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Abstract

This paper studies optimal policy in economies with downward nominal wage rigidity when only prudential instruments are available. The optimal policy reduces labor demand in expansions as this curtails unemployment in recessions. The cost of the intervention is that in expansions, the economy produces below potential. We characterize this trade-off theoretically and quantitatively by applying our model to Greece, 1999-2016. We find that the optimal prudential policy would have significantly reduced Greek unemployment after the downturn in 2008. Furthermore, we find large welfare gains of the optimal prudential policy, removing about one fourth of the total welfare cost of downward wage rigidity.

Keywords: unemployment, fiscal devaluation, euro area, labor market, payroll tax, constrained efficiency, monopsonistic competition
JEL-Codes: E24, E32, F41

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1 Introduction

The sharp rise in wages in some euro countries before the Great Recession, followed by large-scale unemployment and slowly declining wages in the aftermath of the Great Recession, have reinvigorated concerns about the harmfulness of downward nominal wage rigidity for macroeconomic adjustment (Baldwin and Giavazzi, 2015). Optimal policies to address downward wage rigidity are manifold. For example, monetary policy may attempt to raise price inflation to "grease the wheels" of the labor market (Tobin, 1972), or similarly, depreciate the nominal exchange rate (Na et al., 2018). Alternatively, fiscal policy may subsidize the firms' payroll in recessions—so called "fiscal devaluation" (Farhi et al., 2014).

Instead, this paper is concerned with optimal *prudential* policies in economies with downward wage rigidity. The difference is that reactive policies operate in recessions when downward wage rigidity binds, whereas prudential policies operate in expansions when downward wage rigidity is slack. Prudential policies are useful once reactive policies are not available. First, monetary policy may be unable to combat downward wage rigidity, for example if the country has no independent monetary policy. Second, payroll subsidies may be difficult to implement in recessions, for example due to fiscal strain.[1]

To characterize optimal prudential policies, we consider a small open economy model with a Walrasian labor market subject to downward nominal wage rigidity. Because all agents are wage takers, they are too small to internalize that their collective action generates too sharp wage increases in expansions (Schmitt-Grohé and Uribe, 2016). Formally, we show that the competitive equilibrium is *constrained inefficient*. That is, we show that a social planner can improve welfare even by respecting that downward wage rigidity binds in recessions. The fact that the equilibrium is constrained inefficient creates scope for prudential policy intervention (e.g., Bianchi, 2011; Bianchi and Mendoza, 2018; Dávila and Korinek, 2018).

The optimal prudential intervention is to reduce the firms' labor demand in expansions. This can be achieved either by taxing the firms' payroll, or by taxing the firms' sales revenue in those periods. Intuitively, as the labor demand curve shifts in, this curtails wage increases as the economy moves down an upward sloping labor supply curve. While lower wages lead to less unemployment in recessions, the cost is that the economy produces below potential in expansions. We show that this trade-off depends on the slopes of labor supply and demand, the utility penalty suffered from unemployment, the effective degree of downward wage rigidity, as well as on the volatility of shocks underlying the economy. Moreover, we derive a convenient

[1] Once a fiscal and an unemployment crisis occur jointly, it may be counterproductive to raise government spending (give out payroll subsidies) as this adds to sovereign debt (Bianchi et al., 2018). In a monetary union with decentralized fiscal policy, a fiscal crisis may result from short-sighted or free-riding government that over-accumulate sovereign debt (Beetsma and Uhlig, 1999; Chari and Kehoe, 2007).

formula for the optimal prudential tax, as a function of model primitives (such as the labor share and Frisch elasticity of labor supply), and of the magnitude of the unemployment spell expected for next period.

As an extension we depart from the assumption of Walrasian labor market and consider active wage setting. As emphasized in Elsby (2009), firms may actively set wages internalizing that downward wage rigidity affects their workers. We find that, even though firms compress wage increases in expansions which mimics the constrained-efficient outcome, they do so to a socially insufficient extent. In a nutshell, if firms compete with other firms for workers in monopsonistic competition, they have little leeway for prudential wage reductions: following (unilateral) wage reductions, firms lose workers as they substitute to better-paying competitors.[2] This provides an instructive caveat on Elsby (2009)'s finding that under active wage setting, downward wage rigidity is consistent with weak macroeconomic effects. Formally, we show that the competitive equilibrium is still constrained inefficient. Therefore, the case for prudential policy intervention is alive and well.

In a quantitative application we consider the case of Greece, 1999-2016. The Greek cycle has been characterized by a strong increase in wages until 2008, followed by a slow decline in wages after 2008 and record-unemployment. We demonstrate that the competitive equilibrium can replicate the Greek experience rather well. Thereafter, we study the optimal prudential intervention during the initial phase of the cycle 1999-2008, where downward wage rigidity has been slack. We find that the optimal prudential payroll tax reaches up to 13% during this period, significantly reducing wage inflation. As a result, the policy has significant positive effects on unemployment during the following contraction.

Finally, we show that the welfare gain from the optimal prudential policy is large. In our application to Greece, the prudential policy removes about one quarter of the total welfare cost of downward wage rigidity (it removes 0.85 of a total 3.3 percent of permanent consumption). This reflects a reduction in mean unemployment from 6.3 to 1.5 percent.

Related literature.—Modeling downward wage rigidity has recently become very popular. Building on the seminal contribution of Akerlof et al. (1996), recent influential contributions include Benigno and Fornaro (2018), Bianchi et al. (2018) and Na et al. (2018), but also the whole secular stagnation literature (e.g., Fornaro and Romei (2018), Eggertsson et al. (2018) and Corsetti et al. (2018)).

One famous result in the literature on downward wage rigidity is that, once firms actively set wages and internalize downward wage rigidity, downward wage rigidity is consistent with

[2] Therefore, under active wage setting, the problem arises from a strategic complementarity (Woodford, 2003). Were all firms to coordinate on their wage setting, their collective reduction in labor demand and wages during expansions would bring about the constrained-efficient outcome.

weak macroeconomic effects (Elsby, 2009). Here we reach a different conclusion: by studying active firms in general equilibrium and considering competition among firms, downward wage rigidity is as costly as in a Walrasian labor market (see Manning (2003) for details on monopsonistic labor market competition). We show that, even when firms actively compress wage rises which is privately efficient, the resulting allocation is not socially efficient, creating scope for policy intervention.

Specifically, we analyse optimal prudential policy to address downward wage rigidity. This relates with Schmitt-Grohé and Uribe (2016) who show that a currency peg with downward wage rigidity benefits from capital controls. Relative to them, we show that the inefficiency from downward wage rigidity is more general, and in particular does not rely on inefficient movements in the country's capital flows or real exchange rate that hinge on a non-tradable sector (indeed, the inefficiency would arise even in a closed-economy context). Also, here we characterize explicitly the optimal intervention, we additionally study the case of active wage setters, and we study a different set of policies for decentralization: labor-market rather than capital-control policies.[3]

On the theory side, this paper adds to the literature on macro-prudential intervention. This literature is mostly concerned with externalities that arise from financial frictions (e.g., Dávila and Korinek, 2018; Farhi and Werning, 2016; Lorenzoni, 2008). Instead, here we study (macro-) prudential intervention in the presence of pecuniary externalities that arise from a nominal friction: downward nominal wage rigidity.

On the applied side, the differential wage developments across euro area countries have received considerable attention. Schmitt-Grohé and Uribe (2013) and Fahr and Smets (2010) make a case for temporary price inflation in the euro core in order to overcome downward wage rigidity in the euro periphery. Gilchrist et al. (2018) explain the high level of wages in the euro periphery by a combination of customer markets and financial frictions. Kuvshinov et al. (2016) show that debt deleveraging pressure can paralyze the adjustment of relative labor costs across currency union members. We add by showing that a country may benefit from prudential intervention in its labor market.

Outline.—Section 2 presents the baseline model. Sections 3-4 discuss constrained efficiency and decentralization. Section 5 presents the model extension featuring active wage setters. Section 6 presents the quantitative application to Greece. Finally, Section 7 concludes. Appendixes A-B contain analytical derivations and details on the data used in Section 6.

[3] Capital controls would be ineffective to address downward wage rigidity in the present analysis. In Schmitt-Grohé and Uribe (2016) capital controls work because capital flows are indirectly linked to the dynamics of wages, via their effect on the real exchange rate. In the present analysis, we purposefully abstract from a feedback of capital flows into wages, but focus on the inefficiencies that arise from downward wage rigidity directly in the labor market.

2 Model

We consider a simple model of a small currency peg. Households consume, work, and save in (incomplete) international financial markets. Nominal wages are downward rigid. Firms produce a single consumption good, which is freely traded across borders. The economy is buffeted by demand-side shocks and by shocks to labor productivity.

2.1 Households

A representative household maximizes consumption utility net of disutility from work

$$E_0 \sum_{t\geq 0} \beta^t [c_t - V(h_t^f - \delta u_t)], \quad \beta \in (0,1), \quad \delta \in [0,1], \tag{1}$$

subject to the budget constraint

$$p_t c_t + \frac{\Lambda_{t+1}}{R} = w_t h_t + \Pi_t + \Lambda_t. \tag{2}$$

and a no-Ponzi constraint. Here, p_t is the price of consumption, $w_t h_t$ is wage income, Π_t are firm profits and Λ_t are nominal bonds, traded across border at price $1/R$. The household takes prices, wages, profits and the nominal interest rate R as given.

Because the consumption good c_t is traded internationally, its price p_t is exogenous from the vantage point of the small domestic economy (it is here that we use that the country has a fixed nominal exchange rate). In this context, p_t is often interpreted as the country's terms of trade (Schmitt-Grohé and Uribe, 2016).

Taking first order conditions with respect to consumption and bonds gives the consumption Euler equation

$$1 = \beta R E_t \frac{p_t}{p_{t+1}}, \tag{3}$$

where we use that consumption utility is linear, such that the marginal utility of consumption equals one. Because c_t does not appear in (3), the optimal path for consumption and assets is indeterminate in equilibrium. Without loss of generality, we therefore assume that households never save, and set $\Lambda_t = 0$ for all t.

The usefulness of assuming linear consumption utility is *not* that households do not save. What matters is that the labor supply curve (5) is independent of consumption, which can be achieved by assuming a linear consumption utility. This enables us to characterize analytically the inefficiencies associated with downward wage rigidity, as it separates the labor market from the consumption/savings problem.

The Appendix A presents the model by assuming Greenwood-Hercowitz-Huffman (GHH) preferences. As these eliminate the wealth effect on labor supply, the labor supply curve (5) is again independent of consumption (even though now households save). We show that this model has an *identical* equilibrium for output and the labor market as the baseline model, implying that also the optimal prudential intervention is identical. To streamline exposition, in the main text we therefore proceed with linear consumption utility.[4]

In addition, by assuming linear consumption utility (and therefore an intertemporal elasticity of substitution of plus infinity), we may demonstrate that the optimal prudential intervention does not arise from a motive for consumption smoothing.

