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R&D in Trade Networks: The Role of Asymmetry*

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Abstract

Countries differ substantially in their exposure to international trade as determined by the number of their trade partners. This exposure to trade and the asymmetry in trade exposure are anticipated by firms when making their R&D investments. We model a choice of R&D investments by firms in a given trade network focusing on the effects of the network asymmetry. The two large classes of networks considered include asymmetric *hub-and-spoke* networks and symmetric networks. We find that R&D, productivity and welfare are highest in a hub economy and lowest in a spoke, and the larger the degree of network asymmetry, the larger the difference. A country in a symmetric network exhibits intermediate levels of R&D and welfare, even if the number of its trade partners is the same as in a hub or in a spoke. This implies that regional/preferential trade agreements, which result in a highly asymmetric trade network, benefit hub economies but harm spokes. By contrast, multilateral trade agreements, which lead to a symmetric *complete* network, generate equal R&D and welfare benefits for all countries.

JEL Classification: O31, D85, D43, L13, F13

Keywords: Trade, hub-and-spoke network, asymmetry, R&D, oligopolistic competition

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1 Introduction

The last three decades of globalization, expansion of the World Trade Organization (WTO) and unprecedented spread of preferential trade agreements between countries have resulted in international trade network turning progressively large and complex. One of its key features is large asymmetries in countries' exposure to international trade, where some countries trade with many foreign partners, while others trade with only a few.[1] The main message of this paper is that this asymmetry in countries' trade exposure is an important determinant of firms' innovation, productivity and countries' welfare. It benefits countries that trade with many other countries but harms those that trade with a few.

This message has a number of relevant implications. First, it suggests that disregarding asymmetry of the trade system may lead to biased empirical estimates of the impact of trade on innovation, productivity, welfare and – in the long run – on countries' economic growth. This bias is already anticipated by empirical research that emphasizes the importance of trade network topology and country's position in the network for the country's economic growth and trade benefits (Kali and Reyes, 2007; Yang and Gupta, 2005). Secondly, it implies that firms' innovation incentives and productivity as well as countries' welfare depend on the type of their trade agreements with other countries – multilateral or preferential/regional – since each type of trade agreements is associated with a specific type of countries' trade involvement in the overall network of trade. The comparison of multilateral and regional trade agreements in terms of firms' productivity and countries' welfare offers new insights into the reasons for proliferation of regional trade agreements and simultaneous stagnation of multilateral trade talks within the WTO.[2] It also provides a new viewpoint on the question that has attracted much attention in recent public debates and trade policy research on whether allowing for regional trade agreements in the WTO (Article XXIV of the General Agreement on Trade and Tariffs (GATT)) helps or hinders the ultimate goal of multilateral free trade. Finally, by bringing the main message of the paper to the context of inter-regional trade within one country, we obtain that asymmetry in inter-regional trade relations may be an important factor contributing to the commonly observed disparities in firms' productivity and wealth across regions.[3]

The effects of trade on firms' innovation and endogenous productivity growth has been a subject of extensive research in trade literature. In particular, recent empirical studies find that trade openness

[1] In the literature, the international trade network is often described as *core-periphery* or *hub-and-spoke* in structure. Early discussion of hub-and-spoke trade systems can be found in Wonnacott (1990, 1991, 1996) and Kowalczyk and Wonnacott (1992). More recent studies include Baldwin (2004), Yang and Gupta (2005), De Benedictis et al. (2005), Deltas et al. (2006), Horaguchi (2007), Chong and Hur (2008), Kali and Reyes (2007), Fagiolo et al. (2009).

[2] The WTO online statistics asserts that "regional trade agreements (RTAs) have become increasingly prevalent since the early 1990s. As of 31 July 2013, some 575 notifications of RTAs ... had been received by the GATT/WTO." (*www.wto.org/english/tratop_e/region_e/region_e.htm.*) Figure 4 in the Appendix demonstrates this development. At the same time, multilateral trade negotiations have stalled soon after the launch of the Doha Development Round in Qatar in 2001, with most significant disagreement being between the group of developed and developing nations. For details see, for example, Schott (2004), Fergusson (2008, 2011), and Bhagwati (2012).

[3] Strong wealth polarization exists in many countries. For example, according to the statistics reported by The Economist (March 10th, 2011), Britain and the U.S.A. have the widest regional disparities in a group of developed countries covered by the survey. Average GDP per head in central London is more than nine times larger than in parts of Wales; and the District of Columbia in the U.S. is five times as rich as Mississippi. Italy and Germany have the smallest regional spread, yet incomes in their most affluent areas are still almost three times those of the poorest.

increases firms' incentives to innovate and adopt new technologies. For example, Verhoogen (2008) reports that Mexican exporting firms are more likely than non-exporters to be ISO 9000 certified, which is a proxy for the use of more advanced production techniques; and Bustos (2011) finds that Mercosur trade agreement generated an increase in new technology spending by Argentinian exporters.[4] Complementary to empirical work, theoretical studies substantiate the predominantly positive effects of trade on firms' R&D and productivity and identify various channels for these effects. Traditionally, most of these studies analyze the setting with just two countries, home and foreign, or offer numerical results for the setting with more than two countries.

