sudden Stops are defined as periods where capital inflows (net of changes in reserves) fall two standard deviations below their mean and output contracts. Variables are shown in relative deviations from trend, * absolute deviations from mean

Figure 3: Competitive Equilibrium Decision Rules



Note: This figure plots the competitive decision functions for four different levels of current liquidity holdings as a function of current bond holdings when S = 1 and productivity is at its steady state level.

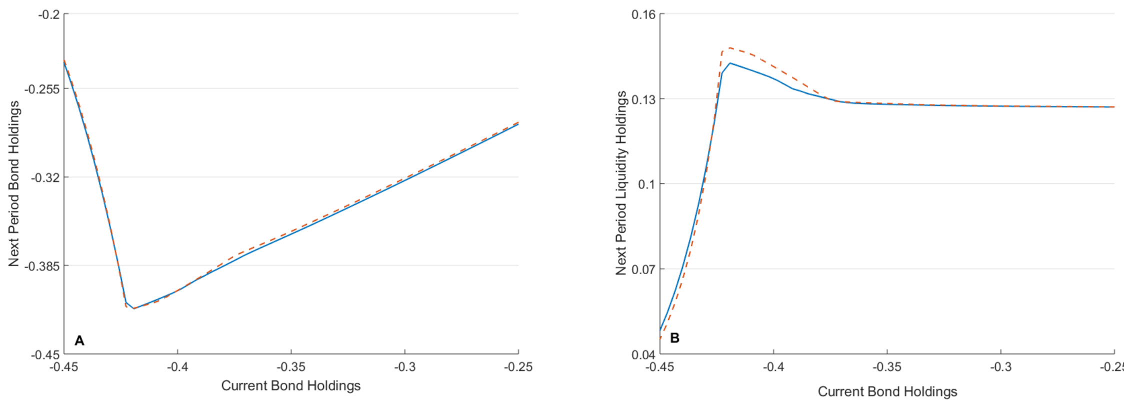
5.3 Borrowing and Liquidity Decisions

In this section, we first characterize the decentralized equilibrium and then show how the decision functions of private agent differ from those of the social planner. Panel A of Figure 3 shows next period bond holdings of private agents as a function of current bond holdings for four different levels of current liquidity holdings. The policy function for bond holdings is V-shaped which is a prominent feature of financial friction models including a Fisherian debt deflation mechanism. [45] In particular, future bond holdings are increasing in current holding in the unconstrained region and decreasing in the constrained region. The change in the slope of the function indicates the point where the collateral constraint starts to bind. Once the collateral constraint binds, higher borrowing levels lead to a fall in the real exchange rate and therefore the borrowing capacity. As a result, the higher current debt is, the lower is future borrowing capacity and the stronger is the forced deleveraging in this region. Higher current liquidity holdings increase the borrowing ability of the economy and shift the policy function to the left as they provide additional collateral.

More interestingly, Panel B of Figure 3 plots the liquidity decision rules for four different levels of current liquidity holdings as a function of current bond holdings. The policy function can be divided in three areas. The left region is the region where the collateral constraint is binding. In this region, the decision rules are sharply decreasing because agents prefer to consume their resources in states where the constraint becomes

[45]See Bianchi (2011)

Figure 4: Competitive Equilibrium vs. Constrained-Efficient Decision Rules



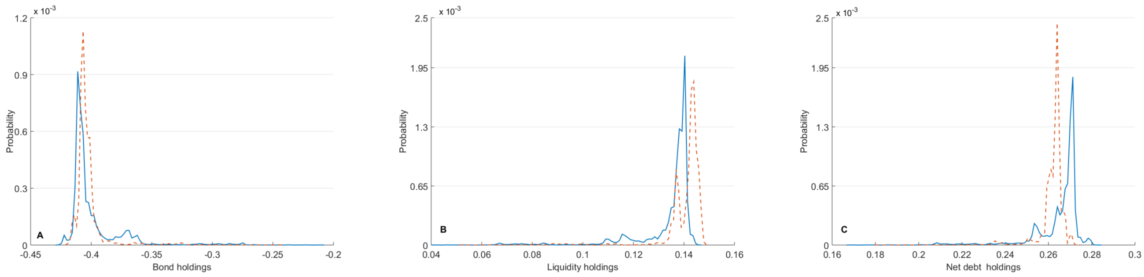
Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient equilibrium. Access to working capital is unrestricted and productivity and liquid assets are fixed at their steady state level.

very tight. In the center, the collateral constraint is nonbinding but there is a positive probability of a binding constraint in the following period. In that sense, the region is characterized by financial instability. The policy function is increasing in current debt holdings as agents understand that more debt today increases the probability of a binding collateral constraint tomorrow and therefore increase their precautionary liquidity holdings. Finally, the right region is the stable region where current bond holdings are so high such that the constraint is nonbinding today and the probability of hitting it in period $t+1$ is zero. In this region, current bond holdings do not affect liquidity decisions, however, agents still hold a positive amount of liquid assets due to the nonzero probability of a liquidity shock. As before, more liquidity holdings shift the policy functions to the left.

The way how the pecuniary externality distorts individual borrowing and liquidity decisions is illustrated in Figure 4 which compares the competitive bond and liquidity decision rules with the social planner's rules. First note that the differences in the bond policy functions are quantitatively small. Interestingly, however, the planner borrows slightly more compared to the unregulated allocation in the region characterized by a positive crises probability. This is in sharp contrast to previous findings which show that the planner reduces borrowing if financial stability is at risk (i.e. close to the constraint).[46] Here, the planner slightly increases gross borrowing, but more than offsets this by accumulating a larger liquidity buffer which becomes evident from Panel B. As

[46]See e.g. Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013), Bianchi and Mendoza (2018).

Figure 5: Ergodic Distributions of Bond and Liquidity Holdings



Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient equilibrium.

a result net debt holdings of the planner are thus strictly below the private optimum in the region characterized by financial instability. Once the constraint becomes binding, the social planner starts to reduce liquid asset holding faster then the private agents. For states where the constraint binds very strongly, we find the inefficient liquidity hoarding mentioned above, since private agents don't internalize the positive effect of extra consumption on the borrowing capacity. Finally, in the right region the probability of a binding collateral constraint goes to zero and the differences in decision rules vanish.

The differences among the two allocations becomes even more evident by comparing the ergodic distributions of bond and liquidity holdings among the unregulated and the constrained-efficient allocation in Figure 5. While the ergodic distribution of current bond holdings in the competitive equilibrium assigns a larger probability to very high levels of debt, on average, debt holdings are slightly below the constrained-efficient allocation (the mean of the competitive allocation is 0.3983 compared to 0.4012 in the constrained-efficient allocation). Further, the competitive equilibrium assigns more mass to very low debt levels, which occur due to forced deleveraging in severe Sudden Stops.

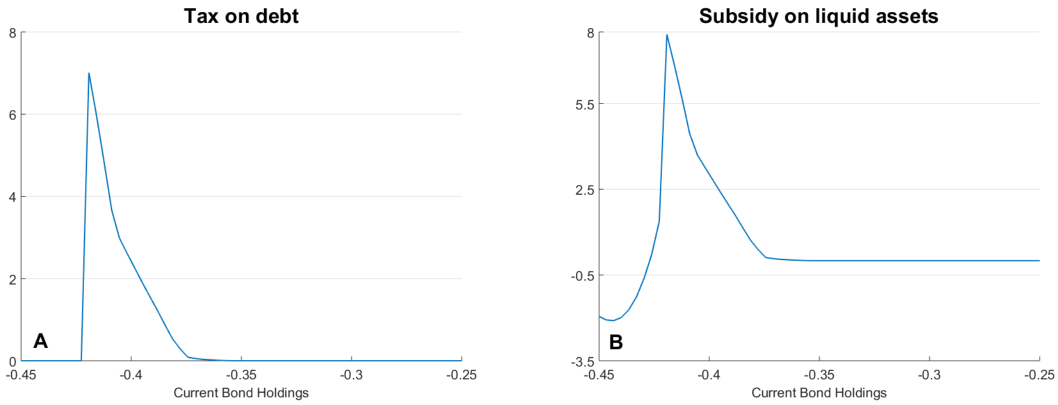
For liquid assets, the ergodic distribution assigns a higher mass to high liquidity holdings in the constrained-efficient allocation as the planner accumulates extra precautionary savings to insure against financial crises. Moreover, liquidity holdings of private agents drop to significantly lower levels compared to the social planner's choices. Finally, to quantify the precautionary differences among the two allocations, Panel C plots the ergodic distribution of net debt.[47] Indeed, the figure shows that the social planner engages in significantly more precautionary behavior in terms of net borrowing.

[47]Note that in contrast to the gross bond holdings, here a higher level corresponds to more borrowing.

5.4 Optimal Policy and Welfare Effects

Figure 6 illustrates the quantitative features of an optimal policy consisting of tax a on debt (Panel A) and a subsidy on liquid assets (Panel B), as a function of current borrowing for a fixed level of liquid assets. For sufficiently low levels of debt (right region), the probability of a binding collateral constraint is zero and the unregulated equilibrium coincides with the constrained-efficient allocation, so no taxes or subsidies are necessary. For higher debt levels the tax on debt increases with the level of debt and falls to zero when the collateral constraint becomes binding. Similarly, the subsidy on liquid assets is increasing in debt levels in the positive crisis probability region. When the collateral constraint binds strongly the subsidy on liquid assets turns negative, reflecting the fact that private agents hold inefficiently large stocks of liquid assets.

Figure 6: Optimal Policy Instruments



Note: The blue solid line corresponds to the tax on debt and the subsidy on liquid assets (in %).

The total welfare gain from implementing the optimal policy in our model is 0.032% of permanent consumption. The small magnitude of welfare gains is common in the literature. Following Bianchi (2011) we argue that this is due to the stylized nature of the model which abstract from many real-world phenomena. In particular, the externality does not distort the efficient use of production resources which could deliver significantly higher welfare gains. We therefore see these welfare gains as a lower bound.

5.5 Sudden Stops and Optimal Policy

In this section, we analyze the effect of optimal policy on the severity of Sudden Stops. As before we simulate the model economy for 100.000 periods and use the resulting data to construct an 8 year event window centered around the Sudden Stop event. Figure 7 shows typical Sudden Stops in the competitive economy compared to the planner economy.[48] We show foreign borrowing and liquid assets in levels to highlight the differences. All other variables are plotted in relative deviations from their long run means. The top two panels show that, compared to constrained efficiency, individuals underborrow in gross debt and underinvest in liquid foreign assets. Furthermore, the competitive economy experiences significantly more severe declines in both variables during periods of financial distress. In both economies, the decline in gross capital inflows exceeds the decline in gross capital outflows, which leads to a reversal of the net foreign asset position. The reversal is much weaker under constrained efficiency at approximately 2% compared to 7% in the competitive equilibrium.

The exchange rate, consumption and GDP also fall more sharply in the unregulated equilibrium with especially large differences in the exchange rate and consumption (the real exchange rate declines by 28% in the competitive and by 15% in the constrained-efficient allocation; consumption declines by 8% in the competitive and by 4% in the constrained-efficient allocation). Particularly interesting is the large difference in the real exchange rate which reflects the strength of the externality and is an important factor for the more pronounced decline in debt levels in the unregulated equilibrium. GDP falls by 23% in the competitive versus 14% in the constraint-efficient economy.

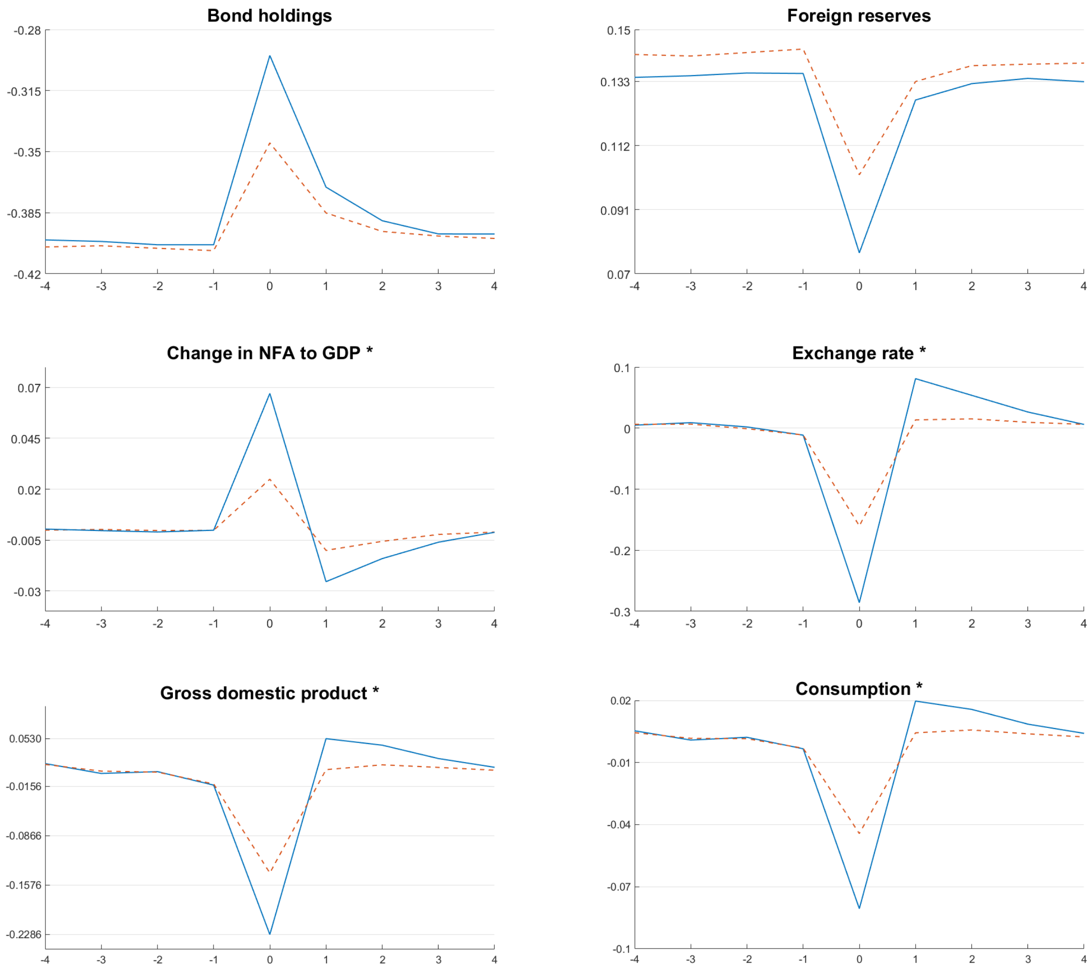
Finally, the macroprudential feature of optimal policy tools becomes evident from Figure 8 which plots the optimal tax on debt and subsidy on liquid assets around a Sudden Stops. Both policy rates increase by about 1% one period before the shock materializes and decline sharply when the crisis hits. Ex-post, both policy rates rise at a fast pace but remain slightly below their pre-crisis level.

5.6 Simple Policies

In practice, optimal policy might be challenging to implement as it requires two information-intense, state dependent policy tools. On the other hand, if macroprudential policy is limited to simple rules it is questionable if theses simple policies can induce welfare gains. In light of these concerns, we now investigate the welfare effects of simple interventions. Figure 9 contrasts the effects of a fixed tax on debt ranging from 0 to .9% with the effects of a fixed subsidy on liquid assets ranging from 0 to 1.2%. Evidently, a simple fixed tax on debt reduces welfare for any tax level. The maximum welfare gain of a fixed subsidy on liquid assets peaks at a subsidy of .7% and induces a welfare gain of 0.0038% which amounts to about 12% of the gain in the constrained-efficient allocation. The best simple policy combining both instruments is given by a .07% subsidy and a .03% tax which

[48] We simulate both economies for the same shocks, identify Sudden Stops in the competitive economy and then pick the same periods from the planner economy.

Figure 7: Competitive equilibrium vs. constrained efficiency



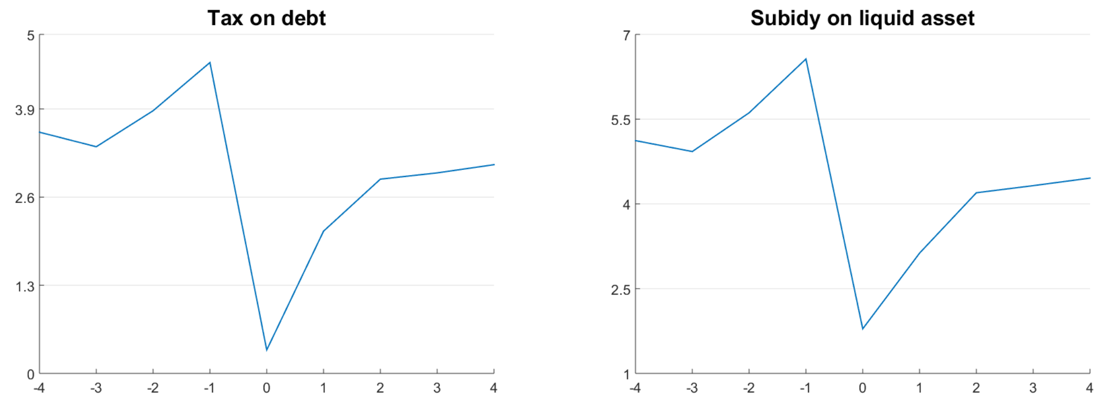
Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient allocation. In order to compare levels, bond holdings and foreign reserves are plotted in absolute values; * are plotted in relative deviations from the mean.

achieves a welfare gain of .00662% (21% of the welfare gain implemented by the optimal policy schedule.).

The fact that a even a small tax on debt reduces welfare compared to the unregulated allocation is in sharp contrast to previous findings.[49] In our framework, this effect is due to the interaction between debt and liquidity holdings. In particular, a fixed tax on debt $\bar{\tau}^b$ reduces individual incentives to invest in liquid assets which increases the costs of crises and induces an aggregate welfare loss. Figure 12 in Appendix F shows Sudden Stops caused by liquidity shocks in the competitive economy compared to an economy where a constant tax of .5% is levied on gross borrowing. With the tax, agents engage in

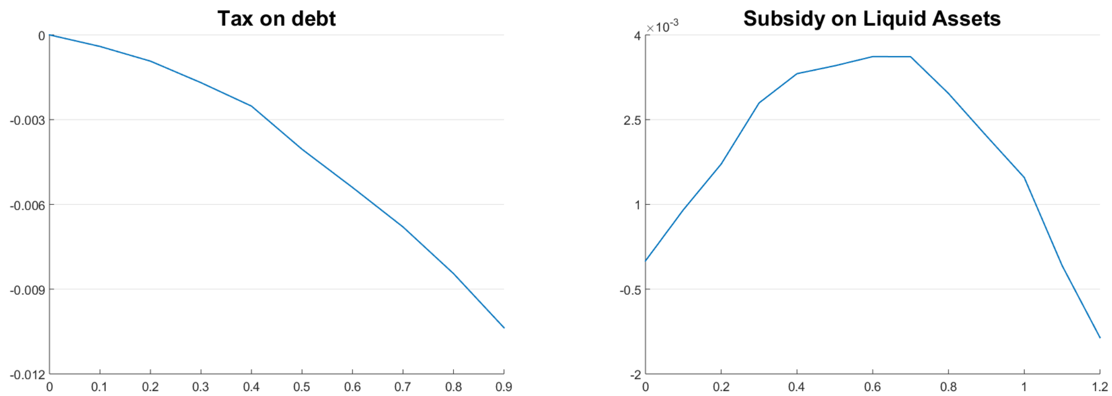
[49]See e.g. Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2018).

Figure 8: Optimal policy during Sudden Stops



Note: Optimal tax on debt and subsidy on liquid assets in % during Sudden Stops.

Figure 9: Welfare gain of simple policies



Note: This figure plots the welfare gain induced by simple policies compared to the unregulated equilibrium in %.

less net borrowing, however, the economy is still experiences worse liquidity crises, since liquid asset holdings are lower. The tax on debt therefore causes welfare losses in two dimensions: agents accumulate more savings, which is costly due to their impatience, but at the same time they experience worse crises.[50]

The intuition for the reduction in precautionary liquidity holdings becomes evident from the Euler conditions for debt and liquid assets of the equilibrium when the regulator

[50]Note that the differences in financial stability are small. The welfare loss therefore mainly comes from the reduction in net borrowing.

implements a fixed tax[51]:

$$\begin{aligned}\frac{\lambda_t}{R(1+\bar{\tau}^b)} &= \beta\mathbb{E}_t\{\lambda_{t+1}\}, &(32)\\ \lambda_t &= \beta\mathbb{E}_t\{\lambda_{t+1}+\xi_{t+1}\}. &(33)\end{aligned}$$

By increasing the cost of borrowing, a tax on debt drives a wedge between the marginal value of funds today and the marginal value of funds tomorrow in equation (32). In particular, agents now value funds today more as λ_t has to increase relative to λ_{t+1}. This in turn increases the costs of investing in liquid assets, as can be seen in equation (33). The only way how this condition can still hold is through an increase in ξ_{t+1}, which means the liquidity constraint is expected to bind more strongly. This can only occur, if liquid asset holdings are reduced. Importantly, this effect is present for any policy intervention targeting gross debt (e.g. quantity restriction on gross capital inflows) due to the resulting wedge in the optimality conditions for borrowing.

