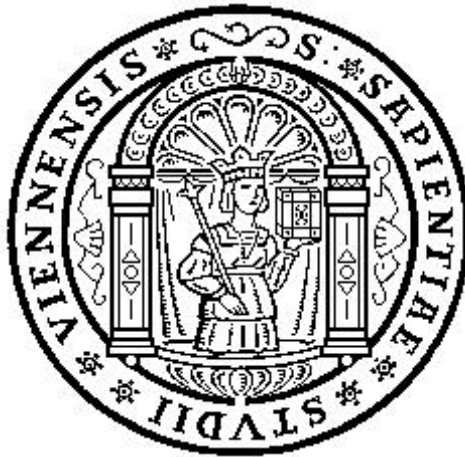# WORKING   PAPERS

Liquidity risk and financial stability regulation
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Liquidity risk and financial stability regulation[*]

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Abstract

We study banks' borrowing and investment decisions in an economy with pecuniary externalities and both aggregate and idiosyncratic liquidity risk. We show that private decisions by profit-maximizing banks always result in socially inefficient outcomes, but the nature of inefficiency depends critically on the structure of liquidity risk. Overborrowing and overinvestment in risky assets arises only if idiosyncratic risk is sufficiently small. By contrast, if idiosyncratic risk is large, unregulated banks underborrow, underinvest and hold insufficient liquidity reserves. A macroprudential regulator can restore constrained efficiency by imposing countercyclical reserve requirements. Pigouvian taxes or bank capital requirements cannot achieve this objective.

JEL **codes:** E44, E58, G21, G28

Key words: liquidity risk, macroprudential policy, reserve requirements, pecuniary externalities, underborrowing

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1 Introduction

The recent financial crisis has forcefully demonstrated the need for policy intervention to contain systemic risk in the financial sector. Macroprudential policy – the use of primarily prudential tools to limit systemic risk – has ever since been established as a new policy area at central banks and other institutions around the globe. The predominant view among policy-makers is that unregulated banks lend too much during booms and too little during recessions, thereby inducing excessive fluctuations in credit, asset prices and aggregate output. This view has been supported by a growing academic literature studying private borrowing and lending decisions in environments where pecuniary externalities give rise to systemic vulnerabilities. A key finding of this literature is that unregulated banks indeed overborrow and overinvest in risky assets during booms, and that this inefficiency renders the economy prone to excessively severe crises. By imposing Pigouvian taxes on borrowing or restricting investment, for example via bank capital requirements, a macroprudential regulator can improve upon the decentralized outcome. This policy prescription has been found robust to variations in the modelling environment, however attention has been restricted to the case of purely aggregate liquidity risk.[1]

In this paper we challenge the predominant view. We study financial stability regulation in a standard model with pecuniary externalities but with both aggregate *and* idiosyncratic liquidity risk.[2] We demonstrate that the model's prescriptions regarding optimal regulatory policy differs markedly from environments with purely aggregate risk. In particular, the overborrowing and overinvestment result stressed in the earlier literature is completely reversed once idiosyncratic risk becomes sufficiently important. The key inefficiency then is that banks hold insufficient liquidity reserves, as they fail to internalize that their own liquidity holdings excert a *positive externality* on the banking system, which further induces them to *underborrow* and *underinvest* in risky assets relative to the constrained-efficient outcome.

Bank capital requirements, outright caps on borrowing, or Pigouvian taxes on debt are no longer appropriate regulatory instruments to improve upon the decentralized outcome under

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[1]See, for example, Lorenzoni (2008), Korinek (2011), Gersbach and Rochet (2012) and Stein (2012).

[2]Allowing for an idiosyncratic component of risk is important on empirical grounds. Real-world banks specialize in different business areas, such as providing firm credit, consumer credit, or residential mortgages, and are thus exposed to different sectoral shocks in the economy. This introduces an idiosyncratic, bank-specific component into their risk structure. Empirical evidence further shows that banks do not fully insure against this type of idiosyncratic risk, for example by holding shares in one another.

idiosyncratic liquidity risk. While they work well to curb excessive borrowing and investment, these instruments lose their bite in an environment characterized by underborrowing and underinvestment. Instead, the macroprudential regulator must ensure sufficient liquidity in the banking system. This can best be achieved by quantity-based measures such as a Minimum Reserve Requirement or a Liquidity Coverage Ratio along the lines of Basel III. Importantly, we show that liquidity regulation must be implemented countercyclically to show its full potential. During the boom, a high liquidity requirement ensures that sufficient reserves are accumulated within the banking system. During the downturn, reducing the requirement releases liquidity in the banking system, which relieves pressure from asset prices and reduces fire-sale losses of distressed banks. Static liquidity requirements that are not adjusted in response to macroeconomic or financial shocks fail to implement constrained efficiency.

As an alternative to quantity-based liquidity regulation, the constrained-efficient allocation in our environment can also be implemented using a mix of price-based regulatory instruments. This policy mix combines paying interest on banks' liquidity reserves with collecting a Pigouvian tax on short-term debt. Compared to quantity-based regulation, however, this approach has an important drawback: it generates financial costs for the regulator, as interest payments on reserves necessarily exceed tax revenues. Against this background, the budget-neutral quantity regulation can arguably be considered the better policy approach.

The policy prescriptions discussed thus far are derived in an environment where the regulator has perfect information. To examine robustness with regard to this informational assumption, we further consider environments where the regulator lacks some of the information that private agents have access to, such as knowledge about banks' lending opportunities, production technologies or endowments. As it turns out, our optimal policy prescriptions carry over to these limited information environments in a straightforward way, as long as there is common knowledge of the size and structure of liquidity risk.

Our paper contributes to several strands of the literature. A first strand is the literature on fire-sales and financial amplification, which dates back to the debt-deflation theory pioneered by Fisher (1933) and, more recently, the seminal contributions by Bernanke and Gertler (1989), Shleifer and Vishny (1992), Kiyotaki and Moore (1997) and Caballero and Krishnamurthy (2003). An important take-away from this literature is that firms and households tend to

borrow and invest too much during booms, as they do not internalize the impact of their asset sales decisions on asset prices during downturns. This financial amplification mechanism exacerbates business cycle fluctuations in the economy and can lead to excessively severe crises.

A second strand examines macroprudential policy as a means to address inefficiencies due to fire-sale externalities and financial amplification. Lorenzoni (2008) studies the trade-off between high investment ex-ante and financial distress ex-post and provides a rationale for macroprudential policy intervention, as private agents underestimate the damage associated with excessive credit booms. Mendoza (2010), Bianchi (2011) and Benigno, Chen, Otrok, Rebucci, and Young (2013) study private borrowing decisions in open economies with credit constraints, where macroprudential policies can mitigate endogenous financial crises. Korinek (2010) provides foundations for prudential capital controls to reduce the risk of financial crises in emerging economies. Jeanne and Korinek (2010) develop a dynamic model of countercyclical macroprudential regulation for credit markets, calling for Pigouvian taxes on debt during booms, whereas Jeanne and Korinek (2013) show that it is optimal for policy-makers to use both ex-ante prudential measures and ex-post stimulus measures to respond to financial crises. While the studies discussed so far examine macroprudential policy in the presence of pecuniary externalities, recent work has explored aggregate demand externalities as a source of inefficiencies (e.g., Schmitt-Grohe and Uribe, 2016; Korinek and Simsek, 2016; Farhi and Werning, 2012).

Most closely related to our paper are the contributions by Gersbach and Rochet (2012), Stein (2012) and Kara and Ozsoy (2016). These papers study externalities and systemic vulnerabilities arising in the banking sector. Gersbach and Rochet (2012) examine aggregate investment externalities in a model where banks can freely reallocate capital across sectors. They argue that banks exacerbate credit, output and asset price fluctuations by excessively reallocating capital in response to productivity shocks. Stein (2012) studies financial stability regulation via traditional monetary policy tools in a framework where commercial banks can finance investment via private money creation. In the absence of regulation, private money creation can lead to intermediaries issuing too much short-term debt and leaving the system excessively vulnerable to financial crises. Kara and Ozsoy (2016) study the optimal design of capital and liquidity regulations, as well as the interaction between the two, in a model with

fire-sale externalities. They show that banks overinvest in risky assets and underinvest in liquid assets in the unregulated competitive equilibrium, and that constrained efficiency can be restored by a regulator who imposes both macroprudential liquidity requirements and capital regulations. Notably, Gersbach and Rochet (2012), Stein (2012) and Kara and Ozsoy (2016) focus on frameworks with purely aggregate risk. We complement their analyses by examining an environment with both aggregate and idiosyncratic risk, and by demonstrating that the risk structure has important consequences for optimal regulatory policy.[3]

Finally, note that our paper also relates to the literature on strategic liquidity hoarding by banks (Acharya, Shin, and Yorulmazer, 2011; Allen and Gale, 2004a, 2004b; Gorton and Huang, 2004), the recent literature studying the role of international foreign-exchange reserves for macroprudential purposes (e.g., Bianchi, Hatchondo, and Martinez, 2013; Jeanne, 2016; Jeanne and Sandri, 2016) as well as recent contributions to bank liquidity regulation (e.g., Farhi, Golosov, and Tsyvinski, 2009; Perotti and Suarez, 2011; Calomiris, Heider, and Hoerova, 2015).

