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Regulating False Disclosure

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Abstract

Firms communicate private information about product quality through a combination of pricing and disclosure where disclosure may be deliberately false. In a competitive setting, we examine the effect of regulation penalizing false disclosure. Stronger regulation reduces the reliance on price signaling, thereby lowering market power and consumption distortions; however, it often creates incentives for excessive disclosure. Regulation is suboptimal unless disclosure itself is inexpensive and even in the latter case, only strong regulation is welfare improving. Weak regulation is always worse than no regulation. Even high quality firms suffer due to regulation.

JEL Classification: L13, L15, D82, D43.

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1 Introduction

On the basis of the information that firms provide, consumers infer which products they should buy. That inference is not always easy as firms may lie about the quality attributes of their own products. Around the world, government agencies have been concerned with firms being able to provide false information and have introduced penalties for false disclosure. For instance, in the US, the FTC Policy Statement Regarding Advertising Substantiation states that "a firm's failure to possess and rely upon a reasonable basis for objective claims constitutes an unfair or deceptive act or practice in violation of Section 5 of the FTC Act".[1] In Canada, section 74.01(1)(a) of the Competition Act prohibits any "representation to the public that is false or misleading in a material respect". A court may order a violating firm to pay significant damages. In Europe in November 2016, the European Commission has updated the 2006 Directive on Misleading and Comparative Advertising, making it unlawful to engage in false statements concerning product quality.[2] The apparent rationale behind these regulations is that they increase the reliability and credibility of communication from firms that ultimately help consumers make better, more informed choices, thereby yielding more efficient market outcomes.

Consumers, however, may also infer hidden information about product quality from other observable actions of firms such as pricing. In a large class of markets, firms are likely to strategically choose a combination of direct (possibly false) disclosure and prices to make consumers buy their product. Making direct communication more credible through regulation of false disclosure may not necessarily yield more information to buyers. It can, however, radically alter the strategic incentives and actions chosen by competing firms and the resulting market outcome. Is there an economic rationale for regulation of false disclosure in markets where buyers can also indirectly infer information about products from other signals (such as, prices)? If so, what is the optimal fine for false disclosure? Is more stringent regulation necessarily better? This paper addresses these questions and in doing so, provides two broad narratives.

First, regulation of false disclosure can lead to *competitive overdisclosure*. As regulation makes disclosure more credible, high quality firms that disclose can be more confident that lowering prices will not trigger adverse beliefs about their products from buyers. This creates a strategic individual incentive for high quality firms to disclose in order to be more

[1] See, e.g., http://www.ftc.gov/bcp/policystmt/ad-decept.htm

[2] See, http://ec.europa.eu/consumers/consumer_rights/unfair-trade/false-advertising/index_en.htm

competitive in the market. Disclosure is, however, costly and must be weighed against the gain in social surplus due to improvements in market allocation; without efficiency gains, disclosure is purely wasteful. In the absence of regulation, direct information provision is not credible and firms will not use it, relying on price signaling instead. The latter is associated with social distortions as low quality firms must have a large market share even when its rival is of high quality to obtain sufficient rents to not imitate the high quality firms. As regulation increases, but is still weak, high quality firms use a combination of disclosure and prices to signal quality, but will do so at comfortably high prices at which they do not want to gain market share from low quality. The net effect is competitive overdisclosure, leaving society strictly worse off than with no regulation.

Second, if the direct disclosure cost are relatively large, high quality firms can only acquire enough rent to cover these costs if consumers buy high quality also when low quality is around. However, if low quality firms only sell in the state where they face low quality rivals, they will compete severely on price, forcing high quality firms to accept such low margins that they are better off not always disclosing their quality, leaving consumers to buy from the low quality firm in case they do not disclose directly. Thus, a fine, no matter how high, will never fully eliminate the consumption distortion if the direct disclosure cost is beyond a minimum threshold level.

These two narratives indicate that regulation can improve welfare only if it is sufficiently strong *and* the disclosure cost is small enough. No regulation of false disclosure is better than intermediate fines and is optimal if the direct disclosure cost is not too small. Profits are negatively impacted by regulation as it intensifies price competition. Even high quality firms may lobby against regulation, and if they do, it could be in line with social welfare.

The market environment we study is a symmetric incomplete information Bertrand duopoly where products differ in quality and consumers have unit demand. A firm knows its own product quality, but it is not observed by the rival firm or the buyers. High quality is more costly to produce (than low quality), but generates more social surplus. Under full information, high quality firms exercise their natural competitive advantage and supply the whole market. Firms can use both pricing and disclosure to convey private information about their own product quality. Disclosure is costly and not necessarily credible as the information provided is not verifiable; a low quality firm may lie and misrepresent its product quality. Firms simultaneously make their disclosure and pricing decisions; buyers use these to update their beliefs about each firm's true product quality, and then make their purchase decisions. The products we study are experience or credence goods where consumers may observe the true quality of the good only after they consume it or the true nature of the good is only discovered much later by a public agency. Regulation takes the form of a future penalty or fine for false disclosure faced by a low quality firm misrepresenting its product as high quality. The expected fine for false disclosure is an exogenous parameter that captures the strength of regulation.

Our model can be viewed as a multidimensional signaling model[3] with competing senders where firms may use price and direct disclosure as signals. By making false disclosure costly, regulation has a profound impact on the nature of equilibrium even though it does *not* lead to *more informed* consumer choices. As mentioned above, in the absence of regulation, firms use prices to signal quality, but the need to provide sufficient rent to low quality firms implies that prices are high and generate a consumption distortion (buyers buy low quality even when high quality is supplied). Regulation affects the structure of equilibria through the kind of beliefs that can be supported to sustain equilibrium outcomes. In particular, adverse out-of-equilibrium beliefs that deter high quality firms from cutting prices become less reasonable when false disclosure is more costly. This makes the market more competitive.

Disclosure in our model may be interpreted as informative advertising, (imperfect) quality certification or rating by third parties. The fact that low quality firms may engage in false advertising or get a high quality certificate may reflect enforcement problems (or other weaknesses) in the regulation of advertisement content, imperfections in the ability of third party certifiers or label providers to verify true product quality (or, even corruption in these institutions). In this vein, increasing the cost of false disclosure may be interpreted as strengthening institutions, stricter regulation of advertisement content (for instance, through "truth in advertising" regulations), or better regulation of third party certifiers and rating agencies.

Our paper is related to several strands of literature. Technically, our model is based on Janssen and Roy (2010) where firms can only use price to signal quality. Janssen and Roy (2015) also analyze a model where firms may disclose their private information, but in that paper (as in much of the disclosure literature) disclosure is assumed to be fully credible and verifiable.[4,5]

[3] See, e.g. Ramey (1996), for an early example of a signaling game with mutiple signals.

[4] In addition, Janssen and Roy (2015) treat disclosure as a long-run decision that firms commit to prior to price competition; in the current paper, disclosure is viewed as direct communication and it is difficult to pre-commit to the content of such communication.

[5] See Daughety and Reinganum (2008) for a monopoly version of that paper.

In terms of content our paper is related to a recent literature that aims to explain why false advertisement may occur in equilibrium. Thus, Rhodes and Wilson (2015) analyze a monopoly model where the production cost is independent of quality and consumers' out-of-equilibrium beliefs depend only on the disclosure statements, but not on price. They show that there exists an equilibrium where low and high quality firms choose the same actions and consumers may be deceived. The possibility of deceptive communication is also explored by Piccolo et al. (2015, 2016) in a duopoly model where it is known that one firm has high and the other low quality, but buyers do not know which firm produces high quality. A similar question is also addressed by Drugov and Troya Martinez (2014) in a persuasion game where a monopolist seller sells at a fixed price, but can choose the precision and the bias of his signal. In contrast to this literature, the aim of our paper is not to explain when or why firms may engage in false advertising. Rather, we show that even if firms do not engage in false disclosure, a policy of increasing the cost of false disclosure may have a real impact on market outcomes through the beliefs that can be sustained.

The questions addressed in our paper can be related to the literature on communication with lying cost (see, e.g. Kartik, Ottaviani and Squintani (2007) and Kartik (2009)) where it is shown that "inflated language" and incomplete separation of types are natural outcomes. An important difference with our setting is that we have multiple senders and we allow firms to have alternative means of signaling (through prices); further, both the sender and the receiver's payoffs depend directly on one of the signals (price) that the competing senders (firms) choose. These differences explain why separation always occurs in our context and that low quality firms will never want to disclose falsely. The two-dimensional nature of signaling and the fact that both the sellers' and buyers' payoffs depend directly on the price signal also distinguishes our framework from the recent literature on competition in Bayesian persuasion games (see, e.g., Gentzkow and Kamenica (2017), Li and Norman (2017)).

Our paper is also related to models where firms advertise content. For instance, Anderson and Renault (2006) consider a monopolist selling to consumers with different match values; the firm may advertise price and/or content. Without advertising the consumers have to incur a search cost to learn both price and match value. It is shown that the firm chooses to convey only limited product information and it is socially harmful to provide full information. In contrast, we have a competitive model with vertical product differentiation where consumers are informed about prices without having to incur a search cost.

There is, of course, a very large literature on signaling quality through price and/or advertising (see, e.g., Milgrom and Roberts (1986) and Bagwell and Riordan (1991)) or on quality disclosure (see, e.g., Viscusi (1978), Grossman (1981), Milgrom (1981) and Jovanovic (1982)). Unlike much of the disclosure literature, disclosure is not verifiable and not fully credible in our paper; a low quality firm may lie and misrepresent its product quality. In contrast to the advertising signaling literature, it is not just the amount of money spent on advertising, but it is also the content that is important as it is more costly for a low, than for a high, quality firm to inform consumers its quality is high.

The rest of the paper is organized as follows. Section 2 presents the details of the model, the equilibrium concept and some basic properties of equilibria. Equilibria can be one of three types: (*i*) pure price signalling, (*ii*) signaling equilibrium where high quality firms disclose for sure, and (*iii*) signaling equilibrium where high quality firms randomize between disclosure and non-disclosure. Sections 3 - 5 discuss fully characterize these equilibria. Section 6 comnbines all of these equilibria to understand the effect of regulation and the implications for optimal regulation. Section 7 concludes and discusses some of the modeling assumptions. The appendix contains the elements of proofs that are not outlined in the main text.

2 The Model and Preliminary Results

There are two firms, $i = 1, 2$, in the market. Each firm's product may be of either high (H) or low (L) quality. The true product quality is known only to the firm that supplies the product; it is not known to the rival firm or to consumers. It is common knowledge that the *ex ante* probability that a firm's product is of high quality is $\alpha \in (0, 1)$. The products of the firms are not differentiated in any dimension other than quality. Firms supply their output at constant unit cost c_s that depends on quality $s \in \{H, L)$, where $c_H > c_L \geq 0$.

There is a unit mass of identical consumers each having unit demand[6]. A consumer's valuation of a product of quality s is given by V_s, $s = H, L$, where

$$V_H > V_L, V_s > c_s, s = L, H.$$

We focus on the more interesting case where the quality premium $V_H - V_L$ that buyers are

[6]Unit demand is a reasonably good approximation of many high end markets (such as cars, TV, computers and other electronic products). In the concluding section we discuss how our results are affected if demand is downward sloping.

willing to pay for the high quality product exceeds the cost difference:

$$\Delta V = V_H - V_L > \Delta c = c_H - c_L. \qquad (1)$$

As the high quality product creates more social surplus than the low quality product, a *consumption distortion* is said to occur if consumers buy low quality even if high quality is provided by some firm in the market. It follows there can only be a consumption distortion in the state of the world where one firm produces low quality and the other high quality, which arises with probability $2\alpha(1-\alpha)$.[7] To ensure full market coverage (i.e., all buyers buy), we assume $\Delta V \leq [(V_H - c_L)/2]$.

Firms have the option of directly disclosing their type to all buyers by sending a message about their type; the (fixed) cost of disclosure is denoted by $D > 0$. As no firm would like to incur a communication cost to say that its product is low quality, a firm either sends no message or sends a message claiming that its product is of high quality. Disclosure is *not* necessarily credible or verifiable. In particular, it is possible for a low quality firm to lie or disclose incorrectly and claim that its product is of high quality. However, when there is regulation of information content communicated by firms in the industry, false disclosure may attract future penalty and the expected regulatory fine for false disclosure is given by $f \geq 0$; the parameter f captures the strength or stringency of regulation. If f is large enough, a firm would never lie and so communication would be equivalent to credible and verifiable disclosure.

Formally, the game proceeds in three stages. First, nature independently draws the type (or quality) τ_i of each firm i from a distribution that assigns probabilities α and $1-\alpha$ to H and L respectively; the realization of τ_i is observed only by firm i. Next, both firms (having observed their own types), simultaneously choose their prices and messages to all buyers; each firm's message $m \in \{0, 1\}$ where 0 means no message and 1 means the message "I produce high quality". After observing the messages sent and the prices set by both firms, consumers decide whether to buy and if so, from which firm. The payoff of each firm is its expected profit net of any disclosure cost and, where relevant, the fine. The payoff of each consumer is her expected net surplus.

The solution concept used is that of symmetric perfect Bayesian equilibrium (PBE)

[7] Note that under complete information, there is no consumption distortion as a high quality producer would use its competitive advantage to limit price a low quality rival and reduce the latter's market share to zero.

where the out-of-equilibrium beliefs satisfy a version of the D1 criterion (Cho and Kreps, 1987). In what follows, we simply refer to this as "equilibrium". The D1 refinement is useful to reduce the number of equilibria in the game, and for many parameter values we show that there is a unique D1 equilibrium. Roughly speaking, the D1 refinement selects the most competitive equilibrium in the set of fully revealing equilibria. However, the qualitative conclusions regarding policy implications remain true if a weaker equilibrium notion is used, but the analysis of comparative statics in terms of the set of equilibria are more complicated. We will elaborate more on this in Section 7.

The D1 criterion has been developed for signaling games with one sender and a one dimensional signaling space. Retaining tractability, a technical innovation this paper makes is to adapt the D1 criterion to environments with multiple senders and two signaling dimensions.[8] Consider firm i unilaterally deviating to a strategy (p, m) outside the support of its equilibrium strategy. Given the (possibly mixed) equilibrium strategies of other firms, each profile of beliefs that buyers may possibly have about the type of firm i (following this deviation) and each profile of best responses of buyers (based on every such belief profile) defines a certain expected quantity sold by firm i at price p and message m. Let $B_i(p, m)$ be the set of of all possible expected quantities sold by firm i at price p and message m that can be generated in this manner by considering all possible beliefs and best responses of buyers. Each $q_i \in B_i(p, m) \subset [0, 1]$ is a quantity that firm i can "expect" to sell at price p and message m for some profile of beliefs of buyers about firm i's type and for some configuration of optimal choices of buyers (that depends on realizations of prices charged by other firms) when other firms play according to their equilibrium strategy.

In the spirit of the D1 criterion, we compare the subsets of expected quantities in $B_i(p, m)$ for which it is gainful for different types of firm i to deviate to price p and message m. More precisely, consider any perfect Bayesian equilibrium where the equilibrium profit of firm i when it is of type τ is given by $\pi_\tau^{i*}, \tau = H, L$. Consider any $p \in [0, V_H]$ and $m \in \{0, 1\}$ outside the support of the equilibrium strategy of firm i and denote by $\pi_\tau^i(p, m; q_i)$ the profit firm i of type τ makes when he sells quantity q_i. For example, $\pi_L^i(p, 1; q_i) = (p - c_L)q_i - D - f$. If for $\tau, \tau' \in \{H, L\}$, $\tau' \neq \tau$,

$$\{q_i \in B_i(p, m) : \pi_\tau^i(p, m; q_i) \geq \pi_\tau^{i*}\} \subset \{q_i \in B_i(p, m) : \pi_\tau^i(p, m; q_i) > \pi_{\tau'}^{i*}\}$$

[8] The notion of undefeated equilibrium, pioneered by Mailath and Sobel (1993) cannot be easily adapted to signaling games with multiple senders.

where "$\subset$ " stands for strict inclusion, then the D1 refinement suggests that the out-of-equilibrium beliefs of buyers (upon observing a unilateral deviation by firm i to price p) should assign zero probability to the event that firm i is of type τ and thus (as there are only two types) assign probability one to firm i being of type τ'. We use this extended D1 crterion in the rest of the paper.

2.1 General Properties of Equilibria

Having defined the equilibrium notion, we now show that equilibrium is always fully revealing, and characterize some other properties of the equilibrium. That equilibria must be fully revealing is important as it implies that consumers are not misled by the content of information disclosed by the firm even though firms have the option of lying. Even if the cost of false advertising f is arbitrarily small the low quality seller will never pretend, through the message it sends, to be of high quality.

To see the main argument, suppose there exists a partially pooling equilibrium. Then with at least some positive probability low and high quality firms pool on disclosing and setting a price p with a certain strictly positive probability. Low and high quality firms earn profit equal to $q(p - c_L) - D - f$, and $q(p - c_H) - D$, respectively, where q is the expected quantity sold by a disclosing firm at price p. As there is a strictly positive probability mass at p, using symmetry it is easy to see that prices slightly above p lie outside the equilibrium strategy of both types. Thus, for $\varepsilon > 0$ small enough, disclosing and setting a price $p + \varepsilon$ is an out-of-equilibrium action for both types. The critical quantity $\widehat{q}_\tau$ that makes a type τ firm indifferent between disclosing with price p and doing so with price $p + \varepsilon$ is given by

$$\widehat{q}_\tau = \frac{q(p - c_\tau)}{p + \varepsilon - c_\tau}, \qquad i = L, H,$$

As $c_H > c_L$, $\widehat{q}_H < \widehat{q}_L$ so that the high quality type has an incentive to deviate for a wider range of quantity responses from buyers.[9] The D1 refinement then requires consumers to believe that it is the high quality firm that has deviated if they observe a price $p + \varepsilon$ with disclosure.[10] If consumers would buy with some positive probability after observing price p

[9] Note that a pooling equilibrium where consumers are deceived by false disclosure exists if $\Delta c = 0$. This clarifies the importance of the common cost assumption in Rhodes and Wilson (2015) and Piccolo et al. (2015a,b) to argue that false advertising may occur in equilibrium. In addition, Piccolo et al. (2015a,b) do not consider equilibrium refinements restricting the out-of-equilibrium beliefs used to punish deviations.

[10] Note that this argument nicely illustrates the idea that (even out-of-equilibrium) higher advertized prices signal higher quality.

being advertised and believing the quality is some weighted average of low and high quality, these same consumers will certainly buy with a strictly higher probability at price $p+\varepsilon$ believing this price is set by a high quality firm. Thus, firms want to deviate and shift the probability mass from price p to price $p+\varepsilon$ (while disclosing). This contradicts the fact that disclosing and setting price $p+\varepsilon$ is an out of equilibrium action.

The next proposition summarizes[11]:

Proposition 1 *There does not exist a pooling or semi-pooling equilibrium.*

Next, we consider the properties that any fully revealing equilibrium has to satisfy. There are three types of fully revealing equilibria: (*i*) a pure price signalling equilibrium, (*ii*) a disclosure equilibrium, and (*iii*) a mixed signalling equilibrium where high quality randomizes between disclosure and pure price signaling.

In a version of the model considered in this paper where firms have no option to disclose or send any message about their type, Janssen and Roy (2010) (hereafter, JR (2010)) characterize the unique (symmetric D1) pure price signaling equilibrium. If there is a pure price signaling equilibrium in the model outlined in this paper (where firms never disclose i.e., always send message 0), then it must be identical to the one in JR(2010). In this equilibrium, low quality firms randomize their prices over an interval $[\underline{p}_L, \overline{p}_L]$, while the high quality firm sets a deterministic price p_H^{ND} with $\overline{p}_L = p_H^{ND} - \Delta V$ and $\underline{p}_L = \alpha\overline{p}_L + (1-\alpha)c_L$. The next Proposition shows that some of these properties of the equilibrium structure can be generalized to hold for all equilibria in our model. We use superscripts D and ND to indicate whether prices are accompanied by disclosure or not.