In this paper, we propose an analytical framework with multiple countries to examine how the impact of trade on firms' R&D and productivity depends on structural features of the trade network. We focus on the effects of asymmetry in the number of countries' trade partners. The key idea is that asymmetry in trade relations creates heterogeneity in aggregate demand faced by firms in different countries, which in turn, leads to different incentives for innovation.

We develop this idea in a model where trade relations between countries are represented by a network. Nodes of the network are countries and links indicate trade agreements or other regulations that allow trade between the linked countries. Given the focus of the paper on the role of trade network asymmetry, we consider the countries as identical in all but their position in the network. In such setting the network formation models of Goyal and Joshi (2006), Furusawa and Konishi (2007) and Mauleon et al. (2010) have shown that asymmetries in countries' trade relations cannot emerge endogenously. Therefore, for the purposes of our analysis here we regard the trade network as *exogenous* but derive comparative static effects of a change in the relative number of country's trade partners on firms' innovation and countries' welfare.[5]

The structure of the trade network determines the degree of country's trade exposure, that is, the number of foreign markets it has access to and the level of competition in every market. Building on the studies of trade agreements in oligopoly setting (Goyal and Joshi, 2006; Saggi, 2006; Chen and Joshi, 2010), we consider Cournot competition between firms in every market and the intra-industry type of trade.[6] For simplicity we also assume that there is a single home producer in each country that can sell its good in the domestic market and in the markets of its trade partner countries. As a result, a firm in the country that has many trade links with other countries is likely to face larger aggregate demand for its good than a firm in the country that has only a few links. On the other hand, it also faces higher competition, at least in the domestic market. The ultimate size of firm's demand is determined by both, the number of its trade links and the level of competition in its domestic and foreign markets, where the latter, in turn, is defined by the number of the foreign

[4]Similar patterns in firm innovation and productivity improvement are shown by Lileeva and Trefler (2010) for Canada, Bernard et al. (2006, 2007) for the U. S., Topalova (2004) for India, Aw et al. (2000) for Korea and Taiwan, Alvarez and Lopez (2005) for Chile, De Loecker (2007) for Slovenia, and Van Biesebroeck (2005) for sub-Saharan Africa.

[5]In the conclusion we discuss a possibility of endogenous formation of asymmetric networks in a model that allows for cross-country differences in firms' demand and cost structure.

[6]We opt for oligopolistic competition, rather than monopolistic competition, which is more commonly assumed in international trade theory, since process R&D merges better with Brander (1981) oligopoly model than with Krugman (1980) monopolistic competition model.

markets' own links. This suggests that asymmetry in the pattern of trade relations between countries is likely to create heterogeneity among firms in terms of their aggregate demand.

The aggregate demand heterogeneity then becomes a key reason for the difference in firms' incentives to innovate. This follows from the interaction between the size of aggregate demand, increasing returns and R&D investment. Before competing in a product market, firms can perform cost-reducing innovations, or *process R&D*. Using the modeling assumptions of D'Aspremont and Jacquemin (1988) and Goyal and Moraga (2001), we consider the cost of R&D to be fixed, that is, independent of the amount produced, while the returns to R&D to be larger the larger the aggregate demand.[7] As a result and consistently with much empirical evidence (Griliches, 1957; Gustavsson, 1999; Kremer, 2002; Acemoglu and Linn, 2004), firm's incentives to innovate increase in its aggregate demand. Then in asymmetric trade networks, where aggregate demand of a firm depends on its network position, the incentives to innovate may differ substantially across firms. The difference in firms' R&D investments, in turn, translates into their productivity differences and eventually generates a disparity in countries' welfare.

We consider two large classes of trade network structures. In the spotlight of our analysis is the class of asymmetric, *hub-and-spoke* or core-periphery type of networks, where some countries (hubs) have a relatively large number of trade partners whereas other countries (spokes) have only a few partners. In the literature, such hub-and-spoke type of structure is considered to be a close fit to the actual international trade network, both in overall terms and at the commodity-specific level.[8] Within this class of networks we allow for a variation in the share of hubs and spokes among trade partners of a given country. For simplicity, we focus on networks where all hub nodes are the same and all spoke nodes are the same, so that each network can be characterized by just four parameters: the number of trade partners of a hub, number of trade partners of a spoke, share of hubs among trade partners of a hub, and share of spokes among trade partners of a spoke. A variation in the first two parameters allows addressing the effects of asymmetry in countries' trade involvement. In addition, a variation in the last two allows studying the effects of the exact pattern of asymmetry. The latter means that we can analyze how for a given degree of asymmetry, the composition of countries' trade partners, and thus the extent of their own trade involvement, matter for firms' incentives to innovate.[9] The results of our analysis for the hub-and-spoke type of networks are then compared with those for the class of symmetric, or regular structures. In a symmetric network all countries have the same number of trade partners, and the special case where any country is linked to every other country is referred to as the *complete* network.

The primary result of this paper is that the impact of trade on firm's R&D and productivity

[7]In contrast to D'Aspremont and Jacquemin (1988) and Goyal and Moraga (2001), the focus of this paper is on the effects on R&D of market access and competition faced by firms, rather than on the role of R&D collaboration or spillovers. Furthermore, while the above papers consider standard oligopoly competition in one market, our model features interaction between firms in several separate markets, where every market is accessible only to those firms that have a trade agreement with it.

[8]See footnote 1 for more detail.

