WORKING PAPERS

Martin Wolf

Pecuniary Externalities in Economies with
Downward Wage Rigidity

June 2019

Working Paper No: 1905

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

Pecuniary Externalities
in Economies with Downward Wage Rigidity

Martin Wolf[*]

June 2019

Abstract

We describe a pecuniary externality in economies with downward nominal wage rigidity that leads firms to hire too many workers in expansions, which leads to too much unemployment in recessions. The externality arises because of competitive behavior in the labor market. When firms hire more workers, they push up market wages for all firms. Firms internalize that with higher wages, it is more likely that they will be constrained by downward nominal wage rigidity in the future themselves; however, they fail to internalize the negative effects over other firms. In the calibrated model, when compared to a benevolent planner who chooses labor allocations on behalf of firms, the externality raises the welfare cost of downward nominal wage rigidity by a factor of 10, as it makes the economy significantly more exposed to unemployment crises.

1 Introduction

A longstanding concern in economics, which dates back to at least Keynes, is that in low inflation environments the labor market may not clear because of downward nominal wage rigidity. This concern has been revived recently. For example, the interaction between downward nominal wage rigidity and fixed nominal exchange rate has recently been suggested as a key driver of the unemployment experience during the Great Recession of some countries in the euro area (Schmitt-Grohé and Uribe, 2016). As another example, informed by globally declining rates of interest and inflation, the recent literature on secular stagnation is built on the assumption of downward nominal wage rigidity (e.g., Benigno and Fornaro, 2018; Corsetti et al., 2019; Eggertsson et al., 2017; Fornaro and Romei, 2018).

However, while the previous literature tends to attribute large adverse effects to downward nominal wage rigidity, this appears puzzling when studying the behavior of individual firms which face downward nominal wage rigidity. Elsby (2009) argues that firms behave prudentially: they increase wages by less following positive technology shocks. As a result, employment hardly falls when technology declines. Elsby concludes that, at the firm level, the economic significance of downward nominal wage rigidity is small.

This paper develops an open economy business cycle model to study wage-setting firms facing downward nominal wage rigidity in general equilibrium. We establish that downward nominal wage rigidity can be consistent with large *macroeconomic* effects, even when firms internalize downward nominal wage rigidity and behave prudentially at the *individual level*. The reason is a pecuniary externality that arises because of competitive behavior in the labor market. When firms hire more workers, they push up market wages for all firms. Firms internalize that with higher wages, it is more likely that they will be constrained by downward nominal wage rigidity in the future (as in Elsby 2009); however, firms fail to internalize that they impose negative effects over other firms.

Our model features a large number of firms that compete in the labor market for workers. Nominal wages are downward rigid. Competition is imperfect, which gives firms market power over wages and implies that they are wage setters (Manning, 2003). This implies that firms internalize downward nominal wage rigidity. We show that the free market equilibrium is *constrained inefficient*. Even if a planner can not intervene during recessions to relax downward nominal wage rigidity ex post, a pecuniary externality calls for policy intervention during expansions. When firms hire more workers, they push up market wages for all firms. As a result, a planner that is constrained by the same friction as the private economy demands less labor in expansions, leading to lower wages and hence to less unemployment in recessions.

We trace the externality to competitive behavior of firms in the labor market. Specifically,

we show that the externality becomes stronger when competition between firms for workers becomes more intense. The externality is strongest under perfect competition, the assumption most commonly made in the literature. Under perfect competition, firms are taking wages as given and fail completely to internalize the effects of downward nominal wage rigidity. In contrast, when firms have some market power, they internalize downward nominal wage rigidity to some extent. However, we show that for reasonable degrees of market power, due to the externality, firms necessarily *underestimate* the true social cost of downward nominal wage rigidity.

We study ways to decentralize the constrained-efficient outcome, i.e., to make firms internalize the pecuniary externality. Let a policy maker choose payroll taxes on firms in a Ramsey-optimal fashion. We show that the allocation chosen by the policy maker and the constrained-efficient allocation coincide. Intuitively, a tax on firms reduces labor demand, and hence if chosen optimally, can restore constrained efficiency. As an alternative, the tax can be applied to firms' sales revenue. We also show that equivalently, labor *supply* can be taxed via labor income taxes, but this only works if downward nominal wage rigidity applies to the (net) wage that workers take home after taxes.

In contrast, by previous arguments, policies which enhance competition in the labor market may be counterproductive. Relatedly, we show that cutting down firms' monopsony rents may backfire. This finding may help inform the debate about structural reforms in the euro area (e.g., Eggertsson et al., 2014).[1] Intuitively, firms internalize downward nominal wage rigidity because a binding rigidity has an effect on their profits. Eliminating firms' profits thus undermines firms' incentives to behave prudentially in the face of downward nominal wage rigidity. On the other hand, reducing firms' mark-ups has the conventional static benefit of increasing labor demand. We show that the latter effect dominates when firms' market power is large. Indeed we show that, in the calibrated model, welfare losses are U-shaped in the degree of labor market competition.[2]

We demonstrate that the pecuniary externality has large negative effects on welfare and unemployment. In the quantitative analysis, we apply the model to a set of countries that either peg to the euro or are members of the euro area, calibrating parameters to match unconditional GDP statistics. We show that the mean welfare loss of the constrained-efficient

---

[1] Here we point at policies which enhance labor market competition or reduce firms' mark-ups, while leaving downward nominal wage rigidity unaffected. To the extent that policies simultaneously alleviate downward nominal wage rigidity, the trade-off that we describe disappears.

[2] That welfare may be hump-shaped in the degree of labor market competition has been emphasized by an earlier literature, which focused on the bargaining power of unions. The initial contribution is Calmfors and Driffill (1988). The arguments made are similar: unions with more market power may better internalize the effects of their actions on the economy, which may be welfare improving.

allocation relative to *first best* is small, a loss of 0.025% of permanent consumption.[3] This result echoes again Elsby (2009): when firms internalize downward nominal wage rigidity "correctly" (which from a social standpoint, is the case under the optimal intervention), the welfare cost of downward nominal wage rigidity is small. In sharp contrast, we show that the mean welfare loss under laissez-faire relative to first best is 0.26% of permanent consumption. This implies that the pecuniary externality raises the welfare cost of downward nominal wage rigidity by a factor of 10.[4] The welfare cost reflects a rise in the frequency of deep crises. Under laissez-faire, rationing unemployment is higher than 10% about once every 3.5 years, whereas such crises are absent under the optimal intervention.

Related literature.— We argued that the pecuniary externality arises because of competitive behavior in the labor market. This begs the question about the generality of our results, since most macroeconomic models assume some form of labor market competition. In the textbook real business cycle model, firms hire workers by taking wages as given. In general equilibrium, this raises market wages because the economy moves along an upward-sloping labor supply curve. However in this case, the externality operates only through agents' budget constraints, and thus does not lead to social inefficiencies in line with the first welfare theorem. In the current analysis, this no longer holds because markets are not frictionless. This allows us to show that, when nominal wages are downward rigid, the externality makes the equilibrium (constrained-) inefficient.[5] One paper that highlights the same externality as the present paper is Bianchi (2016), but in Bianchi's work wages are flexible and the need for prudential intervention arises due to a financial friction (high wages make firms' equity constraints more binding)—whereas in our analysis, intervention is necessary due to downward nominal wage rigidity.

The pecuniary externality that we describe is conceptually *distinct* from the externality described by Schmitt-Grohé and Uribe (2016)—a paper which is otherwise closely related as it is also concerned with constrained efficiency in an open economy model with downward nominal wage rigidity. Schmitt-Grohé and Uribe describe an *aggregate demand externality*, which can be tackled via financial markets intervention (e.g., capital controls). In contrast, we describe a *pecuniary externality* that affects firms' hiring. The aggregate demand externality

---

[3] Under first best, the policy maker has the power to relax wage rigidity ex post, e.g., through raising domestic prices / depreciating the nominal exchange rate, or subsidizing labor demand in recessions (e.g., Friedman, 1953; Schmitt-Grohé and Uribe, 2016; Tobin, 1972).

[4] In the baseline calibration to which these numbers refer, we assume perfect labor market competition such that there is no static distortion from firms' mark-ups. The welfare loss therefore isolates the cost imposed by the pecuniary externality.

[5] The same likely applies in labor search markets. In search markets, by hiring workers, firms in equilibrium raise labor market tightness. Because tightness matters for the outside option of workers, this ends up raising wages in all firms. Because this is not internalized by individual firms, the laissez-faire outcome is likely to be constrained inefficient in a context where wages are downward rigid.

does not arise in our model, and hence constrained efficiency could not be restored via financial markets intervention. In an accompanying Online Appendix, we extend the baseline model to demonstrate that, in a richer model, the two externalities may arise simultaneously. We go on to show that in the extended model, financial markets intervention (capital controls) and a tax on labor demand are *jointly* required in order to decentralize the constrained-efficient allocation.[6]

We also show in the Online Appendix that the externality is at work in a context where the market power is with workers (rather than with firms). This case is quite common in business cycle studies with wage rigidity, mostly in the context of Calvo wages (Galí, 2011; Galí and Monacelli, 2016), but also in the context of downward nominal wage rigidity (Benigno and Ricci, 2011). As we show in the Online Appendix, households (unions) raise wages in expansions, not internalizing the rise in market wages as competition pushes up the wages of other households (unions). In a context of downward nominal wage rigidity, this makes the laissez-faire outcome constrained inefficient.

The labor market intervention we describe is prudential, to be distinguished from those policies that try to relax wage rigidity ex post. Our paper thus adds to the literature on macro-prudential intervention. While this literature is mostly concerned with (pecuniary) externalities interacting with financial frictions (e.g., Bianchi, 2011; Bianchi and Mendoza, 2018; Dávila and Korinek, 2018; Jeanne and Korinek, 2010; Lorenzoni, 2008), some recent papers have shifted attention to nominal frictions. Farhi and Werning (2016) provide a generic treatment of inefficiency in economies with nominal rigidities. Korinek and Simsek (2016) and Fornaro and Romei (2018) study economies with nominal rigidities and a zero-lower-bound constraint on policy rates. All of these studies emphasize aggregate demand externalities and the need for financial markets intervention.[7] We study a different kind of externality which affects firms' hiring and which justifies *labor* market intervention.

The remainder of the paper is structured as follows. Section 2 introduces the model. Section 3 presents the normative analysis. Section 4 presents the quantitative analysis. Section 5 concludes. An accompanying Online Appendix contains proofs and derivations as well as model extensions.

---

[6] As we explain in detail in the Online Appendix, Schmitt-Grohé and Uribe (2016) restrict their attention to capital controls intervention by restricting their social planner to respect all private equilibrium conditions other than aggregate demand. As a result, they do not mention nor does their social planner address the pecuniary externality, even though it is at work in the labor market of their model.

[7] Farhi and Werning (2016) study both aggregate demand and pecuniary externalities. However, the pecuniary externality that they describe is different: it arises from incomplete asset markets in the presence of household heterogeneity. It therefore also justifies financial markets intervention.

2 Model

We study a small open economy model with downward nominal wage rigidity and a fixed nominal exchange rate. The economy is small in the sense that foreign variables are taken as given—there is no feedback of domestic developments on foreign variables. The economy is populated by households and firms. Households consume, work and save in (incomplete) international financial markets, taking prices and wages as given. Firms produce a single consumption good which is freely traded across borders. Firms take prices as given, but have some market power over wages. The business cycle is driven by shocks to total factor productivity (TFP).

The key element of the model is to combine market power of firms in the labor market and downward nominal wage rigidity. The assumption of market power implies that firms are wage setters rather than wage takers. The fact that firms are setting wages implies that they internalize downward nominal wage rigidity (Elsby, 2009). We use the model to show that, even when firms internalize downward nominal wage rigidity and act prudentially, their individual and the social incentives may not be aligned, such that downward nominal wage rigidity can be consistent with large macroeconomic effects.

2.1 Households

The economy is populated by a large number of households, which maximize utility of consumption net of disutility from work

$$E_0 \sum_{t \geq 0} \beta^t U(C_t - G(H_t)), \quad \beta \in (0,1), \tag{1}$$

where U has the constant relative risk aversion form and where $G(H_t) = H_t^{1+\varphi}/(1+\varphi)$, where $1/\varphi > 0$ is the Frisch elasticity of labor supply. The budget constraint is

$$P_t C_t + \frac{B_{t+1}}{R} = \int_0^1 W_t(i) H_t(i) + \Pi_t(i) di + B_t. \tag{2}$$

Here C_t denotes consumption, P_t the domestic price level, $W_t(i) H_t(i)$ and $\Pi_t(i)$ are labor income and profits accruing from firm $i \in [0,1]$, respectively, and B_{t+1} are nominal bonds which are traded across border at price $1/R > 0$. In (1) we assume that households have Greenwood-Hercowitz-Huffman (GHH) preferences. GHH preferences are commonly used in international business cycle models and also in the literature studying macro-prudential intervention (see for example Bianchi, 2016; Bianchi and Mendoza, 2018; Mendoza and Yue, 2012). As is well known, GHH preferences eliminate the wealth effect on labor supply which prevents a counterfactual increase in labor supply during crises. The assumption of a small

income elasticity of labor supply at business cycle frequency is also supported empirically (Galí et al., 2012; Schmitt-Grohé and Uribe, 2012).[8]

We may think of each household as consisting of a large number of workers, and pooling their resources. In the budget constraint (2), we assume that workers supply labor to (and receive profits from) a unit mass of firms, as in Benigno and Ricci (2011). This implies that total income at the household level is $\int_0^1 W_t(i)H_t(i) + \Pi_t(i)di$.

Households are taking wages as given and attempt to direct labor supply to those firms that pay the highest wage. Formally, in each period they maximize

$$\max_{(H_t(i))_{i \in [0,1]}} \int_0^1 W_t(i)H_t(i)di \quad \text{s.t.} \quad H_t \equiv \left(\int_0^1 H_t(i)^{1+\frac{1}{\eta}} di \right)^{1/\left(1+\frac{1}{\eta}\right)}, \quad \eta > 0. \tag{3}$$

As we show in the Appendix, problem (3) has an interior optimum characterized by a set of firm-specific labor supply curves

$$H_t(i) = \left(\frac{W_t(i)}{W_t} \right)^\eta H_t, \quad i \in [0,1], \tag{4}$$

where $W_t \equiv (\int_0^1 W_t(i)^{1+\eta} di)^{1/(1+\eta)}$ is the appropriate wage index. The parameter η captures that jobs offered by different firms are not perfect substitutes for workers (unless $\eta = \infty$). As (4) shows, the parameter η is also measuring the wage elasticity of labor supply that is faced by the individual firm.

As (4) reveals, a firm which pays a higher wage $W_t(i)$ receives a larger labor supply $H_t(i)$. Conversely, when the wage elasticity is less than infinite ($\eta < \infty$), a firm may pay a strictly lower wage than its competitors ($W_t(i) < W_t$) and still not lose all of its workers. Intuitively, this set-up captures the idea that frictions in the labor market exist, whereby workers find it difficult to quickly change their employer (Manning, 2003).[9] The fact that not all workers leave immediately the firm whenever it cuts the wage of its workers by one cent gives the firm market power. The market power is on the labor demand side, making firms *monopsonistic competitors*. The lower elasticity η, the stronger the market power for firms. For $\eta \to \infty$, the model nests the case of perfect labor market competition.

In addition, the labor market is characterized by downward nominal wage rigidity. Following Schmitt-Grohé and Uribe (2016), we assume that nominal wages cannot fall (much)

---

[8] In the Appendix, we present the model with preferences that allow for a wealth effect on labor supply, and we discuss how this changes our conclusions. In a nutshell, the constrained-efficient planner charges capital controls in addition to prudential labor taxes, because wealth effects impact labor supply and thus the labor market which suffers from the pecuniary externality. This is different than in the analysis by Schmitt-Grohé and Uribe (2016), where capital controls are charged to address a demand externality. See the Appendix for further clarification of this difference.

[9] For example, these frictions may include ignorance among workers about labor market opportunities, mobility costs, or firm-specific non-pecuniary benefits.

below their previous-period level

$$W_t(i) \geq \psi W_{t-1}(i), \quad \psi \geq 0, \ i \in [0,1]. \tag{5}$$

Note that because nominal wages are firm-specific, we assume that downward nominal wage rigidity applies at the firm level. In equilibrium, this will imply stickiness also for the aggregate wage W_t.

Replacing $H_t(i)$ by (4) and using the definition of W_t and H_t, we may rewrite total wage income as $\int_0^1 W_t(i) H_t(i) di = W_t H_t$. Using this in the budget constraint (2) allows us to derive aggregate labor supply. It is given by

$$G'(H_t) \leq \frac{W_t}{P_t}, \tag{6}$$

where the marginal utility of consumption does not appear because there is no wealth effect on labor supply by assumption. Equation (6) need only hold with a weak inequality for when downward nominal wage rigidity binds, firms at the aggregate may demand less hours than households are willing to supply.

Finally, taking first order conditions with respect to consumption and bonds gives the consumption Euler equation

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}}, \tag{7}$$

where we define $U'(t) \equiv U'(C_t - G(H_t))$.

2.2 Firms and the labor market

Firms are owned by the households, taking prices as given in the goods market but setting wages in the labor market. This implies that firms internalize downward nominal wage rigidity, because a binding rigidity reduces their (monopsony) profits.

The standard reference for wage setting firms' internalizing downward nominal wage rigidity is Elsby (2009). However, it has to be noted that our set-up differs from the one in Elsby along a few dimensions. In Elsby, single-worker firms face a labor effort supply function that has a kink at $W_t = W_{t-1}$. As a result, downward nominal wage rigidity arises *endogenously* as a part of the firms' optimal choice whereas we impose it exogenously. We assume downward nominal wage rigidity to be exogenous to make the analysis tractable. While Elsby's model is essentially a partial equilibrium framework, this allows us to focus on the general equilibrium response when many firms interact.

We assume that firms face the technology $Y_t(i) = a_t F(H_t(i)) = a_t H_t(i)^\alpha$, where $\alpha \in (0,1)$ is a parameter and where a_t denotes aggregate TFP.

Definition 1. [FIRM PROBLEM] *Firm $i \in [0, 1]$ solves the following dynamic problem*

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i), W_t(i))} \left\{ U'(t) \left(a_t F(H_t(i)) - \frac{W_t(i)}{P_t} H_t(i) \right) + \beta E_t \Gamma_{t+1}(W_t(i)) \right\}$$

subject to the set of constraints

$$i) \qquad\qquad\qquad H_t(i) \leq (W_t(i)/W_t)^\eta H_t,$$

$$ii) \qquad\qquad\qquad W_t(i) \geq \psi W_{t-1}(i),$$

by taking as given the aggregate variables $\{a_t, P_t, H_t, W_t, U'(t)\}$.

In the maximization in Definition 1, the value function Γ_t denotes the present value of utility-weighted real period-profits, which has time index t for it depends on aggregate states. Profits are utility-weighted for firms are owned by the households. Note that firms face both (4) and (5) as a constraint, but (4) holding only with a weak inequality. Firms may face a large individual labor supply but, because of downward nominal wage rigidity (5), decide to employ only a fraction of the workers.[10] In equilibrium, all firms make identical decisions so that index $i \in [0, 1]$ disappears, which implies that in equilibrium, households supply individual labor *with equality*—as we anticipated in (4)—and the rationing of employment arises purely from aggregate labor supply (6).[11]

We now present aggregate labor demand. We solve the firms' problem in the Appendix and present here the optimality conditions after imposing symmetry (assuming symmetric initial conditions, then setting $W_t(i) = W_t$ and $H_t(i) = H_t$ for all $i \in [0, 1]$). Aggregate labor demand is different depending on whether downward nominal wage rigidity (5) is slack, binds "lightly" or binds "strongly". We discuss each case in turn.

Assume first that downward nominal wage rigidity (5) is slack. In this case, the aggregate labor demand curve is

$$a_t F'(H_t) = \frac{\eta + 1}{\eta} \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}. \tag{8}$$

Moreover in this case, aggregate labor supply (6) holds with equality. In labor demand (8), $\lambda_t \geq 0$ is a non-negative multiplier which measures the (shadow) increase in the utility value of the present value of firms' real profits when downward nominal wage rigidity is relaxed by a marginal unit. Formally, λ_t is the Lagrange multiplier attached to constraint ii) in the maximization in Definition 1.

---

[10] This happens whenever $a_t F'(H_t(i)) < W_t(i)/P$, in which case hiring the full labor supply would reduce firms' profits. Instead, an optimizing firm chooses to ration employment according to $a_t F'(H_t(i)) = W_t(i)/P_t$.

[11] In the general case where firms ration labor supply asymmetrically, households' intra-period labor supply problem changes because some firms (but not all) ration labor supply. To save on notation, in (4) we anticipate the symmetric equilibrium and thus specify individual labor supply with equality.
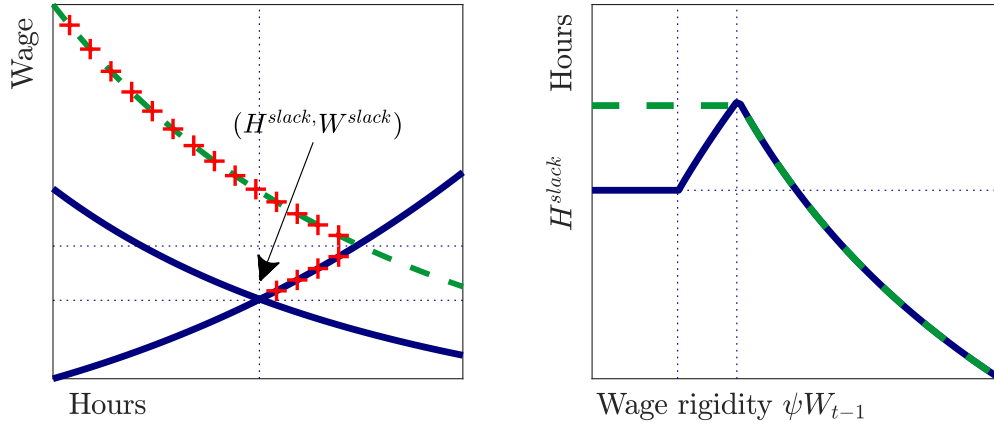
Figure 1: Stylized representation of labor market.