2.2 Labor supply

In the objective (1), h_t^f denotes the amount of hours that the household would like to supply. Because of downward wage rigidity, the amount of hours h_t that are actually supplied may fall below this level, creating unemployment

$$u_t = h_t^f - h_t \geq 0. \tag{4}$$

In the objective (1), we allow for the possibility that voluntary leisure and unemployment are imperfect substitutes, $\delta < 1$. This implies that the disutility of work falls by only little in a recession even as working hours are reduced.[5] This reflects the evidence that the unemployed are not any happier from the additional leisure, but instead suffer a direct utility loss from unemployment (Winkelmann and Winkelmann, 1998). It also reflects the evidence that, while not directly engaged in working-related activities, unemployed workers spend a lot of time searching for new employment (Krueger and Mueller, 2012).

In the absence of downward wage rigidity, unemployment is zero at all times. In this case, we obtain the amount of hours that households would voluntarily supply

$$V'(h_t^f) = \frac{w_t}{p_t}, \tag{5}$$

where consumption c_t does not appear because of our earlier assumptions. Instead, for downward wage rigidity we follow Akerlof et al. (1996) and assume

$$w_t \geq \psi(u_t) w_{t-1}. \tag{6}$$

Nominal wages may fall by an amount $\psi(u_t)$ per period, which may depend on the extent of unemployment. Specifically, we assume that $\psi(0) = \bar{\psi} \leq 1$, and that $\psi' \leq 0$ but $\psi \geq 0$.

[4] Without either linear consumption utility or GHH preferences, consumption c_t would appear in labor supply curve (5). As a result, the labor market and the timing of consumption would become interlinked, the policy problem in Definition 1 would feature the consumption Euler equation as implementability constraint, and the problem would become intractable.

[5] To see this, insert (4) into the disutility of work: $V(h_t^f - \delta u_t) = V(h_t + (1-\delta)u_t)$. In a recession, h_t falls. However, to the extent that this represents unemployment ($u_t > 0$), and if unemployment and leisure are imperfect substitutes ($\delta < 1$), the households' disutility of work falls not to the same extent.

Unemployment can arise only if the wage rigidity binds:

$$u_t(w_t - \psi(u_t)w_{t-1}) = 0. \tag{7}$$

We have $V'(h_t^f - \delta u_t) \leq w_t/p_t$ from (5) and from $u_t \geq 0$; i.e., the marginal rate of substitution may fall strictly below the real wage. Therefore, if downward wage rigidity binds, the labor market is rationed on the labor supply side.

2.3 Firms

A representative firm maximizes profits, taking the nominal wage and the sales price of the consumption good as given. It uses only labor for production

$$y_t = a_t F(h_t), \tag{8}$$

where a_t is the productivity of labor, which is exogenous. Profits are given by $\Pi_t = \max\{p_t y_t - w_t h_t\}$. The labor demand curve is

$$p_t a_t F'(h_t) = w_t. \tag{9}$$

2.4 Competitive equilibrium

A competitive equilibrium can be defined as follows. Given initial conditions w_{-1} and an exogenous process for $\{a_t, p_t\}_{t\geq 0}$, a competitive equilibrium is a process for $\{w_t, h_t, u_t\}_{t\geq 0}$ such that the following conditions are satisfied

i) $\quad w_t/p_t = V'(h_t + u_t)$ (labor supply)

ii $\quad w_t/p_t = a_t F'(h_t)$ (labor demand)

iii) $\quad w_t \geq \psi(u_t)w_{t-1}, \quad u_t \geq 0, \quad \cdot \times \cdot = 0$ (inequality and slackness).

Equilibrium output and consumption follow residually: $y_t = c_t = a_t F(h_t)$. Equilibrium utility (welfare) follows residually: $\mathcal{U} \equiv E_0 \sum_{t\geq 0} \beta^t [c_t - V(h_t + (1-\delta)u_t)]$.

3 Constrained efficiency

Is the competitive equilibrium maximizing welfare? It is clearly not first best, due to downward wage rigidity. However, it may still be second best, or *constrained efficient*, referring to efficiency conditional on downward wage rigidity.

We now show that the competitive equilibrium is *constrained inefficient*. That is, even by respecting that downward wage rigidity binds in recessions, a social planner can increase welfare.

3.1 The planning problem

The planner maximizes utility (1), subject to technology (8), the interest rate (3), and downward wage rigidity (6). The planner also takes as given (2), as this constitutes the economy's resource constraint. To see this, note that from the firm side, profits and labor income must always add to total output, $p_t y_t = w_t h_t + \Pi_t$. Inserting this in (2) yields

$$p_t c_t + \frac{\Lambda_{t+1}}{R} = p_t y_t + \Lambda_t, \tag{10}$$

where Λ_t constitutes the country's net foreign assets.

Because consumption utility is linear, the planner cannot improve household utility by shifting consumption across periods. We therefore again set $\Lambda_t = 0$ for all t, without loss of generality. In this case, the resource constraint (10) collapses to $c_t = y_t$.

Thereafter, the planner faces the following problem

Definition 1. [CONSTRAINED-EFFICIENT EQUILIBRIUM] *The allocation that maximizes household utility in the presence of downward wage rigidity solves*

$$\mathcal{U}(w_{t-1}, a_t, p_t) = \max_{\{h_t, w_t, u_t\}} \{a_t F(h_t) - V(h_t + (1-\delta)u_t) + \beta E_t \mathcal{U}(w_t, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{aligned}
&i) && w_t/p_t \leq a_t F'(h_t)\\
&ii) && w_t/p_t = V'(h_t + u_t)\\
&iii) && u_t \geq 0\\
&iv) && w_t \geq \psi(u_t) w_{t-1}
\end{aligned}$$

for given exogenous $\{a_t, p_t\}$.

The constraints iii) and iv) are easily understood: unemployment cannot be negative, and by definition of constrained efficiency, the planner respects downward wage rigidity. Instead, the constraints i) and ii) deserve further comment.

First, the planner respects the households' choice for voluntary employment (5), which, once combined with (4), is constraint ii). This is because this equation determines the nominal wage as a function of the equilibrium allocation.[6] The planner needs to be able to infer the nominal wage, because he is required to respect downward wage rigidity. In other words, the planner lets labor supply be determined competitively.[7]

[6] Strictly speaking, this equation determines the wage only in an expansion when $u_t = 0$. Instead, in a recession when downward wage rigidity binds, the wage is determined by constraint iv).

[7] The literature studying constrained efficiency commonly assumes that the social planner lets some markets clear competitively, while intervening in others. See for example Bianchi (2011), Dávila and Korinek (2018) or Bianchi and Mendoza (2018).

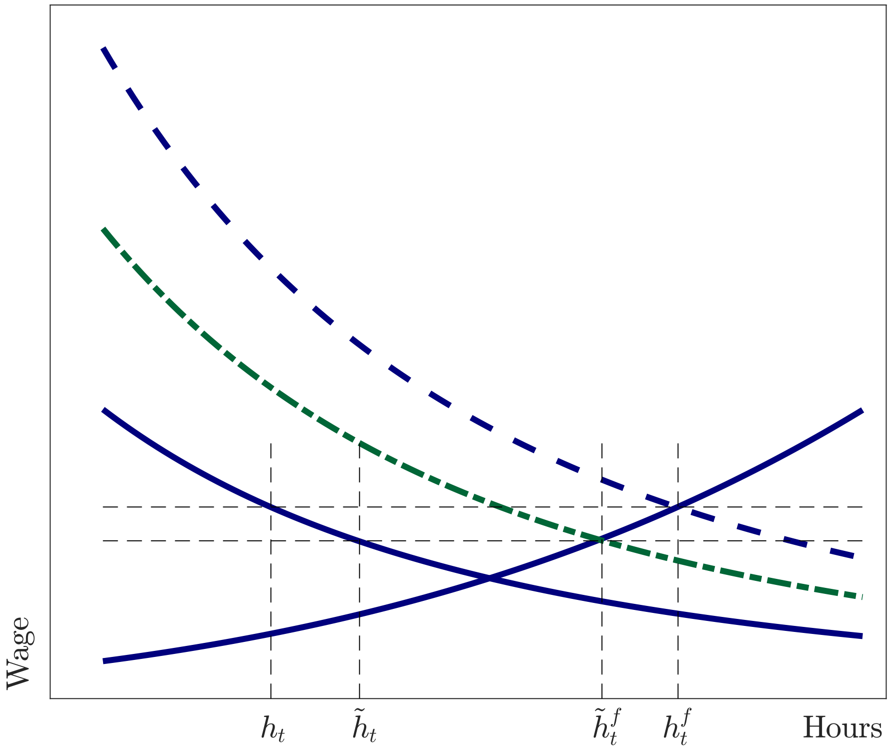

Figure 1: Labor market diagram: labor supply and demand (blue solid). Labor demand shifts rightwards following a rise in technology (blue dashed). Intervention: shift labor demand back to the left (green dashed-dotted). Here it is assumed that wages are fully downward rigid.

Key for the prudential intervention, instead, is constraint i). This constraint says that the planner can intervene in labor demand, but only by *reducing* it relative to the competitive equilibrium, not by raising it. To see the intuition, note that if i) were not a constraint, the planner could always implement the first-best amount of employment: in a period when lagged wages are high, hence w_t is determined by iv), choose h_t according to $a_t F'(h_t) = V'(h_t)$, then let $w_t/p_t > a_t F'(h_t)$. That is, firms hire the first-best amount of workers, even as the marginal product lies below the marginal cost. Implicitly, this is therefore assuming that the planner can *subsidize* the firms' hiring, which from the literature on "fiscal devaluation" is well understood to deliver the first best (Farhi et al., 2014).

Constraint i) therefore imposes that the planner *cannot* intervene in recessions by raising labor demand. Instead, the planner will intervene *ex-ante* (in expansions) by *reducing* labor demand. To see the usefulness of this intervention, consider a labor market diagram (Figure 1). Shown is a temporary rise in technology. Unemployment is zero. Labor demand shifts to the right, inducing a wage rise along the labor supply curve (dashed). Once the shock disappears in the second period, labor demand moves back, but wages remain high. Employment falls to h_t and there is unemployment, reflecting that h_t is low, but also that h_t^f is high (recall that $u_t = h_t^f - h_t$). The intervention is dashed-dotted in green: shift labor demand slightly back to the left in the first period. Wages increase by less, which reduces the cut in employment $\tilde{h}_t$ and unemployment $\tilde{h}_t^f - \tilde{h}_t$ in the second period.

As shown below, reducing labor demand in expansions can be decentralized by taxing the firms' payroll in those periods—or equivalently, by taxing the firms' sales revenue. The benefit of the intervention is that wage inflation is reduced in expansions, which reduces unemployment in subsequent recessions. Instead, the cost of the intervention is that in expansions, the economy produces below potential (see again Figure 1).

To characterize the optimal balance between cost and benefit, one has to solve the dynamic program in Definition 1, to which we turn next.