[9]Of course, not all combinations of the four parameters are feasible. What is important, however, is that many combinations that are feasible can in fact be compared in terms of firms' R&D investments due to monotonicity of equilibrium R&D with respect to each of these four parameters.

depends crucially on structural features of the trade network. In particular, asymmetry in the number of trade partners matters since in any hub-and-spoke trade network, R&D investment of a hub is larger than R&D investment of a spoke. Also, R&D of either a hub or a spoke in the asymmetric network differs from R&D in the symmetric network even if the number of trade partners of a hub (resp., spoke) is the same as the number of trade partners of a country in the symmetric network. Furthermore, the difference between R&D investments of a hub and a spoke tends to increase in the degree of network asymmetry: as the number of hub's trade partners increases and/or the number of spoke's trade partners declines, R&D of a hub rises, while R&D of a spoke falls. Finally, for a given degree of network asymmetry, a larger trade exposure of country's trade partners has a negative effect on firm's incentives to innovate. That is, the larger the share of hubs among trade partners of a country (hub or spoke), the lower its R&D investment.

Intuitively these results can be explained as follows. Note that the aggregate demand of a firm in a hub is larger than the aggregate demand of a firm in the symmetric system even when the number of firm's foreign markets is the same in both cases. This has to do with the fact that some of the hub's foreign markets are spokes, which trade with only a few countries. Therefore, the competition faced by a hub firm in each of its foreign markets is lower and the market share gained is larger than those of a firm in the symmetric network. The reverse is true for spokes: for the same number of trade partners, the aggregate demand of a firm in a spoke is smaller than the aggregate demand of a firm in the symmetric system. This difference in aggregate demand determines the difference in firms' R&D investments: R&D and productivity of a firm in a hub are larger than those of a firm in the symmetric system, which, in turn, are higher than R&D and productivity of a firm in a spoke. Within an asymmetric, hub-and-spoke network, these differences in aggregate demand and R&D of hubs and spokes become larger when either the number of directly accessible markets of a hub increases, or the number of directly accessible markets of a spoke declines, or the share of spokes among hub's trade partners increases, or the opposite occurs at a spoke.

To complete the analysis, we compare welfare implications of trade within a symmetric network and within a *star* network, the simplest type of the hub-and-spoke structure. Consistently with earlier findings in the literature (Kowalczyk and Wonnacott, 1992; Deltas et al., 2006; Chen and Joshi, 2010), we find that for the same number of direct trade partners, welfare of the hub in a star exceeds welfare of a country in the symmetric system. On the other hand, the aggregate welfare benefits in the symmetric system are higher than those in the star for the same total number of countries. Moreover, as soon as trade costs are not too high, a larger degree of asymmetry in the star network, associated with a larger number of hub's trade partners, widens the gap between welfare of the hub and welfare of a spoke. Instead, in the symmetric trade network, implications of a larger number of countries' trade partners (at low trade costs) are positive for every country.

These findings have a number of implications for trade policy. First, we observe that each of the two considered network classes – symmetric and hub-and-spoke networks – can be associated with either multilateral or regional/preferential type of trade. In particular, as we discuss in more detail later, the complete network structure, which is a special case of symmetric networks, captures two basic principles of multilateral trade systems: the "most favoured nation" principle of non-discrimination and "reciprocity"(GATT).[10] By contrast, hub-and-spoke networks emerge when the principle of non-discrimination is violated (Article XXIV of the GATT), and some countries form preferential, or regional trade agreements (customs unions and free-trade areas) with each other. The latter results in a situation where different regional trade agreements "overlap",[11] forming an asymmetric network in which some countries (like the US, countries of the EU and ASEAN) become hubs and others are spokes. Under such an interpretation of symmetric and hub-and-spoke structures, the results of this paper imply that firm's incentives to innovate and promote its country's welfare depend on (a) the type of trade agreements signed by the country – multilateral or regional, and (b) the number of these trade agreements relative to that of the other countries. Firm innovation and productivity are largest in the country that is involved in many regional trade agreements and is, therefore, a hub in the regional trade system, while in a spoke economy firm innovation and productivity are lowest. Multilateral trade induces intermediate levels of firm innovation and productivity: lower than in a hub but higher than in a spoke. As for the welfare, multilateral free trade achieves the highest aggregate welfare for all countries and is also an individually better option for countries that otherwise can only become spokes in the regional trade system. On the other hand, hubs are better off than countries in the multilateral system, and thus, allowing regional trade agreements in the WTO may inhibit the pursuit of multilateral free trade by "powerful" hub economies.

Furthermore, interpreting our model in application to the inter-regional trade within one country, the findings of the paper suggest the benefits of regional integration. In particular, regions where firms have access to a larger number of markets in other parts of the country perform better in terms of both welfare and firm productivity as compared to regions that are less integrated in the inter-regional trade. Moreover, competition tends to play a negative role, reducing firms' incentives to innovate and improve own productivity.

The rest of the paper is organized as follows. Section 2 reviews the closely related literature. Section 3 presents the model and the two-stage game between firms. Section 4 discusses the equilibrium and the assumptions employed. Section 5 presents the results of the equilibrium analysis for the case of hub-and-spoke and symmetric trade networks and provides the comparison of the findings. The last subsection of section 5 presents the welfare analysis, and section 6 concludes.