Proposition 2 *The following properties hold in equilibrium:*

(a) Low quality types do not disclose and in particular, no fine is incurred in equilibrium;

(b) If a high quality type's equilibrium strategy places a strictly positive probability mass on charging price $\widehat{p} > c_H$ and message $\widehat{m} \in \{0,1\}$, then a low quality type must be indifferent between following its equilibrium strategy and imitating price $\widehat{p}$ and message $\widehat{m}$;

(c) If a high quality type does not disclose with strictly positive probability, then in the state where it does not disclose it charges a deterministic price p_H^{ND} and at sells only if the rival firm is a high quality type; further low quality firms randomize over a set of prices whose upper bound $\overline{p}_L$ satisfies $c_L < \overline{p}_L \leq p_H^{ND} - \Delta V$. If, in addition, a high quality type

[11] As is clear from the arguments outlined above, the proposition continues to hold for possibly asymmetric D1 equilibria.

discloses with strictly positive probability, then it charges lower prices when it discloses i.e., $\overline{p}_H^D \leq p_H^{ND}$ (where $\overline{p}_H^D$ is the upper bound of high quality prices under disclosure);

(d) The price distribution of a disclosing high quality firm can have a probability mass point only at its lower bound (in which case the lower bound is an isolated point);

(e) If a low quality type makes strictly positive profit, then it must randomize over prices with no probability mass point.

Part (a) of Proposition 2 points out that low quality types never disclose falsely and pretend to be high quality. It also makes clear that as long as the low quality type makes positive profits, important elements of the structure of the pure price signaling equilibrium remain valid. If a high quality firm does not disclose, it makes positive sales only if the competitor is also a high quality firm: part (c) of the proposition states that a low quality competitor undercuts by a large enough margin to attract all buyers; further, a high quality disclosing competitor sets (weakly) lower prices. Parts (c) and (e) of Proposition 2 also indicate that as long as there is a positive probability that a high quality firm does not disclose, a low quality firm makes positive profit and must randomize over prices. This also implies existence of a consumption distortion as low quality firms must make positive sales when they set a price equal to the upper bound of the price distribution and in that case they sell only if the rival is a high quality firm. When high quality firms randomize between disclosing and not disclosing, they must sell more when they disclose; otherwise they cannot be indifferent between the two actions. Finally, part (b) of Proposition 2 is important as an equilibrium selection tool: not only does the low quality type not have an incentive to imitate the high quality firm's actions, but in a D1 equilibrium the low quality type is actually indifferent between its equilibrium actions and imitating an action to which the high quality type assigns a strictly positive probability mass.

3 Pure Price Signaling

To understand the role of disclosure and what regulation can achieve by making false disclosure more costly, it is important to understand how price signaling works and under what conditions firms abstain from disclosure with probability one. As discussed in the previous section, a pure price signaling equilibrium has to be identical to the equilibrium analyzed in Janssen and Roy (2010) where it is shown that when firms do not have the option to disclose, the unique symmetric D1 equilibrium outcome is a separating equilibrium where high quality firms charge a deterministic high price p_H^{ND} and low quality firms randomize their prices over an interval $[\underline{p}_L, \overline{p}_L]$ according to a distribution function F_L where

$$p_H^{ND} = c_L + 2\Delta V, \overline{p}_L = p_H^{ND} - \Delta V = c_L + \Delta V$$

and for all $p \in [\underline{p}_L, \overline{p}_L]$

$$F_L(p) = 1 - \frac{\alpha}{1-\alpha}\left(\frac{\Delta V}{p - c_L} - 1\right) = \frac{1}{1-\alpha} - \frac{\alpha \Delta V}{(1-\alpha)(p - c_L)}.$$

Further, all consumers buy. If one firm charges p_H^{ND} and the other firm charges a price in $[\underline{p}_L, \overline{p}_L]$, buyers always buy from the latter; when both firms charge prices in $[\underline{p}_L, \overline{p}_L]$, buyers buy from the lower priced firm. Thus, in the price signaling equilibrium there is a consumption distortion in that when both low and high quality firms exist in the market, consumers buy low quality, even though the surplus generated by high quality consumption is larger. Also, the market equilibrium exhibits a fair amount of market power with $\overline{p}_L - c_L = \Delta V$, which is necessary to keep the low quality firm from imitating the high quality price. A high quality firm does not have an incentive to undercut as any firm charging price $p \in (\overline{p}_L, p_H)$, is believed to have low quality with probability one so that no consumer would buy at these prices as they would buy from the competitor instead.

For this equilibrium and for other equilibria discussed in subsequent sections, it is important to understand the role of the D1 refinement. JR(2010) show that there are multiple price signaling equilibria in their model that are identical up to the choice of p_H^{ND} and that any $p_H^{ND} \geq c_L + 2\Delta V$ results in an equilibrium. The D1 equilibrium selects the most competitive of these equilibria and in it the low quality firm is just indifferent between choosing a $p \in [\underline{p}_L, \overline{p}_L]$ and imitating high quality and setting $p_H^{ND} = c_L + 2\Delta V$. To understand this, note that if $p_H^{ND} > c_L + 2\Delta V$, then the low quality type strictly prefers to *not* imitate the high quality type and this implies that it would want to deviate to an out-of-equilibrium price slightly below p_H^{ND} only if there is a significant increase in the quantity it can sell (relative to what it can sell at p_H^{ND}); the high quality type, however, gains from undercutting p_H^{ND} for only a slight increase in quantity sold. The D1 refinement then requires that buyers should believe that an out-of-equilibrium price just below p_H^{ND} can only be charged by a high quality type and this makes it gainful for the high quality type to deviate and undercut p_H^{ND}. This eliminates all signaling equilibria except the one where the low quality type's incentive constraint binds.

As there is a consumption distortion, but no disclosure costs are incurred, the welfare loss, compared to the first-best, equals

$$WL = 2\alpha(1-\alpha)(\Delta V - \Delta c). \tag{2}$$

Consumer surplus can be calculated to be equal to[12]

$$(V_L - c_L) - \alpha(2-\alpha)\Delta V,$$

while firm profits equal $\pi_H^* = \alpha(\Delta V - \Delta c/2)$ and $\pi_L^* = \alpha \Delta V$. It is clear that both total surplus, consumer surplus and firms' profits are independent of f and D whenever the pure price signaling equilibrium exists.

It remains to be seen for which parameter values D and f the pure price signaling equilibrium exists. The main reason why such an equilibrium may not exist is that the high quality firm may have an incentive to deviate by directly disclosing its quality if he is believed to be a high quality firm by consumers. Both the disclosure cost D and the fine f play an important role in this respect: D should not be too high for the deviation to be profitable, whereas f should not be too small for consumers to infer that a low quality firm would have no incentive to engage in such a deviation.

To determine more precisely when a pure price signaling equilibrium exists, define two critical levels of the disclosure cost $\underline{D}$ and $\overline{D}$ as follows:

$$\underline{D} = \alpha(\Delta V - \frac{\Delta c}{2}) \text{ and } \overline{D} = \Delta V - (1 - \frac{\alpha}{2})\Delta c$$

It is easy to check that $0 < \underline{D} < \overline{D}$. For each $D \in (0, \overline{D})$ define $f^*(D)$, a critical level of the fine, by

$$\begin{aligned} f^*(D) &= \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}\Delta c, \text{ for } \underline{D} \leq D < \overline{D} \\ &= \frac{D}{2\Delta V - \Delta c}\Delta c, \text{for } 0 \leq D \leq \underline{D} \end{aligned}$$

[12] Consumer surplus is equal to

$$\begin{aligned} & (1-\alpha)^2(V_L - E\min p_L) + 2\alpha(1-\alpha)(V_L - Ep_L) + \alpha^2(V_H - p_H) \\ = \; & (V_L - c_L) - 2\alpha(1-\alpha)\Delta V \left(1 - \frac{\alpha \Delta V}{1-\alpha}\ln\frac{1}{\alpha}\right) - 2\alpha^2 \Delta V \ln\frac{1}{\alpha} - \alpha^2 \Delta V, \end{aligned}$$

which can be rewritten as the expression in the text.

Observe that $f^*(D)$ is continuous and strictly increasing in D on $(0, \overline{D})$, $f^*(D) \to 0$ as $D \to 0$, and $f^*(\overline{D}) = (1 - \frac{\alpha}{2})\Delta c$.

Our main result in this section is that a pure price signaling equilibrium exists if, and only if, either (*i*) $D \geq \overline{D}$, or (*ii*) $D < \overline{D}$ and $f \leq f^*(D)$. If the disclosure cost D is too large (larger than $\overline{D}$), then it never pays to disclose and undercut the high quality pure price signaling price to gain market share even if the out of equilibrium beliefs are very favorable. $\overline{D}$ is the critical level of disclosure cost such that a high quality firm is just indifferent between sticking to the pure price signaling price (where he only sells when rival is high quality) and deviating to disclosing and reducing his price to $\underline{p}_L + \Delta V$ where it takes over the market regardless of the type of the rival (which is the optimal deviation as long as consumers believe that the deviating firm is of high quality type).

If the disclosure cost is smaller, then it may be gainful for a high quality firm to disclose (and undercut the high quality price), if this double signal is correctly interpreted (by consumers) as coming from a high quality firm. In terms of the D1 logic, this requires that a low quality firm should *not* have a stronger incentive to send such a message than a high quality firm. Intuitively, this depends on the fine f: if the fine for lying is large, then low quality firms will not have an incentive to send such a signal and high quality firms will then find it optimal to deviate; if f is small, then consumers will believe that low quality firms are trying to mislead them, and will therefore not buy from a firm sending this double signal, making it unprofitable for high quality firms to deviate. The critical fine $f^*(D)$ which defines the boundary of the region of pure price signaling is determined by two conditions needed for a gainful deviation from the pure price signaling equilibrium; depending on the parameter values, only one condition is binding. The first is a condition that ensures that there is a neighborhood of prices below the high quality pure price signaling price p_H^{ND} such that charging such prices with disclosure will be interpreted by buyers as coming from a high quality firm (using the D1 criterion). Of course, if this neighborhood of prices is small, then the deviating high quality firm cannot undercut p_H^{ND} sufficiently and therefore, cannot sell much in the state where rival is low quality so that the deviation may not be profitable (taking into account the fixed cost of disclosure); the second condition ensures that there is a profitable deviation within this neighborhood of prices. The first condition binds if D is below $\underline{D}$ and the second condition binds when $D \in [\underline{D}, \overline{D})$.

Figure 1 depicts the region where a pure price signaling equilibrium exist. For $D < \overline{D}$, the bold line represents the function $f^*(D)$. A symmetric pure price signaling equilibrium exists only to the left of and above the bold line.

INSERT FIGURE 1 HERE

For ease of comparison with the characterization of equilibria in subsequent sections, we state the main result regarding pure price signaling equilibria in terms of the "inverse" of $f^*(D)$, where the function $D^*(f)$ is the lowest disclosure cost at which a pure price signaling equilibrium exists (for each value of f) and is given by:

$$D^*(f) = \begin{cases} (2\frac{\Delta V}{\Delta c} - 1)f, \text{ for } 0 \leq f < \frac{\alpha \Delta c}{2} \\ (\frac{\Delta V}{\Delta c} - 1)f + \frac{\alpha}{2}\Delta V, \text{for } \frac{\alpha \Delta c}{2} \leq f \leq (1 - \frac{\alpha}{2})\Delta c \\ \Delta V - (1 - \frac{\alpha}{2})\Delta c = \overline{D}, \text{for } f \geq (1 - \frac{\alpha}{2})\Delta c \end{cases}$$

Proposition 3 *A symmetric equilibrium with pure price signaling i.e., one where firms do not disclose directly, exists if, and only if, the disclose cost $D \geq D^*(f)$.*

4 Pure Disclosure

In the previous section, we characterized the conditions under which the un-reliability of disclosed information implies that firms use only prices to communicate their private information. Price signaling is associated with a consumption distortion. One may expect that if regulation is strengthened and disclosure itself is not too costly, firms will use disclosure (along with pricing) to credibly communicate their private information and this may ameliorate the consumption distortion possibly leading to welfare gains. In this section, we characterize the economic conditions under which high quality firms disclose for sure and examine the welfare implications.

We begin by showing that contrary to what one might expect, even though moderate regulation can increase the credibility of disclosure to the extent that high quality firms always disclose, there is no correction of the consumption distortion so that the disclosure cost incurred by firms is a pure waste from a social point of view. In the equilibrium we analyze first, high quality firms disclose with probability one in equilibrium, while low quality firms make positive profits, capturing the market even if high quality firms are present. Given our characterization result in Proposition 2, it is easy to see that low quality firms must randomize over an interval $[\underline{p}_L, \overline{p}_L]$ if they make positive profits. The pure disclosure equilibrium we characterize first is one where a high quality firm chooses a deterministic price p_H^D and $\overline{p}_L = p_H^D - \Delta V$.[13] The equilibrium structure is very similar to that of the pure price signaling equilibrium, with low quality firms selling to all consumers if the competitor produces high quality and the equilibrium is fully distortionary as disclosure does not reduce the consumption distortion observed in the pure price signaling case. The equilibrium profits are given by

$$\pi_H^* = \frac{\alpha}{2}(p_H^D - c_H) - D = \pi_H^D - D$$

and

$$\pi_L^* = \alpha(p_H^D - \Delta V - c_L).$$

The main remaining issue is how to determine p_H^D. An important consideration is that low quality firms should not have an incentive to imitate the high quality behavior. This implies a lower limit on p_H^D :

$$\frac{\alpha}{2}(p_H^D - c_L - 2\Delta V) + D + f \geq 0.$$

To determine p_H^D we have to consider the other major possibility of deviation, namely for any of the firms to set an out-of-equilibrium price $\widehat{p} \in (\overline{p}_L, p_H^D)$. This can be accompanied both by disclosure and no disclosure. It is clear that if consumers believe that a high quality firm has deviated, then they will buy and this makes such a deviation profitable. Using the D1 logic explained in the previous section and in Proposition 2(b), the requirement that consumers believe that a deviating firm that discloses and chooses $\widehat{p} \in (\overline{p}_L^D, p_H^D)$ is of low quality implies that a low quality firm should be indifferent between its equilibrium strategy and mimicking the high quality behavior. This results in the requirement that

$$p_H^D = c_L + 2\Delta V - \frac{2(D+f)}{\alpha}. \tag{3}$$

Low quality firms make nonnegative profits if, and only if, $\overline{p}_L = p_H^D - \Delta V \geq c_L$ and this is guaranteed if, and only if,

$$D + f \leq \frac{\alpha}{2}\Delta V. \tag{4}$$

In the proof of the next Proposition we show that the requirement that consumers think that a deviating firm that does *not* disclose and chooses $\widehat{p} \in (\overline{p}_L^D, p_H^D)$ is of low quality is

[13] The proof of Proposition 4 in the Appendix explains that there cannot be other equilibria where the high quality firm always discloses and the low quality firm makes positive profit.

consistent with the D1 criterion if, and only if,

$$D \leq \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1 \right) f, \tag{5}$$

while both restrictions together imply that the high quality firms profits are nonnegative as well. Thus, we have

Proposition 4 *A fully distortionary pure disclosure equilibrium exists if, and only if, (4) and (5) hold. In this equilibrium, high quality firms disclose with probability one and charge a deterministic price given by (3), while low quality firms do not disclose and randomize prices over the interval $[\underline{p}_L, \overline{p}_L]$, with $\overline{p}_L = p_H - \Delta V > c_L$. This equilibrium generates lower welfare than a pure price signaling outcome and is the unique equilibrium outcome given the above conditions.*

The main difference between this equilibrium and the pure price signaling equilibrium is that the high quality firm sets a price that is $\frac{2(D+f)}{\alpha}$ lower than in the pure price signaling equilibrium,and that it discloses (which is natural as as the disclosure cost D is relatively low compared to the fine f). It immediately follows that *all* prices are lower than in the pure price signaling equilibrium. This causes consumer surplus to be larger and to be increasing in both D and f over the whole range where this equilibrium exists.

The situation with respect to welfare or total surplus is quite the opposite: as the consumption distortion is unaffected, while high quality firms engage in wasteful disclosure, the total welfare loss equals $[2\alpha(1-\alpha)(\Delta V - \Delta c) + 2\alpha D]$, which is larger than in the pure price signaling equilibrium. Finally, profits of both types of firms are decreasing in f and D. Both high and low quality firms are worse off relative to the pure price signaling outcome and total surplus is also lower.

The general implication is that if D is small, regulation does not necessarily eliminate the consumption distortion of the pure price signaling equilibrium and, in particular, an intermediate level of regulation is worse than no regulation at all. In the next Section we will show that a similar statement holds true when D is larger.

Next, we show that if the disclosure cost is small and regulation is strong enough, the equilibrium outcome involves pure disclosure, but unlike the equilibrium discussed above, consumption distortion is fully eliminated and welfare improves relative to the pure price signaling case. In such a fully non-distortionary equilibrium, low quality firm sells only in case the competitor also sells low quality products; a Bertrand competition argument can then be used to argue that low quality firms earn zero profit and charge a deterministic price $p_L = c_L$. A high quality firm facing a low quality rival will only sell if his maximum price equals $c_L + \Delta V$. One can show that generically, high quality firms must randomize their prices to balance price competition with the high quality rival and the guaranteed market in the state where the rival is low quality[14]. It is clear that by charging the upper bound $\overline{p}_H^D = c_L + \Delta V$ the high quality firm will only sell in case the competitor is of low quality, implying that it makes a profit of $(1-\alpha)(\Delta V - \Delta c) - D$. A first condition for this equilibrium type to exist is that this is nonnegative, i.e.,

$$D \leq (1-\alpha)(\Delta V - \Delta c). \tag{6}$$

A second condition is that low quality firms should not have an incentive to imitate prices that are set in equilibrium by high quality firms. As the high quality firms are indifferent over a set of prices and as $c_H > c_L$ it follows that for the low quality firm the most profitable deviation is to deviate to the lowest price in the equilibrium support. In the proof of the next Proposition we show that the low quality firm does not find it gainful to deviate to this price if,and only if,

$$D + f \geq (1-\alpha)\Delta V + \frac{\alpha}{2}\Delta c. \tag{7}$$

These arguments establish the key parts of the following proposition. The remaining elements of the proof are in the Appendix.

Proposition 5 *A fully non-distortionary (pure disclosure) equilibrium exists, if and only if, (6) and (7) hold. In this equilibrium, high quality firms disclose and (generically) randomize prices with $\overline{p}_H^D = c_L + \Delta V$, while low quality firms set $p_L = c_L$, and only sell in the state where both firms produce low quality. This equilibrium generates higher welfare than a pure price signaling outcome, and is the unique equilibrium in the region of parameters defined by conditions (6) and (7) .*

Thus, regulation can fully eliminate all consumption distortion if it is sufficiently harsh and as long as D is sufficiently small. Strengthening regulation of false disclosure any

[14] High quality firms setting a price $c_L + \Delta V$ for sure can only be part of a non-distortionary equilibrium if consumers buy high quality in the state where both low and high quality firms are active. In that case low quality firms do not have an incentive to imitate the high quality firms if f is such that $D+f \geq \left(1-\frac{\alpha}{2}\right)\Delta V$. To prevent high quality firms to undercut, it should be the case, however, that consumers believe that prices lower than $c_L + \Delta V$ accompanied by a disclosure statement are set by a low quality firm. Using Proposition 2(b), this is consistent with the D1 criterion only if the low quality type is indifferent to imitating the high quality action which is satisfied only for a very specific value of f.

further has no effect on the market outcome. It is clear that the market outcome is fairly competitive. Under full information, high quality firms set $p_H = c_L + \Delta V$ in case there is one high and one low quality firm in the market. With asymmetric information, the high quality firm charges lower prices as with positive probability, it may be in competition with another high quality firm. Also, the high quality prices are strictly smaller and consumer surplus is strictly larger than in any of the other equilibria we have characterized so far. As there is no consumption distortion, the total welfare loss is constant at $2\alpha D$. As the parameter region is such that $D \leq (1-\alpha)(\Delta V - \Delta c)$, it is easy to check that this welfare loss is smaller than the welfare loss in the pure price signaling equilibrium.