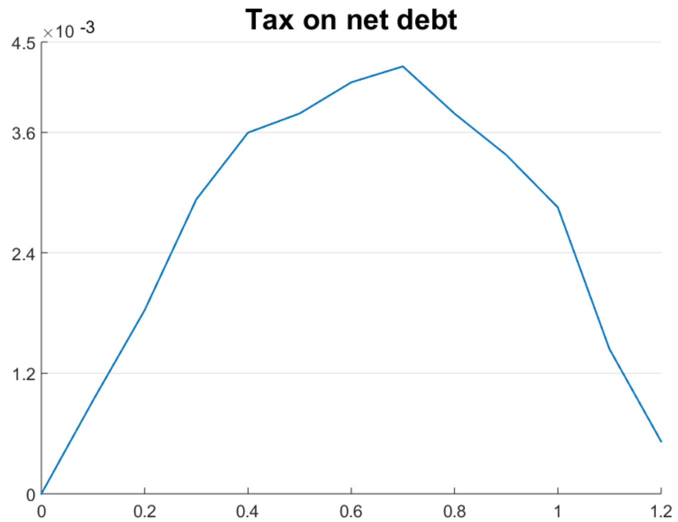
This adverse effect can be offset in different ways. A natural example is to implement a tax $\tilde{\tau}^b$ on net borrowing (b + l), rather than gross borrowing. This yields the following set of inter-temporal optimality conditions:

$$\begin{aligned}\frac{\lambda_t}{R(1+\tilde{\tau}^b)} - \mu_t &= \beta\mathbb{E}_t\{\lambda_{t+1}\}, &(34)\\ \lambda_t\frac{1+(R-1)(1+\tilde{\tau}^b)}{(1+\tilde{\tau}^b)R} - \theta\mu_t - \psi_t &= \beta\mathbb{E}_t\{\lambda_{t+1}+\xi_{t+1}\}. &(35)\end{aligned}$$

Note that the optimality condition for borrowing is unaffected by this change, while the optimality condition for liquid asset now contains a term that lowers the cost of holding liquid assets. This is because, agents understand that borrowing used to finance holdings of liquid assets is essentially tax free. Figure 10 shows the welfare gain associated with such a policy. In contrast to the fixed tax on gross debt, a fixed tax on net debt increases welfare for any tax level below 1.2%. The optimal fixed tax on net debt is given by $\tilde{\tau}^b = .7\%$ and induces a welfare gain of .0043% (13.5% of the welfare gain of the optimal policy).

[51] We consider a situation where the constraint currently doesn't bind

Figure 10: Welfare gain of a tax on net debt



Note: This figure plots the welfare gain induced by a fixed tax on net debt compared to the unregulated equilibrium in %.

6 Conclusions

In the current analysis, we have provided a small open economy model that endogenously generates international borrowing and liquidity holdings, but remains close to a workhorse framework studied in Bianchi (2011). While our model environment remains stylized in many dimensions, it can quantitatively account for the observed behavior of gross goods and capital flows around sudden stops. Furthermore we obtain highly tractable and intuitive results regarding social inefficiencies and optimal policy. In particular, our results show that due to the presence of a pecuniary externality, individual agents overborrow in net debt and underinvest in liquid assets compared to the social optimum. Consequently, a macroprudential regulator needs to intervene in both private borrowing and liquid asset holdings in order to restore constrained efficiency. An optimal policy mix is a combination of macroprudential capital controls and reserve accumulation, combined with suspension of convertibility in severe crises. Importantly, a tax on debt in isolation is detrimental to welfare in our framework, as it reduces agents' incentives to invest in liquid assets and makes the economy more vulnerable to liquidity shocks. We find this result highly relevant for the design of regulatory policy as policies exclusively aiming at gross debt holdings distort incentives to accumulate liquid assets an can thereby increase financial instability.

There are many promising avenues for future research, building on the analysis provided here. Since we have provided a real model, there is no role for liquid reserves in stabilizing the nominal exchange rate. We see explicit exchange rate management by the central bank as an integral part of a theory that aims to fully understand the accumulation of foreign reserves and crises in emerging markets. Another interesting approach would be to model the liquidity dry up in short-term markets endogenously, driven by suppliers expectation of a depreciation and disruption in domestic production. Such a model could endogenize the drop in collateral value of domestic production and thereby address some of the concerns regarding the quantitative realism of the mechanisms proposed here.

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A Microfoundation of the Borrowing Constraints

In this section we show how the constraints 4 and 5 can arise as incentive compatibility constraints in the contracting problem between domestic borrowers and international investors. In principle the contracting friction arises from limited liability coupled with the possibility for borrowers to divert firm output at some cost so that it cannot be observed by creditors. Similar microfoundations are widely used for these types of constraints, see for example Bianchi and Mendoza (2018).

Due to the different debt maturities in our framework, the microfoundation first requires precise definitions of the diversion technology, of the timing of events within a period and of what contracts are possible. While a number of quite technical assumptions are necessary to achieve the result, we nevertheless find the insights of this section interesting. Note that for this microfoundation it is necessary that liquid assets are held by private agents and not the central bank.

Technology and Timing Each household is endowed with tradable h^T and non-tradable labor h^N, both normalized to one. Providing labor causes no disutility to the household so in equilibrium the household always works one unit in each sector. The household can operate two technologies, where non-tradable output is produced according to $Y_t^N = z^N h^N$ and tradable output is produced according to $Y_t^T = z_t (h^T)^{1-\gamma_\nu} \nu_\nu^\gamma$. The household runs separate limited liability firms in each sector, where output is produced. Each household only works in firms it owns, so no wages are paid. These firms hold liquid assets and borrow internationally on behalf of the household. New firms can be opened in the beginning of a period and one household can operate multiple firms in the same sector within a given period. Without loss of generality, we assume that the household owns only one firm in each sector at the beginning of period t. The firm enters the period with inter-temporal debt and a stock of liquid assets. We divide the period into seven sub-periods:

1. The tradable goods firm needs to purchase intermediate inputs to production. At this point it can fund them by taking on trade credit.

2. Before entering production, this firm has an opportunity to divert the purchased inputs at a cost $\kappa_t^B z_t F(\nu_t)$, in terms of the share of potential output. In case of diversion inputs are used in a newly created firm, with no debt outstanding. Within a period, inputs can only be diverted at a very large cost, i.e. $\theta^B = 1$[52]. Diverting firms again face the problem in sub-period 1 and have access to trade credit. In principle resources can be diverted infinitely often within a period, but in equilibrium no diversion ever occurs.

3. Firms in both sectors enter production. Outstanding inter-temporal debt matures and is repaid, defaulting firms are immediately shut down and generate no more cash flows to their owner. Firms can issue new inter-temporal debt to finance repayments and purchase (or sell) liquid assets.[53] Creditors can observe labor and intermediate inputs.

4. Trade-credit matures. At this point firms can already produce enough output to make the payment[54]. A firm that fails to repay trade credit is immediately shut down and has no more opportunity to divert funds.

5. Firms in the tradable sector again decide how many inputs to divert at proportional cost $\kappa^T / R z_t F(\nu_t)$[55] and in the non-tradable sector at cost $\kappa^N Y^N$ of potential firm output. Liquid assets can be diverted at cost θ / R close to 1[56].

6. Diverted funds are transferred to newly set up firms, with no debt outstanding. They face the same problem as a firm in sub-period 3.[57]

[52]For technical reasons it is necessary to assume that in case of diversion the household still retains the asset, but pays an equal cost for diverting it.

[53]Any increases in the net asset position of non defaulting firms at this point have to be financed through equity injections. This issue generally arises with collateral constraints of this type. Technically it is possible to assume that households have enough private liquid funds available to do so. These funds have to be understood as some (small) endowment, which becomes available only in this sub-period. We don't explicitly include these funds in the model however.

[54]Clearly this is preferable to injecting more equity for firms which plan to divert funds, as it lowers the total amount of diversion costs. Firms which plan not to divert are indifferent between using equity or output to repay trade credit.

[55]κ^T and κ^B differ in our setting, since they occur at different stages of production. The first opportunity to divert occurs before production has even started. Generally we would assume that diversion is easier at this point. Dividing by R is just a normalization, as inter-temporal borrowing is in already in units of repayment next period.

[56]The motivation for this assumption is that trade credit is very short-term, which makes even more difficult to divert liquid assets, so $\theta^B = 1$. During production more time passes, which gives firms some opportunity to divert those assets, however, it remains costly, so θ is close to 1.

[57]As above, we assume that the diversion costs don't diminish the quantity of inputs and liquid assets, but accrue as monetary costs to the household that owns the firm

7. If no diversion occurs output is produced and publicly observed.

As will become clear below, as situation, where κ_t^B falls to 0, provides a microfoundation for the liquidity shock.

Contracts We assume that all debt contracts written between domestic borrowers and international lenders have the following features:

1. There is limited liability: if a firm output cannot repay its debt, the owners' private assets cannot be seized by creditors

2. However, if a firm produces enough *oberservable* output to repay the debt in full after the debt contract has been signed, but pays out dividends to its owners and defaults, then this default is considered fraudulent and private assets can be seized and the owners criminally prosecuted. We assume that, if found guilty of fraud, the household receives $-\infty$ utility.

Incentive Compatibility We first analyze a situation where $\kappa_t^B = 1$. In this case no diversion of inputs can be profitable to the household in sub-period 2 and we can focus on the inter-temporal contract. We now consider the decision to divert funds in sub-period 5. We follow Bianchi and Mendoza (2018) and define the continuation values of a diverting $W^d(b, l, tc)$ and a non-diverting firm $W(b, l, tc)$, where b is the level of intertemporal borrowing, l liquid assets and tc outstanding trade credit. Assume that a diverting firm faces the following problem[58]:

$$W^d(b', l', tc) = max_{\{d,b'',l''\}}\tilde{d} + \beta \mathbb{E} V(b'', l'') \tag{36}$$

s.t.

$$\tilde{d} = (F(\nu) - tc)(1 - \kappa^T/R) + (1 - \theta/R)l' + b''/R - (l'' + l') \tag{37}$$

$$b'' \geq -\kappa^T (z_t F(\nu) - tc) - \theta l'' \tag{38}$$

here we have used the fact that, if the firm chooses to divert funds, it will first produce enough to repay trade-credit to avoid immediate liquidation. The owners will therefore divert at most net tradable output. The value in the no default case is given by[59]

$$W(b', l', tc) = max_d d + \beta \mathbb{E} V(b', l') \tag{39}$$

[58] At this point the borrowing constraint is still a guess. We verify that it is consistent below.

[59] Since we only compare intra-period cash flows, weighing by marginal utilities is unnecessary.

s.t.

$$d = (F(\nu) - tc) \tag{40}$$

I.e. dividends are simply given by the output minus repayment of trade credit. We now guess that optimal choices of a firm after diversion are the same as choices before diversion, so $b'' = b'$ and $l'' = l'$. This implies that the two continuation values are equal. Taking the difference between W^d and W under this assumption implies that the firm will chose not to divert funds if $\tilde{d} \leq d$, i.e.:

$$b' \geq -\kappa^T (z_t F(\nu) - tc) - \theta l'. \tag{41}$$

Which is the same constraint faced by the firm after diversion. Given that value is linear in d and under the constraints $b'' = b'$ and $l'' = l'$ is feasible both before and after diversion, the choices will coincide and our guesses are verified. Once the firm produces output in period t, this is observed by creditors. This means any later default on this debt will be considered fraud. Owners therefore always choose to repay their inter-temporal debt in sub-period 3. This establishes that in sub-period 3 creditors will impose the constraint 41. Note that in sub-period 1 ν can only be purchased using trade credit so $tc = \nu$ in equilibrium[60].The case of a non-tradable firm follows analogously, with the only difference that non-tradable output is weighted by the real exchange rate. Aggregating the two constraints at household level delivers exactly the constraint 4 in the main text.

Next we show that the liquidity constraint 5 is imposed of firms in sub-period 2. A firm that diverts inputs in sub-period 3 transfers all resources it can extract to the household, who sets up a new firm and again has the chance to obtain trade credit. The diverting firm will still raise enough equity[61] to repay its outstanding inter-temporal debt to avoid fraud charges, but then cease operations and default on its trade credit in full. We therefore ignore inter-temporal debt here. Consider a firm, which holds liquid assets l and has raised trade credit to purchase inputs ν. If it defaults diverts its inputs, the firm can again raise trade credit to purchase ν' and continue its normal operations. This firm receives cash flow ν' minus diversion costs of old inputs

$$\nu' - \kappa_t^B z_t F(\nu) - \theta^B l, \tag{42}$$

while a non diverting firm generates no cash flows in this sub period. In the same way as the inter-temporal collateral constraint, the choices ν' and ν coincide and continuation

[60]We assume that suppliers cannot act as intermediaries therefore won't lend more than the value of goods that are sold. In any case the firm has no other use for the funds at this point.

[61]Note that production doesn't happen in time to repay inter-temporal debts, so it makes no sense for the firm to leave productive resources in the firm.

values are exactly the same. The firm will therefore divert whenever it can generate a positive cash flow. This yields the constraint:

$$\nu \leq \kappa_t^B z_t F(\nu) + \theta^B l, \tag{43}$$

We assume that in normal times $\kappa^B \geq \nu^* / [z_t F(\nu^*)]$, so no diversion is never profitable and the constraint never binds. In case of a liquidity shock $\kappa^B = 0$, so only liquid assets can serve as collateral. These facts deliver the constraint 5 in the main text. Note that this microfounds the liquidity shock as a situation where domestic assets lose their function as collateral for trade credit. Furthermore this microfoundation also allows for the stochastic process for κ_t^B to take values between 0 and 1 in which case domestic output and the liquidity constraint will be linked endogenously.

B Average share of imported intermediate inputs in the data

We compute the share of imported intermediate inputs in GDP as follows: The average ratio of total imports to GDP in our main dataset is 27.1%. From the OECD Structural Analysis Databases[62], we obtain data on average shares of intermediate inputs (56%), consumption goods (12%), capital goods (15%) and mixed use goods(16%) in total imports for our set of countries from 2005 to 2018. For Venezuela data is missing after 2013 and Thailand has a missing observation in 2017. Since capital goods are not part of our model, we attribute them fully to intermediate inputs to production. We then assume that mixed use goods are used according to the same shares as the rest of the imports. This yields a ratio of imported intermediate goods to total imports of 85.5%. Combining this with the ratio of imports to GDP yields a ratio of imported imports to GDP of 23.5%.

C Competitive Equilibrium with Taxation

We first derive the optimality conditions and optimal tax rates in section 4.2 in the main text and then provide a discussion of how expectations of private Lagrange multipliers in section 4.1 can consistently understood as arising from an economy, where optimal policies are implemented starting in period t+1.

We assume that taxes are introduced in some period t and are expected to remain forever. Private agents therefore expect to be on the constrained efficient equilibrium

[62] http://www.oecd.org/industry/ind/stanstructuralanalysisdatabase.htm

path from now on. We therefore look for a set of taxes, such that the private first order conditions for borrowing and liquid asset holdings are satisfied on the constrained efficient equilibrium path. Note that future taxes affect expected Lagrange multipliers and expectations are taken accordingly.

Consider a set of (state-dependent) taxes τ_t^b and τ_t^l on end of period bond and liquid asset holdings respectively. Bonds are taxed before interest, i.e. the tax applies to $\frac{b}{R}$. Taxes are implemented such that an agent, who holds 1 unit of a particular asset receives a transfer of $\frac{\tau}{1-\tau}$ (This formulation allows us to write taxes like interest rates, see below.). If holdings of the asset are positive, this transfer is positive and the tax becomes a subsidy, if holdings are negative a tax has to be paid. We find this formulation analytically most convenient, note however that these assumptions are without loss of generality.

The private budget constraint, given the taxes becomes:

$$c_t^N p_t^N + c_t^T + \frac{l_{t+1}}{(1+\tau_t^l)} + \frac{b_{t+1}}{R(1+\tau_t^b)} = [z_t F(\nu_t) - p_\nu \nu_t] + p_t^N y_t^N + l_t + b_t + T_t, \qquad (44)$$

where T_t are lump-sum rebates of the tax revenues. No other equations in the private agents' optimization problem are affected, so equations (22) and (23) in the main text follow immediately. The positive part of the tax in equation (24) then follows from equation (18) by (22) where we have used the fact that $\mu_t = \mu_t^{sp} = 0$ and $\lambda_t = \lambda_t^{SP} = u_T(t)$. In cases where the inequality in (24), is not satisfied, the planner chooses to borrow up to the constraint. From equations 9 and 23 it is obvious that in these cases the household LM μ_t is also positive, so no tax is necessary to implement the optimal borrowing level. As said in the main text, however, any tax or subsidy (as long as the household still wants to borrow up to the constraint) implements the constrained efficient borrowing level here. Importantly another choice of the tax rate still affect the level of the multiplier μ_t.

Equation (25) requires more discussion. Note that private expected future LMS shows up in this equation. As just said, however, the private LMs depend on the tax rate on borrowing in states where the constraint binds. The subsidy on liquid assets is therefore not unique, even in states where no constraint currently binds, but depends on future expected taxes.[63] Given a state dependent tax on debt, the multipliers can be computed for every possible state and deliver a unique tax/subsidy on the liquid asset. As above this tax can be found by using equations (20), (17) and (8), (6) and combining equation (19), with (23). Importantly this doesn't imply that the planner can implement the constrained efficient allocation with only one instrument by using the degree of freedom

[63] In this context one might be worried about time consistency of the Ramsey problem. Note, however, that Ramsey planner can implement the constrained efficient solution, so the private inter-temporal optimality conditions are not binding constraints for her and here problem must be time consistent.

once the constraint binds to manipulate the expected private LMs in such a way that (23) holds for $\tau^l = 0$. We conjecture that this is generally impossible, since setting $\tau^l = 0$ pins down τ^b at the constraint to satisfy current the Euler equation for liquid assets. We don't consider verifying this conjecture an interesting exercise, since designing and communicating such a policy would require excessive sophistication.

C.1 Competitive Equilibrium with Reserve Accumulation

If the planner replaces the positive parts of the liquidity subsidy by accumulating reserves at the central bank, the budget constraint of the representative agent becomes:

$$c_t^N p_t^N + c_t^T + \frac{l_{t+1}}{(1+\tau_t^l)} + \hat{l}_{t+1} + \frac{b_{t+1}}{R(1+\tau_t^b)} = [z_t F(\nu_t) - p_\nu \nu_t] + p_t^N y_t^N + l_t + \hat{l}_t + b_t + T_t, \quad (45)$$

The only difference to the budget constraint above is the presence of reserves $\hat{l}$ held by the central bank. These reserves are not chosen by the household itself, but they can serve as collateral for inter-temporal borrowing and will be made available for to households in case of a liquidity shock. In both the collateral and liquidity constraint the relevant amount of liquid assets is therefore $l_{t+1} + \hat{l}_{t+1}$. The presence of central bank reserves reduces private incentives to hold liquid assets. If $\hat{l}_t$ is larger than the competitive choice, households will choose to hold no liquid assets privately. Using similar steps as above, the equations in proposition 2 arise.

C.2 Private LMs in the constrained efficient equilibrium

We now turn to the interpretation of private LMs in section 4.1. To allow comparisons, we evaluate all choices on the constrained efficient equilibrium path throughout that section. Since private agents would make different choices, it is therefore unclear how to interpret their optimality conditions if all decisions are made by the planner. To allow a consistent interpretation, we assume the following:

As long as the inter-temporal borrowing constraint doesn't bind, households are forced to comply with the planners' choices[64]. In these cases we can interpret the (current) residuals in the private Euler equations as household incentives to deviate from constrained efficiency.

Unfortunately we cannot compute meaningful private Lagrange Multipliers μ_t on the inter-temporal borrowing constraint using this approach. If the household is forced to choose the same allocation as the planner, the borrowing constraint doesn't matter to its decisions. This multiplier (and its expected value), however, are central to understanding

[64] Even though we omit the associated Lagrange Multipliers for readability.

the incentives for the liquid asset choice. We therefore assume that, whenever the planner is borrowing constrained, households are allowed to choose borrowing themselves, but liquid assets are still determined by the planner. Due to the presence of the constraint, households choose the same allocation as the planner, but we can compute a Lagrange multiplier on the borrowing constraint μ_t. Under this assumption we can also analyze the differences in incentives for the liquid asset choice, both in the unconstrained and in the constrained case.

It might not be obvious at first sight that, given this set of assumptions, our discussion reflects economically interesting differences in private and social incentives. To alleviate this concern note that it is equally possible to consistently interpret the expectations over μ_{t+1} as private Lagrange Multipliers on the borrowing constraint, assuming that from period t+1 onwards the optimal tax and subsidy defined in equations 24 and 25 are implemented. Given these policies the household consistently expects the constrained efficient equilibrium to be implemented in the future. Moreover, whenever the borrowing constraint actually binds, the tax is set to zero so the multiplier reflects the true shadow value of violating the constraint.

D Computational Solution

Even though our equilibrium conditions are relatively compact, the model has several features which make the computational solution challenging. In particular there are two endogenous states and two occasionally binding constraints. Moreover the model features an amplification mechanism in the price of non-tradables, which introduces a strong non-linearity in the region where the borrowing constraint binds.

We solve both for both competitive and constrained using policy function iteration, using a variant of the algorithm in Wolf (2019). In particular we guess the expectations in the Euler equations 9 and 10 on grids for endogenous states. On the grid we then find the policies that satisfy Euler equations given expectations, which we linearly interpolate between grid points, since optimal choices generally don't lie on the grid. We then update expectations as a convex combination between the previous guess and the value computed from the new policies. We do so by computing equilibria for all possible combinations for binding and non-binding constraints and then choose the equilibrium that satisfies all constraints.

We approximate the AR process for productivity in the tradable sector by a first order Markov process on a 5 point grid using the common Tauchen algorithm. We then choose grids for the endogenous states using 400 points for borrowing and 400 points for liquid asset holdings. Combined with the two states for the liquidity shock, we solve for the policies on 1600 000 grid points in each step. We iterate on the Euler equations until maximum absolute differences in expectations on the grid between iterations are smaller than 10^{-6} in the relevant part of the state space.

We check accuracy by doubling the number of grid points for the endogenous states (the grid becomes 800*800*2*5) and computing differences between the two approximations on the smaller grid. The maximal absolute differences in consumption between the two solutions is smaller than 0.01% while the mean difference is smaller than 0.001%. For the positive analysis of the model, the approximation seems clearly sufficient. Since the welfare gain of the optimal policy is only 0.032% of steady state consumption, one could nevertheless be worried about the accuracy of our normative analysis. Note that by definition the welfare gain of implementing the optimal policy must be weakly positive in the whole state space if the solutions are accurate. As a further robustness check we therefore compare the value function of the competitive and planner solutions on the whole grid. We find that the minimum welfare gain is strictly positive at 0.02% of consumption. We conclude that our solutions are accurate enough for the normative analysis as well.