[10] The reciprocity principle implies that each country agrees to reduce its trade barriers in return for a reciprocal reduction by another; the non-discrimination principle states that all countries in the multilateral trade system should be treated equally, that is, tariffs imposed by a country on imports from *any* other country in the system must be the same.

[11] Examples of such overlapping regional trade agreements include numerous EU free-trade agreements: EU - Chile, EU - Korea, EU - Mexico, EU - South Africa; US free-trade agreements: the North American Free Trade Agreement (NAFTA), US - Israel, US - Australia, US - Chile, US - Korea, together with many proposed trade agreements such as EU - US Transatlantic Free Trade Area (TAFTA) and Free Trade Area of the Americas (FTAA); free-trade agreements of the Asian countries: the Association of Southeast Asian Nations (ASEAN), ASEAN - Australia - New Zealand Free Trade Area (AANZFTA), ASEAN - Korea, ASEAN - China, etc.

2 Related Literature: Trade, Innovation and Trade Agreements

This paper brings together two large and important literatures: the literature on the effects of trade on firms' innovation incentives and productivity and the literature on trade agreements and political economy of trade. Below we briefly discuss the approach and main research directions in each literature. We show that the main contribution of this paper to the former is its focus on the effects of asymmetry of the trade network, while its contribution to the latter is our interest in firm innovation and the difference in innovation levels across different types of trade agreements.

Theoretical studies on the effects of international trade on R&D and/or productivity at the firm level identify different channels for these effects: the improved allocation of resources through specialization in multi-product firms (Grossman and Helpman, 1991; Bernard et al., 2010, 2011; Mayer et al., 2014; Feenstra and Ma, 2008; Dhingra, 2010; Eckel and Neary, 2010), the knowledge spillovers effect (Rivera-Batiz and Romer, 1991; Devereux and Lapham, 1994; Grossman and Helpman, 1991), the increased profitability of lower unit-cost technology due to access to a larger market (Yeapple, 2005; Ekholm and Midelfart, 2005; Atkenson and Burstein, 2010; Van Long et al., 2011), the pro-competitive effect of trade openness (Aghion et al., 2005; Aghion and Griffith, 2008; Peretto, 2003; Licandro and Navas-Ruiz, 2011; Impullitti and Licandro, 2013; Jensen and Thursby, 1987), and others.[12]

Among these studies the most closely related to our paper are oligopoly models of intra-industry trade between countries that are the same or similar in their endowments and technologies.[13] This literature is not large. Early contributions include Jensen and Thursby (1986, 1987), more recent developments are Haaland and Kind (2008), Van Long et al. (2011), Impullitti and Licandro (2013), and Dewit and Leahy (2015). These studies provide the rationale for the impact of trade on firms' innovation incentives and for the effect of R&D subsidies, focusing on a setting with just two countries – home and foreign – or on a setting with more than two countries but such that all countries are absolutely symmetric (including the symmetry in market size). A common assumption in this literature is that firms from all countries trade either in a single "world market", where the demand for good and competition between firms are increased compared to their autarky levels, or that they trade in separate, "segmented markets" but so that *all* firms have access to *all* markets and compete with each other. Clearly, such approach does not allow investigating the effects of asymmetry in countries' trade relations, and evaluate differences in R&D across multilateral and regional trade systems.[14] In addition, Pires (2012) analyzes a two-country trade model with asymmetric markets. However, the market size differences in this model are imposed by assumption rather than implied

[12] In addition, the "new" trade literature with increasing returns in production (Krugman, 1979, 1980; Helpman, 1981) and the firm heterogeneity literature (Melitz, 2003; Bernard et al., 2007; Melitz and Ottaviano, 2008; Falvey et al., 2006; Arkolakis et al., 2014) take firm-level productivity as exogenous and address the effects of trade on productivity at the *industry* level. See World Trade Report 2008 and Melitz and Redding (2013) for surveys.

[13] Similarity in endowments and technologies sets aside the "comparative advantage" explanation of trade asserted in the traditional trade literature (the HeckscherOhlin and the Ricardian models).

[14] There is also a substantial literature on trade under oligopoly that does not address R&D, but examines motives for trade, gains from trade, etc. Classic references include Brander (1981), Brander and Krugman (1983), Venables (1985), Weinstein (1992), Dixit and Grossman (1986), Yomogida (2008), Neary (2003, 2009), and Eckel and Neary (2010).

by differences in countries' exposure to trade. Therefore, the focus of the paper is also different from the one in our paper. It is not on the impact of trade and asymmetries in trade on firms' R&D but in some sense, the other way round: Pires (2012) explores how (assumed) market size differences and the implied differences in firms' R&D affect the amount of firms' export.