The rest of this paper is organized as follows. Section 2 lays out our model economy. Sections 3 and 4 characterize the decentralized competitive equilibrium and the constrained-efficient allocation, respectively. Section 5 describes how private decisions give rise to socially inefficient outcomes in the unregulated competitive equilibrium, and studies the role of idiosyncratic liquidity risk in this context. Section 6 examines whether and how a regulator can restore constrained efficiency, and argues that a countercyclical minimum reserve requirement is a useful macroprudential tool in many environments. Section 7 discusses recent empirical evidence on the use of minimum reserve requirements for financial and macroeconomic stabilization in practice. Section 8 concludes. Proofs are relegated to the Appendix.

2 The Model

We develop a model of a small open economy where agents live for three time periods, $t = 0, 1, 2$. The economy is populated by households, firms, banks, and outside investors. There is a single

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[3]As a robustness exercise, Kara and Ozsoy (2016) also study the financing and investment decisions of unregulated banks under idiosyncratic risk and argue that their findings are robust to this alternative specification. They do not discuss the efficiency losses that arise under idiosyncratic risk or the optimal macroprudential policy in this environment.

perishable consumption good, which also serves as a numeraire. There are two states of the world, $\theta \in \{g, b\}$, which can be interpreted as a good state and a bad state. In what follows, we often refer to the bad state realization as a *crisis*.

2.1 Households

Households are endowed with $X > 0$ consumption goods in period 0 and one unit of labor in period 2. They derive utility from consumption in periods 0 and 2,

$$U = C_0 + \delta \mathbb{E} C_2(\theta), \tag{1}$$

where δ denotes the households' time discount factor. In period 0, households consume C_0 and save by purchasing long-term (maturing at time 2) bonds B^l issued by banks,[4]

$$C_0 + B^l \leq X. \tag{2}$$

In period 2, they work $H(\theta)$ hours and consume $C_2(\theta)$. They also receive income from their ownership of firms and banks, as well as from outside investors who transfer their earnings to the household sector in the end of the period. Denoting by $R^l(\theta)$ the return on bonds and by $w(\theta)$ the wage rate, the budget constraint in period 2 reads

$$C_2(\theta) \leq R^l(\theta) B^l + w(\theta) H(\theta) + \Pi^F(\theta) + \Pi^B(\theta) + \Pi^I(\theta). \tag{3}$$

Households choose $\{B^l, C_0, C_2(\theta), H(\theta)\}$ such as to to maximize (1) subject to (2) - (3), taking $\{X, R^l(\theta), w(\theta), \Pi^B(\theta), \Pi^F(\theta), \Pi^I(\theta)\}$ as given.

2.2 Firms

Firms are perfectly competitive. In period 2, after having observed the state of the world θ, they hire productive capital $\tilde{K}(\theta)$ from outside investors and labor $\tilde{H}(\theta)$ from households to produce consumption goods using a Cobb-Douglas technology. Let total factor productivity be

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[4]Households do not have the technology to store the perishable consumption goods over time.

denoted by A and the capital share by α. Firm profits are then given by

$$\Pi^F(\theta) = A\tilde{K}(\theta)^\alpha \tilde{H}(\theta)^{1-\alpha} - R^K(\theta)\tilde{K}(\theta) - w(\theta)\tilde{H}(\theta), \tag{4}$$

where $R^K(\theta)$ and $w(\theta)$ denote factor prices. Firms maximize (4) taking prices as given.

2.3 Banks

Our modelling of banks builds on the framework developed in Stein (2012). There exists a continuum $[0, 1]$ of banks, each of which has access to a bank-specific investment project with variable scale $I \geq 1$. A project of scale I requires an input of I consumption goods in period 0.

Banks have no intial endowment but must raise all funds externally by issuing long-term debt B^l and short-term (maturing at time 1) debt B^s. They can invest into their bank-specific project, but can also store funds from period 0 to period 2 in the form of non-renumerated reserves, L. A bank's budget constraint in period 0 is hence given by

$$I + L \leq B^l + B^s. \tag{5}$$

In the beginning of period 1, the state of the world is revealed. If the state is *good*, which happens with probability $p \in (0, 1)$, the projects of *all* banks produce a high return $f(I)$, where I is the investment scale and f is a concave function. By contrast, if the state of the world is *bad*, then the projects of only $q \in [0, 1)$ banks have a high return $f(I)$, while the projects of $1 - q$ banks have a low return. Whether an individual bank falls into the first or second category is determined by an idiosyncratic shock, which is i.i.d. across banks and publicly observed together with the state of the world. In period 1, the expected output of low-return projects is equal to $\lambda I \leq I$, and with a small probability this output may drop all the way to zero in period 2.[5] We refer to banks with high-return projects as *sound* banks and to banks with low-return projects as *distressed* banks.

We assume that entirely riskless bonds offer their holders a convenience yield that risky

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[5]A low-return project's period 2 return is $\frac{\lambda I}{1-\varepsilon}$ with probability $1 - \varepsilon$ and 0 with probability ε, where we assume that ε is sufficiently small such that $\frac{\lambda I}{1-\varepsilon} < I$ (cf. Stein, 2012). Whether the output of a failed project collapses to zero or not is privately observed by the project owner in the beginning of period 2.

bonds do not, and thus have a lower rate of return in equilibrium (cf. Stein, 2012). Since there is a positive probability that a bank's investment project yields zero output eventually, long-term debt issued in period 0 can never be made entirely riskless. By contrast, short-term debt issued in period 0 can be made riskless, provided the amount issued is not too large. In particular, the bank must be able to sell its project and use the proceeds together with reserves to pay out short-term creditors in full. Formally,

$$k\lambda I \geq R^s B^s - L, \tag{6}$$

where k is the market price of a share in the bank project delivering one consumption good in period 2, and R^s is the interest rate on riskless short-term debt. The constraint (6) also reflects that distressed banks cannot issue new debt in period 1. Underlying this is the assumption that any claim issued in period 1 must be junior to claims issued in period 0.

There are two types of potential buyers for distressed bank projects: sound banks and outside investors. We assume that both have limited resources in the interim period 1 and cannot raise fresh funds externally. This assumption is starker than necessary, but helps to simplify the analysis.[6] Sound banks are willing to purchase distressed assets if their expected return exceeds the return on reserves, i.e. if $1/k \geq 1$.

Finally, we slightly depart from Stein (2012) and assume that long-term debt is sold exclusively to domestic households, while short-term debt is sold exclusively to international lenders. This form of market segmentation is postulated mainly for simplicity and can easily be microfounded. Moreover, it implies that aggregate domestic welfare is a function of only aggregate consumption, but not the amount of short-term debt issued by banks, which we consider a convenient feature for our welfare analyses.[7]

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[6]What is important is that sound banks and outside investors cannot fully pledge the returns of distressed bank projects to other agents, so that their activities are constrained by their own resources (cf. Acharya, Shin, and Yorulmazer, 2011).

[7]However, as will become clear later, none of our results will depend on the source of bank funding and thus assuming market segmentation is without loss of generality. It is straightforward to extend our model along the lines of Stein (2012) to introduce short-term lending by domestic households, but we do not pursue this extension as it does not provide any new insights.

A bank's ex-ante expected profit is given by

$$
\begin{aligned}
\mathbb{E}\{\Pi(\theta)\} \;=\;& p\left\{f(I) - R^s B^s - R^l(g)B^l + L\right\} \\
&+(1-p)q\left\{f(I) - R^s B^s - R^l(b)B^l + L + \left(\frac{1}{k} - 1\right)L\right\} \\
&+(1-p)(1-q)\left\{\lambda I - R^s B^s - R^l(b)B^l + L - \left(\frac{1}{k} - 1\right)\left(R^s B^s - L\right)\right\}. \quad (7)
\end{aligned}
$$

The first term in curly brackets gives the bank's profit in the good state of the world. The second term denotes the profit in the bad state of the world, but when the bank's own project is of a high-return type. Compared to the good state, the bank makes an additional profit $\left(\frac{1}{k} - 1\right)L$ by using its reserves to purchase distressed assets at a discount. Finally, the third term denotes the profit in the bad state of the world and when the bank's own project is of a low-return type. In this scenario, the bank incurrs fire-sale losses amounting to $\left(\frac{1}{k} - 1\right)\left(R^s B^s - L\right)$.

The bank chooses investment, I, long-term borrowing, B^l, short-term borrowing, B^s, and reserves, L, to maximize (7), subject to the budget constraint (5) and the collateral constraint (6), taking all prices as given. In particular, an individual bank does not take into account its incremental impact on the fire-sale price k, which in equilibrium will be determined by the aggregate asset sales in the banking system. This generates a pecuniary externality which manifests itself in two ways. First, as in Stein (2012), the bank does not internalize that increasing its short-term borrowing B^s excerts a *negative externality* on other banks; a higher B^s implies that a bank must fire-sell more assets when distressed, which has a negative effect on the fire-sale price k and increases the fire-sale losses of other distressed banks. Second, the bank does not internalize that a higher stock of liquidity reserves L excerts a *positive externality* on other banks; if in a crisis the bank remains sound, it can purchase more assets from distressed banks, which has a positive effect on the fire-sale price k and reduces the fire-sale losses of distressed banks. This second effect is present only in an environment with idiosyncratic liquidity risk.