To build intuition, it helps to contrast (8) to the labor demand curve under perfect competition: by taking the limit $\eta \to \infty$, we recover the familiar expression $a_t F'(H_t) = W_t/P_t$. Compared to perfect competition, the labor demand curve is *shifted to the left*. Wages are lower, first, because of a mark-up but more importantly, second, because firms internalize downward nominal wage rigidity (5) (the term $(1/U'(t))(1/\eta)(W_t/H_t)\beta\psi E_t\lambda_{t+1} \geq 0$). This echoes Elsby (2009): internalizing that downward nominal wage rigidity may bind in the future, this leads firms to reduce current wages and hiring.

Figure 1 depicts this fact graphically as it provides a stylized representation of the labor market in this model. The left panel shows the labor demand curve (blue downward sloping) which is located to the left compared to the case of perfect competition (green dashed). The blue upward-sloping line is aggregate labor supply—(6) holding with equality. Correspondingly, the intersection $(H^{\text{slack}}, W^{\text{slack}})$ is the equilibrium when downward nominal wage rigidity is slack. This equilibrium obtains when ψW_{t-1} is sufficiently low (the leftmost part in the right panel).

Assume now that downward nominal wage rigidity (5) binds. Assume first that it binds "lightly", by which we mean that ψW_{t-1} is not much larger than W^{slack}. It is now optimal for firms that hours are determined by labor supply: wages are determined by $W_t = \psi W_{t-1}$ and hours are determined by (6) holding with equality. Intuitively, at $(H^{\text{slack}}, W^{\text{slack}})$ it holds that $a_t F'(H_t) > W_t/P_t$, from (8). As downward nominal wage rigidity binds, this raises wages and increases labor supply. Firms are willing to absorb the additional labor supply, as long as the marginal contribution of workers to profits is still positive.

In Figure 1 the right panel, the intermediate region is depicted by the part of equilibrium hours that slopes upward in wages, between the two vertical lines. This is a classical finding in monopsonies. For example, Manning (2003) points out that "a minimum wage that just

9

binds must raise employment". Empirical evidence for this effect in the context of minimum wages is also discussed in Manning (2003).[12] The size of the intermediate region depends on firms' market power, and it disappears when firms' market power is small ($\eta \to \infty$).

Finally, when (5) binds strongly (i.e., ψW_{t-1} is sufficiently larger than W^{slack}), the labor market is rationed as employment is determined purely by labor demand. In this case, the relevant labor demand curve is

$$a_t F'(H_t) = \frac{W_t}{P_t}, \tag{9}$$

which coincides with the labor demand curve under perfect competition: firms reduce their mark-ups *endogenously* to zero when downward nominal wage rigidity binds. Labor supply is rationed as the weak inequality in (6) becomes strict. Turning back to Figure 1, in the right panel, the region where (5) binds strongly is depicted by the part of equilibrium hours that slopes downward in wages, to the right of the vertical lines.

Mirroring the right panel in Figure 1, the red pluses in the left panel depict how equilibrium hours change—by tracing their movement along the labor demand and supply curves—as downward nominal wage rigidity becomes gradually more binding.

Labor demand by firms is fully characterized once we state an expression for the multiplier λ_t appearing in (8):

$$\lambda_t = -U'(t)\eta \frac{H_t}{W_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{W_t}{P_t}\right) + \beta\psi E_t \lambda_{t+1}. \tag{10}$$

More details on the firms' problem can be found in the Appendix.

2.3 Monetary policy

We assume that all goods are identical and freely traded internationally. Therefore, the law of one price pins down P_t as the price of these goods that prevails internationally $\bar{P}_t$ times the nominal exchange rate $\mathcal{E}_t$ (the price of foreign in terms of domestic currency)

$$P_t = \mathcal{E}_t \bar{P}_t,$$

where $\bar{P}_t$ is exogenous from the vantage point of the domestic economy. Note that monetary policy, by raising the $\mathcal{E}_t$, could raise domestic prices. As this reduces the real value of wages, doing so is useful in an environment where nominal wages are downward rigid (Friedman, 1953). However, we now assume that the nominal exchange rate is fixed

$$P_t = \bar{P}_t. \tag{11}$$

Thus in fixing $\mathcal{E}_t = 1$, our small open economy loses control over its price level.

---

[12] A similar employment effect of minimum wages may also arise in a search-and-matching context, see Flinn (2006).

2.4 Market clearing and definition of equilibrium

In equilibrium, wages and profits correspond to total output: $W_t H_t + \Pi_t = P_t a_t F(H_t)$. As a result, the households' budget constraint (2) becomes

$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t, \tag{12}$$

which constitutes the resource constraint of the domestic economy.

We are now in the position to state the definition of equilibrium.

Definition 2. [FREE MARKET EQUILIBRIUM] *A free market equilibrium is a set of processes* $\{P_t, C_t, H_t, B_{t+1}, W_t, \lambda_t\}_{t \geq 0}$ *such that equations* (7), (10)-(12) *as well as either*

    i) [slack] (6) *with equality and* (8), *if* $W_t \geq \psi W_{t-1}$, *or, if not*

    ii) [binds lightly] (6) *with equality and* $W_t = \psi W_{t-1}$, *if* $a_t F'(H_t) \geq W_t / P_t$, *or else*

    iii) [binds strongly] (9) *and* $W_t = \psi W_{t-1}$,

where $U'(t) \equiv U'(C_t - G(H_t))$, *for given initial conditions* $W_{-1} > 0$ *and* B_0, *and for a given exogenous process* $\{a_t, \bar{P}_t\}_{t \geq 0}$, *are all satisfied.*

We emphasize again that our set-up nests the perfect competition case, in which case the definition of equilibrium reduces to the following. Taking the limit $\eta \to \infty$, the labor demand curves when downward nominal wage rigidity is slack (8) and when it is strongly binding (9) coincide. This implies that firms always equate their marginal product to the real wage and that region ii) in Definition 2 disappears. For labor supply, this implies that (6) holds with equality if $W_t \geq \psi W_{t-1}$ and that, if (6) holds with strict inequality, wages must be constrained by downward nominal wage rigidity.

3 Normative analysis

This section presents the central findings of our analysis, proceeding in three steps. Section 3.1 shows that the free market equilibrium is constrained inefficient by solving the problem of a benevolent planner. Section 3.2 traces the inefficiency to a pecuniary externality. Section 3.3 discusses implications for policy.

3.1 Constrained efficiency

We study a benevolent planner with restricted planning abilities. Specifically, following the analysis in Bianchi (2016), we consider a planner that chooses labor allocations on behalf of firms, but lets all remaining markets clear competitively. The planner is subject to the

same downward nominal wage rigidity constraint as the free market equilibrium. The planner chooses all firms' hiring decisions which are chosen to be identical in equilibrium: $W_t(i) = W_t$ and $H_t(i) = H_t$ for all $i \in [0,1]$.

Definition 3. [CONSTRAINED EFFICIENCY] *The constrained-efficient allocation solves*

$$\max \ E_0 \sum_{t \geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$i) \qquad\qquad\qquad W_t/P_t \leq a_t F'(H_t)$$

$$ii) \qquad\qquad\qquad G'(H_t) \leq W_t/P_t$$

$$iii) \qquad\qquad\qquad W_t \geq \psi W_{t-1}$$

$$iv) \qquad\qquad P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t$$

$$v) \qquad\qquad U'(t)/P_t = \beta R E_t(U'(t+1)/P_{t+1})$$

$$vi) \qquad\qquad\qquad P_t = \bar{P}_t,$$

where $U'(t) \equiv U'(C_t - G(H_t))$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t \geq 0}$.

The planner lets all markets other than labor demand clear competitively—labor supply ii) and borrowing decisions v). The planner respects technology and resource constraints—constraint iv). The planner respects downward nominal wage rigidity, constraint iii).

Constraint vi) imposes that the planner cannot use the nominal exchange rate to raise domestic prices (recall Section 2.3). Constraint i) imposes that the planner cannot subsidize labor demand. Without *either* constraint i) or vi), the planner could implement the first-best amount of employment. Regarding constraint i), when downward nominal wage rigidity binds ($W_t = \psi W_{t-1}$), choose H_t according to $a_t F'(H_t) = G'(H_t)$, then let $W_t/P_t > a_t F'(H_t)$. That is, firms continue to hire even though the marginal product lies strictly below marginal cost. In this case the planner thus subsidizes firms' hiring. Regarding constraint vi), to raise employment the planner could devalue the real value of wages by raising domestic prices, the argument for flexible exchange rates made in Friedman (1953). By imposing constraints i) and vi), we therefore rule out that the planner can use either *external* or *fiscal* devaluation to implement the first-best amount of employment (see Friedman 1953, Farhi et al. 2014 or Schmitt-Grohé and Uribe 2016). We thus impose that the planner is facing the same constraints imposed by downward nominal wage rigidity as the free market equilibrium. The

planner must act prudentially, for she can not intervene ex-post in recessions by raising labor demand.[13]

By construction, the constrained-efficient and free market equilibrium can differ in only one dimension: the labor demand curve. By stating the labor demand curve, we fully characterize the constrained-efficient equilibrium. This yields our main proposition.

Proposition 1. *The free market equilibrium is constrained inefficient.*

Proof. As shown in the Appendix, in the constrained-efficient equilibrium, the labor demand curve when downward nominal wage rigidity is slack is given by

$$a_t F'(H_t) = \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}^{sp}, \tag{13}$$

where the Lagrange multiplier λ_t^{sp} ("*sp*" indicating social planner) associated with downward nominal wage rigidity (constraint iii) in Definition 3) is given by

$$\lambda_t^{sp} = -U'(t) \left(\varepsilon_t^F \frac{H_t}{W_t} \left(\frac{W_t}{P_t} - G'(H_t) \right) + \varepsilon_t^G \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{W_t}{P_t} \right) \right) + \beta \psi E_t \lambda_{t+1}^{sp}. \tag{14}$$

In (13) and (14), $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$ denote the wage elasticities of aggregate labor demand and supply (constraints i) and ii) in Definition 3), respectively.[14]   □

We define the constrained-efficient equilibrium along the lines of Definition 2, replacing, first, labor demand (8) by its constrained-efficient counterpart (13), and second, the multiplier λ_t in (10) by its constrained-efficient counterpart λ_t^{sp} in (14). This implies that the constrained-efficient equilibrium can be visualized according to Figure 1. Indeed, the only difference with the free market equilibrium would be that the labor demand curve when downward nominal wage rigidity is slack lies in a different position.

3.2 Pecuniary externality

When comparing labor demand in the free market equilibrium (8) and constrained-efficient labor demand (13), we note three differences.

The first difference is a mark-up $(\eta + 1)/\eta > 1$ which reflects that firms have some market power over wages. The mark-up is larger, the lower the elasticity of labor supply that is faced

---

[13] Of course, should external or fiscal devaluation in practice be available, these are always to be preferred for they implement the first-best (rather than a constrained-efficient) amount of employment.

[14] Aggregate labor demand is

$$F'^{-1} \left(\frac{W_t}{P_t} \frac{1}{a_t} \right) = H_t = H_t(W_t).$$

The elasticity is defined as $\varepsilon_t^F \equiv H_t'(W_t)(W_t/H_t) < 0$. It is negative because F is assumed to be strictly concave. Elasticity ε_t^G is defined symmetrically for aggregate labor supply.

by the individual firm η. This inefficiency is well understood and would be present even in the absence of downward nominal wage rigidity.

The second difference is that both allocations feature a different shadow value of marginally relaxing downward nominal wage rigidity, $\lambda_t \neq \lambda_t^{sp}$. This difference arises because for firms λ_t represents the utility-value of transferring higher rents to households by relaxing downward nominal wage rigidity by a marginal unit. It is well understood that firms misperceive the social value of their rents from market power.[15]

The third difference represents the pecuniary externality. It is reflected in the fact that firms use the wage elasticity $\eta > 0$ to discount the expected utility loss of downward nominal wage rigidity whereas the planner uses the wage elasticity $\varepsilon_t^G > 0$. In the case of $\eta > \varepsilon_t^G$, firms discount the utility loss more strongly than does the planner, implying that they *underestimate* the true utility cost of downward nominal wage rigidity. We will argue shortly that $\eta > \varepsilon_t^G$ is the case that is most plausible empirically.

To understand the pecuniary externality, take a look at Figure 2.[16] Shown are (aggregate) labor demand (when downward nominal wage rigidity is slack) and labor supply. The constrained-efficient equilibrium corresponds to point D, the intersection of labor supply and constrained-efficient labor demand. The planner finds it optimal to reduce employment below the frictionless level $a_t F'(H_t) = G'(H_t)$, which corresponds to point B. The Harberger triangle represented by the ABD area denotes the second-order welfare loss of restricting employment below the frictionless level. The benefit of doing so is that wages are lower at point D relative to point B. This generates a first-order welfare gain, because it reduces the expected future cost of downward nominal wage rigidity.

The free market equilibrium is given by point C. When firms have market power over wages ($\eta < \infty$), equilibrium hours (and therefore wages) are reduced relative to the frictionless level (point B)—as in the constrained-efficient equilibrium—due to firms' internalizing downward nominal wage rigidity.[17] However, hiring and wages are not reduced as much as in the constrained-efficient equilibrium. This difference reflects the pecuniary externality. Imagine the economy is initially in point D, the constrained-efficient equilibrium. Firms perceive

---

[15] In the Appendix we present the problem of a *single* monopsonist. Compared to the case of monopsonistic competitors that is considered in the main text, the single monopsonist internalizes the pecuniary externality but still exercises market power. This allows us to separate these two effects on the efficiency properties of equilibrium. We find that the monopsonist uses the same λ_t as do the monopsonistic competitors, while he uses elasticity ε_t^G as does the constrained-efficient planner. We conclude that $\lambda_t \neq \lambda_t^{sp}$ represents a distortion due to market power, rather than a distortion due to the pecuniary externality.

[16] The figure is inspired by Bianchi (2016). We write short for $\Psi_t \equiv (1/U'(t))(1/\eta)(W_t/H_t)\beta\psi$, and equivalently for Ψ_t^{sp} under the constrained-efficient allocation, in order to enhance visibility.

[17] In this discussion, to make transparent the effects of the externality, we ignore that changes in η also shift the labor demand curve due to changes in the monopsonistic mark-up $(\eta + 1)/\eta$. Both effects go, in fact, in the same direction: as η falls, labor demand shifts unambiguously to the left.
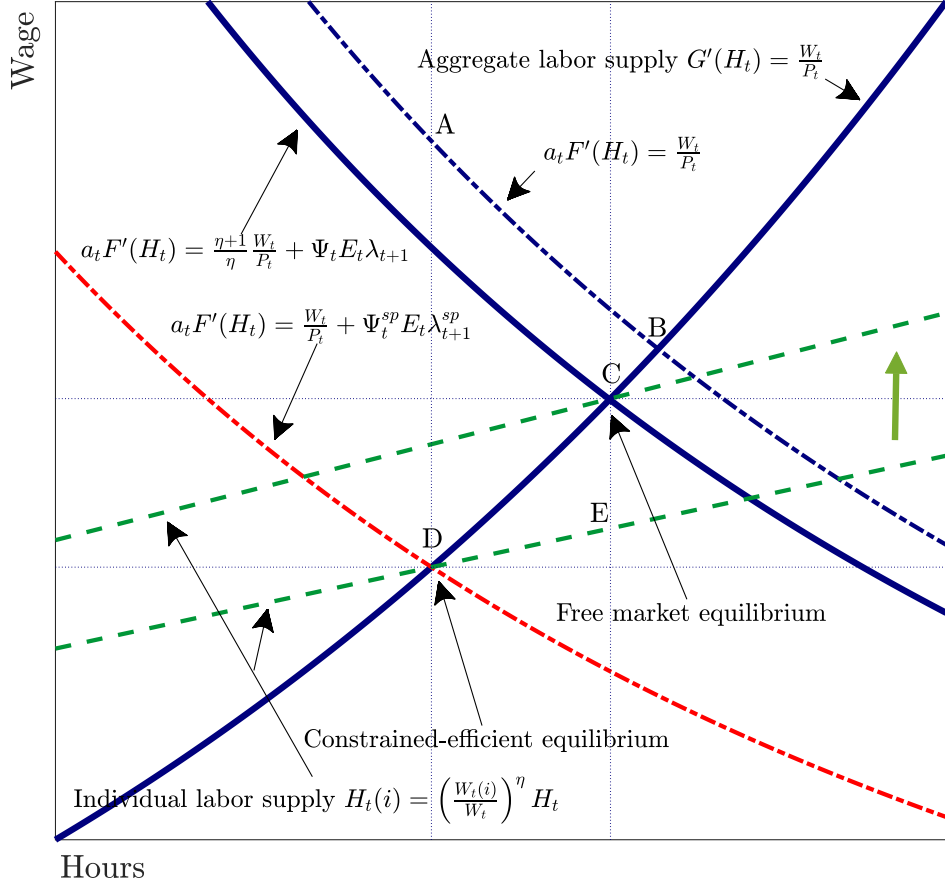
Figure 2: Illustration of pecuniary externality.

that, should they hire additional workers associated with point E, this raises the wage that they pay their workers $W_t(i)$ to the level associated with point E. This is because the green dashed-dotted line denotes labor supply as faced by the individual firm. Firms internalize that when wages are higher at point E, this makes it more likely that they will be constrained by downward nominal wage rigidity in the future.

However, what firms fail to internalize are the negative effects over other firms. In general equilibrium, the market wage rises alongside the wage paid by the individual firm, reflecting that W_t and $W_t(i)$ move in parallel. In the figure, this gives rise to an *upward shift* of firm-specific labor supply (the green arrow pointing upwards), which now passes through the new equilibrium point C. Note that relative to point E, wages have increased further. Because the movement from point E to point C is due to a general equilibrium effect, it is not internalized by individual firms. This implies that firms do also not internalize the greater risk of being constrained by downward nominal wage rigidity in the future that is associated with the higher equilibrium wage at point C.

Point E lies below point C because we have drawn individual labor supply *flatter* than aggregate labor supply. This is because we have assumed that $\eta > \varepsilon_t^G$. It follows immediately that labor market competition is driving the externality, and that the externality becomes stronger when labor market competition becomes more intense. When η increases, point C moves closer to point B because firm-specific labor supply becomes flatter, with two implications. First, firms have incentives to hire even more workers as wages at point E hardly rise relative to point D. Second, a smaller part of the overall wage increase between point D and point C is internalized by individual firms. This also implies that the externality is strongest under perfect labor market competition ($\eta = \infty$), in which case firm-specific labor supply is completely flat.

We now argue that $\eta > \varepsilon_t^G$ is empirically reasonable. Recall that η (ε_t^G) reflects the wage elasticity of firm-specific (aggregate) labor supply. In our model, the latter is simply the Frisch elasticity: $\varepsilon_t^G = 1/\varphi$. The inverse Frisch elasticity φ is a controversial parameter, for micro and macro estimates of this parameter generally do not coincide (Keane and Rogerson, 2012). However, Galí (2011) notes that most of the literature assumes a value for φ in between 1 and 5. If we follow the literature, then the requirement imposed by $\eta > \varepsilon_t^G$ is that η must exceed a number in between 0.2 and 1. Even in the most conservative case of $\eta = 1$, the implied mark-up by firms is $(\eta + 1)/\eta = 200\%$, which appears to be an unreasonably strong degree of market power.[18]

3.3 Implications for policy

Because the externality affects firms' labor demand, we show first that policies which change firms' labor demand can be used to decentralize the constrained-efficient equilibrium. We consider a tax $\tau_t^w \geq 0$ levied on the payroll paid by firms, rebated lump-sum $\mathcal{T}_t$ to firms in equilibrium. Taxing firms' sales revenue is an alternative. Either of the two taxes works, because they reduce labor demand in expansions. If appropriately chosen, this makes firms internalize exactly the pecuniary externality.

---

[18] This being said, some estimates in Manning (2003) of the labor supply elasticity faced by individual firms are as low as $0.75 - 1.5$. Other articles obtain higher estimates, e.g., Ransom and Sims (2010) obtain a value for $\eta = 3.7$. Overall, as summarized in Depew and Srensen (2013), the literature tends to finds values for η in between 1 and 10. Another more recent summary article about estimates of the firm-specific labor supply elasticity is Sokolova and Sorensen (2018).

If a payroll tax is levied on firms, labor demand in the free market equilibrium becomes[19]

$$a_t F'(H_t) = \frac{(\eta+1)}{\eta} \frac{W_t(1+\tau_t^w)}{P_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1} \tag{15}$$

when downward nominal wage rigidity is slack, and

$$a_t F'(H_t) = \frac{(1+\tau_t^w)W_t}{P_t}, \tag{16}$$

when downward nominal wage rigidity is strongly binding. The multiplier λ_t becomes

$$\lambda_t = -U'(t)\eta \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{(\eta+1)}{\eta} \frac{(1+\tau_t^w)W_t}{P_t} \right) + \beta \psi E_t \lambda_{t+1}. \tag{17}$$

In the Appendix we present equilibrium conditions (15)-(17) in case policy makers tax instead firms' sales revenue.

A *regulated* free market equilibrium is defined along the lines of Definition 2, once we replace equations (8)-(10) with (15)-(17). All other equilibrium conditions are unchanged from the economy without intervention. The regulated free market equilibrium depends on the path $\{\tau_t^w \geq 0\}_{t \geq 0}$ that is chosen by policy. This yields our second proposition.

Proposition 2. *[DECENTRALIZATION] Consider the Ramsey problem of maximizing* (1) *over regulated free market equilibria. The outcome of the Ramsey problem coincides with the constrained-efficient equilibrium.*

Proof. In the Appendix. $\qquad\square$

Some remarks regarding the last proposition are in order. First, recall that, if the government could set negative taxes, it would choose to *subsidize* firms in recessions, thereby effectively *undo* downward nominal wage rigidity. Imposing the constraint $\tau_t^w \geq 0$ therefore serves the same purpose as imposing constraint i) in Definition 3 of the constrained-efficient planner. Second, since the problem of the constrained-efficient planner is time consistent, the policy problem in Proposition 2 is also time consistent (Bianchi, 2016).