The second related strand of literature is on trade agreements and political economy of trade policy. Three broad directions can be identified within this literature. The first direction analyzes welfare impact and conditions that determine welfare gains of regional/preferential agreements between countries, where countries within the agreement have "preferential" tariff regime with each other compared to relations with the rest of the world. This literature goes back to Viner (1950), and more recent developments are surveyed in Krishna (2004), Baier et al. (2008) and Feenstra (2004, Chapters 6 and 9). The second direction studies economic rationale and consequences of different trade rules embodied in the GATT and the WTO, such as rules of origin, the principles of reciprocity and non-discrimination (Krishna and Krueger, 1995; Krishna, 2002; Bagwell and Staiger 1997, 2002). Finally, the third group of papers examines the incentives for countries to join regional trade agreements versus multilateral agreements. It focuses on a question that was originally raised by Bhagwati (1993), whether joining a regional trade agreement helps or hinders the ultimate goal of multilateral free trade. Studies that indicate that regional trade agreements may be "stumbling blocks" for multilateral trade include Krishna (1998) and McLaren (2002), while Baldwin (1995) and Ethier (1998) find support on the other side. The same question of incentives for multilateral or regional trade is addressed by network formation models of Goyal and Joshi (2006), Furusawa and Konishi (2007) and Mauleon et al. (2010). They examine the formation of free trade agreements as a network formation game and focus on stability and efficiency of the equilibrium networks.[15] All this literature, while rich and diverse, overlooks the question of firm innovation and productivity. In particular, a potential link between firms' incentives to innovate and its country's position in the network of trade agreements or the type of the trade agreements, has not received any attention. In this paper, we aim to fill this gap.

3 The model

In what follows we define the trade network, the economy and the two-stage game capturing strategic interaction between firms in different countries.

3.1 Trade network

Consider a setting with N countries where at least some of the countries are involved in intra-industry trade with one or more other countries. Trade relations between countries are modeled as a network in which nodes represent countries and links indicate reciprocal trade agreements or other regulations that allow trade between the linked countries. If two countries are connected by a trade link, then

[15] In a non-network setting the incentives for countries to form trade agreements, and in particular, the third country effects, are also examined by Chen and Joshi (2010).

each offers the other an access to its domestic market under a certain level of trade tariffs. This reciprocity in trade relations implies, in particular, that links in the network are *undirected*. On the other hand, if countries do not have a trade link, then tariffs on trade between them are regarded as "too high" and therefore, prohibit trade. This means that only the countries that are directly linked in the network can trade with each other.[16]

For any $i \in 1:N$, N_i denotes the set of *neighbors* of country i, that is, all countries with which i has a trade link in the network. These are *direct trade partners* of i. Similarly, N_i^2 denotes the set of direct trade partners of direct trade partners of i that are different from i itself. We call countries in N_i^2 *two-links-away trade partners* of i. Note that these definitions do not exclude the possibility that some countries are simultaneously direct and two-links-away trade partners of i. In what follows we will denote by $|N_i|$ and $|N_i^2|$ the cardinality of sets N_i and N_i^2, respectively.

3.2 Demand and cost structure

Each country has one firm producing some homogeneous good that can be sold in the domestic market and in the markets of its direct trade partners.[17] Let the output of firm i (from country i) produced for country j be denoted by y_{ij}. Then the total output of firm i is given by $y_i = \sum_{j \in N_i \cup \{i\}} y_{ij}$. Each firm i selling its good in country $j \in N_i \cup \{i\}$ faces the inverse linear demand in country j given by:

$$p_j = a - b\left(y_{ij} + \sum_{k \in N_j \cup \{j\}, k \neq i} y_{kj}\right), \tag{1}$$

where $a, b > 0$ and $\sum_{k \in N_j \cup \{j\}} y_{kj} \leq a/b$. Constants a and b are identical for all countries, so that each country has the same market size, that is, total demand for a given price. This does not only simplify the analysis but also brings attention to the role of the trade network structure, and in particular, the effects of network asymmetry in otherwise symmetric setting.

Following a convention in trade models, we consider that all firms bear the same unit trade costs. Namely, let τ denote the trade costs faced by *every* firm per unit of exports to *any* of its direct trade partners. These costs include tariffs per unit of export, transportation costs, etc.[18] Then the total trade costs of firm i are equal to:

$$t_i(\{y_{ij}\}_{j \in N_i}) = \tau \sum_{j \in N_i} y_{ij}. \tag{2}$$

To reduce its production costs, each firms can invest in R&D. The R&D effort of a firm helps lower its marginal cost of production but since R&D is costly, it also increases firm's fixed costs. We assume linear costs of production and quadratic fixed costs of R&D. The latter implies diminishing

[16]The same representation of trade relations between countries is proposed in the network formation model of trade by Goyal and Joshi (2006).

[17]Implicitly this means that there is no free entry of firms, which gives rise to oligopolistic market structure of international trade. This is, of course, relevant for some but not all industries. The same setting with a single firm per country is considered in Goyal and Joshi (2006), Chen and Joshi (2010), and Mauleon et al. (2010).

[18]The analysis carries over in a setting where $\tau = 0$. The assumption of zero trade costs is standard in the literature that interprets links in the network as free trade agreements and studies the formation of this network. See, for example, Furusawa and Konishi (2007), Goyal and Joshi (2006), and Mauleon et al. (2010).

returns to scale in innovation, which although not uncontroversial, finds support in the empirical literature (see e. g. Fung (2002)). Denote by x_i the R&D effort of firm i. Then the production cost function is given by:

$$c(y_i, x_i) = (\gamma - x_i)y_i, \tag{3}$$

where $0 \leq x_i \leq \gamma \; \forall i \in 1 : N$. The R&D cost function is

$$z(x_i) = \delta x_i^2 \tag{4}$$

for some $\delta > 0$. Thus, when firms choose their innovation efforts, they face a trade-off between investing more in R&D and achieving lower marginal costs at the expense of higher fixed costs, and vice versa. However, firms with a larger overall market (domestic and foreign) can cover their fixed costs of innovation more easily due to increasing returns in production. In fact, this observation is key for the results that follow. The overall market size of a firm, that is, its aggregate demand on the market is a driving force behind firm's incentives to innovate: the larger the aggregate demand, the larger the difference between the returns to R&D, which are increasing in aggregate demand, and costs of R&D, which are independent of the demand. Thus, a larger market size of a firm amplifies its R&D incentives.