3.2 Properties of the optimal intervention

As shown in the Appendix A, in a period when downward wage rigidity is slack, labor demand and supply in the constrained-efficient equilibrium are

$$a_t F'(h_t) = \frac{w_t}{p_t} + \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1} \quad \text{(demand)} \tag{11}$$

$$V'(h_t) = \frac{w_t}{p_t} \quad \text{(supply)} \tag{12}$$

and unemployment is zero ($u_t = 0$). In the labor demand curve, $\varepsilon_t^V > 0$ denotes the elasticity of the labor supply curve V', and $\lambda_t \geq 0$ denotes the non-negative Lagrange multiplier associated with downward wage rigidity.[8]

This is very intuitive. Combining (11)-(12) yields

$$a_t F'(h_t) - V'(h_t) = \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1}. \tag{13}$$

The left hand side is the utility loss from reducing employment below potential in the current period (recall that in the current period, by assumption, downward wage rigidity is slack). Instead, the right hand side is the gain from doing so: the expected reduction in the utility loss suffered from downward wage rigidity in the next period. It is composed of $\varepsilon_t^V (w_t/h_t) > 0$ which measures by how much wages can be reduced by moving labor demand back to the left—by sliding down the labor supply curve (recall again Figure 1). In turn, this is multiplied with the expected loss from downward wage rigidity in the next period, as measured by the compounded term $\beta E_t \psi(u_{t+1}) \lambda_{t+1} \geq 0$.

[8] By denoting a generic variable $\bar{h}_t$ and letting $V'(\bar{h}_t) p_t = w_t$, this is therefore $\varepsilon_t^V \equiv (\partial w_t / \partial \bar{h}_t)(\bar{h}_t / w_t)$.

This implies a stronger intervention when the labor supply curve is steep (w_t is moving strongly with h_t), as in this case the intervention is cheap: a small reduction in employment restrains wage inflation to a strong extent. At the same time, a larger intervention is required when wages are more downward rigid (when $\psi(u_{t+1})$ is expected to be larger), and when the utility loss associated with downward wage rigidity, as measured by multiplier $\lambda_{t+1} \geq 0$, is expected to be larger. Note that, because the function $\beta\psi(u_{t+1})\lambda_{t+1} \geq 0$ is kinked (and thus convex), more uncertainty *per se* also justifies a stronger intervention.[9]

The utility loss $\lambda_t \geq 0$ is an equilibrium object, determined in periods when downward wage rigidity binds. Again as shown in the Appendix A, in this case labor demand and supply in the constrained-efficient equilibrium are

$$a_t F'(h_t) = \frac{w_t}{p_t} \quad \text{(demand)} \tag{14}$$

$$\begin{aligned} V'(h_t + (1-\delta)u_t)\left(\delta + (1-\delta)\frac{\varepsilon_t^F}{\varepsilon_t^V}\frac{h_t + u_t}{h_t}\right) &= \varepsilon_t^F \frac{w_t}{h_t}(\lambda_t - \beta E_t \psi(u_{t+1})\lambda_{t+1}) \\ &+ \frac{w_t}{p_t} + \lambda_t \psi'(u_t) w_{t-1}\left(1 - \frac{\varepsilon_t^F}{\varepsilon_t^V}\frac{h_t + u_t}{h_t}\right) \quad \text{(supply)} \end{aligned} \tag{15}$$

and there is unemployment ($u_t > 0$). This can be read as follows: w_t is determined by downward wage rigidity (constraint iv) in Definition 1). Given w_t, employment h_t is determined by labor demand (14), and unemployment $u_t > 0$ is determined by the condition for voluntary labor supply, constraint ii) in Definition 1. Finally, the utility loss associated with downward wage rigidity $\lambda_t > 0$ is determined residually in (15).

To understand how the utility loss is determined, it helps to focus on the special case of the model where unemployment does not carry a special penalty ($\delta = 1$), and where downward wage rigidity is flat in unemployment ($\psi(u_t) = \bar{\psi}$ such that $\psi'(u_t) = 0$). In this case, (15) simplifies considerably and by combining it with (14) we obtain

$$\lambda_t = -(\varepsilon_t^F)^{-1}\frac{h_t}{w_t}\left(a_t F'(h_t) - V'(h_t)\right) + \beta\bar{\psi}E_t\lambda_{t+1}. \tag{16}$$

When downward wage rigidity binds, employment h_t is inefficiently low. The term $a_t F'(h_t) - V'(h_t) > 0$ measures by how much utility would rise by increasing employment by a marginal unit. By how much must wages decline for firms to be willing to hire the marginal unit? This is given by $-(\varepsilon_t^F)^{-1}(h_t/w_t) > 0$, which is the (negative of the inverse) slope of labor demand F' (here, $\varepsilon_t^F < 0$ denotes the elasticity of labor demand, which is negative because labor demand slopes downwards).[10] Therefore, the product of these two terms is the utility value

[9] In this respect, our analysis is linked to recent contributions on the interaction of risk and nominal rigidities. See for example Basu and Bundick (2017).

[10] Along the lines of the definition of the elasticity of labor supply, for a generic variable $\bar{h}_t$ and $a_t p_t F'(\bar{h}_t) = w_t$, this is given by $\varepsilon_t^F \equiv (\partial w_t/\partial \bar{h}_t)(\bar{h}_t/w_t)$.

of decreasing wages by a marginal unit if downward wage rigidity binds—which is exactly λ_t. Finally, λ_t depends on its own expected value in the next period. This is because when downward wage rigidity binds in the current period, because wages are slow to be reduced, it is likely to also bind in the next period.

One can verify that the multiplier $\lambda_t > 0$ determined in (15) rises once unemployment carries an additional utility penalty (once $\delta < 1$), and that it falls once wages become more downward flexible with unemployment (once $\psi'(u_t) < 0$). Both of these results are intuitive. Then, from (11), a value of $\delta < 1$ strengthens the optimal intervention in a expansion, whereas a rigidity function $\psi'(u_t) < 0$ weakens the intervention in those periods.

The Appendix A contains further details and a step-by-step derivation of the constrained-efficient equilibrium.

4 Decentralization

As discussed above, in the constrained-efficient equilibrium, the firms' hiring is reduced relative to the competitive equilibrium in periods when downward wage rigidity is slack. This points to taxing the firms' payroll as one option to decentralize the constrained-efficient allocation, which we verify next.

Formally, define a regulated competitive equilibrium as

Definition 2. [REGULATED COMPETITIVE EQUILIBRIUM] *The government sets non-negative taxes* $\{\tau_t^w \geq 0\}_{t\geq 0}$ *and rebates them lump-sum to households. Given initial conditions* $w_{-1} > 0$ *and an exogenous process for* $\{a_t, p_t\}$*, a regulated competitive equilibrium is a path for* $\{w_t, h_t, u_t\}_{t\geq 0}$ *such that the following conditions are satisfied*

i) $$w_t/p_t = V'(h_t + u_t) \quad \textit{(labor supply)}$$

ii $$w_t(1+\tau_t^w)/p_t = a_t F'(h_t) \quad \textit{(labor demand)}$$

iii) $$w_t \geq \psi(u_t) w_{t-1}, \quad u_t \geq 0, \quad \cdot \times \cdot = 0 \quad \textit{(inequality and slackness).}$$

This leads to the following Proposition (see the Appendix A for a proof)

Proposition 1. [DECENTRALIZATION] *The regulated competitive equilibrium that maximizes household utility (welfare)* $\mathcal{U} \equiv E_0 \sum_{t\geq 0} \beta^t [a_t F(h_t) - V(h_t + (1-\delta)u_t)]$ *coincides with the constrained-efficient equilibrium.*

Recall that, if the government could set *negative* taxes, it would choose to subsidize firms in recessions, thereby effectively *undo* downward wage rigidity (as discussed above).

However, to the extent that subsidies are not available, the government may use payroll taxes in expansions in order to implement the constrained-efficient allocation.[11]

Clearly, an alternative is to tax the firms' sales revenue, as this implies the labor demand curve $w_t/(p_t(1-\tau_t^p)) = a_t F'(h_t)$. Therefore, in this case the constrained-efficient equilibrium can be decentralized by setting $\tau_t^p = \tau_t^w/(1+\tau_t^w) \geq 0$.

The implied tax τ_t^w can be obtained from (11)

$$\tau_t^w = \left(\frac{w_t}{p_t}\right)^{-1} \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1}) \lambda_{t+1}, \tag{17}$$

whenever the labor market is not rationed, and $\tau_t^w = 0$ else. Here, λ_{t+1} is defined in (15) (or, in the special case $\psi(u_t) = \bar{\psi}$ and $\delta = 1$, in (16)).

To gauge the empirical importance of the optimal tax, consider the following illustration. Assume that downward wage rigidity is slack in the current period, but that it may bind in the next period (and again slack thereafter). Specialize to $\psi(u_t) = \bar{\psi}$ and $\delta = 1$. Denote $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$ unemployment expressed relative to potential, and specialize to the functional forms for production technology and disutility of work introduced in Section 6. The optimal prudential tax can be written as[12]

$$\tau_t^w = \frac{\varphi}{1-\alpha} \beta \bar{\psi} E_t \left(\frac{w_{t+1}}{w_t}\right)^{\frac{1}{\varphi}} \left(\frac{p_t}{p_{t+1}}\right)^{1+\frac{1}{\varphi}} (1-\hat{u}_{t+1})(1-(1-\hat{u}_{t+1})^{\varphi}). \tag{18}$$

The labor share α and the (inverse) Frisch elasticity φ appear in (18), as these determine the elasticities of labor demand and supply (recall the discussion in Section (3.2), and see Section 6 below). The formula in (18) determines the optimal tax as a function only of price inflation and of unemployment expected for the next period.[13]

Assume, for example, that $\varphi = 4$, $\alpha = 2/3$, $\bar{\psi} = 0.96$, $\beta = 0.96$, that there is zero net inflation to next period ($p_{t+1} = p_t$), and that with a probability of 10 percent, a severe crisis is expected for next period whereby unemployment hits 10 percent. In this case, the implied optimal tax is $\tau_t^w = 0.1095$, or about 11 percent.

This example shows that the optimal tax can be quite large. Nonetheless, recall that (18) is only valid for a special case of the model. Moreover, it ignores general equilibrium effects: per effect of charging the prudential tax, the probability (and severity) of the unemployment spell in the next period is reduced. These general equilibrium effects are taken care of in our quantitative application in Section 6.

[11] Note that since the problem of the constrained-efficient planner is time consistent, the Ramsey policy problem is also time consistent (Bianchi, 2016).

[12] Here we have combined (16) and (17), and used (14), (4) and (5). Details are in the Appendix A.

[13] Instead, knowledge of wage inflation is not required. In a period when unemployment opens up, the wage rigidity must bind. Hence, we can replace $w_{t+1}/w_t = \bar{\psi}$. Instead, in states of the world where unemployment is zero, the right hand side of (18) is equal to zero. Therefore, in those states of the world we do not require knowledge of w_{t+1}/w_t.

5 Model extension: The case of active wage setters

As a robustness and extension, we now depart from the assumption of Walrasian labor market but instead consider the case of wage setting firms. Intuitively, once firms are not wage takers they may internalize that downward wage rigidity affects their workers, and act to compress wage rises (perform prudential wage reductions) accordingly as this raises their profits (Elsby, 2009). The case for prudential intervention may then be strongly reduced.

We find that the case for prudential intervention is alive and well. In a nutshell, this is because individual firms compete with other firms for workers. As unilateral prudential wage reductions entail substitution of workers to competing firms, individual firms will not perform (large) unilateral wage reductions. In the limit as competition becomes perfect, firms will set wages always as their competitors, and we are back in a Walrasian labor market.