D.1 Details on the algorithm

The algorithm is complicated by the fact that liquid assets and bonds are perfect substitutes from the point of view of current consumption, as long as the borrowing constraint is not currently binding. This can be seen by the fact that equations 9 and 10 have the same left hand sides in states where no constraint binds. It is therefore not generally possible to satisfy both equations, for a given set of expectations, which makes it impossible to solve for an equilibrium. Note however, that in case of a liquidity shock ν_{t+1} is predetermined by l_t. We use this fact to pull F_ν out of the expectation in the Euler equation for liquid assets, which allows us to solve directly for liquid asset holdings, given expectations.

In some regimes solving for the intra-temporal equilibrium given expectations involves a non-linear system of equations. Solving this system in every iteration would make computation infeasibly slow. We therefore solve the non-linear system in advance on a fine grid, and linearly interpolate in the inner loop. This grid is chosen such that maximal residuals between points are smaller than 10^{-5}.

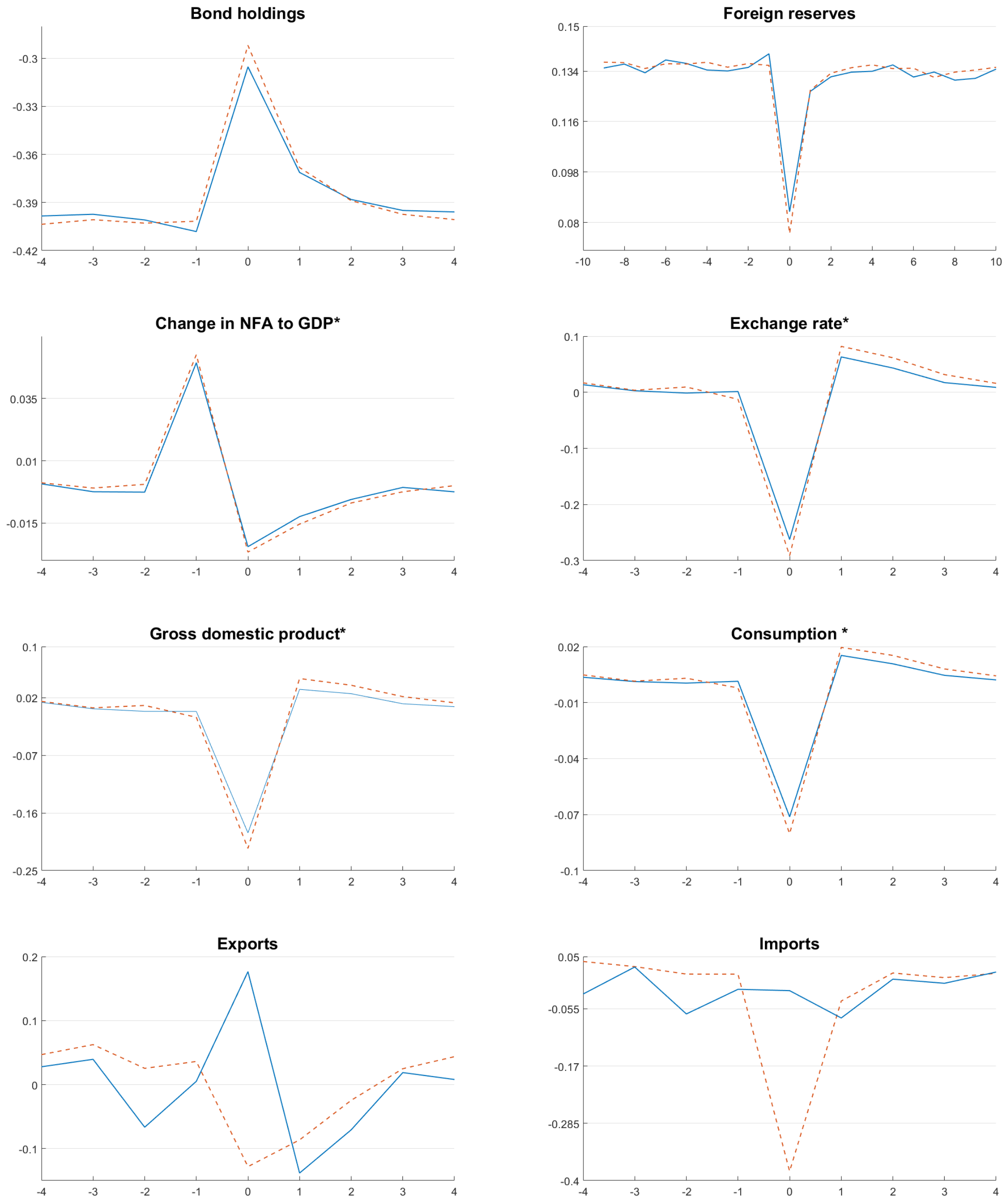
E Sudden stops with and without liquidity shock

Figure 11 compares sudden stops that coincide with a liquidity shock with sudden stops which are purely caused by a decline in productivity. This second type of sudden stops corresponds to the sudden stop events in Bianchi (2011). As can clearly be seen in the figure, imports hardly decline and exports even increase if there is no liquidity shock, as optimal levels of intermediate inputs can still be financed. Liquidity shocks are therefore necessary for the model to generate the observed collapse in gross good flows.

F Sudden stops with a fixed tax on debt

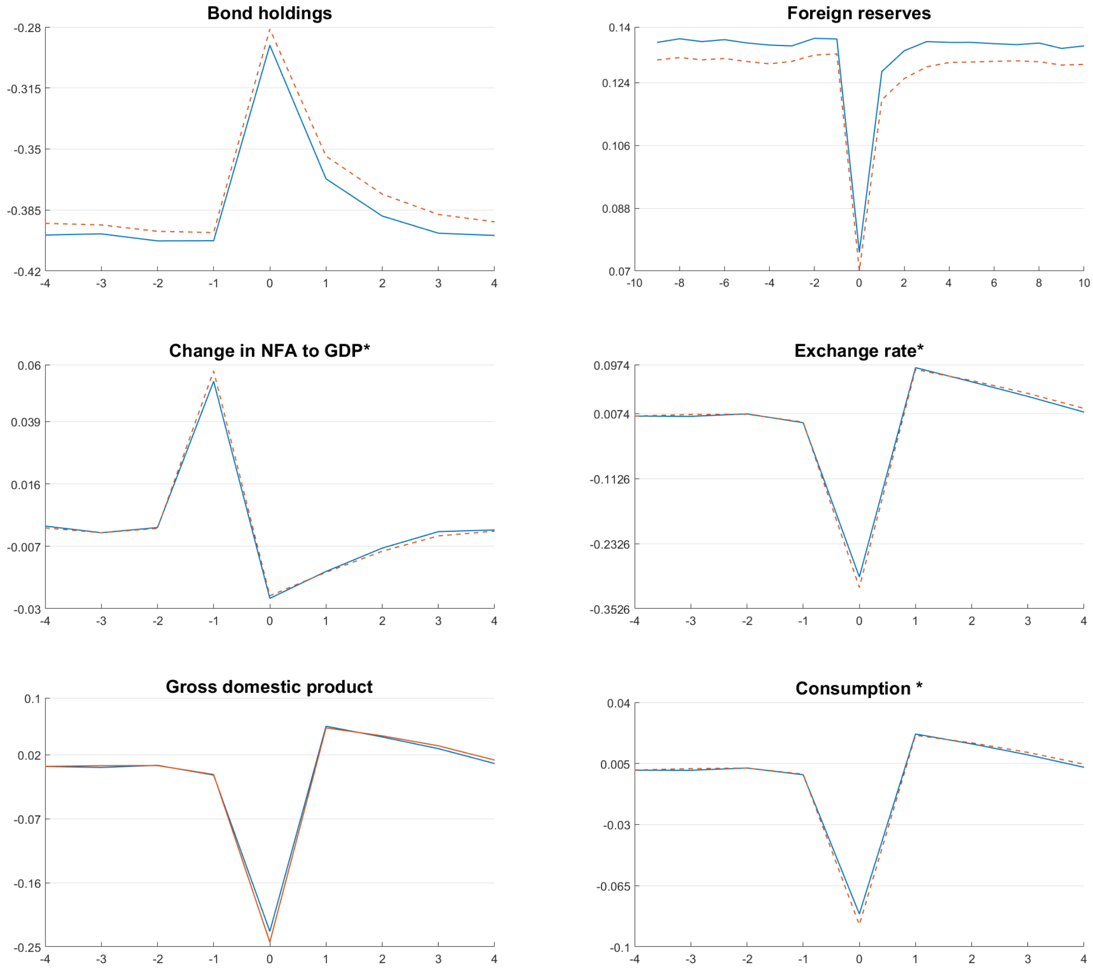
Figure 12 shows a comparison of sudden stops caused by liquidity shocks in the unregulated equilibrium compared to an economy where a 0.5% constant tax on debt is implemented. Again we identify sudden stops in the baseline economy and then average over the paths for the same sets of shocks in the economy with taxation. The graph shows that liquid asset holdings are smaller with the tax in place. Moreover the responses of all variables during the sudden stop are larger in absolute values in the economy where the tax is implemented. This shows that even a small tax is detrimental to financial stability.

Figure 11: Sudden stops without liquidity shocks



Note: The blue solid line corresponds to sudden stop purely caused by declines in total factor productivity, the red dashed line to sudden stops that coincide with a liquidity shock. In order to compare levels, bond holdings and foreign reserves are plotted in absolute values; * are plotted in relative deviations from mean.

Figure 12: Sudden stops with a fixed tax on debt



Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the regulated economy with a .5% tax on debt. In order to compare levels, bond holdings and foreign reserves are plotted in absolute values; * are plotted in relative deviations from mean.

Sudden Stops and Reserve Accumulation in the Presence of International Liquidity Risk*

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Abstract

We propose a small open economy model where agents borrow internationally and invest in liquid foreign assets to insure against liquidity shocks, which temporarily shut out the economy of short-term credit markets. Due to the presence of a pecuniary externality individual agents borrow too much and hold too little liquid assets relative to a social planner. This inefficiency rationalizes macroprudential policy interventions in the form of reserve accumulation at the central bank coupled with a tax on foreign borrowing. Unless combined with other measures, a tax on foreign borrowing is detrimental to welfare; it reduces agents' incentives to invest in liquid assets and thereby increases financial instability. Our model can quantitatively match the simultaneous depreciation of the exchange rate and contractions in output, gross trade flows, foreign liabilities and liquid reserves during Sudden Stop episodes.

JEL **codes: D62, E44, F32, F34, F41**

Key words: international reserves, sudden stops, liquidity, macroprudential policy, pecuniary externalities

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1 Introduction

Over the last three decades many emerging market economies have experienced surging gross capital inflows. At the same time, these economies have accumulated large amounts of liquid foreign assets mainly in the form of foreign exchange reserves. The fact that emerging markets pay high interest rates to foreign investors while earning ultra-low returns on their reserves has sparked an intensive debate about the economic rationale behind this pattern. A prominent empirical literature[1] argues that emerging market economies use liquid foreign assets to insure against sharp reversals in capital flows (Sudden Stops). This literature is successful at explaining observed levels of reserve accumulation, but fails to address the normative question whether these levels indeed reflect an optimal intervention by policymakers. Theoretical quantitative models that study optimal macroeconomic policies, on the other hand, usually abstract from foreign reserve accumulation; accordingly these models, too, are silent about the optimal holdings of foreign reserves by the central bank.[2]

In the present paper we contribute to filling this gap in the literature. We study optimal macroeconomic policy in a small open economy model with emerging market crises, where the central bank's policy toolkit includes foreign reserve accumulation. The central feature of our model is the occurrence of liquidity crises. During these episodes domestic firms lose access to international markets for unsecured short-term debt to finance imports of intermediate inputs. Holdings of foreign reserves are useful for firms, since they can serve as collateral to facilitate transactions with international suppliers. The overall borrowing capacity of the economy is tied to the real exchange rate which causes an inefficiency in the unregulated equilibrium:[3] A *pecuniary externality* arises because individual agents do not internalize the effect of their actions on the exchange rate and thereby borrowing capacity. As a consequence, private agents engage in inefficiently low precautionary behavior and expose the economy to excessive risk of Sudden Stops relative to a (constrained) social planner.

Our theoretical and quantitative analysis yields three main findings: i) Reserve accumulation at the central bank is part of an optimal policy mix, ii) reserve accumulation interacts non-trivially with other macroprudential policies, such as capital controls and iii) the model can jointly account for the level of reserves, foreign borrowing and gross goods flows as well as their dynamics during Sudden Stop episodes.

We show that inefficiencies in private behavior rationalize the observed accumulation

[1] See e.g.Obstfeld, Shambaugh, and Taylor (2010) and Calvo, Izquierdo, and Loo-Kung (2013).

[2] Exceptions include Bianchi, Hatchondo, and Martinez (2018), Arce, Bengui, and Bianchi (2019) and Shousha (2017). We discuss our relation to these studies below.

[3] This type of borrowing constraint has been widely studied since Mendoza (2002). Our model is most similar to Bianchi (2011).

of reserves at central banks. During normal times the social planner accumulates more liquid foreign assets than private agents, but chooses a very similar level of gross borrowing. Private agents therefore *underinvest* in liquid assets and *overborrow* in net debt. By accumulating more reserves the planner reduces the fall in consumption from 8% to 4% and the depreciation in the exchange rate from 30% to 15% in a typical Sudden Stop. An optimal policy mix to decentralize the planner allocation requires an intervention both in private borrowing and liquidity holdings, for example a tax on debt combined with reserve accumulation at the central bank. In a severe crisis, however, the inefficiency in liquid asset choices can change sign. Since agents do not internalize that they further destabilize the exchange rate by purchasing liquid foreign assets, private holdings of these assets can be excessive. In this situation, a policymaker can prevent inefficient liquidity hoarding by limiting convertibility of the domestic currency.

A key insight of our analysis is that the presence of liquid assets fundamentally changes the welfare properties of simple regulatory interventions. A common result in the previous literature is that simple capital controls in the form of a constant tax on foreign debt can be used to reduce overborrowing and improve welfare.[4] Our findings are in sharp contrast: While a tax on debt can be used to reduce both gross and net foreign borrowing, the tax also reduces private holdings of liquid assets. This occurs because the tax on debt makes agents value current consumption more relative to future consumption, which reduces private incentives to accumulate liquid assets. As a result, the tax fails to improve financial stability and causes a welfare loss. The intuition behind this result is general and highly relevant for the design of regulatory interventions: Policies that just aim at reducing gross borrowing, distort incentives for accumulating liquid assets and can therefore lead to financial instability. To avoid unintended consequences such policies have to be combined with liquidity regulation.

To study the quantitative success of our model we present a set of stylized facts on gross goods and capital flows around sudden stop episodes. In particular Sudden Stops are associated with sharp contractions in imports, exports, gross foreign liabilities and international reserves. At the same time the real exchange rate strongly depreciates. We calibrate our model to match selected moments of a set of emerging market economies and find that the model quantitatively replicates the stylized facts on Sudden Stops. Further, the model generates observed levels of international borrowing and liquid asset holdings. We show that the presence of liquidity crises is crucial for the model to match the patterns in the data.[5] In the absence of liquidity crises, agents hold no liquid assets in equilibrium, so neither the level nor the dynamics of international reserves are replicated.

[4] See, for example, Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2018).

[5] Our model nests the model of Bianchi (2011) as a special case, where no liquidity shocks occur.

In addition, the model without liquidity shocks fails to match the contractions in gross goods flows.

Our focus on the role of foreign reserves in facilitating trade in intermediate inputs is motivated by a number of empirical facts.[6] First, intermediate inputs to production dominate overall international trade. At least 60% of international trade is in intermediate inputs. Using OECD data we find that the relevant number in our context is close to 85%. Second, most of trade is not paid for in advance but financed with short-term debt with maturity below one year.[7] Third, there is recent evidence that in times of crisis emerging market firms' ability to finance inputs through borrowing is severely restricted.[8] Sustaining trade in intermediate inputs during crises is hence an important reason for emerging markets to hold liquid reserves, even though clearly not the only one. We nevertheless restrict attention to this single mechanism, which allows us to obtain clear analytical results in a model that is able to quantitatively match empirical regularities. Importantly, however, our normative results are compatible with other explanations for the accumulation of liquid assets, as long as a pecuniary externality is present and liquid assets have some positive effect on domestic output during crises. If this is the case the various inefficiencies in private behavior we identify are present and the result that regulatory interventions in borrowing interfere with incentives to accumulate liquidity applies.

Related literature Our framework builds on the workhorse small open economy model developed by Mendoza (2002) and its normative analysis in Bianchi (2011).[9] Specifically, our model shares the debt-deflation mechanism and the pecuniary externality arising from the fact that the borrowing capacity is tied to the value of domestic output. In the setup of Bianchi (2011), this externality gives rise to overborrowing, which can be addressed by the social planner through a variety of macroprudential measures, for example a tax on foreign borrowing. Importantly, however, there is no explicit role for liquid reserves in these contributions. We show that the introduction of liquidity crises introduces such a role, which allows the model to match not only level and dynamics of reserve holdings, but

[6] The precautionary demand for foreign reserves to hedge against shocks to the trade balance has been studied in the economic literature since Heller (1966). We build on this literature but also stress the role of reserves as insurance against collapse in short-term financing, which is related to modern theories of reserve demand.

[7] IMF (2009) provides evidence that around 40% of trade is intermediated through short-term bank loans and another 40% is direct credit from suppliers to importers.

[8] Swanson (2019) shows that trade credit contracts even more than imports in sudden stop episodes. Interestingly contractions in trade credit supply seem to have played little role in the turmoil at the onset of the Great Recession.

[9] Note that there is a series of related paper that study financial amplification mechanisms and optimal policy in small open-economy models without an explicit role for liquid foreign reserves(see e.g. Benigno, Chen, Otrok, Rebucci, and Young, 2013; Jeanne and Korinek, 2018; Bianchi and Mendoza, 2018).

also gross goods flows during Sudden Stops. Moreover, our model generates novel policy implications which stress the importance of coordinating capital controls with reserve accumulation.

Most closely related to this paper are recent contributions which jointly study reserve accumulation and emerging market crises in quantitative frameworks, in particular Arce, Bengui, and Bianchi (2019), Bianchi, Hatchondo, and Martinez (2018) and Shousha (2017). Arce, Bengui, and Bianchi (2019) show that foreign reserve accumulation can be used to implement the constrained efficient allocation in the model of Bianchi (2011), if reserves carry the same interest as foreign borrowing. In their framework, however, reserves are not useful per se, but are only held by the regulator to offset excessive private borrowing. If the return on reserves is below the international interest rate, reserve accumulation is strictly dominated by other macroprudential policies in their model. In contrast, we provide a framework where reserve accumulation is part of an optimal policy mix, even if reserves pay no interest. Bianchi, Hatchondo, and Martinez (2018) study the optimal accumulation of international reserves in a framework of sovereign default. Sovereigns issue long-term debt and invest in international reserves simultaneously because reserves provide liquid resources in states where borrowing opportunities deteriorate. Our focus on the other hand lies on studying private borrowing and public reserve accumulation, which dominate gross capital flows in the data.[10] Finally Shousha (2017) provides a framework where reserves are held by the central bank for their value as collateral in a liquidity crisis. However, in his framework private decisions are not inefficient per se, but individual agents have no access to liquid assets, so reserves are held at the central bank.[11] Our model on the other hand rationalizes reserve accumulation by the monetary authority as a macroprudential measure, even though private agents could in principle insure themselves through the accumulation of liquid assets.

Our study is further related to a wide empirical literature that studies the idea that reserves are held to provide insurance against Sudden Stops and more generally the dynamics of gross capital flows during emerging market crises.[12] We use the findings from this literature to motivate our modeling approach and to test the quantitative success of the model. Our equilibrium analysis allows us to provide insights for the design of macroprudential policies. We further connect to recent theoretical contributions stressing

[10]See section 2.

[11]A similar argument holds in the models of Benigno and Fornaro (2012), Caballero and Panageas (2008), Jeanne and Rancière (2011) and Jeanne and Sandri (2016) which are more loosely related to our study.

[12]Obstfeld, Shambaugh, and Taylor (2010) Calvo, Izquierdo, and Loo-Kung (2013), Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012), Broner, Didier, Erce, and Schmukler (2013), Avdjiev, Hardy, Kalemli-Ozcan, and Servén (2017)

the precautionary accumulation of foreign reserves in static setups.[13] A common finding in this literature, which relates to our paper, is that agents who may lose access to external credit in future periods are willing to invest in liquid assets with low returns. By providing a dynamic model we can evaluate our theory quantitatively and provide insights into the cyclical nature of policy interventions.

Several other explanations for the accumulation of international reserves, including the presence of growth externalities and facilitation of FDI, have been proposed.[14] We see our study as complementary to this literature. As we argue above, our normative findings are compatible with other explanations for the accumulation of liquid assets, if reserves provide some positive effects on output during a crisis.

Note further that the use of the cash-in-advance constraint on imports follows a well established literature to explain liquidity holdings.[15] Recently, such constraints have been used to explain large liquidity holdings of corporates (Bacchetta, Benhima, and Poilly, 2019) and households (Telyukova, 2013).

Finally, inefficiencies associated with private liquidity choices are also studied in a literature that focuses on systemic vulnerabilities arising in the banking sector, in static models without a focus on emerging markets.[16] Similar to our paper, a key result in this literature is that private agents hold an inefficiently low amount of liquid assets, emphasizing the need for liquidity regulation.

The rest of the article proceeds as follows. Section 2 presents the empirical evidence motivating our analysis and section 3 lays out the modeling framework and characterizes the competitive equilibrium. In section 4 we characterize the constrained-efficient allocation and discuss the inefficiencies present in the competitive equilibrium. Section 5 illustrates the main insights of our model in a quantitative analysis and section 6 concludes.

[13]Important contributions include but are not limited to Corneli and Tarantino (2016), Hur and Kondo (2016), Aizenman and Lee (2008), Jeanne (2016).

[14]See e.g. Aguiar and Amador (2011), Song, Storesletten, and Zilibotti (2011), Korinek and Servén (2016), Coeurdacier, Guibaud, and Jin (2015), Bacchetta and Benhima (2012), Jung and Pyun (2016).

[15]The use of such constraints goes back to Lucas (1982), Lucas and Stokey (1987) and Svensson (1985).