In the trade network, the aggregate demand of a firm depends on the number of its direct trade partners and on the level of competition in every country, where the latter, in turn, is determined by the number of trade partners' own trade partners. This is where a consideration of trade network asymmetry becomes important. In the subsequent analysis we will show that asymmetry in countries' trade relations and the exact pattern of asymmetry (share of trade partners with large and small number of own trade partners) lead to a specific distribution of firm market sizes and large differences in equilibrium R&D investments.

The demand and cost structure described above give rise to the following profit function of firm i:

$$\pi_i = \sum_{j \in N_i \cup \{i\}} \left(a - by_{ij} - b \sum_{k \in N_j \cup \{j\}, k \neq i} y_{kj} \right) y_{ij} - (\gamma - x_i)y_i - \delta x_i^2 - \tau \sum_{j \in N_i} y_{ij}.$$

It is equal to the sum of revenues collected by the firm in the domestic and foreign markets minus the costs of overall production, R&D and transportation.

3.3 Two-stage game

Firms choose their level of R&D activities and the subsequent production plan via interaction in a two-stage non-cooperative game.[19] At the first stage firms simultaneously choose their R&D efforts, which determines their marginal cost of production. Given this cost, at the second stage firms simultaneously choose their production quantities for the domestic market and for the markets of

[19] Under an alternative specification where R&D and production decisions are made at the same time, the results are only *quantitatively* different. The derivations are available from the author.

their direct trade partners.[20] Firms compete as Cournot oligopolists and set their output for each market taking as given the outputs of the rivals in this market.

Note the specific nature of interaction between firms in this game. First, firms compete with each other not in one but in several separate markets. Secondly, since only directly linked countries trade, a firm competes only with its direct and two-links-away trade partners. Furthermore, any direct trade partner of firm i competes with i in its own market and in the market of firm i, while any two-links-away trade partner of i, who is not simultaneously its direct trade partner, competes with i only in the market(s) of their common direct trade partner(s). This two-links-away radius of interaction between firms does not mean, however, that firms located further away from i do not affect its R&D and production choices. As soon as the trade network is connected,[21] firms that are further than two links away from firm i affect its decisions *indirectly*, through the impact they have on R&D and production choices of their own trade partners and trade partners of their partners, etc.

4 Equilibrium

As a solution concept for the game we employ a standard subgame perfect Nash equilibrium. We find it via backward induction and provide the details of derivations in the Appendix. We show that a solution of each stage of the game exists and is interior as long as certain restrictions on parameters hold. Moreover, the Nash-Cournot equilibrium of the second stage is always unique, while the equilibrium of the first stage is unique for some but not all network structures. For the types of networks that will be in focus of our analysis later on, multiple equilibria are possible. In these cases we will consider *symmetric* equilibria, where firms with the same position in the network exert identical R&D effort. Such symmetric equilibrium is unique. Below we discuss these points in more detail.

First, we observe that the profit function of each firm is concave in firm's own choice variables – production quantities at the second stage of the game and R&D effort at the first stage. To be more precise, this is always the case at the second stage of the game and at sufficiently high values of parameter δ at the first stage. Therefore, the equilibrium of each stage is determined by the system of the (linear) first-order optimality conditions of all firms whenever the solution of the system is interior, that is, $y_{ij} > 0$ for all i and $j \in N_i \cup \{i\}$ and $0 < x_i < \gamma$ for all i. At the second, Cournot-competition stage the solution of the system is unique for any vector of firms' R&D efforts $\{x_i\}_{i \in 1:N}$. It is strictly positive and hence, forms the Nash-Cournot equilibrium of the second stage, as soon as the demand in each market is sufficiently high relative to costs of production and exporting. This is guaranteed by the assumption that the demand parameter a is sufficiently large relative to the cost parameters γ and τ:

Assumption 1 $\quad a > \gamma \left(1 + \max_{i \in 1:N} |N_i|\right) + 2\tau.$

[20] Due to trade costs and (potential) asymmetry in the number of firms at every market, markets are segmented, so that quantities to be shipped to each market are chosen independently. This also implies that firms can price discriminate between locations: the export price (net of transport costs) need not be equal to the price charged domestically.

[21] The network is connected if there exists a path of links between any pair of nodes.

