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Abstract

We describe consumers' dynamic decision-making under limited self-control, emphasizing the fatiguing nature of self-regulation. The temptation theory is extended in a two-good setting with tempting and non-tempting goods, where self-regulation in moderating tempting good consumption depreciates mental capital (willpower). The resulting non-homothetic feature of consumer preferences helps describe self-regulatory behavior in such an empirically relevant way that it depends on the nature of the tempting good (luxury or inferior) and on consumer wealthiness. First, richer consumers are more self-indulgent and impatient in consuming tempting luxuries, whereas less so in consuming tempting inferiors: marginal impatience is increasing in wealth for high-end brand wine whereas decreasing for junk foods. Second, self-control fatigue weakens implied patience for tempting good consumption. Third, upon a stressful shock, with the resulting increasing scarcity of willpower, self-indulgence and impatience for tempting good consumption increase over time. Fourth, without substantial difference in wealth holdings, naive consumers, unaware of the willpower constraint, display weaker self-control in the long run than the sophisticated consumers do. (165 wds)

Keywords: Self-control, fatigue, temptation, time preference, willpower, luxury, inferior, wealth.

JEL classification: D90, E21.

1 Introduction

Consumers are tempted by various kinds of attractive goods, either cheap (e.g., junk food) or extravagant (e.g., high-end brand wine). To attain high welfare, they have to regulate themselves not to indulge in consuming tempting goods. However, self-control is stressful and fatiguing (Miller et al., 2015): a self-regulating behavior at a given point in time makes the same behavior more costly thereafter. This would impede consumption smoothing: across goods, e.g., between tempting and non-tempting goods; across time, e.g., before and after a self-regulatory activity; and across states, e.g., across fatigued and unfatigued states. However, economics researchers so far have not discussed how the fatiguing nature of self-control would modify the standard consumer theory based on the permanent income hypothesis. This makes the existing economic theory unable to properly describe actual consumer behavior under temptation and resolve various related issues.

First, many empirical studies report that consumers behave as if they conserve self-control. For example, they become less self-regulatory in consuming tempting goods after an exhausting experience. After the Great East-Japan Earthquake, people in the damaged areas gambled, drank, and overate more (Hanaoka et al., 2015; Ohta et al., 2016). Experimental studies show that an increase in stressful cognitive burdens leads participants to lose self-control in economic decision-making (Shiv and Fedorkhin, 1999; Hinson et al., 2003; Fields et al., 2014). People weaken self-control when they have to make intensive self-regulatory efforts in the near future (Muraven et al., 1998; Lempert et al., 2012). All these behaviors seem to be inconsistent with the implicit assumption in the standard economic theory that consumers can exercise self-regulation to attain consumption smoothing without fatigue and at no cost. A deeper understanding of consumer behavior requires incorporating the constraint of limited self-control under which consumers make decisions.

Another unresolved issue relates to the interacting roles played by wealth and self-control in consumers' decision-making. For example, greater wealth holdings enable people to indulge in consuming tempting goods. Indeed, rich people tend to indulge in extravagant tempting goods, such as champagne, operas, world voyage, etc. In this sense, wealth plays a role as a substitute for self-control. On the other hand, there is evidence that poorer people are more likely to be intemperate towards tempting goods like alcohol, junk food, TV watching etc. (Banerjee et al., 2007). This implies that, in contrast to the case of champagne, poverty and intemperance (therefore wealth and self-control) are complementary with each other. These seemingly asymmetric properties of consumer behavior could be hardly explained by the traditional economic theory.

As self-control relates to patience (Fisher, 1930; Thaler and Shefrin, 1981), this behavioral issue could also have unique implications for how time preference is shaped for tempting good consumption. If self-control is a substitute for wealth, richer people will exhibit a higher time preference for tempting good consumption, whereas the opposite would be true if wealth and self-control are complementary. This would provide a new insight on time preference formation.

Motivated by these unresolved issues, we aim at developing empirically relevant consumer theory that incorporates intra- and inter-temporal trade-offs due to self-control fatigue under temptation. To do so, we propose a model with two unique features. First, by extending the temptation theory, à la Gul and Pesendorfer (2001, 2004) to a two-good framework, we consider an intertemporal utility maximizer that consumes tempting and non-tempting goods. Tempting good consumption x yields commitment and temptation utility, whereas non-tempting good consumption c generates only commitment utility. Although the consumer is tempted to maximize the temptation utility, his goal is to maximize his total lifetime utility by allocating efficiently his resources intratemporally and intertemporally. Such behavior entails self-control and inflicts mental costs proportional to the gap between maximized and realized levels of temptation utility.

Second, we incorporate the fatiguing nature of consumers' self-regulation by assuming that self-regulation under temptation entails a limited and depreciable mental capital, which we refer to as willpower. A certain degree of self-regulation to suppress tempting good consumption at a given point in time reduces the willpower available for self-regulation thereafter. The consumer has to take into account the resulting intertemporal self-control trade-offs when making decisions. This would shape consumption time-profile and, hence, generate time preferences for the two goods depending on the willpower state even when the subjective discount rate equals the real interest rate.

Due to the good-specific temptation and the state-dependent self-control costs, consumer preferences are generically non-homothetic. We begin by showing that if the intertemporal elasticity of substitution for the tempting good is small enough and/or if that for the non-tempting good is large enough, the tempting good has the nature of luxury, since the expenditure share on it goes up as the consumer gets wealthier; otherwise it is a generically inferior good, whose consumption level becomes lower as he gets richer.

The novelty of our model is that it enables us to describe, in an empirically-relevant way, how consumption dynamics and the associate self-regulating activities depend on the nature of the tempting good (luxury or inferior) as well as on consumer wealthiness. This will help better understand, from the viewpoint of efficient self-control allocation: (i) distinct consumption behaviors for tempting and non-tempting goods, (ii) the relationship between wealthiness (or poverty) and self-control (or self-indulgence), (iii) good-specific time preference formation, (iv) the effect of external stressful shocks on consumer behavior, and (v) the behavioral implications of being unaware of the self-control constraint.

Main results are as follows. First, we show that willpower plays a substitute or complementary role to wealth if the tempting good is a luxury or an inferior good, respectively. When the tempting good x is a luxury, a consumer exhibits weaker willpower to resist the temptation from x as he accumulates wealth, whereas when it is an inferior good, he displays stronger willpower when accumulating wealth. Putting otherwise, a wealthier consumer tends to be more self-indulgent in consuming tempting luxury goods, but more self-regulated in consuming tempting inferior ones.

Second, we consider time preferences deriving from tempting and non-tempting consumption behavior to show two implications in typical situation. First, willpower strengthens patience, measured by time preferences, for tempting good consumption, whereas it weakens that for non-tempting good consumption. This implies the occurrence of a domain effect: time preferences differ between tempting and non-tempting goods depending on the willpower state. Second, a wealthier consumer behaves less patiently in consuming a tempting luxury, whereas more patiently in consuming a tempting inferior. This is consistent with the fact that the richer tend to indulge more impatiently in consuming tempting luxuries, whereas the poorer tend to indulge more impatiently in consuming tempting inferiors. It has been controversial whether time preference is increasing (so called increasing marginal impatience, IMI) or decreasing (decreasing marginal impatience, DMI) in wealth (e.g., Uzawa, 1968; Lawrance, 1991; Hirose and Ikeda, 2012). To be noteworthy, our finding is that this depends on the nature of the domain good: time preference for tempting good consumption exhibits increasing or decreasing marginal impatience as it is a luxury or an inferior good, respectively.

Third, consistent with empirical results, upon a permanent stressful shock, a consumer's willpower is weakened due to mental fatigue in steady state. Along with the resulting increasing scarcity of willpower, self-indulgence and impatience for tempting good consumption increase over time.

These results are derived based on the restrictive assumption that consumers are sophisticated, in the sense that they are aware of the intertemporal dependence of self-control costs on willpower and incorporate it into their decisions. However, in reality, they will tend to underestimate the self-control capability (Loewenstein, 2000, p.60). We finally relax this assumption by considering the opposite polar case, in which consumers are completely naive: they are unaware of the presence of the dynamic willpower constraint and change their decisions adaptively to (unconscious) willpower variations over time. Due to the resulting inefficient usage of willpower, naive consumers are shown to have willpower more depleted, and consume more tempting goods in the long run than the sophisticated consumers do, insofar as there is no substantial difference in wealth holdings between the two.

The remainder of the paper proceeds as follows. Section 2 demonstrates relations of the present study to the literature. Section 3 presents the model and characterizes the optimal solutions. Section 4 discusses on the effects of initial values for wealth and willpower. Section 5 examines the effects of shifts in the long-run willpower constraint, such as external stressful shocks. Section 6 considers the case of naive consumers. Section 7 concludes and discusses on the remaining issues.

2 Relations to the literature

2.1 Self-control, willpower, and mental fatigue

In social psychology, Roy Baumeister and his joint researchers postulate the "ego depletion" hypothesis for which there exists a mental resource, measured in terms of the blood glucose level, whose depletion, called "ego depletion," causes failure in self-regulation (Baumeister and Vohs, 2003), and report many experimental results in support of the hypothesis (Hagger et al., 2010; Baumeister and Vohs, 2016).

Whereas there are controversies in psychology over the partial weakness of their experimental evidence for the ego depletion hypothesis (Baumeister et al., 2016; Hagger et al., 2016), research in other science fields, including brain and medical sciences, provide empirical evidence that self-regulatory efforts cause stress or mental fatigue, with detrimental effects on the quality of decision-makings thereafter. For example, stress studies show that after cognitive and/or non-cognitive tasks requiring self-regulation are experimentally imposed, participants' impatience and impulsivity rise with increased stress hormones secreted, e.g., cortisol (for a meta-analysis, see Fields, et al., 2014). Continuous mental burdens cause mental fatigue, giving long-lasting damage to the neuro-system governing the self-regulation capability (Okada et al., 2004; de Lange et al., 2008; Tanaka et al., 2014; Kelly et al., 2015). Self-control efforts in disadvantaged backgrounds lead to greater cardiometabolic risk and faster epidemic aging (Miller et al., 2015; Brody et al., 2016). All these findings imply that self-regulation fatigues mental system, thereby deteriorating the ability for further self-control. Note that fatigue can be mechanically described in models with a stock variable representing a depreciable resource. To capture the fatiguing nature of self-regulation, we thus introduce a depreciable mental capital, the willpower.

Psychology researchers provide the non-resource view of self-control failure: it is postulated that apparent depletion or the "refractory period" takes place in order to trigger a shift in motivation and attention in favor of cheaper behaviors in terms of opportunity costs (Inzlicht et al. 2012, 2014; Kurzban et al., 2013). In their descriptive theory, however, the determination of mental and opportunity costs of self-regulatory activities and the origination of motivations are an open question. In our model, consumers' behavioral motivations are modeled as induced by their intrinsic objects to maximize utility: they are incentivized to behave so that the marginal utilities, which reflect self-control costs, are equalized intratemporally and intertemporally. Self-regulation dynamics, either moderation or indulgence, are generated jointly with the endogenous determination of motivations (i.e., marginal utilities) and opportunity costs (i.e., marginal utilities of other behaviors) occurring under the willpower constraint.

We are also concerned with the finding in psychology for which people's self-control capacities are affected when their implicit theories regarding self-regulation ability are manipulated by beliefs of either limited or unlimited self-control capacity resources. Those made to believe they are limited show impaired self-regulatory performance (Job, et al., 2010, 2015). To reconcile our model to the psychological view of the self-control constraint, we specify self-control costs and the willpower stock in the utility terms, rather than in objective units such as blood glucose assumed by Baumeister in his ego-depletion theory. In our model, willpower availability can thus be affected in a context-dependent way by psychological factors such as "implicit theories," non-economic motivation, and/or reference self-control levels.