Between the regions of relatively weak and strong regulation (and with disclosure cost itself being relatively low), there is an intermediate region of parameter values where Propositions 4 and 5 do not hold. We now show that in this region the equilibrium outcome involves pure disclosure with partial distortion. In this equilibrium, high quality types disclose for sure and set a deterministic price $c_L + \Delta V$ while low quality firms set price equal to c_L. Buyers buy from the low quality firm at price c_L with probability $\beta \in (0,1)$ when the other firm charges $c_L + \Delta V$. This equilibrium is partially distortionary; consumption distortion arises with a positive probability (less than 1).

To guarantee that high quality firms do not have an incentive to undercut, it should be the case that consumers believe that if they observe a price p in the interval $(c_H, c_L + \Delta V)$ it is set by a low quality firm. Using the D1 logic explained in the previous Section, this implies that the low quality should be indifferent between setting $p_L = c_L$ and advertising and setting $p_H = c_L + \Delta V$. Thus, as a low quality firm sells with probability $(1-\alpha)\beta + \frac{\alpha}{2}$ if it imitates the high quality price, we determine

$$\beta = \frac{D+f}{(1-\alpha)\,\Delta V} - \frac{\alpha}{2\,(1-\alpha)} \tag{8}$$

For this equilibrium to exist we should have that $0 < \beta < 1$, which translates into

$$\frac{\alpha}{2}\Delta V < D + f < \left(1 - \frac{\alpha}{2}\right)\Delta V, \tag{9}$$

and that $\pi_H^* \geq 0$, which implies that

$$D \leq (\Delta V - \Delta c)f. \tag{10}$$

These arguments establish the main parts of the following proposition.[15]

Proposition 6 *A partially distortionary pure disclosure equilibrium exists if, and only if, (9) and (10) hold. In this equilibrium* $p_L = c_L$,*and* $p_H = c_L + \Delta V$, *while consumers buy high quality with probability* β *given by (8).*[16]

It is easy to observe from (8) that β is increasing in D and f i.e., the consumption distortion becomes smaller when one of these parameters increases. Further, the total (expected) disclosure cost incurred unaffected is $2\alpha D$. Thus, the welfare loss decreases when f increases. As prices are such that consumers are indifferent between buying high and low quality, consumer surplus remains constant as f increases. As high quality firms make a profit per unit of $\Delta V - \Delta c$ and they sell with probability $(1-\alpha)\beta + \alpha/2$, their profits are increasing in f , while low quality firms always make zero profit. Thus, in this parameter region, firms favor more regulation and social welfare improves.

Having characterized three types of equilibria where high quality firms choose to disclose their information with probability one, we now characterize the full parameter region of such pure disclosure equilibria. The next Proposition shows that the equilibria characterized in Propositions 4 - 6 are the only pure disclosure equilibria.

Define $\widetilde{D}(f)$ as

$$\widetilde{D}(f) = \begin{cases} \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right) f \ , \ \text{if } 0 \leq f \leq \frac{\alpha}{2}\Delta c \\ \left(\Delta V - \Delta c\right) f / \Delta c, \ \text{if } \frac{\alpha}{2}\Delta c < f \leq (1-\frac{\alpha}{2})\Delta c \\ (1-\frac{\alpha}{2})\Delta V - f \ , \ \text{if } (1-\frac{\alpha}{2})\Delta c < f \ \leq \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c \\ (1-\alpha)\left(\Delta V - \Delta c\right), \ \text{if } f > \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c. \end{cases}$$

It is not difficult to see that $\widetilde{D}(f)$ is continuous in f. If $0 \leq f \leq \frac{\alpha}{2}\Delta c$ the value of $\widetilde{D}(f)$ follows from the characterization of the distortionary disclosure equilibrium in Proposition 4.[17] If $f > (1-\frac{\alpha}{2})\Delta c$ the value of $\widetilde{D}(f)$ is determined by conditions for the distortionary disclosure equilibrium in Proposition 5 as $\max\left\{(1-\frac{\alpha}{2})\Delta V - f, (1-\alpha)(\Delta V - \Delta c)\right\}$. This yields the last two components of $\widetilde{D}(f)$.[18] If $\frac{\alpha}{2}\Delta c < f \leq \frac{\alpha}{2}\Delta V$ the distortionary disclosure

[15] The remaining part of the proof is simple: a deviation to a price $p < p_H$ without disclosure is not gainful for either type if, and only if, the out-of-equilibrium beliefs assign sufficiently high probability to the firm being an L-type; as an L- type earns zero profit, such beliefs are always consistent with the D1 criterion.

[16] Considering Proposition 5 and Proposition 7 it is clear that there is a region where $(1-\alpha)\Delta V + \frac{\alpha}{2}\Delta c < D + f < \left(1 - \frac{\alpha}{2}\right)\Delta V$ where the two equilibria with $p_L = c_L$ overlap.

[17] For these values of f, (5) implies that (4) holds and $\widetilde{D}(f)$ is the RHS of (5).

[18] If $f < \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$ the first term is larger, whereas the second term is larger when f is larger.

equilibrium of Proposition 4 still exists, but the upper bound of D where a pure disclosure equilibrium exists is not anymore determined by the partial disclosure equilibrium of Proposition 6.

We depict the function $\widetilde{D}(f)$ in Figure 2 and state the following result.

Proposition 7 *A pure disclosure equilibrium (where high quality firms disclose with probability one) exists if, and only if,* $D \leq \widetilde{D}(f)$.

INSERT FIGURE 2 HERE

5 Mixed disclosure equilibria

In the previous Sections, we have discussed two kinds of equilibria: one pure price signaling equilibrium where firms never disclose, and a class of equilibria where the high quality types discloses for sure. As illustrated in Figure 2, the regions of the parameter space where these equilibria exist are mutually exclusive; the pure price signaling equilibrium exists if, and only if, $D \geq D^*(f)$, whereas the pure disclosure equilibria exist if, and only if, $D \leq \widetilde{D}(f)$, with $\widetilde{D}(f) < D^*(f)$ for any f. As f approaches 0, both $\widetilde{D}(f)$ and $D^*(f)$ converge to 0. In this section, we characterize the equilibria for intermediate values of disclosure cost where $\widetilde{D}(f) < D < D^*(f)$ and show that for these parameter values mixed disclosure equilibria exist where high quality firms are indifferent and randomize between disclosing and not disclosing their private information. Importantly, we show that in this region of the parameter space, welfare is always lower than under no regulation (pure price signaling) regardless of the fine.

We start the analysis when f is relatively small and in the range $0 < f < \frac{\alpha\Delta c}{2}$. For such values of f, results in the previous two sections indicate that the pure price signaling equilibrium and the pure disclosure equilibrium look very similar to each other: the high quality firm sets a deterministic price and the low quality firm randomizes over prices guaranteeing to sell to all consumers if the rival is of high quality. We now show that if $\widetilde{D}(f) < D < D^*(f)$ there exists an equilibrium that naturally transits between these two equilibria, namely a mixed disclosure equilibrium where the high quality firm sets a deterministic price that is independent of whether or not he discloses, *i.e.*, $p_H^D = p_H^{ND} = p_H$, where if both firms set p_H and only one firm discloses, consumers buy from the disclosing firm with probability $\widetilde{\beta}$. Thus, high quality firms randomize their disclosure decision, relying on both signaling mechanisms we have encountered before. As the equilibrium is separating and consumers anyway infer that the firm is of high quality even if it does not disclose, they are indifferent between firms that charge p_H and hence may randomize their purchasing decision. As disclosure comes at an additional cost, it is clear that by disclosing a high quality firm sells more than a non-disclosing high quality firm, *i.e.*, $\widetilde{\beta} > .5$, to make the high quality type indifferent. The other features are the same as in the pure price signaling equilibrium and the pure disclosure equilibrium of Proposition 4: a low quality type randomizes over the interval $[\underline{p}_L, \overline{p}_L]$, with $\overline{p}_L = p_H - \Delta V$.

In the proof of the next Proposition we show that all of the conditions for equilibrium can be fulfilled and that they imply that

$$\widetilde{\beta} = \frac{1}{2} + \frac{f}{\alpha \Delta c}, \quad \gamma_H = \alpha \frac{\Delta V}{D+f} - \frac{\alpha \Delta c}{2f}$$

and

$$p_H = c_H + \Delta c \frac{D}{f}.$$

It follows that indeed $0.5 < \widetilde{\beta} < 1$ for $0 < f < \frac{\alpha}{2}\Delta c$ and that high quality firms make positive profit.[19]

Proposition 8 *A fully distortionary mixed disclosure equilibrium exists if, and only if,* $\widetilde{D}(f) < D < D^*(f)$ *and* $0 < f \leq \frac{\alpha \Delta c}{2}$. *In this equilibrium high quality firms choose to directly disclose with probability* $0 < \gamma_H < 1$ *and chooses a price* $p_H = [c_H + (D\Delta c)/f]$ *independent of whether or not it discloses, while low quality firms randomize over the interval* $[\underline{p}_L, \overline{p}_L]$, *with* $\overline{p}_L = p_H - \Delta V$. *This equilibrium generates lower welfare than a pure price signaling outcome and is the unique equilibrium outcome (given the conditions). The expected profit earned by both low and high quality firms are lower than in the pure price signaling outcome and decreasing in* f.

In terms of welfare properties, this equilibrium shares many features with the fully distortionary pure disclosure equilibrium in Proposition 4. The consumption allocation is characterized by the same level of distortion as in the pure price signaling equilibrium while high quality firms engage in wasteful disclosure with strictly positive probability (that

[19] Also, the gradual transition to the pure disclosure and the pure price signaling equilibria becomes transparent from these equations. Substituting $D = D^*(f) = (2\lambda - 1)f$, one gets $\gamma_H = 0$ and $p_H = c_L + 2\Delta V$, which characterizes the pure price signaling equilibrium. Substituting $D = \widetilde{D}(f) = \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right) f$, one gets $\gamma_H = 1$ and $p_H = c_L + 2\Delta V - \frac{2(D+f)}{\alpha}$, which characterizes the pure disclosure equilibrium.

approaches 1 as f increases to $\frac{\alpha\Delta c}{2}$). The total welfare loss given by

$$[2\alpha(1-\alpha)(\Delta V-\Delta c)+2\alpha\gamma_H D]$$

is higher than in the pure price signaling equilibrium. However, as in the pure disclosure equilibrium in Proposition 4, the price charged by the high quality firm is decreasing in f implying that *all* prices are decreasing in f (and lower than in the pure price signaling equilibrium). Consumer surplus is therefore increasing in f. Finally, profits of both types of firms are decreasing in f. Thus, in terms of profits and total surplus, this mixed disclosure equilibrium is in between the pure price signaling equilibrium and the pure disclosure equilibrium in Proposition 4.

It remains to be seen how the market performs if $\widetilde{D}(f) < D < D^*(f)$ and regulation is sufficiently strong so that $f > \frac{\alpha\Delta c}{2}$. It turns out that in this region too, the mixed disclosure equilibria transit nicely between the pure price signaling equilibria and the pure disclosure equilibria. When it discloses the high quality firm takes over the whole market and randomizes its prices with $\overline{p}_H^D = \underline{p}_L + \Delta V$ (eliminating all consumption distortion as in the pure disclosure equilibrium of Proposition 5), while it leaves the market to the low quality firm if it does not disclose and sets $p_H^{ND} = \overline{p}_L + \Delta V = c_L + 2\Delta V$ (generating full distortion in that state as in the pure price signaling equilibrium). The randomization probability γ_H ranges between 1 and 0 when D increases from the upper bound $(1-\alpha)(\Delta V - \Delta c)$ of the pure disclosure equilibria to $\overline{D}$ (the boundary of the pure price signaling equilibrium) and $\overline{p}_L$ moves accordingly from c_L to $c_L + \Delta V$.

Proposition 9 *A partially distortionary mixed disclosure equilibrium exists if, and only if,*

$$D^*(f) > D > \widetilde{D}(f) \text{ and } f \in (\frac{\alpha}{2}\Delta c, (1-\frac{\alpha}{2})\Delta c] \tag{11}$$

or

$$D^*(f) > D > (1-\alpha)(\Delta V - \Delta c) \text{ and } f > (1-\frac{\alpha}{2})\Delta c. \tag{12}$$

All these equilibria generate lower welfare than the pure price signaling equilibrium. The expected profit earned by both low and high quality firms are lower than in the pure price signaling outcome.

To understand the equilibrium structure and welfare result in the above proposition, note that Proposition 2(c) establishes that if high quality does not disclose with positive probability, then a low quality rival captures the market in the state where it does not disclose. By leaving the market to low quality firms when it does not disclose, a high quality firm softens price competition, creating enough rent to cover the intermediate level of disclosure cost. As there is a positive probability that high quality does not disclose and when it does so, cedes the market to the low quality competitor, the consumption distortion is never fully eliminated. Denoting by q^e_L, respectively $q^e_{L/D}$, the expected quantity sold by a low quality firm if the competitor is high quality, respectively a high quality disclosing firm, the expression for welfare loss in this mixed disclosure equilibrium can be written as

$$\begin{aligned} WL &= 2\alpha[\gamma_H D + (1-\alpha)q^e_L(\Delta V - \Delta c)] \\ &= 2\alpha[D\gamma_H + (1-\alpha)(\Delta V - \Delta c)\{(1-\gamma_H) + \gamma_H q^e_{L/D}\}] \\ &= -2\alpha\gamma_H[(1-\alpha)(\Delta V - \Delta c)(1 - q^e_{L/D}) - D] + 2\alpha(1-\alpha)(\Delta V - \Delta c) \end{aligned}$$

so that the welfare loss is strictly larger than that in the pure price signaling equilibrium if, and only if,

$$2\alpha\gamma_H[(1-\alpha)(\Delta V - \Delta c)(1 - q^e_{L/D}) - D] < 0.$$

As for any $D < D^*(f)$ we have $\gamma_H > 0$ and $D > (1-\alpha)(\Delta V - \Delta c)$, it follows that this inequality holds. In other words, for intermediate levels of disclosure cost, no matter how large the fine (or, how strong the regulation), the equilibrium outcome is welfare dominated by the pure price signaling equilibrium.

Using Proposition 9 and the fact that $\widetilde{D}(f) \geq (1-\alpha)(\Delta V - \Delta c)$ if $f > (1-\frac{\alpha}{2})\Delta c$, it follows that a mixed disclosure equilibrium exists if $f > \frac{\alpha}{2}\Delta c$ and $D^*(f) > D > \widetilde{D}(f)$.Combining this with Propositions 3 and 7, we have

Corollary 10 *An equilibrium always exists.*

In our model, the signaling space consists of a combination of a continuous price signal and a binary signal of whether or not to send a direct disclosure message. Existence of a D1 equilibrium is non-trivial and, as we have shown, the overall proof is based on constructing equilibria for various subsets of the parameter space. A symmetric D1 equilibrium is not always uniquely determined, however. The region of partially distortionary mixed disclosure equilibria and full disclosure equilibria overlaps for D and f values such that $(1-\alpha)(\Delta V - \Delta c) < D < \widetilde{D}(f)$ and $(1-\frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$. However, there is no other overlap in the parameter regions for the different kinds of equilibria. This is important for the analysis we perform in the next Section about the regulatory implications of our model.

6 Regulatory Implications

Having characterized the equilibrium structure for the entire parameter space, we are now in a position to address the implications for optimal regulatory policy. In doing so, we also comment on the extent to which firms' incentives are aligned with total surplus or welfare. In particular, we focus on the comparative statics of changes in f on welfare and profits for different ranges of the disclosure cost $D < \overline{D}$.[20]

In the absence of regulation ($f = 0$) only the pure price signaling equilibrium exists. As there is no cost of disclosure, any disclosure by high quality firms can be imitated by low quality firms, and is therefore not believed by consumers. Initially, as regulation increases f, the equilibrium outcome remains unchanged (a pure price signaling equilibrium). For somewhat larger values of f (in particular, when $f > f^*(D)$ as defined in Section 3) we transit to equilibria where disclosure occurs with strictly positive probability.

In a pure price signaling equilibrium, by ceding the entire market to the rival if he is of low quality, a high quality firm generates an expected welfare loss (conditional on his own type and relative to the first best) equal to $(1 - \alpha)(\Delta V - \Delta c)$. If regulation induces a high quality firm to disclose and correct some of this distortion, the maximum possible expected welfare gain (conditional on his own type) is given by $(1 - \alpha)(\Delta V - \Delta c) - D$. So if the disclosure cost D is somewhat large and in particular, $D > (1 - \alpha)(\Delta V - \Delta c)$ then disclosure never improves net welfare and the equilibrium outcome is at least weakly welfare dominated by the pure price signaling outcome that occurs under no regulation; further, if regulation is moderately strong ($f > f^*(D)$) and high quality firms disclose with strictly positive probability, whie the consumption distortion is not eliminated if it does not discloses, then welfare is strictly lower than under no regulation. Thus, we have:

Proposition 11 *If $D > (1-\alpha)(\Delta V - \Delta c)$, total surplus is maximized at $f = 0$. If, further, $D < \overline{D}$ then any level of regulation such that $f > f^*(D)$ generates strictly lower welfare than no regulation.*

[20] If $D > D_2$ the pure price signaling equilibrium is the only equilibrium that exists, independent of the policy parameter f. For high disclosure costs, firms will rely on price signaling only and therefore regulation does not have any effect.

Next, consider the situation where the disclosure cost is small and in particular, $D < (\Delta V - \Delta c)\min\{\frac{\alpha}{2}, 1-\alpha\}$.[21] Using the argument outlined above, as the disclosure cost $D < (1-\alpha)(\Delta V - \Delta c)$, one can see that if regulation induces high quality firms to disclose for sure and sell to the entire market when facing a low quality rival (thereby fully correcting the consumption distortion associated with pure price signaling), then it strictly improves welfare over no regulation. As regulation increases f one transits from the pure price signaling equilibrium to the fully distortionary mixed disclosure equilibrium described in Proposition 8, and then to the fully distortionary pure disclosure equilibrium of Proposition 4. Throughout this process and until f equals $\frac{\alpha}{2}\Delta V - D$, there is no reduction in the consumption distortion though at that level of regulation, high quality firms disclose for sure; the welfare generated is strictly lower than under no regulation and in fact, the welfare loss (compared to the first best) is at its highest at this level of f and equals $2\alpha[(1-\alpha)(\Delta V - \Delta c) + D]$. When f increases beyond $\frac{\alpha}{2}\Delta V - D$, welfare starts increasing; we first enter the region of the partially distortionary pure disclosure equilibrium described in Proposition 7 and the consumption distortion gradually disappears when f increases further as consumers shift to buying high quality with higher probability when both low and high quality firms exist in the market; the total welfare loss reaches $2\alpha D$ when $f = (1-\frac{\alpha}{2})\Delta V - D$ at which point buyers buy only high quality as long as it is supplied in the market; at this "strong" level of regulation, welfare is higher than under no regulation. When f increases beyond $(1-\frac{\alpha}{2})\Delta V - D$, equilibrium is fully non-distortionary and net surplus or welfare does not change anymore. Thus, we have:

Proposition 12 *If $D < (\Delta V - \Delta c)\min\{\frac{\alpha}{2}, 1-\alpha\}$, it is optimal to impose a fine $f \geq (1-\frac{\alpha}{2})\Delta V - D$. Weak regulation, i.e., $f^*(D) < f < \frac{\alpha}{2}\Delta V - D$, is strictly worse than no regulation.*

It is interesting to ask whether firms have an incentive to lobby in favor of or against regulation of false disclosure. Indeed, at first glance, one might think that a penalty on lying about product quality (by low quality firms) should hurt low quality firms and possibly benefit high quality firms. What we find is that for a large segment of the parameter space, regulation affects the profits of high and low quality firms in the same direction and further, the interests of both types of firms appear to be broadly aligned with welfare.

[21] When $\frac{\alpha}{2} < 1-\alpha$, then the comparative statics for $D \in [\frac{\alpha}{2}(\Delta V - \Delta c), (1-\alpha)(\Delta V - \Delta c)]$ is a somewhat complicated combination of the comparative statics of the two regions of disclosure cost analyzed here (there may be several transitions between the pure disclosure and mixed disclosure equilibria).