The same condition turns out to be sufficient for the solution of the first, R&D stage to be positive. That is, as soon as the demand for the good in each market is sufficiently large, the returns to R&D turn out to be high enough relative to costs to induce strictly positive R&D investments in all countries. On the other hand, if R&D is sufficiently costly, the amount of R&D investments will not exceed the upper threshold of γ. This is ensured by

Assumption 2 $\quad \delta > \frac{1}{\gamma b} \max_{i \in N} \left[\sum_{j \in N_i} \frac{|N_j|+1}{(|N_j|+2)^2} \left(\gamma |N_j| + a - 2\tau \right) + \frac{|N_i|+1}{(|N_i|+2)^2} (\gamma |N_i| + a + |N_i| \tau) \right].$

In fact, Assumption 2 is also sufficient for the concavity of the profit function at the second stage of the game, at least as soon as Assumption 1 holds. Thus, both assumptions together imply that the system of the first-order optimality conditions at each stage of the game determines the equilibrium production quantities and R&D efforts of firms, where all production quantities and R&D efforts are strictly positive and R&D does not exceed γ.[22]

We also observe that the solution of the first, R&D stage might not be unique. In fact, for hub-and-spoke and symmetric networks that will be central to our subsequent analysis multiple solutions are possible. In these cases, we think it is reasonable to focus on symmetric equilibria, such that in a symmetric network all firms exert the same R&D effort, and in a hub-and-spoke network, R&D efforts of all hub firms are the same, and R&D efforts of all spoke firms are the same. Such symmetric equilibrium is unique. Moreover, we notice some interesting features of equilibria at both stages. First, the equilibrium output of firm i produced for any country $j \in N_i \cup \{i\}$ is increasing in firm's own R&D effort and decreasing in R&D efforts of i's rivals in market j. That is, the higher the equilibrium R&D of firm i and the lower the equilibrium R&D of every other firm competing with i in market j, the higher the share of market j gained by i. Second, the equilibrium R&D efforts of firm i and its direct and two-links-away trade partners are *strategic substitutes* from i's perspective. Intuitively, by exerting higher R&D efforts, firm i's rivals capture larger market shares, and this reduces returns to R&D for firm i.

5 Results

In this section, we derive the equilibrium R&D efforts of firms in hub-and-spoke and symmetric trade networks and study the role of network asymmetry. Asymmetry of the trade network means heterogeneity in firms' exposure to trade and in the level of competition faced in every market. We will show that this then leads to heterogeneity in firms' overall market size, which in turn results in different incentives for innovation. Note, that while the role of firm's market size for innovation follows from the specification of firms' R&D and production in section 3.2, the impact of trade network asymmetry on firms' market size is not immediately clear. For example, a larger exposure to trade means access to a larger number of markets and hence, larger aggregate demand (*scale* effect), but it also implies higher competition in the domestic market. Competition, on the the other

[22] In the Appendix we also show that under Assumptions 1 and 2, the equilibrium profits of all firms are strictly positive.

hand, may itself affect firm's aggregate demand in opposite directions. A larger competition reduces each firm's individual market share (*market-share* effect of competition) but it also reduces price markups, which increases firm's overall demand (*markups* effect of competition). Moreover, in the network setting with multiple countries, the size of the scale and competition effects of trade between any pair of countries depends on the number of trade partner's own trade partners and on strategic interaction among them.

In what follows we analyze the impact of trade network asymmetry on firms' overall market size and R&D in detail. First, we study the role of asymmetry and its exact pattern for R&D in asymmetric, hub-and-spoke networks. Then we complement this analysis by the comparison with the case of simple, symmetric networks. We conclude by studying the implications of asymmetry for countries' welfare.

5.1 Asymmetric trade networks

An asymmetric trade network in our model is hub-and-spoke in structure. It involves two types of countries: *hubs*, which have a relatively large number of direct trade partners, and *spokes*, which have only a few partners. Anecdotal evidence and numerous accounts in the literature suggest that such type of structure is, in fact, a close representation of actual asymmetries in the international trade network.

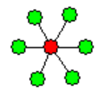
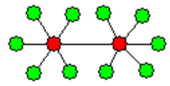
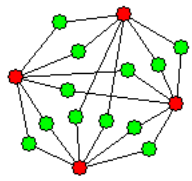
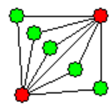
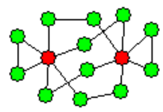
For simplicity, we focus on hub-and-spoke structures where all hub nodes are the same and all spoke nodes are the same. This means that the network can be described by just four parameters: n_h, the number of direct trade partners of a hub; n_s, the number of direct trade partners of a spoke; ψ, the share of hubs among direct trade partners of a hub; and φ, the share of spokes among direct trade partners of a spoke. Asymmetry of the network is captured by considering $1 \leq n_s < n_h$, and the larger the difference between n_h and n_s, the larger the degree of asymmetry. The values of ψ and φ, on the other hand, determine the composition of countries' direct trade partners, that is, the exact pattern of asymmetry in the network. We consider $0 \leq \psi, \varphi < 1$, where $\varphi = \psi = 0$ represents the case in which hubs trade *exclusively* with spokes and vice versa.[23] Clearly, not all combinations of the four parameters are feasible but those that are feasible form a rich set of different hub-and-spoke architectures.[24] Some examples are presented in Table 1, where Types 1 - 5 of the hub-and-spoke network differ either in their degree of network asymmetry (the difference between n_h and n_s) or in the pattern of asymmetry (the values of ψ and φ).

In trade policy, hub-and-spoke networks can be viewed as a representation of regional/preferential type of trade. When signed by members of the WTO, regional trade agreements signify a major departure from the basic WTO principle of non-discrimination, according to which all WTO countries are obliged to grant each other unconditionally any benefit or privilege affecting custom duties and

[23] The examples of $\varphi = \psi = 0$ are Type 1 and 3 networks in Table 1. We also note that setting $n_s = n_h$ eliminates the difference between hubs and spokes and turns the network into a symmetric network of degree $n_s = n_h$ (ψ and φ are meaningless in this case). Similarly, setting $\varphi = \psi = 1$ effectively splits the network into two symmetric components of degree n_s and n_h.