In sum, we abandon the dichotomy between the pure resource view and the pure psychological view, and instead attempt to integrate the competing key factors into an economic model for utility-maximizers. Such a synthetic approach is consistent with recent scientific research on brain and fatigue (Posner and Rothbart, 2012; Tanaka et al., 2014; Vohs et al., 2013; Evans et al., 2015), which emphasize the necessity of integrating the resource/biological view and the non-resource/psychological view to describe human behavior under limited self-control.

2.2 Economic theory of consumer willpower

Irrespective of the cumulative studies on self-control depletion in psychology and other fields, there are only a few articles in economics, to our best knowledge, which examine implications of limited self-control capacity for consumers' behavior. Loewenstein (2000) provides deep insights on the theoretical implications of the willpower constraint for decision-making in a descriptive way. He suggests the necessity to understand human behavior from the viewpoint of decisions on efficient willpower allocation. We pursue this approach in this study. Our research is strongly inspired by the article by Ozdenoren et al. (2012), which is the first analytical research on consumer behavior under limited willpower. However, as they limit their attention to a problem on how people eat a whole cake in a finite horizon, they do not consider the intratemporal choice between tempting and non-tempting goods or the consumption/saving decisions. Moreover, they assume that the utility function for tempting consumption is satiable, so that the tempting consumption level is constant over time. In our more general setting, the tempting consumption level is endogenously determined by wealth holdings, which enables us to describe luxurious and inferior temptations under non-homothetic preferences.

Loewenstein et al. (2015) develop a dual process model for human behavior in which deliberative and affective processes interactively determine human behavior under limited willpower. Although they show several testable predictions including those for intertemporal choice, the discussions are not analytical but limited to expositions based on illustrative models with exogenous willpower state. Consumers' decision-making in our model has a similar dual structure: one is temptation utility maximization, which could be regarded as an affective process, and the other is total utility maximization, which could be taken as a deliberative process. By incorporating endogenous willpower dynamics into the model, we shall characterize consumer behaviors through analytical solutions

2.3 Endogenous time preference theory

It has been pointed out that self-control has a critical importance in the determination of time preferences (e.g., Fisher, 1930; Thaler and Shefrin, 1981). Following the insight, this study attempts to explicitly describe how self-control determines time preferences in the framework of dynamic utility maximizers.

Our model has an important implication for a long controversy on whether consumers' time preference is increasing in their wealth (increasing marginal impatience: IMI) or decreasing (decreasing marginal impatience: DMI) (e.g., Uzawa, 1968; Epstein, 1987; Lawrance, 1991; Das, 2003; Hirose and Ikeda, 2012). We show that good-specific time preferences can be of either type, depending on the type of the good (tempting or not, luxury or inferior). Under weak conditions, consumer behavior is shown to imply IMI for a tempting luxury good, whereas DMI for a tempting inferior good. In the previous literature on endogenous time preferences, IMI or DMI is assumed to be exogenously given. Our model provides a micro-foundation to the shape of the time-preference schedule.

Frederick et al. (2002) suggest the possibility of a domain effect of time preference. By using the IMI-type endogenous time preference model, Ikeda (2006) shows that the level of time preference depends on whether the domain good is luxury or not. In this study, time preference displays IMI for luxuries, whereas DMI for inferiors.

3 Tempted consumers with limited self-control

3.1 The model with tempting and non-tempting goods

Consider a consumer who lives in infinite time $[0, \infty)$. There are two distinct consumption goods: a tempting good, x, and a non-tempting good, c. Good x (e.g., a sweet) is tempting in the sense that, as discussed more explicitly later, the consumer is tempted to consume a greater amount than he would once he can commits to maximizing long-run welfare. Thus, he has to exercise self-control to restrain himself from consuming too much. In contrast, good c (e.g., a vegetable) is not tempting: the choice to consume a small quantity of the good does not require any self-control effort.

Taking non-tempting good c as numeraire, let q denote the price of the tempting good x, assumed to be constant. Let a be consumer financial wealth. He is endowed with constant income y at each point in time. At the constant interest rate r, his flow budget constraint at time t ($t \in [0, \infty)$) is given by

$$\dot{a}_t = ra_t + y - qx_t - c_t, a_0 = \text{given} \tag{1}$$

where the overdot represents the time derivative, that is, $\dot{a}_t = da_t/dt$; and the initial financial asset stock a_0 is exogenously given.

Let $u(x)$ and $v(c)$ denote the commitment felicity from consumption x and c, respectively. Both functions are assumed to be twice continuously differentiable, strictly increasing, and strictly concave. As good x is tempting, the consumer has to make self-control efforts to attain higher long-run welfare. The associated self-control costs in each instant are specified as the product of required self-control efforts σ_t, and the utility costs per self-control effort $\gamma_t : \sigma_t \gamma_t$. To focus on the effect of limited self-control, assume that the self-control effort cost γ is a decreasing and convex function of the stock of a mental resource W that enables self-control effort: $\gamma = \gamma(W), \gamma'(W) < 0, \gamma''(W) > 0$. As in Ozdenoren et al. (2012) and Loewenstein et al. (2015), we refer to this mental resource as willpower. In this setting, the lifetime utility U_t is specified as the discounted sum of net felicity flows, that is, the commitment felicity minus self-control effort costs, $u(x_s) + v(c_s) - \gamma(W_s)\sigma_s - f$, where f is the independent self-control cost capturing exogenous needs for self-control effort.[1] For simplicity, assume that the subjective discount rate equals the interest rate r. Then, the lifetime utility is given by

$$U_t = \int_t^{\infty} \{u(x_s) + v(c_s) - \gamma(W_s)\sigma_s - f\} \exp(-r(s-t))\, ds \tag{2}$$

To describe the tempting nature of good x consumption and thereby specify self-control efforts in the simplest manner, assume that the consumer is tempted to maximize his temptation utility $\int_t^{\infty} u(x_s) \exp(-r(s-t))\, ds$ from good x consumption under the budget constraint (1) and the Non-Ponzi game condition (NPGC hereafter), where the temptation felicity function is assumed to be the same as the commitment felicity function u.[2] Define the associated indirect temptation utility as

$$V(a_t) = \max_{\{x_s\}_{s=t}^{\infty} \text{ s.t. (1) and NPGC}} \int_t^{\infty} u(x_s) \exp(-r(s-t))\, ds \tag{3}$$

Since the subjective discount rate is assumed to equal the interest rate r as for the commitment utility, the solution to the right-hand side of (3) is the flat time-profile $x_s = x_t^T$ for $s \in [t, \infty]$, where

$$x_t^T = \frac{ra_t + y}{q} \tag{4}$$

[1] It would be more natural to define the independent self-control needs f as a constant term in the function of self-control costs $\gamma\{f + \sigma_t\}$. However, we can show that the specification does not substantially change the main results below.

[2] Thus, we are assuming that a tempted consumer does not care about good c consumption. Alternatively we could assume that he is attracted by the maximization of the temptation utility under (1) and $c \geq \underline{c}$, where $\underline{c}$ is a constant subsistence level. However, this specification does not affect our model and the related results, except that the temptation consumption level x_t^T, given by (4), depends on the disposable wealth $a_t - \underline{c}/r$, rather than a_t only.

We refer to x_t^T as the temptation consumption level. With $x_s = x_t^T$ for $s \in [t, \infty)$, the indirect temptation utility function is given by

$$V(a_t) = \frac{u\left(\frac{ra_t + y}{q}\right)}{r} \tag{5}$$

To consider the required self-control efforts σ_t for the consumer at time t, imagine a *tempted self* or the affective system (Loewenstein et al., 2015) inside him who aims to maximize his own affective satisfaction (3). Although the most desirable behavior for the tempted self is to consume the amount x_t^T in each instant $s \in [t, \infty)$ without any saving, the consumer as the deliberate system actually behaves differently, i.e., consumes x_t ($\neq x_t^T$) and saves possibly non-zero $\dot{a}_t$. The resulting felicity loss for the tempted self is the required self-control effort. Formally, this is the difference between (i) the optimal felicity level that would be attained with the most tempting behavior, $u\left(x_t^T\right)$, and (ii) the suboptimal felicity level that the tempted self obtains when his time-t consumption and saving deviate from x_t^T and 0 to, respectively, the actual amounts x_t and $\dot{a}_t$, $u(x_t) + V'(a_t)\dot{a}_t$.[3] As $V'(a_t) = u'\left(x_t^T\right)/q$ from (5), the required self-control effort at time t is given by

$$\sigma_t = u\left(x_t^T\right) - \left\{u(x_t) + \left(u'\left(x_t^T\right)/q\right)\dot{a}_t\right\} \tag{6}$$

The first two terms on the right-hand side represent self-control necessity due to the difference between the temptation and actual consumption levels. The third term captures the negative effect that actual saving (for $\dot{a}_t > 0$) has on self-control need: saving increases the tempted self's felicity by $V'\dot{a}_t$, thereby reducing the need for self-control.

The relevancy of this specification for self-control effort flows can be checked by computing the discounted sum of the σ stream (6) by partial integration to obtain

$$\int_t^\infty \sigma_s \exp(-r(s-t))\,ds = V(a_t) - \int_t^\infty u(x_s)\exp(-r(s-t))\,ds, \tag{7}$$

which means that, as in Gul and Pesendorfer (2004), the discounted sum of the required self-control effort flows equals the maximized temptation utility

[3]Generally, the time-t felicity from the temptation consumption flow also equals $u\left(x_t^T\right) + V'\dot{a}_t^T$, where a_t^T denotes wealth holdings along the temptation consumption plan. However, $\dot{a}_t^T = 0$ holds along the optimal temptation consumption path. It follows that the time-t felicity from the temptation consumption flow is $u\left(x_t^T\right)$.

minus realized temptation utility.[4]

Willpower dynamics are generated by two factors. First, the consumer is endowed with a constant willpower recovery ψ at each point in time. Second, self-control effort $\gamma_t \sigma_t + f$ depletes the mental resource and, hence, decreases the willpower stock available in the next instant. In sum, starting from an exogenously given initial stock W_0, willpower evolves according to

$$\dot{W}_t = \psi - \alpha \left(f + \gamma_t \sigma_t\right), W_0 = \text{ given} \tag{8}$$

where $\alpha (> 0)$ denotes the strength of the willpower-exhausting effect of self-control efforts. We assume $\psi - \alpha f > 0$.

After Substituting (6) into (2) and (8), we summarize the consumer's problem as follows:

Problem (P): Choose the time profile $\{c_s, x_s, a_s, W_s\}_{t=0}^{\infty}$ that maximizes U_t:

$$\int_t^{\infty} \left[u\left(x_s\right) + v\left(c_s\right) - f - \gamma\left(W_s\right) \left\{ u\left(x_s^T\right) - u\left(x_s\right) - u'\left(x_s^T\right) \frac{\dot{a}_s}{q} \right\} \right] e^{-r(s-t)} ds \tag{9}$$

subject to (1), the initial conditions for (a_0, W_0), the NPGC, and

$$\dot{W}_t = \psi' - \alpha\gamma\left(W_t\right) \left\{ u\left(x_t^T\right) - u\left(x_t\right) - u'\left(x_t^T\right) \frac{\dot{a}_t}{q} \right\}, \tag{10}$$

where $\psi' \equiv \psi - \alpha f > 0$.