To see this, first focus on the fully distortionary mixed disclosure equilibrium described in Proposition 8 that arises under relatively weak regulation when f is increased beyond $f^*(D)$. Once we are in the region of this equilibrium, both high and low quality profits are decreasing in f and always lower than in the case of no regulation. So, if there is a proposal to create or marginally strengthen such an intermediate level regulation, both high and low quality firms lobby against it. The fact that even high quality firms may be opposed to penalizing low quality firms for lying is based on the desire to protect the market power created by use of price signaling and the pro-competitive effects of making disclosure more credible. Further, as noted above, the total surplus generated with such regulation is always smaller than under no regulation so that an industry lobby would be aligned with social welfare.

If disclosure cost is large and, in particular $D > \frac{\alpha}{2}(\Delta V - \Delta c)$ then as regulation is further strengthened, we transit from the fully distortionary mixed disclosure equilibrium of Proposition 8 to a partially distortionary mixed disclosure equilibrium outlined in Proposition 9 where both types of firms make less profit than in the pure price signaling equilibrium. Thus, it is in the interest of both types of firms to oppose regulation. If D also exceeds $(1-\alpha)(\Delta V - \Delta c)$, then (from Proposition 11) this is in line with social welfare as it is socially optimal to have no regulation.

If disclosure cost is small and in particular, $D < \min\{\frac{\alpha}{2}, 1-\alpha\}(\Delta V - \Delta c)\}$, then as regulation is strengthened we transit from the fully distortionary mixed disclosure equilibrium to the fully distortionary pure disclosure equilibrium of Proposition 4 and profits of both types decline with f until the low quality firm earns zero profit at $f = \frac{\alpha}{2}\Delta V - D$. Once again both types lobby against this "intermediate" level of regulation which, as noted above, minimizes welfare. Further strengthening of regulation by increasing f implies that we are in the region of the partially distortionary equilibrium of Proposition 6 with continuing zero profit for low quality firms but higher profits for high quality firms until $f = (1 - \frac{\alpha}{2})\Delta V - D$ where welfare is maximized and becomes constant; this is also the level of f at which high quality profit is maximized and equals $(1-\frac{\alpha}{2})(\Delta V - \Delta c) - D$ which (depending on the parameters) may be larger than the high quality profit in the pure price signaling equilibrium. If it is larger[22], high quality firms may lobby for and low quality firms will lobby against a socially optimal level of "strong" regulation; if not, both types of firms lobby against strong regulation even though that is socially optimal. Note that

[22]This occurs if $D < (1-\alpha)(\Delta V - \Delta c) - \frac{\alpha}{2}\Delta V$.

stronger regulation where f is increased beyond $(1 - \frac{\alpha}{2})\Delta V - D$ is also socially optimal, but in that region high quality profits decline with f because of competition between high quality firms so that even if high quality firms lobby for strong regulation they do not favor further strengthening of regulation.

Thus, we conclude that the interest of firms may be aligned with the social objectives of the regulator and firms' lobbying activities may be aligned with the objective of maximizing total surplus. This is certainly true for higher disclosure cost, where firms' profits and total surplus reach their global maximum at $f = 0$. For smaller disclosure cost, low quality profits are decreasing in f, whereas high quality profits are non-monotonic in roughly the same way as total surplus.

7 Discussion and Conclusion

This paper explores the economic implications of policies aimed at making it more costly for firms to misrepresent their product quality to buyers. In markets where firms can communicate their private information about product quality indirectly through price (or other forms of) signaling, direct communication or disclosure is only one of the channels of communication. While regulation that increases the cost of lying can improve the credibility and increase the use of disclosure as a means of communication, it can also alter the conduct of firms in other dimensions such as pricing that firms may also use to signal information. This can significantly alter the strategic interaction of competing firms and make the market more competitive as well as reduce some of the consumption distortions associated with signaling. However, improving the credibility of disclosure through regulation can lead to excessive disclosure in a competitive setting. Whether or not such regulation is socially desirable critically depends on the disclosure cost itself. If the disclosure cost is small, imposing a sufficiently high fine for false disclosure is welfare optimal. The welfare effects are, however, non-monotonic and an intermediate fine is worse than no regulation. If the disclosure cost is larger, no regulation is socially optimal as regulation always leads to (socially) excessive disclosure because of strategic competition between firms.

Our results are not driven by the fact that we focus on the D1 equilibrium which, in fact, selects the most competitive signaling equilibrium. The qualitative conclusions outlined above would also hold if we used a weaker restriction on beliefs such as the Intuitive Criterion (IC). That would, however, require us to admit larger sets of equilibrium outcomes thereby complicating the comparative statics of regulation. It is easy to see that if we replace the D1 criterion by IC, pure price signaling cannot be an equilibrium outcome if the fine is large enough as a high quality firm would want to disclose and undercut; also, equilibria with disclosure do not exist if the disclosure cost is high enough. Interestingly, pooling equilibria (that are ruled out with the D1 criterion) may exist under IC; however, when the fine f is large enough there is no pooling equilibrium with disclosure (no deception). These are also among the main findings of our analysis using D1. The precise parameter values for which different types of equilibria exist under IC differ from that in our analysis; further, we are more liklely to have coexistence of different types of equilibria in certain regions.

We have chosen a very simple framework that allows us to capture strategic competition, multidimensional signaling and the welfare analysis of regulation in a tractable fashion. In our model, consumers are identical and have unit demand. If we would allow consumers to differ in their relative valuations for high quality, firms would face downward sloping market demand. In that case, in the pure price signaling equilibrium some consumers would buy high quality even if low quality is provided. This would reduce, but not eliminate, the consumption distortion associated with price signaling (see, Janssen and Roy (2010)). It is, however, unlikely to add new insights about the qualitative effects of information regulation. The assumption of unit demand is certainly restrictive but very useful in obtaining an explicit characterization of the signaling equilibria; if individual demand is sufficiently inelastic the qualitative features of our conclusions should continue to hold.

We have assumed that consumers are sophisticated enough to understand how signaling works. When all buyers are not sophisticated, regulation of false disclosure may be a means to protect naive consumers. Our model can be extended to allow for some naive buyers. However, there are different ways of modeling naivety within our framework. For example, naivety could mean the inability to engage in Bayesian updating after observing signals. It could also mean that consumers are gullible, *i.e.*, take disclosure at face value. It may also imply that consumers are skeptical and think that firms always lie (independent of the cost of false disclosure and the direct disclosure cost). Depending on exactly how one wants to model naivety, our conclusions are robust to allowing some fraction of consumers to be naive.

We focus on situations where disclosure takes the form of direct communication or advertising so that it is more natural to model firms' disclosure as a short-run decision simultaneously with pricing. If one wishes to capture third party certification or ratings that require prior testing one could model disclosure and pricing as occurring sequentially (and thereby view disclosure as a long-run decision). Our way of modeling disclosure and pricing decisions allows us to focus on disclosure as communication to a single audience of uninformed consumers (rather than as a means to influence rival firms and the intensity of price competition); it allows us to analyze the interaction in terms of a signaling game with multiple signals and multiple senders. Some of our conclusions remain valid, however, if disclosure precedes price setting. For example, it will remain true that there exists an intermediate region of parameter values where high quality firms will want to randomize their disclosure decision to keep the competitor unsure about quality thus weakening price competition. Also, for the limiting case where the cost of false disclosure is prohibitively large and firms can only truthfully disclose their private information, a pure disclosure equilibrium exists if the disclosure cost is small enough (see, Janssen and Roy 2015).

Finally, competition plays an important role in our main results. The monopoly version of our model is easier to analyze and results in the conclusion that a high quality firm may want to disclose if, and only if, the fine for false disclosure is above a critical threshold value. Moreover, the incentives of the high quality firm to disclose are aligned with that of a social planner. It follows that it is always socially optimal to have strong regulation i.e., a high fine for false disclosure in a monopolistic market. Our paper shows that deriving policy conclusions by ignoring the competitive interaction of firms and the possibility of excessive disclosure can be quite misleading.

APPENDIX

In this appendix, let λ be defined by

$$\lambda = \frac{\Delta V}{\Delta c}$$

Proof of Proposition 2: (a) Suppose an $L-$type firm discloses (falsely) with strictly positive probability in a fully revealing equilibrium. Let for any price p charged in equilibrium by this firm when it discloses, $(p - c_L)q \geq D$ where q is the expected quantity sold at price p. Then, $p > c_L$. If the firm does not disclose and charges $p - \epsilon$ for $\epsilon > 0$ small enough, the expected quality of buyers cannot be any worse and the firm sells an expected quantity of at least q. As $D > 0$, this deviation is strictly gainful for ϵ small enough.

(b) Suppose that a high quality type's equilibrium strategy places a strictly positive probability mass on price $\widehat{p} > c_H$ and a message $\widehat{m} \in \{0, 1\}$. Incentive compatibility requires that an L-type's equilibrium profit $\pi_L^* \geq \pi_L^d$, where $\pi_L^d = (\widehat{p} - c_L)q_H^* - (D + f)I_{\{\widehat{m}=1\}}$is L-type's profit from deviation to $(\widehat{p}, \widehat{m})$, q_H^* is the expected quantity sold by the H type at $(\widehat{p}, \widehat{m})$ and I is an indicator function. Note that π_H^*, the equilibrium profit of H type, is given by $\pi_H^* = (\widehat{p} - c_H)q_H^* - DI_{\{\widehat{m}=1\}}$so that

$$\frac{(\widehat{p} - c_L)}{(\widehat{p} - c_H)} = \frac{\pi_L^d + (D + f)I_{\{\widehat{m}=1\}}}{\pi_H^* + DI_{\{\widehat{m}=1\}}}. \tag{13}$$

Using symmetry of equilibrium, prices slightly below $\widehat{p}$ cannot be in the support of $H-$type's equilibrium strategy in the event that it chooses message $\widehat{m}$ as such a price would undercut rival H type when the latter chooses $(\widehat{p}, \widehat{m})$ and therefore yield higher payoff. Further, to deter the $H-$type from deviating to an out-of-equilibrium price $\widehat{p} - \epsilon$ (for $\epsilon > 0$ small enough) while sending message $\widehat{m}$, beliefs must assign sufficiently high probability that the deviating firm is of $L-$type. We now claim that if

$$\pi_L^* > \pi_L^d, \tag{14}$$

then the D1 criterion implies that after observing a deviation to a price $\widehat{p} - \epsilon$ (for $\epsilon > 0$ small enough) with message $\widehat{m}$, buyers must believe that the deviating firm is an H type with probability 1, a contradiction. To establish this claim, let $q_H(\widehat{p} - \epsilon)$, $q_L(\widehat{p} - \epsilon)$ be the expected quantity that an H and an L type firm must sell respectively in order to be indifferent between this deviation and their equilibrium strategies:

$$\begin{aligned}(\widehat{p}-\epsilon-c_H)q_H(\widehat{p}-\epsilon)-DI_{\{\widehat{m}=1\}} &= \pi_H^*, \\ (\widehat{p}-\epsilon-c_L)q_L(\widehat{p}-\epsilon)-(D+f)I_{\{\widehat{m}=1\}} &= \pi_L^*\end{aligned}$$

Using the D1 criterion, it is sufficient to show that $q_H(\widehat{p}-\epsilon) < q_L(\widehat{p}-\epsilon)$ for ϵ small enough. Note that $q_H(\widehat{p}-\epsilon) \geq q_L(\widehat{p}-\epsilon)$ if, and only if,

$$\frac{\widehat{p}-\epsilon-c_L}{\widehat{p}-\epsilon-c_H} \geq \frac{\pi_L^*+(D+f)I_{\{\widehat{m}=1\}}}{\pi_H^*+DI_{\{\widehat{m}=1\}}} \tag{15}$$

and as the left hand side of (15) is continuous and strictly increasing in ϵ, (15) holds for all ϵ arbitrarily close to 0, if, and only if,

$$\frac{\widehat{p}-c_L}{\widehat{p}-c_H} \geq \frac{\pi_L^*+(D+f)I_{\{\widehat{m}=1\}}}{\pi_H^*+DI_{\{\widehat{m}=1\}}} \tag{16}$$

and using (14) in (16) we obtain a contradiction to (13). Thus, $\pi_L^* = \pi_L^d$.

(c) Consider an equilibrium where $H-$types disclose with probability $\gamma_H \in [0,1)$. Let $\overline{p}_H^D, \underline{p}_H^D$ ($\overline{p}_H^{ND}, \underline{p}_H^{ND}$) be the supremum and the infimum of the essential support of prices charged by an $H-$type firm when it discloses (does not disclose). We first show that if $\gamma_H > 0$, $\overline{p}_H^D \leq \overline{p}_H^{ND}$. Suppose to the contrary that $\overline{p}_H^D > \overline{p}_H^{ND}$. As $D > 0$, in order to cover the disclosure cost, an $H-$type firm must sell strictly positive expected quantity at price $\overline{p}_H^D$ and (therefore) at price $\overline{p}_H^{ND} < \overline{p}_H^D$. As $L-$type firm can always imitate price $\overline{p}_H^{ND} \geq c_H > c_L$ without disclosing, it follows that L-type's equilibrium profit $\pi_L^* > 0$. Thus $\overline{p}_L > c_L$. Standard undercutting arguments (using symmetry of equilibrium) can be used to show that there is no probability mass point at $\overline{p}_L$ and so, at $\overline{p}_L$ the (limiting) expected quantity sold by $L-$type firm in the event where rival is of $H-$type is strictly positive. As $\overline{p}_H^D$ is the upper bound of high quality prices, $\overline{p}_L \leq \overline{p}_H^D - \Delta V$. Thus, at price $\overline{p}_H^D$ an $H-$type firm that discloses sells zero when its rival is of $L-$type. Therefore, the only way $H-$type can sell strictly positive expected quantity at $\overline{p}_H^D$ is if there is a strictly positive probability mass $\sigma > 0$ at price $\overline{p}_H^D$ and the equilibrium profit of H-type must be $\pi_H^* = (\overline{p}_H^D - c_H)\frac{\alpha\sigma}{2} - D$ and further, using part (b) of this proposition, $L-$type must be indifferent between its equilibrium strategy and deviating to disclosing and charging $\overline{p}_H^D$

i.e.,

$$\begin{aligned}\pi_L^* &= (\overline{p}_H^D - c_L)\frac{\alpha\sigma}{2} - D - f = \pi_H^* + \Delta c\frac{\alpha\sigma}{2} - f \\ &= (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c\frac{\alpha\sigma}{2} - f \qquad (17)\end{aligned}$$

where $q(\underline{p}_H^{ND})$ is the expected quantity sold by the high quality type at price $\underline{p}_H^{ND}$ when it does not disclose. Observe that as $\underline{p}_H^{ND} < \overline{p}_H^D, q(\underline{p}_H^{ND}) \geq \alpha\sigma$. Further, the incentive constraint of an $L-$type implies:

$$\pi_L^* \geq (\underline{p}_H^{ND} - c_L)q(\underline{p}_H^{ND}) = (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c q(\underline{p}_H^{ND}) \geq (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c\alpha\sigma.$$

Thus, from (17)

$$\pi_L^* = (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}) + \Delta c\frac{\alpha\sigma}{2} - f \leq \pi_L^* - \Delta c\frac{\alpha\sigma}{2} - f,$$

a contradiction. Thus, if $\gamma_H > 0$, $\overline{p}_H^D \leq \overline{p}_H^{ND}$.

Next, we show that if $\gamma_H \in [0,1)$, then $\overline{p}_H^{ND} = \underline{p}_H^{ND}$. Suppose not. Then, $\overline{p}_H^{ND} > \underline{p}_H^{ND}$; this implies that $H-$type sells strictly positive expected quantity at price $\underline{p}_H^{ND} \geq c_H > c_L$ (for instance, when rival is of $H-$type and does not disclose) and so $\pi_H^* > 0$ (note $\overline{p}_H^{ND} > \underline{p}_H^{ND} \geq c_H$). To deter imitation of $H-$type's non-disclosure action, $\pi_L^* > 0$. Then, using identical arguments as above, $\overline{p}_L \leq \overline{p}_H^{ND} - \Delta V$ and there is no probability mass point at $\overline{p}_L$ and at price $\overline{p}_H^{ND}$, the high quality type sells zero where rival is $L-$type. Let $q(p, ND)$ and $q(p, D)$ denote respectively the expected quantity sold by the high quality type at p when it does not disclose and when it discloses. As $\pi_H^* > 0$, $q(\overline{p}_H^{ND}, ND) > 0$. Let $\xi_{ND} \geq 0$ and $\xi_D \geq 0$ be the respective probability probability masses, if any, placed by the high quality type at the price $\overline{p}_H^{ND}$ in the states where it does not disclose and where it discloses ($\xi_D = 0$ if $\overline{p}_H^D < \overline{p}_H^{ND}$). Using symmetry of the equilibrium, there exists $\widehat{\beta} \in [0,1]$ such that

$$q(\overline{p}_H^{ND}, ND) = \alpha\left[\frac{1-\gamma_H}{2}\xi_{ND} + \widehat{\beta}\gamma_H\xi_D\right], q(\overline{p}_H^{ND}, D) = \alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\widehat{\beta})(1-\gamma_H)\xi_{ND}\right].$$

Then

$$\pi_H^* = (\overline{p}_H^{ND} - c_H)\alpha\left[\frac{1-\gamma_H}{2}\xi_{ND} + \beta\gamma_H\xi_D\right] = (\underline{p}_H^{NA} - c_H)q(\underline{p}_H^{NA}, NA). \qquad (18)$$

Note that as $\underline{p}_H^{ND} < \overline{p}_H^{ND}$,

$$q(\underline{p}_H^{ND}, ND) \geq \alpha((1-\gamma_H)\xi_{ND} + \gamma_H \xi_D). \tag{19}$$

Consider the case where $\gamma_H \xi_D > 0$. This implies that $\overline{p}_H^D = \overline{p}_H^{ND}$ and $\xi_D > 0$. Using part (b) of this proposition, $L-$type is then indifferent between its equilibrium strategy and deviating to disclosing (falsely) and charging $\overline{p}_H^{ND}$:

$$\begin{aligned}
\pi_L^* &= (\overline{p}_H^{ND} - c_L)\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - D - f \\
&= \pi_H^* - D + \Delta c\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - f, \text{ using (18)} \\
&= (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}, ND) + \Delta c\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - f \\
&\leq (\underline{p}_H^{ND} - c_L)q(\underline{p}_H^{ND}, ND) - \Delta c\alpha((1-\gamma_H)\xi_{ND} + \gamma_H\xi_D) \\
&\quad + \Delta c\alpha\left[\frac{1}{2}\gamma_H\xi_D + (1-\beta)(1-\gamma_H)\xi_{ND}\right] - f, \text{ using (19)} \\
&\leq \pi_L^* - \Delta c\alpha(\frac{1}{2}\gamma_H\xi_D + (1-\gamma_H)\beta\xi_{ND}) - f,
\end{aligned}$$

a contradiction (the last inequality uses the incentive constraint of the low quality type to not imitate $\underline{p}_H^{ND}$ without disclosing). Now, consider the case where $\gamma_H \xi_D = 0$. As $q(\overline{p}_H^{ND}, ND) > 0$, we have $\xi_{ND} > 0$ and $q(\overline{p}_H^{ND}, ND) = \frac{\alpha}{2}(1-\gamma_H)\xi_{ND}$ and

$$q(\underline{p}_H^{ND}, ND) > \frac{\alpha}{2}(1-\gamma_H)\xi_{ND}\,. \tag{20}$$

Using part (b) of this proposition, $L-$type must be indifferent between its equilibrium strategy and deviating to $\overline{p}_H^{ND}$ without disclosing:

$$\begin{aligned}
\pi_L^* &= (\overline{p}_H^{ND} - c_L)\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} = (\overline{p}_H^{ND} - c_H)\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} + \Delta c\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} \\
&= (\underline{p}_H^{ND} - c_H)q(\underline{p}_H^{ND}, ND) + \Delta c\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} \\
&= (\underline{p}_H^{ND} - c_L)q(\underline{p}_H^{ND}, ND) - \Delta c q(\underline{p}_H^{ND}, ND) + \Delta c\frac{\alpha(1-\gamma_H)\xi_{ND}}{2} \\
&\leq \pi_L^* - \Delta c\left[q(\underline{p}_H^{ND}, ND) - \frac{\alpha(1-\gamma_H)\xi_{ND}}{2}\right] < \pi_L^*
\end{aligned}$$

a contradiction (the last inequality follows from the incentive constraint of $L-$quality type to not imitate $\underline{p}_H^{ND}$ and (20)). Thus, if $\gamma_H \in [0,1)$, then $\overline{p}_H^{ND} = \underline{p}_H^{ND}$ and when it discloses, $H-$type charges a deterministic price $p_H^{ND} \geq \overline{p}_H^D$.