2.4   Outside investors

Outside investors are endowed with $W > 0$ units of the consumption good in period 1. They invest their entire endowment and distribute their earnings to the household sector in the end of period 2. There are two investment opportunities: outside investors can either purchase shares in bank projects, or transform their resources costlessly into productive capital and rent it out to firms at the rental rate R^K. Among these two options, they choose the one that yields the higher return. They do not take into account that, by purchasing bank projects rather than providing capital to the real sector, they reduce the wage rate and hence the households' labor income. This gives rise to an aggregate investment externality along the lines of Gersbach and Rochet (2012).

3   The decentralized competitive equilibrium

In the decentralized competitive equilibrium, households, firms, banks and outside investors maximize their objectives individually and are unaware of their incremental influence on market prices and aggregate outcomes.

Let R^l denote the expected return on long-term debt, i.e. $R^l = \mathbb{E}R^l(\theta)$. From the households' utility maximization problem, we obtain that households supply their labor endowment inelastically, $H(\theta) = 1$, and purchase long-term debt only if the expected return compensates them for postponing consumption,

$$R^l \geq \frac{1}{\delta}.$$

From the firms' profit maximization problem under perfect competition, it follows that factors are paid their marginal products,

$$w(\theta) = (1 - \alpha)A\tilde{K}(\theta)^{\alpha}\tilde{H}(\theta)^{-\alpha},$$
$$R^K(\theta) = \alpha A\tilde{K}(\theta)^{\alpha-1}\tilde{H}(\theta)^{1-\alpha},$$

and firms make zero profits, $\Pi^F(\theta) = 0$. From the outside investors' portfolio problem, it follows

that

$$\frac{1}{k} = R^K(\theta)$$

whenever investment into bank projects and real capital is positive, and that $\frac{1}{k} < R^K(\theta)$ when investment into bank projects is zero.

Let us finally consider the representative bank's optimization problem. Recall that the bank chooses investment, I, long-term borrowing, B^l, short-term borrowing, B^s, and reserves, L, to maximize (7), subject to the budget constraint (5) and the collateral constraint (6). The Lagrangian function for this problem can be written as

$$
\begin{aligned}
\mathcal{L}^B &= f(I) - R^s B^s - R^l(I + L - B^s - E) + L \\
&\quad + (1-p)\left\{ q\left(\frac{1}{k} - 1\right) L + (1-q)\{\lambda I - f(I) - \left(\frac{1}{k} - 1\right)(R^s B^s - L)\} \right\} \\
&\quad - \kappa \left\{ \frac{R^s B^s - L}{k} - \lambda I \right\},
\end{aligned}
$$

where we use $B^l = I + L - B^s$ and κ denotes a Lagrangian multiplier. The first-order condition with respect to the investment choice, I, yields

$$f'(I)(p + (1-p)q) + (1-p)(1-q)\lambda + \kappa\lambda = R^l. \tag{8}$$

The left-hand side gives the expected marginal benefit of an increase in investment, whereby the term $\kappa\lambda$ reflects that an increase in I relaxes the collateral constraint. Note that we assume that $f'(1)$ is sufficiently large to ensure $I > 1$, such that (8) holds as a strict equality. The first-order condition with respect to the short-term borrowing, B^s, yields

$$R^l - R^s \leq (1-p)(1-q)\left(\frac{1}{k} - 1\right) R^s + \frac{\kappa R^s}{k}, \tag{9}$$

which has to hold with equality if $B^s > 0$. The left-hand side gives the marginal benefit of short-term debt financing relative to long-term financing of investment, the right-hand side gives the marginal cost (resulting from higher fire-sale losses when distressed and a tighter

collateral constraint). The first-order condition with respect to reserves, L, yields

$$(1-p)\left(\frac{1}{k}-1\right) + \frac{\kappa}{k} \leq R^l - 1, \tag{10}$$

which has to hold with equality if $L > 0$. The left-hand side gives the marginal benefit of holding an additional unit of reserves, the right hand side gives the marginal opportunity cost. Finally, the following complementary slackness condition has to hold:

$$0 = \kappa\{\frac{R^s B^s - L}{k} - \lambda I\} \tag{11}$$

In the competitive equilibrium, all agents behave optimally and markets clear, i.e. $\tilde{H}(\theta) = H(\theta)$, $\tilde{K}(\theta) = K(\theta)$, $K(g) = W$ and $K(b) = W + L - (1-q)R^s B^s$. Accordingly, the no-arbitrage condition for outside investors is given by

$$\frac{1}{k} = \alpha A(W + L - (1-q)R^s B^s)^{\alpha-1}. \tag{12}$$

Let us denote by B^{s*} the optimal level of short-term borrowing in the competitive equilibrium. Independent of the structure of liquidity risk, the following result obtains:

Proposition 1. *Banks issue a strictly positive amount of short-term debt in the competitive equilibrium, $B^{s*} > 0$.*

The intuition behind this result is readily seen. Assume that banks would not issue any short-term debt in the competitive equilibrium. Then there would be no fire-sales in the bad state of the world, and the equilibrium price of bank projects would be equal to their fundamental value, $k = 1$. As individual banks do not take into account their own incremental effect on this equilibrium price, they would take $k = 1$ as parametrically given. In this situation, and since $R^s < R^l$ by assumption, every individual bank would have an incentive to issue short-term debt and choose $B^s > 0$, because the liquidity risk generated by the issuance of short-term debt is not perceived as costly. $B^s = 0$ can therefore not hold in equilibrium.

We next examine whether banks hold positive reserves in the competitive equilibrium, $L^* > 0$. Clearly, they do so if and only if the expected return on liquidity reserves exceeds the opportunity cost of accumulating reserves. As it turns out, this depends critically on the

structure of liquidity risk, as reflected by q, and on the quantity of resources available to outside investors, W. The parameter q is a key determinant of the probability that a bank can use its reserves profitably to purchase assets at a fire-sale discount; when q is small, then this probability is low because the liquidity shock hits most banks simultaneously. The parameter W, in turn, is a key determinant of the size of the fire-sale discount during crises. In particular, when W is small, then the marginal product of productive capital in the real sector, R^K, is high, which translates into a large fire-sale discount via the outside investors' no-arbitrage condition. The expected return on liquidity reserves is thus high when q is large and W is small. This reasoning leads to the following proposition.

Proposition 2. *Banks hold positive liquidity reserves in the competitive equilibrium, $L^* > 0$, if and only if the idiosyncratic component of liquidity risk, q, is sufficiently large and the outside investors' resources, W, are sufficiently small.*

In the proof of Proposition 2 we establish that there exist unique threshold levels $\overline{W}$ and $\bar{q}$ such that liquidity reserves are positive if and only if

$$W \leq \overline{W} \; = \; \left(\frac{\delta q(1-p)R^s \alpha A}{\delta q(1-p)R^s + R^s - 1} \right)^{\frac{1}{1-\alpha}} - \left(\frac{\delta q(1-p)R^s}{\delta q(1-p)R^s + R^s - 1} \right) \tag{13}$$
$$\times (q-1)\lambda(f')^{-1} \left(\frac{\frac{1}{\delta} - \lambda \frac{(1-p)(qR^s - \delta qR^s - R^s + 1)}{(1-p)\delta qR^s + R^s - 1} - (1-p)(1-q)\lambda}{p + (1-p)q} \right)$$

and

$$q \geq \bar{q} = \frac{(R^s - 1)R^l}{(R^l - 1)R^s} > 0. \tag{14}$$

An immediate consequence of Proposition 2 is hence that liquidity reserves are necessarily zero in the competitive equilibrium under purely aggregate risk, $q = 0$. When banks can never use their liquidity reserves profitably to purchase fire-sold assets of other banks at a discount, the expected net return on reserves is zero and hence strictly dominated by the opportunity cost of reserves storage (cf. Stein, 2012).

We now explore whether reserve holdings are ever sufficiently large to absorb all fire-sold assets within the banking sector during crises, and hence to prevent fire-sale losses for distressed banks altogether. Our findings are summarized in the following proposition.

Proposition 3. *Banks in the competitive equilibrium never hold sufficient reserves to absorb all fire-sold assets within the banking sector, i.e. $L^* < (1-q)R^s B^{s*}$.*

Proposition 3 shows that private banks' reserves holdings are insufficient in the following sense. In the bad state of the world, sound banks run out of liquid resources to absorb all the assets sold by distressed banks. Outside investors come in as buyers of distressed bank assets, but they are only willing to purchase these assets at a discount. Distressed banks therefore incur fire-sale losses. Moreover, purchases of bank assets crowd out capital investment in the real sector which reduces the marginal productivity of workers. The wage rate falls, which reduces the households' labor income, and output production in the real sector is eventually depressed relative to its potential level.