3.2 Optimal consumer behavior

Let λ and η denote the current-value shadow prices of willpower increments and saving, respectively. Then, the optimality conditions for problem (P)

[4] For the derivation of (7), see Appendix 1. As seen from (7), if γ were constant, the lifetime utility (2) would reduce to

$$\int_t^{\infty} \left\{u\left(x_s\right) + v\left(c_s\right)\right\} \exp\left(-r\left(s-t\right)\right) ds - \gamma \left\{ V\left(a_t\right) - \int_t^{\infty} u\left(x_s\right) \exp\left(-r\left(s-t\right)\right) ds \right\}$$

which is a typical specification of the intertemporal temptation model discussed by Gul and Pesendorfer (2004). However, we cannot rewrite (2) in such a simple form, because here self-control cost γ is not constant, but varies endogenously over time depending on the willpower fluctuation.

are obtained from the following current-valued Hamiltonian function

$$\begin{aligned} H_t &= u(x_t) + v(c_t) - f \\ &\quad -\gamma(W_t)\left\{u\left(x_t^T\right) - u(x_t) - u'\left(x_t^T\right)\frac{ra_t + y - qx_t - c_t}{q}\right\} \\ &\quad +\lambda_t\left[\psi' - \alpha\gamma(W_t)\left\{u\left(x_t^T\right) - u(x_t) - u'\left(x_t^T\right)\frac{ra_t + y - qx_t - c_t}{q}\right\}\right] \\ &\quad +\eta_t(ra_t + y - qx_t - c_t), \end{aligned} \tag{11}$$

as

$$\{1 + (1 + \alpha\lambda_t)\gamma(W_t)\}u'(x_t) - (1 + \alpha\lambda_t)\gamma(W_t)u'\left(x_t^T\right) = q\eta_t, \tag{12}$$

$$v'(c_t) - \frac{(1 + \alpha\lambda_t)\gamma(W_t)u'\left(x_t^T\right)}{q} = \eta_t, \tag{13}$$

$$\dot{\lambda}_t - r\lambda_t = (1 + \alpha\lambda_t)\gamma(W_t)\left\{u\left(x_t^T\right) - u(x_t) - \frac{u'\left(x_t^T\right)(ra_t + y - qx_t - c_t)}{q}\right\}, \tag{14}$$

$$\dot{\eta}_t = -\frac{r(1 + \alpha\lambda_t)\gamma(W_t)u''\left(x_t^T\right)(ra_t + y - qx_t - c_t)}{q^2}, \tag{15}$$

together with the NPGC for a and the transversality conditions for W.

Equation (12) requires that the marginal utility from the tempting good consumption x (LHS) to be equal to the shadow price of saving (RHS). The marginal utility of x is composed of the direct marginal commitment utility ($u'(x_t)$) plus the marginal net benefits of economizing self-control costs ($\{1 + (1 + \alpha\lambda_t)\gamma(W_t)\}u'(x_t) - (1 + \alpha\lambda_t)\gamma(W_t)u'\left(x_t^T\right)$). A similar condition for the non-tempting good consumption c is given by (13). Because of this, the optimal good c consumption is affected by the temptation consumption level x^T and willpower level W, even though good c is non-tempting. Combining (12) and (13) yields the usual condition, that is,

$$\frac{\{1 + (1 + \alpha\lambda_t)\gamma(W_t)\}u'(x_t)}{v'(c_t)} = q, \tag{16}$$

which equates the intratemporal marginal rate of substitution between x and c to their relative prices.

Equation (14) represents the Euler condition for the shadow price of willpower savings $\dot{W}$. From the assumption that the discount rate equals the interest rate, the Euler condition (15) for savings requires their shadow price to increase over time in proportion to the negative of the marginal benefits from saving.

We now discuss the properties of the optimum consumer behavior by linearizing the above dynamic system around a steady state point. As we assume the equality of the primitive discount rate and the interest rate r in utility function (2), the dynamic system has a zero root, so that the steady state solution depends on the initial values for the two state valuables, a_0 and W_0 (Giavazzi and Wyplosz, 1985). In what follows, we fix the initial state to an (a_0, W_0) value and discuss linearlized dynamics around the corresponding steady state. The sensitivity of the solution to the initial state is discussed in Section 4.

3.2.1 Tempting luxury or tempting inferior

Assuming that a steady-state solution (x^*, c^*, a^*, W^*) exists, we consider local dynamics around it. By combining the linearized versions of (16) and (13) through (15), tempting and non-tempting good consumption can be linked as follows (see Appendix 2 for the derivation):

$$\frac{c_t - c^*}{c^*} = -\frac{\theta^c}{\theta^x}\left(\frac{u'\left(x^T\right)}{u'\left(x\right) - u'\left(x^T\right)}\right)\frac{x_t - x^*}{x^*}, \tag{17}$$

where, as in what follows, the coefficients are all evaluated at the steady state; and θ^i (> 0) $(i = c, x)$ represent the intertemporal elasticity of substitution (IES) in terms of good i's felicity functions: $\theta^x = -\left(xu''/u'\right)^{-1}$ and $\theta^c = -\left(cv''/v'\right)^{-1}$. By construction, the optimal level of tempting good consumption is always smaller than its temptation level, $x < x^T$ hence $u'\left(x\right) > u'\left(x^T\right)$. Thus, (17) implies that x_t and c_t move in opposite directions over time.

By using (17), we can relate total consumption expenditures, $E \equiv c + qx$, to x by

$$E_t - E^* = \varepsilon q\left(x_t - x^*\right), \tag{18}$$

where

$$\varepsilon = \frac{q\left\{u'\left(x\right)x - u'\left(x^T\right)x^T\right\} + cu'\left(x^T\right)\left(\theta^x - \theta^c\right)/\theta^x}{qx\left\{u'\left(x\right) - u'\left(x^T\right)\right\}}. \tag{19}$$

As seen from (18), ε equals the inverse of the marginal propensity to spend on x from total expenditures $\left(\left(d\left(qx\right)/dE\right)^{-1}\right)$. We call it the marginal total expenditure-tempting good consumption ratio. Its sign decides the nature of each good. When $\varepsilon > 0$, an increase in E_t necessarily co-occurs with an increase in x and from (17), a decrease in c. This means that the tempting good x is marginally a luxury good: its share in consumption expenditures rises when the budget expands over time. Examples could be jewels, high-end brand wines, classical music concerts, extravagant liner voyages, etc. We call this case ($\varepsilon > 0$) the *tempting luxury* case. In contrast, when $\varepsilon < 0$, the tempting good consumption x decreases as the budget expands over time, meaning that x is marginally an inferior tempting good, such as junk food, cigars, karaoke, back-pack trips, etc. The case in which $\varepsilon < 0$ is referred to as the *tempting inferior* case.

Property 1: *Tempting good x is marginally a luxury or an inferior good as ε is positive or negative, respectively.*

When $\varepsilon = 0$, the effect of any over-time change in qx on total spending E is cancelled by the opposite change in c by exactly the same amount, so that E and, consequently, wealth holdings a are time-invariant. In what follows, we focus on the generic case $\varepsilon \neq 0$ by disregarding this dynamically trivial case.

Assumption 1: $\varepsilon \neq 0$.

Whether ε is positive or negative and, hence, whether the tempting good x is marginally a luxury or an inferior good depends on two properties, as seen from the two bracketed terms in the numerator of the RHS of (19): (i) whether the IES for good x, θ^x, is smaller than one or not (note that $u'(x)\,x - u'\left(x^T\right)x^T > 0 \Leftrightarrow \theta^x < 1$); and ($ii$) whether θ^x is greater than θ^c or not. For example, if $\theta^c < \theta^x < 1$, ε is positive; and hence, x_t is marginally a luxury good. If $\theta^c > \theta^x > 1$, ε is negative; and hence, x_t is an inferior good.

Remark 1: In the case of the standard time-additive utility, luxury consumption is characterized by a higher IES (Browning and Crossley, 2000; Ikeda, 2006). Property (ii) reflects this relationship: If x is easier to postpone than c, i.e., $\theta^x > \theta^c$, x is a luxury good as long as θ^x is not too large compared to one. However, unlike the time-additive utility case, this is not necessary: even when x's IES is smaller than c's, x can be a luxury good by property (i) if θ^x is sufficiently smaller than one.

3.2.2 Self-control, consumption, and wealth

By linearizing the optimality conditions (1), (4), and (10) though (15), we can obtain the three-dimensional autonomous system with a recursive structure (for the derivation, see Appendix 3):

$$\begin{pmatrix} \dot{x}_t \\ \dot{W}_t \end{pmatrix} = M \begin{pmatrix} x_t - x^* \\ W_t - W^* \end{pmatrix}, \tag{20}$$

$$\dot{a}_t = r\left(a_t - a^*\right) - q\varepsilon\left(x_t - x^*\right), \tag{21}$$

where

$$M = \begin{pmatrix} r + \alpha\gamma'\left\{u\left(x^T\right) - u\left(x\right)\right\} & \Delta\left\{u'\left(x^T\right) - u'\left(x\right)\right\} \\ \alpha\gamma\left\{u'\left(x\right) - \varepsilon u'\left(x^T\right)\right\} & -\alpha\gamma'\left\{u\left(x^T\right) - u\left(x\right)\right\} \end{pmatrix};$$

$$\Delta = \frac{-\gamma' r + \alpha\left(1 + \alpha\lambda\right)\left\{u\left(x^T\right) - u\left(x\right)\right\}\left(\gamma\gamma'' - \gamma'^2\right)}{-\left\{1 + \gamma\left(1 + \alpha\lambda\right)\right\}u''\left(x\right)}.$$

As x is jumpable and W is not, the local dynamics of (20) is stable and uniquely determined if and only if it is saddle-point stable, and hence, the determinant Φ of matrix M is strictly negative:

$$\Phi < 0$$

where

$$\begin{aligned} \Phi \;=\; & -\alpha\gamma'\left[r + \alpha\gamma'\left\{u\left(x^T\right) - u\left(x\right)\right\}\right]\left\{u\left(x^T\right) - u\left(x\right)\right\} \\ & -\alpha\gamma\Delta\left\{u'\left(x\right) - u'\left(x^T\right)\right\}\left\{u'\left(x\right) - \varepsilon u'\left(x^T\right)\right\}. \end{aligned} \tag{22}$$

Assumption 2: The dynamic system of (20) and (21) is saddle-point stable: $\Phi < 0$.

With Assumption 2, matrix M has one stable root and one unstable root. Let $\chi\,(< 0)$ denote the stable root. Stable root χ specifies a saddle-path trajectory for the optimal consumption dynamics. In particular, as the optimal time-path of willpower satisfies $\dot{W}_t = \chi\left(W_t - W^*\right)$, the relation between W_t and x_t is obtained by substituting it into the linearized version of (8) and rearranging the result as

$$x_t - x^* = \frac{\left\{u'\left(x\right) - u'\left(x^T\right)\right\}\left[\chi + \alpha\gamma'\left\{u\left(x^T\right) - u\left(x\right)\right\}\right]}{\alpha\gamma\left[\left\{u'\left(x\right) - u'\left(x^T\right)\right\}^2 + \frac{c}{qx}\frac{\theta^c}{\theta^x}u'\left(x^T\right)^2\right]}\left(W_t - W^*\right). \tag{23}$$

As the coefficient on the right-hand side of (23) is negative, the tempting good consumption is negatively associated with the willpower stock. From (17), this implies that the non-tempting goods and the willpower stock are positively associated. This can be summarized as follows.

Property 2: *A decrease over time in willpower is associated with consumption shift over time from non-tempting good c in favor of tempting good x.*

Property 2 shows that when a consumer's willpower depreciates over time, his self-control to overcome temptation from x gradually weakens to conserve the willpower. However, the resulting increase in tempting good consumption may or may not be associated with dissaving. It depends crucially on whether x is a luxury or an inferior good, as we shall show below.

To relate optimal wealth holdings with willpower, set $a_t - a^*$ as linear in $W_t - W^*$, and determine its coefficient such that it validates the linearized equation of (1) under (23). Then, we can obtain

$$a_t - a^* = -\Omega \left(W_t - W^* \right), \tag{24}$$

where

$$\Omega = \frac{q\varepsilon \left\{ u'(x) - u'\left(x^T\right) \right\} \left[\chi + \alpha\gamma' \left\{ u\left(x^T\right) - u(x) \right\} \right]}{\alpha\gamma \left(\chi - r \right) \left[\left\{ u'(x) - u'\left(x^T\right) \right\}^2 + \frac{c}{qx} \frac{\theta^c}{\theta^x} u'\left(x^T\right)^2 \right]} \geq 0 \Leftrightarrow \varepsilon \geq 0$$

Equation (24) implies that wealth holdings comove or inversely move with willpower, as ε is negative or positive, and hence, as the tempting good is a luxury or an inferior, respectively. In the tempting luxury case ($\varepsilon > 0$), wealth accumulation is positively associated with an increase in tempting good consumption, which reduces requirement of the self-control, and hence, the needs for willpower over time. On the contrary, an increase in willpower leads to reduction in tempting good consumption, and hence, in wealth size. Thus, the two budget variables, wealth a and willpower W, are *marginally substitutable* for each other in the sense that an increase over time in one reduces the need for the other.