For ease of exposition, in this section we specialize to the version of the model where unemployment does not carry a special penalty ($\delta = 1$) and where downward wage rigidity does not depend on unemployment ($\psi(u_t) \equiv \bar{\psi}$). To introduce active wage setting, we break the assumption of firms being wage takers, but instead grant them some monopoly power. Specifically, as in Galí (2011), Galí and Monacelli (2016) and others, we assume that the labor market is characterized by monopolistic competition.[14]

We let aggregate employment be a CES-composite of firm-specific employment. By denoting $i \in [0, 1]$ the firm index on the unit interval and $\eta > 0$ the elasticity of substitution of firm-specific employment, this is

$$h_t = \left(\int_0^1 h_t(i)^{1+\frac{1}{\eta}} di \right)^{1/(1+\frac{1}{\eta})}. \tag{19}$$

In the households' budget (2), the wage income $w_t h_t$ is replaced by the integral expression $\int_0^1 w_t(i) h_t(i) di$. Households attempt to divide employment between firms so as to maximize their wage income, subject to aggregate employment being h_t. This yields a set of firm-specific labor supply curves

$$h_t(i) \leq \left(\frac{w_t(i)}{w_t} \right)^{\eta} h_t, \tag{20}$$

for all $i \in [0, 1]$. A firm which pays a higher wage than its competitors, $w_t(i) > w_t$, receives a larger share of the aggregate labor supply. Conversely, a firm may pay less than its competitors and still not lose all of its workers. This is the source of monopoly power which makes firms active wage setters. Notice that equation (20) holds only with a weak inequality. Because of

[14] More precisely, we consider *monopsonistic* competition as the market power is on the labor demand side. The standard reference for monopsonistic labor market competition is Manning (2003). Only recently, Paul Krugman has argued that the combination of monopsony power and downward nominal wage rigidity may be useful also to explain the recent wage experience in the US (Krugman, 2018).

downward wage rigidity, an individual firm may face a very large labor supply, yet decide to only employ a fraction of the workers (see below).

The households' intertemporal problem is unchanged from the baseline model. In particular, we again obtain the (aggregate) labor supply curve

$$V'(h_t) \leq \frac{w_t}{p_t}, \tag{21}$$

inequality which may be strict when downward wage rigidity binds.[15]

The heart of the extended model is the problem of the firms. As in the baseline model, we assume that the firms' technology is $a_t F(h_t(i))$, where firm index $i \in [0,1]$ is now made explicit. We have the following dynamic program

Definition 3. [FIRM PROBLEM EXTENDED MODEL] *In the extended model, the problem of individual firm $i \in [0,1]$ is to solve the following dynamic program*

$$\mathcal{P}_t(w_{t-1}(i)) = \max_{\{h_t(i), w_t(i)\}} \{a_t F(h_t(i)) - \frac{w_t(i)}{p_t} h_t(i) + \beta E_t \mathcal{P}_{t+1}(w_t(i))\}$$

subject to the set of constraints

$$\begin{aligned} &i) && h_t(i) \leq (w_t(i)/w_t)^{\eta} h_t, \\ &ii) && w_t(i) \geq \bar{\psi} w_{t-1}(i), \end{aligned}$$

for given exogenous variables $\{a_t, p_t, h_t, w_t\}$.

In Definition 3, the value function $\mathcal{P}_t$ denotes the present value of (real) period-profits, which has time index t as it depends on aggregate states.[16] We focus on symmetric equilibria and therefore, after solving the problem in Definition 3, impose the symmetry condition $w_t(i) = w_t$ and $h_t(i) = h_t$ for all firms $i \in [0,1]$.

Before turning to the characterization of equilibrium, it is important to recognize that the extended model has the *same* constrained-efficient equilibrium as the baseline model. This is because the planner will impose symmetry $w_t(i) = w_t$ and $h_t(i) = h_t$ for all firms $i \in [0,1]$ from the start, such that the "new" equations (19)-(20) disappear. Then, regarding aggregate variables the planner's problem is unchanged from the baseline model.

[15] In equilibrium, all firms are identical and index i disappears. Therefore, in equilibrium, equation (20) always holds with equality, whereas the rationing of employment arises from equation (21) (as in the baseline model). However, specifying (20) as a weak inequality is important for correctly specifying the problem of the firms, see Definition 3.

[16] Notice that the real stochastic discount factor multiplying future period-profits is β, reflecting that consumption utility is linear.

Turn now to equilibrium in the extended model. As shown in the Appendix A, in a period when downward wage rigidity is slack, labor demand and supply are

$$a_t F'(h_t) = \frac{\eta+1}{\eta}\frac{w_t}{p_t} + \frac{1}{\eta}\frac{w_t}{h_t}\beta\bar{\psi}E_t\lambda_{t+1} \quad \text{(demand)} \tag{22}$$

$$V'(h_t) = \frac{w_t}{p_t} \quad \text{(supply)}. \tag{23}$$

In equation (22), the variable $\lambda_t \geq 0$ denotes the multiplier associated with downward wage rigidity. By combining equations (22)-(23) we obtain

$$a_t F'(h_t) - \frac{\eta+1}{\eta}V'(h_t) = \frac{1}{\eta}\frac{w_t}{h_t}\beta\bar{\psi}E_t\lambda_{t+1} \tag{24}$$

which has to be compared with (13) in the constrained-efficient equilibrium.

We notice that the firms in the extended model charge a monopolistic mark-up, reflecting monopolistic competition. Besides this conventional effect, two results are noteworthy. First, once firms actively set wages they reduce their labor demand in expansions, which mimics the constrained-efficient outcome and therefore the optimal prudential intervention (the right hand side of (24)). This echoes the main result in Elsby (2009).

However, second, the extent to which labor demand and wages are reduced is not maximizing social welfare. The reason is that elasticity ε_t^V enters equation (13), whereas elasticity $1/\eta$ enters equation (24). The role of elasticity ε_t^V for the optimal intervention was discussed in Section 3.2, and is briefly repeated here for convenience. A steep (aggregate) labor supply curve, reflected in a large ε_t^V, warrants a large prudential intervention because by reducing wages, the resulting drop in employment below potential is only small.

Yet, the labor supply curve which is relevant for the individual firm is (20), having elasticity $1/\eta$, rather than (21), having elasticity ε_t^V. To the extent that $\varepsilon_t^V > 1/\eta$, the individual firm faces a labor supply curve that is *flat* compared with the aggregate labor supply curve. Then, the individual firm will find it optimal to compress wage rises only *weakly* compared to the optimal intervention. It is easy to see that this is the empirically relevant case. In the application below, we will specialize to the disutility of labor supply $V(h) = h^{1+\varphi}/(1+\varphi)$, implying an elasticity of aggregate labor supply $\varepsilon_t^V = \varphi$. A plausible value for the (inverse) Frisch elasticity φ is 4. Thus, $\varepsilon_t^V > 1/\eta$ imposes the restriction $\eta > 1/4$. Clearly, $\eta = 0.25$ reflects very strong market power for the firms.[17] Conversely, for more reasonable values of η such as 5, elasticity ε_t^V is about 20 times as large as elasticity $1/\eta$.

Intuitively, the reason why wage setting firms behave almost as under perfect competition is that, by lowering wages unilaterally, they face a strong substitution of their workers to competing firms. Indeed, note that as $\eta \to \infty$ (perfect competition), we recover the baseline

[17] From (22), the implied monopolistic mark-up $(\eta+1)/\eta$ is 500 percent.

model as (22) reduces to (9). This effect arises because, in choosing $w_t(i)$, individual firms take the aggregate wage w_t as given. Instead, were individual firms to agree on collective wage compressions, they would do so to a stronger extent because the crowding out in aggregate employment is comparatively small.

To finish, we state the equation determining the multiplier λ_t in the extended model in periods when downward wage rigidity binds.[18] As shown in the Appendix A, this is

$$\lambda_t = \frac{1}{p_t} h_t + \beta \bar{\psi} E_t \lambda_{t+1}, \tag{25}$$

which has to be compared with (16) in the constrained-efficient equilibrium. The multiplier λ_t is different because here it reflects the (shadow) increase in the firms' real profits by relaxing downward wage rigidity by a marginal unit, whereas for the planner it reflects the (shadow) rise in households' utility. An unambiguous ranking of the size of the two multipliers is not possible. However, in simulations one can verify that the effect of elasticity $1/\eta$ versus ε_t^V by far dominates any differences in multiplier λ_t.

The Appendix A contains further details and a step-by-step derivation of the extended-model equilibrium.

6 Quantitative analysis

Here we assess the quantitative relevance of the optimal prudential intervention by applying the model to Greece, 1999-2016. We ask the following questions: How large is the optimal prudential tax during 1999-2008? How much does the tax reduce wage inflation during this period? How much is unemployment reduced after 2008 due to the policy intervention? And what are the welfare gains of the optimal prudential policy?

In this section, for simplicity, we focus on the baseline model from Section 2. This reflects that, as explained above, for reasonable values of firm competition $\eta > 0$ the baseline and extended model produce very similar dynamics.

6.1 Functional forms, parameters, and data

We solve the non-linear model numerically by using a global solution method.[19] To do so, we choose functional forms as well as specialize to a set of parameters. While some parameters

[18] Instead, labor demand in periods when downward wage rigidity binds is given by (14), as in the baseline model and the constrained-efficient equilibrium. See the Appendix A for details.

[19] The constrained-efficient equilibrium and the equilibrium in the extended model can be solved by using a fixed-point iteration algorithm. Alternatively, the constrained-efficient equilibrium can be obtained by using value function iteration, see Definition 1. The competitive equilibrium of the baseline model is static, and therefore can be obtained by solving a system of non-linear equations.

are standard and therefore will be taken from earlier studies, other parameters are tailored to our specific application.

We specialize to the following conventional functional form for production technology

$$F(h_t) = h_t^{\alpha}$$

and set the standard value of $\alpha = 0.66$ to capture a labor share of two thirds. For the disutility of labor supply we assume that

$$V(h_t) = h_t^{1+\varphi}/(1+\varphi)$$

and set the (inverse) Frisch elasticity to the conventional value of $\varphi = 4$. Notice that both functional forms imply constant elasticities of labor demand and supply, given by $\varepsilon^F = \alpha - 1 = -0.33 < 0$ and $\varepsilon^V = \varphi = 4 > 0$, which had been used in Sections 4-5 above.

The degree of downward wage rigidity is governed by function $\psi(u_t)$, which we set to

$$\psi(u_t) = \bar{\psi} - \kappa u_t.$$

Here, $\kappa \geq 0$ determines how downward wage rigidity is affected by unemployment, and $\bar{\psi} \leq 1$ determines downward wage rigidity when unemployment is zero.

For the parameter governing downward wage rigidity in the absence of unemployment, we set $\bar{\psi} = 0.96$ such that wages can fall by at most one percent per quarter. This corresponds to estimates in Schmitt-Grohé and Uribe (2016) for Greece.

Below we will employ annual data and therefore set for the time discount factor $\beta = 0.96$.