Third, for very low values of the elasticity η the externality flips, as firms compress wage increases in expansions more strongly than does the social planner (in Figure 2, this happens when individual labor supply is steeper than aggregate labor supply). Similarly, the fact that firms charge mark-ups $(\eta+1)/\eta$ may also interfere with the requirement that $\tau_t^w \geq 0$, for

---

[19] The problem of firms changes as follows:

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i), W_t(i))} \left\{ U'(t) \left(a_t F(H_t(i)) - \frac{(1+\tau_t^w)W_t(i)}{P_t} H_t(i) + \frac{\mathcal{T}_t}{P_t} \right) + \beta E_t \Gamma_{t+1}(W_t(i)) \right\}.$$

Constraints i) and ii) in Definition 1 remain unchanged from the economy without intervention. In equilibrium, $\mathcal{T}_t = \tau_t^w W_t(i) H_t(i)$. More details can be found in the Appendix.

the mark-up itself calls for labor subsidies. The requirement that $\tau_t^w \geq 0$ would need to be violated in such a case, for policy makers would need to *subsidize* firms' hiring in expansions to implement the constrained-efficient allocation. In such a case, Proposition 2 continues to be valid once we additionally assume that subsidies are available in expansions, but not to outright support labor demand in recessions.

Fourth, in Proposition 2 we have implicitly assumed that downward nominal wage rigidity applies to the wage received by workers W_t rather than to the cost faced by firms $(1 + \tau_t^w)W_t$. This appears a natural assumption if wage stickiness derives from the worker side, e.g., a loss in worker morale/productivity after a wage cut (Bewley, 1999). However, it should be noted that the constrained-efficient allocation cannot be decentralized with payroll taxes on firms if downward nominal wage rigidity applies to the labor cost faced by firms (in this case, a tax on firms' sales revenue would still be feasible).

We next ask if labor supply (rather than demand) policies can be used to decentralize the constrained-efficient allocation. This is a natural question, because taxing labor supply and demand are commonly seen as equivalent. The answer is yes and no. Consider a payroll tax $\tilde{\tau}_t^w \geq 0$ that is levied on households, rebated lump-sum to households in equilibrium. In budget constraint (2), households' wage income would need to be replaced by $\int_0^1 (1 - \tilde{\tau}_t^w)W_t(i)H_t(i)di$. This tax changes aggregate labor supply, as equation (6) in the free market equilibrium needs to be replaced by

$$G'(H_t) \leq \frac{(1 - \tilde{\tau}_t^w)W_t}{P_t}. \tag{18}$$

All other equilibrium conditions are unchanged from the economy without intervention.[20]

We find that this policy cannot be used to decentralize the constraint-efficient allocation. Intuitively, while this policy successfully reduces hiring in expansions, the rise in W_t would be reinforced whereas W_t declines in case the tax is levied on firms. This matters, because downward nominal wage rigidity applies exactly to W_t.[21] This being said, this policy *can* be used in case downward nominal wage rigidity applies to the *take-home* wage received by households $(1 - \tilde{\tau}_t^w)W_t$, rather than to the gross wage W_t. Intuitively, in this case inflation in W_t does not matter, because W_t is not directly affected by downward nominal wage rigidity. We show this formally in the Appendix. To summarize, the conventional wisdom that the economic incidence of a labor tax is independent of the formal incidence, holds up in case downward nominal wage rigidity applies to the wage that households take home after taxes.[22]

---

[20] Firm-specific labor supply (4) is not affected, because the tax affects labor income derived from all firms symmetrically. Formally, the tax cancels in $W_t(i)/W_t$ appearing in equation (4).

[21] Symmetrically, *subsidizing* labor supply would successfully reduce wage inflation, but it would also lead to higher employment, whereas equilibrium hiring falls in the constrained-efficient allocation.

[22] Poterba et al. (1986) argue that, because work contracts are commonly denominated in terms of gross

Turn back to the case where the tax is levied on firms. In the general case, obtaining an analytical expression for τ_t^w is tedious. Yet, one special case of the model allows for a convenient analytical characterization: the case $\eta \to \infty$ (perfect labor market competition). To see this, we first define *potential employment* H_t^p as solving

$$G'(H_t^p) = \frac{W_t}{P_t}, \tag{19}$$

implying that $H_t = H_t^p$ whenever labor supply is not rationed. We define *unemployment* u_t as the shortfall of hours due to rationing relative to potential

$$u_t \equiv \frac{H_t^p - H_t}{H_t^p} \geq 0. \tag{20}$$

Assume now that downward nominal wage rigidity is slack in the current period, binding in the next period, and again slack thereafter. As we show in the Appendix, the optimal tax under perfect competition has a closed-form representation

$$\tau_t^w = \frac{\varphi}{1-\alpha} \psi^{\frac{\varphi+1}{\varphi}} E_t \xi_{t,t+1} \left(\frac{P_t}{P_{t+1}} \right)^{\frac{1}{\varphi}} (1 - u_{t+1})(1 - (1 - u_{t+1})^{\varphi}). \tag{21}$$

The tax depends negatively on the wage elasticity of (aggregate) labor supply: as we argued earlier, $\varepsilon_t^G = 1/\varphi$. When labor supply is elastic, taxing is costly as this crowds out strongly employment (see the income tax literature, e.g., Saez 2001). Conversely, note that inelastic labor supply $\varphi \to \infty$ would imply the trivial policy implication $\tau_t^w = \infty$: tax labor as much as possible in expansions. This is because (aggregate) labor supply in Figure 2 would be vertical, implying that taxing labor demand can costlessly curb wage inflation as there are no repercussions on equilibrium employment.[23] Moreover, note that τ_t^w in (21) depends positively on the wage elasticity of labor demand: under our assumed production function: $|\varepsilon_t^F| = 1/(1 - \alpha)$. When labor demand is elastic, employment is rationed strongly when downward nominal wage rigidity binds in recessions, justifying a larger intervention when it is slack in expansions.

Equation (21) conveniently determines τ_t^w as a function of the stochastic discount factor $\xi_{t,t+1} \equiv \beta(U'(t+1)/U'(t))(P_t/P_{t+1}) \geq 0$, of price inflation and of unemployment expected for next period. As an illustrative example for yearly frequency, assume that $\varphi = 4$, $\alpha = 2/3$, that wages can fall four percent before downward nominal wage rigidity becomes binding ($\psi = 0.96$), that there is no price inflation $P_t = P_{t+1}$, that the stochastic discount factor

---

wages, it is natural to assume stickiness at the level of gross wages rather than at the level of net wages. If we follow their assessment, then payroll taxes levied on households could not be used to decentralize the constrained-efficient allocation.

[23] The assumption of inelastic labor supply is in fact made often in the literature (e.g., Eggertsson et al., 2017; Fornaro and Romei, 2018). In such a case, taxing labor demand is thus a "free lunch", for it can be used to costlessly prevent wage inflation.

is $\xi_{t,t+1} = 0.96$, and that with a probability of 10 percent, a crisis is expected for next year in which the labor market is rationed by 10 percent. In this case, the implied tax is $\tau_t^w = 0.1095$, or about 11 percent. This example shows that the optimal tax can be quite large. Nonetheless, recall that (21) is only valid for a special case of the model. Moreover, it ignores general equilibrium effects: per effect of charging the tax, the probability of the unemployment spell in the next period is reduced. These general equilibrium effects are taken care of in our quantitative application in Section 4.

Another important aspect of the model is that firms' market power as reflected in their monopsonistic mark-ups, and the extent to which firms internalize downward nominal wage rigidity, are inherently intertwined. To see this, consider a policy which aims at eliminating firms' mark-ups. From (15), this requires setting $\tau_t^w = -1/(1+\eta)$ when downward nominal wage rigidity is slack in expansions.[24] Policies aimed at reducing "market concentration" have frequently been discussed in the context of the euro crisis (e.g. Cacciatore and Fiori, 2016; Eggertsson et al., 2014). We obtain our third proposition.

Proposition 3. *[DEREGULATION] Assume that policy makers set $\tau_t^w = -1/(1+\eta)$ when downward nominal wage rigidity is slack. The free market equilibrium now coincides with the equilibrium allocation under perfect labor market competition ($\eta = \infty$). As a result, firms now fail completely to internalize the effects of downward nominal wage rigidity.*

Proof. We set $\tau_t^w = -1/(1+\eta)$ in (15) and (17), and $\tau_t^w = 0$ in (16). As can be readily verified, (15)-(17) are now solved by $a_t F'(H_t) = W_t/P_t$ as well as $\lambda_t = 0$ for all t. Thus, the equilibrium allocation is identical as in the case of perfect labor market competition. $\qquad\square$

As Proposition 3 shows, eliminating firms' mark-ups has the effect of *eliminating firms' incentives* to internalize downward nominal wage rigidity. The intuition for this result is that, as we explained earlier, λ_t represents the utility value of firms' transferring higher rents to households when downward nominal wage rigidity is relaxed by a marginal unit. But when mark-ups are zero, monopsony rents are zero, and so $\lambda_t = 0$ as well. In our setting, structural reforms which aim at reducing firms' mark-ups can therefore be counterproductive. On the one hand, they raise welfare as they reduce a static distortion in the conventional way. On the other hand, they *reduce* welfare as they intensify the pecuniary externality. In the next section we will explore this trade-off numerically.[25] Specifically we will show that, due to the pecuniary externality, welfare losses are U-shaped in the degree of labor market competition.

---

[24] In contrast, in recessions τ_t^w can be set to zero. This is because, as explained in Section 2, firms endogenously reduce their mark-ups to zero when downward nominal wage rigidity binds in recessions.

[25] This trade-off disappears once structural reforms simultaneously affect downward nominal wage rigidity directly, which we ruled out by assuming that ψ was independent of either η or τ_t^w.

We conclude this section by discussing one potential caveat of the prudential tax on labor explored in this section. In the present model, the price level is exogenous by the law of one price and because we assumed that all goods are internationally traded. This rules out that firms can pass the higher labor cost implied by the tax on to domestic prices. If, by way of contrast, the price level were endogenous, the intervention could end up being inflationary. As we show in a model extension in the Appendix, our conclusions are robust in this dimension, in the following sense: the externality is still operative in the extended model, and it may be addressed via a prudential tax on labor. We also derive the analogue of equation (21) in the extended model to show that i) policy makers should raise τ_t^w more aggressively when prices are endogenous, because firms' real labor cost becomes less sensitive to changes in τ_t^w and ii) there is an offsetting feedback on τ_t^w of the labor demand and supply elasticities, such that τ_t^w should be raised less aggressively. In general, which of the two effects dominates is ambiguous.

4   Quantitative analysis

In this section we demonstrate that the pecuniary externality has large negative effects on welfare and unemployment. We first calibrate the model's key parameters in Section 4.1. Here we follow the bulk of the literature by assuming perfect labor market competition. Assuming perfect competition has the benefit of making the analysis transparent, because the pecuniary externality is the *only* inefficiency vis-à-vis to the constrained-efficient equilibrium. Section 4.2 presents results of this analysis. In Section 4.3, we discuss in depth how model predictions change under imperfect labor market competition.

4.1   Calibration and numerical implementation

We target a set of 12 countries that either peg to the euro or are part of the euro area. The countries are Bulgaria, Estonia, Ireland, Greece, Spain, Italy, Cyprus, Latvia, Lithuania, Portugal, Slovenia and Slovakia. The time-span we consider is 2000Q1-2018Q4, at a quarterly frequency.

We choose this set of countries because Schmitt-Grohé and Uribe (2016) provide an estimate for parameter ψ for these countries during the Great Recession in the euro area. Recall that ψ measures by how much nominal wages can decline before downward nominal wage rigidity binds, making it the key parameter for the impact of this friction quantitatively. By using aggregate wage dynamics, Schmitt-Grohé and Uribe's estimate is $\psi = 0.993$ at a quarterly frequency after accounting for technology growth, implying that nominal wages can decline up to 2.8 percent per year.

We view this estimate of parameter ψ as suggestive. In a recent survey, Elsby and Solon (2018) point out that nominal wages appear quite downward flexible when looking at administrative data. Moreover, it has been argued that aggregate wage data may not be informative about wage rigidity for what matters for employment adjustment is the wage rigidity of new hires (Pissarides, 2009). In this regard, there is evidence that wages of new hires are quite flexible (e.g., Haefke et al., 2013, for the US). On the other hand, Gertler et al. (2016) argue that composition effects due to workers moving to better jobs in expansions lead to an understatement of the true degree of wage rigidity, and that after controlling for this composition effect, wages appear quite sticky at the relevant margin of new hires. To take account of this debate, we will explore in a sensitivity analysis how our results change when ψ is changed to a different value.

One parameters that matters quantitatively for the pecuniary externality is the aggregate labor supply elasticity $1/\varphi$. As we discussed in Section 3.2, there is considerable uncertainty regarding plausible values for this elasticity. However, Galí (2011) notes that much of the literature sets φ to a number in between 1 and 5. In our baseline calibration, we use the midpoint of this range by setting $\varphi = 3$. Moreover, we will explore in a sensitivity analysis the implications for our results of using a different φ.

We turn to the model's stochastic structure. The cycle is driven by shocks to total factor productivity, for which we assume a log-Normal AR(1) structure

$$\log(a_t) = \rho_a \log(a_{t-1}) + \sigma_a v_t, \tag{22}$$

where $v_t \sim^{iid} \mathcal{N}(0,1)$, $\sigma_a > 0$ and $\rho_a \in [0,1)$.

We pick (ρ_a, σ_a) to match the volatility and autocorrelation of real GDP of the countries in our sample. As in Schmitt-Grohé and Uribe (2016), we use OECD data on manufacturing output to proxy for the fact that in our model, all goods are internationally tradable. We first HP-filter the series and compute the standard deviation and autocorrelation of the cyclical component for all countries.[26] We then take the arithmetic average. The result is $\sigma(y) = 7.1\%$ and $\rho(y) = 0.77$. The calibrated parameters are $\rho_a = 0.9$ and $\sigma_a = 0.023$. Given quarterly frequency, the value for σ_a appears quite high, but this reflects the high measured standard deviation of tradable output in the countries in our sample.[27]

For price inflation in the nominal-anchor country we assume that $\bar{P}_t = \bar{\pi}\bar{P}_{t-1}$. The average HICP inflation in the euro area during our sample period has been 1.7% yearly. We therefore set $\bar{\pi} = 1.00425$. Accounting for trend inflation is important, because the "greasing the wheels

---

[26] There is no data available for Cyprus and Bulgaria. For this part of the calibration, where therefore omit these two countries from the sample.

[27] However, these numbers are in line with Schmitt-Grohé and Uribe (2016), who estimate a quarterly standard deviation of (de-trended) tradable output $\sigma(y) = 6.5\%$ during 1981-2011 for Greece.

Parameter and description of parameter		Value assigned
β	Time discount factor	0.9926
$\bar{\pi}$	Trend inflation	1.00425
R	Nominal gross borrowing rate	1.0116
ψ	Downward nominal wage rigidity	0.993
α	Labor share	2/3
η	Elasticity of substitution firm employment	∞
φ	Inverse Frisch elasticity	3
σ_a	Volatility TFP innovations	0.023
ρ_a	Autocorrelation TFP	0.90

Table 1: Baseline calibration.

effect" dampens the impact of downward nominal wage rigidity quantitatively (Tobin, 1972).[28] We use EMU-convergence-criterion bond yields to proxy for the nominal borrowing rate R. We average across time and countries, then convert the number to quarterly frequency. The result is $R = 1.0116$ (a yearly nominal rate of 4.6%). Under perfect competition, firms make zero profits such that α equals the labor share of income. Here we use the standard value $\alpha = 2/3$. For U we assume a coefficient of relative risk aversion $\sigma = 2$, a value commonly used in international business cycle studies (e.g., Mendoza and Yue, 2012). Finally, we calibrate the time discount factor $\beta = 0.9926$, to obtain a mean ratio of foreign assets to annual GDP of -52 percent, in line with the average foreign asset to GDP ratio of the countries in our sample. In order to obtain a well-defined asset distribution, we also specify a borrowing limit of 150% foreign debt to GDP, which however in equilibrium is almost never binding. The calibrated parameters are summarized in Table 1.

The model is solved globally by using a version of fixed point iteration which can deal with occasionally binding constraints. To implement the TFP process (22) we use the routine developed by Rouwenhorst (1995). The Rouwenhorst's routine is superior to the more common Tauchen algorithm when the approximated process has a high autocorrelation—as is the case in (22) since we implement an autocorrelation of $\rho_a = 0.9$. Because of the presence of trend inflation, our model is not stationary. Therefore, we first define all variables in stationary terms before applying the solution procedure. The model's equilibrium conditions in terms of stationary variables can be found in the Appendix.
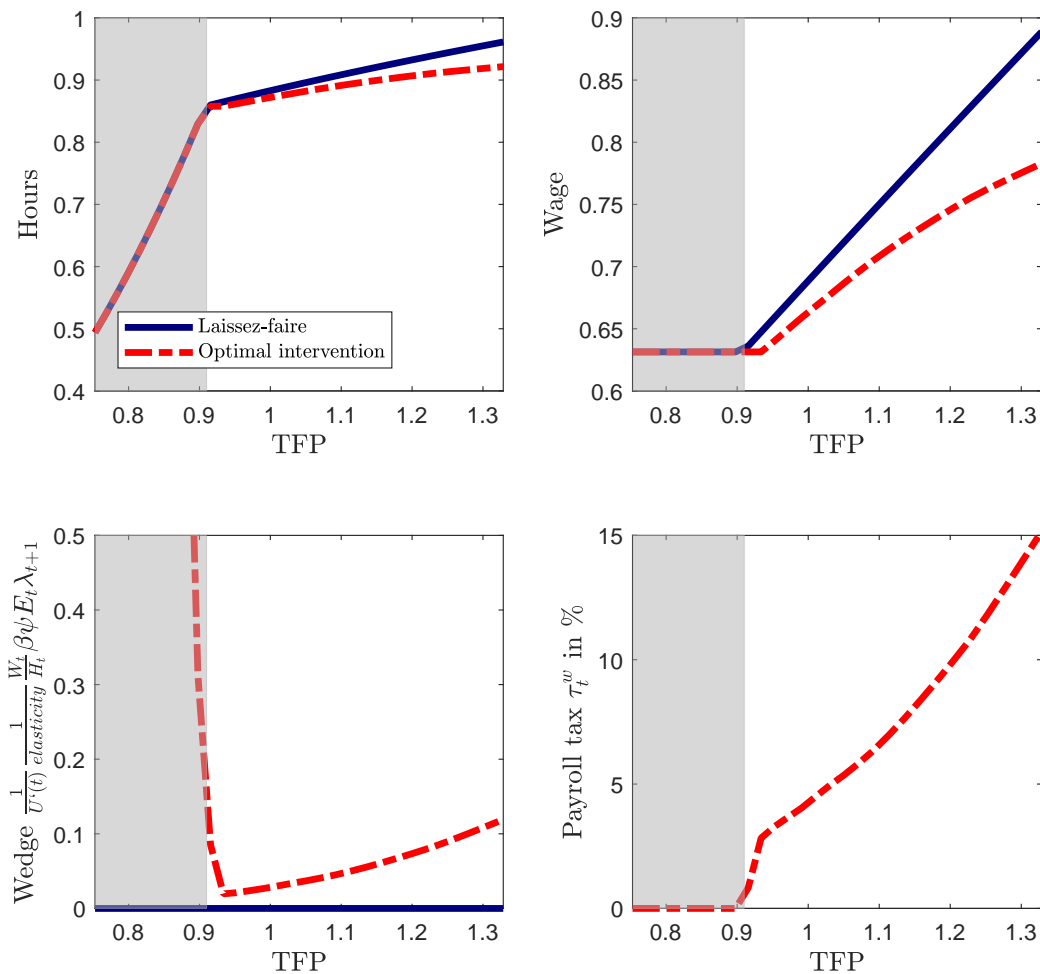
Figure 3: Policy functions. The gray area indicates the region where downward nominal wage rigidity binds in the free market equilibrium ("laissez-faire"). The constrained-efficient equilibrium is indicated by "Optimal intervention". Lagged wages as well as foreign assets are set one standard deviation below the steady state.

4.2 Results of the quantitative analysis

Figure 3 shows policy functions for hours, wages, the wedge term appearing in equations (8) and (13), and the prudential payroll tax τ_t^w needed to decentralize the constrained-efficient allocation. In the wedge term, *elasticity* denotes the relevant labor supply elasticity, corresponding to *elasticity* $= \infty$ in the free market equilibrium (since we assume perfect competition), and to *elasticity* $= 1/\varphi = 1/3$ under the optimal intervention.

As Figure 3 shows, hours and wages rise in expansions, and hours fall sharply and wages are bounded below in recessions. In recessions, the constrained-efficient and the free market

---

[28] A higher value for $\bar{\pi}$ has an identical effect as a lower value for ψ. By assessing robustness with respect to ψ, we are thus indirectly assessing the effects of a higher trend inflation.

equilibrium coincide, reflecting that the planner respects the same frictions as the private economy. However, in expansions hiring is reduced in the constrained-efficient relative to the free market equilibrium, inducing less wage inflation. This represents an endogenous wedge term (lower left panel) affecting labor demand in the constrained-efficient equilibrium, which becomes larger the larger is the expansion.[29] This can be achieved via a tax on labor, which is positive during expansions, and set to zero in recessions.

Figure 4 shows stationary distributions. Focus first on the upper row. The left panel shows unemployment as defined in (20), the right panel shows the implied payroll tax τ_t^w. The left panel reveals that rationing in the labor market absent the prudential intervention is very frequent and volatile. Conditional on unemployment being strictly positive, the mean unemployment rate implied by rationing is 5.6%, with a standard deviation of 5%.[30] This compares with a mean unemployment rate of 10.9% and standard deviation of 4% of the countries in our sample during the time span that we consider. Therefore, the model is able to explain a large chunk of the mean unemployment rate as well as of its standard deviation, even though the model abstracts from other frictions that generate unemployment, and most notably from search frictions.[31] The probability mass to the right of a 10% unemployment rate is still 7.4%. Given quarterly calibration, this implies that once every 3.5 years, the labor market is rationed by at least 10%, which appears sizable. The probability mass to the right of 2% unemployment is 34.3%.