[16]Recent contributions include Malherbe (2014), Calomiris, Heider, and Hoerova (2015), Kara and Ozsoy (2016), Walther (2016), Lutz and Pichler (2017).

2 Sudden Stops and Capital Flows in the Data

Our work is motivated by a recent empirical literature emphasizing the importance of gross capital flows.[17] These studies document (i) that gross capital flows are large and volatile compared to net flows, (ii) gross as well as net capital flows are highly pro-cyclical and (iii) gross capital outflows of emerging markets mainly take the form of international reserves.

Avdjiev, Hardy, Kalemli-Ozcan, and Servén (2017) further decompose capital flows by sectors. They find that the cyclical behavior of capital flows is mainly driven by private sector borrowing and international reserves, which are both pro-cyclical. Furthermore, sovereign borrowing is negatively correlated with domestic output. These results provide support for our focus on private sector borrowing and a regulator intervening in the accumulation of liquid assets.

Following the empirical literature, we consider patterns of gross capital and trade flows around sudden stop episodes in the data.[18] The data we use is taken from the updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2006) and the Word Developement Indicators.

The patterns discussed above can clearly be seen in Figure 1. We show all non-stationary variables as log-deviations from their trend.[19] The ratios of the trade balance and the change in NFA to GDP are already stationary and are therefore shown in levels. By definition output strongly contracts and the trade balance sharply increases during the sudden stop. Both gross flows fall strongly, but imports contract more than exports. Moreover, there are is a decline in gross capital outflows, driven by a reduction in foreign reserves. Still the net foreign asset position increases, since gross liabilities contract even more strongly. Since gross liabilities are much larger than reserve holdings, even though they contract less in relative terms, the net foreign asset position strongly increases. Note that for consistency all variables are measured in dollars, which explains the large contractions in output. This contraction partly arises from the depreciation in the exchange rate, while in purchasing power parity it is significantly smaller.

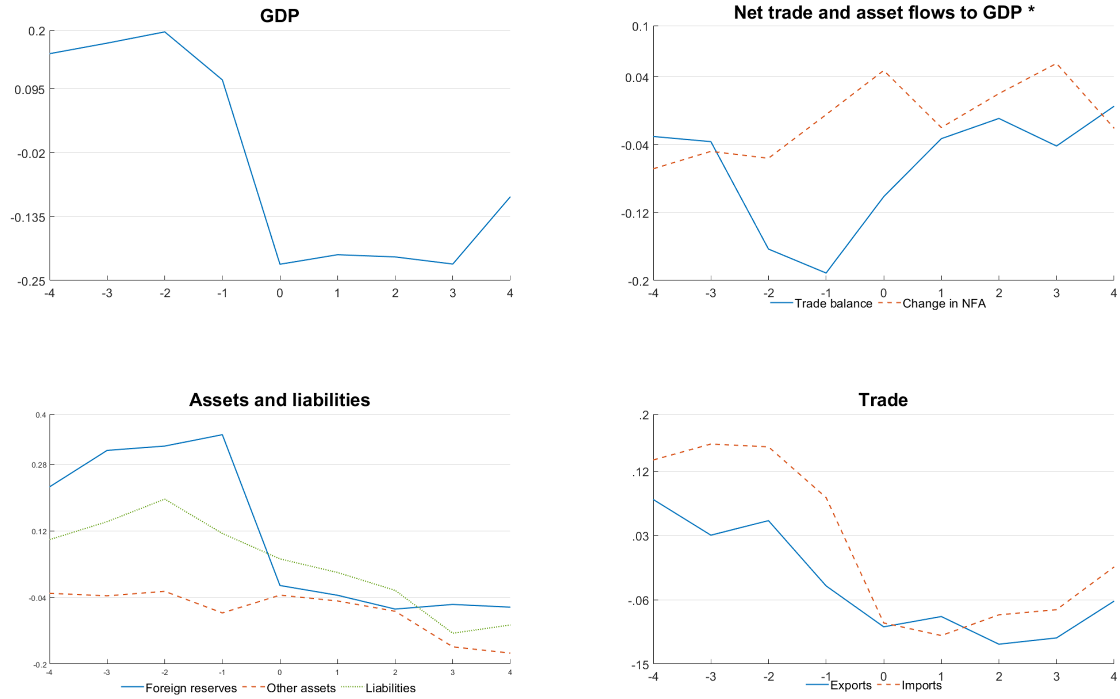
Foreign assets other than reserves do not seem to be correlated with the sudden stop event in an obvious way and fluctuate much less compared to foreign reserve holdings. Reserves and other assets are in similar magnitudes, so overall fluctuations in foreign

[17]Contributions include Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012), Broner, Didier, Erce, and Schmukler (2013), Avdjiev, Hardy, Kalemli-Ozcan, and Servén (2017).

[18]The dates for sudden stop episodes are taken from Calvo, Izquierdo, and Mejía (2004) who identify sudden stops as periods where year-on-year fall in capital flows (net of changes in reserves) fall two standard deviations below their mean and output simultaneously contracts. They use data between 1990 and 2004.

[19]We use a standard hp filter with parameter 100, which is widely used for yearly data.

Figure 1: Sudden Stops in the Data



Note: Variables are shown in relative deviations from trend; * is shown in absolute values.

assets during sudden stops are mostly driven by reserves. We therefore use international reserves as a proxy for total liquid assets held by domestic agents in our quantitative analysis and use the terms 'foreign liquid assets' and 'reserves' interchangeably in the rest of this paper. To the extent that these other assets are also used to provide liquidity to domestic agents, we underestimate the importance of the channels we study. Note that even in the data it is difficult to disentangle which agent can actually use the liquidity associated with foreign reserves, as monetary authorities in emerging markets deposit significant shares of their reserve holdings in their banking systems.[20]

In the following section we develop a model that can quantitatively match the facts presented here.

[20]Emerging market banks hold large amounts of international interbank deposits. Moreover, central banks engage in trading forward contracts on foreign currency with their domestic banking system. If, for example, a central bank buys dollars forward from a domestic bank, this bank will then hold dollar liquidity until the contract matures (see, for example, Wooldridge, 2006 and McCauley and Zukunft, 2008).

3 The Model

We develop a two-sector small-open economy model based on Bianchi (2011) and introduce liquidity risk. The economy is populated by an infinitely lived representative bank-firm-household who consumes tradable and nontradable goods. Agents can borrow long-term on international debt markets subject to a standard collateral constraint.[21] Furthermore they can invest in a non-interest bearing liquid foreign asset. Production of tradable output requires imported goods as an essential input. In normal times imports can be financed through short-term international loans, such as trade credit or international bank loans. There are times, however, when these short-term debt markets shut down and only safe liquid assets can serve as collateral for these transactions.[22] These are episodes when global financial markets are in turmoil and liquid funding dries up for reasons exogenous to the domestic economy. In what follows, we often refer to such an event as a *liquidity shock*. As we show below, the shut down in short-term debt markets can turn into a full Sudden Stop, with an associated contraction of domestic production and a sharp reversal in the current account, if agents hold too little liquid assets.

Note that we don't model a separate balance sheet for the central bank, but assume that liquid assets are held directly by the domestic agent. Due to the simplicity of our model, central bank reserves are perfect substitutes to privately held liquid foreign assets, if it is commonly understood by domestic agents and international lenders that the central bank will use its reserves to act as a lender-of-last-resort in the event of a liquidity crisis.[23] We maintain this assumption throughout this analysis and therefore don't take a strong stand on which agent actually holds these assets on their balance sheet. What matters in this section is that on the margin the amount of liquid assets held in the domestic economy is determined by private decisions. In section 4 we study an economy where liquid assets are chosen by a central authority.

3.1 Bank-Firm-Household Optimization Problem

The representative agent maximizes life-time utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

[21] Long-term refers to an inter-temporal loan, which lasts for one model period, as opposed to a short-term loan, which has to be repaid in the same period.

[22] Alternatively the assets could directly be used for the transaction

[23] This equivalence is trivial if one assumes, that the central bank accepts domestic non-tradable output as collateral and sets a haircut exactly, such that all its reserves are alloted to domestic agents. This means each agent will have access to exactly the same amount of reserves. Based on Uribe (2006) we conjecture that the equivalence still holds, if a market clearing interest rate is set.

where $\mathbb{E}_0$ is the expectation operator conditional on information available at date 0 and β is the discount factor. The consumption basket c_t is a CES aggregator with elasticity of substitution $1/(\eta+1)$ between tradable goods c_t^T and nontradable goods c_t^N given by

$$c_t = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta}. \tag{2}$$

In each period agents receive a fixed endowment of nontradable goods y^N. We extend the model of Bianchi (2011) by assuming that tradable output y_t^T is produced using intermediate input goods ν_t. Tradable output is given by $y_t^T = z_t F(\nu_t)$, where z_t denotes total factor productivity in the tradable sector, which follows a finite-state Markov process and $F(.)$ denotes a twice continuously differentiable, concave production function. Net output is thus given by $[z_t F(\nu_t) - p_\nu \nu_t]$, where p_ν denotes the exogenous price of intermediate inputs. In principle the framework can be extended to include endogenous production using capital and labor inputs in both sectors.[24] To keep both theoretical and quantitative analysis concise, we exclude those extensions.

All prices and contracts are written in terms of the tradable good, which acts as a numeraire with a price of one. Agents have access to a one-period non-state contingent foreign bond b_{t+1} that pays a fixed interest rate $R > 1$ and to a non-remunerated liquid foreign asset l_{t+1}, i.e. essentially a storage technology. In equilibrium agents will choose negative amounts of b, which corresponds to foreign borrowing.[25] We denote the price of nontradables by p_t^N. Following the literature we refer to p_t^N as the real exchange rate in the remainder of the text. The budget constraint of the representative agent reads

$$c_t^N p_t^N + c_t^T + l_{t+1} + \frac{b_{t+1}}{R} = [z_t F(\nu_t) - p_\nu \nu_t] + p_t^N y_t^N + l_t + b_t, \tag{3}$$

where b_{t+1} are foreign bond holdings chosen at time t and negative values of b indicate foreign debt of domestic agents.

Inter-period debt is subject to a collateral constraint limiting total debt to not exceed a fraction κ of the market value of eligible collateral:

$$b_{t+1} \geq -\kappa^N p_t^N y_t^N - \kappa^T [z_t F(\nu_t) - \nu_t p_\nu] - \theta l_{t+1}, \tag{4}$$

where κ^N and κ^T denote the shares of nontradable and tradable net output, respectively, which can be pledged to foreign investors.[26] The crucial feature of this constraint is that

[24] Extensions to endogenous labor and capital accumulation are numerous in the literature.

[25] Note that we implicitly assume that private agents directly borrow on international markets in foreign currency. See Mendoza and Rojas (2019) for a model where domestic agents borrow from domestic banks in dollar denominated debt.

[26] Constraints of this type have widely been used in the international macroeconomic literature since,

nontradable goods are part of the collateral and are valued at their endogenous price p_t^N. Since individual agents do not anticipate their incremental effect on equilibrium prices, this may lead to an inefficiently low precautionary behavior (see e.g. Lorenzoni, 2008; Korinek, 2010; Jeanne and Korinek, 2010; Bianchi, 2011).

Constraint 4 is a natural extension of the constraint in Bianchi (2011) to account for endogenous production of tradable goods and the presence of liquid assets, which can also serve as collateral. Their use as collateral here is not necessary for liquid assets to be held in equilibrium and our normative results apply for any $\theta \in [0, 1]$. The share of liquid assets that can serve as collateral determines how much of an extra unit of liquid assets can be financed through debt issuance and how much has to be financed through reduced consumption. We provide a microfoundation of this constraint in Appendix A.

We now turn to the central, new feature of our model. At the beginning of the period intermediate inputs ν_t have to be imported before production takes place. Agents can only finance these imports through unsecured short-term credit.[27] In normal times end-of-period production provides sufficient collateral to finance desired levels of imports, but during liquidity crises lenders are only willing to accept liquid assets. This gives rise to the following stochastic constraint:

$$S_t \nu_t p_\nu \leq l_t. \tag{5}$$

Note that effectively this constraint is a cash-in-advance constraint on imported inputs, where S_t is a stochastic variable that governs the state of the short-term debt market and follows a first-order two-state Markov process. In normal times ($S_t = 0$) agents have access to unlimited short-term credit and the constraint is never binding. However, in case of a liquidity shock ($S_t = 1$) only liquid assets can serve as collateral for short-term credit. If the stock of liquid assets is low, imported inputs have to be reduced, so output, tradable consumption and the price of nontradable goods fall. This price decline reduces the capacity to borrow inter-temporally, which further lowers the price of nontradables and triggers a Fisherian debt-deflation mechanism. We provide a microfoundation of this constraint along with the inter-temporal borrowing constraint in Appendix A. We

as far as we know, Mendoza (2002) and are generally understood to capture financial frictions associated with borrower default in a reduced form manner. As pointed out by Schmitt-Grohé and Uribe (2018) it is possible that, while this constraint binds on the individual level, the effect of trade-able consumption on the exchange rate is so strong, that on aggregate higher consumption and borrowing is possible. Policy implications in such economies are very different, as generally there is underborrowing in net debt. We assume that this is not the case throughout our normative analysis. In our calibrated economy, the constraint binds on aggregate, whenever it binds individually.

[27] This assumption is not as stark as it might seem. IMF (2009) presents evidence that worldwide only 20% of imported goods are paid in advance, while 80% are credit financed. Half of these 80% is direct trade credit from exporters to importers while the other half is intermediated.

discuss the role of the liquidity constraint in more detail in section 3.2, after defining a competitive equilibrium in our model.

3.2 Competitive Equilibrium

The representative household maximizes life-time utility (1) subject to the budget constraint (3), the collateral constraint (4), the liquidity constraint (5) and the non-negativity constraint on liquid assets $l_{t+1} \geq 0$ by choosing $\{b_{t+1}, l_{t+1}, c_t^T, c_t^N, \nu_t\}$, taking prices as given. This maximization problem yields the following optimality conditions

$$\lambda_t = u_T(t), \tag{6}$$

$$p_t^N = (\frac{1-\omega}{\omega})(\frac{c_t^T}{c_t^N})^{\eta+1}, \tag{7}$$

$$\xi_t S_t p_\nu = (\lambda_t + \mu_t \kappa^T)[z_t F_{\nu_t}(.) - p_\nu], \tag{8}$$

$$\frac{\lambda_t}{R} - \mu_t = \beta \mathbb{E}_t\{\lambda_{t+1}\}, \tag{9}$$

$$\lambda_t - \theta\mu_t - \psi_t = \beta \mathbb{E}_t\{\lambda_{t+1} + \xi_{t+1}\}, \tag{10}$$

where λ is the nonnegative multiplier associated with the budget constraint, μ is the nonnegative multiplier of the collateral constraint, ξ is the nonnegative multiplier of the liquidity constraint and ψ is the nonnegative multiplier of the non-negativity constraint on liquid assets.[28] Further, $u_T(t)$ denotes the derivative of the utility function with respect to tradable consumption.

Equation (6) is the optimality condition for tradable consumption equating the marginal utility to the shadow value of current wealth. Condition (7) equates the relative price of tradable and nontradable goods with their marginal rate of substitution. Note that an increase in c^T generates an increase in the price of nontradable goods thereby increasing the collateral value of nontradables. Equation (8) is the intra-temporal optimality condition for intermediate inputs. If the liquidity constraint is nonbinding in period t, the optimality condition simplifies to $z_t F_{\nu_t}(.) = p_\nu$. Otherwise, intermediate inputs are determined by last periods liquidity holdings and $z_t F_{\nu_t}(.) > p_\nu$. Equation (9) is the intertemporal optimality condition for foreign bonds, which is completely standard. When the collateral constraint binds, the marginal benefit of borrowing exceeds the expected marginal costs of repayment by the shadow price of the constraint. Finally, the optimality condition for liquid assets is given by equation (10), which captures the central

[28] This constraint is not really necessary. As long as there is a positive probability of a liquidity shock, household endogenously hold positive liquid assets in equilibrium. Otherwise they would run the risk of producing 0 output, as intermediate inputs are an essential input to production. We nevertheless state it here to make clear that agents cannot borrow in the liquid asset.

mechanisms in our model. While liquid assets do not carry any interest, they are still useful to private agents because they anticipate that liquid assets can be used to buy intermediate inputs in case of a liquidity shock as captured by the term ξ_{t+1}. Since all agents are identical, market clearing conditions are given by

$$c_t^N = y^N, \tag{11}$$

$$c_t^T = z_t F(\nu_t) - p_\nu \nu_t + l_t + b_t - l_{t+1} - \frac{b_{t+1}}{R}. \tag{12}$$

Finally, the following complementary slackness conditions have to hold

$$b_{t+1} + \kappa^N p_t^N y_t^N + \kappa^T [z_t F(\nu_t) - p_\nu \nu_t] + \theta l_{t+1} \geq 0, \quad \mu \geq 0, \tag{13}$$

$$l_t - S_t p_\nu \nu_t \geq 0, \quad \xi \geq 0, \tag{14}$$

$$l_{t+1} \geq 0, \quad \psi \geq 0, \tag{15}$$

as well as the laws of motion for the exogenous variables $\{S_t, z_t\}$. The unregulated competitive equilibrium is defined as follows

Definition 1. *A unregulated recursive competitive equilibrium is a set of*

i a pricing function $\hat{p}^N(b, l, S, z)$

ii decision functions $\{\hat{b}(b, l, S, z), \hat{l}(b, l, S, z), \hat{c}^T(b, l, S, z), \hat{c}^N(b, l, S, z), \hat{\nu}(b, l, S, z)\}$

iii and multipliers $\{\hat{\lambda}(b, l, S, z), \hat{\mu}(b, l, S, z), \hat{\xi}(b, l, S, z), \hat{\psi}(b, l, S, z)\}$

that satisfies the equations (6) - (15) given the laws of motion for the exogenous states S *and* z.

3.3 Discussion of the Liquidity Constraint

The liquidity constraint (5), is central to the mechanism studied in this paper, as introduces a separate role for liquid assets. In comparison in the model of Bianchi (2011) the equilibrium is not affected, if beginning of period debt and liquid asset holdings are increased by the same amount. In that model the extra liquid asset can only be used to repay the extra debt.[29] The same is true in our model if $S_t = 0$. However, if $S_t = 1$ an extra unit of liquid assets allows agents to produce more than one unit of output on the margin, while they have to repay just one extra unit of debt[30]. Agents anticipate this and

[29]Since liquid assets earn zero return, while agents have to pay interest on debt, liquid assets are not held in equilibrium.

[30]It is straightforward to see that the marginal product in case of a liquidity shock will always be strictly larger than 1 in equilibrium

build up precautionary liquidity holdings. Note that in the absence of liquidity shocks, intermediate inputs are always at their unconstrained level and tradable production only depends on exogenous productivity. As a result, no liquid assets are held in equilibrium and the model reduces to Bianchi (2011), with a stochastic endowment of tradable goods. This shows that the model with liquidity shocks nests the model by Bianchi (2011) as a special case.

Importantly, the liquidity constraint does not introduce a new pecuniary externality to the model, as there is no price involved.[31] Liquidity choices are still potentially inefficient because of the interaction of the two constraints: If a liquidity shock hits the economy, the level of liquid assets affects the amount of tradeable output and consumption. Tradeable consumption in turn affects the real exchange rate, which is the source of the pecuniary externality. This effect is not internalized by individual agents when making their decisions, which, as we show in the following analysis, generally leads to underinvestment in liquid assets during normal times.

A potential concern is that trade flows over short periods of time are not large enough to warrant the large observed amounts of liquid asset holdings in emerging economies. Obstfeld, Shambaugh, and Taylor (2010) point out that trade flows alone could at most account for reserve outflows of 1/2% of GDP per week, which would give both policy makers and private agents time to adjust their liquid asset positions. They argue that only rapid withdrawals of domestic bank deposits in order to purchase foreign currency, coupled with unwillingness of foreign investors to buy domestic currency can explain the large precauntionary liquid asset holdings. We would like to point out that our model can be understood this way as well. In the event of a liquidity shock, private agents expect a depreciation of the exchange rate and a shortage of foreign liquid assets. In this situation even firms, which have no immediate use for them, will hoard foreign liquid assets. This leads to an instant outflow of liquid assets from the domestic banking system that by far exceeds the actual flow of imports. As a result, firms which have actual liquidity needs will have access to insufficient amounts of reserves.

Finally, note that access to short-term debt markets is determined by a purely exogenous process which is unrelated to economic fundamentals. One can think of this as the risk aversion of international investors. Empirical studies show that global risk aversion is among the most robust predictors for sudden capital flow reversals in emerging markets (see e.g. Eichengreen, Gupta, and Mody, 2016; Calvo, 2013). Furthermore, the assumption that domestic output completely loses its value as collateral for international short-term credit might seem very stark. In principle, the model can easily be extended by introducing a time-varying share of domestic output entering in the liquidity constraint.

[31] We show this point formally in the section 4.1.

In such a model, access to short-term debt markets would indeed be related to domestic output. While this might add to the quantitative realism of the model it would come at the cost of analytical clarity. We prefer the simplest possible version of the model, which can match the key features in the data we are interested in.