Corollary 1. *Outside investors purchase fire-sold bank assets in crisis times. This crowds out capital investment in the real sector, $K^* < W$, which leads to output losses, $Y^* < \bar{Y} = AW^\alpha$.*

The following numerical example illustrates our findings. To facilitate comparison, our choice for functional forms and parameters is largely based on Stein (2012).

Example 1. *Assume $f(I) = \xi log(I) + I$, $\xi = 3.5$, $R^s = 1.01$, $R^l = 1.04$, $\lambda = 1 - \varepsilon$, $W = 140$, $X = 100$, $\alpha = 0.4$, $A = \frac{1}{\alpha W^{\alpha-1}}$, $q = 1/3$, and $p = 0.955$. Note that these parameters satisfy (14) but not (13). In the competitive equilibrium, $B^{s*} = 88.56$, $B^{l*} = 36.22$, $L^* = 0$, $I^* = 124.79$, $k^* = 0.72$, $K^* = 80.37$ and $Y^*/\bar{Y} = 0.80$. Banks do not hold any liquidity reserves. By contrast, if the outside investors' endowment is only $W = 60$, in which case (13) is satisfied, the competitive equilibrium is given by $B^{s*} = 69.19$, $B^{l*} = 40.63$, $L^* = 11.70$, $I^* = 98.12$, $k^* = 0.59$, $K^* = 25.11$ and $Y^*/\bar{Y} = 0.71$. Banks do hold liquidity reserves, but these are not sufficient to prevent fire-sale losses.*

4   The constrained-efficient allocation

The presence of externalities and the output drop caused by asset fire-sales during crises suggests that the competitive equilibrium is socially inefficient. To examine this conjecture we now contrast the competitive equilibrium with the allocation that would be chosen by a social planner. In line with the literature, we assume the planner is subject to the same collateral

and technology constraints as private agents, but internalizes the impact of his choices on aggregate outcomes. The allocation chosen by the planner is referred to as the constrained-efficient allocation. Formally, it maximizes aggregate welfare subject to the resource constraint (5), the collateral constraint (6), and $K \leq W$. The latter constraint reflects our assumption that only outside investors, but not banks, can rent out productive capital to firms.

Aggregate welfare in our economy is given by

$$
\begin{aligned}
U &= \{X - B^l\} + \delta\{p\{f(I) + Y(W) - R^s B^s + L\} \\
&+ (1-p)\{qf(I) + (1-q)\lambda I + Y(W + L - (1-q)R^s B^s) - R^s B^s + L\}\},
\end{aligned}
$$

so that the Lagrangian associated with the planner's problem can be written as

$$
\begin{aligned}
\mathcal{L}^{CEA} &= R^l(X - I - L + B^s) + f(I) + Y(W) - R^s B^s + L \\
&+ (1-p)\{(1-q)(\lambda I - f(I)) + Y(W + L - (1-q)R^s B^s) - Y(W)\} \\
&- \kappa\{(R^s B^s - L)R^K(W + L - (1-q)R^s B^s) - \lambda I\} - \eta(L - (1-q)R^s B^s),
\end{aligned}
$$

where κ and η denote Lagrangian multipliers and where we have made use of $\frac{1}{k} = R^K(W + L - (1-q)R^s B^s)$. Note also that we have replaced $K \leq W$ with $L \leq (1-q)R^s B^s$, which is perfectly equivalent in our environment. Differentiating the Lagrangian with respect to the planner's choice variables $\{I, B^s, B^l, L\}$, it is straightforward to show that the constrained-efficient allocation is characterized by (8), (11), and (12) together with

$$
\frac{R^l}{R^s} - 1 \leq (1-p)(1-q)\frac{1}{k} + \frac{\kappa}{k}\left(1 - (1-q)(R^s B^s - L)\frac{R^{K'}(K)}{R^K(K)}\right) - \eta(1-q), \qquad (15)
$$

$$
(1-p)\frac{1}{k} + \frac{\kappa}{k}\left(1 - (R^s B^s - L)\frac{R^{K'}(K)}{R^K(K)}\right) - \eta \leq R^l - 1, \qquad (16)
$$

and $K \leq W$. Equations (15) and (16) are the planner's optimality conditions for short-term borrowing, B^s, and reserves, L. Compared to (9) and (10), these conditions reflect that the planner takes into account the effect of its choices on the fire-sale price k as well as on the crowding out of productive investment, while ignoring the purely redistributive consequences of fire-sales.

15

Unlike private banks, the constrained planner takes into account that even small amounts of outstanding short-term debt per bank can force relatively large aggregate fire-sales, and can hence cause significant output losses during crises. Against this background, the planner may choose not to issue any short-term debt at all, despite the lower borrowing costs short-term debt brings about. This is particularly relevant when both the probability of a crisis, $p^c = 1 - p$, is large and the idiosyncratic risk component, q, is small. In this case, liquidity crisis are both likely and severe, as most banks become distressed simultaneously. This reasoning leads to the following proposition.

Proposition 4. *If the probability of a crisis is sufficiently high, $p^c \geq \overline{p}^c$, and the idiosyncratic component of risk is sufficiently small, $q \leq \overline{q}$, short-term debt is zero in the constrained-efficient allocation, $B^{s***} = 0$. Liquidity reserves are then also zero, $L^{**} = 0$.*

In the proof of Proposition 4 we establish that, for any given $q < \overline{q}$, there exists a unique threshold level $\overline{p}^c$ for the crisis probability such that $q < \overline{q}$ and

$$p^c \geq \frac{R^l - R^s}{R^s(1 - q)} \equiv \overline{p}^c \tag{17}$$

together are sufficient conditions for $B^{s***} = 0$. The planner thus abstains from issuing short-term debt altogether, eliminating the risk of fire-sales. Absent fire-sale risk, liquidity reserves have no useful purpose and $L^{**} = 0$ follows trivially.

We next consider the situation where the probability of a crisis is again large, but the crisis does not hit as many banks simultaneously because liquidity risk has a large idiosyncratic component, $q > \overline{q}$. In this situation, the planner can benefit from the lower borrowing costs associated with short-term debt, and at the same time insure against fire-sale risk by holding sufficient aggregate liquidity reserves within the banking system.

Proposition 5. *If the probability of a crisis is sufficiently high, $p^c \geq \overline{p}^c$, and the idiosyncratic component of risk is sufficiently large, $q > \overline{q}$, short-term debt is positive in the constrained-efficient allocation, $B^{s***} > 0$. Liquidity reserves are positive and sufficient to absorb all fire-sold assets within the banking sector, $L^{**} = (1 - q)R^s B^{s***}$.*

Proposition 5 shows that, under the maintained parametric assumptions, the planner is again not willing to accept any fire-sale risk. Sound banks are ensured to have sufficient idle liquidity

during crises to purchase all distressed bank assets at their fundamental price $k = 1$. This leads directly to the following corollary.

Corollary 2. *Let (14) and (17) be satisfied. In the constrained-efficient allocation, distressed banks' assets are fully absorbed within the banking system. There is no crowding-out of capital investment in the real sector, $K^{**} = W$, and no output loss is incurred, $Y^{**} = \bar{Y}$.*

We again illustrate these findings with a numerical example.

Example 2. *Assume the same parameters as in Example 1. Note that these parameters satisfy both (14) and (17). In the constrained-efficient allocation, $B^{s***} = 326.44$, $B^{l**} = 3.26$, $L^{**} = 219.80$, $I^{**} = 109.90$, $k^{**} = 1$, $K^{**} = 140$ and $Y^{**}/\bar{Y} = 1$. Liquidity reserves are sufficient to prevent fire-sale losses.*

Propositions 4 and 5 show that the planner is not willing to take fire-sale risk if the probability of crises is sufficiently large, $p^c \geq \bar{p}^c$. If the crisis probability is small, however, this result is no longer true. The planner is then willing to accept fire-sale risk to benefit from the lower costs of short-term borrowing. Example 3 provides a numerical illustration.

Example 3. *Assume the same parameters as in Example 1, with the exception that $q = 1/4$ and $p = 0.97$. Note that the parameters neither satisfy (14) nor (17). In the constrained-efficient allocation, $B^{s***} = 68.48$, $B^{l**} = 17.05$, $L^{**} = 0$, $I^{**} = 85.53$, $k^{**} = 0.76$, $K^{**} = 88.13$ and $Y^{**}/\bar{Y} = 0.83$. Liquidity reserves are insufficient to prevent fire-sale losses.*

5 Social inefficiencies and the role of idiosyncratic risk

Our findings thus far have shown that in environments where idiosyncratic liquidity risk is important, the constrained-efficient allocation features a significantly higher level of short-term borrowing and liquidity reserves as compared to the decentralized competitive equilibrium. Private banks therefore *underborrow*, which is in sharp contrast to models with pecuniary externalities and purely aggregate risk, where *overborrowing* emerges a prevalent phenomenon. In the following, we further examine the efficiency of banks' borrowing and investment decisions, and its relationship to idiosyncratic risk.