Another parameter to be determined is δ, measuring the utility penalty from unemployment. This parameter is hard to pin down as it lacks a clear empirical counterpart. We take a pragmatic approach and set $\delta = 0.5$, implying that leisure derived from being unemployed carries half as much utility value as voluntary leisure. Notice that, under an (equally plausible) value of $\delta = 0$, whereby households do not receive any pleasure from unemployment, the optimal prudential intervention would be correspondingly larger.

A remaining set of parameters are κ and the parameters governing the stochastic properties of the two stochastic processes $\{a_t, p_t\}$. These parameters will be determined as part of the empirical strategy, to be described below.

The empirical application will rely on a set of four time series from 1999-2016, used to proxy for the hourly wage w_t, labor productivity a_t, unemployment u_t and real GDP y_t.

To proxy for w_t we use Eurostat's *labor cost index*. The labor cost index measures hourly wage costs of firms, inclusive of taxes and minus subsidies. A second time series used is *hourly productivity*, from OECD. This series is used to proxy for the variable a_t. From Eurostat we also extract *total unemployment* as a fraction of the participating population to proxy for u_t, and from OECD we also extract a time series for *real GDP* to capture y_t. All series (except for the unemployment series) are de-trended by using a euro-zone average, to take account of technology growth as well as trend inflation both of which are not modeled. Further details on the data are provided in the Appendix B.

6.2 Empirical strategy and model performance

Recall that in this class of models, p_t is commonly interpreted as the terms of trade. In turn, a_t may be interpreted as labor productivity. Therefore, wage dynamics in the model can be driven by the "demand side" following terms of trade shocks, or by the "supply side" following technology shocks. In what follows, we will interpret p_t more broadly than just the terms of trade. Rather, p_t will be used to capture demand shocks more generally, or any variation in wages that is not productivity-related.[20]

Specifically, we use the following empirical strategy. From solving the competitive equilibrium we obtain a policy function for wages $w(w_{t-1}, a_t, p_t)$. As the sequences for $\{w_t\}$ and $\{a_t\}$ are observed, at each time t, this policy function can be *inverted* to solve for the implied value of p_t.[21] That is, we back out a series of demand shocks in order to explain the observed wage series perfectly. Importantly, in the Greek crisis after 2008 when w_t falls and downward wage rigidity binds, the parameter κ determines to what extent wage deflation translates into unemployment. We therefore adjust parameter κ so as to generate a rise in unemployment of 18 percentage points, which, as shown below, corresponds to the rise in unemployment in Greece during the crisis period.

Before turning to the results, two remarks are in order. First, the previous exercise requires that the series for $\{w_t, a_t\}$ is observable in levels, not merely as an index. Yet, this is not the case. Therefore, to be able to continue we make the following assumption[22]

Assumption 1. *When joining the euro in 1999, Greece was in its steady state.*

While admittedly a strong assumption, it is also not entirely unreasonable. In the present model, the single endogenous state variable is the lagged wage w_{t-1}. Therefore, assuming that the country is in steady state amounts to assuming that the country's wage is in steady state.

[20] A similar strategy is pursued by Berka et al. (2017), who decompose real exchange rate movements into productivity-related and other (demand-side) components which capture real exchange rate variation that is independent of productivity.

[21] This inversion step is only possible if $\kappa > 0$. Otherwise, the policy function for w_t would be flat in the region where downward wage rigidity binds, and therefore could not be inverted.

[22] Without this assumption, the initial level for wages in the estimation is not determined. One could attempt to estimate it by using a non-linear filter. However, the computational burden is likely to become overwhelming given the numerical complexities involved in solving the (non-linear) model.

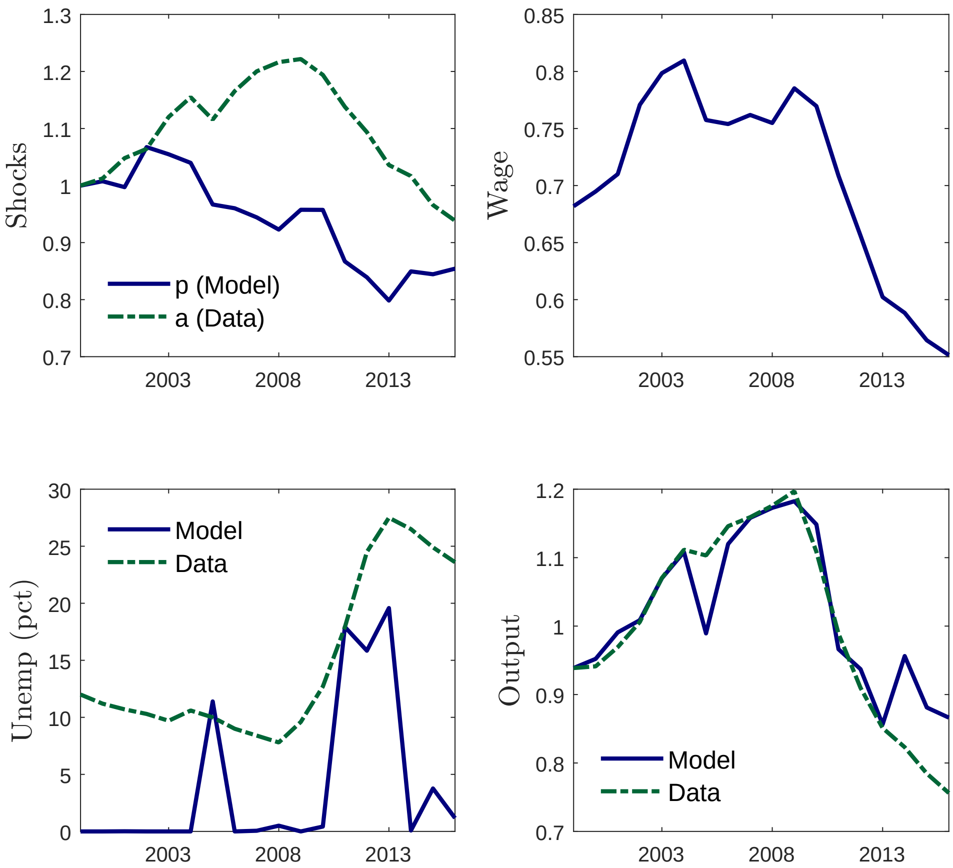

Figure 2: Model performance. Data green dashed, model predictions solid blue. Data sources, methodology and parameters are described in the text. Unemployment shown in the figure is $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$, expressed in percent.

When the euro was incepted, real exchange rates had been set so as to minimize the distance of each country's real exchange rate from purchasing power parity (e.g., Berka et al., 2017). Therefore, it appears not unreasonable that, in the year 1999, Greece did not experience any severe up- or downward pressure on its nominal wages.

Second, to carry out the empirical exercise, no information about the process governing $\{a_t, p_t\}$ is needed. This is because the competitive equilibrium is *static*. As a consequence, the policy functions are identical for any stochastic process governing the two variables.

Turn now to the result (Figure 2). The upper left panel shows that productivity follows hump-shaped dynamics during the sample period, and that a declining path for demand shocks is required to capture the hump-shaped dynamics for hourly wages (upper right panel). The lower left panel shows the evolution of unemployment in the model and data. By construction, model-implied unemployment rises by 18 percentage points in the crisis (the implied $\kappa = 0.23$). Finally, as an external validation, the lower right panel shows that the model captures the evolution of real output quite well.

It should be noted that the model fails to capture the unemployment dynamics before the crisis. Clearly, this is because we abstract from any frictional unemployment, but only consider "rationing" unemployment. In this regard, the model successfully predicts an unemployment spike after 2008.[23] A decomposition of unemployment into frictional versus rationing for the US during the Great Recession is done in Michaillat (2012).

6.3 Optimal prudential intervention

We now study counterfactual dynamics of Greek variables induced by the optimal prudential policy, as well as the implied prudential payroll tax. To do so, we specify a stochastic process for the two exogenous variables $\{a_t, p_t\}$. This is because the constrained-efficient equilibrium is dynamic and non-linear, and therefore depends not only on the shocks' realization, but also on their stochastic properties.

As is common in the international business cycle literature, we assume that the shocks obey a first-order bivariate autoregressive process in logs (e.g., Bianchi, 2011)

$$\log([a_t, p_t]') = \boldsymbol{\rho} \times \log([a_{t-1}, p_{t-1}]') + v_t,$$

where $v_t \sim \mathcal{N}([0,0]', \boldsymbol{\Sigma})$, with $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$ conformable matrices.

From the earlier analysis, a time series for $\{a_t, p_t\}$ is directly available. Thus, the matrices $\boldsymbol{\rho}$ and $\boldsymbol{\Sigma}$ can be estimated by using standard econometric methods. The result is

$$\boldsymbol{\rho} = \begin{pmatrix} 1.0312 & -0.1667 \\ 0.2097 & 0.9161 \end{pmatrix}, \quad \boldsymbol{\Sigma} = 0.0001 \times \begin{pmatrix} 8.8148 & 7.0084 \\ 7.0084 & 16.0398 \end{pmatrix}.$$

As can be cross-checked against Figure 2, the shocks are strongly autocorrelated as well as positively correlated. To implement this process numerically, we use the quadrature-based procedure of Tauchen and Hussey (1991).

After solving the constrained-efficient equilibrium we proceed as follows. We assume that the constrained-efficient planner takes over the economy in the initial period. By facing the same sequence of shocks as the competitive equilibrium, we study the resulting path for

[23] The 2005-spike in unemployment that the model predicts results from the fact that wages decline strongly in this year—and hence downward wage rigidity binds (see the upper right panel). At the same time, because the rate of wage deflation subsides after 2013, the model fails to predict that unemployment remains high during this period. Recall that all data is de-trended by using a euro-zone average.