The stationary distribution for unemployment is shifted to the left under the optimal intervention. While small rates of unemployment below 2% become in fact more frequent, the probability mass to the right of 2% unemployment drops from 34.3% to 2.3%. The probability mass to the right of 10% unemployment drops all the way to zero. The mean unemployment rate is reduced to 0.2%, with a standard deviation of 1%. Overall, the prudential intervention thus makes the economy significantly less exposed to unemployment crises.[32] The right panel reveals that the tax on labor underlying the intervention is in fact quite small. The distribution is tightly centered around a mean tax rate of 3.9%.

---

[29] The wedge term also shoots up when downward nominal wage rigidity binds, due to the multiplier λ_t turning positive. However, in this region, labor demand is determined by (9), such that the wedge has no effect on the equilibrium allocation.

[30] The stationary distribution for unemployment has a mass point at zero, due to firms' hiring the full labor supply when downward nominal wage rigidity is slack. By including the mass point, the mean unemployment rate drops to 2.7%, and the standard deviation to 4.5%.

[31] Michaillat (2012) considers a model where unemployment due to search and rationing may arise simultaneously.

[32] It is important to notice that the payroll tax and thus the intervention itself does not lead to unemployment. While the tax reduces employment, it does so via a reduction in wages. This implies that lower employment is not measured as unemployment according to our definition (20). Intuitively, when wages are lower workers are less willing to work—i.e., workers are still "on their labor supply curve".
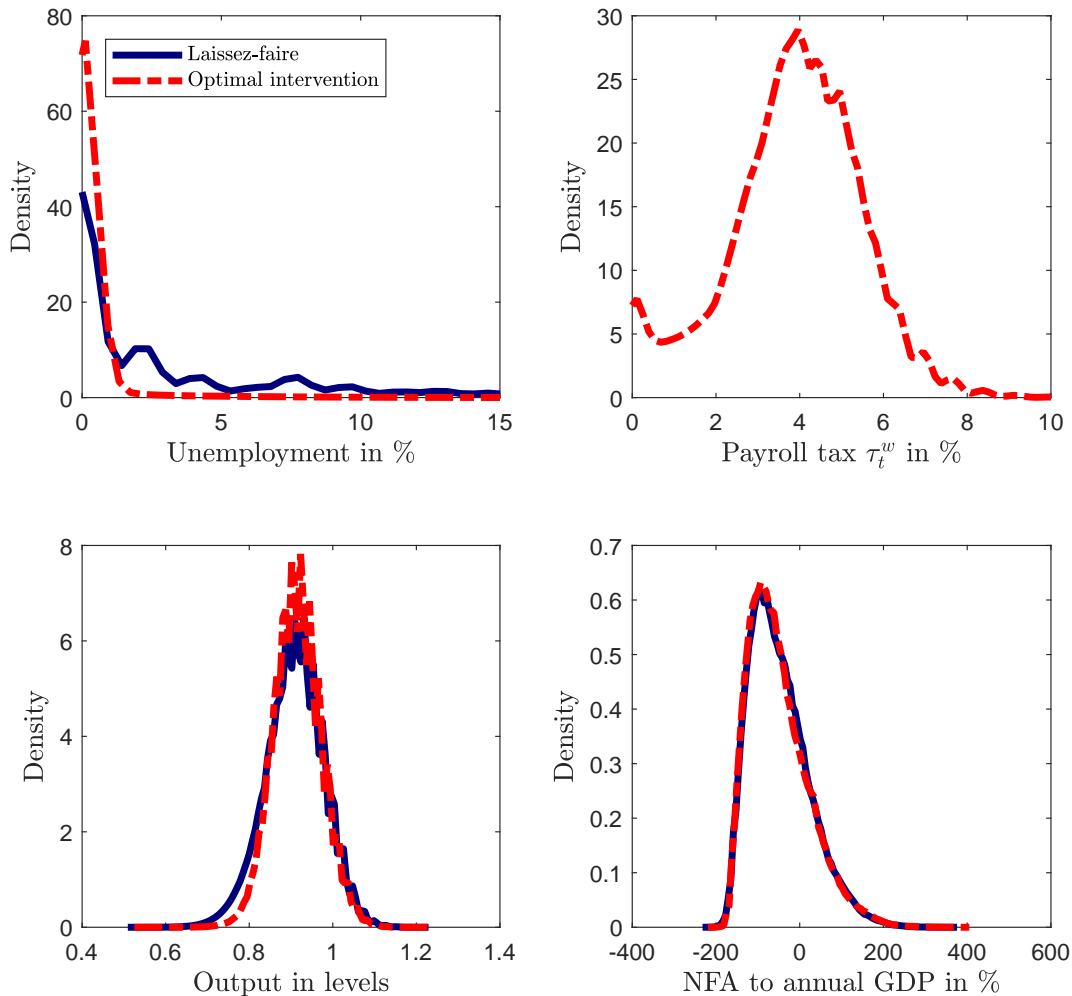
Figure 4: Stationary distributions.

Focus now on the lower row, which shows stationary distributions for output in levels and net foreign assets to GDP $B_{t+1}/(4P_tY_t)$ in percent. The distribution of output has less mass on the left under the constrained-efficient allocation, reflecting that deep recessions with high unemployment are less frequent. The right panel, in turn, shows that the stationary distribution of external assets is not much affected by the intervention. This is noteworthy in light of earlier studies on macro-prudential intervention, which emphasized shifts in the stationary distribution of external assets reflecting that the private equilibrium "over-borrows" (e.g., Bianchi, 2011; Schmitt-Grohé and Uribe, 2016). As emphasized in the introduction, here we study a different type of externality which operates through firms' hiring, implying that intervention is required in the labor market whereas capital markets work efficiently. This also implies that the stationary distribution of external assets is hardly affected by the intervention.
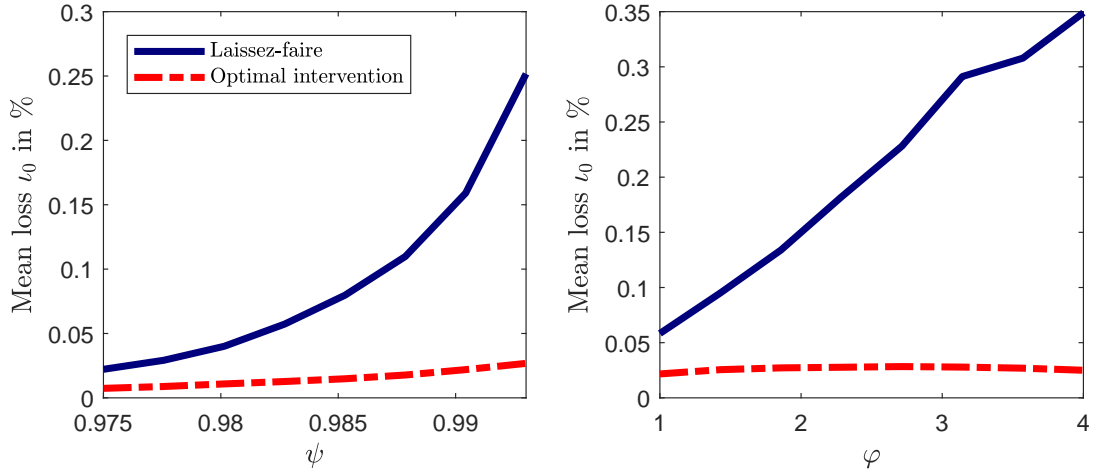
Figure 5: Welfare effects and sensitivity vis-à-vis variation in ψ and φ. Shown are welfare losses relative to first best, respectively.

We next study the welfare implications of the intervention. We are interested in the following two statistics. First, what are welfare losses under the optimal intervention compared to a benchmark where downward nominal wage rigidity is absent in the first place? Second, what are welfare losses relative to this benchmark absent the intervention? We compute losses in terms of consumption equivalents according to

$$E_0 \sum_{t \geq 0} \beta^t U(C_t(1 + \iota_0) - G(H_t)) \equiv E_0 \sum_{t \geq 0} \beta^t U(C_t^{fb} - G(H_t^{fb})), \tag{23}$$

where "fb" denotes policy functions in the constrained-efficient equilibrium where we additionally impose that $\psi = 0$ (first best, no downward nominal wage rigidity).

The first welfare statistic can be obtained by evaluating the constrained-efficient equilibrium against the first-best benchmark via equation (23). The second statistic can be obtained by evaluating the free market equilibrium against the benchmark. In both cases, we report the mean of the stationary distribution for ι_0. The result is in Figure 5. We show the mean welfare loss under the baseline calibration, but additionally by varying the two parameters ψ and φ. Recall that the baseline calibration is $\psi = 0.993$ and $\varphi = 3$.

In the baseline calibration, losses of the constrained-efficient equilibrium amount to 0.025% of permanent consumption. In contrast, losses absent the intervention are 0.26% of permanent consumption. From this we draw two conclusions. First, the welfare cost of downward nominal wage rigidity *per se* is small. This echoes again Elsby (2009): to the extent that firms internalize downward nominal wage rigidity "correctly" (which from a social standpoint, is the case under the optimal intervention), the welfare cost of downward nominal wage rigidity is small. Second, the welfare cost of the pecuniary externality is large, as it raises the welfare

cost of downward nominal wage rigidity by a factor of 10.

The left panel in Figure 5 varies the degree of downward nominal wage rigidity ψ. We note that welfare losses decrease as wages become more flexible. More importantly, the relative distance between the two welfare losses remains roughly the same when ψ is lowered. Therefore, the pecuniary externality is still relevant, in the sense that it strongly *exacerbates* the welfare cost of downward nominal wage rigidity. The right panel varies the wage elasticity of aggregate labor supply $1/\varphi$. We find that the relative distance between the two welfare losses increases as the elasticity drops, indicating that the externality becomes stronger as aggregate labor supply becomes steeper. This is line with the intuition provided in Section 3.2.

4.3   The effects of imperfect labor market competition

We next depart from the polar case of perfect labor market competition by assuming $\eta < \infty$. As noted in Section 3.2, there is considerable uncertainty regarding plausible values for the wage elasticity faced by individual firms. Sokolova and Sorensen (2018) point out that micro estimates of the wage elasticity vary among countries and even among industries. Depew and Srensen (2013) note that most micro estimates for η lie in between 1 and 10.

We first reproduce the policy functions of Figure 3 by assuming a wage elasticity $\eta = 5$. Figure 6 shows the result. Wages and hiring are now *lower* under laissez-faire compared to the optimal intervention, which turns results from Figure 3 on its head. We argued earlier that the pecuniary externality leads firms to underestimate the social cost of downward nominal wage rigidity, even when firms have *substantial* market power (the discussion in Section 3.2). How is this compatible with Figure 6?

When firms have market power, differences vis-à-vis the constrained-efficient equilibrium arise both due to the pecuniary externality and due to firms' charging monopsonistic mark-ups. Under $\eta = 5$, firms' mark-ups are substantial as firms reduce wages below their marginal product by $(\eta + 1)/\eta = 1.2$, that is 20%. In Figure 6, this leads firms to reduce hiring more strongly in expansions than does the constrained-efficient planner. However, the externality is still at work. This can be seen from the lower left panel, which displays the wedge in the labor demand curve that arises from the expected utility loss due to downward nominal wage rigidity. In the relevant (slack) region, firms' wedge is less than half its counterpart under constrained efficiency, which establishes that firms substantially underestimate the welfare cost of downward nominal wage rigidity even when $\eta = 5$.

In Figure 6, we also display how policy functions change as policy makers regulate firms' mark-ups in an attempt to break labor market power. In line with Proposition 3, policy
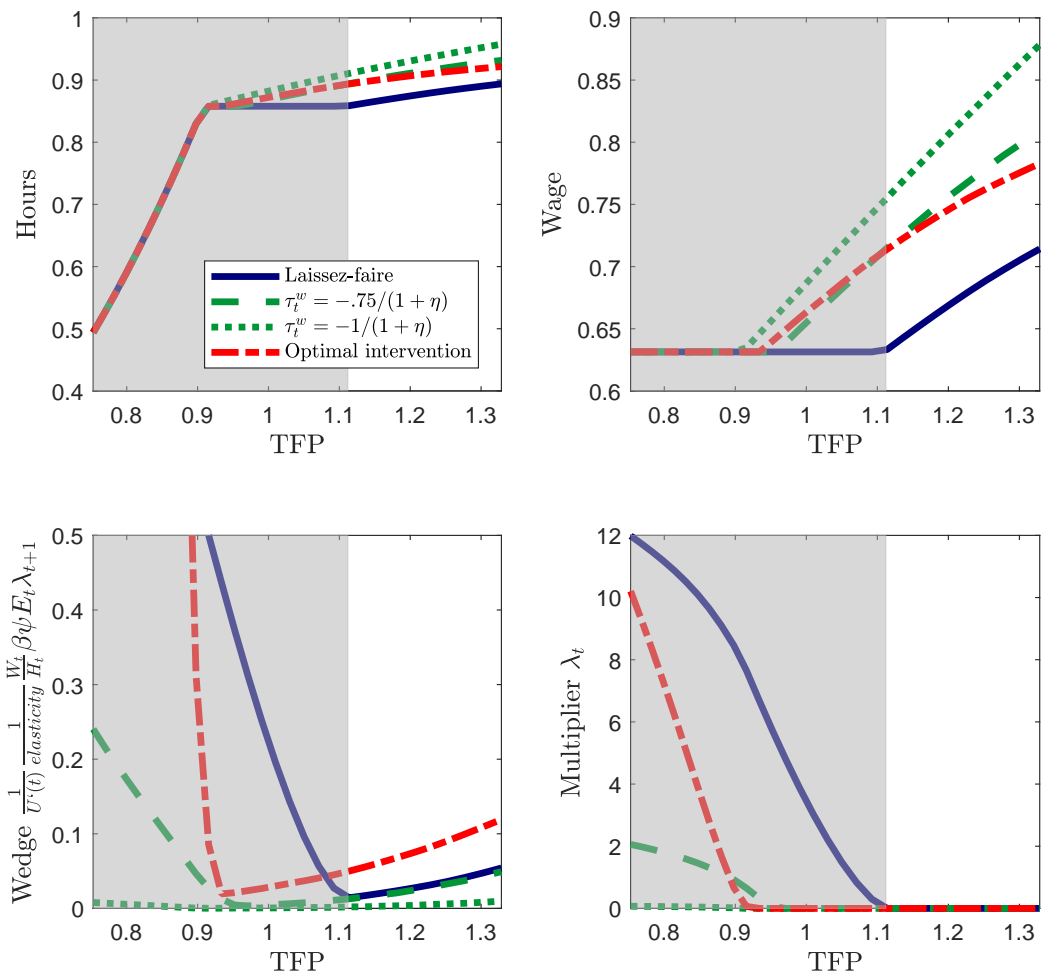
Figure 6: Policy functions under imperfect labor market competition. The gray area indicates the region where downward nominal wage rigidity binds under laissez-faire. The green dashed and dotted lines represent non-optimal regulatory interventions.

functions now converge to those under perfect competition: as τ_t^w approaches $-1/(1+\eta)$ from above, the multiplier λ_t and therefore the wedge term $(1/U'(t))(1/\eta)(W_t/H_t)\beta\psi E_t\lambda_{t+1}$ gradually converge to zero. (Fully) breaking firms' monopsony power may therefore be counterproductive in the present environment.

More intuition can be gained from Figure 7, which varies mean welfare losses in $1/\eta$ under laissez-faire and constrained efficiency relative to first best, as in Figure 5. The origin thus corresponds to perfect labor market competition. Note first that welfare losses under constrained-efficiency are independent of η, because this parameter does not appear in the constrained-efficient allocation. In contrast, welfare losses under laissez-faire have a U-shape. When market power is strong, welfare losses are dominated by large mark-ups, as can be seen in the right panel of the figure. Welfare losses drop as competition increases. However, the
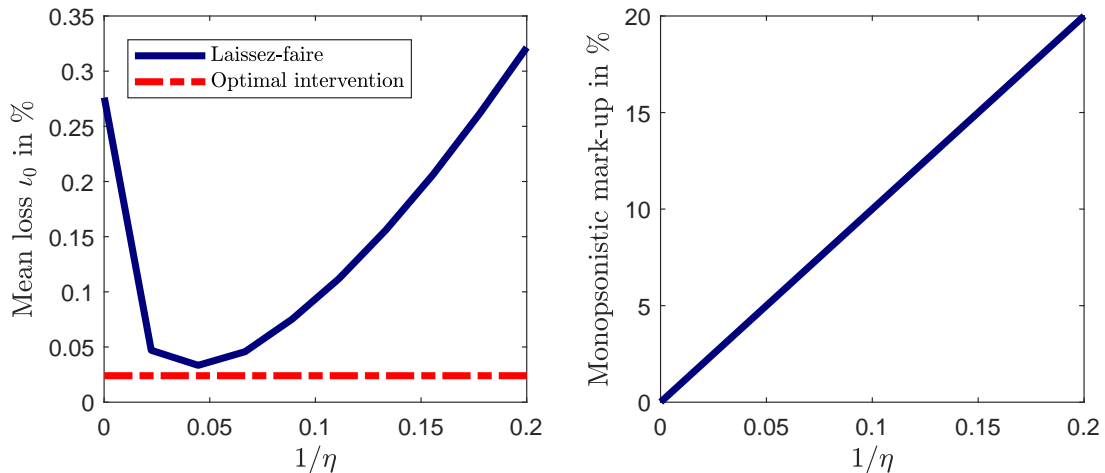
Figure 7: Welfare effects and sensitivity vis-à-vis variation in η.

result flips when $1/\eta$ becomes too low: welfare losses are dominated by the externality, and start to rise in the degree of labor market competition.

5   Conclusion

We describe a pecuniary externality in economies with downward nominal wage rigidity that leads firms to hire too many workers in expansions, which leads to too much unemployment in recessions. The externality arises because of competitive behavior in the labor market. We have shown that it can be addressed via a prudential tax on labor, and that policies which attempt to raise labor market competition may backfire due to the externality. We also show that welfare losses implied by the externality are large.

The current analysis hints at a number of interesting open questions. First, while in the main text we have shown that the externality affects the labor demand side, in the Appendix we show that it also affects the labor supply side: the externality also arises in a context with wage-setting households (unions). Studying the interaction of firms' and unions' wage setting in a context of downward nominal wage rigidity therefore provides an interesting aspect for future research.

Similarly, in the Appendix we show that the pecuniary externality and aggregate demand externalities of the type studied in Schmitt-Grohé and Uribe (2016) generally interact. Exploring the precise nature of this interaction and how this shapes prescriptions for macro-prudential regulation in a quantitative setting hence provides another interesting avenue for future research.

References

Benigno, G. and Fornaro, L. (2018). Stagnation traps. *The Review of Economic Studies*, 85(3):1425–1470.

Benigno, P. and Ricci, L. A. (2011). The inflation-output trade-off with downward wage rigidities. *American Economic Review*, 101(4):1436–66.

Bewley, T. F. (1999). *Why Wages Don't Fall During a Recession.* Harvard University Press, Cambridge, MA.

Bianchi, J. (2011). Overborrowing and Systemic Externalities in the Business Cycle. *American Economic Review*, 101(7):3400–3426.

Bianchi, J. (2016). Efficient bailouts? *American Economic Review*, 106(12):3607–59.

Bianchi, J. and Mendoza, E. G. (2018). Optimal time-consistent macroprudential policy. *Journal of Political Economy*, 126(2):588–634.

Cacciatore, M. and Fiori, G. (2016). The macroeconomic effects of goods and labor markets deregulation. *Review of Economic Dynamics*, 20:1 – 24.

Calmfors, L. and Driffill, J. (1988). Bargaining structure, corporatism and macroeconomic performance. *Economic Policy*, 3(6):13–61.

Corsetti, G., Mavroeidi, E., Thwaites, G., and Wolf, M. (2019). Step away from the zero lower bound: Small open economies in a world of secular stagnation. *Journal of International Economics*, 116:88 – 102.

Dávila, E. and Korinek, A. (2018). Pecuniary externalities in economies with financial frictions. *The Review of Economic Studies*, 85(1):352–395.

Depew, B. and Srensen, T. A. (2013). The elasticity of labor supply to the firm over the business cycle. *Labour Economics*, 24:196 – 204.

Eggertsson, G., Ferrero, A., and Raffo, A. (2014). Can structural reforms help Europe? *Journal of Monetary Economics*, 61(C):2–22.

Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2017). A Model of Secular Stagnation: Theory and Quantitative Evaluation. NBER Working Papers 23093, National Bureau of Economic Research, Inc.

Elsby, M. W. (2009). Evaluating the economic significance of downward nominal wage rigidity. *Journal of Monetary Economics*, 56(2):154 – 169.

Elsby, M. W. and Solon, G. (2018). How Prevalent Is Downward Rigidity in Nominal Wages? International Evidence from Payroll Records and Pay Slips. NBER Working Papers 25393, National Bureau of Economic Research, Inc.

Farhi, E., Gopinath, G., and Itskhoki, O. (2014). Fiscal Devaluations. *Review of Economic Studies*, 81(2):725–760.

Farhi, E. and Werning, I. (2016). A theory of macroprudential policies in the presence of nominal rigidities. *Econometrica*, 84(5):1645–1704. Lead article.

Flinn, C. J. (2006). Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates. *Econometrica*, 74(4):1013–1062.

Fornaro, L. and Romei, F. (2018). The paradox of global thrift. Working Papers 1845, Banco de Espaa;Working Papers Homepage.

Friedman, M. (1953). The case for flexible exchange rates. In *Essays in Positive Economics*. University of Chicago Press, Chicago.

Galí, J. (2011). The Return Of The Wage Phillips Curve. *Journal of the European Economic Association*, 9(3):436–461.

Galí, J. and Monacelli, T. (2016). Understanding the gains from wage flexibility: The exchange rate connection. *American Economic Review*, 106(12):3829–68.

Galí, J., Smets, F., and Wouters, R. (2012). Unemployment in an estimated new keynesian model. *NBER Macroeconomics Annual*, 26(1):329–360.

Gertler, M., Huckfeldt, C., and Trigari, A. (2016). Unemployment fluctuations, match quality, and the wage cyclicality of new hires. Working Paper 22341, National Bureau of Economic Research.

Haefke, C., Sonntag, M., and van Rens, T. (2013). Wage rigidity and job creation. *Journal of Monetary Economics*, 60(8):887–899.

Jeanne, O. and Korinek, A. (2010). Excessive volatility in capital flows: A pigouvian taxation approach. *The American Economic Review*, 100(2):403–407.