The tempting inferior case ($\varepsilon < 0$) is characterized in a contrasting way. Wealth accumulation co-occurs with a decreasing process of tempting good consumption, which in turn requires willpower accumulation. On the contrary, willpower accumulation leads to a decrease in x and an increase in c by a greater amount. The resulting increase in total expenditures, in turn enhances the need for wealth, thereby leading to savings. Thus, when the tempting good is an inferior good, a and W comove over time: the two budget variables are *marginally complementary* in that an increase over time in one enhances the need for the other.

Property 3: *In the case of tempting luxury* ($\varepsilon > 0$)*, two budget variables, willpower and wealth, are marginally substitutable* ($sign\left(\dot{a}\right) = -sign\left(\dot{W}\right)$)*, whereas in the tempting inferior case* ($\varepsilon < 0$) *the two are marginally complementary* ($sign\left(\dot{a}\right) = sign\left(\dot{W}\right)$).

Intuitively, restraining tempting good consumption induces demand for willpower. When the tempting good is a luxury, a consumer who accumulates wealth indulges more easily in savoring it, and hence, decumulates unnecessary willpower. Similarly, when willpower is under accumulation, he will restrain himself from consuming the tempting luxury good and decumulate unnecessary wealth. Therefore, wealth and willpower play substitutable roles as budget variables. When the tempting good is an inferior good, poorer people eat more junk food having less willpower and vice versa. Thus, the two budget variables are marginally complementary.

This property helps characterize the consumer behavior in terms of wealth and willpower as follows:

Proposition 1: *The wealthier a consumer becomes, the less self-regulated he becomes in consuming a tempting luxury good, but more self-regulated in consuming a tempting inferior good.*

We can reinterpret this proposition in terms of the comparison between two consumers who differ only in the wealth-willpower states: a rich consumer is more self-indulgent in luxury wine and more self-regulated in junk food than a poor one.

3.2.3 Time preferences

From Property 2 and Proposition 1, we can conjecture that time preference, and hence, the degrees of impatience that shape the consumption time-profile would differ between tempting and non-tempting good consumption. For explicit discussion, we follow the literature (e.g., Epstein, 1987; Obstfeld, 1990) in defining the pure rate of time preference for good i $(i = x, c)$, δ^i, as the negative of the logarithmic time-derivative of the present-value marginal utility of i in the Voltera sense, evaluated at $\dot{i} = 0$. As the present-value marginal utility of good i consumption is expressed in terms of the current-value marginal utility as $\exp(-rt)\, MU_{it}$, where the current-value marginal utility MU_{xt} and MU_{ct} represent the LHS of (12) and (13), respectively, the time preference rate for each consumption good is given by

$$\delta_t^x = r - (\ln \dot{M}U_{xt})\Big|_{\dot{x}=0}, \quad \delta_t^c = r - (\ln \dot{M}U_{ct})\Big|_{\dot{c}=0} \tag{25}$$

From the expressions of MU_{xt} and MU_{ct}, the time preference rates depend on all the endogenous variables, $c_t, x_t, a_t, W_t, \eta_t$, and λ_t. Instead of characterizing the time preference functions in terms of partial derivatives with respect to each variable, we follow the literature (e.g., Becker and Mulligan, 1997) in focusing on the rates of time preference along the optimum consumption time path. They are the time preferences implied from the optimum consumer behavior. By substituting the optimality conditions into

(25), Appendix 4 derives the time preference rates as

$$\delta_t^i = r + \Lambda^i \left(W_t - W^*\right), i = x, c; \tag{26}$$

$$\begin{aligned}
\Lambda^x &= -\frac{r\chi\gamma\left(1+\alpha\lambda\right)u''\left(x^T\right)\left[\chi+\alpha\gamma'\left\{u\left(x^T\right)-u\left(x\right)\right\}\right]}{q\alpha\eta\left(\chi-r\right)\left\{u'\left(x\right)-u'\left(x^T\right)\varepsilon\right\}}\left(\varepsilon-\varepsilon_-\right) \\
\Lambda^c &= \frac{r\chi\gamma\left(1+\alpha\lambda\right)u''\left(x^T\right)\left[\chi+\alpha\gamma'\left\{u\left(x^T\right)-u\left(x\right)\right\}\right]}{q\alpha\eta\left(\chi-r\right)\left\{u'\left(x\right)-u'\left(x^T\right)\varepsilon\right\}}\left(\varepsilon_+-\varepsilon\right).
\end{aligned}$$

where, letting $\theta^T (= -\left[x^T u''\left(x^T\right)/u'\left(x^T\right)\right]^{-1} (>0))$ denote the IES for temptation felicity function, ε_- and ε_+ are defined as

$$\varepsilon_- = -\left(\frac{x^T}{x}\right)\left(\frac{1+\gamma\left(1+\alpha\lambda\right)}{\left(1+\alpha\lambda\right)\gamma}\right)\left(\frac{u'\left(x\right)}{u'\left(x^T\right)}\right)\left(\frac{r-\chi}{r}\right)\left(\frac{\theta^T}{\theta^x}\right)$$

$$\varepsilon_+ = -\left(\frac{u'\left(x^T\right)}{u'\left(x\right)-u'\left(x^T\right)}\right)\varepsilon_-$$

On the right hand sides of these equations, all the parenthesized factors except for θ^T/θ^x are greater than one, so that $\varepsilon_- < -\theta^T/\theta^x$ and $\varepsilon_+ > \theta^T/\theta^x$. We also have $\varepsilon \leq 1$. From these properties, we can show that under weak conditions, the marginal total expenditures-tempting good consumption ratio ε satisfies

$$\varepsilon_+ > \varepsilon > \varepsilon_-, \tag{27}$$

around the steady state, so that we have $\Lambda^x < 0$ and $\Lambda^c > 0$. Appendix 4 derives the sufficient conditions for (27), which include:

Example:

- $\varepsilon > \varepsilon_-$ if consumers are weakly prudent for x ($u''' \geq 0$) and $\theta^c \leq 1$;
- $\varepsilon < \varepsilon_+$ if $\theta^T \geq \theta^x$, or the IES for $u(x)$ is weakly increasing in x,

both of which are simultaneously met, for example, if u is of the CRRA type and $\theta^c \leq 1$.

In what follows, we assume (27) to focus on the normal situation:

Assumption 3: The marginal total expenditures-tempting good consumption satisifies (27).

Remark 2: If $\varepsilon < \varepsilon_-$, consumers would paradoxically increase (or decrease) x_t over time when the corresponding time preference δ_t^x is *higher* (or *lower*) than its steady state value r: he would restrain (or indulge in) x when he is impatient (or patient). This could occur because in that case the marginal return of saving is high (or low) under $W_t > W^*$ (or $W_t < W^*$), which leads him to restrain (or indulge in) x irrespective of a high (or low) time preference.[5] Similarly, if $\varepsilon > \varepsilon_+$, c increases (or decreases) over time when $\delta_t^c > r$ (or $\delta_t^c < r$) as the return of savings becomes dominantly high (or low) under $W_t < W^*$ (or $W_t > W^*$). However, unlike the normal case that we focus on, it is difficult to show explicitly the parametric regions in which these paradoxical cases occur.

With Assumption 3, (26) implies that a greater willpower is associated with a lower rate of time preference for tempting good consumption ($\Lambda^x < 0$). This reflects consumers' behavior toward efficient self-regulation. Consumers that are too fatigued mentally to exercise strong self-control required for the long-run optimum allow themselves to impatiently shift consumption from non-tempting to tempting goods, and thereby, save the self-control resource. In contrast, with willpower exceeding the long-run need, they patiently restrain themselves from consuming tempting goods by incurring the self-control costs. We summarize the property of impatience for x as follows:

Property 4: *Impatience for tempting good consumption, measured by its optimal time preference, is (i) higher or lower than its steady-state value (r), as willpower is currently more or less scarce than it would be in the future steady-state; and (ii) weakened by self-control fatigue.*

The literature has sometimes noted the possibility that time preference depends on its type of consumption domain (e.g., Frederick et al., 2002). Equation (26) reveals that such a "domain effect" on time preference occurs when the consumer preferences for individual goods depend on their degrees of temptingness and the willpower state.[6]

[5] Formally, note that the Euler condition is written in terms of time preference δ^x as

$$\dot{x}_t = -(MU_x^x / MU_{xx}^x)(H_{at} - \delta_t^x),$$

where the marginal return of saving is given by H_a, i.e., the first derivative of the currrent-value Hamiltonian by a. The derivative H_a equals the negative of the RHS of (15). We can show that H_a positively depends on $W - W^*$ when $\varepsilon < \varepsilon_-$, whereas it negatively depends on $W - W^*$ when $\varepsilon > \varepsilon_+$.

[6] See Ikeda (2006), which shows a domain effect on time preference that occurs between luxury and necessity goods.

This domain-dependent nature of time preferences has an interesting implication if we relate δ^i to wealth holdings. Substituting (24) into (26) yields

$$\delta_t^i = r - \frac{\Lambda^i}{\Omega}\left(a_t - a^*\right), i = x, c. \tag{28}$$

Recall that Ω has the same sign as ε. Thus, coefficient $-\Lambda^i/\Omega$ has the same sign as ε for x and a different sign from ε for c. In the literature (e.g., Epstein, 1987; Obstfeld, 1990), time preference is said to exhibit increasing marginal impatience (IMI) or decreasing marginal impatience (DMI), as it increases or decreases in wealth holdings. With Assumption 3, (28) implies that time preference for tempting good x can exhibit IMI or DMI depending on whether x is a luxury or an inferior good, as summarized below.

Proposition 2: *The wealthier consumers become, the less patient they are in consuming a tempting luxury, whereas the more patient they are in consuming a tempting inferior good. That is, consumer behavior exhibits IMI for a tempting luxury good but DMI for a tempting inferior good.*

Proposition 2 describes anecdotal contrasting behaviors of consumers for luxurious tempting goods and for inferior tempting goods in terms of impatience. Consumers tend to indulge more in consuming high-end brand wine impatiently, as they become richer, whereas they are likely to be more impatient for junk food as they become *poorer*.

Two points are noteworthy. First, in previous theoretical studies on endogenous time preference, IMI or DMI is exogenously specified. Proposition 2 shows that a good-specific time preference can be of either type, depending whether the domain good is tempting or not, and if it is a luxury or an inferior good. Therefore, this model provides a micro-foundation to the shape of time preference.

Second, from (28) for the non-tempting good consumption c, the time preference for c is of the DMI type when it is an inferior whereas it is of the IMI type when it is a luxury. Combining this with Proposition 2 yields the following corollary:

Corollary 1: *Time preference is of the IMI type for luxury good consumption, whereas it is of the DMI type for inferior good consumption.*

This is a natural consequence of the definitions of luxury and inferior goods, because the consumption propensities increase in wealth (permanent income) for luxury goods and decrease for inferior goods.