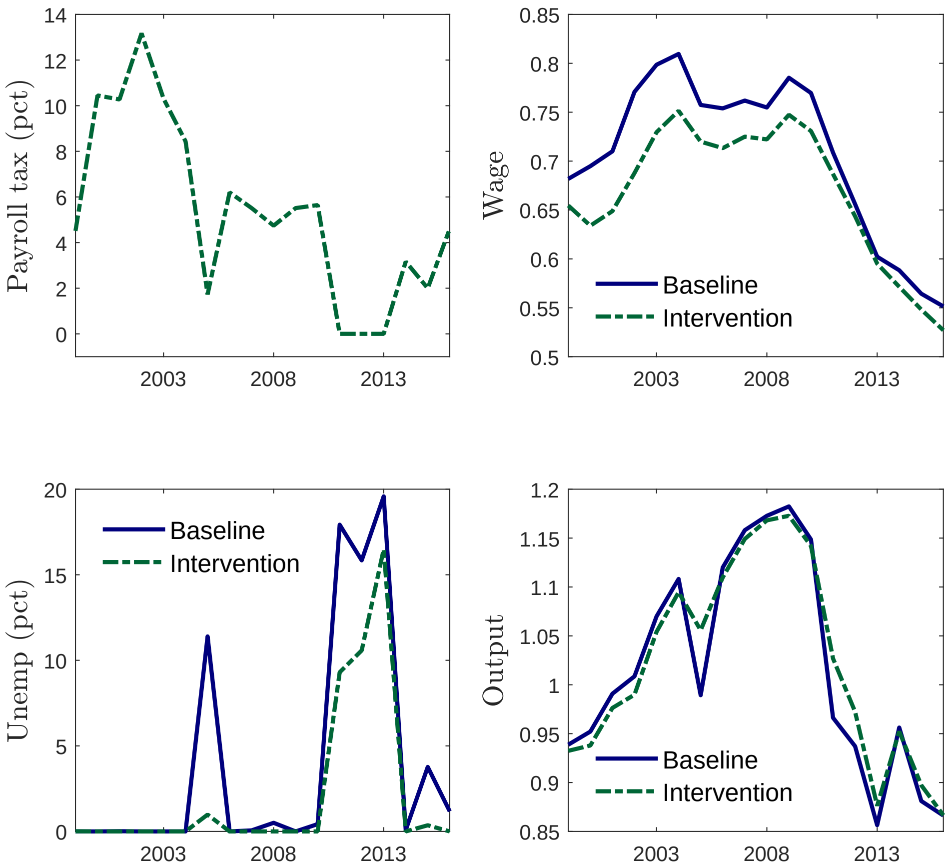

Figure 3: Model counterfactual. The policy intervention is green dashed, the baseline dynamics are solid blue. The methodology and parameters are described in the text. Unemployment shown in the figure is $\hat{u}_t \equiv (h_t^f - h_t)/h_t^f$, expressed in percent.

wages, unemployment, real GDP, and also the payroll tax that is needed to decentralize the constrained-efficient allocation.

The result is shown in Figure 3. We find that the prudential tax climbs to 13 percent, which distinctly reduces wages during the early years of the Greek cycle. As wages are lower at the onset of the Greek crisis, the unemployment spike post 2008 is markedly reduced—in the years 2011-2012 by about half. Instead, in the year 2013 Greek unemployment is still very high. This reflects that the shocks which hit Greece in the crisis were, in fact, quite severe and low-probability events. As a consequence, even under the optimal prudential intervention, unemployment could not be avoided entirely.

Notice also from Figure 3 that the payroll tax is reduced to zero once the crisis hits. This

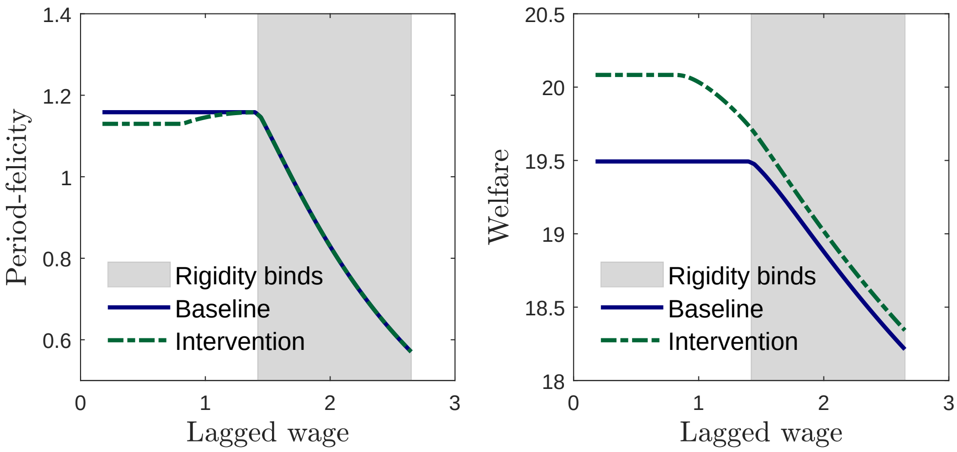

Figure 4: Period felicity $c_t - V(h_t^f - \delta u_t)$ and total welfare $\mathcal{U}_t$ among the two allocations. Shown are policy functions against the lagged wage w_{t-1}, by keeping the exogenous disturbances a_t and p_t constant at +1 standard deviation. "Rigidity binds" refers to the states w_{t-1} where downward wage rigidity binds in the competitive equilibrium ("Baseline").

reflects that the policy is inherently *prudential*. It is never used during contractions, but only during expansions.

6.4 Welfare effects and summary statistics

Under the optimal policy, the country produces below potential in expansions, but it faces less frequent and shallower recessions. This balancing act between "static losses" and "dynamic gains" is visualized in Figure 4, which compares policy functions for period-felicity $c_t - V(h_t^f - \delta u_t)$ and total welfare $\mathcal{U}_t$ among the two allocations.

Observe first that period-felicity in the two allocations coincides when downward wage rigidity binds. Again, this reflects that the optimal policy is *prudential*: conditional on being in a recession, the policy is powerless. The second observation is that felicity is *lower* under the optimal intervention when downward wage rigidity is slack. This is the "static loss" of the optimal policy: the fact that the economy produces below potential.

Still, welfare under the optimal policy is higher, reflecting "dynamic gains": while conditional on state variables, the economy under the optimal policy performs worse, the stationary distribution of state variables (nominal wages) is shifted towards more favorable states, such that overall welfare increases. This gain is more pronounced in the region where downward wage rigidity is slack, for this is the region where policy actively intervenes.[24]

[24] Instead, in the region where downward wage rigidity binds, the welfare gain arises from *expectations* that in the future, once downward wage rigidity turns slack, the policy will optimally intervene.

$mean(\hat{u}_t)\%$	$frac(\hat{u}_t > 0)$	$mean(\hat{u}_t > 0)\%$	$mean(\hat{u}^*_t)\%$	$frac(\hat{u}^*_t > 0)$
6.3	0.42	15	1.5	0.11
	$mean(\hat{u}^*_t > 0)\%$	$mean(\tau^w_t)\%$	$mean(\iota_t)\%$	$mean(\iota^*_t)\%$
	13.4	16.3	0.85	3.3

Table 1: Summary statistics. Reported are: mean unemployment, frequency of unemployment being strictly positive and mean unemployment conditional on unemployment being strictly positive, competitive equilibrium (columns 1-3) and constrained-efficient equilibrium (columns 4-6). Column 7 contains the mean optimal prudential payroll tax, in percent. Mean consumption equivalent: columns 8-9, column 9 relative to the constrained-efficient equilibrium where it is additionally imposed that $\psi(u_t) \equiv 0$.

Table 1 contains summary statistics. The first three columns are unemployment statistics in the competitive equilibrium: mean unemployment, the fraction of periods in which unemployment is strictly positive, and unemployment conditional on unemployment being strictly positive. As in Figures 2-3, here we define unemployment as $\hat{u}_t \equiv (h^f_t - h_t)/h^f_t$, and express it in percent. The mean unemployment rates are 6.3 and 15 percent respectively, and the fraction of periods in which the labor market is rationed is 0.42.

Columns 4-6 show the same unemployment statistics in the constrained-efficient equilibrium. In this case, the means drop to 1.5 and 13.4 percent, respectively, and the fraction of periods in which the labor market is rationed is only 0.11. Thus, the optimal prudential intervention reduces unemployment fluctuations significantly, and this operates mainly through a reduction in the frequency of periods in which the labor market is rationed.

This also implies that in most periods, the economy bears the "static loss" of the intervention (whereas the periods of crisis in which the economy actually benefits are relatively rare). Column 7 contains the source of this static loss: the mean payroll tax, expressed in percent, that is required in order to decentralize the constrained-efficient equilibrium. It amounts to 16.3 percent, and therefore is in fact higher than in the particular episode preceding the Greek crisis that was considered earlier above (recall Figure 3).

Even so, the welfare gain from the intervention is large. The last two columns contain permanent consumption statistics. Here we compute the percentage increase in period-consumption that is necessary in order to make the household indifferent between staying in the competitive equilibrium and moving to the constrained-efficient equilibrium.[25] We report

[25] The consumption equivalent in percent $\iota_t\%$ is defined as

$$\tilde{\mathcal{U}}(w_{t-1}, a_t, p_t) = c_t(1 + \iota_t\%/100\%) - V(h_t + (1-\delta)u_t) + \beta E_t \tilde{\mathcal{U}}(w_t, a_{t+1}, p_{t+1}),$$

where $\tilde{\mathcal{U}}$ denotes welfare in the constrained-efficient equilibrium, and where $[c_t, h_t, u_t, w_t]'$ are policy functions in the competitive equilibrium.

the mean of the stationary distribution of the consumption equivalent. It is 0.85 percent—a large number compared to traditional estimates of the cost of business cycles (see also Schmitt-Grohé and Uribe, 2016).

In turn, the last column shows the consumption equivalent in case downward wage rigidity is absent in the constrained-efficient equilibrium (that is, the policy maker has the ability to subsidize labor demand in recessions which, as explained above, restores the first best). This number therefore isolates the cost of downward wage rigidity *per se*. The loss is 3.3 percent of permanent consumption. Therefore, the optimal prudential intervention removes about one quarter of the total welfare cost of downward wage rigidity.

7 Conclusion

This paper studies optimal policy in economies with downward nominal wage rigidity when only prudential instruments are available. It examines a Walrasian labor market, and a labor market with active wage setters. The prudential intervention strikes a balance between reducing labor demand and therefore output and wages in expansions, and reducing unemployment in subsequent recessions.

We have explored this trade-off both theoretically and numerically. The theoretical analysis reveals how the prudential intervention depends on the economy's characteristics, and provides guidance as to how the optimal prudential tax should be set. The numerical analysis suggests that the welfare gains from the prudential intervention are large. In an application to Greece, the optimal policy removes about one fourth of the total welfare loss due to downward wage rigidity, and it reduces mean unemployment significantly.

We hence conclude that prudential labor market policies are an important tool in a policy maker's toolkit in order to reduce the economic cost of downward wage rigidity.

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A Appendix: Technical Appendix

This Appendix contains additional proofs and derivations. It presents the model with GHH preferences and shows that the equilibrium in this model and the baseline model are identical. We derive the special case of the optimal tax (18). And we provide analytical derivations of the constrained-efficient equilibrium and the equilibrium in the extended model, as well as the proof of Proposition 1.

A.1 GHH preferences

With GHH preferences, the household maximizes

$$E_0 \sum_{t \geq 0} \beta^t U(c_t - V(h_t^f - \delta u_t)), \quad \beta \in (0,1), \quad \delta \in [0,1],$$

subject to (2) and a no-Ponzi constraint, where $U(\cdot)$ has the usual properties of consumption felicity functions (it is positive, increasing and concave).

First order conditions with respect to consumption and assets yield the consumption Euler equation

$$1 = \beta R E_t \frac{U'(t+1)}{U'(t)} \frac{p_t}{p_{t+1}},$$

where we define $U'(t) \equiv U'(c_t - V(h_t^f - \delta u_t))$. First order conditions with respect to (voluntary) leisure and consumption yields

$$\frac{U'(t)V'(h_t^f)}{U'(t)} \frac{w_t}{p_t},$$

which, canceling the $U'(t)$, is exactly (5). This is the well-known property of GHH preferences of eliminating the wealth effect on labor supply.

Therefore, the difference with respect to the baseline model is that households now face a non-trivial intertemporal consumption/savings problem. However, their labor supply decision is unchanged.