Keane, M. and Rogerson, R. (2012). Micro and macro labor supply elasticities: A reassessment of conventional wisdom. *Journal of Economic Literature*, 50(2):464–476.

Korinek, A. and Simsek, A. (2016). Liquidity trap and excessive leverage. *American Economic Review*, 106(3):699–738.

Lorenzoni, G. (2008). Inefficient Credit Booms. *Review of Economic Studies*, 75(3):809–833.

Manning, A. (2003). *Monopsony in Motion: Imperfect Competition in Labor Markets*. Princeton University Press, Princeton, New Jersey.

Mendoza, E. G. and Yue, V. Z. (2012). A General Equilibrium Model of Sovereign Default and Business Cycles. *The Quarterly Journal of Economics*, 127(2):889–946.

Michaillat, P. (2012). Do matching frictions explain unemployment? not in bad times. *American Economic Review*, 102(4):1721–50.

Ottonello, P., Presno, I., and Bianchi, J. (2017). Fiscal Policy, Sovereign Risk, and Unemployment. Technical report.

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, 77(5):1339–1369.

Poterba, J. M., Rotemberg, J. J., and Summers, L. H. (1986). A Tax-Based Test for Nominal Rigidities. *American Economic Review*, 76(4):659–675.

Ransom, M. R. and Sims, D. P. (2010). Estimating the firms labor supply curve in a new monopsony framework: Schoolteachers in missouri. *Journal of Labor Economics*, 28(2):331–355.

Rouwenhorst, G. (1995). *Asset pricing implications of equilibrium business cycle models. In: Frontiers of business cycle research: Thomas F. Cooley, editor*. Princeton University Press, Princeton, New Jersey.

Saez, E. (2001). Using Elasticities to Derive Optimal Income Tax Rates. *Review of Economic Studies*, 68(1):205–229.

Schmitt-Grohé, S. and Uribe, M. (2012). What's News in Business Cycles. *Econometrica*, 80(6):2733–2764.

Schmitt-Grohé, S. and Uribe, M. (2016). Downward nominal wage rigidity, currency pegs, and involuntary unemployment. *Journal of Political Economy*, 124(5):1466–1514.

Sokolova, A. and Sorensen, T. A. (2018). Monopsony in Labor Markets: A Meta-Analysis. IZA Discussion Papers 11966, Institute for the Study of Labor (IZA).

Tobin, J. (1972). Inflation and Unemployment. *American Economic Review*, 62(1):1–18.

A Proofs and derivations

This Appendix contains proofs and derivations. The Appendix is structured as follows. A.1 contains details on the free market equilibrium. A.2 contains details on the constrained-efficient equilibrium and presents the proof of Proposition 1. A.3 presents the problem of a single monopsonist. A.4 contains details on the policy implications, and presents the proof of Proposition 2. A.5 presents the model's equilibrium definition in terms of stationary variables.

A.1 More details on the free market equilibrium

A.1.1 Deriving firm-specific labor supply

Here we derive the labor supply curves (4) by solving the maximization problem (3).

Set up the Lagrangian

$$\mathcal{L} = \int_0^1 W_t(i) H_t(i) di + W_t \left(H_t - \left(\int_0^1 H_t(i)^{1+\frac{1}{\eta}} di \right)^{1/\left(1+\frac{1}{\eta}\right)} \right),$$

where we denote W_t the Lagrange multiplier (which, by the Envelope theorem, measures the increase in $\int_0^1 W_t(i) H_t(i) di$ following a marginal rise in H_t).

The first order conditions for hours $H_t(i)$ are

$$W_t(i) - W_t H_t^{-\frac{1}{\eta}} H_t(i)^{\frac{1}{\eta}} = 0, \quad i \in [0,1].$$

To determine the multiplier W_t, use that

$$W_t(i)^{1+\eta} = W_t^{1+\eta} \left(\frac{H_t(i)}{H_t} \right)^{\frac{1+\eta}{\eta}}$$

$$\Rightarrow \int_0^1 W_t(i)^{1+\eta} di = W_t^{1+\eta} \int_0^1 \left(\frac{H_t(i)}{H_t} \right)^{\frac{1+\eta}{\eta}} di = W_t^{1+\eta}$$

where the integral in the second equation is one by the definition of the aggregator H_t.

Provided that $\eta > 0$ the second order condition is negative,

$$-W_t H_t^{-\frac{1}{\eta}} \frac{1}{\eta} H_t(i)^{\frac{1}{\eta}-1} < 0,$$

which verifies that we study a local maximum.

A.1.2 Deriving firms' labor demand

We repeat the dynamic program of the firms from Definition 1

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i), W_t(i))} \left\{ U'(t) \left(a_t F(H_t(i)) - \frac{W_t(i)}{P_t} H_t(i) \right) + \beta E_t \Gamma_{t+1}(W_t(i)) \right\}$$

subject to the set of constraints

i) $\qquad\qquad (W_t(i)/W_t)^\eta H_t \geq H_t(i), \qquad$ (multiplier: $\xi_t(i)$)

ii) $\qquad\qquad W_t(i) \geq \psi W_{t-1}(i), \qquad$ (multiplier: $\lambda_t(i)$)

for given aggregate states $\{a_t, P_t, H_t, W_t, U'(t)\}$. We are writing the constraints i) and ii) as greater-or-equal inequalities, because this implies that the multipliers $\xi_t(i)$ and $\lambda_t(i)$ must be non-negative. The first order conditions are

$$U'(t)\left(a_t F'(H_t(i)) - \frac{W_t(i)}{P_t}\right) - \xi_t(i) = 0 \qquad\qquad \text{(Equ A.1)}$$

for hours $H_t(i)$ as well as

$$-U'(t)\frac{1}{P_t}H_t(i) - \beta\psi E_t\lambda_{t+1}(i) + \lambda_t(i) + \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^\eta)H_t = 0 \qquad\qquad \text{(Equ A.2)}$$

for wages $W_t(i)$, where we have already used the Envelope condition

$$\frac{\partial}{\partial W_{t-1}(i)}\Gamma_t(W_{t-1}(i)) = -\lambda_t(i)\psi.$$

We proceed by distinguishing the cases where downward wage rigidity is slack and binding, respectively, then by studying the symmetric equilibrium.

Case 1: Downward wage rigidity is slack

Assume that constraint i) in the maximization is slack. In this case, it must be that downward wage rigidity, constraint ii), is binding. Namely, if not, it were always possible to choose the same $H_t(i)$ but a strictly lower $W_t(i)$, which is feasible because constraint i) is slack, and which raises $\Gamma_t(W_{t-1}(i))$ because current profits increase ($a_t F(H_t(i))$ is the same but the wage bill $W_t(i)H_t(i)/P_t$ is reduced) and because there is a non-negative effect on the continuation value, because $\Gamma_{t+1}(W_t(i))$ is weakly decreasing in the individual state $W_t(i)$.

The contra-position of this statement is that, once constraint ii) is slack, it must be that constraint i) is binding. By using that $\lambda_t(i) = 0$ once constraint ii) is slack, we solve for multiplier $\xi_t(i)$ from (Equ A.2)

$$\xi_t(i) = U'(t)\frac{1}{\eta}\frac{W_t(i)}{P_t} + \frac{1}{\eta}\frac{W_t(i)}{H_t(i)}\beta\psi E_t\lambda_{t+1}(i),$$

where we have used that constraint i) is binding to replace H_t in (Equ A.2). This expression shows that the multiplier $\xi_t(i) > 0$ is strictly positive. Combining this with (Equ A.1) yields

$$a_t F'(H_t(i)) = \frac{\eta+1}{\eta}\frac{W_t(i)}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{W_t(i)}{H_t(i)}\beta\psi E_t\lambda_{t+1}(i) \qquad\qquad \text{(Equ A.3)}$$

which determines labor demand.

Case 2: Downward wage rigidity is binding

Assume now that constraint ii) binds. Assume also that constraint i) is binding. In this case, $H_t(i)$ is determined by constraint i) which holds with equality. Since $W_t(i)$ is determined by constraint ii), $W_t(i)$ and $H_t(i)$ both are determined by $\psi W_{t-1}(i)$. The multiplier $\xi_t(i)$ is determined in (Equ A.1):

$$\xi_t(i) = U'(t) \left(a_t F'(H_t(i)) - \frac{W_t(i)}{P_t} \right). \tag{Equ A.4}$$

Recall that $\xi_t(i) > 0$ when constraint ii) is slack, implying that $a_t F'(H_t(i)) > W_t(i)/P_t$ from (Equ A.1). Furthermore, both $W_t(i)$ and $H_t(i)$ increase in $\psi W_{t-1}(i)$, implying that $a_t F'(H_t(i))$ falls and that $W_t(i)/P_t$ increases in $\psi W_{t-1}(i)$. This implies that $\xi_t(i)$ falls as $\psi W_{t-1}(i)$ increases. However, $\xi_t(i)$ may remain positive as long as constraint ii) binds only "mildly" (i.e., as long as $\psi W_{t-1}(i)$ is not far above the frictionless $W_t(i)$, described in the last subsection). Instead, $\xi_t(i)$ will hit zero when constraint ii) binds strong enough.

Therefore, from (Equ A.4), when constraint ii) binds strong enough employment is determined purely by labor demand:

$$a_t F'(H_t(i)) = \frac{W_t(i)}{P_t} \tag{Equ A.5}$$

and the multiplier $\xi_t(i) = 0$.

The multiplier $\lambda_t(i)$ can be inferred from (Equ A.2):

$$\lambda_t(i) = U'(t) \frac{1}{P_t} H_t(i) + \beta \psi E_t \lambda_{t+1}(i) - \xi_t(i) \eta (W_t(i)^{\eta-1}/W_t^{\eta}) H_t. \tag{Equ A.6}$$

Notice that, when constraint ii) binds strongly, implying that $\xi_t(i) = 0$, $\lambda_t(i)$ simplifies to the following:

$$\lambda_t(i) = U'(t) \frac{1}{P_t} H_t(i) + \beta \psi E_t \lambda_{t+1}(i). \tag{Equ A.7}$$

Symmetric equilibrium

We now study the symmetric equilibrium. All firms are identical, hence we set $W_t(i) = W_t$, $H_t(i) = H_t$, $\xi_t(i) = \xi_t$ and $\lambda_t(i) = \lambda_t$. Note that this implies that constraint i) always holds with equality.

When constraint ii) is slack, employment is determined from (Equ A.3)

$$a_t F'(H_t) = \frac{\eta + 1}{\eta} \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}, \tag{8}$$

which is equation (8) in the main text.

When downward nominal wage rigidity binds, wages are determined by constraint ii), in equilibrium: $W_t = \psi W_{t-1}$. When it binds mildly, equilibrium employment is determined by aggregate labor supply (6), implying for the multiplier λ_t

$$\lambda_t = -U'(t)\eta\frac{H_t}{W_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{W_t}{P_t}\right) + \beta\psi E_t\lambda_{t+1},$$

where we have combined (Equ A.4) and (Equ A.6) in the symmetric equilibrium.

Instead, when downward wage rigidity binds strongly, equilibrium employment is determined by labor demand

$$a_t F'(H_t) = \frac{W_t}{P_t} \tag{9}$$

which is equation (9) in the main text. Inserting this the multiplier λ_t reduces to

$$\lambda_t = U'(t)\frac{1}{P_t}H_t + \beta\psi E_t\lambda_{t+1} \tag{Equ A.8}$$

which is (Equ A.7) after imposing the symmetric equilibrium.

A.2 Constrained-efficient equilibrium and Proposition 1

From Definition 3, the constrained-efficient equilibrium solves

$$\max E_0 \sum_{t\geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$i) \qquad\qquad\qquad\qquad P_t a_t F'(H_t) \geq W_t \qquad \text{(multiplier: } \gamma_t\text{)}$$

$$ii) \qquad\qquad\qquad\qquad W_t \geq P_t G'(H_t) \qquad \text{(multiplier: } \zeta_t\text{)}$$

$$iii) \qquad\qquad\qquad\qquad W_t \geq \psi W_{t-1} \qquad \text{(multiplier: } \lambda_t\text{)}$$

$$iv) \qquad P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t \qquad \text{(multiplier: } \iota_t\text{)}$$

$$v) \qquad (U'(t)/P_t) = \beta R E_t(U'(t+1)/P_{t+1}) \qquad \text{(multiplier: } \nu_t\text{)}$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t\geq 0}$.

We show first that constraint v) is slack, implying that we can omit this constraint from the maximization. We proceed as in Bianchi (2016): we consider the maximization without constraint v) and show that constraint v) is implied as an optimality condition.

Assume v) is never binding ($\nu_t = 0$). Taking first order conditions with respect to C_t and B_{t+1} gives

$$U'(t) - \iota_t P_t = 0$$

$$-\iota_t/R + \beta E_t\iota_{t+1} = 0.$$

Combining both yields constraint v). Thus we have verified that constraint v) is never binding in equilibrium.

Now we take first order conditions with respect to W_t and H_t

$$-\gamma_t + \zeta_t + \lambda_t - \beta\psi E_t\lambda_{t+1} = 0 \qquad \text{(Equ A.9)}$$

$$-U'(t)G'(H_t) + \gamma_t\frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} - \zeta_t\frac{1}{\varepsilon_t^G}\frac{W_t}{H_t} + U'(t)a_tF'(H_t) = 0 \qquad \text{(Equ A.10)}$$

where we define $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$ as the wage elasticities of the labor demand and supply curve, respectively. The demand elasticity is negative under our assumptions imposed on F (labor demand slopes downward in wages), whereas the supply elasticity is positive under our assumptions imposed on G (labor supply slopes upwards).[33]

We proceed by distinguishing the cases where downward wage rigidity is slack and binding, respectively.

Case 1: Downward wage rigidity is slack

If downward nominal wage rigidity is slack ($W_t > \psi W_{t-1}$) then $\lambda_t = 0$ by complementary slackness. We show that in this case, constraint ii) must hold with equality. Assume not. In this case, $\zeta_t = 0$. Using $\zeta_t = 0$ in (Equ A.10) gives

$$\gamma_t = \varepsilon_t^F\frac{H_t}{W_t}U'(t)\left(G'(H_t) - a_tF'(H_t)\right).$$

Because of constraint i) and because constraint ii) holds with strict inequality by assumption, the expression in brackets is strictly negative. Because also $\varepsilon_t^F < 0$, it follows that $\gamma_t > 0$. Imposing $\zeta_t = 0$ in (Equ A.9) yields $-\gamma_t = \beta\psi E_t\lambda_{t+1} \geq 0$. This is a contradiction.

Constraint ii) thus holds with equality. We next show that $\gamma_t = 0$. Assume not, $\gamma_t > 0$. In this case, constraint i) holds with equality. Because constraints i) and ii) both hold with equality, equation (Equ A.10) yields

$$\gamma_t\frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} = \zeta_t\frac{1}{\varepsilon_t^G}\frac{W_t}{H_t}.$$

Because $\gamma_t > 0$ and $\varepsilon_t^F < 0$, the left hand side is strictly negative. But the right hand side is weakly positive ($\zeta_t \geq 0$ and $\varepsilon_t^G > 0$). This is a contradiction.

---

[33] As written in the main text, labor demand is

$$F'^{-1}\left(\frac{W_t}{P_t}\frac{1}{a_t}\right) = H_t = H_t(W_t).$$

The demand elasticity is defined as $\varepsilon_t^F \equiv H_t'(W_t)(W_t/H_t) < 0$. Elasticity ε_t^G is defined symmetrically for labor supply.

We summarize: when downward nominal wage rigidity is slack, $\lambda_t = \gamma_t = 0$ and constraint ii) holds with equality. Combining (Equ A.10) and (Equ A.9) by replacing ζ_t therefore yields

$$a_t F'(H_t) = \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}. \tag{13}$$

This is equation (13) in the main text.

Case 2: Downward wage rigidity is binding

Assume now that constraint iii) is binding. Assume also that constraint ii) is binding. This implies that W_t is determined by downward nominal wage rigidity whereas H_t is determined by labor supply.

We first show that, in this case $\gamma_t = 0$. Assume not, $\gamma_t > 0$. When $\gamma_t > 0$, constraint i) must hold with equality. Both constraints i) and ii) therefore hold with equality. Using this in (Equ A.10) yields

$$\gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} = \zeta_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t}.$$

Because $\gamma_t > 0$ and $\varepsilon_t^F < 0$, the left hand side is strictly negative. But the right hand side is weakly positive ($\zeta_t \geq 0$ and $\varepsilon_t^G > 0$). This is a contradiction.

Hence $\gamma_t = 0$. Because W_t and H_t are both determined, and because of $\gamma_t = 0$, the two equations (Equ A.9) and (Equ A.10) determine the multipliers λ_t and ζ_t.

Specifically we obtain for ζ_t from (Equ A.10)

$$\zeta_t = \varepsilon_t^G \frac{H_t}{W_t} U'(t) \left(a_t F'(H_t) - \frac{W_t}{P_t} \right),$$

where we have used that constraint ii) is binding. When downward nominal wage rigidity is slack, $a_t F'(H_t) > W_t/P_t$, in which case $\zeta_t > 0$ is strictly positive (recall that $\varepsilon_t^G > 0$). The previous expression shows that ζ_t can still be positive when downward nominal wage rigidity starts to be binding. In this region downward nominal wage rigidity thus binds "mildly", as in the free market equilibrium analyzed above. However, W_t/P_t rises and $a_t F'(H_t)$ falls when $W_t = \psi W_{t-1}$ gradually rises and hours are determined by labor supply. At some point, ζ_t will therefore become equal to zero.

At this point, $\zeta_t = 0$ and constraint ii) holds with strict inequality. As labor supply is rationed, labor demand is given by constraint i) which holds with equality

$$a_t F'(H_t) = \frac{W_t}{P_t}.$$

From (Equ A.10), γ_t turns strictly positive

$$\gamma_t = \varepsilon_t^F \frac{H_t}{W_t} U'(t) \left(G'(H_t) - \frac{W_t}{P_t} \right) > 0$$

where we use that constraint i) holds with equality and that constraint ii) holds with strict inequality (recall that $\varepsilon_t^F < 0$).

The multiplier λ_t when downward nominal wage rigidity binds can be inferred from combining (Equ A.9) and (Equ A.10)

$$\lambda_t = -U'(t) \left(\varepsilon_t^F \frac{H_t}{W_t} \left(\frac{W_t}{P_t} - G'(H_t) \right) + \varepsilon_t^G \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{W_t}{P_t} \right) \right) + \beta \psi E_t \lambda_{t+1} \qquad (14)$$

which is equation (14) in the main text.

A.3   A single monopsonist

A single monopsonist faces the dynamic problem

$$\Gamma_t(W_{t-1}) = \max_{\{H_t, W_t\}} \left\{ U'(t) \left(a_t F(H_t) - \frac{W_t}{P_t} H_t \right) + \beta E_t \Gamma_{t+1}(W_t) \right\}$$

subject to the set of constraints

$$i) \qquad\qquad W_t \geq P_t G'(H_t), \qquad \text{(multiplier: } \xi_t)$$

$$ii) \qquad\qquad W_t \geq \psi W_{t-1}, \qquad \text{(multiplier: } \lambda_t)$$

by taking as given the aggregate variables $\{a_t, P_t, U'(t)\}$.[34]

We take first order conditions with respect to W_t and H_t

$$-U'(t) \frac{1}{P_t} H_t + \xi_t + \lambda_t - \beta \psi E_t \lambda_{t+1} = 0$$

$$U'(t) \left(a_t F'(H_t) - \frac{W_t}{P_t} \right) - \xi_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} = 0,$$

where we have already used the Envelope condition

$$\frac{\partial}{\partial W_{t-1}} \Gamma_t(W_{t-1}) = -\lambda_t \psi$$

and where the wage elasticity of labor supply $\varepsilon_t^G > 0$ is defined analogously as in A.2 earlier above.

Here we only derive the labor demand curve when downward nominal wage rigidity is slack, as well as the multiplier λ_t when it is strongly binding.

In the former case, $\lambda_t = 0$ and $\xi_t > 0$. Combining the two first order conditions yields

$$a_t F'(H_t) = \frac{\varepsilon_t^G + 1}{\varepsilon_t^G} \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}. \qquad \text{(Equ A.11)}$$

---

[34] We assume the single monopsonist takes $U'(t)$ as given even though H_t enters through G: $U'(t) = U'(C_t - G(H_t))$. Nothing material changes if we assume the monopsonist internalizes the effect on $U'(t)$.

When comparing this labor demand curve with (8) in the main text, we note two important differences. First, the monopsonist discounts the cost of downward nominal wage rigidity by using the wage elasticity of *aggregate* labor supply ε_t^G. This shows that the single monopsonist is not subject to the pecuniary externality. Second, the mark-up charged by the monopsonist is $(\varepsilon_t^G + 1)/\varepsilon_t^G$. This reflects that the monopsonist exploits the aggregate labor supply curve when maximizing profits.

When downward nominal wage rigidity is strongly binding, $\xi_t = 0$ and $\lambda_t > 0$. Using this in the first order condition with respect to W_t yields

$$\lambda_t = U'(t)\frac{1}{P_t}H_t + \beta\psi E_t\lambda_{t+1}, \qquad \text{(Equ A.12)}$$

which is the same as in the case of monopsonistic competitors (see (10) in the main text after imposing $a_tF'(H_t) = W_t/P_t$, reflecting that downward nominal wage rigidity binds strongly; or see (Equ A.8) in the Appendix).

A.4 Implications for policy

A.4.1 Free market equilibrium with a payroll tax levied on firms

The households' problem is unchanged relative to the economy without intervention: in particular, the first order conditions (4) and (6) are unchanged. Downward nominal wage rigidity is still given by (5).

The firms' problem changes as follows

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i),W_t(i))} \left\{ U'(t)\left(a_tF(H_t(i)) - \frac{(1+\tau_t^w)W_t(i)}{P_t}H_t(i) + \frac{\mathcal{T}_t}{P_t} \right) + \beta E_t\Gamma_{t+1}(W_t(i)) \right\}$$

subject to the set of constraints

$$i) \qquad\qquad (W_t(i)/W_t)^\eta H_t \geq H_t(i),$$

$$ii) \qquad\qquad W_t(i) \geq \psi W_{t-1}(i),$$

by taking as given the aggregate variables $\{a_t, P_t, H_t, W_t, U'(t), \tau_t^w, \mathcal{T}_t\}$.