3.2.4 Steady-state solution

The steady-state solution $(x^*, c^*, a^*, W^*, \lambda^*, \eta^*)$ is obtained by setting $\left(\dot{W}, \dot{a}, \dot{\lambda}\right) = 0$ in the optimality conditions (8) and (1) through (15). Explicitly, it is determined by the following six equations:

$$\alpha\gamma(W^*)\left\{u\left(\frac{ra^*+y}{q}\right) - u(x^*)\right\} = \psi', \tag{29}$$

$$ra^* + y = qx^* + c^*, \tag{30}$$

$$\frac{\{1 + (1+\alpha\lambda^*)\gamma(W^*)\}u'(x^*)}{v'(c^*)} = q, \tag{31}$$

$$\lambda^* = -\frac{\gamma'(W^*)\left\{u\left(\frac{ra^*+y}{q}\right) - u(x^*)\right\}}{r + \alpha\gamma'(W^*)\left\{u\left(\frac{ra^*+y}{q}\right) - u(x^*)\right\}}, \tag{32}$$

$$a^* - a_0 = -\Omega(W^* - W_0), \tag{33}$$

$$\eta^* = v'(c^*) - (1+\alpha\lambda^*)\frac{\gamma(W^*)}{q}u'\left(\frac{ra^*+y}{q}\right). \tag{34}$$

To understand the determination of the steady state solution, eliminate λ^* from (31) by substituting (32). Using the resulting equation and (30), we can express consumption x^* and c^* as

$$x^* = X\left(\underset{(-)}{W^*}, \underset{(+)}{ra^*+y}\right) \text{ and } c^* = C\left(\underset{(+)}{W^*}, \underset{(+)}{ra^*+y}\right), \tag{35}$$

where signs below the arguments indicate the signs of the corresponding partial derivatives. Substitution of X for x^* in (29) yields

$$\alpha\gamma(W^*)\left\{u\left(\frac{ra^*+y}{q}\right) - u(X(W^*, ra^*+y; q))\right\} = \psi'. \tag{36}$$

As (36) constrains W^* and a^* such that self-control costs (LHS) equals the constant rate of willpower net recovery (RHS), we call it the *long-run willpower constraint* (LWC).

We refer to (33) as the *long-run saddle trajectory* (LST). The LST goes through the initial value points (W_0, a_0). The long-run willpower W^* and wealth holdings a^* are jointly determined at the intersection of LWC and LST. As illustrated graphically in Appendix 5, the slopes of the two schedules depend on whether x is a luxury ($\varepsilon > 0$) or an inferior ($\varepsilon < 0$) in two ways. First, from (24) and Property 3, the LST is negatively or positively sloping as ε is positive or negative. Second, the relative magnitudes of the slopes of the two schedules depend on the sign of ε. The slope of LWC can be either positive or negative even when ε takes a certain sign. It depends on the relative magnitudes of direct and indirect effects that willpower has on self-control costs: an increase in willpower directly reduces self-control costs (i.e., $\alpha\gamma' < 0$), which we call the *SCC-reducing effect*, hereafter, while it enlarges the costs indirectly by decreasing tempting good consumption, and hence, enlarging self-control requirement (i.e., $-\alpha\gamma X_W > 0$), which we call the *SCR-enlarging effect*, hereafter. When ε is positive, the slope of the LWC is negative or positive, as the negative SCC-reducing effect dominates or is dominated by the positive SCR-enlarging effect. The opposite is true when ε is negative.

In sum, the properties of (W^*, a^*) depend on whether x is a luxury or an inferior good and whether the SCC-reducing effect dominates, or is dominated by, the SCR-enlarging effect. With (W^*, a^*) determined at the intersections of the two schedules, the steady-state consumption basket (x^*, c^*) is determined by (35), and the shadow prices λ^* and η^* are decided by (32) and (34), respectively. Table 1 summarizes the long-run properties of the optimum solution by conducting comparative statics (see Appendix 6 for the analytical results of the comparative statics). We shall discuss them briefly in the following two sections.

Insert Table 1.

4 Initial values of wealth and willpower

As the long-run saddle trajectory (LST) depends on the initial values of W_0 and a_0, the consumer's long-run behavior is affected by the strength of willpower in earlier life stages and the wealth he was born into.

4.1 Initial willpower

An increase in initial willpower W_0 shifts the LST schedule to the right, and thereby, increases the long-run willpower W^*, as summarized in Table 1. This is consistent with an empirical finding in psychology that self-control in one's early childhood predicts cognitive and self-regulatory competence in later stages (e.g., Shoda, et al., 1990; Moffitt, et al., 2011). The result theoretically justifies the importance of upbringing and education in early childhood stressed in the empirical literature (e.g., Heckman, 2006; Bucciol, et al., 2010).

Note, however, that in our model with endogenous wealth holdings, stronger self-regulation owing to greater initial willpower may not result in moderation in tempting good consumption x^*, as the steady-state wealth holdings a^*, and hence, disposable income, may increase.

4.2 Initial wealthiness

As an increase in initial wealth a_0 shifts the LST schedule upward, its effect on the long-run self-control depends crucially on whether the tempting good is a luxury or an inferior good. When the tempting good is a luxury, a larger a_0 allows the consumer to indulge more in enjoying the tempting luxury in early periods and save the self-control resource for later. It follows that initially wealthy consumers will exhibit *ceteris paribus* stronger willpower in consuming tempting luxuries in the long run. This relationship might be more intuitive if restated as follows: an initially poor consumer will exhibit weaker willpower in consuming tempting luxuries in the long run. This is similar to the story of "parvenus", who indulge in consuming tempting luxuries that they had to moderate in their earlier poor life stages of relative poverty.

In contrast, when the tempting good is an inferior, initially poor consumers will be more self-regulated in consumption of the good in the long run because they indulge in consuming the inferior good in their earlier poor life stages, thereby saving self-control in the long run.

Property 5: *In the long run, initially poorer consumers display weaker self-control in consuming tempting luxuries (parvenu effect), whereas exhibit stronger self-control in consuming tempting inferiors.*

5 Shifts in the long-run willpower constraint

Let us next examine the effects that shifts in the LWC schedule have. To do so, we consider an exogenous increase in external needs for self-control f, and hence, a decrease in net recovery rate $\psi'(= \psi - \alpha f)$, and a negative income shock to endowed income level y. Subsequently, the steady-state point moves along the LST schedule. Unlike the shift in the LST-schedule shifting, the substitutability/complementarity relationships between W_t and a_t that hold marginally along the transition path (see Property 3) are retained in the long run between W^* and a^*. With a shift in the LWC, two budget variables W^* and a^* change in the opposite directions, as if they were substitutes, if $\varepsilon > 0$, whereas they comove in the same directions as if they were complementary, if $\varepsilon < 0$.

5.1 External stressful shocks

A typical nature of consumer behavior under limited self-control is that it is affected by outside stressful events, for example, troubles in human relationships, loss of family members, heart-break, etc., that are supposedly irrelevant to consumption choices. This property is seen by supposing an exogenous increase in external needs for self-control f, and hence, a decrease in net recovery rate $\psi'(=\psi-\alpha f)$. As summarized in Table 1, the outside stressful event due to a decrease in ψ' leads to reduction in long-run willpower. Given the reduction in W^*, wealth holdings a^* increase or decrease, as the tempting good is a luxury ($\varepsilon > 0$) or an inferior ($\varepsilon < 0$), and hence, as a^* is substitutable with, or complementary to W^*.

As tempting good consumption x^* ($= X(W^*, ra^*+y)$) depends on a^*, the signs of the effect on x^* also depend on whether it is a luxury or an inferior good. When it is a luxury, x^* necessarily goes up with the stressful event. When it is an inferior good, in contrast, we cannot exclude the paradoxical possibility that the same shock leads to reduction in x^* through decrease in a^*. Intuitively, in this case, the consumer reduces the self-control requirement σ, instead of reducing its unit cost, by reducing wealth holdings a^*, and hence, the temptation consumption level x^{T*} (see (4)).

Note that irrespective of whether the effect on x^* is positive or negative, x_t necessarily exhibits an increasing time-path in the interim run. This occurs because a once-and-for-all stressful shock makes future willpower more scarce ($W_t > W^*$), and thereby, makes the consumer temporarily patient in consuming x in the interim run (see (i) in Property 4). As the scarcity of willpower gradually increases due to mental fatigue over time, the consumer loses patience in consuming x.

Proposition 3: *A permanent stressful shock weakens a consumer's willpower in the long run. Along with the increasing scarcity of willpower, self-indulgence, and hence, impatience for tempting good consumption increases over time.*

Remark 3: The reduction in willpower due to an exogenous stressful shock is consistent with the existing empirical reports that an increase in external burden on self-control results in loss of self-regulatory power in decision-making (Shiv and Fedorkhin, 1999). On the other hand, a low δ^x, compared to r, in the interim run appears to be inconsistent with the empirical results that experimentally imposed stress tasks lead to rises in participants' subjective discount rates for money (Hinson et al., 2003; Fields et al., 2014), if money is a tempting good. This apparent contradiction occurs because the stressful shock under consideration here is a once-and-for-all shock, which makes current self-control less scarce than in future. In contrast, experimentally imposed stress shocks are temporary, in which case current self-control would become scarcer than in future, and impatience temporarily increases.

5.2 Negative income shock

Consider a permanent decrease in endowment income y. From (35) and (36), it shifts the LWC schedule downward and definitely increases a^*, implying that the consumer mitigates the effect of the negative income shock on the budget by increasing wealth holdings through savings. The effect on W^* is negative or positive as x is a luxury or an inferior, and hence, as W^* is substitutable with, or complementary to a^*.

Paradoxically, Table 1 suggests the possibility that x^* increases after the negative income shock even when the tempting good is marginally a luxury ($\varepsilon > 0$). This can happen for two reasons. First, a decrease in y can increase the steady-state disposable income $ra^* + y$ through the enhancing effect on a^*. It can be shown that $d\left(ra^* + y\right)/dy$ is positive or negative, as the SCC-reducing effect of self-control is smaller or larger than the SCR-enlarging effect. Thus, with the dominating SCC-reducing effect, the negative income shock leads to an increase in the long-run disposable income, which has a positive effect on x^*. Second, when $\varepsilon > 0$, W^* falls after the shock, which again has a positive effect on x^*. It follows that a negative income shock increases tempting good consumption if the SCC-reducing effect of self-control dominates the SCR-enlarging effect and if the tempting good is a luxury.

6 Naive consumers

We have so far considered a *sophisticated* consumer who incorporates the willpower constraint completely in decision-making. However, actual consumers often display the naivety and do not incorporate the willpower constraint completely, failing to efficiently self-control. We shall examine the effect of such naiveté.

6.1 Naive dynamics

Consider a completely naive consumer who does not incorporate the flow budget equation for willpower (8) into his decision-making at all. We call him consumer N. He derives utility from the same utility function (2) as the sophisticated consumer (consumer S, hereafter) would do, but is not aware that it actually depends on his willpower level. He takes the influences of variations in willpower on utility as external preference shifts.

We denote naive consumer N by superscript N. The optimality conditions for the time-t self are given by

$$\frac{\{1+\gamma(W_t^N)\}u'(x_t^N)}{v'(c_t^N)} = q, \tag{37}$$

together with (1), (8), (13), (15), and the transversality condition for a^N, with superscript N added to the variables. Based on these conditions, consumer N makes a lifetime consumption plan at each point in time t, and consumes (x_t^N, c_t^N) according to the time-t plan. However, as willpower evolves over time, the plan is overwritten. From (13), (15), and (37), he plans consumption of both tempting and non-tempting goods as constant over time, and hence, keeps saving zero to satisfy the transversality condition for a_t^N. The asset holdings stay at the initial value a_0^N, so that the consumption basket (x_t^N, c_t^N) always satisfies the zero-net saving condition:

$$ra_0^N + y = qx_t^N + c_t^N. \tag{38}$$

Using (38), eliminate c_t from (37) to obtain

$$\frac{\{1+\gamma(W_t^N)\}u'(x_t^N)}{v'(ra_0^N + y - qx_t^N)} = q, \tag{39}$$

which can be solved for x_t^N. Substituting it to (38) yields the solution for c_t^N. We express the results as

$$x_t^N = X^N\left(\underset{(-)}{W_t^N}; \underset{(+)}{ra_0^N + y}\right) \text{ and } c_t^N = X^N\left(\underset{(+)}{W_t^N}; \underset{(+)}{ra_0^N + y}\right). \tag{40}$$

Note that different from (35), behavioral relation (40) does not represent optimal strategic responses to willpower changes: however, it captures unexpected adaptive adjustments to preference shifting due to unconscious willpower changes. We obtain the autonomous willpower dynamics that are external to consumer N by substituting (40) into (8) as

$$\dot{W}_t^N = \psi' - \alpha\gamma(W_t^N)\left\{u\left(\frac{ra_0^N + y}{q}\right) - u\left(X^N\left(W_t^N; ra_0^N + y\right)\right)\right\}. \tag{41}$$

Letting Π denote

$$\begin{aligned}\Pi \equiv\ & \gamma(W^N)u'(x^N)^2 \\ & + \left\{u\left(\frac{ra_0^N + y}{q}\right) - u(x^N)\right\}\left[\{1+\gamma(W^N)\}u''(x^N) + q^2 v''(c^N)\right],\end{aligned}$$

the necessary and sufficient condition for the stability $d\dot{W}/dW < 0$ is

$$\Pi > 0. \tag{42}$$

Assumption 4: The external willpower dynamics for the naive consumer (41) are stable: $\Pi > 0$.