4 Constrained Efficient Equilibrium

We now consider a constrained social planner that faces the same collateral constraint and liquidity constraint but internalizes how prices are determined in equilibrium. In particular, the planner chooses b_{t+1}, l_{t+1}, c_t^T, c_t^N and ν_t to maximize aggregate welfare (1) subject to the resource constraints (11) and (12), the collateral constraint (4), the working capital constraint (5) and the pricing rule of the competitive equilibrium allocation (7).[32]

As opposed to private agents, the planner internalizes the effect of debt and liquid assets on the price of nontradable goods. Critically, the planner anticipates that higher outstanding debt reduces tradable consumption, which lowers the exchange rate and borrowing capacity in states where the collateral constraint binds. Conversely, more liquid assets increase tradable consumption in adverse states of the world, which raises the exchange rate and prevents strong declines in the borrowing ability of the economy. In recursive form the social planner's problem can be expressed as

$$
\begin{aligned}
V(b,l,S,z) &= \max_{\{b',l',c^T,\nu_t\}} U(c(c^T,y^N)) + \beta\mathbb{E}V(b',l',s'), \qquad (16)\\
\text{s.t.} & \\
c^T &+ \frac{b'}{R} + l' = zF(\nu) - p_\nu\nu + b + l,\\
b' &\geq -\kappa^N \frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{(\eta+1)} y^N - \kappa^T[zF(\nu) - p_\nu\nu] - \theta l', \quad \mu \geq 0,\\
l &\geq Sp_\nu\nu, \quad \xi \geq 0,\\
l' &\geq 0,
\end{aligned}
$$

where we have used the pricing rule of the competitive equilibrium to replace the price p_t^N in the collateral constraint of the social planner. The first order conditions in sequential

[32] By constraining the social planner problem to the pricing rule of the competitive equilibrium we follow the constrained-efficiency concept of Kehoe and Levine (1993), which has been widely used for example in Bianchi (2011). Note that not allowing the social planner to directly intervene in the exchange rate is a non-trivial constraint. As shown by Benigno, Chen, Otrok, Rebucci, and Young (2016) the planner can actually completely undo the effects of the borrowing constraint by using consumption taxes to manage the real exchange rate in a similar model.

form are given by

$$\lambda_t^{SP} = u_T(t) + \mu_t^{SP}\Psi_t, \tag{17}$$

$$\frac{\lambda_t^{SP}}{R} - \mu_t^{SP} = \beta\mathbb{E}_t\{\lambda_{t+1}^{SP}\}, \tag{18}$$

$$\lambda_t^{SP} - \theta\mu_t^{SP} - \psi_t^{SP} = \beta\mathbb{E}_t\{\lambda_{t+1}^{SP} + \xi_{t+1}^{SP}\}, \tag{19}$$

$$\xi_t^{SP} S_t p_\nu = (\lambda_t^{SP} + \kappa^T\mu_t^{SP})[z_t F_\nu(.) - p_\nu], \tag{20}$$

where we use superscript sp to distinguish the multipliers of the competitive equilibrium from the multipliers of the constrained-efficient allocation and $\Psi_t = \kappa(p_t^N c_t^N / c_t^T)(1+\eta) > 0$. This term summarizes the equilibrium effect on the collateral value of nontradable goods for a marginal change in tradable consumption.[33]

It is important to note that the optimality condition for ν implies that the planner makes the same choice for imported inputs as agents in the competitive economy given the states. In particular the equation implies that ν is pinned down by the condition $z_t F_\nu(.) - p_\nu = 0$ if possible and by the constraint $S_t p_\nu \nu_t \leq l_t$ otherwise.

4.1 Comparison of Private and Planner Optimality conditions

The key difference between the unregulated competitive equilibrium and the constrained-efficient allocation becomes evident by comparing equation (6) with equation (17). In particular, the social planner's marginal utility gain from tradable consumption is composed of a direct effect on utility $u_T(t)$ and an indirect effect as the planner understands that an increase in tradable consumption relaxes the collateral constraint. This effect on the price of tradable goods and thereby on the aggregate borrowing ability of the economy is not internalized by individual agents. Individual agents thus value wealth less than the social planner in states where the collateral constraint binds. Importantly, in our framework the difference in the valuation of wealth distorts both individual borrowing and liquidity choices, which we now discuss in more detail.

Macroprudential interventions Following Bianchi (2011) we first consider states where the collateral constraint is currently not binding i.e. $\mu_t^{SP} = 0$.[34] This simplifies equations and allows us to develop important insights into the nature of macroprudential interventions, which the planner uses in states where there is currently no debt crisis. We turn to the case where the constraint currently binds below. As noted before, the

[33] This term is equivalent to Bianchi (2011).

[34] For consistency we assume in the remainder of this section that both optimality conditions are evaluated at the constrained-efficient allocation. Agents form their expectations knowing this. We discuss how to consistently interpret private LMs in this setting in Appendix C.

planner's choice for purchases of imports coincides with the competitive one. We can therefore focus on the consumption-borrowing-liquidity choices.

The overborrowing result becomes evident by using (6) and (17) with $\mu_t = 0$ to rewrite the competitive and constrained-efficient Euler equations for inter-temporal debt holdings respectively

$$\begin{aligned} u_T(t) &= \beta R \mathbb{E}_t\{u_T(t+1)\}, \\ u_T(t) &= \beta R \mathbb{E}_t\{u_T(t+1) + \mu^{SP}_{t+1}\Psi_{t+1}\}. \end{aligned}$$

Clearly the planner faces higher future costs of current debt holdings given by $\mu^{SP}_{t+1}\Psi_{t+1}$ as she anticipates the negative effect on prices. This implies that the private first order condition cannot be satisfied, and the agents have an incentive to marginally increase borrowing.[35] Private agents therefore overborrow (taking liquid asset holdings as given) compared to constrained-efficiency leaving the economy excessively prone to financial distress. The farther away the economy is from the borrowing constraint ($\mathbb{E}_t\mu^{SP}_{t+1}$ close to 0), the smaller the difference between the two allocations becomes.

Critically, in our model the wedge in the valuation of wealth also distorts optimal liquidity choices. Using the fact that $\lambda_t = u_T(t)$ and $\xi_t = \frac{(\lambda_t + \mu_t\kappa^T)[zF_\nu(.) - p_\nu]}{p_\nu}$ in the competitive equilibrium and t $\lambda^{sp}_t = u_T(t) + \mu^{sp}_t\Psi_t$ and $\xi^{sp}_t = \frac{(\lambda^{sp}_t + \mu^{sp}_t\kappa^T)[zF_\nu(.) - p_\nu]}{p_\nu}$ in the constrained-efficient allocation, the Euler conditions for liquid assets can be rewritten to

$$\begin{aligned} u_T(t) &= \beta\mathbb{E}_t\Big\{u_T(t+1)\frac{z_{t+1}F_\nu(t+1)}{p_\nu} + \mu_{t+1}\kappa^T\frac{z_{t+1}F_\nu(t+1) - p_\nu}{p_\nu}\Big\}, \\ u_T(t) &= \beta\mathbb{E}_t\Big\{u_T(t+1)\frac{z_{t+1}F_\nu(t+1)}{p_\nu} + \mu^{SP}_{t+1}\kappa^T\frac{z_{t+1}F_\nu(t+1) - p_\nu}{p_\nu} \\ &\quad + \mu^{SP}_{t+1}\Psi_{t+1} + \mu^{SP}_{t+1}\Psi_{t+1}\frac{z_{t+1}F_\nu(t+1) - p_\nu}{p_\nu}\Big\}. \end{aligned}$$

Here, the competitive and planner optimality conditions differ in three ways. Firstly, it is clear that the gain of holding liquid assets for the social planner exceeds the gain of private agents by $\mu^{SP}_{t+1}\Psi_{t+1}$. This difference arises for exactly the same reason as in the optimality conditions for borrowing, since the planner values savings more than individual agents. Note that the valuation of wealth is unrelated to the actual use of the liquid asset in liquidity crises and therefore not the central part of our analysis. Moreover, in the absence of liquidity shocks, this term on its own would not cause the planner to hold positive amounts of liquid assets because the benefit from holding liquid assets is still strictly smaller than the interest rate on borrowing in a model without liquidity shocks.

[35]Formally we could introduce the planners' choice as a constraint in the decentralized optimization problem. This constraint would then have a positive LM, so that the Euler equation is satisfied

The other differences are more interesting, as they are new in our model and more subtle. The last term in the planner's Euler equation $\mu_{t+1}^{SP}\Psi_{t+1}\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu}$ captures the benefit the planner attaches to the positive effect of the additional net output on the real exchange rate.[36] In a liquidity crisis, a higher stock of liquid assets increases output, which allows for higher tradable consumption. This in turn raises the real exchange rate, which makes the borrowing constraint less tight. Since private agents do not internalize this effect, the term unambiguously raises the planner's incentives to hold liquid assets relative to the competitive equilibrium.

Finally, in both optimality conditions the term $\mu_{t+1}^i\kappa^T\frac{z_{t+1}F_\nu(t+1)-p_\nu}{p_\nu}$ enters. This term captures the shadow value of liquid assets in relaxing the borrowing constraint through raising net tradable output. This effect is separate from the one above, as it is unrelated to the exchange rate, but reflects the fact that the additional output can be collateralized which directly relaxes the constraint. It is therefore understood by both planner and individual agents. Importantly, however, the values that planner and individuals attach to relaxing the constraint do not coincide in general. Understanding the relationship between the two, however, requires us to consider situations where the borrowing constraint is currently binding, i.e. $\mu_t^{SP}>0$.[37] Rearranging the planner and competitive optimality conditions we get the following relationship between the two multipliers

$$\mu_t^{SP}=\mu_t+\frac{\mu_t^{SP}\Psi_t}{R}-\beta\mathbb{E}_t\{\mu_{t+1}^{SP}\Psi_{t+1}\}. \tag{21}$$

Since these are endogenous, general equilibrium objects we cannot show whether one or the other multiplier is larger in general. Nevertheless, this equation is very useful to develop intuition about the difference in how the planner and individual households respond to the presence of the constraint. In particular, both understand the benefit from relaxing the constraint and increasing consumption, captured by their respective μ_t^i, however, the planner makes two more considerations. First, she understands that relaxing the constraint and increasing consumption raises the real exchange rate, which in turn further relaxes the constraint. This effect is captured by the term $\frac{\mu_t^{SP}\Psi_t}{R}$. Moreover, the planner also understands that relaxing the constraint today increases the borrowing, which tightens the constraint tomorrow, as given by the term $\beta\mathbb{E}_t\{\mu_{t+1}^{SP}\Psi_{t+1}\}$. This precautionary motive lowers the planner's shadow value relative to the individual agent. While in general it is unclear which effect dominates, note that the negative term is discounted with β which is smaller than $\frac{1}{R}$, but is otherwise the same expression shifted by one period. On average, the planner thus values relaxing the constraint more than

[36] Note that the last two terms in the equation can be simplified to $\mu_{t+1}^{SP}\Psi_{t+1}\frac{z_{t+1}F_\nu(t+1)}{p_\nu}$. We keep them separate to conceptually distinguish the incentives at play.

[37] It is obvious from the two optimality conditions that $\mu_t>0$ if $\mu_t^{SP}>0$. See Appendix C for details.

the individual agent.

Furthermore μ_t^{SP} is much larger than μ_t in states where μ_t^{SP} is currently large because a binding collateral constraint forces the economy to deleverage which implies a slack borrowing constraint in the following period, i.e. $\mathbb{E}_t\{\mu_{t+1}^{SP}\}$ is close to 0. Especially in a crisis, the planner hence values relaxing the borrowing constraint clearly more than individual agents.

Returning to the optimality conditions for liquid asset holdings, the presence of the shadow values is ambiguous in general. However, in our numerical solution we find that the planner always chooses to hold (weakly) more liquid assets than agents in the competitive equilibrium in situations where the constraint does currently not bind. This indicates that the negative term is strictly dominated by the positive term discussed above. Finally, it is important to note here, that all three differences in the optimality conditions for liquid assets between private agents and the social planner are related to the value of μ_{t+1}^{SP}. In particular, if the economy is very far from the constraint so $\mu^i = 0$ and $\mathbb{E}_t\{\mu_{t+1}^i\} = 0$, the optimality conditions coincide. This shows that the motive for liquidity regulation in our model only arises because of the interaction of the liquidity and collateral constraints and not because of a new externality.

Ex-post interventions We close this section by analyzing the optimal choice of liquid assets in a situation where the collateral constraint currently binds,[38] i.e. $\mu_t^{SP} > 0$. This is interesting in our case, because the planner can still meaningfully affect the equilibrium, by choosing liquid assets and borrowing subject to the constraint. In Bianchi (2011) on the other hand, the equilibrium under a binding constraint is already fully determined and the competitive and planner choices coincide. In our model there is effectively only one inter-temporal choice when the constraint binds, since the choice of liquid assets also pins down borrowing. We therefore do not discuss borrowing separately.

In contrast to the first part of the section differences in incentives here do not reflect macroprudential interventions by the planner, but differences in how she responds to a crisis. The Euler equations for the liquidity choice at the constraint are given by

$$\begin{aligned} u_T(t) - \theta\mu_t &= \beta\mathbb{E}_t\big\{\lambda_{t+1} + \xi_{t+1}\big\}, \\ u_T(t) + \mu_t^{SP}\Psi_t - \theta\mu_t^{SP} &= \beta\mathbb{E}_t\big\{\lambda_{t+1}^{SP} + \xi_{t+1}^{SP}\big\}. \end{aligned}$$

For readability we have not plugged in the terms on the right hand side, but in principle all insights from above still apply here. However, in cases where the constraint strongly binds (μ_t^{SP} is large), the economy is forced to deleverage. This means that borrowing is

[38] Note that we have already discussed the differences in optimality conditions for borrowing in this case above, by relating the two Lagrange multipliers. We use this relation again below.

low, so the probability of a binding constraint tomorrow and therefore $\mathbb{E}_t\mu^i_{t+1}$ is small. As we just discussed, for small $\mathbb{E}_t\mu^i_{t+1}$ the right hand sides of the two equations become very close, so we can focus on differences on the left hand side.

We have pointed out above that in a crisis μ^{SP}_t is likely much larger than μ_t. This has two effects. Firstly, since a share θ of liquid assets can be collateralized, a higher multiplier on the borrowing constraint makes holding liquid assets more attractive to the planner. This effect pushes her towards borrowing more and accumulating more liquid assets. At the same time, however, the planner understands, that by selling liquid assets, she can increase current consumption of tradable goods which boosts the real exchange rate and thereby also increases borrowing capacity. We find this effect highly interesting, as it captures a form of inefficient liquidity hoarding: Individual agents do not understand that they can increase the aggregate borrowing capacity by reducing their accumulation of liquid assets. Note that this is more likely to occur in economies where the share of liquid assets that can be collateralized is low, as the opposing effect is smaller. Intuitively this occurs because the planner has to give up more consumption today for holding an extra marginal unit of liquidity, if its collateral value is low.

While our analytical results do not allow us to characterize one dominating effect, our numerical results show that both are possible. In our calibrated economy we find that in parts of the state space, where the constraint binds very strongly, the planner prefers to reduce liquid assets more drastically than individual agents. We now turn to a discussion of which instruments the planner can use to implement the constrained-efficient allocation.

4.2 Decentralizing the planner allocation

We first characterize optimal price based policy and show that, in order to restore constrained-efficiency, a policy maker needs to impose both, a state dependent tax on foreign debt and a state dependent subsidy on liquid assets.[39] We then continue by showing that in the states where the subsidy is positive, the planner can implement the constraint efficient allocation without paying out subsidies, by accumulating reserves at the central bank and committing to act as a lender of last resort in a liquidity crisis.

4.2.1 Price based policy tools

Assume that the policy maker imposes a state contingent tax on debt, τ^b_t, and a state contingent subsidy on liquid assets, τ^l_t, on debt and liquidity holdings chosen in period

[39] For simplicity we refer to the two instruments as tax and subsidy, even though the subsidy on liquid assets turns negative in some states, which effectively makes it a tax.

t. Details on the implementation of these taxes and a derivation of the equations below can be found in Appendix C.

With the tax and subsidy the price of bonds becomes $\frac{1}{R(1+\tau_t^b)}$ and the price of liquid assets becomes $\frac{1}{(1+\tau_t^l)}$ in the budget constraint of private agents. Note that τ_t^b is always and τ_t^l is (usually) positive. The reason why we consider τ_t^b a tax and τ_t^l a subsidy is that bond holdings are always negative in equilibrium, while liquid assets holdings are by definition positive. Net tax revenues are rebated lump-sum to the household. The Euler equation for debt holdings and liquid assets of private agents are given by

$$\frac{\lambda_t}{R(1+\tau_t^b)} - \mu_t \quad = \quad \beta \mathbb{E}_t\{\lambda_{t+1}\}, \tag{22}$$

$$\frac{\lambda_t}{(1+\tau_t^l)} - \theta\mu_t - \psi_t \quad = \quad \beta \mathbb{E}_t\{\lambda_{t+1} + \xi_{t+1}\}. \tag{23}$$

The planner allocation can be implemented by setting the tax and subsidy such that the Euler equations of the private agent coincide with the planner's optimality conditions. This results is summarized in the following proposition.

Proposition 1. *The constrained-efficient allocation can be decentralized using a state contingent tax on debt* τ_t^b *and a state contingent subsidy on liquid assets* τ_t^l *set to satisfy*

$$\tau_t^b = \begin{cases} \frac{\mathbb{E}_t\{\mu_{t+1}^{SP}\Psi_{t+1}\}}{\mathbb{E}_t\{u_T(t+1)\}}, & > 0, \quad \textit{if } \beta\mathbb{E}_t\{u_T(t+1) + \mu_{t+1}^{SP}\Psi_{t+1}\} > \frac{u_T(t)}{R}, \\ 0, & \textit{otherwise.} \end{cases} \tag{24}$$

$$\tau_t^l = \frac{\beta\mathbb{E}_t\{[\kappa^T\mu_{t+1}^{SP} - \kappa^T\mu_{t+1}]\frac{F_\nu(t+1)-p_\nu}{p_\nu}\} + \mu_{t+1}^{SP}\Psi_{t+1}\frac{F_\nu(t+1)}{p_\nu} + \theta(\mu_t^{SP} - \mu_t) - \Psi_t\mu_t^{SP}}{\beta\mathbb{E}_t\{u_T(t+1) + \mu_{t+1}\kappa^T\frac{F_\nu(t+1)}{p_\nu}\} + \theta\mu_t}, \tag{25}$$

and tax revenues rebated as lump-sum transfers.

Note that the private Lagrange multipliers in the expressions above are different from the competitive ones as they arise from optimality conditions in an economy where the optimal policies are already implemented. Details can be found in Appendix C. To economize on notation we omit this distinction.

The optimal tax rate for foreign debt is identical to the expression in Bianchi (2011). This tax is weakly positive representing the uninternalized marginal cost of borrowing of private agents. Further, the tax rate increases as the probability of a binding collateral constraint increases and is zero if the probability of a binding collateral constraint is zero. Note that we follow a convention in the literature and set the tax rate to zero in cases where individual agents would like to borrow, but are prevented from doing so by the constraint. This is slightly arbitrary, but necessary, because in this case infinitely many combinations of τ^l and τ^b could implement the constrained-efficient allocation. Note that a positive tax in such a situation, would lower the multiplier μ_t, which would raise the optimal subsidy on liquidity. This is the case, because effectively there is only one inter-temporal choice, if borrowing and liquidity holdings are linked by the constraint, so the planner needs only one instrument.[40] We omit a discussion of the subsidy on liquid assets, as we have investigated the underlying incentives in the previous section. Note that exactly the same terms as discussed above enter the numerator of the expression.

4.2.2 Quantity based policy tools

We now show that as an alternative, the planner can implement the constraint efficient allocation without paying out subsidies, by accumulating reserves at the central bank and committing to act as a lender of last resort in a liquidity crisis. This exercise is interesting, not only because reserve accumulation is widely used in practice, but also because as the sovereign might not have the fiscal capacity to pay out subsidies during a crisis.

Under the the optimal policy private agents will not hold any liquid assets and the non-negativity constraint (15) binds during normal times . In states of severe crisis where the planner chooses less liquid assets than the competitive equilibrium, she needs to impose a tax on liquid assets to prevent liquidity hoarding. The optimal tax on borrowing is unaffected by these liquidity policies. We summarize these findings in the following proposition:

Proposition 2. *Alternatively, the constrained-efficient allocation can be decentralized using a state contingent tax on debt $\hat{\tau}_t^b$, a tax on liquid assets $\hat{\tau}_t^l$ and by accumulating liquid reserves $\hat{l}_{t+1}$ at the central bank:*

$$\hat{l}_{t+1} = l^*_{t+1} \tag{26}$$

where l^*_{t+1} denotes the socially optimal level of liquid assets and

$$\hat{\tau}_t^l = \begin{cases} 0, & if \psi_t > 0 \\ \tau_t^l & otherwise \end{cases} \tag{27}$$

And the optimal tax on debt is given by equation (24), i.e. $\hat{\tau}_t^b = \tau_t^b$ Tax revenues and proceeds from trading liquid assets are rebated as lump-sum transfers.

[40]In models where there is only one inter-temporal choice, like Bianchi (2011), the tax rate at the constraint doesn't affect the equilibrium allocation at all.

In all states where $\hat{\tau}_t^l$ is different form zero, it is strictly negative, since it is used to discourage liquidity hoarding. This means that no subsidies are paid out by the planner. However, private liquidity holdings are zero everywhere, so no tax is actually collected in equilibrium either. In fact $\hat{\tau}_t^l$ is not unique, as any larger tax would also achieve zero private liquidity holdings. The planner could also impose an infinite tax on liquid assets, which effectively corresponds to prohibiting purchases of foreign reserves. This could be achieved by suspending convertibility of the domestic currency.

While no subsidies are paid out, the planner still needs resources to accumulate liquid assets. We show in the quantitative analysis, that she strongly reduces liquid asset holdings during crises and only accumulates liquid assets during expansions. Therefore fiscal constraints should not impede the implementation of this policy. Finally, note that a wide range of alternative instruments can be used to implement the constrained-efficient allocation. For example a reserve requirement can replace the subsidy on liquid assets in states where it is positive. For brevity we omit the explicit formula here, as it adds little economic insight.

5 Quantitative Analysis

In this section, we describe the calibration of the model and evaluate implications regarding Sudden Stops, optimal policy and welfare. We solve the model globally using fixed point iteration on the conditional expectations.[41]

5.1 Calibration and functional forms

We chose the following standard functional forms:

$$u(c_t) = \frac{c_t^{1-\varepsilon} - 1}{1-\varepsilon}, \tag{28}$$

$$c_t = [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta}, \tag{29}$$

$$z_t F(\nu_t) = z_t \nu_t^{\gamma_\nu}. \tag{30}$$

Our calibration uses standard values from the literature and data for 16 economies classified as emerging markets and developing economies by the IMF's World Economic Outlook[42]. The data is taken from the updated version of the dataset provided by Lane and Milesi-Ferretti (2006) and from the Word Developement Indicators (WDI) One period

[41] We use a variation of the algorithm in Wolf (2019).