Following the steps in A.1.2, the first order conditions are given by

$$U'(t)\left(a_tF'(H_t(i)) - \frac{(1+\tau_t^w)W_t(i)}{P_t} \right) - \xi_t(i) = 0$$

for hours $H_t(i)$ as well as

$$-U'(t)\frac{(1+\tau_t^w)}{P_t}H_t(i) - \beta\psi E_t\lambda_{t+1}(i) + \lambda_t(i) + \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^\eta)H_t = 0$$

for wages $W_t(i)$. We proceed as in A.1.2 and impose the symmetry condition $W_t(i) = W_t$ and $H_t(i) = H_t$. This yields labor demand curves (15) and (16) and the multiplier (17) shown in the main text.

41

Profits are given by $\Pi_t = P_t a_t F(H_t(i)) - (1 + \tau_t^w) H_t(i) W_t(t) + \mathcal{T}_t$. By using that $\mathcal{T}_t = \tau_t^w W_t(i) H_t(i)$ in equilibrium, and using that $W_t(i) = W_t$ and $H_t(i) = H_t$ in equilibrium, this implies that the country's resource constraint is still given by (12).

A.4.2 Proof of Proposition 2

To prove Proposition 2, we show first that the policy maker facing the regulated free market equilibrium can choose τ_t^w to replicate the constrained-efficient allocation. That is, we show that this allocation is feasible for the policy maker. We argue, second, that the constraints faced by the policy maker are at least as stringent as those faced by the constrained-efficient planner. Because both the policy maker and the constrained-efficient planner are maximizing welfare, both together implies that the policy maker, among all feasible allocations, chooses to implement the constrained-efficient allocation.

We state the equilibrium conditions of the regulated free market equilibrium. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t\}_{t \geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}}$$

$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{\eta + 1}{\eta} \frac{W_t(1 + \tau_t^w)}{P_t} + \frac{1}{U'(t)} \frac{1}{\eta} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}$$

$$G'(H_t) = \frac{W_t}{P_t}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$

$$G'(H_t) = \frac{W_t}{P_t}$$

if $a_t F'(H_t) \geq \frac{(1+\tau_t^w)W_t}{P_t}$ (*binds lightly*), or else

$$W_t = \psi W_{t-1}$$

$$a_t F'(H_t) = \frac{(1 + \tau_t^w)W_t}{P_t},$$

(*binds strongly*), where the multiplier λ_t is given by

$$\lambda_t = -U'(t)\eta \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{\eta + 1}{\eta} \frac{(1 + \tau_t^w)W_t}{P_t} \right) + \beta \psi E_t \lambda_{t+1},$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t \geq 0}$ and $\{\tau_t^w \geq 0\}_{t \geq 0}$, are all satisfied.

We state the equilibrium conditions of the constrained-efficient planner. An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t^{sp}\}_{t \geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}}$$

$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{W_t}{P_t} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}^{sp}$$

$$G'(H_t) = \frac{W_t}{P_t}$$

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$

$$G'(H_t) = \frac{W_t}{P_t}$$

if $a_t F'(H_t) > \frac{W_t}{P_t}$ (*binds lightly*), or else

$$W_t = \psi W_{t-1}$$

$$a_t F'(H_t) = \frac{W_t}{P_t},$$

(*binds strongly*), where the multiplier λ_t is given by

$$\lambda_t^{sp} = -U'(t) \left(\varepsilon_t^F \frac{H_t}{W_t} \left(\frac{W_t}{P_t} - G'(H_t) \right) + \varepsilon_t^G \frac{H_t}{W_t} \left(a_t F'(H_t) - \frac{W_t}{P_t} \right) \right) + \beta \psi E_t \lambda_{t+1}^{sp}$$

where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t \geq 0}$, are all satisfied.

Comparing the two allocations reveals that the policy maker can replicate the constrained-efficient allocation by i) setting

$$\tau_t^w = \frac{\eta}{\eta+1} \left(\left(\frac{W_t}{P_t} \right)^{-1} \frac{1}{U'(t)} \frac{W_t}{H_t} \beta \psi \left(\frac{1}{\varepsilon_t^G} E_t \lambda_{t+1}^{sp} - \frac{1}{\eta} E_t \lambda_{t+1} \right) - \frac{1}{\eta} \right) \qquad \text{(Equ A.13)}$$

whenever downward nominal wage rigidity is slack, by ii) setting τ_t^w such that

$$a_t F'(H_t) = \frac{(1 + \tau_t^w) W_t}{P_t}$$

if downward nominal wage rigidity is binding as long as this implies $\tau_t^w \geq 0$[35] and by iii), setting

$$\tau_t^w = 0$$

otherwise. The tax in (Equ A.13) is positive as long as η is sufficiently larger than ε_t^G (as argued in the main text).

Furthermore, we note that the constraints faced by the policy maker all satisfy i) $W_t/P_t \leq a_t F'(H_t)$ (because of $\tau_t^w \geq 0$), ii) $G'(H_t) \leq W_t/P_t$ and iii) $W_t \geq \psi W_{t-1}$, which are constraints i)-iii) faced by the constrained-efficient planner (see Definition 3). Because all other constraints are identical, we conclude that the constraints faced by the policy maker are at least as stringent as those faced by the constrained-efficient planner.

A.4.3   A closed-form expression for the payroll tax τ_t^w

Here we derive formula (21) from the main text.

We use equation (Equ A.13) and take the limit as $\eta \to \infty$

$$\tau_t^w = \left(\frac{W_t}{P_t}\right)^{-1} \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}^{sp}.$$

We assume that downward nominal wage rigidity is binding strongly in $t+1$, and again slack thereafter (in periods $t+2, t+3$ etc). This implies for the multiplier λ_{t+1}^{sp}, from (14)

$$\lambda_{t+1}^{sp} = -U'(t+1)\varepsilon_{t+1}^F \frac{H_{t+1}}{W_{t+1}} \left(\frac{W_{t+1}}{P_{t+1}} - G'(H_{t+1})\right),$$

where we use that downward nominal wage rigidity is binding strongly to replace $a_{t+1}F'(H_{t+1}) - W_{t+1}/P_{t+1} = 0$, and that is it slack thereafter to replace $\lambda_{t+2}^{sp} = 0$.

Using our functional forms $G(H_t) = H_t^{1+\varphi}/(1+\varphi)$ and $F(H_t) = H_t^\alpha$, we can replace $\varepsilon_{t+1}^F = -1/(1-\alpha)$ and $G'(H_t) = H_t^\varphi$. Using this, and using (19) and (20) we rewrite the multiplier λ_{t+1}^{sp}

$$\lambda_{t+1}^{sp} = U'(t+1)\frac{1}{1-\alpha}\frac{H_{t+1}^p}{W_{t+1}}(1-u_{t+1})(H_{t+1}^p)^\varphi(1-(1-u_{t+1})^\varphi).$$

We again replace $(H_{t+1}^p)^\varphi = W_t/P_t$ by using (19) and cancel the W_{t+1} to arrive at

$$\lambda_{t+1}^{sp} = U'(t+1)\frac{1}{1-\alpha}\frac{H_{t+1}^p}{P_{t+1}}(1-u_{t+1})(1-(1-u_{t+1})^\varphi).$$

We insert this into the equation for τ_t^w

$$\tau_t^w = \frac{\varphi}{1-\alpha}\psi E_t \beta \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}} \frac{H_{t+1}^p}{H_t^p}(1-u_{t+1})(1-(1-u_{t+1})^\varphi)$$

---

[35] This is the region where downward nominal wage rigidity binds lightly.

where we use $\varepsilon_t^G = 1/\varphi$ and replace $H_t = H_t^p$ because by assumption, labor supply in the current period is not rationed.

Finally, we use again (19) to replace H_t^p and H_{t+1}^p, and we use that downward nominal wage rigidity binds in the next period to replace $W_{t+1} = \psi W_t$

$$
\tau_t^w = \frac{\varphi}{1-\alpha}\psi^{1+\frac{1}{\varphi}}E_t\beta\frac{U'(t+1)}{U'(t)}\frac{P_t}{P_{t+1}}\left(\frac{P_t}{P_{t+1}}\right)^{\frac{1}{\varphi}}(1-u_{t+1})(1-(1-u_{t+1})^\varphi),
$$

which is equation (21) in the main text.

A.4.4 Decentralization through sales taxes

The households' problem is unchanged from the baseline model, see A.4.1. Here we only present the firms' problem. Denoting the sales tax $\tau_t^p \geq 0$, we have

$$
\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i),W_t(i))} \left\{ U'(t)\left((1-\tau_t^p)a_tF(H_t(i)) - \frac{W_t(i)}{P_t}H_t(i) + \frac{\mathcal{T}_t}{P_t}\right) + \beta E_t\Gamma_{t+1}(W_t(i)) \right\}
$$

subject to the set of constraints

$$
i) \qquad\qquad\qquad (W_t(i)/W_t)^\eta H_t \geq H_t(i),
$$

$$
ii) \qquad\qquad\qquad W_t(i) \geq \psi W_{t-1}(i),
$$

by taking as given the aggregate variables $\{a_t, P_t, H_t, W_t, U'(t), \tau_t^p, \mathcal{T}_t\}$. The tax is rebated lump-sum to firms in equilibrium, $\mathcal{T}_t = P_t\tau_t^p a_tF(H_t(i))$.

Following the steps in A.1.2, the first order conditions are given by

$$
U'(t)\left((1-\tau_t^p)a_tF'(H_t(i)) - \frac{W_t(i)}{P_t}\right) - \xi_t(i) = 0
$$

for hours $H_t(i)$ as well as

$$
-U'(t)\frac{1}{P_t}H_t(i) - \beta\psi E_t\lambda_{t+1}(i) + \lambda_t(i) + \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^\eta)H_t = 0
$$

for wages $W_t(i)$. Proceed as in A.1.2, then impose the symmetry condition $W_t(i) = W_t$ and $H_t(i) = H_t$. This yields the labor demand curves

$$
(1-\tau_t^p)a_tF'(H_t) = \frac{\eta+1}{\eta}\frac{W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}
$$

when downward nominal wage rigidity is slack, and

$$
(1-\tau_t^p)a_tF'(H_t) = \frac{W_t}{P_t},
$$

when downward nominal wage rigidity is strongly binding. The multiplier λ_t is

$$
\lambda_t = -U'(t)\eta\frac{H_t}{W_t}\left((1-\tau_t^p)a_tF'(H_t) - \frac{\eta+1}{\eta}\frac{W_t}{P_t}\right) + \beta\psi E_t\lambda_{t+1}.
$$

These replace equilibrium conditions (15)-(17) in the main text. All other equilibrium conditions are identical.

By comparing these conditions with (15)-(17) in the main text, we see that payroll taxes and sales taxes are equivalent once we set $\tau_t^p = \tau_t^w/(1 + \tau_t^w)$.

A.4.5 Decentralization through income taxes on households

Assume that a tax $\tilde{\tau}_t^w$ applies to households' wage income, rebated lump-sum to households in equilibrium. We show that, in case downward nominal wage rigidity applies to the *net* wage $(1 - \tilde{\tau}_t^w)W_t(i)$, this instrument can be used to decentralize the constrained-efficient allocation.

The budget constraint (2) becomes

$$P_t C_t + \frac{B_{t+1}}{R} = \int_0^1 (1 - \tilde{\tau}_t^w)W_t(i)H_t(i) + \Pi_t(i)di + B_t + \mathcal{T}_t,$$

where $\mathcal{T}_t$ denotes the lump-sum transfer.

The maximization problem (3) becomes

$$\max_{(H_t(i))_{i \in [0,1]}} \int_0^1 (1 - \tilde{\tau}_t^w)W_t(i)H_t(i)di \quad \text{s.t.} \quad H_t \equiv \left(\int_0^1 H_t(i)^{1 + \frac{1}{\eta}} di \right)^{1/\left(1 + \frac{1}{\eta}\right)}.$$

The labor supply curves (4) remain unchanged for $1 - \tilde{\tau}_t^w$ cancels in $W_t(i)/W_t$.

For downward nominal wage rigidity we assume that

$$(1 - \tilde{\tau}_t^w)W_t(i) \geq \psi(1 - \tilde{\tau}_{t-1}^w)W_{t-1}(i),$$

which replaces (5).

The aggregate labor supply curve (6) becomes

$$G'(H_t) \leq \frac{(1 - \tilde{\tau}_t^w)W_t}{P_t}.$$

The firms' problem in Definition 1 changes to

$$\Gamma_t(W_{t-1}(i)) = \max_{(H_t(i),W_t(i))} \left\{ U'(t) \left(a_t F(H_t(i)) - \frac{W_t(i)}{P_t}H_t(i) \right) + \beta E_t \Gamma_{t+1}(W_t(i)) \right\}$$

subject to

$$i) \qquad\qquad (W_t(i)/W_t)^\eta H_t \geq H_t(i),$$

$$ii) \qquad\qquad (1 - \tilde{\tau}_t^w)W_t(i) \geq \psi(1 - \tilde{\tau}_{t-1}^w)W_{t-1}(i),$$

by taking as given the aggregate variables $\{a_t, P_t, H_t, W_t, U'(t), \tilde{\tau}_t^w\}$.

Following the steps in A.1.2, the first order conditions are given by

$$U'(t) \left(a_t F'(H_t(i)) - \frac{W_t(i)}{P_t} \right) - \xi_t(i) = 0$$

for hours $H_t(i)$ as well as

$$-U'(t)\frac{1}{P_t}H_t(i) - \beta\psi E_t(1-\tilde{\tau}_t^w)\lambda_{t+1}(i) + (1-\tilde{\tau}_t^w)\lambda_t(i) + \xi_t(i)\eta(W_t(i)^{\eta-1}/W_t^\eta)H_t = 0$$

for wages $W_t(i)$. Proceed as in A.1.2, then impose the symmetry condition $W_t(i) = W_t$ and $H_t(i) = H_t$. The tax is rebated lump-sum to the households in equilibrium $\mathcal{T}_t = \tilde{\tau}_t^w \int_0^1 W_t(i)H_t(i)di$.

An equilibrium is a path $\{C_t, H_t, B_{t+1}, W_t, \lambda_t\}_{t\geq 0}$ such that

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)}\frac{P_t}{P_{t+1}}$$

$$P_t C_t + \frac{B_{t+1}}{R} = P_t a_t F(H_t) + B_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{\eta+1}{\eta}\frac{W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{(1-\tilde{\tau}_t^w)W_t}{H_t}\beta\psi E_t\lambda_{t+1}$$

$$G'(H_t) = \frac{(1-\tilde{\tau}_t^w)W_t}{P_t}$$

if $(1-\tilde{\tau}_t^w)W_t \geq \psi(1-\tilde{\tau}_{t-1}^w)W_{t-1}$ (*slack*), or else

$$(1-\tilde{\tau}_t^w)W_t = \psi(1-\tilde{\tau}_{t-1}^w)W_{t-1}$$

$$G'(H_t) = \frac{(1-\tilde{\tau}_t^w)W_t}{P_t}$$

if $a_t F'(H_t) \geq \frac{W_t}{P_t}$ (*binds lightly*), or else

$$(1-\tilde{\tau}_t^w)W_t = \psi(1-\tilde{\tau}_{t-1}^w)W_{t-1}$$

$$a_t F'(H_t) = \frac{W_t}{P_t},$$

(*binds strongly*), where the multiplier λ_t is given by

$$\lambda_t = -U'(t)\eta\frac{H_t}{(1-\tilde{\tau}_t^w)W_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{W_t}{P_t}\right) + \beta\psi E_t\lambda_{t+1},$$

where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$ and $\{\tilde{\tau}_t^w \geq 0\}_{t\geq 0}$, are all satisfied.

To see the equivalence to the case discussed in the main text (the payroll tax on firms), define $\tilde{W}_t \equiv (1-\tilde{\tau}_t^w)W_t$ and rewrite all optimality conditions accordingly. This yields for the labor market market conditions

$$a_t F'(H_t) = \frac{\eta+1}{\eta}\frac{\tilde{W}_t/(1-\tilde{\tau}_t^w)}{\bar{P}_t} + \frac{1}{U'(t)}\frac{1}{\eta}\frac{\tilde{W}_t}{H_t}\beta\psi E_t\lambda_{t+1}$$

$$G'(H_t) = \frac{\tilde{W}_t}{\bar{P}_t}$$

if $\tilde{W}_t \geq \psi \tilde{W}_{t-1}$ (*slack*), or else

$$\tilde{W}_t = \psi \tilde{W}_{t-1}$$

$$G'(H_t) = \frac{\tilde{W}_t}{\bar{P}_t}$$

if $a_t F'(H_t) \geq \frac{\tilde{W}_t/(1-\tilde{\tau}_t^w)}{\bar{P}_t}$ (*binds lightly*), or else

$$\tilde{W}_t = \psi \tilde{W}_{t-1}$$

$$a_t F'(H_t) = \frac{\tilde{W}_t/(1-\tilde{\tau}_t^w)}{\bar{P}_t},$$

(*binds strongly*), where the multiplier λ_t is given by

$$\lambda_t = -U'(t)\eta \frac{H_t}{\tilde{W}_t}\left(a_t F'(H_t) - \frac{\eta+1}{\eta}\frac{\tilde{W}_t/(1-\tilde{\tau}_t^w)}{\bar{P}_t}\right) + \beta\psi E_t \lambda_{t+1}.$$

This corresponds to equations (15)-(17) in the main text, once we replace W_t by $\tilde{W}_t$ and set $1+\tau_t^w = 1/(1-\tilde{\tau}_t^w)$. Therefore, the allocation induced by using payroll taxes on firms $1+\tau_t^w$, or by using income taxes on households $1-\tilde{\tau}_t^w = 1/(1+\tau_t^w)$, are identical.

A.5   Equilibrium definition in terms of stationary variables

We define the real wage $w_t \equiv W_t/P_t$ and real foreign assets $b_{t+1} \equiv B_{t+1}/P_{t+1}$. We also define the shadow value of relaxing downward nominal wage rigidity in real terms $\tilde{\lambda}_t \equiv \lambda_t P_t$ (and analogously $\tilde{\lambda}_t^{sp} \equiv \lambda_t^{sp} P_t$). The real interest rate can be defined as $R^r \equiv R/\bar{\pi}$.

A.5.1   Free market equilibrium

The free market equilibrium is a path $\{C_t, H_t, b_{t+1}, w_t, \tilde{\lambda}_t\}_{t \geq 0}$ such that

$$1 = \beta R^r E_t \frac{U'(t+1)}{U'(t)}$$

$$C_t + \frac{b_{t+1}}{R^r} = a_t F(H_t) + b_t$$

as well as the labor market conditions

$$a_t F'(H_t) = \frac{\eta+1}{\eta}w_t + \frac{1}{\bar{\pi}}\frac{1}{U'(t)}\frac{1}{\eta}\frac{w_t}{H_t}\beta\psi E_t \tilde{\lambda}_{t+1}$$

$$G'(H_t) = w_t$$

if $w_t \geq (\psi/\bar{\pi})w_{t-1}$ (*slack*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$

$$G'(H_t) = w_t$$

if $a_t F'(H_t) \geq w_t$ (*binds lightly*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$

$$a_t F'(H_t) = w_t,$$

(*binds strongly*), where the multiplier $\tilde{\lambda}_t$ is given by

$$\tilde{\lambda}_t = -U'(t)\eta\frac{H_t}{w_t}(a_t F'(H_t) - \frac{\eta+1}{\eta}w_t) + \frac{1}{\bar{\pi}}\beta\psi E_t\tilde{\lambda}_{t+1},$$

where $U'(t) \equiv U'(C_t - G(H_t))$, for given initial conditions $w_{-1} > 0$ and b_0, and for given exogenous $\{a_t\}_{t\geq 0}$, are all satisfied.

A.5.2 Constrained-efficient equilibrium

The constrained-efficient equilibrium is a path $\{C_t, H_t, b_{t+1}, w_t, \tilde{\lambda}_t^{sp}\}_{t\geq 0}$ such that

$$1 = \beta R^r E_t\frac{U'(t+1)}{U'(t)}$$

$$C_t + \frac{b_{t+1}}{R^r} = a_t F(H_t) + b_t$$

as well as the labor market conditions

$$a_t F'(H_t) = w_t + \frac{1}{\bar{\pi}}\frac{1}{U'(t)}\frac{1}{\varepsilon_t^G}\frac{w_t}{H_t}\beta\psi E_t\tilde{\lambda}_{t+1}^{sp}$$

$$G'(H_t) = w_t$$

if $w_t \geq (\psi/\bar{\pi})w_{t-1}$ (*slack*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$

$$G'(H_t) = w_t$$

if $a_t F'(H_t) \geq w_t$ (*binds lightly*), or else

$$w_t = (\psi/\bar{\pi})w_{t-1}$$

$$a_t F'(H_t) = w_t,$$

(*binds strongly*), where the multiplier $\tilde{\lambda}_t^{sp}$ is given by

$$\tilde{\lambda}_t^{sp} = -U'(t)\left(\varepsilon_t^F\frac{H_t}{w_t}(w_t - G'(H_t)) + \varepsilon_t^G\frac{H_t}{w_t}(a_t F'(H_t) - w_t)\right) + \frac{1}{\bar{\pi}}\beta\psi E_t\tilde{\lambda}_{t+1}^{sp}$$

where $U'(t) \equiv U'(C_t - G(H_t))$, for given initial conditions $w_{-1} > 0$ and b_0, and for given exogenous $\{a_t\}_{t\geq 0}$, are all satisfied.

B   Model extensions

This Appendix contains three model extensions. B.1 presents the model with wealth effects on labor supply. B.2 presents the model with market power on the labor *supply* side, i.e., a model with wage-setting unions. B.3 presents the model with a non-tradable sector.

The last extension is used to show that aggregate demand externalities of the kind discussed in Schmitt-Grohé and Uribe (2016) and pecuniary externalities are distinct, but may arise simultaneously. It is also used to illustrate that endogenous price inflation does not invalidate the use of prudential labor taxes.

B.1   Model with wealth effect on labor supply

Here we study how our results change once we allow for wealth effects on labor supply.