As this is a symmetric equilibrium, at price p_H^{ND} (and without disclosing), the high quality firm sells in the state where rival is of high quality and does not disclose and as $p_H^{ND} \geq c_H > c_L$, and to deter imitation by $L-$type of this action, $\pi_L^* > 0$. Using very similar arguments as above, $L-$types must randomize over prices and there cannot be a mass point at $\overline{p}_L$ and at that price L-type must sell with strictly positive probability in the state where rival is $H-$type; as p_H^{ND} is the upper bound of the support of high quality prices,

$$\overline{p}_L \leq p_H^{ND} - \Delta V. \tag{21}$$

Therefore, at price p_H^{ND}, $H-$type sells zero expected quantity in the event that rival is $L-$type. Note that $\pi_L^* > 0$ implies that $\overline{p}_L > c_L$ so that $p_H^{ND} \geq c_L + \Delta V > c_H$ so that $\pi_H^* > 0$.

(d) We now show that if an $H-$type charges a price $\widetilde{p}$ with strictly positive probability in the event that it discloses, then $\widetilde{p} = \underline{p}_H^D$. From above, discussion we know that $\underline{p}_H^{ND} \geq \widetilde{p}$. Suppose to the contrary that $\underline{p}_H^D < \widetilde{p}$. Then there exists $\widehat{p} \in [\underline{p}_H^D, \widetilde{p})$ such that

$$\pi_H^* = (\widehat{p} - c_H)q(\widehat{p}, D) - D = (\widetilde{p} - c_H)q(\widetilde{p}, D) - D. \tag{22}$$

As $\widehat{p} < \widetilde{p}$, $q(\widehat{p}, D) > q(\widetilde{p}, D)$. As there is a strictly positive probability mass at $\widetilde{p}$, out-of-equilibrium beliefs must deter H-type from undercutting $\widetilde{p}$ by assigning sufficient probability to the deviant being of L-type and using part (b) of this proposition

$$\begin{aligned}
\pi_L^* &= (\widetilde{p} - c_L)q(\widetilde{p}, D) - D - f \\
&= (\widehat{p} - c_L)q(\widehat{p}, D) - D + \Delta c(q(\widetilde{p}, D) - q(\widehat{p}, D)) - f, \text{using (22)} \\
&< (\widehat{p} - c_L)q(\widehat{p}, D) - D \leq \pi_L^*
\end{aligned}$$

a contradiction (the last inequality follows from the incentive constraint of an $L-$type).

(e) This follows readily from standard undercutting arguments (using the symmetry of the equilibrium and the fact that in a revealing equilibrium, adverse beliefs cannot deter an L-type firm from undercutting its rival).

Proof of Proposition 3.

We focus on a candidate pure price signaling equilibrium as described in the text preceding the proposition. To establish the existence of such an equilibrium we fix out-of-equilibrium beliefs of buyers to be such that any firm that does *not* disclose and deviates to a price in $(\overline{p}_L, p_H)$ is perceived as being a low quality type for sure. As this has been shown to be a D1 perfect Bayesian equilibrium in the special case where firms are not allowed to disclose (Janssen and Roy, 2010), to show that this is an equilibrium in our model the main step is to check that neither type has any incentive to deviate by *disclosing* and charging a price $\widehat{p} \in (\overline{p}_L, p_H^{ND})$. Let $q_H(\widehat{p})$,$q_L(\widehat{p})$ be the expected quantity that an H and an L type firm must sell respectively in order to be indifferent between this deviation and not deviating from their equilibrium strategies:

$$q_H(\widehat{p}) = \frac{\pi_H^* + A}{(\widehat{p} - c_H)}, q_L(\widehat{p}) = \frac{\pi_L^* + D + m}{(\widehat{p} - c_L)}$$

Note that $q_H(\widehat{p}) \geq q_L(\widehat{p})$ if, and only if,

$$\frac{\widehat{p} - c_L}{\widehat{p} - c_H} \geq \frac{\pi_L^* + D + f}{\pi_H^* + A} \tag{23}$$

and as the left hand side of (23) is continuous and strictly decreasing in $\widehat{p}$, (23) hold for all $\widehat{p} \in (\overline{p}_L, p_H^{ND})$ if, and only if, (23) holds for $\widehat{p} = p_H^{ND}$ which (using the fact that $\pi_H^* = \frac{\alpha}{2}(p_H^{ND} - c_H)$ and $\pi_L^* = \alpha \Delta V = \frac{\alpha}{2}(p_H^{ND} - c_L))$ reduces to,

$$f \leq \frac{D}{2\lambda - 1} \tag{24}$$

Under (24), $q_H(\widehat{p}) \geq q_L(\widehat{p})$ so that the D1 refinement is consistent with beliefs associating disclosure and price $\widehat{p} \in (\overline{p}_L, p_H^{ND})$ as coming from an $L-$type firm with probability one so that the candidate pure price signaling equilibrium is a D1 equilibrium in our model. Next, suppose that (24) does not hold. Then, one can check that exists a unique $p_0 \in (c_H, p_H)$ such that (23) holds with equality a $\widehat{p} = p_0$ and, in particular

$$p_0 = \frac{(\alpha \Delta V + A + f)\, \Delta c}{f + \frac{\alpha}{2} \Delta c} + c_L \tag{25}$$

and D1 refinement requires that a firm deviating to disclosure and price $\widehat{p} \in (p_0,\ p_H^{ND}\ )$ should be regarded as H-type with probability one. It is easy to see that if $p_0 \geq \underline{p}_L + \Delta V$, a deviating firm will not set price below $\underline{p}_L + \Delta V$ because the firm sells to all buyers with probability one at price $\underline{p}_L + \Delta V$. Without loss of generality, we confine attention to deviations to a price $\widehat{p}$ satisfying

$$\widehat{p} \geq r = \max\{p_0, \underline{p}_L + \Delta V\}. \tag{26}$$

Deviation to any price $\widehat{p}$ satisfying (26) is strictly gainful for a type H firm if, and only if,

$$\begin{aligned} \pi_H^* &< (\widehat{p} - c_H)[\alpha + (1-\alpha)(1 - F(\widehat{p} - \Delta V))] - D \\ &= \left[\frac{\widehat{p} - c_H}{\widehat{p} - \Delta V - c_L}\right] \alpha \Delta V - D \end{aligned} \tag{27}$$

and (as the right hand side of (27) is strictly decreasing in $\widehat{p}$) this holds if, and only if,

$$\pi_H^* < \left[\frac{r - c_H}{r - \Delta V - c_L}\right] \alpha \Delta V - D. \tag{28}$$

Similarly, deviation to any price $\widehat{p}$ satisfying (26) is strictly gainful for an L type firm if, and only if,

$$\pi_L^* < \alpha \Delta V \left[\frac{r - c_L}{r - \Delta V - c_L}\right] - D - f. \tag{29}$$

Using (25), one can show that: $p_0 \geq \underline{p}_L + \Delta V$ if, and only if,

$$D \geq f\left[(1+\alpha)\lambda - 1\right] - \left(\frac{\alpha(1-\alpha)}{2}\right) \Delta V. \tag{30}$$

First, suppose that (30) does not hold so that $r = \underline{p}_L + \Delta V$. In this case, (28) reduces to

$$D < \Delta V - (1 - \frac{\alpha}{2}) \Delta c = \overline{D} \tag{31}$$

and (29) reduces to

$$D < \Delta V - f. \tag{32}$$

It is easy to check that if (30) does not hold, (32) implies (31) so that a pure price signaling equilibrium exists if, and only if, (31) does not hold i.e.,

$$D \geq \overline{D}. \tag{33}$$

Next, suppose that (30) holds so that $r = p_0$. In this case, one can check that both (28) and (29) reduce to

$$D < \frac{\alpha}{2}\Delta V + f[\lambda - 1]. \tag{34}$$

Thus, if (30) holds then a pure price signaling equilibrium exists if, and only if, (34) does not hold i.e.,

$$f \leq \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}. \tag{35}$$

We are now ready to establish the proposition as stated. First, we argue that if (33) holds, then a pure price signaling equilibrium always exists. If (24) does not hold, then we know that a pure price signaling equilibrium exists. If (24) and (30) do not hold, then we have shown (33) implies that a pure price signaling equilibrium exists. If (24) holds and (30) does not hold, then one can show that (33) implies (35) so that once again a pure price signaling equilibrium exists. Next, suppose that $D \in [\underline{D}, \overline{D})$. As $f^*(.)$ is increasing, $f \leq f^*(D)$ implies $f \leq \lim_{D \uparrow \overline{D}} f^*(D) = (1 - \frac{\alpha}{2})\Delta c$ and using this,

$$f\left[(1+\alpha)\lambda - 1\right] - \frac{\alpha(1-\alpha)}{2}\Delta V \leq \frac{\alpha}{2}\Delta V + f[\lambda - 1]. \tag{36}$$

Further, for $D \in [\underline{D}, \overline{D}), f^*(D) = \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}$ so that

$$f \leq f^*(D) \Leftrightarrow D \geq f(\lambda - 1) + \frac{\alpha}{2}\Delta V. \tag{37}$$

Combining (36) and (37), we see that (30) and (35) hold so that a pure price signaling equilibrium exists (whether or not (24) holds). On the other hand, if $f > f^*(D)$, then $f > f^*(D) \geq f^*(\underline{D}) = \frac{\alpha}{2}\Delta c$ and further from (37), $D < f(\lambda - 1) + \frac{\alpha}{2}\Delta V$ which together implies that (24) does not hold. As $D < \overline{D}$ and further, $f > f^*(D) = \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}$ implies (35) does not hold, a pure price signaling equilibrium does not exist when (24) does not hold. Finally, consider $D \in (0, \underline{D})$. If $f \leq f^*(D) = \frac{D}{2\lambda - 1}$, then (24) holds so that a pure signaling equilibrium exists. Now, consider $f > f^*(D)$. Then, (24) does not hold. As $D < \underline{D} < \overline{D}$, a pure price signaling equilibrium does not exist if (30) does not hold. On the other hand, using $D < \underline{D} = \alpha(\lambda - \frac{1}{2})\Delta c$ one can show that

$$f^*(D) = \frac{D}{2\lambda - 1} > \frac{D - \frac{\alpha}{2}\Delta V}{\lambda - 1}$$

so that $f > f^*(D)$ implies (35) does not hold. Thus, a symmetric pure price signaling equilibrium does not exist for $f > f^*(D)$.

Proof of Proposition 4.

Given the arguments outlined in the main text preceding the proposition, it remains to rule out any incentive to deviate to out-of-equilibrium price $\widehat{p} \in (\overline{p}_L^D, p_H^D)$ without disclosing and this requires that out-of-equilibrium beliefs assign sufficiently high probability to the deviating firm being of L-type Using an argument similar to that at the beginning of the proof of Proposition 3, such a belief is consistent with the D1 criterion if, and only if,

$$\frac{p_H^D - c_L}{p_H^D - c_H} \geq \frac{\pi_L^*}{\pi_H^*} = \frac{\alpha(p_H^D - \Delta V - c_L)}{\frac{\alpha}{2}(p_H^D - c_H) - D}.$$

which reduces to

$$f\left(2\Delta V - \Delta c - \frac{2(D+f)}{\alpha}\right) \geq D\Delta c$$

i.e., condition (5).

Proof of Proposition 5.

To deter an $H-$type from deviating to not disclosing out-of-equilibrium beliefs must regard any such deviating firm as low quality with sufficiently high probability, and this can be shown to be consistent with the D1 criterion as $L-$types earn zero profit in this equilibrium. Following the arguments outlined in the main text preceding the proposition, it remains to show that an $L-$type firm has no strict incentive to disclose (falsely) and charge the lower bound of the support of an H-type's equilibrium prices. First, consider the equilibrium where the H-type's price distribution has no mass point; in this case, the distribution function F of $H-$type's price satisfies:

$$(p_H - c_H)[(1-\alpha) + \alpha(1 - F(p_H)] = (\Delta V - \Delta c)(1-\alpha), p_H \in [\underline{p}_H^D, \overline{p}_H^D] \tag{38}$$

where $\underline{p}_H^D = (\Delta V - \Delta c)(1-\alpha) + c_H$. If an $L-$type firm discloses (falsely) and charges $\underline{p}_H^D$ (the optimal deviation price in the interval $[\underline{p}_H^D, \overline{p}_H^D]$), then his deviation profit equals

$$\underline{p}_H^D - c_L - (D+f) = \Delta V(1-\alpha) + \alpha\Delta c - (D+f)$$

so that the deviation is not gainful if, and only if,

$$D + f \geq \Delta V(1-\alpha) + \alpha\Delta c. \tag{39}$$

Under (39), an $L-$type firm will also not gain by disclosing and deviating to a price less than $\underline{p}_H^D$ even if it is thought of as a high quality firm with probability one. It is easy to check that no other deviation is gainful (with sufficiently pessimistic belief).

Next, consider the equilibrium where H-type's price distribution has a mass point. From Proposition 2, the mass point can only be at the at the lower bound of its price distribution and this lower bound price must be an isolated point in the support. In particular, $H-$types randomize over prices in the interval $[\underline{p}_H^D, c_L+\Delta V)$ with some probability κ, and with probability $1-\kappa$, they charge a deterministic price equal to $\widehat{p}_H^D < \underline{p}_H^D$. As the H-type must be indifferent between prices in its equilibrium strategy, it follows that

$$\underline{p}_H^D = c_H + \frac{1-\alpha}{1-\alpha+\alpha\kappa}(\Delta V - \Delta c), \widehat{p}_H^D = c_H + \frac{1-\alpha}{1-\alpha+\alpha\frac{1+\kappa}{2}}(\Delta V - \Delta c).$$

Using part (b) of Proposition 2, $L-$types must be indifferent between their equilibrium strategy and imitating $\widehat{p}_H$ with disclosure which yields

$$\left(1-\alpha+\alpha\frac{1+\kappa}{2}\right)\Delta c + (1-\alpha)(\Delta V - \Delta c) - D - f = 0,$$

so that

$$\kappa = \frac{2\left(D+f-(1-\alpha)\Delta V - \frac{\alpha}{2}\Delta c\right)}{\alpha \Delta c}.$$

$\kappa \in [0,1]$ if, and only if,

$$(1-\alpha)\Delta V + \alpha\Delta c \geq D+f \geq [(1-\alpha)\lambda + \frac{\alpha}{2}]\Delta c \tag{40}$$

As the $H-$type is indifferent over prices in the range $[\underline{p}_H^D, c_L+\Delta V)$ and the price $\widehat{p}_H$, it is easy to show that the $L-$type (with lower marginal cost) cannot gain by deviating to a price in the range $[\underline{p}_H, c_L+\Delta V)$. It is easy to check now that no other deviation can be gainful for either type. Combining (39) and (40) we obtain condition (7) as one of the two necessary and sufficient conditions for this equilibrium to exist (the other condition being (6)).

Proof of Proposition 7

Consider a pure disclosure equilibrium. There are two kinds of such equilibria: (i) Low quality type makes strictly positive profit; (ii) Low quality type makes zero profit. Consider an equilibrium of type (i). We first show that $H-$types must charge a deterministic price. Suppose to the contrary that $H-$type randomizes. Then, $L-$type must sell with strictly positive probability in the state where rival is of $H-$type and randomize its price between some lower bound $\underline{p}_L$an upper bound $\overline{p}_L$. From Proposition 2, there is no mass point at $\overline{p}_L$ so that at price $\overline{p}_L$, $L-$type sells only in the state where rival is $H-$ type which implies that $\overline{p}_L \leq \overline{p}_H - \Delta V$. Also from Proposition 2, $H-$type has no mass point at the upper bound $\overline{p}_H$ of the support of its price distribution and so at that price, it must sell strictly positive quantity (to cover disclosure cost) and as it sells only in the state where rival is L-type we must that $\overline{p}_H \leq \overline{p}_L + \Delta V$. Thus, $\overline{p}_H = \overline{p}_L + \Delta V$. As there is no mass point at $\overline{p}_H$ or $\overline{p}_L$, $L-$type is undercut with probability one at price $\overline{p}_L$ and must earn zero expected profit in equilibrium, a contradiction. Thus, $H-$type cannot randomize and must charge a deterministic price $\widetilde{p}_H = \overline{p}_L + \Delta V$. This is exactly the equilibrium outcome described in Proposition 4 and it exists if, and only if, conditions (4) and (5) hold; it is easy to check that these are equivalent to the following conditions:

$$0 < f \leq \frac{\alpha}{2}\Delta V \tag{41}$$

and

$$\begin{aligned} D &\leq \left[\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right] f \text{ , if } 0 \leq f < \frac{\alpha}{2}\Delta c & (42) \\ &\leq \frac{\alpha}{2}\Delta V - f \text{ , if } \frac{\alpha}{2}\Delta c \leq f \leq \frac{\alpha}{2}\Delta V. & (43) \end{aligned}$$

Now consider an equilibrium of type (ii) where $L-$types make zero profit. It easy to check (use symmetry), that $L-$types must charge a deterministic price c_L. The upper bound $\overline{p}_H$ of the support of the distribution of $H-$types prices must satisfy

$$\overline{p}_H \leq c_L + \Delta V \tag{44}$$

for otherwise, an $L-$type can deviate to a price above c_L and sell strictly positive expected quantity. There are two sub-cases: (ii.a) $L-$types sell strictly positive quantity in the state where rival is high quality; (ii.b) $L-$types sell only in the state where rival is low quality. Consider (ii.a). If $H-$types randomize over prices in such an equilibrium, then using Proposition 2(d) there is no probability mass at $\overline{p}_H$ so that (44) implies that $L-$ type is undercut with probability one in the state where rival is $H-$type. Therefore, $H-$types must charge a deterministic price $\widetilde{p}_H = c_L + \Delta V$. Buyers are then indifferent between buying high quality at price $\widetilde{p}_H$ and low quality at price c_L. In the state where one firm is of high quality and the other is of low quality, a fraction $\beta \in [0,1)$ of buyers buy high quality and the rest buy low quality. This is identical to the equilibrium outcome described in Proposition 6 and such an equilibrium exists if, and only if, conditions (9) and (10) hold; it is easy to check that the latter conditions are equivalent to the following::

$$\frac{\alpha}{2}\Delta c \leq f \leq \left(1-\frac{\alpha}{2}\right)\Delta V \tag{45}$$

and

$$\begin{aligned}\frac{\alpha}{2}\Delta V - f &\leq D \leq f(\lambda-1), \text{ if } \frac{\alpha}{2}\Delta c \leq f \leq \left(1-\frac{\alpha}{2}\right)\Delta c &(46)\\ \frac{\alpha}{2}\Delta V - f &\leq D < \left(1-\frac{\alpha}{2}\right)\Delta V - f, \text{ if } \left(1-\frac{\alpha}{2}\right)\Delta c \leq f < \left(1-\frac{\alpha}{2}\right)\Delta V. &(47)\end{aligned}$$

Now, consider an equilibrium of type (ii.b) There are two further sub-cases: (ii.b.1) $H-$types charge a deterministic price; (ii.b.2) $H-$types randomize over prices. It is easy to check that an equilibrium of type (ii.b.1) must be essentially identical to a type (ii.a) equilibrium with $\beta = 1$ and using (8) such an equilibrium exists if and only if (45)-(46) hold and further,

$$D = \left(1-\frac{\alpha}{2}\right)\Delta V - f, \ \left(1-\frac{\alpha}{2}\right)\Delta c \leq f \leq \left(1-\frac{\alpha}{2}\right)\Delta V. \tag{48}$$