We first consider an environment where investor resources are scarce, $W < \overline{W}$, and both the idiosyncratic risk and the crisis probability are high, $q > \overline{q}$ and $p^c > \overline{p^c}$. The following Proposition shows that in such an environment, private banks always underborrow and underinvest in risky projects. Moreover, they hold insufficient liquidity reserves.

Proposition 6. *Assume that (13), (14), and (17) are satisfied. Then banks in the competitive equilibrium hold too little liquidity reserves relative to the constrained-efficient allocation, $L^* < L^{**}$, invest too little, $I^* < I^{**}$, and issue too little short-term debt, $B^{s*} < B^{s**}$. Moreover, these findings not only hold in absolute terms, but also in relative terms, $\frac{L^*}{I^*} < \frac{L^{**}}{I^{**}}$, $\frac{B^{s*}}{I^*} < \frac{B^{s**}}{I^{**}}$ and $\frac{L^*}{B^{s*}} < \frac{L^{**}}{B^{s**}}$.*

Key to this result is the presence of idiosyncratic liquidity risk. It allows the planner to essentially redistribute liquidity from sound banks to distressed banks in crises times, which provides insurance against fire-sale driven output losses. In the presence of this insurance, short-term debt is cheap and hence the planner relies mostly on this source of financing. By contrast, private banks do no internalize that their liquidity holdings excert a positive externality, and thus aggregate liquidity reserves are inefficiently low. This underinsurance gives rise to fire-sale risk in equilibrium, which in turn increases the cost of short-term borrowing. In the face of this higher cost private banks choose a lower amount of short-term debt compared to the planner, and they invest less.

By contrast, in an environment with mostly aggregate risk, $q < \overline{q}$, and where the crisis probability is again high, $p^c > \overline{p^c}$, private banks overborrow. This follows directly from Propositions 1 and 4, which together imply that $B^{s*} > B^{s**} = 0$. Intuitively, when idiosyncratic risk is low, using reserves to insure against liquidity risk is costly.[8] Due to the high probability of crises the planner is not willing to take fire-sale risk and chooses not to issue any short-term debt altogether. Private banks, which do not internalize their incremental impact on fire-sale risk, do issue a positive amount and hence overborrow relative to the planner solution.

Figure 1 further illustrates the role of idiosyncratic risk for the efficiency of private borrowing and investment decisions. It shows equilibrium allocations for different choices of the parameter $q \in [0, 0.97)$, holding an individual bank's probability of experiencing an adverse shock constant

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[8]The more banks become distressed during crises, the higher are the aggregate liquidity needs to insure a given amount of short-term debt per bank.

Figure 1: The role of idiosyncratic liquidity risk
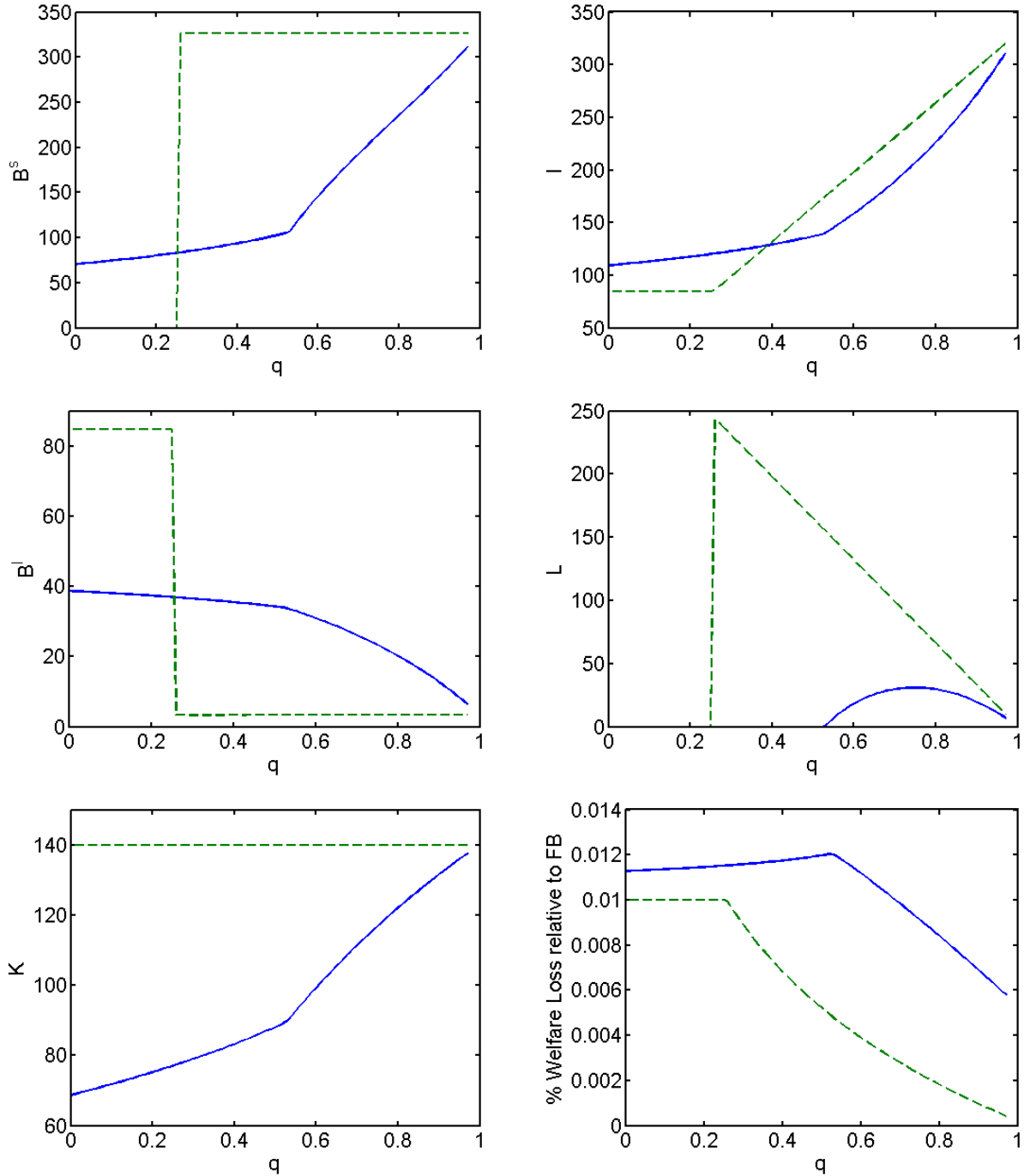
Note: The blue line shows the competitive equilibrium, the green dashed line the constrained-efficient allocations for varying levels of the idiosyncratic risk component q. The crisis probability is adjusted with q such that the probability for an individual bank to experience an adverse shock is fixed at $(1 - q)(1 - p) = 0.03$. The remaining parameters are as in Example 1. The capital investment panel is conditional on the realization of the bad state.

at 3% as in Example 1. The crisis probability is hence adjusted with q according to $p^c = 0.03/(1-q)$. Note that $\bar{q} = 0.26$ and by construction $p^c > \overline{p^c}$.

When liquidity risk is mostly of an aggregate type, $q \leq \bar{q}$, banks in the competitive equilibrium overborrow and overinvest relative to the constrained-efficient allocation, while reserve holdings are zero. When the bad state of the world materializes, distressed banks fire-sell their assets and productive capital investment in the real sector is severely crowded out, which leads to large output – and eventually welfare – losses. These results are well in line with the existing literature (see, for example, Bianchi, 2011; Stein, 2012; Jeanne and Korinek, 2013).

As the idiosyncratic risk component becomes more important, $q > \bar{q}$, the picture is significantly reversed. The constrained-efficient allocation then features a large amount of short-term borrowing combined with a large amount of liquidity holdings, such that the banking sector is fully insured against the risk of fire-sale losses. By contrast, the competitive equilibrium features underborrowing and underinvestment by banks, as well as significant output losses.

Finally, the last panel of Figure 1 illustrates the welfare losses relative to the first-best allocation for both the competitive equilibrium and the constrained-efficient allocation.[9] These welfare losses are relatively large for small q, and small for large q. More interestingly, though, the welfare gap between the decentralized equilibrium and the constrained-efficient allocation increases notably as the idiosyncratic risk component surpasses the threshold $\bar{q}$. This suggests that regulatory policies, which aim at closing this welfare gap, are particularly important in environments with idiosyncratic risk.

6   Macroprudential regulation

We now examine whether and how a macroprudential regulator can implement the constrained-efficient allocation as a competitive equilibrium. For environments with private overborrowing and overinvestment, it is well known from the existing literature that this can be achieved using instruments that prevent excessive borrowing, such as Pigouvian taxes on debt issuance or bank capital requirements. We therefore focus our attention on environments that give rise

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[9]In the first-best allocation, the planner is not subject to the collateral constraint but can commit to repay debt. Investment is thus financed via cheap riskless short-term debt only, up to the point where the expected return equals the risk-free rate R^s.

to underborrowing and underinvestment by private banks, which are less well understood.[10]

As for now, we assume that the regulator and all other economic agents have perfect information about the economic environment. In particular, there is common knowledge of all model parameters such as f, p, q, etc. In this situation, the regulator can implement constrained efficiency by imposing appropriate liquidity buffers on banks, as established by the following proposition.