The problem of the firms is unchanged. By using that $p_t y_t = \Pi_t + w_t h_t$ in equation (2), we obtain a competitive equilibrium as follows. Given initial conditions w_{-1} and an exogenous process for $\{a_t, p_t\}_{t \geq 0}$, a competitive equilibrium is a process for $\{w_t, h_t, u_t\}_{t \geq 0}$ such that the following conditions are satisfied

i) $$w_t/p_t = V'(h_t + u_t) \quad \text{(labor supply)}$$

ii $$w_t/p_t = a_t F'(h_t) \quad \text{(labor demand)}$$

iii) $$w_t \geq \psi(u_t) w_{t-1}, \quad u_t \geq 0, \quad \cdot \times \cdot = 0 \quad \text{(inequality and slackness).}$$

Equilibrium output follows residually: $y_t = a_t F(h_t)$.

Equilibrium consumption and assets $\{c_t, \Lambda_{t+1}\}$ are determined residually from the following equations (as well as from a terminal condition)

$$iv) \qquad 1 = \beta R E_t(U'(t+1)/U'(t))(p_t/p_{t+1}) \quad \text{(consumption Euler)}$$

$$v) \qquad p_t c_t + \Lambda_{t+1}/R = p_t y_t + \Lambda_t. \quad \text{(resource)}$$

Equilibrium utility follows residually: $\mathcal{U} \equiv E_0 \sum_{t \geq 0} \beta^t U(c_t - V(h_t + (1-\delta)u_t))$.

The key property to note is that the competitive equilibrium is recursive. Equations i)-iii) pin down equilibrium in the labor market. In turn, equations iv)-v) determine consumption and assets *residually*. In other words, to obtain equilibrium in the labor market, and therefore to analyse optimal prudential intervention in the labor market, we may ignore the distribution of consumption and assets over time.

Consequently, nothing is lost by assuming that households have linear consumption utility, as equilibrium in the labor market is identical. To streamline exposition, this assumption is therefore made in the main text.

A.2 Constrained-efficient equilibrium

The maximization problem is

$$\mathcal{U}(w_{t-1}, a_t, p_t) = \max_{\{h_t, w_t, u_t\}} \{a_t F(h_t) - V(h_t + (1-\delta)u_t) + \beta E_t \mathcal{U}(w_t, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$i) \qquad w_t \leq p_t a_t F'(h_t) \qquad \text{(multiplier: } \gamma_t)$$

$$ii) \qquad w_t = p_t V'(h_t + u_t) \qquad \text{(multiplier: } \zeta_t)$$

$$iii) \qquad u_t \geq 0 \qquad \text{(multiplier: } \kappa_t)$$

$$iv) \qquad w_t \geq \psi(u_t) w_{t-1} \qquad \text{(multiplier: } \lambda_t)$$

for given exogenous $\{a_t, p_t\}$.

The first order conditions are

$$a_t F'(h_t) - V'(h_t + (1-\delta)u_t) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} = 0$$

$$\beta E_t \frac{\partial}{\partial w_t} \mathcal{U}(w_t, a_{t+1}, p_{t+1}) - \gamma_t + \zeta_t + \lambda_t = 0$$

$$-V'(h_t + (1-\delta)u_t)(1-\delta) - \zeta_t \varepsilon_t^V \frac{w_t}{h_t + u_t} + \kappa_t - \lambda_t \psi'(u_t) w_{t-1} = 0$$

and the Envelope condition is

$$\frac{\partial}{\partial w_{t-1}} \mathcal{U}(w_{t-1}, a_t, p_t) = -\lambda_t \psi(u_t).$$

Using the Envelope condition and the fact that multiplier ζ_t is always non-zero, we may combine these conditions to

$$\begin{aligned} a_t F'(h_t) - V'(h_t + (1-\delta)u_t) + \gamma_t \varepsilon_t^F \frac{w_t}{h_t} &= \varepsilon_t^V \frac{w_t}{h_t + u_t}(\gamma_t - \lambda_t + \beta E_t \psi(u_{t+1})\lambda_{t+1}) \qquad (*) \\ &= -V'(h_t + (1-\delta)u_t)(1-\delta) + \kappa_t - \lambda_t \psi'(u_t) w_{t-1}. \end{aligned}$$

We proceed as follows. First, we derive the labor demand and supply curves when downward wage rigidity is slack. Second, we derive both curves when it is binding.

A.2.1 Downward wage rigidity is slack

We first show that, whenever downward wage rigidity is slack, there can be no unemployment. That is, we show that the complementary slackness condition (7) from the competitive equilibrium is also an equilibrium condition in the constrained-efficient allocation.

Assume that condition iv) is slack, such that $\lambda_t = 0$. Assume also that $u_t > 0$ such that $\kappa_t = 0$. From (*), this implies that

$$\varepsilon_t^V \frac{w_t}{h_t + u_t}(\gamma_t + \beta E_t \psi(u_{t+1})\lambda_{t+1}) = -V'(h_t + (1-\delta)u_t)(1-\delta).$$

The right hand side is strictly negative. However, the left hand side is weakly positive ($\gamma_t \geq 0$ and $E_t \psi(u_{t+1})\lambda_{t+1} \geq 0$; recall that $\varepsilon_t^V > 0$ because labor supply is upward sloping)—a contradiction.

Therefore, when $w_t > \psi(u_t)w_{t-1}$ is slack such that $\lambda_t = 0$, it must be that $u_t = 0$. From condition ii), this yields equation (12) in the main text. Using $u_t = 0$ and condition ii), we can express (*) as

$$a_t F'(h_t) = \frac{w_t}{p_t} + \gamma_t \left(\varepsilon_t^V \frac{w_t}{h_t} - \varepsilon_t^F \frac{w_t}{h_t} \right) + \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1})\lambda_{t+1}.$$

Is the multiplier γ_t strictly positive or equal to zero? Assume it is strictly positive. In this case, from condition i), it must be that $w_t/p_t = a_t F'(h_t)$. Inserting this in the previous equation we obtain

$$-\gamma_t \left(\varepsilon_t^V \frac{w_t}{h_t} - \varepsilon_t^F \frac{w_t}{h_t} \right) = \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1})\lambda_{t+1}.$$

Because $\gamma_t > 0$ by assumption and $\varepsilon_t^F < 0$ (recall that $\varepsilon_t^F < 0$ because labor demand is downward sloping), the left hand side is strictly negative. However, the right hand side is weakly positive—a contradiction. Therefore, it must be that $\gamma_t = 0$. Using this we obtain the final expression for the labor demand curve when downward wage rigidity is slack

$$a_t F'(h_t) = \frac{w_t}{p_t} + \varepsilon_t^V \frac{w_t}{h_t} \beta E_t \psi(u_{t+1})\lambda_{t+1},$$

which is equation (11) in the main text.

In turn, the multiplier κ_t is implied as

$$\kappa_t = (1-\delta)\frac{w_t}{p_t} + a_t F'(h_t) - \frac{w_t}{p_t} > 0,$$

and therefore is strictly positive.

A.2.2 Downward wage rigidity is binding

We now turn to the case where downward wage rigidity is binding. That is, we turn to the region where $\lambda_t > 0$. In fact, here we need to make a case distinction that is not discussed in the main text. Namely, we distinguish the two cases $u_t = 0$ versus $u_t > 0$.

In the main text, we have only discussed the case $u_t > 0$, because this holds in the major part of the state space where $\lambda_t > 0$. Intuitively, the case of $\lambda_t > 0$ but $u_t = 0$ can arise in an intermediate region: if the wage rigidity just about binds, by reducing the optimal intervention (i.e., by shifting labor demand rightwards)—but still remaining in the region $w_t/p_t < a_t F'(h_t)$—the planner can maintain full employment.

Formally, $\lambda_t > 0$ implies that condition iv) holds with equality. Therefore, w_t is determined from this condition. As additionally $u_t = 0$, h_t is determined from condition ii). Since both w_t and h_t are determined, condition i) can only hold with an inequality.[26] Therefore, $\gamma_t = 0$. Using this, the fact that $u_t = 0$ and condition ii) we obtain from (*)

$$a_t F'(h_t) = \frac{w_t}{p_t} + \varepsilon_t^V \frac{w_t}{h_t}(\beta E_t \psi(u_{t+1})\lambda_{t+1} - \lambda_t).$$

As explained above, while policy still intervenes $w_t/p_t < a_t F'(h_t)$, the intervention is weakened reflected in the positive multiplier $\lambda_t > 0$. In this part of the state space, multiplier κ_t is still positive (unless $\psi'(0)$ is very negative, which we rule out by assumption):

$$\kappa_t = (1-\delta)\frac{w_t}{p_t} + \lambda_t \psi'(0) w_{t-1} + a_t F'(h_t) - \frac{w_t}{p_t} > 0.$$

Now turn to $u_t > 0$. This happens once the multiplier $\lambda_t > 0$ grows too large (the recession is too deep). In this case, policy stops to intervene and labor demand, condition i), holds with equality (this is equation (14) from the main text). In turn, γ_t turns strictly positive. From the equation above, this occurs once

$$\lambda_t > \beta E_t \psi(u_{t+1})\lambda_{t+1}.$$

[26] The case where it holds with equality is the knife-edge case where $w_t = \psi(u_t) w_{t-1}$ corresponds exactly to the wage under full employment. Therefore, $\lambda_t = 0$, and this case was dealt with earlier above.

In turn, once condition i) holds with equality, condition ii) implies that unemployment turns strictly positive, $u_t > 0$, implying $\kappa_t = 0$ from condition iii). Using this in (*), we obtain

$$V'(h_t + (1-\delta)u_t)\left(\delta + (1-\delta)\frac{\varepsilon_t^F}{\varepsilon_t^V}\frac{h_t+u_t}{h_t}\right) = \varepsilon_t^F \frac{w_t}{h_t}(\lambda_t - \beta E_t \psi(u_{t+1})\lambda_{t+1}) + \frac{w_t}{p_t} + \lambda_t \psi'(u_t) w_{t-1}\left(1 - \frac{\varepsilon_t^F}{\varepsilon_t^V}\frac{h_t+u_t}{h_t}\right),$$

which is condition (12) in the main text.

A.3 Proof of Proposition 1

The maximization in Proposition 1 can be written as

$$\mathcal{U}(w_{t-1}, a_t, p_t) = \max_{\{h_t, w_t, u_t, \tau_t^w \geq 0\}} \{a_t F(h_t) - V(h_t + (1-\delta)u_t) + \beta E_t \mathcal{U}(w_t, a_{t+1}, p_{t+1})\}$$

subject to the set of constraints

$$\begin{aligned}
&i) && w_t(1+\tau_t) = p_t a_t F'(h_t) \\
&ii) && w_t = p_t V'(h_t + u_t) \\
&iii) && u_t \geq 0 \\
&iv) && w_t \geq \psi(u_t) w_{t-1} \lambda_t \\
&v) && u_t(w_t - \psi(u_t) w_{t-1}) = 0
\end{aligned}$$

for given exogenous $\{a_t, p_t\}$.