[42] We use data for the following countries: Argentina, Brazil, Chile, Croatia, Columbia, India, Indonesia, Mexico, Malaysia, Poland, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela.

in the model represents a year and the values assigned to the model parameters are summarized in Table 1. The risk aversion is set to 2 and the world interest rate to 4% which are standard values in the DSGE-SOE literature. The domestic discount factor is set to .92 to target the average net foreign asset position of -28% of GDP between 1990-2011.

Table 1: Calibration

Parameter	Value	Source/Target
Risk aversion	σ =2	Standard value DSGE-SOE
World interest rate	$R = 1.04$	Standard value DSGE-SOE
Discount factor	$\beta = .92$	NFA=-30%
Elasticity of substitution	$1/(1+\eta)$ =.83	Bianchi (2011)
Weight on tradables in CES	ω=.3	Share of tradable output = 30%
Share of pledgeable liquidity	$\theta = .95$	Reserve Outflows during Sudden Stops
Intermediate inputs share	$\gamma_\nu = .45$	Bianchi and Mendoza (2018) & imported inputs/GDP
Share of pledgeable output, tradable	$\kappa^T = .3$	Probability of Sudden Stop=5.4%
Share of pledgeable output, nontradable	$\kappa^N/\kappa^T = 1$	Bianchi (2011)
Probability of liquidity shock	$P_{gb} = 10\%$	International reserves/GDP = 14%
TFP Process	$\rho_z = .537, \sigma_s = .0134$	Mendoza (2010)

The rate of substitution between tradable and nontradable goods determines the sensitivity of price of nontradables, with respect to relative changes in consumption of the two goods. Empirical estimates for this parameter range from 0.4 to 0.83. Following Bianchi (2011) we choose a conservative value and set $1/(\eta + 1) = .83$. The parameters ω determines the tradable share in the CES aggregator and is calibrated to match a 30% share of tradable output which is the average value for the set of countries from 1990-2011. In line with previous studies we define tradable output as the sum of primary and manufacturing goods.

The share of pledgeable liquid assets is calibrated to match the decline in foreign reserves during a Sudden Stop episode. As pointed out in section 3.1 the share of liquid assets that can serve as collateral determines how much of an extra unit of liquid assets has to be financed through reduced consumption. Effectively, θ therefore determines how strongly reserve holdings respond to changes in the marginal utility of tradable consumption. In a crisis tradable consumption is low and its marginal utility is high, which triggers a fall in reserves. To match the observed magnitude of decline in reserves during a Sudden Stop we set $\delta = .95$. A value close to one seems reasonable since liquid assets can be easily pledged as collateral. The production share of intermediate inputs γ_ν is set to .45 as in Bianchi and Mendoza (2018), who target the average share of total intermediate inputs to gross output for all OECD members. While it is not clear whether this target is relevant for emerging markets, setting $\gamma_\nu = .45$ generates an average ratio of imported intermediate inputs to GDP of 23.5%. This value is consistent with the data for our set of countries.[43]

The coefficient of the collateral constraint is set to $\kappa^T = .3$ to match the yearly observed frequency of Sudden Stops reported by Eichengreen, Gupta, and Mody (2008). In line with Calvo, Izquierdo, and Mejía (2004), Sudden Stops are defined as periods where capital inflows (net of changes in reserves) fall two standard deviations below their mean and output contracts. Following Bianchi (2011) we set $\kappa^T = \kappa^N$.

The probability of a liquidity shock is set to 10% which yields an international reserves to GDP ratio of 14% consistent with the cross-country average observed between 1990 and 2011. The transition probability matrix is symmetric which yields an average duration of a Sudden Stop slightly above one year.[44]

Finally, the stochastic process for tradable sector total factor productivity is specified as

$$log(z_t) = \rho log(z_{t-1}) + \epsilon_t, \tag{31}$$

where ϵ_t is an iid N(0,σ_z^2) shock. The parameters of this process are set to $\rho_z = 0.537$ and $\sigma_s = 0.0134$ which are the first autocorrelation and the standard deviation of aggregate total factor productivity reported by Mendoza (2010).

5.2 Sudden Stops

In this section, we compare a typical Sudden Stop generated by our model with the characteristics of actual Sudden Stops observed in the data. To this end, we simulate the model economy for 100.000 periods. We then use the same procedure to identify Sudden Stop events as Calvo, Izquierdo, and Mejía (2004) on the simulated paths. In particular Sudden Stops are defined as periods where the trade balance increases two standard deviation above its mean and output falls one standard deviation below trend. We then average across all identified Sudden Stop episodes.

Figure 2 contains the central quantitative findings of this paper. In particular, it compares a typical Sudden Stop in the model with the data counterpart. We show all non-stationary variables as log deviations from trend, so that magnitudes can be understood

[43]We compute this number based on data from the OECD Structural Analysis Databases. For details see Appendix B. If this number seems high, note that we include imported capital goods in imports to production, given that capital goods are not part of our model.

[44]This is in line with the estimated average duration of a Sudden Stop reported by Eichengreen, Gupta, and Mody (2008).

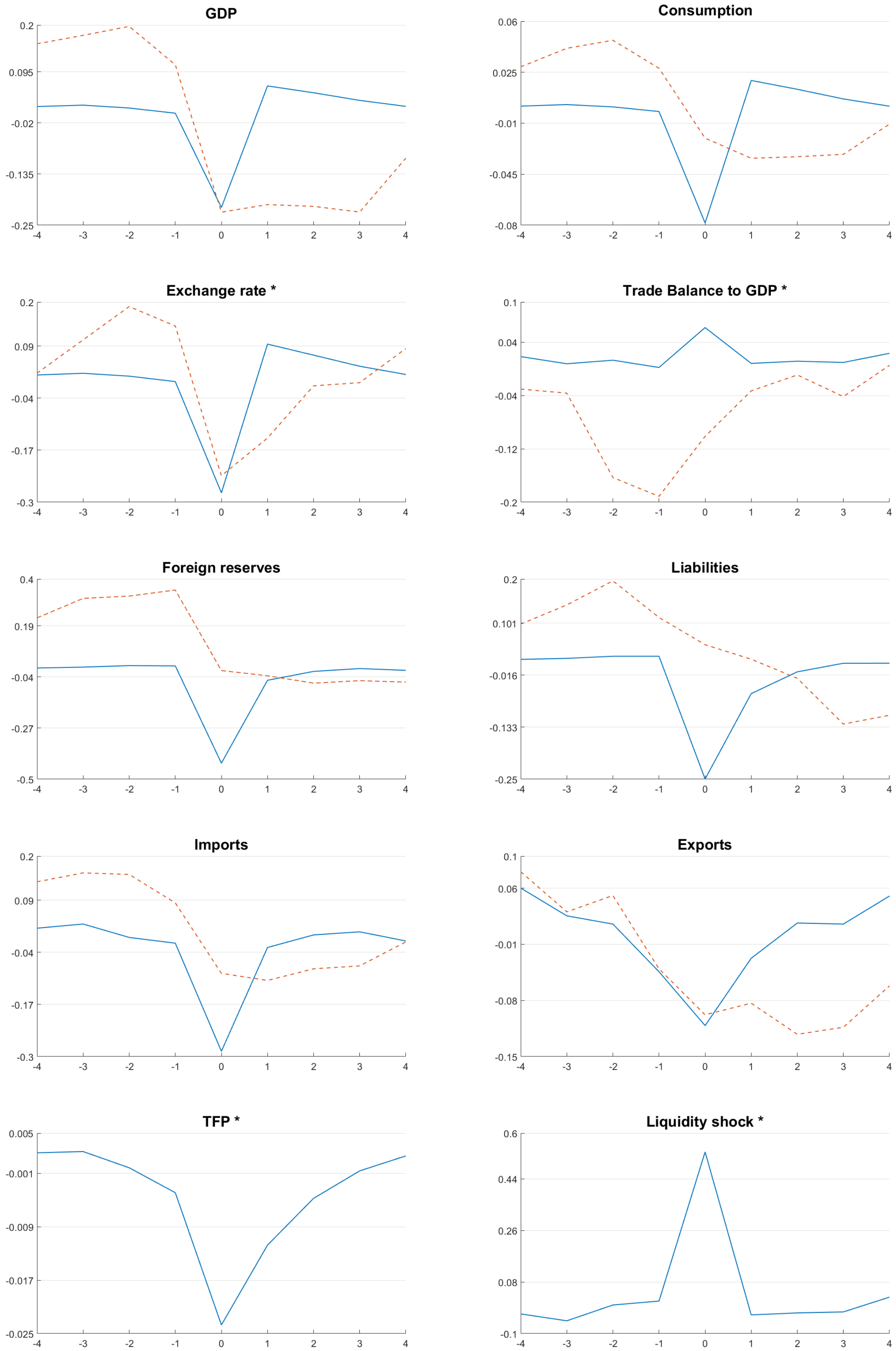
as relative deviations. The trade balance relative to GDP and the real exchange rate are already stationary and their magnitudes have natural interpretations. We therefore show them in terms of deviations from their long-run means.

Given the simplicity of our model, we find that it replicates Sudden Stop episodes surprisingly well across many dimensions. Firstly, the decline in foreign reserves was targeted and is matched well. The responses in consumption, GDP, imports and exports are all in similar magnitudes to the data. Finally, the model somewhat underpredicts the reversal in the trade balance and overpredicts the fall in gross liabilities. The lowest left panel shows that typically Sudden stops occur in the model when tradable sector productivity is around 2% below trend. Importantly, note that the model qualitatively accounts for all Sudden stop characteristics observed in the data. However, the model fails to account for the trends observed before Sudden Stops. In particular, Sudden stops occur in the data, when GDP, consumption, the exchange rate, foreign reserves and gross liabilities are above trend and the trade balance below its mean.

Note further that the model fails to match the persistent response of most variables following Sudden Stops observed in the data. This is a common feature of models that don't feature investment, since there is no endogenous mechanism that could generate persistence. Moreover, the liquidity shock in our model only takes two extreme values and has a symmetric transition matrix, so exogenous persistence could only arise through productivity. Further, the strong deleveraging process induced by the binding collateral constraint leads to an increase in the net foreign asset position. As a result, the exchange rate increases above its mean already in the period after the Sudden Stop which raises consumption and GDP in the model. In reality, other mechanisms, such as externalities in the production sector, default of domestic firms and declines in foreign direct investment are likely to contribute to the observed persistence of real variables. A model where access to international short-term markets follows a somewhat persistent AR process might also be able to get closer to the data in this dimension.

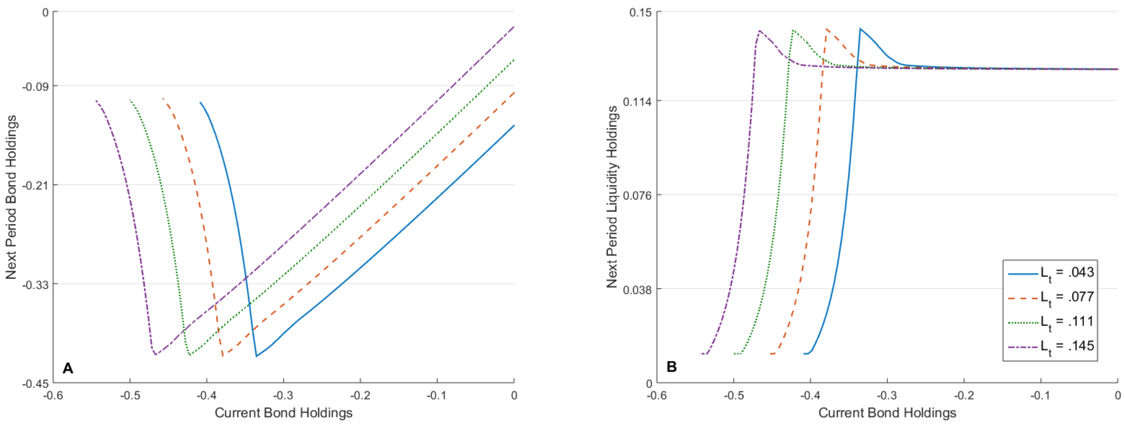
Finally, note that around half of the Sudden Stops in our model economy are caused by a liquidity shock. In section E of the Appendix we show a comparison between Sudden Stops caused by liquidity shocks and those purely caused by declining productivity. The contractions in output, the exchange rate and asset positions are very similar. Sudden stops that are purely due to declines in productivity, however, lead to negligible falls in gross goods flows. This shows that the inclusion of liquidity shocks is necessary for the model to match the dynamics in goods flows observed in the data.

Figure 2: Sudden Stops in the Model and the Data



Note: The blue solid line corresponds to the model simulations, the red dashed line to the data. Dates for Sudden stops are taken from Calvo, Izquierdo, and Mejía (2004). Accordingly in the model Sudden Stops are defined as periods where capital inflows (net of changes in reserves) fall two standard deviations below their mean and output contracts. Variables are shown in relative deviations from trend, * absolute deviations from mean

Figure 3: Competitive Equilibrium Decision Rules



Note: This figure plots the competitive decision functions for four different levels of current liquidity holdings as a function of current bond holdings when S = 1 and productivity is at its steady state level.

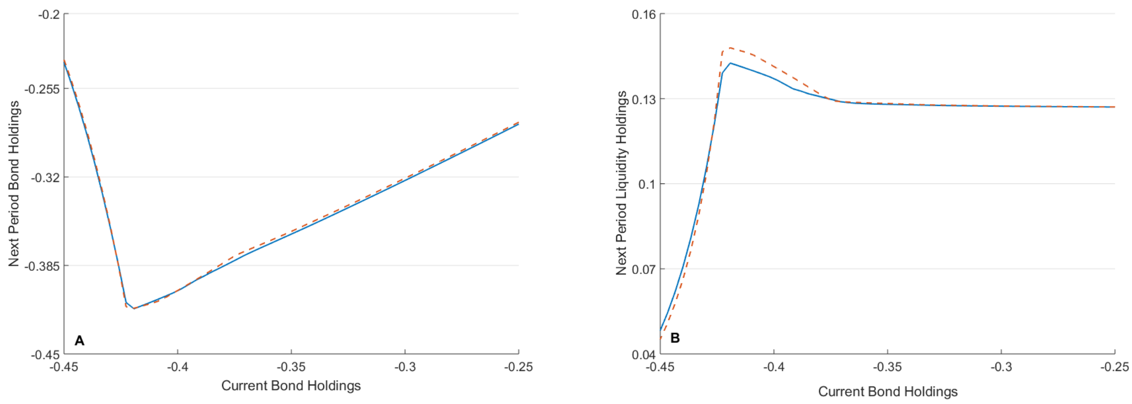
5.3 Borrowing and Liquidity Decisions

In this section, we first characterize the decentralized equilibrium and then show how the decision functions of private agent differ from those of the social planner. Panel A of Figure 3 shows next period bond holdings of private agents as a function of current bond holdings for four different levels of current liquidity holdings. The policy function for bond holdings is V-shaped which is a prominent feature of financial friction models including a Fisherian debt deflation mechanism. [45] In particular, future bond holdings are increasing in current holding in the unconstrained region and decreasing in the constrained region. The change in the slope of the function indicates the point where the collateral constraint starts to bind. Once the collateral constraint binds, higher borrowing levels lead to a fall in the real exchange rate and therefore the borrowing capacity. As a result, the higher current debt is, the lower is future borrowing capacity and the stronger is the forced deleveraging in this region. Higher current liquidity holdings increase the borrowing ability of the economy and shift the policy function to the left as they provide additional collateral.

More interestingly, Panel B of Figure 3 plots the liquidity decision rules for four different levels of current liquidity holdings as a function of current bond holdings. The policy function can be divided in three areas. The left region is the region where the collateral constraint is binding. In this region, the decision rules are sharply decreasing because agents prefer to consume their resources in states where the constraint becomes

[45]See Bianchi (2011)

Figure 4: Competitive Equilibrium vs. Constrained-Efficient Decision Rules



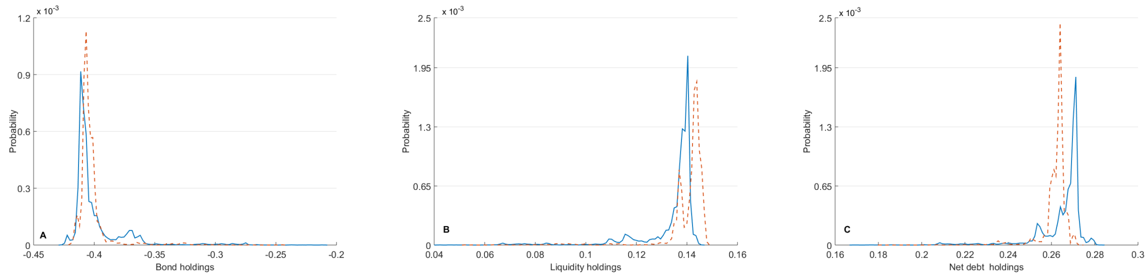
Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient equilibrium. Access to working capital is unrestricted and productivity and liquid assets are fixed at their steady state level.

very tight. In the center, the collateral constraint is nonbinding but there is a positive probability of a binding constraint in the following period. In that sense, the region is characterized by financial instability. The policy function is increasing in current debt holdings as agents understand that more debt today increases the probability of a binding collateral constraint tomorrow and therefore increase their precautionary liquidity holdings. Finally, the right region is the stable region where current bond holdings are so high such that the constraint is nonbinding today and the probability of hitting it in period $t+1$ is zero. In this region, current bond holdings do not affect liquidity decisions, however, agents still hold a positive amount of liquid assets due to the nonzero probability of a liquidity shock. As before, more liquidity holdings shift the policy functions to the left.

The way how the pecuniary externality distorts individual borrowing and liquidity decisions is illustrated in Figure 4 which compares the competitive bond and liquidity decision rules with the social planner's rules. First note that the differences in the bond policy functions are quantitatively small. Interestingly, however, the planner borrows slightly more compared to the unregulated allocation in the region characterized by a positive crises probability. This is in sharp contrast to previous findings which show that the planner reduces borrowing if financial stability is at risk (i.e. close to the constraint).[46] Here, the planner slightly increases gross borrowing, but more than offsets this by accumulating a larger liquidity buffer which becomes evident from Panel B. As

[46]See e.g. Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013), Bianchi and Mendoza (2018).

Figure 5: Ergodic Distributions of Bond and Liquidity Holdings



Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient equilibrium.

a result net debt holdings of the planner are thus strictly below the private optimum in the region characterized by financial instability. Once the constraint becomes binding, the social planner starts to reduce liquid asset holding faster then the private agents. For states where the constraint binds very strongly, we find the inefficient liquidity hoarding mentioned above, since private agents don't internalize the positive effect of extra consumption on the borrowing capacity. Finally, in the right region the probability of a binding collateral constraint goes to zero and the differences in decision rules vanish.

The differences among the two allocations becomes even more evident by comparing the ergodic distributions of bond and liquidity holdings among the unregulated and the constrained-efficient allocation in Figure 5. While the ergodic distribution of current bond holdings in the competitive equilibrium assigns a larger probability to very high levels of debt, on average, debt holdings are slightly below the constrained-efficient allocation (the mean of the competitive allocation is 0.3983 compared to 0.4012 in the constrained-efficient allocation). Further, the competitive equilibrium assigns more mass to very low debt levels, which occur due to forced deleveraging in severe Sudden Stops.

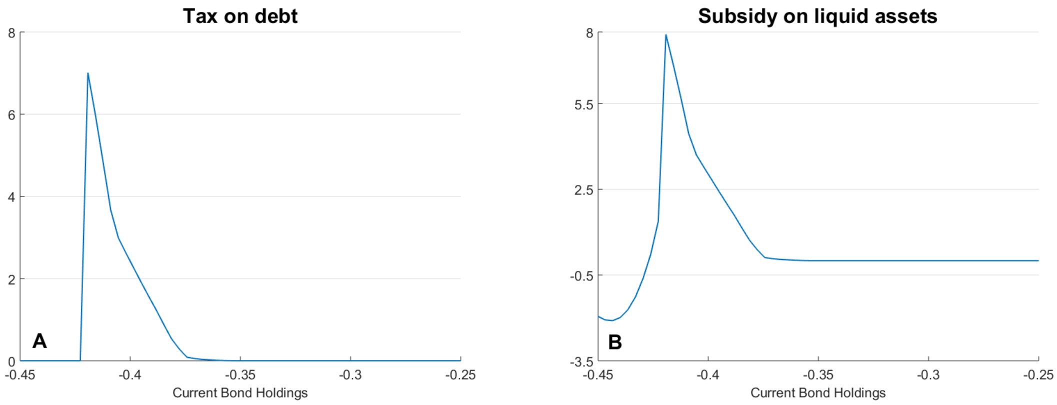
For liquid assets, the ergodic distribution assigns a higher mass to high liquidity holdings in the constrained-efficient allocation as the planner accumulates extra precautionary savings to insure against financial crises. Moreover, liquidity holdings of private agents drop to significantly lower levels compared to the social planner's choices. Finally, to quantify the precautionary differences among the two allocations, Panel C plots the ergodic distribution of net debt.[47] Indeed, the figure shows that the social planner engages in significantly more precautionary behavior in terms of net borrowing.

[47]Note that in contrast to the gross bond holdings, here a higher level corresponds to more borrowing.

5.4 Optimal Policy and Welfare Effects

Figure 6 illustrates the quantitative features of an optimal policy consisting of tax a on debt (Panel A) and a subsidy on liquid assets (Panel B), as a function of current borrowing for a fixed level of liquid assets. For sufficiently low levels of debt (right region), the probability of a binding collateral constraint is zero and the unregulated equilibrium coincides with the constrained-efficient allocation, so no taxes or subsidies are necessary. For higher debt levels the tax on debt increases with the level of debt and falls to zero when the collateral constraint becomes binding. Similarly, the subsidy on liquid assets is increasing in debt levels in the positive crisis probability region. When the collateral constraint binds strongly the subsidy on liquid assets turns negative, reflecting the fact that private agents hold inefficiently large stocks of liquid assets.

Figure 6: Optimal Policy Instruments



Note: The blue solid line corresponds to the tax on debt and the subsidy on liquid assets (in %).