Proposition 7. *Assume (13), (14) and (17) hold such that $B^{s**} > B^{s*} > 0$ and $L^{**} > L^* > 0$. A macroprudential regulator can implement the constrained-efficient allocation as a competitive equilibrium by imposing a minimum reserve requirement of $\mu = 1 - q$ units for each unit of short-term liabilities $R^s B^s$ in $t = 0$, and reducing this requirement to $\mu = 0$ in $t = 1$ if the bad state is realized.*

It is important to emphasize that the regulator needs only one instrument, the minimum reserve requirement, but this instrument must be employed actively. Only by reducing the reserve requirement in the bad state for *all* banks, the regulator eventually prevents fire-sales by inducing sound banks to purchase assets from distressed banks. A *static* liquidity requirement cannot restore constrained efficiency. An important take-away is therefore, that liquidity requirements, such as the liquidity coverage ratio (LCR) introduced in Basel III[11], need to be managed actively and changed in response to macroeconomic and financial shocks to show their full potential.

The quantity-based minimum reserves policy described above is not the only way to implement the constrained-efficient allocation. This is also possible using a price-based regulation in the form of a Pigouvian tax on short-term borrowing combined with interest on reserves. However, the price-based approach has a serious disadvantage: it is not budget neutral but generates a financial cost for the regulator, as the interest subsidies paid on reserves necessarily exceed the Pigouvian tax revenues.

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[10]Formally, we proceed under the assumption that the conditions (13), (14), and (17) are jointly satisfied.

[11]The LCR requires banks to hold liquid assets against the expected outflows within the next 30 days. Its aim is to "[...] improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source, thus reducing the risk of spillover from the financial sector to the real economy" (BIS, 2013).

Proposition 8. *Assume (13), (14) and (17) hold such that $B^{s***} > B^{s*} > 0$ and $L^{**} > L^{*} > 0$. A macroprudential regulator can implement the constrained-efficient allocation as a competitive equilibrium with a tax on short-term borrowing, combined with interest on reserves. However, this price-based regulation is not budget neutral but generates a cost for the regulator.*

Consider now a situation where the regulator is imperfectly informed. Specifically, as in Stein (2012), the regulator has limited knowledge about banks' lending opportunities and does not know the function f. From Proposition 7 it follows immediately that the regulator can still implement the constrained-efficient allocation using a countercyclical minimum reserve requirement, as no knowledge about f is necessary to set the correct level of the requirement, $\mu = 1 - q$. By contrast, the price-based approach becomes more difficult to implement, because the correct levels of the interest rate on reserves and the Pigouvian tax do depend on banks' productivity f. Imperfect information about bank lending opportunities thus further strengthens the case for using minimum reserve requirements as a macroprudential policy tool. It is immediate that this also holds true for other sources of imperfect information, such as limited knowledge of firms' production technologies or outside investors' endowments, as long as there is perfect (and common) knowledge of the size and structure of liquidity risk.

Finally, note that bank capital adequacy ratios, or other macroprudential tools that seek to curb excessive borrowing and risky investment by banks, are not suitable instruments in our environment. The simple reason is that, as the competitive equilibrium is characterized by underborrowing and underinvestment rather than overborrowing and overinvestment, any restrictions on borrowing and investment would not bind in the competitive equilibrium.

7   Reserve requirements in practice - a macroprudential tool?

Our theoretical analysis has found that minimum reserve requirements are a useful instrument for financial and macroeconomic stabilization. This is well in line with recent empirical studies, which argue that many central banks –mostly in developing countries– use minimum reserve requirements, together with interest and exchange rate policies, to smooth business cycles.

Federico, Vegh, and Vuletin (2014) construct a quarterly dataset for legal reserve requirements in 52 countries, 15 industrial and 37 developing, for the time period 1970-2011. They find that approximately 2/3 of the countries covered use reserve requirements actively, i.e. change their legal requirements more than once over the business cycle. Most, but not all, of these countries are developing economies, which employ a countercyclical reserve requirement as a prudential stabilization tool.

Cerutti, Correa, Fiorentino, and Segalla (2016) construct a database for various widely-used prudential tools, including minimum reserve requirements for both local-currency and foreign-currency denominated deposits. Their database covers a total of 64 countries for the time period 2000-2014. They argue that changes in local-currency and foreign currency reserve requirements are hardly ever observed in industrial economies, but frequent in developing economies. Moreover, they are typically positively and significantly correlated with real credit growth, which signals a countercyclical use of these instruments. Figures 2 and 3, which are constructed from the data provided by Cerutti, Correa, Fiorentino, and Segalla (2016), illustrate the use of reserve requirements in selected industrial and developing economies.

Finally, note that it is beyond the scope of the present analysis to formally test the theory developed in this paper against the data. To this end, we would need precise information on the structure of liquidity risk in both advanced and emerging economies, which is extremely difficult to collect. We do think, however, that this is a promising avenue for future research.

8   Conclusion

In this paper we have studied financial stability regulation in a simple model economy with pecuniary externalities, and where banks face both aggregate and idiosyncratic liquidity risk. We have argued that, when liquidity risk is not purely aggregate but has an important idiosyncratic component, banks underborrow and underinvest relative to the constrained-efficient allocation. Key to this result is that banks hold insufficient liquidity reserves, because they do not internalize that their reserves holdings excert a positive externality on other banks. A macroprudential regulator can restore constrained efficiency by ensuring sufficient liquidity in the banking system. This can best be achieved with a countercyclical minimum reserve requirement.

Figure 2: Frequency of changes in minimum reserve requirements, 2000-2014
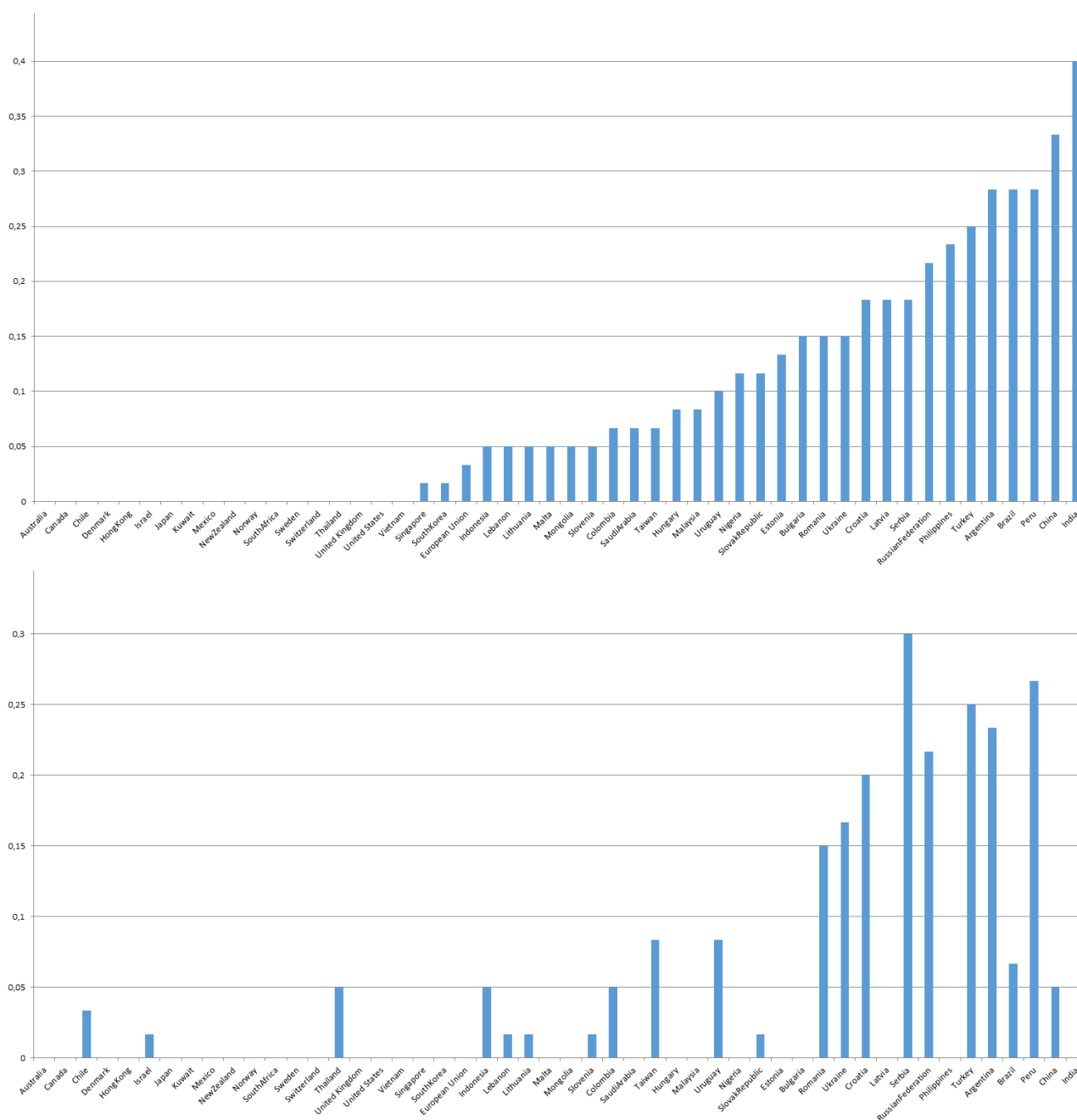
Note: The figure plots the frequency of changes in the minimum reserve requirement for selected countries from 2000 to 2014. The top panel refers to minimum reserve requirements on domestic currency deposits, and the bottom panel to reserve requirements on foreign currency deposits. Data Source: Cerutti, Correa, Fiorentino, and Segalla (2016).