Consider now an equilibrium of type (ii.b.2) We first show that the upper bound $\overline{p}_H$ of the support of the distribution of high quality prices must satisfy (44) with equality:

$$\overline{p}_H = c_L + \Delta V. \tag{49}$$

Suppose to the contrary that $\overline{p}_H < c_L + \Delta V$. Using Proposition 2, there is no mass point at price $\overline{p}_H$ and so at this price an H type can sell only if its rival is an L type. We now claim that D1 refinement implies that a firm disclosing and charging a price $\widehat{p} \in (\ \overline{p}_H, c_L + \Delta V)$ must be regarded as an $H-$type. To see this consider such a $\widehat{p}$. Let $q^{\tau}(\widehat{p})$ be the (expected) quantity that must be sold by a type τ at price $\widehat{p}$ to be indifferent between its equilibrium strategy and deviation to disclosing and charging $\widehat{p}.t$Then, $(\overline{p}_H - c_H)(1-\alpha) - A = (\widehat{p} - c_H)q^H(\widehat{p}) - D$ and $0 = (\widehat{p} - c_L)q^L(\widehat{p}) - D - f$ so that $\frac{q^H(\widehat{p})}{q^L(\widehat{p})} < 1$ if, and only if,.

$$(1-\alpha)((\overline{p}_H - c_H) < (D+f)\left(\frac{\widehat{p}-c_H}{\widehat{p}-c_L}\right). \tag{50}$$

As $\left(\frac{\widehat{p}-c_H}{\widehat{p}-c_L}\right)$ is strictly increasing in $\widehat{p}$, (50) must hold for some $\widehat{p} \in (\overline{p}_H, c_L + \Delta V)$ if $(1-\alpha)((\overline{p}_H - c_H) \leq (D+f)\left(\frac{\overline{p}_H - c_H}{\overline{p}_H - c_L}\right)$ which holds as long as $(1-\alpha)((\overline{p}_H - c_L) - (D+f) \leq 0$; this last inequality must hold in this equilibrium to ensure that $L-$type does not deviate to disclosing and imitating high quality price $\overline{p}_H$. Thus, $q^H(\widehat{p}) < q^L(\widehat{p})$ for some $\widehat{p} \in (\overline{p}_H, c_L + \Delta V)$ D1 criterion implies that out-of-equilibrium belief should regard any firm that discloses and charges such $\widehat{p}$ $H-$type with probability one. But this kind of belief makes deviation to price $\widehat{p}$ while disclosing strictly gainful for H type (sell the same expected quantity as at $\overline{p}_H$ and earn strictly higher profit). Hence, (49) holds. As there can only be a mass point at the lower bound of $H-$type's price distribution, it is easy to check that the $H-$type must randomize with a continuous distribution over an interval whose upper bound is $c_L + \Delta V$ with or without a positive mass point at an isolated price strictly below this interval. This is exactly the equilibrium outcome described in Proposition 5 and it exists if, and only if, (6) and (7); it is easy to check that the latter conditions are equivalent to the following:

$$f \geq \left(1 - \frac{\alpha}{2}\right) \Delta c \tag{51}$$

and

$$\max\left\{\left((1-\alpha)\Delta V + \frac{\alpha \Delta c}{2} - f\right)\ ,0\right\} \leq D \leq (1-\alpha)(\Delta V - \Delta c). \tag{52}$$

We have now covered all possible pure disclosure equilibria. Using the necessary and sufficient conditions for all possible pure disclosure equilibria outlined above (in particular, conditions (41)-(43), (45)-(47), (48), (51)-(52)) one can check that a pure disclosure equilibrium exists if, and only if, $D \leq \widetilde{D}(f)$. The details for various ranges of values of f are as follows:

(a) $0 \leq f < \frac{\alpha}{2}\Delta c$: only an equilibrium of type (i) can hold and it does (for that range of f) if, and only if, (42) holds i.e., $D \leq \left(\frac{2\alpha\lambda\Delta c}{\alpha\Delta c + 2f} - 1\right) f$

(b) $\frac{\alpha}{2}\Delta c \leq f < (1 - \frac{\alpha}{2})\Delta c$: a type (i) equilibrium exists if, and only if (43) holds i.e., $D \leq \frac{\alpha}{2}\Delta V - f$ while a type (ii.a) equilibrium holds if and only if (46) holds i..e, $\frac{\alpha}{2}\Delta V - f \leq D \leq f(\lambda - 1)$; a type (ii.b) equilibrium does not exist. As $f \geq \frac{\alpha}{2}\Delta c$ implies $\frac{\alpha}{2}\Delta V - f \leq f(\lambda - 1)$, a pure disclosure equilibrium exists if, and only if, $D \leq f(\lambda - 1)$.

(c) $(1 - \frac{\alpha}{2})\Delta c \leq f \leq \frac{\alpha}{2}\Delta V$: a type (i) equilibrium exists if, and only if (43) holds i.e., $D \leq \frac{\alpha}{2}\Delta V - f$ and types (ii.a) or (ii.b.1) equilibria exist if and only if (47) holds i.e., $\frac{\alpha}{2}\Delta V - f \leq D \leq \left(1 - \frac{\alpha}{2}\right)\Delta V - f$; thus, a pure disclosure equilibrium exists for all $D \leq \left(1 - \frac{\alpha}{2}\right)\Delta V - f$. A type (ii.b.2) equilibrium exists, if and only if (52) holds i.e.,

$(1-\alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \le D \le (1-\alpha)(\Delta V - \Delta c)$; as $f \le \frac{\alpha}{2}\Delta V$ implies $(1-\alpha)(\Delta V - \Delta c) \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$, this kind of equilibrium occurs only for $D \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$. Thus, a pure disclosure equilibrium exists if, and only if, $D \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$

(d) $\frac{\alpha}{2}\Delta V < f \le \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$: a type (i) equilibrium does not exist, types (ii.a) or (ii.b.1) equilibria exist if and only if (47) holds which reduces to $D \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$; type (ii.b.2) equilibrium exists, if and only if (52) holds i.e., $(1-\alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \le D \le (1-\alpha)(\Delta V - \Delta c)$. As $f \le \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$ implies $(1-\alpha)(\Delta V - \Delta c) \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$, this kind of equilibrium occurs only for $D \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$. Thus, a pure disclosure equilibrium exists if, and only if, $D \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$

(e) for $f > \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$: a type (i) equilibrium does not exist, types (ii.a) or (ii.b.1) equilibria exist if and only if (47) holds which reduces to $D \le \left(1-\frac{\alpha}{2}\right)\Delta V - f$; while;type (ii.b.2) equilibrium exists, if and only if (52) holds i.e., $(1-\alpha)\Delta V + \frac{\alpha\Delta c}{2} - f \le D \le (1-\alpha)(\Delta V - \Delta c)$. As for this range of f, $\left(1-\frac{\alpha}{2}\right)\Delta V - f < (1-\alpha)(\Delta V - \Delta c)$ we have that a pure disclosure equilibrium exists if, and only if, $D \le (1-\alpha)(\Delta V - \Delta c)$.

This concludes the proof of the proposition.

Proof of Proposition 8.

In a fully distortionary mixed disclosure equilibrium, a high quality type does not sell in the state where its rival has low quality. It follows then that $\underline{p}_H^D \ge \overline{p}_L + \Delta V$. Suppose the disclosing $H-$type randomizes over prices with distribution function F_H. Then it cannot have a probability mass point at $\overline{p}_H^D$ (see Proposition 2(d)) and $\overline{p}_H^D \le p_H^{ND}$, it sells to all buyers with probability one in the state where the rival firm is also of high quality but does not disclose. Further, $\overline{p}_H^D > \underline{p}_H^D \ge \overline{p}_L + \Delta V$. A disclosing $H-$type's profit at any $p \in (\overline{p}_L + \Delta V, \overline{p}_H^D)$ is

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(p))]\,(p - c_H) - D = \alpha(1-\gamma_H)(\overline{p}_H^D - c_H) - D, \qquad (53)$$

while setting a price equal to $p - \Delta V > \overline{p}_L$ the low quality firm would make a profit of

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(p))]\,(p - \Delta V - c_L),$$

which using (53) can be rewritten as

$$\alpha(1-\gamma_H)(\overline{p}_H^D - c_H)\frac{p - \Delta V - c_L}{\overline{p}_H^D - c_H},$$

and this is increasing in p, implying a low quality firm would gain by deviating to prices larger than $\overline{p}_L$, a contradiction. Thus, the disclosing high quality type must charge a deterministic price p_H^D. There are two possibilities (a) $p_H^D < p_H^{ND}$ and all buyers buy from the disclosing high quality type when both firms are of high quality and only one discloses, (b) $p_H^D = p_H^{ND} = p_H$ (say) and buyers randomize between the disclosing and non-disclosing firms when both are high quality type and only one discloses.

First, we derive the necessary and sufficient condition for an equilibrium of type (b). Using Proposition 2, at price p_H, H types sell only if the rival firm is a high quality type and $p_H \geq \overline{p}_L + \Delta V$. If $p_H > \overline{p}_L + \Delta V$, L type will always earn strictly higher profit by deviating to a price slightly higher than $\overline{p}_L$. Therefore, $p_H = \overline{p}_L + \Delta V$. As this is a symmetric equilibrium, buyers buy from each firm with equal probability when they both disclose or when neither discloses. When both firms charge p_H and only one discloses, let $\widetilde{\beta} \in (0, 1)$ denotes the probability that each buyer buys from the disclosing firm in that state (i.e., the expected market share of the disclosing firm is $\widetilde{\beta}$). L types should randomize over the interval $[\underline{p}_L, \overline{p}_L]$, with $p_H = \overline{p}_L + \Delta V$ and no mass points; it is easy to check that $\underline{p}_L = \alpha\overline{p}_L + (1 - \alpha)c_L$ and the equilibrium profits are given by

$$\pi_H^* = \alpha(\frac{\gamma_H}{2} + \widetilde{\beta}(1 - \gamma_H))(p_H - c_H) - D = \alpha(\frac{1 - \gamma_H}{2} + \gamma_H(1 - \widetilde{\beta}))(p_H - c_H) \tag{54}$$

and

$$\pi_L^* = \alpha(p_H - \Delta V - c_L), \tag{55}$$

respectively. From Proposition 2(b), $L-$type must be indifferent between its equilibrium strategy and and setting p_H with and without disclosure, we have

$$\pi_L^* = \alpha(\frac{\gamma_H}{2} + \beta(1 - \gamma_H))(p_H - c_L) - D - f = \alpha(\frac{1 - \gamma_H}{2} + \gamma_H(1 - \beta))(p_H - c_L). \tag{56}$$

Using this in (54), we get

$$\widetilde{\beta} = \frac{1}{2} + \frac{f}{\alpha\Delta c} \tag{57}$$

Note that $\widetilde{\beta} > 0.5$. Further, $\widetilde{\beta} \leq 1$ if, and only if,

$$f \leq \frac{\alpha}{2}\Delta c \tag{58}$$

From (56) and (57) we have $p_H = c_L + \Delta c(\frac{D+f}{f}) = c_H + \frac{D\Delta c}{f}$ and $H-$type makes strictly positive profit. Substituting the expressions for $\widetilde{\beta}$ and p_H into the first equation in (56) and using (55) we have $\gamma_H = \alpha\frac{\Delta V}{D+f} - \frac{\alpha\Delta c}{2f}$.so that $0 < \gamma_H < 1$ if, and only if,

$$f > \frac{D}{2\lambda - 1} \tag{59}$$

and

$$D > f\left(\frac{2\Delta V}{\Delta c + \frac{2f}{\alpha}} - 1\right), \tag{60}$$

Note that (60) is equivalent to $D > \widetilde{D}(f)$under constraint (58). Further, $\frac{D}{2\lambda-1} \geq \frac{\alpha}{2}\Delta c$ for $D \geq \frac{\alpha}{2}\Delta c(2\lambda - 1) = \alpha(\Delta V - \Delta c)$ so that (59) and (58) can be jointly satisfied only if $D < \alpha(\Delta V - \frac{\Delta c}{2}) = \underline{D}$and for this range of D, (59) is equivalent to $D < D^*(f)$.Thus, $0 < f \leq \frac{\alpha}{2}\Delta c$, and $\widetilde{D}(f) < D < D^*(f)$ are necessary and sufficient for the existence of an equilibrium of type (b).

In an equilibrium if type (a), the equilibrium profits are given by

$$\pi_H^* = \alpha(\frac{\gamma_H}{2} + (1 - \gamma_H))(p_H^D - c_H) - D = \alpha(\frac{1 - \gamma_H}{2})(p_H^{ND} - c_H)$$

$$\pi_L^* = \alpha(p_H^D - \Delta V - c_L),$$

From Proposition 2(b), $L-$type must be indifferent between its equilibrium strategy and and setting p_H with and without disclosure, we have

$$\pi_L^* = \alpha(\frac{\gamma_H}{2} + (1 - \gamma_H))(p_H^D - c_L) - D - f = \alpha(\frac{1 - \gamma_H}{2})(p_H^{ND} - c_L).$$

and it is easy to verify that these conditions can be jointly satisfied only if $f = \frac{\Delta c\alpha}{2}$ and $\widetilde{D}(f) < D < D^*(f) = \underline{D}$ at that specific value of f. This completes the proof.

Proof of Proposition 9

In a partially distortionary mixed disclosure equilibrium, high quality type must sell with positive probability in the state where its rival is of low quality type. From Proposition 2(c) , we know that when it does not disclose, high quality type sells only in the state where rival is high quality type. Therefore, in a partially distortionary mixed disclosure equilibrium, high quality type must sell with strictly positive probability when it discloses and the rival firm is of low quality type. It follows then that $\underline{p}_H^D < p_H^{ND}$. As $\overline{p}_H^D \leq p_H^{ND}$, in the state where it discloses an H-type cannot have a probability mass point at $\overline{p}_H^D$ if it randomizes over prices (see Proposition 2(d)). Thus, with probability one all buyers buy from the disclosing high quality firm when both firms are of high quality and only one discloses. We now argue that either the disclosing high quality firm sets a deterministic price, or it serves the entire market if the rival firm produces low quality. To see this suppose that with strictly positive probability a disclosing high quality firm randomizes over prices but does not sell to all buyers when rival is low quality. Then, $\overline{p}_H^D > \underline{p}_L + \Delta V$ (no mass point at $\overline{p}_H^D$ conditional on disclosure). First, suppose that

$$\underline{p}_L + \Delta V < \overline{p}_H^D \leq \overline{p}_L + \Delta V.$$

Then, $\overline{p}_H^D - \Delta V \in (\underline{p}_L, \overline{p}_L]$. Further, there exists $\epsilon > 0$ such that $\overline{p}_H^D - \epsilon$ is the in the interior of the support of the distribution of high quality prices with disclosure and

$$\underline{p}_L < \overline{p}_H^D - \epsilon - \Delta V < \overline{p}_H^D - \Delta V \leq \overline{p}_L \tag{61}$$

The high quality equilibrium profit must be equalized at prices $\overline{p}_H^D - \epsilon$ and $\overline{p}_H^D$

$$\begin{aligned}
& \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\overline{p}_H^D - \epsilon - c_H) \\
= & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - c_H)
\end{aligned}$$

However, this implies that

$$\begin{aligned}
& \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\overline{p}_H^D - \epsilon - \Delta V - c_L) \\
= & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\overline{p}_H^D - \epsilon - c_H) \\
& - \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\
= & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - c_H) \\
& - \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\
= & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - \Delta V - c_L) \\
& + \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\Delta V - \Delta c) \\
& - \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha(1-\gamma_H) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\
= & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - \Delta V - c_L) \\
& - \left[(1-\alpha)(F_L(\overline{p}_H^D - \Delta V) - F_L(\overline{p}_H^D - \epsilon - \Delta V)) + \alpha\gamma_H(1-F_H(\overline{p}_H^D - \epsilon))\right](\Delta V - \Delta c) \\
< & \left[(1-\alpha)(1-F_L(\overline{p}_H^D - \Delta V)) + \alpha(1-\gamma_H)\right](\overline{p}_H^D - \Delta V - c_L)]
\end{aligned}$$

so that low quality type strictly prefers to charge $\overline{p}_H^D - \Delta V$ than $\overline{p}_H^D - \epsilon - \Delta V$ which contradicts (61). Next, suppose that $\overline{p}_H^D > \overline{p}_L + \Delta V$. The the disclosing high quality firm's profit at any $p \in (\overline{p}_L + \Delta V, \overline{p}_H^D)$ is

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1 - F_H(p))]\,(p - c_H) - D = \alpha(1-\gamma_H)(\overline{p}_H^D - c_H) - D \qquad (62)$$

While setting a price $p - \Delta V > \overline{p}_L$ the low quality firm would make a profit of

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1 - F_H(p))]\,(p - \Delta V - c_L),$$

which using (62) can be rewritten as

$$\alpha(1-\gamma_H)(\overline{p}_H^D - c_H)\frac{p - \Delta V - c_L}{\overline{p}_H^D - c_H},$$

and this is increasing in p, implying low quality firm would gain by deviating to prices larger than $\overline{p}_L$, a contradiction. Thus, if the disclosing high quality firm randomizes, it serves the entire market when the rival firm produces low quality. We now consider two possibilities: the disclosing high quality firm sets a deterministic price or randomizes over prices.

First, consider a partially distortionary mixed disclosure equilibrium where high quality discloses and sets a deterministic price p_H^D. Then, as we have assumed $\underline{p}_H^D < p_H^{ND}$, it must the case that $p_H^D < p_H^{ND}$. As this equilibrium is partially distortionary, low quality type must sell to all buyers with probability one when rival is a non-disclosing high quality firm but share the market with strictly positive probability when rival is a disclosing high quality firm. It is easy to see that in that case the upper bound of low quality price distribution must be exactly equal $p_H^{ND} - \Delta V$ (if it is lower, low quality type would gain by increasing its price above the upper bound). Further, as the disclosing high quality firm facing a low quality rival sells with strictly positive probability, $p_H^D - \Delta V \in (\underline{p}_L, \overline{p}_L)$. As there would be discontinuity in expected quantity sold by low quality firm at price $p_H^D - \Delta V$, the support of low quality price distribution must consist of two disjoint intervals $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ with $\overline{p}_L^1 = p_H^D - \Delta V$ and $\overline{p}_L^2 = p_H^{ND} - \Delta V$ (note that if $\overline{p}_L^1 < p_H^D - \Delta V$, low quality firm can gain by deviating to a price slightly above $\overline{p}_L^1$). When a high quality firm discloses, he serves the entire market if the low quality rival sets prices in the upper interval, but not when low quality sets prices in the lower interval as for all $p, \widetilde{p}$ with

$p < \overline{p}_L^1 < \widetilde{p}$, $V_L - p > V_H - p_H^D > V_L - \widetilde{p}$. Consider such a candidate equilibrium where high quality chooses p_H^{ND} with probability $1-\gamma_H$ and price p_H^D with probability γ_H, where $p_H^D < p_H^{ND}$ and low quality randomizes over two disconnected sets $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ with probabilities γ_L and $1-\gamma_L$, respectively. Let's call this an E1 equilibrium. We will show that:

R.1. An E1 equilibrium exists if, and only if,

$$\frac{\alpha}{2}\Delta c < f \leq (1-\frac{\alpha}{2})\Delta c \tag{63}$$

and

$$\widetilde{D}(f) < D < D^*(f) \tag{64}$$

Next, consider a partially distortionary mixed disclosure equilibrium where when the high quality type discloses it randomizes its pricing decision. As argued above, such a disclosing high quality firm sells to the entire market if the competitor is of low quality. This requires $\overline{p}_H^D \leq \underline{p}_L + \Delta V$. If this behavior constitute part of an equilibrium, it must be the case that $\overline{p}_L = p_H^{ND} - \Delta V$ so that $\pi_L^* = \alpha(1-\gamma_H)(\overline{p}_L - c_L)$ and $\pi_H^* = \frac{\alpha(1-\gamma_H)}{2}(p_H^{ND} - c_H)$. Let's call this E2 equilibrium. We will show that

R.2 An E2 equilibrium exists if, and only if,

$$f > \left[1-\frac{\alpha}{2}\right]\Delta c. \tag{65}$$

$$(\lambda-1)(1-\alpha)\Delta c < D < \overline{D} = D^*(f) \tag{66}$$

Combining the conditions in **R.1** and **R.2,** we obtain Proposition 9. In the rest of the proof we prove **R.1** and **R.2.**