As in the case of sophisticated consumers, an increase in W^N has a direct negative SCC-reducing effect on the self-control cost through $\alpha\gamma\left(W_t^N\right)$ and an indirect positive SCR-enlarging effect on it through $-u\left(X^N\left(W_t^N\right)\right)$. Assumption 4 requires that the SCR-enlarging effect dominates the SCC-reducing effect.

6.2 The naiveté effect

We obtain the long-run optimal solution for the naive consumer by setting $\dot{W}^N = 0$ in (41). The resulting long-run self-control constraint

$$\alpha\gamma\left(W^{N*}\right)\left\{u\left(\frac{ra^{N*}+y}{q}\right) - u\left(X^N\left(W^{N*}; ra^{N*}+y\right)\right)\right\} = \psi' \tag{43}$$

determines W^{N*}, in which we set $a^{N*} = a_0^N$ from the zero net-saving condition, (38). Given the W^{N*} value, x^{N*} and c^{N*} are given from (40).

To observe how self-control inefficiency due to naiveté affects consumer behavior, compare consumers N and S who are identical in preferences and income y. A close look at (39) and (31) reveals

$$X^N\left(W^{N*}; ra^{N*}+y\right) < X\left(W^*; ra^*+y\right) \text{ if } \left(W^{N*}, a^{N*}\right) = \left(W^*, a^*\right), \tag{44}$$

where the sophisticate's tempting good consumption function X is given by (35). That is, in steady state, with the same willpower stock and the same wealth holdings, consumer N would keep the level of the tempting good consumption lower than consumer S would do, meaning that consumer N unconsciously exercises too much self-control.

Let us define the left-hand side of (43) as the steady-state self-control cost function for the naive consumer $Z^N\left(W^{N*}, a^{N*}\right)$. The corresponding function $Z\left(W^*, a^*\right)$ for the sophisticated consumer is given by the left-hand side of (36). From inequality (44), if $\left(W^{N*}, a^{N*}\right) = \left(W^*, a^*\right)$, we have

$$\begin{aligned} & Z\left(W^*, a^*\right) - Z^N\left(W^{N*}, a^{N*}\right) \\ = \; & \alpha\gamma\left(W^*\right)\left\{u\left(X^N\left(W^{N*}; ra^{N*}+y\right)\right) - u\left(X\left(W^*, ra_0+y\right)\right)\right\} \\ < \; & 0, \end{aligned}$$

meaning that without difference in willpower and wealth between consumers S and N, N should incur greater self-control costs in the long run than S would. As the values of Z and Z^N should commonly equal ψ' in the steady state, this inequality should be cleared by the adjustments of steady-state values of willpower and/or wealth holdings. From (42), we have $dZ^N/dW^N > 0$. It follows that, if $a^{N*} = a^*$, W^{N*} should be smaller than W^*. From (36) and (43), this in turn implies that x^{N*} is larger than x^*.[7]

Proposition 4: *With the same asset holdings, naive consumer N would have weaker willpower and consume more tempting goods than sophisticated consumer S would in the steady state:*

$$W^{N*} < W^* \text{ and } x^{N*} > x^* \text{ if } a^{N*} = a^*.$$

Similarly, we can consider the effect of being naive on wealth holdings by comparing consumers N and S with the same willpower stock.[8] We can easily show that an increase in a^{N*} decreases or increases the self-control requirement $u\left(x^{TN}\right) - u\left(x^{N*}\right)$, and hence, self-control cost Z^N as x is a luxury ($\varepsilon > 0$) or an inferior ($\varepsilon < 0$). Under stability condition (42), this implies the following result.

Proposition 5: *The steady-state solutions for naive consumer N and sophisticated consumer S satisfy the following:*

1. *Tempting luxury case ($\varepsilon > 0$): With the same willpower stock, naive consumer N would be richer and consume more tempting goods than sophisticated consumer S would.*

$$a^{N*} > a^* \textit{ and } x^{N*} > x^* \text{ if } W^{N*} = W^*.$$

2. *Tempting inferior case ($\varepsilon < 0$): With the same willpower stock, naive consumer N would be poorer and consume less tempting goods than sophisticated consumer S would.*

$$a^{N*} < a^* \text{ and } x^{N*} < x^* \text{ if } W^{N*} = W^*.$$

[7] By choosing intial values a_0^N and a_0, we can construct consumers N and S with the same utility function and the same steady-state wealth holding: $a^{N*} = a^*$.

[8] Consumers N and S with the same utility function can be constructed such that $W^{N*} = W^*$ by choosing intial values W_0^N and W_0.

When x is a luxury, the loss of efficiency in self-control leads to weakened self-regulation in moderating tempting good consumption. The resulting increase in x^{N*} is financed by greater wealth holdings, which is attained by reducing non-tempting good consumption, and thereby, increasing savings. When x is an inferior, the conservation of willpower to reduce self-control cost Z^N takes place in the form of wealth decumulation and the resulting decrease in temptation consumption level $x^{TN}(= \left(ra^{N*} + y\right)/q)$.

7 Conclusions and discussions

Using a model with fatiguing self-regulation, we describe consumer behavior from the integrated viewpoint of intratemporal choices between tempting and non-tempting goods, intertemporal consumption/saving choices, and intertemporal allocation of self-regulation. The degree of impatience implied from optimal consumption choices depends on how tempting the domain good is; whether the tempting good is a luxury or an inferior; and whether the consumer is sophisticated or naive. The solution describes asymmetric consumer behaviors toward tempting luxuries and tempting inferiors in an empirically-relevant way. In particular, we show that richer consumers are more self-indulgent and, under weak conditions, more impatient for tempting luxuries, whereas this is less so for tempting inferiors. Naive consumers that are unaware of the willpower constraint will exhaust their willpower and indulge in tempting consumption in the long run more than the sophisticated consumers would.

In the analysis, we implicitly assume that self-regulatory efforts just reduce willpower, but do not enhance it. This assumption may be somewhat limited compared to the evidence that willpower is malleable (Muraven et al., 1999). In Appendix 7, we weaken it by considering that self-regulation in moderating tempting good consumption has a willpower-toughening effect as well as a willpower-depreciating effect. In this case, if the willpower-toughening effect were much greater than the willpower-depleting effect, consumers with weak willpower might moderate their consumption of tempting goods patiently to enhance their future willpower; therefore, consumer behavior relates to willpower in opposite ways to the case of the dominant willpower-depreciating effect. Whether or not such an implausible prediction actually occurs is an empirical issue.

We leave several important tasks undone. First, we have assumed that the primitive discount rates are identical for both commitment and temptation utility. However, as discussed by Loewenstein et al. (2015), consumers may be more impatient for temptation utility than for commitment utility.

It would be interesting to describe consumers by solving the resulting time-inconsistent utility-maximization problem. Second, the theoretical predictions should be tested using empirical and/or experimental data. Third, Appendix 7 specifies an extended model by simply assuming that the willpower-depleting and -enhancing effects of self control commonly occur instantly. A longer delay is more plausible for the willpower-enhancing effect than for the willpower-depleting effect. In such a realistic setting, a self-regulatory behavior would exert non-monotonic effects on consumers' decision-making, leading to cyclical behaviors. Our next step will be to tackle these issues.

Appendices

Appendix 1. Derivation of (7)

Substituting (6) into the left-hand sides of (7), we have

$$\begin{aligned}\int_t^\infty \sigma_s \exp\left(-r\left(s-t\right)\right) ds &= \int_t^\infty u(x_s^T) \exp\left(-r\left(s-t\right)\right) ds \\ &\quad - \int_t^\infty u(x_s) \exp\left(-r\left(s-t\right)\right) ds \\ &\quad - \int_t^\infty u'(x_s^T) \frac{\dot{a}_s}{q} \exp\left(-r\left(s-t\right)\right) ds. \quad (45)\end{aligned}$$

By using partial integration, the last term on the right-hand side of (45) can be expressed as

$$\begin{aligned}\int_t^\infty u'\left(x_s^T\right) \frac{\dot{a}_s}{q} \exp\left(-r\left(s-t\right)\right) ds &= 0 - \frac{u(x_t^T)}{r} + \int_t^\infty u(x_s^T) \exp\left(-r\left(s-t\right)\right) ds \\ &= \int_t^\infty u(x_s^T) \exp\left(-r\left(s-t\right)\right) ds - V(a_t).\end{aligned}$$

Substituting this equation into (45) yields (7):

$$\int_t^\infty \sigma_s \exp\left(-r\left(s-t\right)\right) ds = V\left(a_t\right) - \int_t^\infty u\left(x_s\right) \exp\left(-r\left(s-t\right)\right) ds.$$

Appendix 2. Derivation of (17)

Linearizing (16) around the steady state gives us

$$c_t - c^* = \frac{\{1+(1+\alpha\lambda)\gamma\}u''(x)}{qv''}(x_t - x^*) + \frac{u'(x)}{qv''}\{(1+\alpha\lambda)\gamma'(W_t - W^*) + \alpha\gamma(\lambda_t - \lambda^*)\}. \quad (46)$$

On the other hand, (13) and (15) are linearized to, respectively,

$$\begin{aligned}\{1+(1+\alpha\lambda)\gamma\}u''(x)(x_t - x^*) &= -\{u'(x) - u'(x^T)\}(1+\alpha\lambda)\gamma'(W_t - W^*) \\ &\quad -\{u'(x) - u'(x^T)\}\alpha\gamma(\lambda_t - \lambda^*) \quad (47) \\ &\quad +(1+\alpha\lambda)\gamma u''(x^T)\frac{r}{q}(a_t - a^*) + q(\eta_t - \eta^*), \\ q\dot{\eta}_s &= -(1+\alpha\lambda)\gamma u''(x^T)\frac{r}{q}\dot{a}_s. \quad (48)\end{aligned}$$

For the sake of convenience, we replace subscript t with s in (48). Integrating (48) with respect to s from t to infinity, we have

$$q(\eta_t - \eta^*) = -(1+\alpha\lambda)\gamma u''(x^T)\frac{r}{q}(a_t - a^*).$$

Substituting this for $q(\eta_t - \eta^*)$ in (47) results in

$$\begin{aligned} & \{1+(1+\alpha\lambda)\gamma\}u''(x)(x_t - x^*) \\ = \quad & -\{u'(x) - u'(x^T)\}\{(1+\alpha\lambda)\gamma'(W_t - W^*) - \alpha\gamma(\lambda_t - \lambda^*)\}. \end{aligned} \tag{49}$$

Combining (46) with (49) to eliminate the term $(1+\alpha\lambda)\gamma'(W_t - W^*) - \alpha\gamma(\lambda_t - \lambda^*)$, we obtain

$$c_t - c^* = -\frac{\{1+(1+\alpha\lambda)\gamma\}u''(x)u'(x^T)}{qv''\{u'(x) - u'(x^T)\}}(x_t - x^*).$$

Using (31), the coefficient of x_t in the above equation can be expressed as

$$\begin{aligned} -\frac{\{1+(1+\alpha\lambda)\gamma\}u''(x)u'(x^T)}{qv''\{u'(x) - u'(x^T)\}} &= -\frac{v'u''(x)u'(x^T)}{u'(x)v''\{u'(x) - u'(x^T)\}} \\ &= -\frac{\theta^c c^*}{\theta^x x^*}\left(\frac{u'(x^T)}{u'(x) - u'(x^T)}\right), \end{aligned}$$

from which (17) follows.