The total welfare gain from implementing the optimal policy in our model is 0.032% of permanent consumption. The small magnitude of welfare gains is common in the literature. Following Bianchi (2011) we argue that this is due to the stylized nature of the model which abstract from many real-world phenomena. In particular, the externality does not distort the efficient use of production resources which could deliver significantly higher welfare gains. We therefore see these welfare gains as a lower bound.

5.5 Sudden Stops and Optimal Policy

In this section, we analyze the effect of optimal policy on the severity of Sudden Stops. As before we simulate the model economy for 100.000 periods and use the resulting data to construct an 8 year event window centered around the Sudden Stop event. Figure 7 shows typical Sudden Stops in the competitive economy compared to the planner economy.[48] We show foreign borrowing and liquid assets in levels to highlight the differences. All other variables are plotted in relative deviations from their long run means. The top two panels show that, compared to constrained efficiency, individuals underborrow in gross debt and underinvest in liquid foreign assets. Furthermore, the competitive economy experiences significantly more severe declines in both variables during periods of financial distress. In both economies, the decline in gross capital inflows exceeds the decline in gross capital outflows, which leads to a reversal of the net foreign asset position. The reversal is much weaker under constrained efficiency at approximately 2% compared to 7% in the competitive equilibrium.

The exchange rate, consumption and GDP also fall more sharply in the unregulated equilibrium with especially large differences in the exchange rate and consumption (the real exchange rate declines by 28% in the competitive and by 15% in the constrained-efficient allocation; consumption declines by 8% in the competitive and by 4% in the constrained-efficient allocation). Particularly interesting is the large difference in the real exchange rate which reflects the strength of the externality and is an important factor for the more pronounced decline in debt levels in the unregulated equilibrium. GDP falls by 23% in the competitive versus 14% in the constraint-efficient economy.

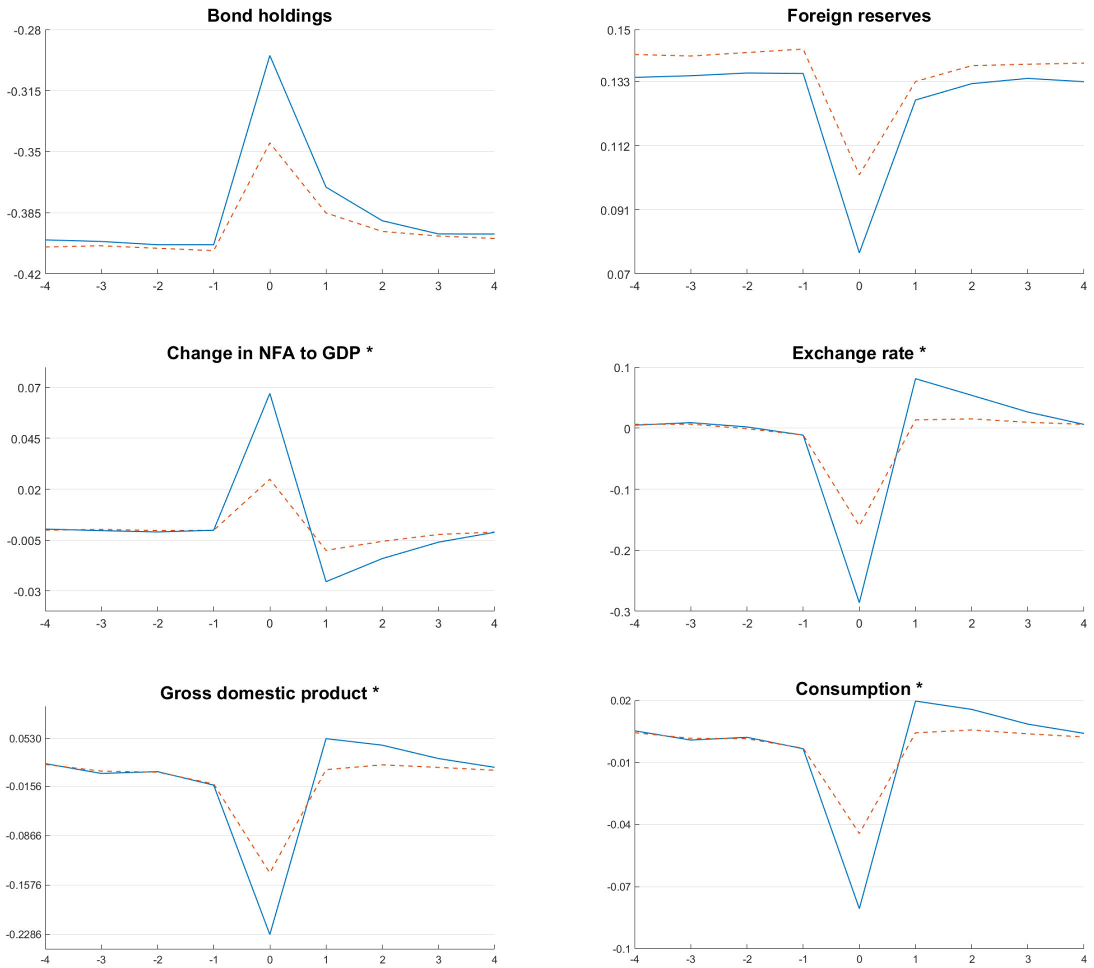
Finally, the macroprudential feature of optimal policy tools becomes evident from Figure 8 which plots the optimal tax on debt and subsidy on liquid assets around a Sudden Stops. Both policy rates increase by about 1% one period before the shock materializes and decline sharply when the crisis hits. Ex-post, both policy rates rise at a fast pace but remain slightly below their pre-crisis level.

5.6 Simple Policies

In practice, optimal policy might be challenging to implement as it requires two information-intense, state dependent policy tools. On the other hand, if macroprudential policy is limited to simple rules it is questionable if theses simple policies can induce welfare gains. In light of these concerns, we now investigate the welfare effects of simple interventions. Figure 9 contrasts the effects of a fixed tax on debt ranging from 0 to .9% with the effects of a fixed subsidy on liquid assets ranging from 0 to 1.2%. Evidently, a simple fixed tax on debt reduces welfare for any tax level. The maximum welfare gain of a fixed subsidy on liquid assets peaks at a subsidy of .7% and induces a welfare gain of 0.0038% which amounts to about 12% of the gain in the constrained-efficient allocation. The best simple policy combining both instruments is given by a .07% subsidy and a .03% tax which

[48] We simulate both economies for the same shocks, identify Sudden Stops in the competitive economy and then pick the same periods from the planner economy.

Figure 7: Competitive equilibrium vs. constrained efficiency



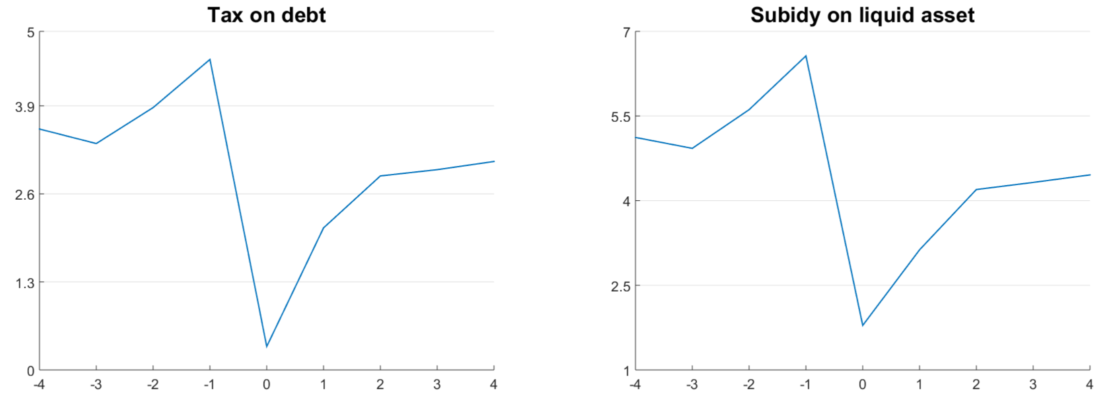
Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient allocation. In order to compare levels, bond holdings and foreign reserves are plotted in absolute values; * are plotted in relative deviations from the mean.

achieves a welfare gain of .00662% (21% of the welfare gain implemented by the optimal policy schedule.).

The fact that a even a small tax on debt reduces welfare compared to the unregulated allocation is in sharp contrast to previous findings.[49] In our framework, this effect is due to the interaction between debt and liquidity holdings. In particular, a fixed tax on debt $\bar{\tau}^b$ reduces individual incentives to invest in liquid assets which increases the costs of crises and induces an aggregate welfare loss. Figure 12 in Appendix F shows Sudden Stops caused by liquidity shocks in the competitive economy compared to an economy where a constant tax of .5% is levied on gross borrowing. With the tax, agents engage in

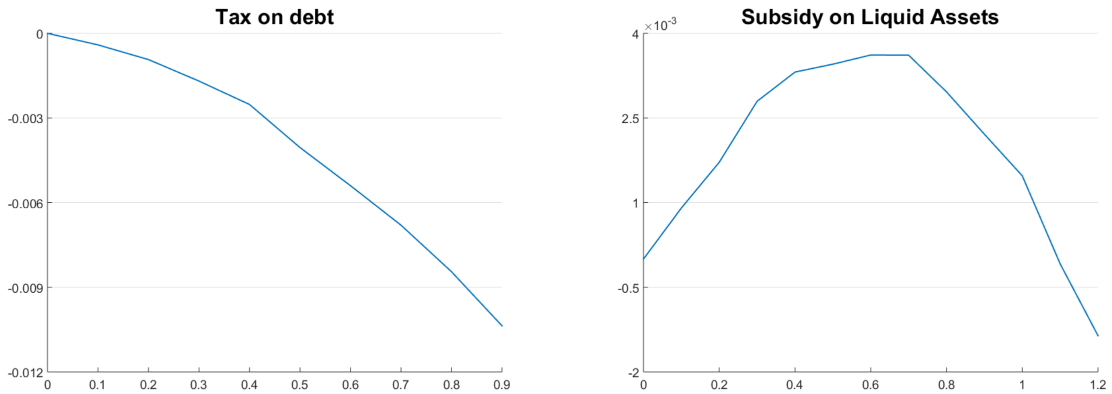
[49] See e.g. Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2018).

Figure 8: Optimal policy during Sudden Stops



Note: Optimal tax on debt and subsidy on liquid assets in % during Sudden Stops.

Figure 9: Welfare gain of simple policies



Note: This figure plots the welfare gain induced by simple policies compared to the unregulated equilibrium in %.

less net borrowing, however, the economy is still experiences worse liquidity crises, since liquid asset holdings are lower. The tax on debt therefore causes welfare losses in two dimensions: agents accumulate more savings, which is costly due to their impatience, but at the same time they experience worse crises.[50]

The intuition for the reduction in precautionary liquidity holdings becomes evident from the Euler conditions for debt and liquid assets of the equilibrium when the regulator

[50]Note that the differences in financial stability are small. The welfare loss therefore mainly comes from the reduction in net borrowing.

implements a fixed tax[51]:

$$\frac{\lambda_t}{R(1+\bar{\tau}^b)} = \beta\mathbb{E}_t\{\lambda_{t+1}\}, \quad (32)$$

$$\lambda_t = \beta\mathbb{E}_t\{\lambda_{t+1}+\xi_{t+1}\}. \quad (33)$$

By increasing the cost of borrowing, a tax on debt drives a wedge between the marginal value of funds today and the marginal value of funds tomorrow in equation (32). In particular, agents now value funds today more as λ_t has to increase relative to λ_{t+1}. This in turn increases the costs of investing in liquid assets, as can be seen in equation (33). The only way how this condition can still hold is through an increase in ξ_{t+1}, which means the liquidity constraint is expected to bind more strongly. This can only occur, if liquid asset holdings are reduced. Importantly, this effect is present for any policy intervention targeting gross debt (e.g. quantity restriction on gross capital inflows) due to the resulting wedge in the optimality conditions for borrowing.

This adverse effect can be offset in different ways. A natural example is to implement a tax $\tilde{\tau}^b$ on net borrowing (b + l), rather than gross borrowing. This yields the following set of inter-temporal optimality conditions:

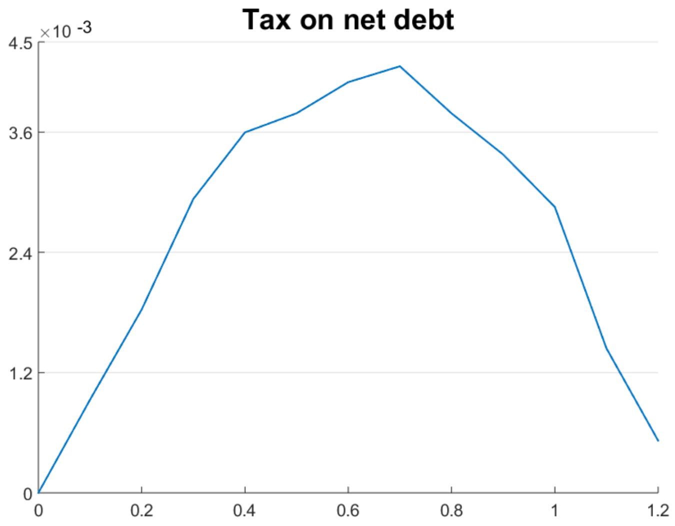
$$\frac{\lambda_t}{R(1+\tilde{\tau}^b)} - \mu_t = \beta\mathbb{E}_t\{\lambda_{t+1}\}, \quad (34)$$

$$\lambda_t\frac{1+(R-1)(1+\tilde{\tau}^b)}{(1+\tilde{\tau}^b)R} - \theta\mu_t - \psi_t = \beta\mathbb{E}_t\{\lambda_{t+1}+\xi_{t+1}\}. \quad (35)$$

Note that the optimality condition for borrowing is unaffected by this change, while the optimality condition for liquid asset now contains a term that lowers the cost of holding liquid assets. This is because, agents understand that borrowing used to finance holdings of liquid assets is essentially tax free. Figure 10 shows the welfare gain associated with such a policy. In contrast to the fixed tax on gross debt, a fixed tax on net debt increases welfare for any tax level below 1.2%. The optimal fixed tax on net debt is given by $\tilde{\tau}^b = .7\%$ and induces a welfare gain of .0043% (13.5% of the welfare gain of the optimal policy).

[51] We consider a situation where the constraint currently doesn't bind

Figure 10: Welfare gain of a tax on net debt



Note: This figure plots the welfare gain induced by a fixed tax on net debt compared to the unregulated equilibrium in %.

6 Conclusions

In the current analysis, we have provided a small open economy model that endogenously generates international borrowing and liquidity holdings, but remains close to a workhorse framework studied in Bianchi (2011). While our model environment remains stylized in many dimensions, it can quantitatively account for the observed behavior of gross goods and capital flows around sudden stops. Furthermore we obtain highly tractable and intuitive results regarding social inefficiencies and optimal policy. In particular, our results show that due to the presence of a pecuniary externality, individual agents overborrow in net debt and underinvest in liquid assets compared to the social optimum. Consequently, a macroprudential regulator needs to intervene in both private borrowing and liquid asset holdings in order to restore constrained efficiency. An optimal policy mix is a combination of macroprudential capital controls and reserve accumulation, combined with suspension of convertibility in severe crises. Importantly, a tax on debt in isolation is detrimental to welfare in our framework, as it reduces agents' incentives to invest in liquid assets and makes the economy more vulnerable to liquidity shocks. We find this result highly relevant for the design of regulatory policy as policies exclusively aiming at gross debt holdings distort incentives to accumulate liquid assets an can thereby increase financial instability.

There are many promising avenues for future research, building on the analysis provided here. Since we have provided a real model, there is no role for liquid reserves in stabilizing the nominal exchange rate. We see explicit exchange rate management by the central bank as an integral part of a theory that aims to fully understand the accumulation of foreign reserves and crises in emerging markets. Another interesting approach would be to model the liquidity dry up in short-term markets endogenously, driven by suppliers expectation of a depreciation and disruption in domestic production. Such a model could endogenize the drop in collateral value of domestic production and thereby address some of the concerns regarding the quantitative realism of the mechanisms proposed here.

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A Microfoundation of the Borrowing Constraints

In this section we show how the constraints 4 and 5 can arise as incentive compatibility constraints in the contracting problem between domestic borrowers and international investors. In principle the contracting friction arises from limited liability coupled with the possibility for borrowers to divert firm output at some cost so that it cannot be observed by creditors. Similar microfoundations are widely used for these types of constraints, see for example Bianchi and Mendoza (2018).

Due to the different debt maturities in our framework, the microfoundation first requires precise definitions of the diversion technology, of the timing of events within a period and of what contracts are possible. While a number of quite technical assumptions are necessary to achieve the result, we nevertheless find the insights of this section interesting. Note that for this microfoundation it is necessary that liquid assets are held by private agents and not the central bank.

Technology and Timing Each household is endowed with tradable h^T and non-tradable labor h^N, both normalized to one. Providing labor causes no disutility to the household so in equilibrium the household always works one unit in each sector. The household can operate two technologies, where non-tradable output is produced according to $Y_t^N = z^N h^N$ and tradable output is produced according to $Y_t^T = z_t (h^T)^{1-\gamma_\nu} \nu_\nu^\gamma$. The household runs separate limited liability firms in each sector, where output is produced. Each household only works in firms it owns, so no wages are paid. These firms hold liquid assets and borrow internationally on behalf of the household. New firms can be opened in the beginning of a period and one household can operate multiple firms in the same sector within a given period. Without loss of generality, we assume that the household owns only one firm in each sector at the beginning of period t. The firm enters the period with inter-temporal debt and a stock of liquid assets. We divide the period into seven sub-periods:

1. The tradable goods firm needs to purchase intermediate inputs to production. At this point it can fund them by taking on trade credit.

2. Before entering production, this firm has an opportunity to divert the purchased inputs at a cost $\kappa_t^B z_t F(\nu_t)$, in terms of the share of potential output. In case of diversion inputs are used in a newly created firm, with no debt outstanding. Within a period, inputs can only be diverted at a very large cost, i.e. $\theta^B = 1$[52]. Diverting firms again face the problem in sub-period 1 and have access to trade credit. In principle resources can be diverted infinitely often within a period, but in equilibrium no diversion ever occurs.

3. Firms in both sectors enter production. Outstanding inter-temporal debt matures and is repaid, defaulting firms are immediately shut down and generate no more cash flows to their owner. Firms can issue new inter-temporal debt to finance repayments and purchase (or sell) liquid assets.[53] Creditors can observe labor and intermediate inputs.

4. Trade-credit matures. At this point firms can already produce enough output to make the payment[54]. A firm that fails to repay trade credit is immediately shut down and has no more opportunity to divert funds.

5. Firms in the tradable sector again decide how many inputs to divert at proportional cost $\kappa^T/Rz_tF(\nu_t)$[55] and in the non-tradable sector at cost $\kappa^N Y^N$ of potential firm output. Liquid assets can be diverted at cost θ/R close to 1[56].

6. Diverted funds are transferred to newly set up firms, with no debt outstanding. They face the same problem as a firm in sub-period 3.[57]

[52] For technical reasons it is necessary to assume that in case of diversion the household still retains the asset, but pays an equal cost for diverting it.

[53] Any increases in the net asset position of non defaulting firms at this point have to be financed through equity injections. This issue generally arises with collateral constraints of this type. Technically it is possible to assume that households have enough private liquid funds available to do so. These funds have to be understood as some (small) endowment, which becomes available only in this sub-period. We don't explicitly include these funds in the model however.

[54] Clearly this is preferable to injecting more equity for firms which plan to divert funds, as it lowers the total amount of diversion costs. Firms which plan not to divert are indifferent between using equity or output to repay trade credit.

[55] κ^T and κ^B differ in our setting, since they occur at different stages of production. The first opportunity to divert occurs before production has even started. Generally we would assume that diversion is easier at this point. Dividing by R is just a normalization, as inter-temporal borrowing is in already in units of repayment next period.

[56] The motivation for this assumption is that trade credit is very short-term, which makes even more difficult to divert liquid assets, so $\theta^B = 1$. During production more time passes, which gives firms some opportunity to divert those assets, however, it remains costly, so θ is close to 1.

[57] As above, we assume that the diversion costs don't diminish the quantity of inputs and liquid assets, but accrue as monetary costs to the household that owns the firm

7. If no diversion occurs output is produced and publicly observed.

As will become clear below, as situation, where κ_t^B falls to 0, provides a microfoundation for the liquidity shock.

Contracts We assume that all debt contracts written between domestic borrowers and international lenders have the following features:

1. There is limited liability: if a firm output cannot repay its debt, the owners' private assets cannot be seized by creditors

2. However, if a firm produces enough *oberservable* output to repay the debt in full after the debt contract has been signed, but pays out dividends to its owners and defaults, then this default is considered fraudulent and private assets can be seized and the owners criminally prosecuted. We assume that, if found guilty of fraud, the household receives $-\infty$ utility.