Proof of **R.1**

First note that for the range of values of f satisfying (63), $\widetilde{D}(f) = (\lambda-1)f$ and $D^*(f) = \frac{\alpha}{2}\Delta V + (\lambda-1)f$ so that (64) is equivalent to the following condition:

$$(\lambda-1)f < D < \frac{\alpha}{2}\Delta V + (\lambda-1)f \tag{67}$$

Consider the following partially distortionary mixed disclosure equilibrium where high quality type sets a deterministic price $p_H^D < p_H^{ND}$. A low quality type randomizes over two disconnected intervals of prices $[\underline{p}_L^1, \overline{p}_L^1]$ and $[\underline{p}_L^2, \overline{p}_L^2]$ with probabilities γ_L and $1-\gamma_L$ respectively and a continuous distribution over each interval, where $\overline{p}_L^1 < \underline{p}_L^2$ and $\gamma_L \in [0, 1)$, ($\gamma_L < 1$ reflects the partially distortionary nature of this equilibrium). Further,

$$\overline{p}_L^1 = p_H^D - \Delta V,\ \overline{p}_L^2 = p_H^{ND} - \Delta V. \tag{68}$$

The equilibrium expected profits π_H^* and π_L^* then satisfy:

$$\pi_L^* = (\alpha + (1-\alpha)(1-\gamma_L))\,(\overline{p}_L^1 - c_L) = \alpha(1-\gamma_H)(\overline{p}_L^2 - c_L) \tag{69}$$

$$\pi_H^* = \left(\frac{\alpha\gamma_H}{2} + \alpha(1-\gamma_H) + (1-\alpha)(1-\gamma_L)\right)(p_H^D - c_H) - D = \frac{\alpha(1-\gamma_H)}{2}(p_H^{ND} - c_H) \tag{70}$$

Out of equilibrium beliefs regard any firm charging price $\in (\overline{p}_L^2, p_H^{ND})$ without disclosure or price $\in (\overline{p}_L^1, p_H^{ND})$ with disclosure as being of of low type. From Proposition 2(b), low quality type is indifferent between following its equilibrium strategy and deviating to disclosing and charging p_H^D or not disclosing and charging p_H^{ND} :

$$\pi_L^* = \left(\frac{\alpha\gamma_H}{2} + \alpha(1-\gamma_H) + (1-\alpha)(1-\gamma_L)\right)(p_H^D - c_L) - D - f = \frac{\alpha(1-\gamma_H)}{2}(p_H^{ND} - c_L) \tag{71}$$

From (68), (69) and (71) we obtain

$$p_H^{ND} = c_L + 2\Delta V, \overline{p}_L^2 = c_L + \Delta V \tag{72}$$

Using (70), (71) reduces to

$$\gamma_L = \frac{1 - \frac{\alpha}{2} - \frac{f}{\Delta c}}{1-\alpha} \tag{73}$$

and this lies in $[0, 1)$ if and only if condition (63) holds. From (69) and (70), we obtain:

$$p_H^D - c_H = \Delta V - \Delta c + \frac{\alpha(1-\gamma_H)\Delta V}{\frac{\alpha}{2} + \frac{f}{\Delta c}} \tag{74}$$

and

$$p_H^D - c_H = \frac{2D + \alpha(1-\gamma_H)\,(2\Delta V - \Delta c)}{2\left(\frac{\alpha(1-\gamma_H)}{2} + \frac{f}{\Delta c}\right)} \tag{75}$$

that simultaneously determine p_H^D and γ_H.These yield:

$$\Delta V\left(\alpha(1-\gamma_H)\right)^2+\left(\frac{f}{\Delta c}-\frac{\alpha}{2}\right)\Delta V\alpha(1-\gamma_H)+2\left(\frac{\alpha}{2}+\frac{f}{\Delta c}\right)\left((\lambda-1)f-D\right)=0$$

It is easy to check that for $D=(\lambda-1)f, \gamma_H=1$, that γ_H is decreasing in D, and that at $D=(\lambda-1)f+\frac{\alpha\Delta V}{2}\gamma_H=0$. Thus, (67) or equivalently, condition (64), is necessary and sufficient to ensure that there is some $\gamma_H\in(0,1)$ and therefore (using (74) or (75)) there is some $p_H^D>c_H$ that meets the conditions for an equilibrium. Note that (74) implies that p_H^D is strictly decreasing in γ_H so we have

$$p_H^D\leq c_L+\Delta V+\frac{\alpha\Delta V}{\frac{\alpha}{2}+\frac{f}{\Delta c}}<c_L+2\Delta V=p_H^{ND}$$

using the first inequality in condition (63). Using (68), we obtain $\overline{p}_L^1$. The values of $\underline{p}_L^1$ and $\underline{p}_L^2$ are determined by:

$$(\alpha+(1-\alpha)(1-\gamma_L))\left(\overline{p}_L^1-c_L\right)=\left(\underline{p}_L^1-c_L\right)$$

$$\alpha(1-\gamma_H)(\overline{p}_L^2-c_L)=((1-\alpha)(1-\gamma_L)+\alpha(1-\gamma_H))(\underline{p}_L^2-c_L)$$

and using the previous equations one can check that $\overline{p}_L^1<\underline{p}_L^2$. The distribution of low quality prices over the two segments $[\underline{p}_L^1,\overline{p}_L^1]$ and $[\underline{p}_L^2,\overline{p}_L^2]$ can now be determined in the usual manner by equalizing the expected profit earned at various prices and it can be shown that the distribution is continuous over each interval. Finally, the out of equilibrium beliefs can be used to show that no type of any firm has a unilateral incentive to deviate. This completes the proof of **R.1.**

Proof of **R.2**

We first consider a version of this equilibrium where the disclosing high quality firm randomizes with no probability mass point. Let $\gamma_H\in(0,1)$ denote the probability of disclosure by a high quality type.As in any mixed disclosure equilibrium, when it does not disclose, the high quality firm charges p_H^{ND} and at this price it only sells in the state where the rival is H type and does not disclose i.e., it sells with probability $\frac{\alpha(1-\gamma_H)}{2}$. Further, a low quality type does not disclose and sells in the state where rival is low quality as well as the state in which rival is high quality and charges p_H^N.In the specific equilibrium we construct, the low quality firm randomizes over an interval $[\underline{p}_L,\overline{p}_L]$ where $\overline{p}_L=p_H^{ND}-\Delta V$.

When it discloses, the high quality firm randomizes prices over an interval $[\underline{p}_H^D, \overline{p}_H^D]$ where $\overline{p}_H^D = \underline{p}_L + \Delta V < p_H^{ND}$ i.e., buyers are indifferent between buying low quality at the lower bound of low quality prices $\underline{p}_L$ and the upper bound of high quality prices when the firm discloses. It is easy to see that $\overline{p}_H^D > \overline{p}_L$. At price $\overline{p}_L$ a low quality firm sells with probability $\alpha(1-\gamma_H)$ i.e., only when rival is high quality but does not disclose. At price $\underline{p}_L$ a low quality firm sells with probability $\alpha(1-\gamma_H)+(1-\alpha) = 1-\alpha\gamma_H$. When it discloses and charges price $\overline{p}_H^D$, a high quality firm also sells with probability $1-\alpha\gamma_H$,and it sells with probability 1 when it charges $\underline{p}_H^D$. The only restriction on out of equilibrium beliefs is that a firm that does not disclose and charges any price below p_H^{ND} is deemed to be low quality with probability one. Using Proposition 2(b), the low quality firm must be indifferent between charging p_H^{ND} (without disclosing) and sticking to its equilibrium strategy i.e.., $(p_H^{ND} - c_L)\frac{\alpha(1-\gamma_H)}{2} = (\overline{p}_L - c_L)\alpha(1-\gamma_H)$ and this yields:

$$p_H^{ND} = 2\Delta V + c_L \tag{76}$$

The equilibrium profit of the high quality firm is therefore:

$$\pi_H^* = (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} \tag{77}$$

Further:

$$\overline{p}_L = p_H^{ND} - \Delta V = \Delta V + c_L \tag{78}$$

and therefore, the equilibrium profit of the low quality firm is

$$\pi_L^* = \Delta V \alpha(1-\gamma_H) \tag{79}$$

Further, as

$$(\underline{p}_L - c_L)(1-\alpha\gamma) = \pi_L^* \tag{80}$$

we have

$$\underline{p}_L = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right]\Delta V + c_L \tag{81}$$

The upper bound of prices for a high quality firm that discloses is now:

$$\overline{p}_H^D = \underline{p}_L + \Delta V = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H} + 1\right]\Delta V + c_L \tag{82}$$

which is decreasing in γ_H and converges to $\overline{p}_L$ as $\gamma \to 1$. The profit of the high quality firm when it discloses and charges price $\overline{p}_H^D$ is given by

$$(\overline{p}_H^D - c_H)(1 - \alpha\gamma_H) - D \tag{83}$$

and this is equal to π_H^* if, and only if,

$$\gamma_H = \frac{1}{\alpha}\left[\frac{\Delta V - \Delta c(1 - \frac{\alpha}{2}) - D}{\Delta V - \frac{\Delta c}{2}}\right] \tag{84}$$

It can be checked that condition (66) is exactly what is needed to ensure that $\gamma_H \in (0, 1)$. Indeed, $\gamma_H \to 0$ as $D \to \left[\lambda - (1 - \frac{\alpha}{2})\right]\Delta c$ and $\gamma_H \to 1$ as $D \to (\lambda - 1)(1 - \alpha)\Delta c$. The lower bound $\underline{p}_H^D$ for high quality price when the firm discloses satisfies:

$$(\underline{p}_H^D - c_H) = (\overline{p}_H^D - c_H)(1 - \alpha\gamma_H) = \pi_H^* + D \tag{85}$$

and this yields:

$$\begin{aligned} \underline{p}_H^D &= \left[\left(\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H} + 1\right)\Delta V - \Delta c\right](1 - \alpha\gamma_H) + c_H \\ &= 2D + c_H - (1-\alpha)(\lambda - 1)\Delta c \end{aligned} \tag{86}$$

The distribution function $F(.)$ for low quality price satisfies:

$$(p_L - c_L)[\alpha(1 - \gamma_H) + (1 - \alpha)(1 - F(p_L))] = \pi_L^* = \alpha(1 - \gamma_H)\Delta V, p_L \in [\underline{p}_L, \overline{p}_L] \tag{87}$$

The distribution function $G(.)$ for high quality price when the firm discloses satisfies:

$$\begin{aligned} & (p_H^D - c_H)[(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^D))] \\ = \; & \pi_H^* + D \qquad (88) \\ = \; & \Delta V(1 + \alpha - 2\alpha\gamma_H) - \Delta c(1 - \alpha\gamma), p_H^D \in [\underline{p}_H^D, \overline{p}_H^D] \qquad (89) \end{aligned}$$

This completes the description of the equilibrium. Next, we show that there is no incentive to deviate from this equilibrium. It is easy to check that given the out of equilibrium belief, no high quality firm can strictly gain by deviating from its equilibrium strategy without disclosing. As the high quality firm gets the entire market at price $\underline{p}_H^D$ when it discloses, it has no incentive to disclose and charge price below $\underline{p}_H^D$. Nor can it gain by charging price above p_H^{ND} (sells zero). It remains to check that a high quality firm cannot gain by disclosing and charging an out of equilibrium price $p_H \in (\overline{p}_H^D, p_H^{ND})$.For any such deviation price p_H, there exists $p_L = p_H - \Delta V \in (\underline{p}_L, \overline{p}_L)$. The deviation profit is given by:

$$\begin{aligned}
& [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_H - \Delta V))](p_H - c_H) - D \\
= \; & [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))](p_L + \Delta V - c_H) - D \\
= \; & \left[\frac{p_L + \Delta V - c_H}{p_L - c_L}\right] \pi_L^* - D, \text{ using (87)},
\end{aligned}$$

and since $\frac{p_L+\Delta V - c_H}{p_L - c_L}$ is strictly decreasing in p_L (as $\Delta V > \Delta c$) this is

$$\begin{aligned}
\leq \; & \left[\frac{\underline{p}_L + \Delta V - c_H}{\underline{p}_L - c_L}\right] \pi_L^* - D = \left[\underline{p}_L + \Delta V - c_H\right](1-\alpha\gamma_H) - D \\
= \; & \left[\overline{p}_H^D - c_H\right](1-\alpha\gamma_H) - D = \pi_H^*
\end{aligned}$$

using (83) and (84). Therefore, the deviation cannot be strictly gainful. We now look at the incentive of a low quality firm to deviate. Whether or not it discloses, the firm will sell zero if it charges price above p_H^{ND} (even if it is thought of as a high quality firm). Given the out of equilibrium beliefs, if a low quality firm deviates without disclosing and charges price $\in (\overline{p}_L, p_H^{ND})$ it will be thought of as a low quality firm and will sell zero. If it charges price $p_L < \underline{p}_L$ (without disclosing) it will be perceived as a low quality firm but may be able to attract more buyers in the state where rival is high quality and discloses; without loss of generality, consider deviation to $p_L \in [\underline{p}_H^D - \Delta V, \underline{p}_L)$. The deviation profit is then given by

$$\begin{aligned}
& [(1-\alpha\gamma_H) + \alpha\gamma_H(1-G(p_L + \Delta V))](p_L - c_L) \\
= \; & [(1-\alpha\gamma_H) + \alpha\gamma_H(1-G(p_H^D))](p_H^D - \Delta V - c_L) \text{ where } p_H^D = p_L + \Delta V \\
= \; & \left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right](\pi_H^* + D), \text{using (88)}
\end{aligned}$$

and since $\left[\frac{p_H^D - \Delta V - c_L}{p_H^D - c_H}\right]$ is strictly increasing in p_H^D (as $\Delta V > \Delta c$) this is

$$
\begin{aligned}
&\leq \left[\frac{\overline{p}_H^D - \Delta V - c_L}{\overline{p}_H^D - c_H}\right](\pi_H^* + D) \\
&= \left[\overline{p}_H^D - \Delta V - c_L\right](1 - \alpha\gamma_H) \;\; \text{(use (83) and (84))} \\
&= \Delta V \alpha(1 - \gamma_H), \text{ using (82)} \\
&= \pi_L^* \;\; \text{(see (79))}
\end{aligned}
$$

and thus the deviation is not strictly gainful. We now consider deviation by a low quality firm where it discloses (falsely). If it does so, it cannot gain by charging price below $\underline{p}_H^D$ as it sells to the entire market at that price. So, consider deviation price $p_H^D \in [\underline{p}_H^D, \overline{p}_H^D]$ with disclosure. The deviation profit is given by

$$
\begin{aligned}
&[(1 - \alpha\gamma_H) + \alpha\gamma_H(1 - G(p_H^D))](p_H^D - c_L) - D - f \\
= &\left[\frac{p_H^D - c_L}{p_H^D - c_H}\right](\pi_H^* + D) - D - f, \text{ using (88)}
\end{aligned}
$$

and as $\left[\frac{p_H^D - c_L}{p_H^D - c_H}\right]$ is strictly decreasing in p_H^D, this is

$$
\begin{aligned}
&\leq \left[\frac{\underline{p}_H^D - c_L}{\underline{p}_H^D - c_H}\right](\pi_H^* + D) - D - f \\
&= \underline{p}_H^D - c_L - D - f, \text{ using (85)} \qquad (90) \\
&= (\underline{p}_H^D - c_H) + \Delta c - D - f = \pi_H^* + D + \Delta c - D - f \\
&= (2\Delta V - \Delta c)\frac{\alpha(1 - \gamma_H)}{2} + \Delta c - f = \Delta V \alpha(1 - \gamma_H) + \Delta c(1 - \frac{\alpha(1 - \gamma_H)}{2}) - f \\
&= \pi_L^* + \Delta c(1 - \frac{\alpha(1 - \gamma_H)}{2}) - f
\end{aligned}
$$

which is $\leq \pi_L^*$ if $f \geq \Delta c(1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2})$ and the latter (using (84)) holds if, and only if :

$$
f \geq \left[1 - \frac{\alpha}{2} + \frac{\lambda - (1 - \frac{\alpha}{2})}{2\lambda - 1}\right]\Delta c - \frac{D}{2\lambda - 1} \qquad (91)
$$

Thus, under (91), deviation by a low quality type to advertising and charging price in $[\underline{p}_H^D, \overline{p}_H^D]$ is not gainful. Finally, consider deviation by the same firm to disclosing and setting price $p \in (\overline{p}_H^D, p_H^{ND})$. The maximum possible deviation profit (i.e., even if the firm is perceived as high quality with probability 1) is given by:

$$\begin{aligned}
& [\alpha(1-\gamma_H) + (1-\alpha)(1 - F(p - \Delta V))](p - c_L) - D - f \\
= & [\alpha(1-\gamma_H) + (1-\alpha)(1 - F(p_L))](p_L + \Delta V - c_L) - D - f, \text{ where } p_L = p - \Delta V \\
= & \left[\frac{p_L + \Delta V - c_L}{p_L - c_L}\right] \pi_L^* - D - f, \text{ using (87)},
\end{aligned}$$

and as $\left[\frac{p_L + \Delta V - c_L}{p_L - c_L}\right]$ is strictly decreasing in p_L, this is

$$\begin{aligned}
\leq & \left[\frac{\underline{p}_L + \Delta V - c_L}{\underline{p}_L - c_L}\right] \pi_L^* - D - f \\
= & \left[\underline{p}_L + \Delta V - c_L\right](1 - \alpha\gamma) - D - f, \text{ using (80)} \\
= & \left[\overline{p}_H^D - c_L\right](1 - \alpha\gamma) - D - f = \left[\overline{p}_H^D - c_H\right](1 - \alpha\gamma) + \Delta c - D - f \\
= & (\underline{p}_H^D - c_H) + \Delta c - D - f = (\underline{p}_H^D - c_L) - D - f
\end{aligned}$$

$\leq \pi_L^*$ under condition (91) as was shown above (see arguments following (90)). Thus, we have shown that under (91) is necessary and sufficient for ruling out any incentive to deviate.