Appendix 3. Derivation of dynamic system (20)

- $\dot{x}_t$ equation

  Differentiating (47) by t yields

$$\begin{aligned} & \{1+(1+\alpha\lambda)\gamma\}u''(x)\dot{x}_t + (1+\alpha\lambda)\left\{u'(x) - u'\left(x^T\right)\right\}\gamma'\dot{W}_t \\ & +\alpha\gamma\left\{u'(x) - u'\left(x^T\right)\right\}\dot{\lambda}_t - q\dot{\eta} - (1+\alpha\lambda)\gamma u''\left(x^T\right)\frac{r}{q}\dot{a}_t \\ = \quad & 0 \end{aligned} \tag{50}$$

From the linealized version of this equation, we shall show that $\dot{x}_t$ depends only on $x_t - x^*$ and $W_t - W^*$. First, because of (48), the terms of $\dot{\eta}_t$ and $\dot{a}_t$ on the LHS of (50) are cancelled out. Second, substitute (10) and (14) for $\dot{W}_t$ and $\dot{\lambda}_t$ in (50), respectively. Although the two equations are expressed as functions of x_t, W_t, λ_t and a_t, their linealized versions do not depend on $a_t - a^*$, because, around the steady state,

$$\begin{aligned}
\left.\frac{\partial \dot{W}_t}{\partial a_t}\right|_{(10)} &= -\alpha\gamma \left\{ u'\left(x^T\right)\frac{r}{q} - u'\left(x^T\right)\frac{r}{q} \right\} = 0 \\
\left.\frac{\partial \dot{\lambda}_t}{\partial a_t}\right|_{(14)} &= \left(1+\alpha\lambda\right)\gamma' \left\{ u'\left(x^T\right)\frac{r}{q} - u'\left(x^T\right)\frac{r}{q} \right\} = 0
\end{aligned}$$

Third, we can eliminate $\lambda_t - \lambda^*$ by using (49). Consequently, we have the $\dot{x}$ equation in (20).

- $\dot{W}_t$ equation

  Eq. (10) is linearized as

$$\begin{aligned}
\dot{W}_t &= \alpha\gamma \left\{ u'\left(x\right) - u'\left(x^T\right) \right\} \left(x_t - x^*\right) \\
&\quad -\alpha\gamma' \left\{ u'\left(x\right) - u'\left(x^T\right) \right\} \left(W_t - W^*\right) - \alpha\gamma u'\left(x\right)\frac{1}{q}\left(c_t - c^*\right)
\end{aligned}$$

Eliminate $c_t - c^*$ by substituting (17). Rearrangement yields the $\dot{W}_t$ equation in (20).

Appendix 4. Good-specific time preferences

- Derivation of (26)

  By definition (25), time preference δ_t^x is given by

$$\delta_t^x = r - \frac{MU_{xa}}{MU_x}\dot{a}_t - \frac{MU_{xW}}{MU_x}\dot{W}_t - \frac{MU_{x\lambda}}{MU_x}\dot{\lambda}_t$$

where MU_x is the RHS of (12). Substitute

$$\dot{a}_t = \chi\left(a_t - a^*\right), \dot{W}_t = \chi\left(W_t - W^*\right), \dot{\lambda}_t = \chi\left(\lambda_t - \lambda^*\right)$$

into the above. Eliminating $\lambda_t - \lambda^*$ by using the linearized version of (12) and substituting (23) and (18) yields the expression of δ_t^x in (26). In the same way, we can derive δ_t^c in (26).

- Sufficient conditions for (27)

To discuss sufficient conditions for $\varepsilon > \varepsilon_{-}$, we start with the following property:

Lemma 1 *If $u'''(x) \geq 0$, there is a lower bound for ε around the steady state, such that*

$$\varepsilon \geq 1 - \frac{\theta^c \theta^T}{\theta^x} \frac{x^T}{x}$$

Proof: Parameter ε can be re-expressed as

$$\varepsilon = 1 - \frac{cu'\left(x^T\right)\frac{\theta^c}{\theta^x}}{qx\left\{u'(x) - u'\left(x^T\right)\right\}} = 1 + \frac{u'\left(x^T\right)\frac{\theta^c}{\theta^x}}{x\frac{\left\{u'\left(x^T\right) - u'(x)\right\}}{x^T - x}}$$

where the last equality follows as $c/q = (ra + y - qx)/q = x^T - x$ around the steady state. Thus, if $u'''(x) \geq 0$ and hence $u'(x)$ is weakly convex, we have $\frac{\left\{u'\left(x^T\right) - u'(x)\right\}}{x^T - x} \leq u''\left(x^T\right)$, so that

$$\varepsilon \geq 1 + \frac{u'\left(x^T\right)\frac{\theta^c}{\theta^x}}{xu''\left(x^T\right)} = 1 - \frac{\theta^c \theta^T}{\theta^x} \frac{x^T}{x}$$

□

With the lemma, we propose the following sufficient condition.

Sufficient condition 1: Inequality (27) holds valid if

$$u'''(x) \geq 0 \text{ and } \theta^c < \frac{\theta^x}{\theta^T} \frac{x}{x^T} + \left(\frac{1 + \gamma(1 + \alpha\lambda)}{(1 + \alpha\lambda)\gamma}\right)\left(\frac{u'(x)}{u'\left(x^T\right)}\right)\left(\frac{r - \chi}{r}\right)$$

Proof: Suppose that $u'''(x) \geq 0$. Then, from the definition of ε_{-} and the above lemma, we have $\varepsilon > \varepsilon_{-}$ if

$$1 - \frac{\theta^c \theta^T}{\theta^x} \frac{x^T}{x} > -\left(\frac{x^T}{x}\right)\left(\frac{1 + \gamma(1 + \alpha\lambda)}{(1 + \alpha\lambda)\gamma}\right)\left(\frac{u'(x)}{u'\left(x^T\right)}\right)\left(\frac{r - \chi}{r}\right)\left(\frac{\theta^T}{\theta^x}\right),$$

which can be rearranged to the second inequality of sufficient condition 1.□

As $\left(\frac{1+\gamma(1+\alpha\lambda)}{(1+\alpha\lambda)\gamma}\right)\left(\frac{u'(x)}{u'\left(x^T\right)}\right)\left(\frac{r-\chi}{r}\right)$ is greater than one, the above condition is sufficiently met if $u'''(x) \geq 0$ and either of the following conditions are met:

1. $\theta^c \leq 1$
2. u is of the CRRA type (and, hence, $\theta^x = \theta^T$) and $\theta^c \leq 1 + x/x^T$

3. u is of the CARA type (and, hence, $x\theta^x = x^T\theta^T$) and $\theta^c \leq 2$

where condition 1 is the first item of the example in section 3.2.3.

Inequality $\varepsilon < \varepsilon_+$ holds valid if $\theta^T \geq \theta^x$, because $\varepsilon \leq 1$ and $\varepsilon_+ > \theta^T/\theta^x$, as referred to as the second item of the example shown in section 3.2.3.

Appendix 5. The LWC and LST schedules

Under Assumption 2, we can show from (24) and (36) that:

- When x is marginally a luxury ($\varepsilon > 0$),

$$\left.\frac{da}{dW}\right|_{LST} < 0, \left.\frac{da}{dW}\right|_{LST} < \left.\frac{da}{dW}\right|_{LWC}$$

- When x is marginally an inferior ($\varepsilon < 0$),

$$\left.\frac{da}{dW}\right|_{LST} > 0, \left.\frac{da}{dW}\right|_{LST} > \left.\frac{da}{dW}\right|_{LWC}$$

meaning that the LST is negatively or positively sloping as ε is positive or negative; and that the relative magnitudes of the slopes of the two schedules depend on the sign of ε. Note that when $\varepsilon > 0$, the LWC has a negative or positive slope, as the negative SCC-reducing effect of self-control dominates or is dominated by the positive SCR-enlarging effect, whereas the opposite is true when $\varepsilon < 0$. It follows that we can illustrate the typical LWC and LST schedules in four ways, depending on whether ε is positive or negative and whether the negative SCC-reducing effect dominates or is dominated by the positive SCR-enlarging effect, as shown in Figure A1.

Insert Figure A1.

Appendix 6. The results of comparative statics

- Initial wealth and willpower

$$\frac{da^*}{da_0} = \frac{SC}{\kappa_1},$$

$$\frac{da^*}{dW_0} = \frac{SC}{\kappa_1}\Omega,$$

$$\frac{dW^*}{da_0} = \frac{-\alpha r\gamma q \left\{u'(x) - u'\left(x^T\right)\right\}^2 \left\{u'(x) - u'\left(x^T\right)\varepsilon\right\} v''\varepsilon}{\kappa_1},$$

$$\frac{dW^*}{dW_0} = \frac{-\alpha r\gamma q \left\{u'(x) - u'\left(x^T\right)\right\}^2 \left\{u'(x) - u'\left(x^T\right)\varepsilon\right\} v''\varepsilon}{\kappa_1}\Omega > 0,$$

$$\begin{aligned}
\frac{dx}{da_0} &= X_W \frac{dW^*}{da_0} + X_a \frac{da^*}{da_0}, \\
\frac{dx}{dW_0} &= X_W \frac{dW^*}{dW_0} + X_a \frac{da^*}{dW_0}, \\
\frac{dc}{da_0} &= C_W \frac{dW^*}{da_0} + C_a \frac{da^*}{da_0}, \\
\frac{dc}{dW_0} &= C_W \frac{dW^*}{dW_0} + C_a \frac{da^*}{dW_0},
\end{aligned}$$

where

$$SC = -\alpha\gamma' \left\{u\left(x^T\right) - u(x)\right\} q^2 \left\{u'(x) - u'\left(x^T\right)\right\}^2 v''\varepsilon^2 + \frac{\{1+(1+\alpha\lambda)\gamma\}u''(x)u'(x)^2}{r+\alpha\gamma'\{u(x^T)-u(x)\}}\Phi \geq 0$$

$$\Leftrightarrow \text{SCC-reducing effect} \leq \text{SCR-enlarging effect},$$

$$\kappa_1 = \frac{\left[r+\alpha\gamma'\left\{u\left(x^T\right)-u(x)\right\}\right]q^2\left\{u'(x)-u'\left(x^T\right)\right\}^2 v''\chi\varepsilon^2}{r-\chi} + \frac{\{1+(1+\alpha\lambda)\gamma\}u''(x)u'(x)^2}{r+\alpha\gamma'\{u(x^T)-u(x)\}}\Phi > 0,$$

$$X_W = \frac{\left[-\gamma'\left(r+\alpha\gamma'\left(u\left(x^T\right)-u(x)\right)\right)+\alpha\gamma\gamma''\left(u\left(x^T\right)-u(x)\right)\right]u'(x)(1+\alpha\lambda)^2}{r^2(1+(1+\alpha\lambda)\gamma)u''(x)+r\alpha\gamma\gamma'u'(x)^2(1+\alpha\lambda)^2+q^2r^2v''} < 0,$$

$$X_a = \frac{\left\{\alpha\gamma\gamma'u'\left(x^T\right)u'(x)(1+\alpha\lambda)^2+q^2rv''\right\}}{r(1+(1+\alpha\lambda)\gamma)u''(x)+\alpha\gamma\gamma'u'(x)^2(1+\alpha\lambda)^2+q^2rv''}\frac{1}{q} > 0,$$

$$C_W = \frac{q\left[\gamma'\left(r+\alpha\gamma'\left(u\left(x^T\right)-u(x)\right)\right)-\alpha\gamma\gamma''\left(u\left(x^T\right)-u(x)\right)\right]u'(x)(1+\alpha\lambda)^2}{r^2(1+(1+\alpha\lambda)\gamma)u''(x)+r\alpha\gamma\gamma'u'(x)^2(1+\alpha\lambda)^2+q^2r^2v''} > 0,$$

$$C_a = \frac{\alpha\gamma\gamma'u'(x)(1+\alpha\lambda)^2\left(u'(x)-u'\left(x^T\right)\right)r+r^2(1+(1+\alpha\lambda)\gamma)u''(x)}{r(1+(1+\alpha\lambda)\gamma)u''(x)+\alpha\gamma\gamma'u'(x)^2(1+\alpha\lambda)^2+q^2rv''(c)} > 0.$$