Households' welfare (1) is now given by

$$E_0 \sum_{t \geq 0} \beta^t (U(C_t) - G(H_t)), \quad \beta \in (0,1)$$

where U and G are specified as before. In the free market equilibrium, the only equilibrium condition that is affected by this change is aggregate labor supply

$$\frac{G'(H_t)}{U'(C_t)} \leq \frac{W_t}{P_t},$$

which replaces (6).

We study a constrained social planner as in Definition 3, however, in addition to assuming that the planner chooses labor allocations on behalf of firms, also assuming that the planner chooses consumption demand on behalf of households.

The problem of the planner becomes

$$\max \; E_0 \sum_{t \geq 0} \beta^t \{U(C_t) - G(H_t)\}$$

subject to the set of constraints

$$i) \qquad\qquad\qquad\qquad P_t a_t F'(H_t) \geq W_t \qquad \text{(multiplier: } \gamma_t\text{)}$$

$$ii) \qquad\qquad\qquad W_t \geq P_t G'(H_t)/U'(C_t) \qquad \text{(multiplier: } \zeta_t\text{)}$$

$$iii) \qquad\qquad\qquad\qquad\qquad W_t \geq \psi W_{t-1} \qquad \text{(multiplier: } \lambda_t\text{)}$$

$$iv) \qquad\qquad P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t \qquad \text{(multiplier: } \iota_t\text{)}$$

where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t \geq 0}$.

The first order conditions with respect to C_t and B_{t+1} are

$$U'(C_t) - P_t \iota_t - \zeta_t \nu_t^G \frac{C_t}{W_t} = 0$$

$$-\iota_t/R + \beta E_t \iota_{t+1} = 0,$$

where $\nu_t^G > 0$ denotes the elasticity of wages with respect to a change in C_t. It is positive, because a higher C_t implies a positive wealth effect which shifts labor supply upwards.

Combining both gives the constrained-efficient Euler equation

$$1 = \beta R E_t \frac{U'(C_{t+1}) - \zeta_{t+1} \nu_{t+1}^G (C_{t+1}/W_{t+1})}{U'(C_t) - \zeta_t \nu_t^G (C_t/W_t)} \frac{P_t}{P_{t+1}}, \tag{B.1}$$

which needs to be compared with

$$1 = \beta R E_t \frac{U'(C_{t+1})}{U'(C_t)} \frac{P_t}{P_{t+1}},$$

in the free market equilibrium.

This is the main difference of the model with wealth effects relative to the model presented in the main text: the planner has an incentive to regulate aggregate demand, for aggregate demand matters for wealth effects which impact labor supply. In turn, regulating labor supply matters due to the pecuniary externality in the labor market.

The first order conditions for W_t and H_t are (largely) unchanged from the baseline model (compare equations (Equ A.9) and (Equ A.10) in the Appendix)

$$-\gamma_t + \zeta_t + \lambda_t - \beta \psi E_t \lambda_{t+1} = 0$$

$$-G'(H_t) + \gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} + \iota_t P_t a_t F'(H_t) = 0.$$

These can be combined to yield

$$a_t F'(H_t) = \frac{G'(H_t)}{\iota_t} \frac{1}{P_t} + \frac{1}{\varepsilon_t^G} \frac{1}{P_t} \frac{1}{\iota_t} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}$$

when downward nominal wage rigidity is slack, which compares with (8) in the baseline model.

In sum, in the model with wealth effects, in addition to regulating labor demand, the social planner also regulates aggregate demand.

One can show that capital controls in this model are prudential. As argued in A.2, $\zeta_t > 0$ when downward nominal wage rigidity is slack, but $\zeta_t = 0$ when downward nominal wage rigidity is (strongly) binding reflecting that in this case, labor supply is rationed. As an illustration, assume that downward nominal wage rigidity is binding with certainty next period (i.e., $\zeta_{t+1} = 0$). This implies for the Euler equation (B.1)

$$U'(C_t) = \beta R E_t U'(C_{t+1}) \frac{P_t}{P_{t+1}} + \zeta_t \nu_t^G (C_t/W_t),$$

where $\zeta_t > 0$ if downward nominal wage rigidity is slack in the current period. Recalling that $\nu_t^G > 0$, we see that the planner regulates today's marginal utility of consumption upwards, such that capital controls are indeed prudential (in the sense that the planner decreases current borrowing in anticipation of the binding constraint).

It is important to note that, even though capital controls are now required to decentralize the constrained-efficient equilibrium, the motive for doing so is different than in the model of Schmitt-Grohé and Uribe (2016). In the current model, curbing aggregate demand contains the wealth effect on labor supply shifting labor supply downwards, which reduces wage inflation in expansions, which in turn helps to alleviate the pecuniary externality. In the model of Schmitt-Grohé and Uribe (2016), aggregate demand plays a role of its own, due to the presence of an aggregate demand externality. We explore this in detail in B.3.

B.2 Market power on the labor supply side

In the main text, we argued that the pecuniary externality affects labor demand. We now show that it affects also labor supply. We use a framework where the market power is with households (unions), rather than with firms. We show that households (unions) raise wages in expansions, not internalizing the rise in market wages as competition pushes up the wages of other households (unions). In a context of downward nominal wage rigidity, this makes the laissez-faire outcome constrained inefficient.

B.2.1 Economic environment

An economy is populated by a unit mass of households indexed by $j \in [0, 1]$. Each household j maximizes

$$E_0 \sum_{t \geq 0} \beta^t U(C_t(j) - G(H_t(j))) \tag{B.2}$$

subject to the budget constraint

$$P_t C_t(j) + \frac{B_{t+1}(j)}{P_t} = W_t(j) H_t(j) + \Pi_t + B_t(j).$$

Here we assume that households receive symmetric firm profits Π_t. All variables and the functions U and G are defined as in the main text.

Following Benigno and Ricci (2011), we assume consumption risk sharing across households through a set of state-contingent claims to monetary units. This yields the Euler equation

$$1 = \beta R \frac{U'(t+1)}{U'(t)} \frac{P_t}{P_{t+1}},$$

where $U'(t) = U'(C_t(j) - G(H_t(j)))$ for all $j \in [0,1]$.[36]

Households supply differentiated labor types $H_t(j)$ which for firms are imperfectly substitutable. This gives market power to households. Note the difference to the baseline model: there households could not easily substitute their employer, which gave market power to firms.

Specifically, a representative firm operates the technology $a_t F(H_t)$, where F is defined as in the main text, and where H_t is a composite

$$H_t = \left(\int_0^1 H_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

Here $H_t(j)$ is the demand for labor supplied by household j. Every household sells labor to the representative firm. The demand for household-specific labor on the part of wage-taking firms is the solution of

$$\min_{\{H_t(j)\}_{j \in [0,1]}} \int_0^1 W_t(j) H_t(j) dj \quad \text{s.t.} \quad H_t \equiv \left(\int_0^1 H_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

This yields a set of labor demand curves

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\theta} H_t, \tag{B.3}$$

where W_t is the Lagrange multiplier given by $W_t \equiv (\int_0^1 W_t(j)^{1-\theta} dj)^{1/(1-\theta)}$.

Moreover, aggregate labor demand is

$$a_t F'(H_t) = \frac{W_t}{P_t}$$

and profits are given by $\Pi_t = P_t a_t F(H_t) - W_t H_t$.

Turn now to labor supply. Each household j maximizes utility (B.2), subject to household-specific labor demand (B.3), subject to the budget constraint, and subject to downward nominal wage rigidity $W_t(j) \geq \psi W_{t-1}(j)$.

As shown in Benigno and Ricci (2011), this is equivalent to solving the following problem

$$\Gamma_t(W_{t-1}(j)) = \max_{(H_t(j), W_t(j))} \left\{ U'(t) \left(\frac{W_t(j) H_t(j)}{P_t} - G(H_t(j)) \right) + \beta E_t \Gamma_{t+1}(W_t(j)) \right\}$$

subject to the set of constraints

$$i) \qquad\qquad H_t(j) = (W_t(j)/W_t)^{-\theta} H_t,$$

$$ii) \qquad\qquad W_t(j) \geq \psi W_{t-1}(j),$$

---

[36] Strictly speaking, the consumption-risk-sharing assumption is not required in our analysis, because households are identical and downward nominal wage rigidity is symmetric (and we assume symmetric intitial conditions, i.e., $B_0(j) = B_0$ and $W_{-1}(j) = W_{-1}$). In Benigno and Ricci (2011), this assumption is strictly required because households are subject to idiosyncratic preference shocks. In the literature studying Calvo wages, this assumption is also commonly made because households have ex-post different wage incomes even though they are ex-ante identical.

by taking as given the aggregate variables $\{H_t, W_t, U'(t), P_t\}$.

Inserting constraint $i)$ to replace $H_t(j)$ and denoting the Lagrange multiplier with respect to constraint $ii)$ $\lambda_t(j)$, we obtain the first order condition

$$U'(t)\left((1-\theta)\frac{H_t(j)}{P_t} - G'(H_t(j))(-\theta)\frac{H_t(j)}{W_t(j)}\right) + \lambda_t(j) - \beta\psi E_t\lambda_{t+1}(j) = 0,$$

where we have already used the Envelope condition

$$\frac{\partial}{\partial W_{t-1}(j)}\Gamma_t(W_{t-1}(j)) = -\psi\lambda_t(j).$$

When downward nominal wage rigidity is slack $\lambda_t(j) = 0$, this can be rearranged to

$$G'(H_t(j)) = \frac{\theta-1}{\theta}\frac{W_t(j)}{P_t} + \frac{1}{U'(t)}\frac{1}{\theta}\frac{W_t(j)}{H_t(j)}\beta\psi E_t\lambda_{t+1}(j),$$

where $\lambda_t(j)$ turns positive in periods when downward nominal wage rigidity binds

$$\lambda_t(j) = -U'(t)\left((1-\theta)\frac{H_t(j)}{P_t} - G'(H_t(j))(-\theta)\frac{H_t(j)}{W_t(j)}\right) + \beta\psi E_t\lambda_{t+1}(j).$$

Notice that under perfect labor market competition $(\theta \to \infty)$, labor supply reduces to the familiar expression

$$G'(H_t(j)) = \frac{W_t(j)}{P_t}.$$

Conversely, when labor market competition is imperfect, labor supply is increased (wages are reduced) as households internalize that downward nominal wage rigidity may bind in future recessions.

B.2.2 Definition of equilibrium

We study the symmetric equilibrium where $W_t(j) = W_t$ and $H_t(j) = H_t$ for all $j \in [0,1]$. We also assume symmetric initial conditions $(B_0(j) = B_0$ and $W_{-1}(j) = W_{-1})$.

A free market equilibrium is a set of processes $\{C_t, H_t, B_{t+1}, W_t, \lambda\}_{t \geq 0}$ such that

$$1 = \beta E_t\frac{U'(t+1)}{U'(t)}\frac{P_t}{P_{t+1}}$$

$$P_tC_t + \frac{B_{t+1}}{R} = P_ta_tF(H_t) + B_t$$

as well as the labor market conditions

$$G'(H_t) = \frac{\theta-1}{\theta}\frac{W_t}{P_t} + \frac{1}{U'(t)}\frac{1}{\theta}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}, \tag{B.4}$$

$$a_tF'(H_t) = \frac{W_t}{P_t}$$

54

if $W_t \geq \psi W_{t-1}$ (*slack*), or else

$$W_t = \psi W_{t-1}$$
$$a_t F'(H_t) = \frac{W_t}{P_t}$$

(*binding*), where the multiplier λ_t is given by

$$\lambda_t = -U'(t)\left((1-\theta)\frac{H_t}{P_t} - G'(H_t)(-\theta)\frac{H_t}{W_t}\right) + \beta\psi E_t\lambda_{t+1},$$

where $U'(t) = U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, \bar{P}_t\}_{t\geq 0}$, are all satisfied.

There is a very close similarity of this economy with the economy in the main text. The definition of equilibrium resembles Definition 2 in the main text. The difference is that wage setting is now performed by households rather than by firms. Moreover, households internalize downward nominal wage rigidity, because a wedge term containing $E_t\lambda_{t+1}$ appears in the labor supply curve (B.4).

B.2.3 Constrained efficiency

We show that the free market equilibrium is constrained inefficient.

Consider the planning problem

$$\max E_0 \sum_{t\geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$i) \qquad\qquad\qquad\qquad W_t = P_t a_t F'(H_t) \qquad \text{(multiplier: } \gamma_t)$$

$$ii) \qquad\qquad\qquad\qquad W_t \geq \psi W_{t-1} \qquad \text{(multiplier: } \lambda_t)$$

$$iii) \qquad\qquad P_t C_t + B_{t+1}/R = P_t a_t F(H_t) + B_t \qquad \text{(multiplier: } \iota_t)$$

$$iv) \qquad\qquad (U'(t)/P_t) = \beta R E_t(U'(t+1)/P_{t+1}) \qquad \text{(multiplier: } \nu_t)$$

where $U'(t) \equiv U'(C_t - G(H_t))$ and where $P_t = \bar{P}_t$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, \bar{P}_t\}_{t\geq 0}$.

This is the same as in the main text (Definition 3), except that the planner chooses labor allocations on behalf of *households* (rather than on behalf of firms, as in the main text), while letting all other markets clear competitively.

We proceed as in A.2. First, $\nu_t = 0$, because the Euler equation is not a binding constraint in equilibrium. We can therefore omit it from the above maximization. Moreover, $\iota_t = U'(t)/P_t$.

First order conditions with respect to W_t and H_t are given by

$$\gamma_t + \lambda_t - \beta\psi E_t\lambda_{t+1} = 0$$

$$-U'(t)G'(H_t) - \gamma_t\frac{1}{\varepsilon_t^F}\frac{W_t}{H_t} + U'(t)a_tF'(H_t) = 0,$$

where $\varepsilon_t^F < 0$ continues to denote the wage elasticity of (aggregate) labor demand. Combining both by replacing γ_t yields

$$U'(t)G'(H_t) = \frac{1}{\varepsilon_t^F}\frac{W_t}{H_t}(\lambda_t - \beta\psi E_t\lambda_{t+1}) + U'(t)a_tF'(H_t)$$

In a period when downward nominal wage rigidity is slack $\lambda_t = 0$, this yields

$$G'(H_t) = \frac{W_t}{P_t} - \frac{1}{U'(t)}\frac{1}{\varepsilon_t^F}\frac{W_t}{H_t}\beta\psi E_t\lambda_{t+1}, \tag{B.5}$$

where we used that $a_tF'(H_t) = W_t/P_t$, and where λ_t turns positive in periods when downward nominal wage rigidity binds

$$\lambda_t = U'(t)\varepsilon_t^F\frac{H_t}{W_t}\left(G'(H_t) - \frac{W_t}{P_t}\right) + \beta\psi E_t\lambda_{t+1}.$$

Comparing labor supply (B.4) and constrained-efficient labor supply (B.5) reveals the same externality as in the baseline economy: the expected cost of downward nominal wage rigidity is discounted using the wage elasticity of household-specific labor demand θ in the free market, but the wage elasticity of aggregate labor demand $-\varepsilon_t^F$ in the constrained-efficient equilibrium (note $\varepsilon_t^F < 0$, because labor demand slopes downwards in wages).

B.3   Price effects and aggregate demand

We now augment the baseline model by introducing a non-tradable sector. Introducing non-tradables makes the consumer price index endogenous, as it fluctuates with the relative price of tradables to non-tradables. This allows us to study if our results change when domestic prices are endogenous.

Introducing non-tradables also gives a role to aggregate demand, because the price elasticity of aggregate demand for consumption ceases to be infinity.[37] This allows us to demonstrate that the pecuniary externality is conceptually *distinct* from the (demand) externality that is described in Schmitt-Grohé and Uribe (2016).

To ease exposition, in this section we only present the case $\eta \to \infty$, which corresponds to perfect labor market competition.

---

[37] In our baseline model, aggregate demand has no effect on the domestic price level for the latter is exogenously fixed at $P_t = \bar{P}_t$, independently of domestic developments.

B.3.1 Economic environment

Welfare is still given by (1), but $C_t = A(C_t^T, C_t^N)$ now denotes a composite between tradable and non-tradable consumption. The aggregator A is increasing, concave and linearly homogeneous. We assume that tradable output Y_t^T is an endowment, and that labor income $W_t H_t$ and profits Π_t derive from firms which operate in the non-tradable sector exclusively, as in Schmitt-Grohé and Uribe (2016).

The budget constraint (2) thus becomes

$$P_t^T C_t^T + P_t^N C_t^N + \frac{B_{t+1}}{R} = P_t^T Y_t^T + W_t H_t + \Pi_t + B_t.$$

We continue to assume that the price of tradables $P_t^T = \bar{P}_t^T$ is exogenously fixed. However, the price index of non-tradables P_t^N is endogenous.

The Euler equation of households is still given by (7) if we replace P_t by P_t^T and $U'(t) \equiv U'(C_t - G(H_t))$ by $U'(t)A_1(t)$, where $A_i(t), i \in \{1, 2\}$ denotes the partial derivative of function A with respect to its first and second argument, respectively:

$$1 = \beta R E_t \frac{U'(t+1)A_1(t+1)}{U'(t)A_1(t)} \frac{P_t^T}{P_{t+1}^T}. \tag{B.6}$$

By the same token, labor supply is still given by (6):

$$G'(H_t) \leq \frac{W_t}{P_t^A}, \tag{B.7}$$

with the key difference that P_t has been replaced by P_t^A. The price level P_t^A is an expenditure-based measure of the consumer price index (CPI), linked to the price index of non-tradables via $P_t^A = P_t^N / A_2(t)$.[38] It is therefore also endogenous.

Underlying the weak inequality in (B.7) is the assumption of downward nominal wage rigidity, as before:

$$W_t \geq \psi W_{t-1}.$$

A novel equilibrium condition is the optimal allocation of expenditure across tradable and non-tradable consumption, given by

$$A_2(t) = \frac{P_t^N}{P_t^T} A_1(t). \tag{B.8}$$

---

[38] To see this, consider the problem of optimally allocating expenditure

$$\max \{P_t^T C_t^T + P_t^N C_t^N\} \quad \text{s.t.} \quad C_t \equiv A(C_t^T, C_t^N).$$

The Lagrange multiplier of this problem, which by the Envelope theorem represents an expenditure-based measure of the consumer price index, is equal to $P_t^N / A_2(t)$.

This equation implies that the price of non-tradables P_t^N and therefore the consumer price index P_t^A is an increasing function in tradable consumption C_t^T. Intuitively, when tradables and non-tradables are imperfect substitutes, households increase their demand for non-tradables as they consume more tradables, which raises the price of non-tradables. This is a key channel through which aggregate demand matters for labor market outcomes.

Firms are owned by the households. They operate in the non-tradable sector, maximizing period-profits $\Pi_t = P_t^N a_t F(H_t) - W_t H_t$ by taking prices and wages as given (recall that we assume $\eta \to \infty$). This yields labor demand

$$a_t F'(H_t) = \frac{W_t}{P_t^N}. \tag{B.9}$$

With respect to the baseline model, the important difference is that the sales price of firms' output P_t^N is now endogenous.

The market clears when all non-tradables are consumed in each period domestically. In symbols, $a_t F(H_t) = C_t^N$. Using this and replacing $W_t H_t + \Pi_t = P_t^N a_t F(H_t)$ in the budget constraint yields the equilibrium resource constraint

$$P_t^T C_t^T + \frac{B_{t+1}}{R} = P_t^T Y_t^T + B_t. \tag{B.10}$$

We state the following definition of equilibrium. A free market equilibrium is a set of processes $\{P_t^N, C_t^T, H_t, B_{t+1}, W_t\}_{t \geq 0}$ such that equations (B.6) and (B.8)-(B.10) as well as either

$\quad i$) [slack] (B.7) with equality if $W_t \geq \psi W_{t-1}$, or else

$\quad ii$) [binds] $W_t = \psi W_{t-1}$,

where $U'(t) \equiv U'(C_t - G(H_t))$, $A_i(t) = A_i(C_t^T, a_t F(H_t))$ for $i \in \{1, 2\}$, $P_t^A = P_t^N / A_2(t)$, $C_t = A(C_t^T, a_t F(H_t))$, and where $P_t^T = \bar{P}_t^T$, for given initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, Y_t^T, \bar{P}_t^T\}_{t \geq 0}$, are all satisfied.

B.3.2 Constrained efficiency

The model described in the previous subsection corresponds exactly to the model in Schmitt-Grohé and Uribe (2016), except that we assume labor supply to be endogenous.[39] As they emphasize, this economy suffers from an externality. As they write in their introduction (pp. 1468-69): "The nature of the externality is that expansions in aggregate demand drive up

---

[39] As we explain in the main text in Section 3, inelastic labor supply is a special case of the model where the pecuniary externality leads to trivial policy implications: tax labor at an infinite rate in expansions, for this has no repercussions on equilibrium employment. In their Online Appendix, Schmitt-Grohé and Uribe (2016) perform robustness checks in a model with endogenous labor supply, which is almost identical to the model described in this chapter. The only difference is that we use GHH preferences, whereas Schmitt-Grohé and Uribe (2016) use preferences that allow for a wealth effect on labor supply.

wages, putting the economy in a vulnerable situation.... Agents understand this mechanism but are too small to internalize the fact that their individual expenditure decisions collectively cause inefficiently large increases in wages during expansions."

Formally, Schmitt-Grohé and Uribe show that a social planner regulates aggregate demand. They describe a *demand* externality which affects *households*, which can be addressed by manipulating households' Euler equation (e.g., via capital controls). In contrast, the externality that we describe is a *pecuniary* externality that arises from competitive behavior of *firms*. It can be addressed via a prudential tax on labor.

The two externalities are related, but distinct. To establish this, we consider a planner that chooses labor allocations on behalf of firms, as in the main text. To take account of Schmitt-Grohé and Uribe's finding, we additionally let the planner choose aggregate demand (demand for tradables C_t^T) on behalf of households. All remaining markets clear competitively. The constrained-efficient allocation solves

$$\max \ E_0 \sum_{t \geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$i) \qquad\qquad W_t \leq P_t^T \frac{A_2(t)}{A_1(t)} a_t F'(H_t)$$

$$ii) \qquad\qquad G'(H_t)\frac{P_t^T}{A_1(t)} \leq W_t$$

$$iii) \qquad\qquad W_t \geq \psi W_{t-1}$$

$$iv) \qquad\qquad P_t^T C_t^T + B_{t+1}/R = P_t^T Y_t^T + B_t$$

where $A_i(t) = A_i(C_t^T, a_t F(H_t))$ for $i \in \{1, 2\}$, $C_t = A(C_t^T, a_t F(H_t))$, and where $P_t^T = \bar{P}_t^T$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, Y_t^T, \bar{P}_t^T\}_{t \geq 0}$.