Next, we consider a variation of the above equilibrium with the only difference that when a high quality firm discloses, it randomizes prices over an interval $[\underline{p}_H^D, \overline{p}_H^D]$ with probability $1 - \kappa \in (0, 1)$ and chooses a price $\widetilde{p}_H^D \in (c_H, \underline{p}_H^D)$ with probability κ. As before, $\overline{p}_H^D = \underline{p}_L + \Delta V < p_H^{ND}$. Also, as before, at price $\overline{p}_L$ a low quality firm sells with probability $\alpha(1 - \gamma_H)$ i.e., only when rival is high quality but does not disclose. At price $\underline{p}_L$ a low quality firm sells with probability$1 - \alpha\gamma_H$. When it discloses and charges price $\overline{p}_H^D$, a high quality firm also sells with probability $1 - \alpha\gamma_H$,and it sells with probability $1 - \alpha\kappa\gamma_H$ when it charges $\underline{p}_H^D$. When it discloses and charges $\widetilde{p}_H^D$, the high quality firm sells with probability $\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right)$. Only restrictions on out of equilibrium beliefs are that : (a) a firm that does not disclose and charges any price below p_H^{ND} is deemed to be low quality with probability one and (b) any firm disclosing and charging price below $\widetilde{p}_H$ is deemed to be low quality with probability one. From Proposition 2(b), a low quality firm should be indifferent between charging p_H^{ND} without disclosing and sticking to its equilibrium strategy which yields the same expressions for

$$p_H^{ND} = 2\Delta V + c_L. \tag{92}$$

and the equilibrium profit of the high quality firm :

$$\pi_H^* = (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} \tag{93}$$

Further, as before,

$$\overline{p}_L = p_H^N - \Delta V = \Delta V + c_L \tag{94}$$

$$\pi_L^* = \Delta V \alpha(1-\gamma_H) \tag{95}$$

$$\underline{p}_L = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H}\right]\Delta V + c_L \tag{96}$$

$$\overline{p}_H^D = \underline{p}_L + \Delta V = \left[\frac{\alpha(1-\gamma_H)}{1-\alpha\gamma_H} + 1\right]\Delta V + c_L \tag{97}$$

$$\gamma_H = \frac{1}{\alpha}\left[\frac{\Delta V - \Delta c(1-\frac{\alpha}{2}) - D}{\Delta V - \frac{\Delta c}{2}}\right] \tag{98}$$

and condition (66) is necessary and sufficient for $\gamma_H \in (0,1)$. $\underline{p}_H^D$ satisfies:

$$(\underline{p}_H^D - c_H)(1-\alpha\kappa\gamma_H) = (\overline{p}_H^D - c_H)(1-\alpha\gamma_H) = \pi_H^* + D \tag{99}$$

and this yields:

$$\begin{aligned} \underline{p}_H^D &= [\Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma_H)]\frac{1}{(1-\alpha\kappa\gamma_H)} + c_H \\ &= \frac{\pi_H^* + D}{1-\alpha\kappa\gamma_H} + c_H \end{aligned} \tag{100}$$

As before, the distribution function $F(.)$ for low quality price satisfies:

$$(p_L - c_L)[\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))] = \pi_L^* = \alpha(1-\gamma_H)\Delta V, p_L \in [\underline{p}_L, \overline{p}_L] \tag{101}$$

The distribution function $G(.)$ for high quality price on the interval $[\underline{p}_H^D, \overline{p}_H^D]$ when the firm discloses satisfies:

$$
\begin{aligned}
& (p_H^D - c_H)[(1-\alpha\gamma_H) + \alpha\gamma_H(1-\kappa)(1-G(p_H^D))] \\
= \;& \pi_H^* + D && (102)\\
= \;& \Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma_H), p_H^D \in [\underline{p}_H^D, \overline{p}_H^D] && (103)
\end{aligned}
$$

Proposition 2(b) implies that a low quality firm should be indifferent between deviating to advertising and charging $\widetilde{p}_H^D$ and sticking to its equilibrium strategy i.e..,

$$
(\widetilde{p}_H^D - c_L)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) - D - f = \pi_L^* = \Delta V\alpha(1-\gamma_H) \tag{104}
$$

Further, high quality type must be indifferent between choosing $\widetilde{p}_H^D$ while disclosing and other actions in the support of its equilibrium strategy which requires:

$$
\begin{aligned}
(\widetilde{p}_H^D - c_H)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) &= \pi_H^* + D && (105)\\
&= (2\Delta V - \Delta c)\frac{\alpha(1-\gamma_H)}{2} + D && (106)\\
&= \Delta V(1+\alpha-2\alpha\gamma_H) - \Delta c(1-\alpha\gamma_H) && (107)
\end{aligned}
$$

Comparing (105) and (99), we can see that $\widetilde{p}_H^D < \underline{p}_H^D$. From (104)

$$
\begin{aligned}
& \pi_L^* + D + f = (\widetilde{p}_H^D - c_L)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) \\
= \;& (\widetilde{p}_H^D - c_H)\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) + \Delta c\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) \\
= \;& \pi_H^* + D + \Delta c\left(1 - \frac{\alpha\kappa\gamma}{2}\right), \text{ using (105)}
\end{aligned}
$$

so that

$$
\left(1 - \frac{\alpha\kappa\gamma_H}{2}\right) = \frac{1}{\Delta c}(\pi_L^* - \pi_H^* + f) \tag{108}
$$

which yields:

$$
\kappa = \frac{2}{\alpha\gamma_H}\left[1 - \frac{1}{\Delta c}(\pi_L^* - \pi_H^* + f)\right] \tag{109}
$$

Further, using (108) in (105) we have:

$$
\widetilde{p}_H^D = \left[\frac{\pi_H^* + D}{\pi_L^* - \pi_H^* + f}\right]\Delta c + c_H \tag{110}
$$

We need to ensure that $\kappa \in (0,1)$ which is satisfied as long as

$$(1-\frac{\alpha\gamma_H}{2})\Delta c - (\pi_L^* - \pi_H^*) < f < \Delta c - (\pi_L^* - \pi_H^*) \tag{111}$$

Note that

$$(\pi_L^* - \pi_H^*) = \frac{\alpha(1-\gamma_H)}{2}\Delta c < \Delta c \tag{112}$$

and (111) is satisfied as long as

$$\left[1-\frac{\alpha}{2}\right]\Delta c < f < \left[1-\frac{\alpha}{2}+\frac{\alpha\gamma_H}{2}\right]\Delta c \tag{113}$$

which (using (98)) reduces to

$$\left[1-\frac{\alpha}{2}\right]\Delta c < f < \left[1-\frac{\alpha}{2}+\frac{\lambda-(1-\frac{\alpha}{2})}{2\lambda-1}\right]\Delta c - \frac{D}{2\lambda-1} \tag{114}$$

Observe that the inequalities in (114) can be written as

$$D + f(2\lambda-1) \quad < \quad [(2-\alpha)(\lambda-1)+\lambda]\Delta c \tag{115}$$

$$f \quad > \quad \left[1-\frac{\alpha}{2}\right]\Delta c \tag{116}$$

This completes the description of the equilibrium. Next, we show that there is no incentive to deviate from this equilibrium. Observe that a high quality firm can never strictly gain by disclosing and choosing a price $p \in (\widetilde{p}_H^D, \underline{p}_H^D)$ as it sells the same expected quantity in that case as it would at $\underline{p}_H^D$. As the high quality firm gets the entire market at price $\widetilde{p}_H^D$ when it discloses, it has no incentive to disclose and charge price below $\widetilde{p}_H^D$. Using identical arguments to that in the first part of the proof, one can check that there is no other gainful deviation for a high quality type. We now look at the incentive of a low quality firm to deviate. Whether or not it discloses, the firm will sell zero if it charges price above p_H^{ND} (even if it is thought of as a high quality firm). Given the out of equilibrium beliefs, if a low quality firm deviates without disclosing and charges price $\in (\overline{p}_L, p_H^{ND})$ it will be thought of as a low quality firm and will sell zero. If it charges price $p_L < \underline{p}_L$ (without advertising) it will be perceived as a low quality firm but may be able to attract more buyers in the state where rival is high quality and discloses. First, consider deviation to $p_L \in [\underline{p}_H^D - \Delta V, \underline{p}_L)$.

The deviation profit is then given by

$$\begin{aligned}
& [(1-\alpha\gamma_H)+\alpha\gamma_H(1-\kappa)(1-G(p_L+\Delta V))](p_L-c_L) \\
= & [(1-\alpha\gamma_H)+\alpha\gamma_H(1-\kappa)(1-G(p_H^D))](p_H^D-\Delta V-c_L) \text{ where } p_H^D=p_L+\Delta V \\
= & \left[\frac{p_H^D-\Delta V-c_L}{p_H^D-c_H}\right](\pi_H^*+D) \text{ (using (102))} \\
\leq & \left[\frac{\overline{p}_H^D-\Delta V-c_L}{\overline{p}_H^D-c_H}\right](\pi_H^*+D) \text{ (as } \left[\frac{p_H^D-\Delta V-c_L}{p_H^D-c_H}\right] \text{ is strictly increasing in } p_H^D) \\
= & \left[\overline{p}_H^D-\Delta V-c_L\right](1-\alpha\gamma_H) \text{ (using (97) and (98))} \\
= & \Delta V\alpha(1-\gamma_H)=\pi_L^*, \text{ using (97) and (95)}
\end{aligned}$$

and thus the deviation is not strictly gainful. We now consider deviation by a low quality firm where it discloses (falsely). Consider deviation to price $p_H^D \in [\underline{p}_H^D, \overline{p}_H^D]$ with disclosure. The deviation profit is given by

$$\begin{aligned}
& [(1-\alpha\gamma_H)+\alpha\gamma_H(1-\kappa)(1-G(p_H^D))](p_H^D-c_L)-D-f \\
= & \left[\frac{p_H^D-c_L}{p_H^D-c_H}\right](\pi_H^*+D)-D-f, \text{ using (102)}
\end{aligned}$$

and as $\left[\frac{p_H^D-c_L}{p_H^D-c_H}\right]$ is strictly decreasing in p_H^D, this is

$$\begin{aligned}
\leq & \left[\frac{\underline{p}_H^D-c_L}{\underline{p}_H^D-c_H}\right](\pi_H^*+D)-D-f \\
= & (\underline{p}_H^D-c_L)(1-\alpha\kappa\gamma_H)-D-f, \text{ using (98)} \\
= & (\underline{p}_H^D-c_H)(1-\alpha\kappa\gamma_H)+\Delta c(1-\alpha\kappa\gamma_H)-D-f \qquad (117) \\
= & \pi_H^*+D+\Delta c(1-\alpha\kappa\gamma_H)-D-f \\
= & (2\Delta V-\Delta c)\frac{\alpha(1-\gamma_H)}{2}+\Delta c(1-\alpha\kappa\gamma_H)-f \\
= & \Delta V\alpha(1-\gamma_H)+\Delta c(1-\alpha\kappa\gamma_H-\frac{\alpha(1-\gamma_H)}{2})-f \\
= & \pi_L^*+\Delta c(1-\alpha\kappa\gamma_H-\frac{\alpha(1-\gamma_H)}{2})-f
\end{aligned}$$

which is $\leq \pi_L^*$ if

$$\begin{aligned}
f &\geq \Delta c(1-\alpha\kappa\gamma_H - \frac{\alpha(1-\gamma_H)}{2}) \\
&= \Delta c\left[\frac{2}{\Delta c}\{\pi_L^* - \pi_H^* + f\} - 1 - \frac{\alpha(1-\gamma_H)}{2}\right], \text{ using (109)} \\
&= \Delta c\left[\frac{2}{\Delta c}\{\frac{\alpha(1-\gamma_H)}{2}\Delta c + f\} - 1 - \frac{\alpha(1-\gamma_H)}{2}\right], \text{ using (112)} \\
&= \Delta c\left[\frac{\alpha(1-\gamma_H)}{2} - 1\right] + 2f
\end{aligned}$$

which reduces to $f \leq \left[1 - \frac{\alpha}{2} + \frac{\alpha\gamma_H}{2}\right]\Delta c$ and the latter follows from condition (114). It is obvious that deviation to disclosing and setting any price in the segment $(\widetilde{p}_H^D, \underline{p}_H^D)$ cannot be strictly gainful as the maximum amount it can sell (even if it is perceived as high quality) is identical to that at $\underline{p}_H^D$.Given restriction (b) on out of equilibrium beliefs, deviating to disclosing and charging a price below $\widetilde{p}_H^D$ will make buyers believe that it is a low quality firm and therefore the firm will and so the deviating firm will earn strictly less profit than it would if it did not disclose and charged the same price; we have already seen that the latter kind of deviation cannot be gainful. Finally, consider deviation by the low quality firm to disclosing and charging price $p_H^D \in (\overline{p}_H^D, p_H^{ND})$. The maximum possible deviation profit (i.e., even if the firm is perceived as high quality with probability 1) is given by:

$$\begin{aligned}
& [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p-\Delta V))](p-c_L) - D - f \\
= & [\alpha(1-\gamma_H) + (1-\alpha)(1-F(p_L))](p_L + \Delta V - c_L) - D - f, \text{ where } p_L = p - \Delta V \\
= & \left[\frac{p_L + \Delta V - c_L}{p_L - c_L}\right]\pi_L^* - D - f, \text{ using (101)}, \\
\leq & \left[\frac{\underline{p}_L + \Delta V - c_L}{\underline{p}_L - c_L}\right]\pi_L^* - D - f, \text{ (as } \left[\frac{p_L + \Delta V - c_L}{p_L - c_L}\right] \text{ is strictly decreasing in } p_L \\
= & \left[\underline{p}_L + \Delta V - c_L\right](1-\alpha\gamma_H) - D - f, \text{ using (??)} \\
= & \left[\overline{p}_H^D - c_L\right](1-\alpha\gamma_H) - D - f = \left[\overline{p}_H^D - c_H\right](1-\alpha\gamma_H) + \Delta c(1-\alpha\gamma_H) - D - f \\
= & \left[\underline{p}_H^D - c_H\right](1-\alpha\kappa\gamma_H) + \Delta c(1-\alpha\gamma_H) - D - f \\
\leq & \left[\underline{p}_H^D - c_H\right](1-\alpha\kappa\gamma_H) + \Delta c(1-\alpha\kappa\gamma_H) - D - f, \text{ as } \kappa \in (0,1) \\
= & \left[\underline{p}_H^D - c_L\right](1-\alpha\kappa\gamma_H) - D - f
\end{aligned}$$

which is $\leq \pi_L^*$ under condition (114) as shown above (see arguments following (117)). Thus, condition (114) is necessary and sufficient for ruling out any incentive to deviate. Finally, note that (65) implies that either (91) or (114) holds. This completes the proof **R.2.** Finally, one can easily show that for both types of equilibria analyzed above, the profit of a high quality firms is not larger than $\frac{\alpha}{2}(1-\gamma_H)(p_H^{ND}-c_H) = \alpha(1-\gamma_H)(\Delta V - \Delta c/2)$, while the profit of a low quality firm is not larger than $\alpha(1-\gamma_H)(\overline{p}_L - c_L) = \alpha(1-\gamma_H)\Delta V$. This completes the proof.

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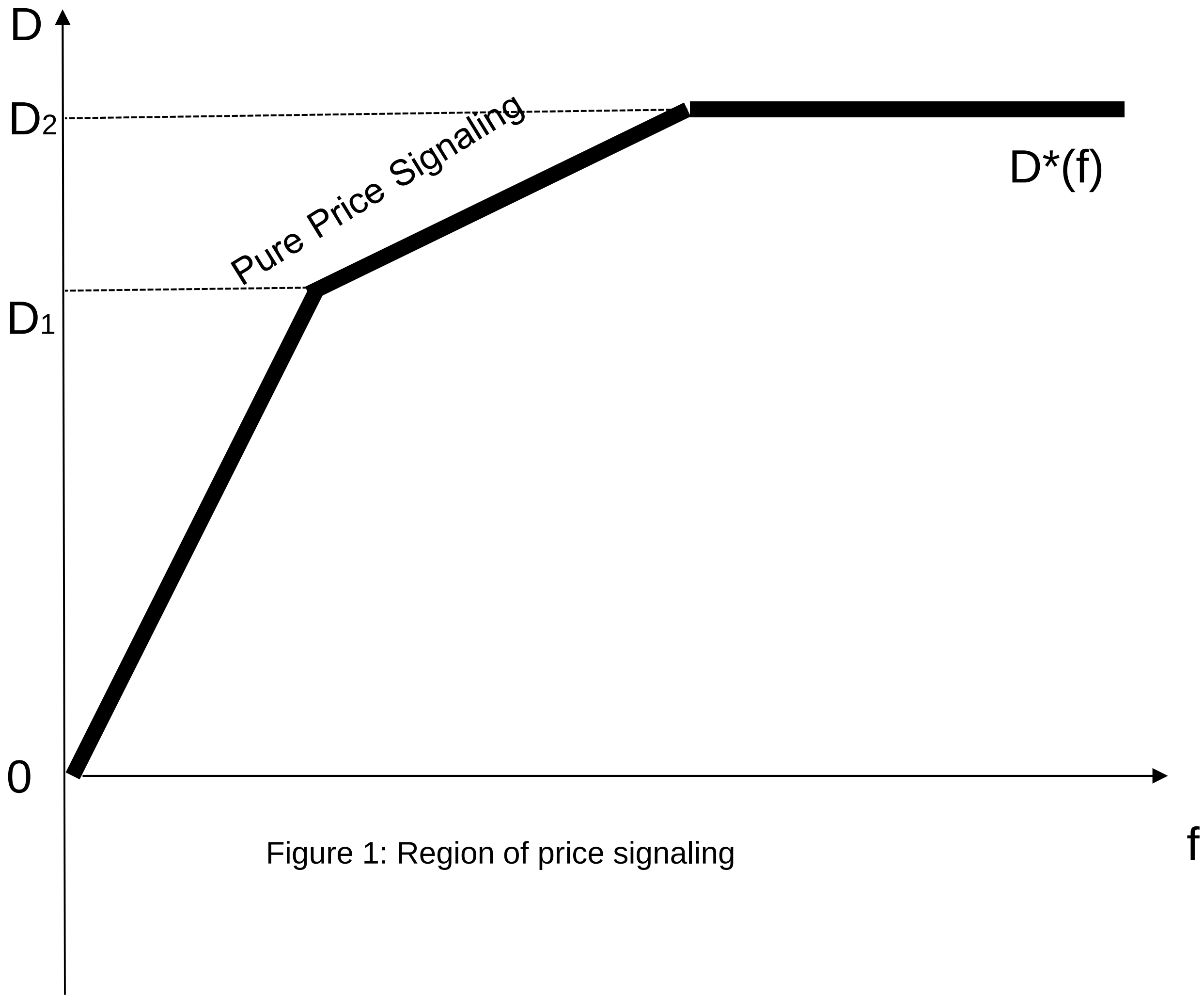

Figure 1: Region of price signaling

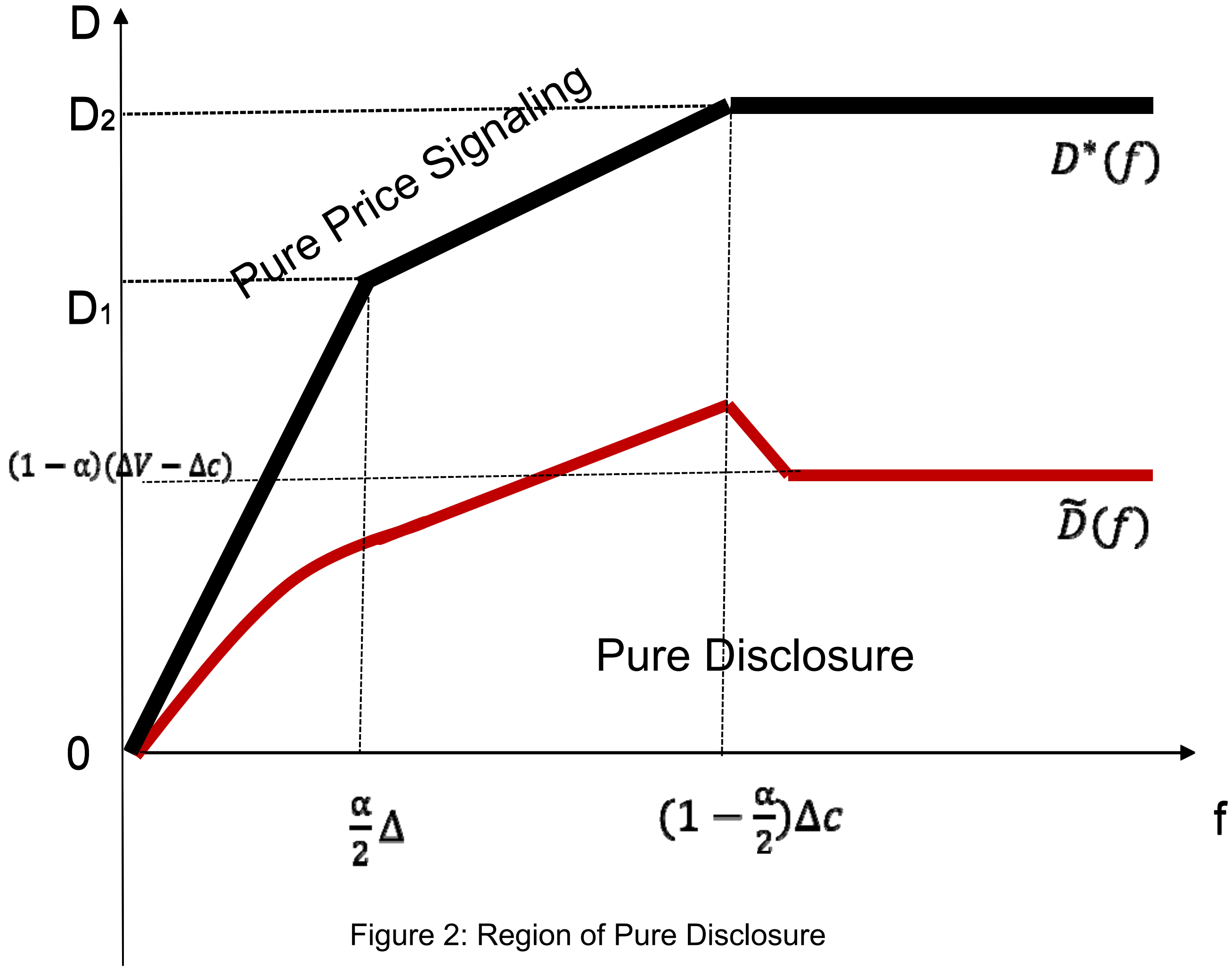

Figure 2: Region of Pure Disclosure