Figure 3: Minimum reserve requirements, 2000-2014
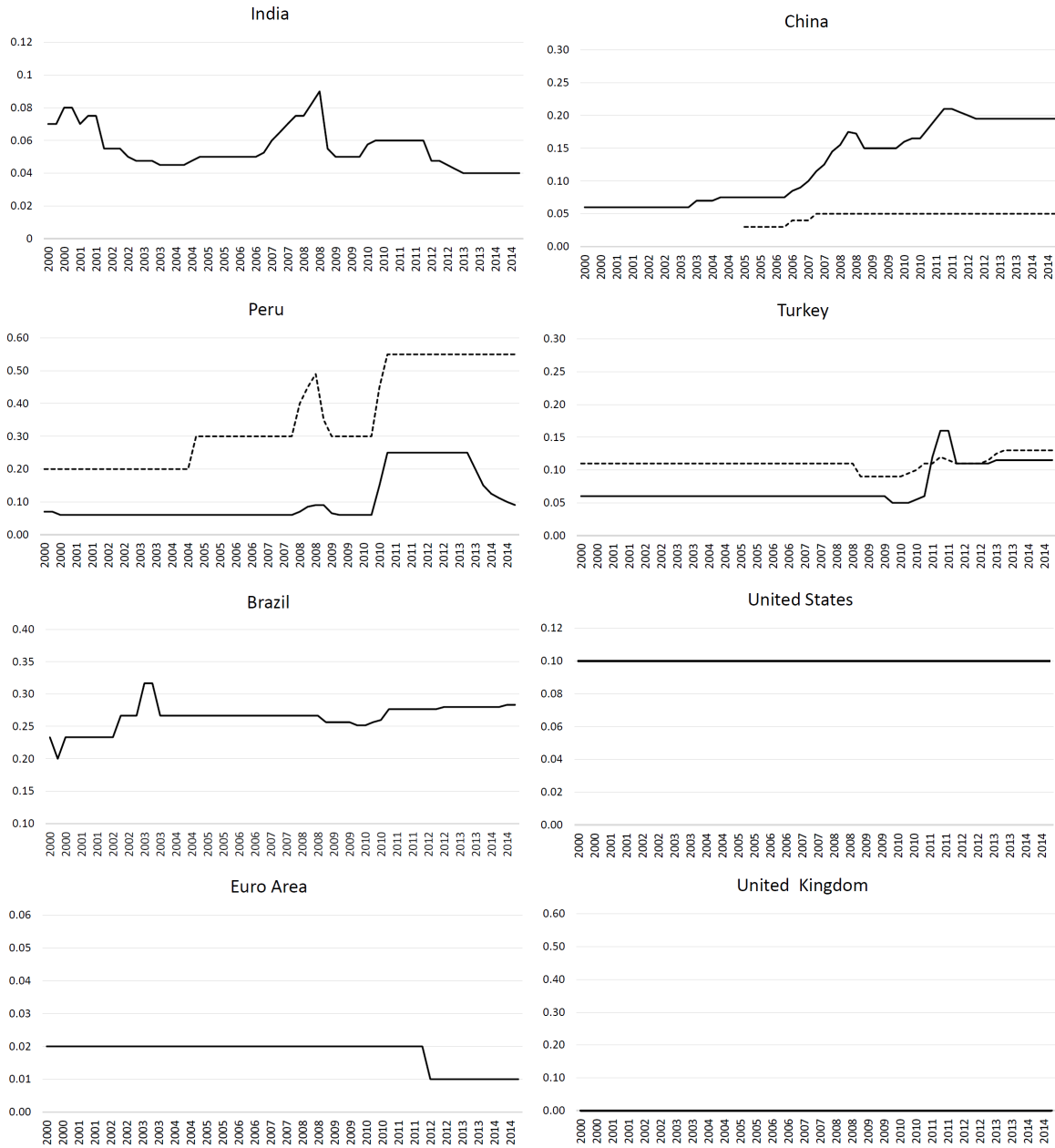
Note: The figure plots the minimum reserve requirement for selected countries from 2000 to 2014. The solid line displays the minimum reserve requirement on domestic currency deposits; the dotted line displays the minimum reserve requirement on foreign currency deposits. Data Source: Cerutti, Correa, Fiorentino, and Segalla (2016).

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A   Proofs

Proof of Proposition 1

By contradiction. Assume that banks would not issue short-term debt in the decentralized equilibrium, $B^s = 0$. Then $\kappa = 0$, as the collateral constraint would necessarily be non-binding, and $k = 1$ as there would not be any fire sales. Hence (9) would boil down to $R^l - R^s \leq 0$, which cannot hold under our maintained assumption $R^l > R^s$. Hence in the competitive equilibrium, $B^s > 0$.

Proof of Proposition 2

Consider a candidate equilibrium allocation with positive reserve holdings, $L > 0$. When $L > 0$ is optimal, equation (10) must hold with strict equality. The equilibrium conditions can then be solved for the allocation

$$
\begin{aligned}
\frac{1}{k} &= 1 + \frac{R^s - 1}{\delta q(1-p)R^s} > 1, \\
\kappa &= \left(\frac{1}{\delta} - 1 - \frac{R^s - 1}{\delta q R^s} \right) k, \\
I &= (f')^{-1} \left(\frac{\frac{1}{\delta} - \kappa\lambda - (1-p)(1-q)\lambda}{p + (1-p)q} \right), \\
K &= (k\alpha A)^{\frac{1}{1-\alpha}}, \\
B^s &= \frac{K + \lambda I k - W}{q R^s}, \\
L &= R^s B^s - \lambda I k.
\end{aligned}
$$

This allocation is a competitive equilibrium if and only if the implied value for L is non-negative and the Lagrangian multiplier κ is non-negative. It is straightforward to show that this is equivalent to the conditions

$$
q \geq \frac{(R^s - 1)R^l}{(R^l - 1)R^s} \equiv \bar{q} > 0
$$

and

$$W \leq \left(\frac{\delta q (1-p) R^s \alpha A}{\delta q (1-p) R^s + R^s - 1} \right)^{\frac{1}{1-\alpha}} - \left(\frac{\delta q (1-p) R^s}{\delta q (1-p) R^s + R^s - 1} \right)$$

$$\times (q-1) \lambda (f')^{-1} \left(\frac{\frac{1}{\delta} - \lambda \frac{(1-p)(qR^s - \delta q R^s - R^s + 1)}{(1-p)\delta q R^s + R^s - 1} - (1-p)(1-q)\lambda}{p + (1-p)q} \right) \equiv \overline{W}.$$

Conversely, a competitive equilibrium with positive reserve holdings does not exist when either of these conditions is violated.

Proof of Proposition 3

Recall that $B^s > 0$ in any competitive equilibrium. In an equilibrium with zero reserve holdings, $L = 0$, this implies that $K = W - (1-q)R^s B^s < W$ in times of crises. Similarly, in an equilibrium with positive reserve holdings, $L > 0$, the fire-sale price satisfies $k < 1$ as established in the proof of Proposition 2. This again implies $K < W$ and thus $L < (1-q)R^s B^s$.

Proof of Proposition 4

We start by showing that $L^{**} = 0$. Assume that $B^{s**} > 0$ and $L^{**} > 0$. Then both (15) and (16) hold with equality, and the Lagrangian multiplier in the constrained-efficient allocation can be computed from these conditions as

$$\kappa^{**} = \frac{1}{\delta} - 1 - \frac{R^s - 1}{\delta q R^s}.$$

When (14) is violated, as is the case under our maintained assumptions, the implied value for κ^{**} is negative. This cannot be the case in a constrained-efficient equilibrium. Rather, $L^{**} = 0$ must hold when $B^{s**} > 0$. Finally assume that $B^{s**} = 0$. Then $L^{**} = 0$ trivially, since there is no role for reserves. We have thus established that $L^{**} = 0$ in the constrained efficient allocation when (14) is violated.

It remains to show that $B^{s**} = 0$. Assume this was not the case. Then the optimality condition for B^s would hold with equality and, since $L^{**} = 0$, the constraint $K \leq W$ would be

non-binding such that $\eta = 0$. The optimality condition for B^s would thus boil down to

$$\frac{R^l - R^s}{R^s} = (1-p)(1-q)\frac{1}{k} + \frac{\kappa}{k}\left(1 - (1-q)(R^s B^s - L)\frac{R^{K'}(K)}{R^K(K)}\right).$$

Note that $\frac{\kappa}{k}\left(1 - (1-q)(R^s B^s - L)\frac{R^{K'}(K)}{R^K(K)}\right) \geq 0$, since $R^{K'}(K) < 0$. The above condition can thus only hold when (17) is violated. Accordingly, when (14) is violated and at the same time (17) is satisfied, $B^{s**} = 0$ and $L^{**} = 0$.