- Net willpower recovery rate

$$\frac{dW^*}{d\psi'} = \frac{\kappa_2 \left[u'(x) - u'\left(x^T\right)\right]\left\{u'(x) - u'\left(x^T\right)\varepsilon\right\}}{\kappa_1} > 0,$$

$$\frac{da^*}{d\psi'} = \frac{-\kappa_2 \left[u'(x) - u'\left(x^T\right)\right]\left\{u'(x) - u'\left(x^T\right)\varepsilon\right\}}{\kappa_1}\Omega,$$

$$\frac{dx^*}{d\psi} = (X_W - X_a\Omega)\frac{\kappa_2 \left[u'(x) - u'\left(x^T\right)\right]\left\{u'(x) - u'\left(x^T\right)\varepsilon\right\}}{\kappa_1},$$

$$\frac{dc^*}{d\psi} = (C_W - C_a\Omega)\frac{\kappa_2 \left[u'(x) - u'\left(x^T\right)\right]\left\{u'(x) - u'\left(x^T\right)\varepsilon\right\}}{\kappa_1},$$

where

$$\kappa_2 = -\tfrac{\alpha\gamma\gamma' u'(x)^2 (1+\alpha\lambda)^2}{r} - (1 + (1+\alpha\lambda)\gamma)\, u''(x) - q^2 v'' > 0.$$

- Endowment income

$$\frac{dW^*}{dy} = \frac{-\alpha r\gamma q \left\{u'(x) - u'\left(x^T\right)\right\}^2 \left\{u'(x) - u'\left(x^T\right)\varepsilon\right\} v''\varepsilon}{r\kappa_1}$$

$$\frac{da^*}{dy} = \frac{\alpha r\gamma q \left\{u'(x) - u'\left(x^T\right)\right\}^2 \left\{u'(x) - u'\left(x^T\right)\varepsilon\right\} v''\varepsilon}{r\kappa_1}\Omega < 0$$

$$\frac{dx^*}{dy} = \frac{-\alpha r\gamma q \left\{u'(x) - u'\left(x^T\right)\right\}^2 \left\{u'(x) - u'\left(x^T\right)\varepsilon\right\} v''\varepsilon}{r\kappa_1}X_W + \frac{SC}{r\kappa_1}X_a$$

$$\frac{dc^*}{dy} = \frac{-\alpha r\gamma q \left\{u'(x) - u'\left(x^T\right)\right\}^2 \left\{u'(x) - u'\left(x^T\right)\varepsilon\right\} v''\varepsilon}{r\kappa_1}C_W + \frac{SC}{r\kappa_1}C_a$$

Appendix 7. The case with willpower-enhancing effects of self-control

In the text, we have focused on the willpower-depleting effect of self-control in tempting good consumption. As is shown in psychological experiments (e.g., Muraven et al., 1999; Baumeister et al., 2016), and as is casually believed, self-control toughens future willpower as if training strengthened one's muscle. In psychological theory, the model of toughenable willpower is called the muscle model (e.g., Baumeister and Vohs, 2003). Here we incorporate the willpower-enhancing effect of self-control in the simplest way and thereby discuss on its implication.

Specify the willpower-enhancing effect of self-control by assuming that the willpower recovery rate ψ is an increasing and concave function of self-control costs F: $\psi(F), \psi' > 0, \psi'' < 0$, where

$$F(x, W, a) = f_0 + \gamma(W)\left\{u\left(x^T\right) - u(x) - u'\left(x^T\right)\left(\frac{ra + y - qx - c}{q}\right)\right\}.$$

The willpower flow constraint (8) is then changed to

$$\dot{W}_t = \psi\left(F\left(x_t, W_t, a_t\right)\right) - \alpha F\left(x_t, W_t, a_t\right), \tag{51}$$

From (51), a marginal increase in self-control F at time t affects willpower at next instant $\dot{W}$ in two ways: it depletes the willpower by α, and enhances the recovery rate by ψ'. The net effect depends crucially on whether $\bar{\alpha} \equiv \alpha - \psi'$ is positive or negative.

Formally, the local dynamic behavior under (51) is described by the system that is obtained by replacing α with $\bar{\alpha}$ in (20). Re-define Φ ((22)), χ, and Ω ((24)) by using $\bar{\alpha}$, instead of α, and denote them by $\bar{\Phi}, \bar{\chi}$, and $\bar{\Omega}$, respectively. In the same way as in (22), the necessary and sufficient condition for saddle-point stability is given by $\bar{\Phi} < 0$. Suppose that the condition is met. Then, the optimal relations (23) and (24) in the previous model are changed to, respectively,

$$x_t - x^* = \frac{\left\{u'(x) - u'\left(x^T\right)\right\}\left[\bar{\chi}/\bar{\alpha} + \gamma'\left\{u\left(x^T\right) - u(x)\right\}\right]}{\gamma\left[\left\{u'(x) - u'\left(x^T\right)\right\}^2 + \frac{c}{qx}\frac{\theta^c}{\theta^x}u'\left(x^T\right)^2\right]}\left(W_t - W^*\right),$$

$$a_t - a^* = -\bar{\Omega}\left(W_t - W^*\right),$$

In these equations, note that the signs of the coefficients of $W_t - W^*$ depend crucially on the sign of $\bar{\chi}/\bar{\alpha} + \gamma'\left(u\left(x^T\right) - u(x)\right)$. In particular, when the willpower-enhancing effect is strong enough that $\bar{\chi}/\bar{\alpha} + \gamma'\left(u\left(x^T\right) - u(x)\right)$ is positive, the coefficients of $W_t - W^*$ have the opposite signs to those in Section 3, so that the optimal consumption of tempting good x becomes smaller with weaker willpower ($dx_t/dW_t > 0$): consumers with weaker willpower keep x lower so as to toughen their future willpower.

Which of the willpower-depleting effect and -enhancing effect is actually dominant should be discussed based on empirical facts. As long as concerned with short-term behaviors, the prediction in the willpower-enhancing effect case seems not only implausible but also inconsistent with empirical facts. For example, in that case, when the independent self-control requirement f increases in the tempting luxury ($\varepsilon > 0$), tempting good consumption decreases and willpower level becomes higher over time. This is inconsistent with the fact that increases in external burden on self-control fatigue consumers, and thereby, reduces their self-regulatory power (see, e.g., Shiv and Fedorkhin, 1999; Hinson et al., 2003).

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Table 1. Long-run properties of the optimal behavior

	Luxury temptation ($\varepsilon > 0$)				Inferior temptation ($\varepsilon < 0$)			
	W^*	a^*	x^* $=X(ra^*+y, W^*)$	c^* $=C(ra^*+y, W^*)$	W^*	a^*	x^* $=X(ra^*+y, W^*)$	c^* $=C(ra^*+y, W^*)$
W_0	+	(-, +)	(-, ?)	(?, +)	+	(+, -)	(?, -)	(+, ?)
a_0	+	(-, +)	(-, ?)	(?, +)	-	(-, +)	(?, +)	(-, ?)
ϕ'	+	-	-	?	+	+	?	+
y	+	-	(-, ?)	(?, +)	-	-	(?, +)	(-, ?)

Note: The question marks indicate that the signs of the corresponding effects are not uniquely determined. When two signs are bracketed in a cell, the first sign represents the effect when the SCC-reducing effect dominates the SCR-enhancing effect (see Section 3.2.4 for the two effects). The second sign represents the effect when the latter effect dominates the former. For example, (-, +) in the (W_0, a^*) cell in the luxury-temptation case indicates that an increase in W_0 decreases or increases a^* if the SCC-reducing or the SCR-enhancing effect is dominating, respectively.

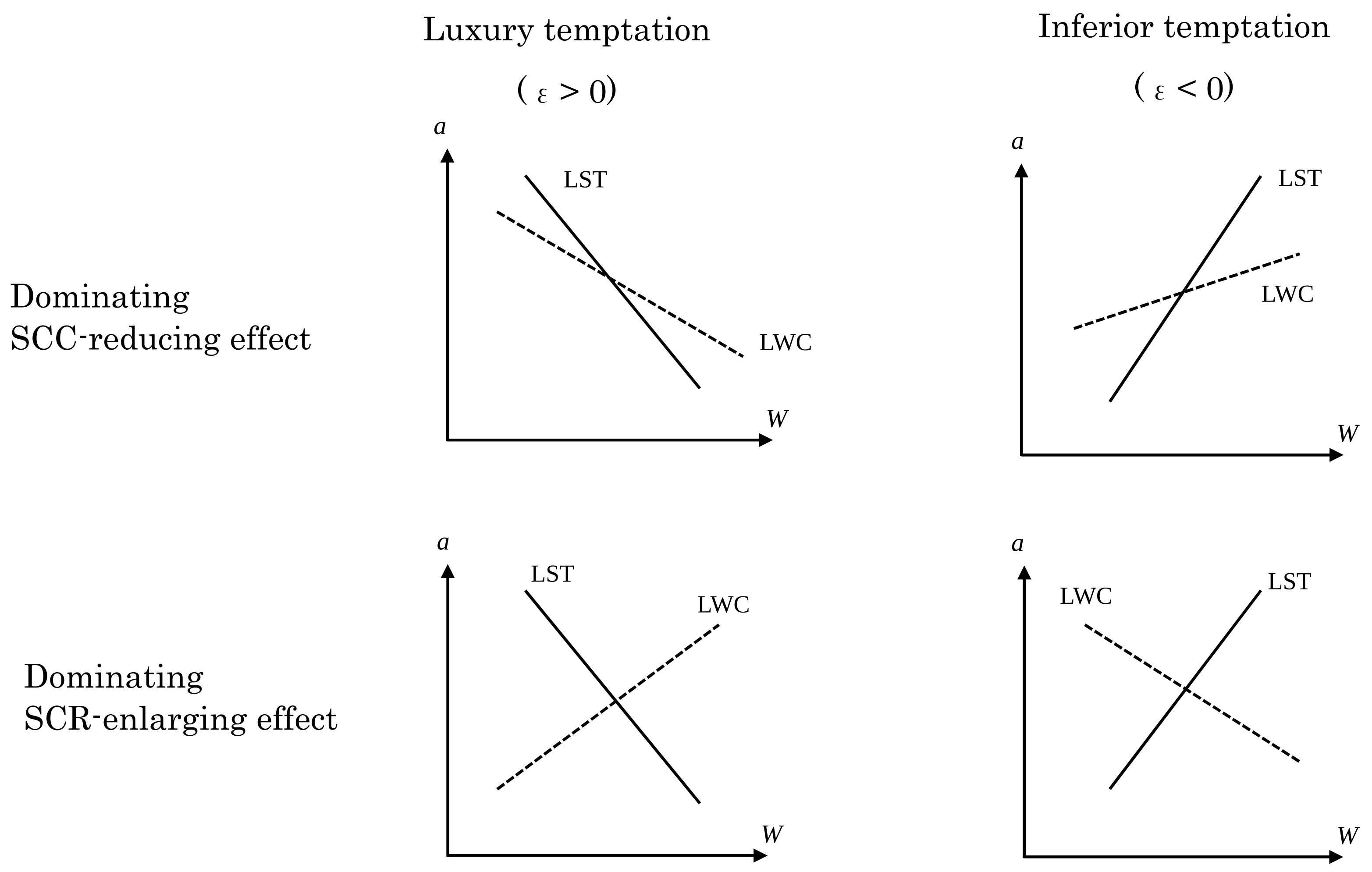

Figure A1. Steady-state optimum solutions