The planning problem reveals the benefit of regulating aggregate demand. Because C_t^T enters labor demand (and supply) through $A_1(t)$ and $A_2(t)$, aggregate demand matters for wages, and therefore for downward nominal wage rigidity. In B.3.5 below, we thus recover Schmitt-Grohé and Uribe's result that the planner chooses a different Euler equation than the free market equilibrium.[40,41]

---

[40] Another way to regulate aggregate demand (other than through capital controls) might be through regulating government spending. In expansions, by reducing government spending, aggregate demand would be reduced which could help address the aggregate demand externality. This aspect would be worth exploring, because government spending drove much of the pre-crisis boom in Greece. Ottonello et al. (2017) take some steps in this direction.

[41] In the model from the main text, aggregate demand does not matter for wages, see Definition 3. The planner therefore has no incentive to regulate aggregate demand. Formally, as we show in A.2, constraint v) in Definition 3 is always slack in equilibrium. This implies that an unrestricted planner would *choose* the same Euler equation (7) as the free market equilibrium.

In addition to regulating aggregate demand, the planner in this model regulates labor demand by firms, as in the main text. If the inequality in constraint i) is strict, the planner reduces firms' hiring relative to the free market equilibrium. The planner has an incentive to do so due to the pecuniary externality. The pecuniary externality has gone unnoticed by Schmitt-Grohé and Uribe, for they impose that constraint i) must always hold with equality: the constrained planner that they consider does not have the power to intervene directly in the labor market.[42]

Here we state the labor demand curve when downward nominal wage rigidity is slack. In B.3.5 we derive this equation formally. It is

$$\frac{P_t^N a_t F'(H_t)}{P_t^A} = \frac{W_t}{P_t^A} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1}, \tag{B.11}$$

where $\varepsilon_t^G > 0$ continues to denote the wage elasticity of labor supply and $\lambda_t \geq 0$ the Lagrange multiplier associated with downward nominal wage rigidity. Notice that the labor demand curve (B.11) closely resembles its counterpart in the baseline model (13), which shows that the pecuniary externality is relevant in the extended model.

B.3.3 Implications for regulation

In the extended model, two externalities interact. What are the implications for regulation? How can the constrained-efficient equilibrium be decentralized?

We show in B.3.5 below that the constrained-efficient allocation can be decentralized by using prudential payroll taxes as in the main text—however, augmented by capital controls which are needed to address the aggregate demand externality. In this model, *two* instruments are needed to decentralize the constrained-efficient allocation.

Are the two instruments substitutes or complementary? The previous analysis suggests that they are rather substitutes. For example, with the payroll tax in place, constraint i) in the planning problem is slack in expansions. This implies that in expansions, *at the margin*, aggregate demand does not impact wage inflation through labor demand, which reduces the need for capital controls in expansions. However, aggregate demand still matters through labor supply, constraint ii) in the planning problem, which holds with equality in expansions. Therefore, prudential capital controls can not be dispensed with entirely to decentralize the constrained-efficient allocation.

---

[42] In Schmitt-Grohé and Uribe (2016), this can be seen by noting that the constrained planner (their equation (32)) takes as a constraint the competitive labor demand curve holding with equality (their equation (16)).

B.3.4 Assessing endogenous price effects

To what extent should the tax on labor be set differently when there is an endogenous response of domestic inflation? Intuitively, the payroll tax in the extended model is inflationary, because firms (partially) pass the higher labor cost implied by the tax through to domestic prices. Does this imply that the tax should be set higher or lower compared to the model without endogenous inflation?

To answer this question, we derive a closed-form expression for the optimal payroll tax, the analogue of equation (21) in the main text. To do so, as we did in the main text, we again assume that downward nominal wage rigidity is slack in the current period, strongly binding in the following period, and again slack thereafter. As we show in B.3.5 below, the optimal tax satisfies

$$\tau_t^w = -\frac{1}{\varepsilon_t^G}\psi^{1+\frac{1}{\varphi}}E_t\xi_{t,t+1}\left(\frac{P_t^A}{P_{t+1}^A}\right)^{\frac{1}{\varphi}}\varepsilon_{t+1}^F(1-u_{t+1})(1-(1-u_{t+1})^\varphi), \qquad \text{(B.12)}$$

where $\xi_{t,t+1} \equiv \beta(U'(t+1)/U'(t))(P_t^A/P_{t+1}^A)$ is a stochastic discount factor. The tax resembles equation (21) from the main text, with three differences.

First, expected price inflation is now endogenous due to the consumer price index P_t^A being endogenous. Second, the labor supply and demand elasticities $\varepsilon_t^G > 0$ and $\varepsilon_{t+1}^F < 0$ have not been substituted out like in the main text (recall the discussion about these elasticities in Section 3.3). This is because in the extended model, both do not have closed-form expressions. However, we can characterize the properties of both elasticities, and how they change relative to the model from the main text.

As we show in B.3.5 below, an endogenous feedback of prices *reduces* (in absolute value) the wage elasticity of labor demand ε_t^F, whereas it *raises* the wage elasticity of labor supply ε_t^G. Intuitively, a change in W_t affects labor demand by less if firms pass it through to a change in P_t^N. This implies that labor demand becomes less responsive to wage changes. Conversely, a change in W_t that induces households to work more (produce more non-tradables) reduces P_t^N and therefore the consumer price index P_t^A, which raises the real wage, which raises labor supply even further. This implies that labor supply becomes more responsive to wage changes.

This implies that introducing endogenous price inflation into the model has an ambiguous effect on the optimal tax. By the endogenous response of the two elasticities, the tax should be set strictly lower. However, if the tax is inflationary in the period of implementation which raises P_t^A/P_{t+1}^A, the tax should be set strictly higher. Intuitively, this is because the response of domestic prices makes the tax less effective at increasing real wages. Overall, whether the tax should be set higher or lower relative to the baseline model can not be answered

unambiguously: it may depend on the state of the economy, as well as on calibration.

B.3.5   Formal derivations

We repeat the problem of the constrained-efficient planner

$$\max \; E_0 \sum_{t \geq 0} \beta^t U(C_t - G(H_t))$$

subject to the set of constraints

$$
\begin{aligned}
&i) & P_t^T a_t F'(H_t) \frac{A_2(t)}{A_1(t)} &\geq W_t & \text{(multiplier: } \gamma_t) \\[2mm]
&ii) & W_t &\geq \frac{P_t^T}{A_1(t)} G'(H_t) & \text{(multiplier: } \zeta_t) \\[2mm]
&iii) & W_t &\geq \psi W_{t-1} & \text{(multiplier: } \lambda_t) \\[2mm]
&iv) & P_t^T C_t^T + B_{t+1}/R &= P_t^T Y_t^T + B_t & \text{(multiplier: } \iota_t)
\end{aligned}
$$

where $A_i(t) = A_i(C_t^T, a_t F(H_t))$ for $i \in \{1,2\}$, $C_t = A(C_t^T, a_t F(H_t))$, and where $P_t^T = \bar{P}_t^T$, for given initial $W_{-1} > 0$ and B_0, and for the given exogenous process $\{a_t, Y_t^T, \bar{P}_t^T\}_{t \geq 0}$.

We take first order conditions with respect to C_t^T and B_{t+1}

$$U'(t)A_1(t) - P_t^T \iota_t + \gamma_t \nu_t^F \frac{C_t^T}{W_t} - \zeta_t \nu_t^G \frac{C_t^T}{W_t} = 0$$

$$-\iota_t/R + \beta E_t \iota_{t+1} = 0,$$

where $\nu_t^F > 0$ ($\nu_t^G > 0$) are the elasticity of wages W_t in labor demand (supply) with respect to tradable consumption C_t^T. Both are positive because a higher C_t^T shifts rightwards labor demand (demand for non-tradables increases, which leads firms to hire on more workers), and because it shifts leftwards labor supply (the consumer price index rises, which discourages agents from working at given nominal wages).

We take first order conditions with respect to W_t and H_t

$$-\gamma_t + \zeta_t + \lambda_t - \psi \beta E_t \lambda_{t+1} = 0$$

$$U'(t)(A_2(t)a_t F'(H_t) - G'(H_t)) + \gamma_t \frac{1}{\varepsilon_t^F} \frac{W_t}{H_t} - \zeta_t \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} = 0,$$

where as before, $\varepsilon_t^F < 0$ and $\varepsilon_t^G > 0$ denote the wage elasticities of labor demand and supply, respectively. Compared with the model from the main text, these are now more complicated expressions for they also depend on the properties of $A_1(t)$ and $A_2(t)$, see constraints i) and ii) above. We will explore this in detail below.

We proceed by distinguishing the cases where downward wage rigidity is slack and binding, respectively.

Case 1: Downward wage rigidity is slack

Along the lines of the derivation in A.2, when downward nominal wage rigidity is slack $\lambda_t = 0$ and $\gamma_t = 0$. Using this in the above first order conditions we obtain for the Euler equation

$$1 = \beta R E_t \frac{U'(t+1)A_1(t+1) + (C_{t+1}^T/W_{t+1})(\gamma_{t+1}\nu_{t+1}^F - \zeta_{t+1}\nu_{t+1}^G)}{U'(t)A_1(t) - \zeta_t\nu_t^G(C_t^T/W_t)} \frac{P_t^T}{P_{t+1}^T},$$

and for the labor demand curve

$$\frac{P_t^N a_t F'(H_t)}{P_t^A} = \frac{W_t}{P_t^A} + \frac{1}{U'(t)} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta\psi E_t\lambda_{t+1}, \tag{27}$$

which is equation (B.11) shown above. To derive the last equation, we have performed the substitution $P_t^A = P_t^N/A_2(t) = P_t^T/A_1(t)$, see equation (B.8), and we have used that constraint ii) in the planing problem holds with equality when downward nominal wage rigidity is slack.

Case 2: Downward wage rigidity is binding

When downward nominal wage rigidity binds lightly, λ_t turns positive but still $\gamma_t = 0$ (see the A.2). In this region, firms hire the full labor supply as long as the marginal product of the additional workers net of the wage remains positive.

When downward nominal wage rigidity binds strongly, labor supply is rationed implying $\zeta_t = 0$. Moreover, constraint i) in the maximization of the planner starts to bind, implying

$$P_t^N a_t F'(H_t) = W_t$$

and $\gamma_t > 0$. Therefore, the Euler equation in this region becomes

$$1 = \beta R E_t \frac{U'(t+1)A_1(t+1) + (C_{t+1}^T/W_{t+1})(\gamma_{t+1}\nu_{t+1}^F - \zeta_{t+1}\nu_{t+1}^G)}{U'(t)A_1(t) + \gamma_t\nu_t^F(C_t^T/W_t)} \frac{P_t^T}{P_{t+1}^T}.$$

Thus in this region, the planner ceases to regulate labor demand as in the baseline model from the main text, but the planner still regulates aggregate demand.

Decentralization

We next show that the constrained-efficient equilibrium can be decentralized with capital controls (which are needed to address the demand externality) as well as labor demand taxes (which are needed to address the pecuniary externality).

We assume households' savings are taxed at rate τ_t^R and firms' labor cost is taxed at rate $\tau_t^w \geq 0$. Both taxes are rebated lump-sum to households in equilibrium. The regulated free market equilibrium is a sequence $\{P_t^N, C_t^T, H_t, B_{t+1}, W_t\}_{t \geq 0}$ such that

$$1 = \beta R(1 + \tau_t^R) E_t \frac{U'(t+1)A_1(t+1)}{U'(t)A_1(t)} \frac{P_t^T}{P_{t+1}^T}$$

$$P_t^T \frac{A_2(t)}{A_1(t)} a_t F'(H_t) = (1 + \tau_t^w) W_t$$

$$P_t^T C_t^T + B_{t+1}/R = P_t^T Y_t^T + B_t$$

as well as the labor market conditions

$$\frac{P_t^T}{A_1(t)} G'(H_t) = W_t$$

if $W_t \geq \psi W_{t-1}$ or else

$$W_t = \psi W_{t-1},$$

where $U'(t) \equiv U'(C_t - G(H_t))$, $A_i(t) = A_i(C_t^T, a_t F(H_t))$ for $i \in \{1, 2\}$, $P_t^A = P_t^N/A_2(t)$, $C_t = A(C_t^T, a_t F(H_t))$, and where $P_t^T = \bar{P}_t^T$, for given initial conditions $W_{-1} > 0$ and B_0, for given exogenous $\{a_t, Y_t^T\}_{t \geq 0}$, and for given $\{\tau_t^R, \tau_t^w \geq 0\}_{t \geq 0}$ chosen by policy, are all satisfied. The policy problem is to maximize welfare (1) over regulated free market equilibria.

To show that the welfare-optimal regulated free market equilibrium and the constrained-efficient equilibrium coincide, we proceed as in A.4.2. We first show that the constrained-efficient equilibrium is feasible for the policy maker. Secondly, we show that the constraints faced by the policy maker are at least as stringent as those faced by the constrained-efficient planner. Both together implies that the policy maker chooses to implement the constrained-efficient allocation.

Feasibility can be seen as follows. Whenever downward nominal wage rigidity is slack, set τ_t^w equal to

$$\tau_t^w = \left(\frac{W_t}{P_t^A}\right)^{-1} \frac{1}{\varepsilon_t^G} \frac{W_t}{H_t} \beta \psi E_t \lambda_{t+1} \geq 0 \tag{B.13}$$

and τ_t^R equal to

$$1 + \tau_t^R = \left(E_t \frac{U'(t+1)A_1(t+1)}{U'(t)A_1(t)} \frac{P_t^T}{P_{t+1}^T} \right)^{-1}$$

$$E_t \frac{U'(t+1)A_1(t+1) + (C_{t+1}^T/W_{t+1})(\gamma_{t+1}\nu_{t+1}^F - \zeta_{t+1}\nu_{t+1}^G)}{U'(t)A_1(t) - \zeta_t \nu_t^G (C_t^T/W_t)} \frac{P_t^T}{P_{t+1}^T}.$$

When downward nominal wage rigidity starts to bind, W_t is determined by downward nominal wage rigidity and H_t is determined by labor supply. In this case, set τ_t^w to satisfy

$$P_t^T \frac{A_2(t)}{A_1(t)} a_t F'(H_t) = (1 + \tau_t^w) W_t$$

as long as τ_t^w remains positive. This implies that firms hire the additional labor supply and do not ration employment (compare with A.4.2).

Instead, when downward nominal wage rigidity binds strongly, set $\tau_t^w = 0$ and set τ_t^R equal to

$$
1 + \tau_t^R = \left(E_t \frac{U'(t+1)A_1(t+1)}{U'(t)A_1(t)} \frac{P_t^T}{P_{t+1}^T} \right)^{-1}
$$
$$
E_t \frac{U'(t+1)A_1(t+1) + (C_{t+1}^T/W_{t+1})(\gamma_{t+1}\nu_{t+1}^F - \zeta_{t+1}\nu_{t+1}^G)}{U'(t)A_1(t) + \gamma_t \nu_t^F (C_t^T/W_t)} \frac{P_t^T}{P_{t+1}^T}.
$$

All constraints faced by the policy maker satisfy i) $W_t \leq P_t^T a_t F'(H_t)(A_2(t)/A_1(t))$ (because of $\tau_t^w \geq 0$), ii) $W_t \geq G'(H_t)(P_t^T/A_1(t))$ as well as iii) $W_t \geq \psi W_{t-1}$, which are constraints i-iii) faced by the constrained-efficient planner. Because all other constraints are identical, we conclude that the constraints faced by the policy maker are at least as stringent as those faced by the constrained-efficient planner.

A closed-form expression for τ_t^w

We now derive equation (B.12). We proceed as in the derivation of equation (21) from the main text, see A.4.3. We thus assume that downward nominal wage rigidity is slack in the current period, binding strongly in the next period, and again slack thereafter. Moreover, we define potential employment as solving

$$
G'(H_t^p) = \frac{W_t}{P_t^A}, \tag{B.14}
$$

and we define unemployment according to equation (20).

Under our assumptions λ_{t+1} is given by

$$
\lambda_{t+1} = -U'(t+1)\varepsilon_{t+1}^F \frac{H_{t+1}}{W_{t+1}} \left(\frac{W_{t+1}}{P_{t+1}^A} - G'(H_{t+1}) \right),
$$

where we used that $\lambda_{t+2} = 0$, that $P_t^A = W_t/A_1(t)$, and that constraint i) holds with equality when downward nominal wage rigidity binds strongly. We can rewrite this term along the lines of A.4.3. We first use that G is of the constant-Frisch-elasticity type to obtain

$$
\lambda_{t+1} = -U'(t+1)\varepsilon_{t+1}^F \frac{H_{t+1}^p}{W_{t+1}} (1 - u_{t+1})(H_{t+1}^p)^{\varphi}(1 - (1 - u_{t+1})^{\varphi}).
$$

We substitute equation (B.14) to obtain

$$
\lambda_{t+1} = -U'(t+1)\varepsilon_{t+1}^F \frac{H_{t+1}^p}{P_{t+1}^A} (1 - u_{t+1})(1 - (1 - u_{t+1})^{\varphi}).
$$

We insert this equation into (B.13) to obtain

$$\tau_t^w = -\frac{1}{\varepsilon_t^G}\psi E_t\beta\frac{U'(t+1)}{U'(t)}\frac{P_t^A}{P_{t+1}^A}\frac{H_{t+1}^p}{H_t^p}\varepsilon_{t+1}^F(1-u_{t+1})(1-(1-u_{t+1})^\varphi),$$

where we use that downward nominal wage rigidity is slack in period t to replace $H_t = H_t^p$. Finally, we use again equation (B.14) to rewrite

$$\tau_t^w = -\frac{1}{\varepsilon_t^G}\psi^{1+\frac{1}{\varphi}}E_t\xi_{t,t+1}\left(\frac{P_t^A}{P_{t+1}^A}\right)^{\frac{1}{\varphi}}\varepsilon_{t+1}^F(1-u_{t+1})(1-(1-u_{t+1})^\varphi), \qquad \text{(B.12)}$$

where $\xi_{t,t+1} \equiv \beta(U'(t+1)/U'(t))(P_t^A/P_{t+1}^A)$ is a stochastic discount factor and where we use that $W_{t+1} = \psi W_t$ because by assumption, downward nominal wage rigidity in period $t+1$ is binding.

Assessing wage elasticities

The optimal tax (B.12) contains the wage elasticities $\varepsilon_t^G > 0$ as well as $\varepsilon_{t+1}^F < 0$. Here we analyze the two wage elasticities in more detail and contrast them to the baseline model in which domestic prices are exogenous.

Start with labor demand

$$W_t = a_t F'(H_t)P_t^T\frac{A_2(t)}{A_1(t)}.$$

Wages W_t depend on hours directly, through the marginal product term $a_t F'(H_t)$, and indirectly, through the price term $P_t^N = P_t^T(A_2(t)/A_1(t))$. Computing the elasticity of W_t with respect to H_t thus yields

$$\epsilon_t^{W,H} = \epsilon_t^{aF',H} + \epsilon_t^{P,H}.$$

The first elasticity is negative for the marginal product slopes downward in hours $\epsilon_t^{aF',H} < 0$. Indeed under the assumed production function, this term is $\epsilon_t^{aF',H} = \alpha - 1$, where $\alpha \in (0,1)$. In the baseline model, the price term is P_t which is exogenous, so that $\epsilon_t^{P,H} = 0$.

H_t enters $A_i(t) = A_i(C_t^T, a_t F(H_t))$, for $i \in \{1,2\}$, positively through the second argument. Because A is increasing, concave and linearly homogeneous, this implies that $A_1(t)$ slopes upward in H_t whereas $A_2(t)$ slopes downward in H_t. This implies that the ratio $A_2(t)/A_1(t)$, and therefore P_t^N, slopes downward in H_t.

The elasticity $\epsilon_t^{P,H} < 0$ is therefore negative in the extended model.

Turning to the wage elasticity of labor demand, note that $\varepsilon_t^F = 1/\epsilon_t^{W,H}$. Thus

$$|\varepsilon_t^F| = \frac{1}{|\epsilon_t^{aF',H}| + |\epsilon_t^{P,H}|} < \frac{1}{|\epsilon_t^{aF',H}|},$$

where we use that both $\epsilon_t^{aF',H}$ and $\epsilon_t^{P,H}$ are negative. This expression shows that the wage elasticity of labor demand is *reduced* by the presence of endogenous prices.

Turn now to labor supply, which is

$$W_t = \frac{P_t^T}{A_1(t)} G'(H_t).$$

Wages W_t depend on hours directly, through the marginal disutility term $G'(H_t)$, and indirectly, through the price term $P_t^A = P_t^T/A_1(t)$. Computing the elasticity with respect to H_t thus yields

$$\epsilon_t^{W,H} = \epsilon_t^{G',H} + \epsilon_t^{P,H}.$$

The first elasticity is positive for the marginal disutility slopes upward in hours $\epsilon_t^{G',H} > 0$: under our assumed utility function, $\epsilon_t^{G',H} = \varphi$, $\varphi > 0$. In the baseline model, the price term is P_t which is exogenous, so that $\epsilon_t^{P,H} = 0$.

As explained above, in the extended model $A_1(t)$ slopes upward in H_t, implying that P_t^A slopes downward in H_t. This implies that elasticity $\epsilon_t^{P,H}$ is negative in the extended model.

The wage elasticity of labor supply is given by $\varepsilon_t^G = 1/\epsilon_t^{W,H}$. As a result

$$\varepsilon_t^G = \frac{1}{\epsilon_t^{G',H} + \epsilon_t^{P,H}} > \frac{1}{\epsilon_t^{G',H}},$$

where we use that $\epsilon_t^{G',H}$ is positive but that $\epsilon_t^{P,H}$ is negative. Therefore, the wage elasticity of labor supply is *raised* by the presence of endogenous prices.