Proof of Proposition 5

We first show that $B^{s**} > 0$ if (14) and (17) are both satisfied. Assume this was not the case. Then $L = 0$, as there is no role for reserve accumulation, and $\kappa = 0$, as the collateral constraint is certainly non-binding when $B^s = 0$. The planner's optimality conditions then boil down to

$$\frac{R^l}{R^s} - 1 < (1-q)\left\{(1-p)\frac{1}{k} - \eta\right\},$$

$$(1-p)\frac{1}{k} - \eta < R^l - 1.$$

Combining these conditions, we obtain

$$\frac{R^l - R^s}{R^s} < (1-q)(R^l - 1),$$

which can be rearranged as

$$R^s > \frac{1}{1 - q(1-\delta)}.$$

This condition is *necessary* (but not sufficient) for $B^s = 0$ in the constrained-efficient allocation. Hence (14) is *sufficient* (but not necessary) for $B^s > 0$ in the constrained-efficient allocation.

When (14) holds, $B^{s**} > 0$ in the constrained-efficient allocation and the corresponding optimality condition holds as an equality,

$$\frac{R^l}{R^s} - 1 = (1-p)(1-q)\frac{1}{k} + \frac{\kappa}{k}\left(1 - (1-q)(R^s B^s - L)\frac{R^{K'}(K)}{R^K(K)}\right) - \eta(1-q).$$

Note that $\frac{\kappa}{k}\left(1 - (1-q)(R^s B^s - L)\frac{R^{K'}(K)}{R^K(K)}\right) \geq 0$, since $R^{K'}(K) < 0$ and $\kappa = 0$ whenever $L > R^s B^s$. Moreover, note that since $k \in (0, 1]$,

$$0 < (1-p)(1-q) \leq (1-p)(1-q)\frac{1}{k}.$$

Condition (17) is hence a sufficient condition for $\eta > 0$ and thus $K^{**} = W$. Since $B^{s**} > 0$, this implies that $L^{**} = (1-q)R^s B^{s**} > 0$.

Proof of Proposition 6

Under our maintained assumption, the competitive equilibrium allocation is given by

$$
\begin{aligned}
\frac{1}{k^*} &= 1 + \frac{R^s - 1}{\delta q (1-p) R^s}, \\
\kappa^* &= \left(\frac{1}{\delta} - 1 - \frac{R^s - 1}{\delta q R^s}\right) k^*, \\
I^* &= (f')^{-1}\left(\frac{\frac{1}{\delta} - \kappa^* \lambda - (1-p)(1-q)\lambda}{p + (1-p)q}\right), \\
K^* &= (k^* \alpha A)^{\frac{1}{1-\alpha}} < W, \\
B^{s*} &= \frac{\lambda I^* k^*}{q R^s} - \frac{W - K^*}{q R^s}, \\
L^* &= \left(\frac{1}{q} - 1\right) \lambda I^* k^* - \frac{W - K^*}{q R^s}
\end{aligned}
$$

whereas the constrained-efficient allocation is given by

$$
\begin{aligned}
\frac{1}{k^{**}} &= 1, \\
\kappa^{**} &= \frac{1}{\delta} - 1 - \frac{R^s - 1}{\delta q R^s}, \\
I^{**} &= (f')^{-1}\left(\frac{\frac{1}{\delta} - \kappa^{**} \lambda - (1-p)(1-q)\lambda}{p + (1-p)q}\right), \\
K^{**} &= W, \\
B^{s**} &= \frac{\lambda I^{**} k^{**}}{q R^s}, \\
L^{**} &= \left(\frac{1}{q} - 1\right) \lambda I^{**}
\end{aligned}
$$

Comparing the two allocations, it is evident that $I^{**} > I^*$, $B^{s**} > B^{s*}$ and $L^{**} > L^*$, $k^{**} > k^*$, $\kappa^{**} > \kappa^*$, and $K^{**} > K^*$. Finally, note that

$$
\begin{aligned}
\frac{B^{s**}}{I^{**}} &= \frac{\lambda}{qR^s} > \frac{\lambda k^*}{qR^s} + \frac{K^* - W}{qR^s I^*} = \frac{B^{s*}}{I^*}, \\
\frac{L^{**}}{I^{**}} &= \left(\frac{1}{q} - 1\right)\lambda > \left(\frac{1}{q} - 1\right)\lambda k^* + \frac{K^* - W}{qR^s I^*} = \frac{L^*}{I^*}, \\
\frac{L^{**}}{B^{s**}} &= (1-q)R^s > \frac{L^*}{B^*}.
\end{aligned}
$$

Proof of Proposition 7

Suppose that banks, in addition to the collateral constraint, also face a minimum reserve requirement of the form

$$
L \geq \mu R^s B^s,
$$

where μ is the reserve ratio chosen by the macroprudential regulator. The banks' Lagrangian then reads

$$
\begin{aligned}
\mathcal{L}^B &= f(I) - R^s B^s - R^l(I + L - B^s - E) + L \\
&+ (1-p)q\left(\frac{1}{k} - 1\right)L \\
&+ (1-p)(1-q)\{\lambda I - f(I) - \left(\frac{1}{k} - 1\right)(R^s B^s - L)\} \\
&- \kappa\left\{\frac{R^s B^s - L}{k} - \lambda I\right\} - \xi\{\mu R^s B^s - L\}.
\end{aligned}
$$

The first-order conditions with respect to B^s and L yield

$$
\begin{aligned}
R^l - R^s &\leq (1-p)(1-q)\left(\frac{1}{k} - 1\right)R^s + \frac{\kappa R^s}{k} + \xi\mu R^s, \\
R^l - 1 &\geq (1-p)(\frac{1}{k} - 1) + \frac{\kappa}{k} + \xi.
\end{aligned}
$$

These conditions coincide with the corresponding expressions in the constrained social planner's problem when

$$\xi \;=\; (1-p)\frac{1}{k} - \frac{\kappa}{k}(R^s B^s - L)\frac{R'^K(K)}{R^K(K)} - \eta - (1-p)(\frac{1}{k} - 1).$$

and

$$\mu\xi \;=\; (1-q)\left\{(1-p)\frac{1}{k} + \frac{\kappa}{k}(R^s B^s - L)\frac{R'^K(K)}{R^K(K)} - \eta - (1-p)(\frac{1}{k} - 1)\right\} = (1-q)\xi.$$

Accordingly, the contrained-efficient allocation can be implemented as a competitive equilibrium by setting $\mu = 1-q$. Finally, in times of crises, the regulator must reduce the reserve requirement to zero, such that successful banks can use their reserves to purchase other banks' assets.

Proof of Proposition 8

Suppose that banks have to pay taxes τ on short-term borrowing and earn interest on reserves, with the interest rate being equal to r. The banks' Lagrangian is then given by

$$
\begin{aligned}
\mathcal{L}^B \;=\;& f(I) - R^s B^s - R^l(I + L + \tau B^s - B^s - E) - \tau B^s + L(1+r) \\
+\;& (1-p)q\{\left(\frac{1}{k} - 1\right)L - rL\} \\
+\;& (1-p)(1-q)\{\lambda I - f(I) - rL - \left(\frac{1}{k} - 1\right)(R^s B^s - L)\} \\
-\;& \kappa\left\{\frac{R^s B^s - L}{k} - \lambda I\right\}.
\end{aligned}
$$

Taking the derivative with respect to B^s and L yields

$$
\begin{aligned}
(1-p)(1-q)(\frac{1}{k} - 1)R^s + \kappa R^s \frac{1}{k} + \tau(1+R^l) \;&\geq\; R^l - R^s, \\
(1-p)(\frac{1}{k} - 1) + pr + \kappa\frac{1}{k} \;&\leq\; R^l - 1.
\end{aligned}
$$

The constrained-efficient allocation can thus be implemented by setting τ and r as

$$\tau = \frac{R^s}{(1+R^l)}(1-q)\{(1-p) - \frac{\kappa}{k}(R^s B^s - L)\frac{R'^K(K)}{R^K(K)} - \eta\}, \tag{18}$$

$$r = \frac{1}{p}\{(1-p) - \frac{\kappa}{k}(R^s B^s - L)\frac{R'^K(K)}{R^K(K)} - \eta\}. \tag{19}$$

It still remains to show that this policy is not budget neutral but entails a net cost to the regulator, i.e.

$$\tau B^s < rL.$$

To see this, note that (18) and (19) can be combined to yield

$$\tau = \frac{(1-q)R^s}{(1+R^l)}pr.$$

Multiplying by B^s gives

$$\tau B^s = \frac{p}{1+R^l}r(1-q)R^s B^s = \frac{p}{1+R^l}rL < rL,$$

since in the constrained-effcient allocation $L = (1-q)R^s B^s$.