Incentive Compatibility We first analyze a situation where $\kappa_t^B = 1$. In this case no diversion of inputs can be profitable to the household in sub-period 2 and we can focus on the inter-temporal contract. We now consider the decision to divert funds in sub-period 5. We follow Bianchi and Mendoza (2018) and define the continuation values of a diverting $W^d(b, l, tc)$ and a non-diverting firm $W(b, l, tc)$, where b is the level of intertemporal borrowing, l liquid assets and tc outstanding trade credit. Assume that a diverting firm faces the following problem[58]:

$$W^d(b', l', tc) = max_{\{d, b'', l''\}} \tilde{d} + \beta \mathbb{E} V(b'', l'') \tag{36}$$

s.t.

$$\tilde{d} = (F(\nu) - tc)(1 - \kappa^T/R) + (1 - \theta/R)l' + b''/R - (l'' + l') \tag{37}$$

$$b'' \geq -\kappa^T(z_t F(\nu) - tc) - \theta l'' \tag{38}$$

here we have used the fact that, if the firm chooses to divert funds, it will first produce enough to repay trade-credit to avoid immediate liquidation. The owners will therefore divert at most net tradable output. The value in the no default case is given by[59]

$$W(b', l', tc) = max_d d + \beta \mathbb{E} V(b', l') \tag{39}$$

[58] At this point the borrowing constraint is still a guess. We verify that it is consistent below.
[59] Since we only compare intra-period cash flows, weighing by marginal utilities is unnecessary.

s.t.

$$d = (F(\nu) - tc) \tag{40}$$

I.e. dividends are simply given by the output minus repayment of trade credit. We now guess that optimal choices of a firm after diversion are the same as choices before diversion, so $b'' = b'$ and $l'' = l'$. This implies that the two continuation values are equal. Taking the difference between W^d and W under this assumption implies that the firm will chose not to divert funds if $\tilde{d} \leq d$, i.e.:

$$b' \geq -\kappa^T(z_t F(\nu) - tc) - \theta l'. \tag{41}$$

Which is the same constraint faced by the firm after diversion. Given that value is linear in d and under the constraints $b'' = b'$ and $l'' = l'$ is feasible both before and after diversion, the choices will coincide and our guesses are verified. Once the firm produces output in period t, this is observed by creditors. This means any later default on this debt will be considered fraud. Owners therefore always choose to repay their inter-temporal debt in sub-period 3. This establishes that in sub-period 3 creditors will impose the constraint 41. Note that in sub-period 1 ν can only be purchased using trade credit so $tc = \nu$ in equilibrium[60].The case of a non-tradable firm follows analogously, with the only difference that non-tradable output is weighted by the real exchange rate. Aggregating the two constraints at household level delivers exactly the constraint 4 in the main text.

Next we show that the liquidity constraint 5 is imposed of firms in sub-period 2. A firm that diverts inputs in sub-period 3 transfers all resources it can extract to the household, who sets up a new firm and again has the chance to obtain trade credit. The diverting firm will still raise enough equity[61] to repay its outstanding inter-temporal debt to avoid fraud charges, but then cease operations and default on its trade credit in full. We therefore ignore inter-temporal debt here. Consider a firm, which holds liquid assets l and has raised trade credit to purchase inputs ν. If it defaults diverts its inputs, the firm can again raise trade credit to purchase ν' and continue its normal operations. This firm receives cash flow ν' minus diversion costs of old inputs

$$\nu' - \kappa_t^B z_t F(\nu) - \theta^B l, \tag{42}$$

while a non diverting firm generates no cash flows in this sub period. In the same way as the inter-temporal collateral constraint, the choices ν' and ν coincide and continuation

[60]We assume that suppliers cannot act as intermediaries therefore won't lend more than the value of goods that are sold. In any case the firm has no other use for the funds at this point.

[61]Note that production doesn't happen in time to repay inter-temporal debts, so it makes no sense for the firm to leave productive resources in the firm.

values are exactly the same. The firm will therefore divert whenever it can generate a positive cash flow. This yields the constraint:

$$\nu \leq \kappa_t^B z_t F(\nu) + \theta^B l, \tag{43}$$

We assume that in normal times $\kappa^B \geq \nu^*/[z_t F(\nu^*)]$, so no diversion is never profitable and the constraint never binds. In case of a liquidity shock $\kappa^B = 0$, so only liquid assets can serve as collateral. These facts deliver the constraint 5 in the main text. Note that this microfounds the liquidity shock as a situation where domestic assets lose their function as collateral for trade credit. Furthermore this microfoundation also allows for the stochastic process for κ_t^B to take values between 0 and 1 in which case domestic output and the liquidity constraint will be linked endogenously.

B Average share of imported intermediate inputs in the data

We compute the share of imported intermediate inputs in GDP as follows: The average ratio of total imports to GDP in our main dataset is 27.1%. From the OECD Structural Analysis Databases[62], we obtain data on average shares of intermediate inputs (56%), consumption goods (12%), capital goods (15%) and mixed use goods(16%) in total imports for our set of countries from 2005 to 2018. For Venezuela data is missing after 2013 and Thailand has a missing observation in 2017. Since capital goods are not part of our model, we attribute them fully to intermediate inputs to production. We then assume that mixed use goods are used according to the same shares as the rest of the imports. This yields a ratio of imported intermediate goods to total imports of 85.5%. Combining this with the ratio of imports to GDP yields a ratio of imported imports to GDP of 23.5%.

C Competitive Equilibrium with Taxation

We first derive the optimality conditions and optimal tax rates in section 4.2 in the main text and then provide a discussion of how expectations of private Lagrange multipliers in section 4.1 can consistently understood as arising from an economy, where optimal policies are implemented starting in period t+1.

We assume that taxes are introduced in some period t and are expected to remain forever. Private agents therefore expect to be on the constrained efficient equilibrium

[62] http://www.oecd.org/industry/ind/stanstructuralanalysisdatabase.htm

path from now on. We therefore look for a set of taxes, such that the private first order conditions for borrowing and liquid asset holdings are satisfied on the constrained efficient equilibrium path. Note that future taxes affect expected Lagrange multipliers and expectations are taken accordingly.

Consider a set of (state-dependent) taxes τ_t^b and τ_t^l on end of period bond and liquid asset holdings respectively. Bonds are taxed before interest, i.e. the tax applies to $\frac{b}{R}$. Taxes are implemented such that an agent, who holds 1 unit of a particular asset receives a transfer of $\frac{\tau}{1-\tau}$ (This formulation allows us to write taxes like interest rates, see below.). If holdings of the asset are positive, this transfer is positive and the tax becomes a subsidy, if holdings are negative a tax has to be paid. We find this formulation analytically most convenient, note however that these assumptions are without loss of generality.

The private budget constraint, given the taxes becomes:

$$c_t^N p_t^N + c_t^T + \frac{l_{t+1}}{(1+\tau_t^l)} + \frac{b_{t+1}}{R(1+\tau_t^b)} = [z_t F(\nu_t) - p_\nu \nu_t] + p_t^N y_t^N + l_t + b_t + T_t, \tag{44}$$

where T_t are lump-sum rebates of the tax revenues. No other equations in the private agents' optimization problem are affected, so equations (22) and (23) in the main text follow immediately. The positive part of the tax in equation (24) then follows from equation (18) by (22) where we have used the fact that $\mu_t = \mu_t^{sp} = 0$ and $\lambda_t = \lambda_t^{SP} = u_T(t)$. In cases where the inequality in (24), is not satisfied, the planner chooses to borrow up to the constraint. From equations 9 and 23 it is obvious that in these cases the household LM μ_t is also positive, so no tax is necessary to implement the optimal borrowing level. As said in the main text, however, any tax or subsidy (as long as the household still wants to borrow up to the constraint) implements the constrained efficient borrowing level here. Importantly another choice of the tax rate still affect the level of the multiplier μ_t.

Equation (25) requires more discussion. Note that private expected future LMS shows up in this equation. As just said, however, the private LMs depend on the tax rate on borrowing in states where the constraint binds. The subsidy on liquid assets is therefore not unique, even in states where no constraint currently binds, but depends on future expected taxes.[63] Given a state dependent tax on debt, the multipliers can be computed for every possible state and deliver a unique tax/subsidy on the liquid asset. As above this tax can be found by using equations (20), (17) and (8), (6) and combining equation (19), with (23). Importantly this doesn't imply that the planner can implement the constrained efficient allocation with only one instrument by using the degree of freedom

[63] In this context one might be worried about time consistency of the Ramsey problem. Note, however, that Ramsey planner can implement the constrained efficient solution, so the private inter-temporal optimality conditions are not binding constraints for her and here problem must be time consistent.

once the constraint binds to manipulate the expected private LMs in such a way that (23) holds for $\tau^l = 0$. We conjecture that this is generally impossible, since setting $\tau^l = 0$ pins down τ^b at the constraint to satisfy current the Euler equation for liquid assets. We don't consider verifying this conjecture an interesting exercise, since designing and communicating such a policy would require excessive sophistication.

C.1 Competitive Equilibrium with Reserve Accumulation

If the planner replaces the positive parts of the liquidity subsidy by accumulating reserves at the central bank, the budget constraint of the representative agent becomes:

$$c_t^N p_t^N + c_t^T + \frac{l_{t+1}}{(1+\tau_t^l)} + \hat{l}_{t+1} + \frac{b_{t+1}}{R(1+\tau_t^b)} = [z_t F(\nu_t) - p_\nu \nu_t] + p_t^N y_t^N + l_t + \hat{l}_t + b_t + T_t, \quad (45)$$

The only difference to the budget constraint above is the presence of reserves $\hat{l}$ held by the central bank. These reserves are not chosen by the household itself, but they can serve as collateral for inter-temporal borrowing and will be made available for to households in case of a liquidity shock. In both the collateral and liquidity constraint the relevant amount of liquid assets is therefore $l_{t+1} + \hat{l}_{t+1}$. The presence of central bank reserves reduces private incentives to hold liquid assets. If $\hat{l}_t$ is larger than the competitive choice, households will choose to hold no liquid assets privately. Using similar steps as above, the equations in proposition 2 arise.

C.2 Private LMs in the constrained efficient equilibrium

We now turn to the interpretation of private LMs in section 4.1. To allow comparisons, we evaluate all choices on the constrained efficient equilibrium path throughout that section. Since private agents would make different choices, it is therefore unclear how to interpret their optimality conditions if all decisions are made by the planner. To allow a consistent interpretation, we assume the following:

As long as the inter-temporal borrowing constraint doesn't bind, households are forced to comply with the planners' choices[64]. In these cases we can interpret the (current) residuals in the private Euler equations as household incentives to deviate from constrained efficiency.

Unfortunately we cannot compute meaningful private Lagrange Multipliers μ_t on the inter-temporal borrowing constraint using this approach. If the household is forced to choose the same allocation as the planner, the borrowing constraint doesn't matter to its decisions. This multiplier (and its expected value), however, are central to understanding

[64] Even though we omit the associated Lagrange Multipliers for readability.

the incentives for the liquid asset choice. We therefore assume that, whenever the planner is borrowing constrained, households are allowed to choose borrowing themselves, but liquid assets are still determined by the planner. Due to the presence of the constraint, households choose the same allocation as the planner, but we can compute a Lagrange multiplier on the borrowing constraint μ_t. Under this assumption we can also analyze the differences in incentives for the liquid asset choice, both in the unconstrained and in the constrained case.

It might not be obvious at first sight that, given this set of assumptions, our discussion reflects economically interesting differences in private and social incentives. To alleviate this concern note that it is equally possible to consistently interpret the expectations over μ_{t+1} as private Lagrange Multipliers on the borrowing constraint, assuming that from period t+1 onwards the optimal tax and subsidy defined in equations 24 and 25 are implemented. Given these policies the household consistently expects the constrained efficient equilibrium to be implemented in the future. Moreover, whenever the borrowing constraint actually binds, the tax is set to zero so the multiplier reflects the true shadow value of violating the constraint.

D Computational Solution

Even though our equilibrium conditions are relatively compact, the model has several features which make the computational solution challenging. In particular there are two endogenous states and two occasionally binding constraints. Moreover the model features an amplification mechanism in the price of non-tradables, which introduces a strong non-linearity in the region where the borrowing constraint binds.

We solve both for both competitive and constrained using policy function iteration, using a variant of the algorithm in Wolf (2019). In particular we guess the expectations in the Euler equations 9 and 10 on grids for endogenous states. On the grid we then find the policies that satisfy Euler equations given expectations, which we linearly interpolate between grid points, since optimal choices generally don't lie on the grid. We then update expectations as a convex combination between the previous guess and the value computed from the new policies. We do so by computing equilibria for all possible combinations for binding and non-binding constraints and then choose the equilibrium that satisfies all constraints.

We approximate the AR process for productivity in the tradable sector by a first order Markov process on a 5 point grid using the common Tauchen algorithm. We then choose grids for the endogenous states using 400 points for borrowing and 400 points for liquid asset holdings. Combined with the two states for the liquidity shock, we solve for the policies on 1600 000 grid points in each step. We iterate on the Euler equations until maximum absolute differences in expectations on the grid between iterations are smaller than 10^{-6} in the relevant part of the state space.

We check accuracy by doubling the number of grid points for the endogenous states (the grid becomes 800*800*2*5) and computing differences between the two approximations on the smaller grid. The maximal absolute differences in consumption between the two solutions is smaller than 0.01% while the mean difference is smaller than 0.001%. For the positive analysis of the model, the approximation seems clearly sufficient. Since the welfare gain of the optimal policy is only 0.032% of steady state consumption, one could nevertheless be worried about the accuracy of our normative analysis. Note that by definition the welfare gain of implementing the optimal policy must be weakly positive in the whole state space if the solutions are accurate. As a further robustness check we therefore compare the value function of the competitive and planner solutions on the whole grid. We find that the minimum welfare gain is strictly positive at 0.02% of consumption. We conclude that our solutions are accurate enough for the normative analysis as well.

D.1 Details on the algorithm

The algorithm is complicated by the fact that liquid assets and bonds are perfect substitutes from the point of view of current consumption, as long as the borrowing constraint is not currently binding. This can be seen by the fact that equations 9 and 10 have the same left hand sides in states where no constraint binds. It is therefore not generally possible to satisfy both equations, for a given set of expectations, which makes it impossible to solve for an equilibrium. Note however, that in case of a liquidity shock ν_{t+1} is predetermined by l_t. We use this fact to pull F_ν out of the expectation in the Euler equation for liquid assets, which allows us to solve directly for liquid asset holdings, given expectations.

In some regimes solving for the intra-temporal equilibrium given expectations involves a non-linear system of equations. Solving this system in every iteration would make computation infeasibly slow. We therefore solve the non-linear system in advance on a fine grid, and linearly interpolate in the inner loop. This grid is chosen such that maximal residuals between points are smaller than 10^{-5}.

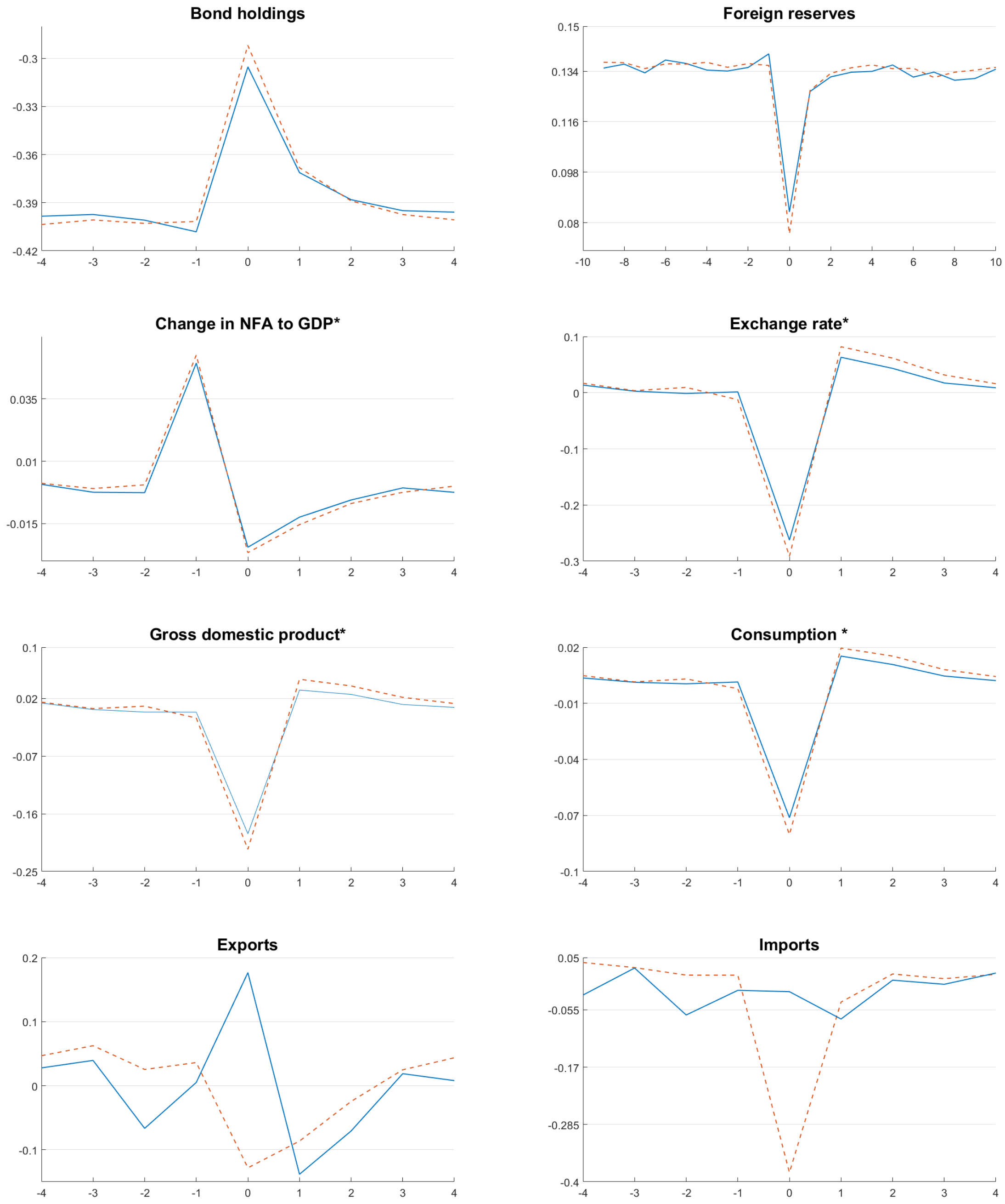
E Sudden stops with and without liquidity shock

Figure 11 compares sudden stops that coincide with a liquidity shock with sudden stops which are purely caused by a decline in productivity. This second type of sudden stops corresponds to the sudden stop events in Bianchi (2011). As can clearly be seen in the figure, imports hardly decline and exports even increase if there is no liquidity shock, as optimal levels of intermediate inputs can still be financed. Liquidity shocks are therefore necessary for the model to generate the observed collapse in gross good flows.

F Sudden stops with a fixed tax on debt

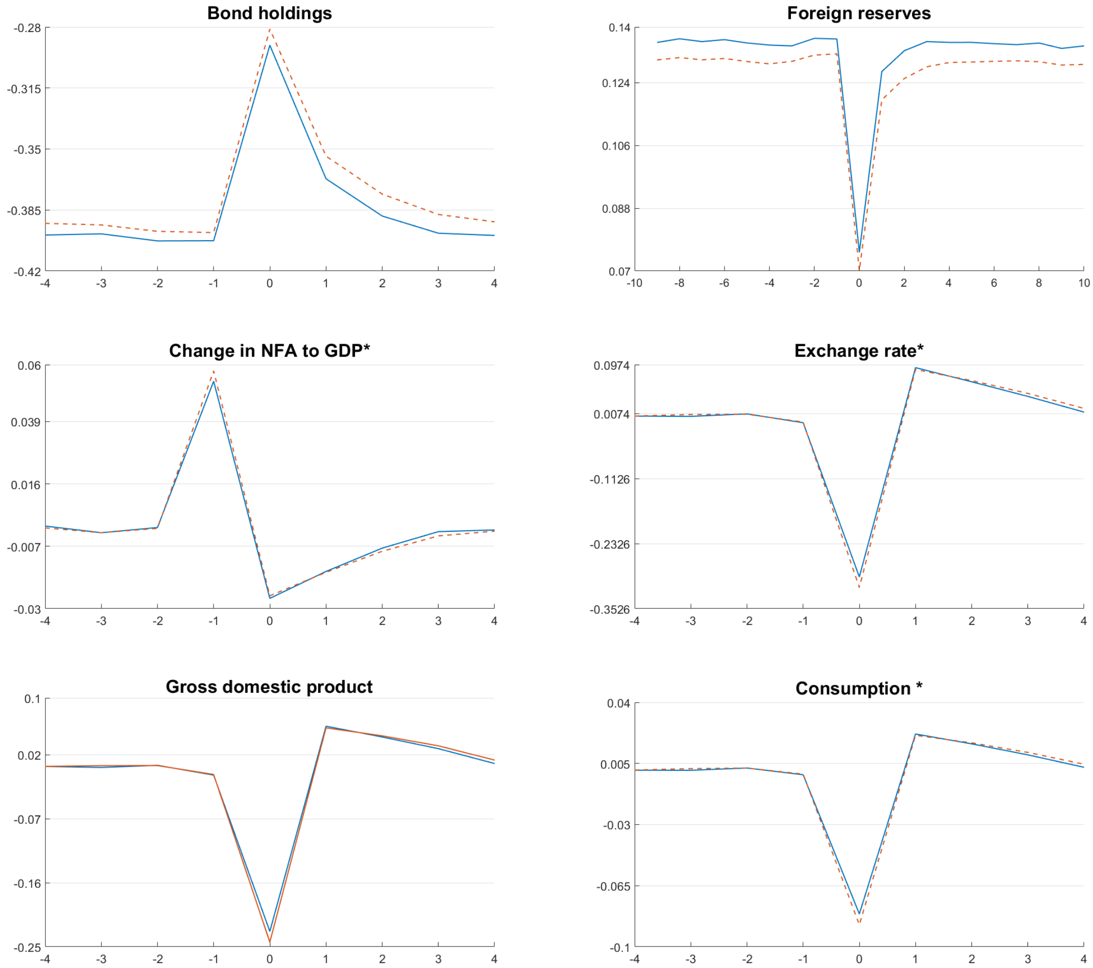
Figure 12 shows a comparison of sudden stops caused by liquidity shocks in the unregulated equilibrium compared to an economy where a 0.5% constant tax on debt is implemented. Again we identify sudden stops in the baseline economy and then average over the paths for the same sets of shocks in the economy with taxation. The graph shows that liquid asset holdings are smaller with the tax in place. Moreover the responses of all variables during the sudden stop are larger in absolute values in the economy where the tax is implemented. This shows that even a small tax is detrimental to financial stability.

Figure 11: Sudden stops without liquidity shocks



Note: The blue solid line corresponds to sudden stop purely caused by declines in total factor productivity, the red dashed line to sudden stops that coincide with a liquidity shock. In order to compare levels, bond holdings and foreign reserves are plotted in absolute values; * are plotted in relative deviations from mean.

Figure 12: Sudden stops with a fixed tax on debt



Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the regulated economy with a .5% tax on debt. In order to compare levels, bond holdings and foreign reserves are plotted in absolute values; * are plotted in relative deviations from mean.