Because the tax τ_t^w appears only in the labor demand curve, and by recognizing that $\tau_t^w \geq 0$, we note that constraint i) can equivalently be replaced by $w_t \geq p_t a_t F'(h_t)$. Therefore, the maximization problem is identical as the maximization problem of the constrained planner (see Definition 1 and the previous subsection), except for slackness condition v), which constitutes an additional constraint.

We proceed as in Bianchi (2016), by showing that constraint v) is always slack. That is, while v) formally constitutes an additional constraint in the maximization, at the optimum, this constraint is never binding. To do so, we consider the maximization without constraint v), and show that constraint v) is *implied as an optimality condition.* We had done so in the previous subsection, where we had shown that v) is an equilibrium condition.

This completes the proof of Proposition 1.

A.4 Optimal prudential tax

Here we show how to obtain the special case for the optimal prudential tax (18).

Start with (17) which we write as

$$\tau_t^w = \varphi \frac{p_t}{h_t^f} \beta \bar{\psi} E_t \lambda_{t+1},$$

where we have canceled the w_t, used our assumption $\psi(u_t) = \bar{\psi}$, used that $h_t = h_t^f$ when downward wage rigidity is slack ((4) when $u_t = 0$), and replaced $\varepsilon_t^V = \varphi$.[27]

To replace λ_{t+1}, we use (16) and set $\lambda_{t+2} = 0$ (by assumption, downward wage rigidiy is slack from period $t+2$ onwards). Furthermore, we replace $a_t F'(h_t) = w_t/p_t$ from (14) and use that $V'(\bar{h}) = \bar{h}^\varphi$, the latter from our assumption on the functional form for the dis-utility of labor supply. We also replace $-(\varepsilon_t^F)^{-1} = 1/(1-\alpha)$

$$\lambda_{t+1} = \frac{1}{1-\alpha} \frac{h_{t+1}}{w_{t+1}} \left(\frac{w_{t+1}}{p_{t+1}} - h_{t+1}^\varphi \right).$$

Now use (5) and (4) to re-write the last equation as

$$\lambda_{t+1} = \frac{1}{1-\alpha} \frac{h_{t+1}^f - u_{t+1}}{w_{t+1}} \left((h_{t+1}^f)^\varphi - (h_{t+1}^f - u_{t+1})^\varphi \right).$$

Now use the definition of $\hat{u}_t \equiv (h_t^f - h_t)/h_t = u_t/h_t^f \Leftrightarrow u_t = \hat{u}_t h_t^f$ to obtain

$$\lambda_{t+1} = \frac{1}{1-\alpha} \frac{h_{t+1}^f}{w_{t+1}} (1-\hat{u}_{t+1})(h_{t+1}^f)^\varphi \left(1 - (1-\hat{u}_{t+1})^\varphi\right).$$

Use again (5) to replace $(h_{t+1}^f)^\varphi$ and cancel the w_{t+1} to arrive at

$$\lambda_{t+1} = \frac{1}{1-\alpha} \frac{h_{t+1}^f}{p_{t+1}} (1-\hat{u}_{t+1}) \left(1 - (1-\hat{u}_{t+1})^\varphi\right).$$

Now combine the two key equations to obtain

$$\tau_t^w = \varphi \beta \bar{\psi} E_t \frac{1}{1-\alpha} \frac{h_{t+1}^f}{h_t^f} \frac{p_t}{p_{t+1}} (1-\hat{u}_{t+1}) \left(1 - (1-\hat{u}_{t+1})^\varphi\right).$$

Use (5) one last time to replace h_{t+1}^f/h_t^f to end up with (18) from the main text.

A.5 Model extension: The case of active wage setters

We repeat the dynamic program of the firms from Definition 3

$$\mathcal{P}_t(w_{t-1}(i)) = \max_{\{h_t(i), w_t(i)\}} \left\{ a_t F(h_t(i)) - \frac{w_t(i)}{p_t} h_t(i) + \beta E_t \mathcal{P}_{t+1}(w_t(i)) \right\}$$

[27] As shown in Section 6, we specialize to $V(h_t) = h_t^{1+\varphi}/(1+\varphi)$ and $F(h_t) = h_t^\alpha$. As a result, we obtain labor supply and demand $w_t = p_t h_t^\varphi$ and $w_t = p_t a_t \alpha h_t^{\alpha-1}$. Hence, the elasticities of labor supply and demand are $\varepsilon_t^V = \varphi$ and $\varepsilon_t^F = \alpha - 1$.

subject to the set of constraints

$$i) \qquad h_t(i) \le (w_t(i)/w_t)^{\eta} h_t, \qquad \text{(multiplier: } \kappa_t)$$
$$ii) \qquad w_t(i) \ge \bar{\psi} w_{t-1}(i), \qquad \text{(multiplier: } \lambda_t)$$

for given aggregate state variables $\{a_t, p_t, h_t, w_t\}$.

The first order conditions are

$$a_t F'(h_t(i)) - \frac{w_t(i)}{p_t} - \kappa_t = 0 \qquad (**)$$

for hours $h_t(i)$ as well as

$$-\frac{1}{p_t} h_t(i) - \beta\bar{\psi} E_t \lambda_{t+1} + \lambda_t + \kappa_t \eta (w_t(i)^{\eta-1}/w_t^{\eta}) h_t = 0 \qquad (***)$$

for wages $w_t(i)$, where we have already used the Envelope condition $(\partial/\partial w_{t-1}(i))\mathcal{P}_t(w_{t-1}(i))) = -\lambda_t \bar{\psi}$ (as in the constrained-efficient equilibrium above).

We proceed by distinguishing the cases where downward wage rigidity is slack and binding, respectively, then by studying the symmetric equilibrium.

A.5.1 Downward wage rigidity is slack

Assume that constraint i) in the maximization is slack. In this case, it must be that downward wage rigidity, constraint ii), is binding. Namely, if not, it were always possible to choose the same $h_t(i)$ but a lower $w_t(i)$, which is feasible because constraint i) is slack, and which raises $\mathcal{P}_t$ because production is the same, but the wage bill is reduced.[28]

Therefore, once downward wage rigidity ii) is slack, it must be that constraint i) is binding. By using that $\psi_t = 0$ in (**) and that $\kappa_t > 0$, we can combine (**) and (***) to obtain

$$a_t F'(h_t(i)) = \frac{\eta+1}{\eta}\frac{w_t(i)}{p_t} + \frac{1}{\eta}\frac{w_t(i)}{h_t(i)}\beta\bar{\psi} E_t \lambda_{t+1},$$

where we have used that constraint i) is binding to rewrite (***).

A.5.2 Downward wage rigidity is binding

Turn now to the case where downward wage rigidity ii) binds, such that $\psi_t > 0$.

Here we have to make a case distinction which is identical as the one in the constrained-efficient equilibrium (see the Appendix earlier above). Namely, we have to distuinguish the case where constraint i) is binding (as in the case where downward wage rigidity is slack),

[28] And there would be a non-negative effect on the continuation value, because $\mathcal{P}_t$ is weakly decreasing in the individual state $w_{t-1}(i)$.

versus the case where constraint i) is slack. In the main text we have only mentioned the second case, as this is the case that matters for the most part of the state space.

Intuitively, when the wage rigidity just about binds, by eating into their monopoly rents and reducing their precaution against downward wage rigidity, firms can still face $a_t F'(h_t(i)) > w_t(i)/p_t$ (that is, the marginal worker still adds to current profits):

$$\lambda_t < \frac{1}{p_t} h_t(i) + \beta\bar{\psi} E_t \lambda_{t+1},$$

where we have imposed $\kappa_t > 0$ in equation (***).[29]

Instead, if downward wage rigidity binds too strongly, $\lambda_t > 0$ becomes large enough such that the implied κ_t would turn negative. In this case, we set $\kappa_t = 0$ which yields for λ_t

$$\lambda_t = \frac{1}{p_t} h_t(i) + \beta\bar{\psi} E_t \lambda_{t+1}.$$

In turn, imposing $\kappa_t = 0$ in (**) we obtain the labor demand curve

$$a_t F'(h_t(i)) = \frac{w_t(i)}{p_t}.$$

To sum up, in this region, $w_t(i)$ is determined from downward wage rigidity ii), $h_t(i)$ is determined from labor demand (the previous equation), $\kappa_t = 0$ and λ_t is determined from the second to last equation.

A.5.3 Symmetric equilibrium

We now impose that $w_t(i) = w_t$ and that $h_t(i) = h_t$ for all $i \in [0, 1]$.

In the region where downward wage rigidity is slack, this straightforwardly yields labor demand (22) from the main text. In turn, if downward wage rigidity is slack, households will choose to sell hours according to (23).

Turn now to the case where downward wage rigidity binds. As shown above, in an intermediate region where downward wage rigidity binds only lightly, an equilibrium obtains with h_t determined by labor supply (23), and λ_t determined in

$$a_t F'(h_t) = \frac{\eta + 1}{\eta} \frac{w_t}{p_t} + \frac{1}{\eta} \frac{w_t}{h_t} [\beta\bar{\psi} E_t \lambda_{t+1} - \lambda_t],$$

which holds as long as $a_t F'(h_t) > w_t/p_t$. Intuitively, in this region firms eat into their monopoly rents and reduce their precuation against downward wage rigidity by shifting rightwards labor demand. They are willing to do so as long as $a_t F'(h_t) > w_t/p_t$, as in this case the marginal worker still adds to current profits.

[29] Thus, in this region, $w_t(i)$ is determined by downward wage rigidity ii), $h_t(i)$ is determined by constraint i), and κ_t and λ_t are determined residually from (**) and (***), and are both strictly positive.

Instead, if the implied $a_t F'(h_t) < w_t/p_t$, firms would stop hiring the additional workers. In this case, hours h_t are determined by labor demand

$$a_t F'(h_t) = w_t/p_t,$$

and labor supply is strictly rationed: $V'(h_t) < w_t/p_t$. In turn, the implied λ_t is determined as shown above:

$$\lambda_t = \frac{1}{p_t} h_t + \beta\bar{\psi} E_t \lambda_{t+1},$$

which is equation (25) in the main text.

B Appendix: Data

Here we provide further details on the data used in the empirical application.

1. Labor cost index

   Here we use Eurostat's *Labour cost index by NACE Rev. 2 activity - nominal value, annual data [lc_lci_r2_a].* We are using the version *Index* 2012 = 100, *Business Economy, Labour cost for LCI (compensation of employees plus taxes minus subsidies).* The data for Greece only starts in 2001. Therefore, we augment the series by adding *Labour cost index by NACE Rev. 1.1 activity - nominal value, annual data* [*lc_lci_r*1*_a*] in the first two years of the sample.

2. Real GDP

   Here we are using the series *B1_GA: Gross domestic product (output approach), Current Prices*, from OECD.stat.

3. Unemployment

   Here we are using Eurostat's series *Unemployment by sex and age - annual average [une_rt_a].* We are using the series *Total*, *Percentage of active population.*

4. Productivity

   Here we are using the series *Level of GDP per capita and productivity* from OECD.stat. We are using the series *GDP per hour worked, National currency, Current prices.*

The series labor cost index, real GDP and productivity are de-trended by assembling the same series for all other euro area countries, then removing a GDP-weighted average (GDP weights from 1999).