[24] Below we will show that the dependence of equilibrium R&D efforts of all firms on each of these four parameters is monotone. This then allows a comparison of R&D across many of the feasible hub-and-spoke structures.

Table 1: Examples of hub-and-spoke trade networks

Network characteristics	Network
Type 1: Single star (n_h bilaterals of a hub with spokes) $n_h > 1$, $n_s = 1$, $\psi = 0$, $\varphi = 0$	
Type 2: Stars with linked hubs $n_h > 1$, $n_s = 1$, $\psi > 0$, $\varphi = 0$	
Type 3: Stars sharing spokes $n_h > 1$, $n_s > 1$, $\psi = 0$, $\varphi = 0$	
Type 4: Stars with linked hubs, sharing spokes $n_h > 1$, $n_s > 1$, $\psi > 0$, $\varphi = 0$	
Type 5: Stars where some spokes are linked with each other $n_h > 1$, $n_s > 1$, $\psi = 0$, $\varphi > 0$	

Remark: Red nodes indicate hubs, green nodes indicate spokes.

charges on equal, non-discriminatory basis.[25] In regional trade agreements (customs unions and free-trade areas), countries within the agreement are treated more favorably than those outside: import tariffs are completely eliminated between the members of the agreement, but kept on imports from the rest of the world. Regional trade agreements have become prevalent in the last two decades, with some countries being parties to several agreements.[26] This results in the network where different regional trade agreements "overlap", forming the hub-and-spoke structure of the type considered here.

In what follows, we focus on our key question of interest and study the effects of asymmetry in the hub-and-spoke system on R&D decisions of firms. To this end, we first derive the equilibrium levels of R&D in the hub-and-spoke system, focusing on the symmetric equilibrium, where all hubs exert identical R&D effort x_h^* and all spokes exert identical R&D effort x_s^*. Those can be found from the system of the first-order conditions (10), which in case of the hub-and-spoke network reduces to two equations. The equations and their unique closed-form solution are provided in the Appendix.[27] With this solution in hand, we then do comparative statics and examine the impact on R&D of

[25]See section 5.2 for further description of the main WTO principles.
[26]See Figure 4 and examples of regional trade agreements in footnote 11.
[27]See the proof of Proposition 1 in Appendix A.

a change in each of the four parameters describing the hub-and-spoke network. First, we examine the effects of a change in n_s, n_h or both, which allows evaluating the effects of asymmetry in the trade network. Then we study the implications of a change in ψ or ϕ, which for a given degree of asymmetry describe how the composition of countries' trade partners matters for firms' incentives to innovate.

Our main findings from this comparative statics analysis can be summarized as follows. First, the larger the degree of asymmetry associated with an increase in n_h or a decrease in n_s or both, the larger the equilibrium R&D of a hub. On the contrary, for a spoke the larger the degree of asymmetry, the lower the equilibrium R&D, at least when certain conditions hold. As a result, a larger asymmetry in the trade network leads to a larger disparity in innovation efforts of hubs and spokes. Moreover, for a given degree of asymmetry, R&D of both a hub and a spoke increases in the share of spokes among their direct trade partners. Formally, these findings are stated below, and they are illustrated with Figures 5 and 6 in the Appendix.[28]

Proposition 1 (**hub-and-spoke network**). *Consider hub-and-spoke trade networks, where $n_s < n_h \leq \bar{n}$ for some $\bar{n} > 1$. Suppose that demand in each country is sufficiently large (Assumption 1) and R&D is sufficiently costly (Assumption 2) for all $n_s < n_h \leq \bar{n}$. Then there exists $\Delta > 0$ such that for any $\delta \geq \Delta$ and for any $n_s < n_h < \bar{n}$ and $0 \leq \psi, \varphi < 1$, the following statements are fulfilled:*

1. *the equilibrium R&D effort x_h^* of a hub is monotonically increasing in the number of hub's direct trade partners, n_h, and monotonically decreasing in the number of spoke's direct trade partners, n_s;*

2. *the equilibrium R&D effort x_s^* of a spoke is monotonically decreasing in the number of hub's direct trade partners, n_h;*

3. *the equilibrium R&D effort x_s^* of a spoke is monotonically increasing in the number of spoke's direct trade partners, n_s, if at least one of the conditions holds:*

   (a) the trade costs are sufficiently high: $\tau \geq \frac{1-\varphi}{3-2\varphi}(a-\gamma)$,

   (b) the share of other spokes among direct trade partners is at least 1/3: $\varphi \geq \frac{1}{3}$,

   (c) the gap between n_h and n_s is relatively small: $n_h \leq n_s^2$, that is, $1 < \frac{n_h}{n_s} \leq n_s$.

4. *Equilibrium R&D efforts of both a hub and a spoke are monotonically increasing in the share of spokes among their direct trade partners: x_h^* is monotonically decreasing in ψ, and x_s^* is monotonically increasing in φ.*

Parts 1. and 3. of the proposition state that the larger the number of directly accessible markets of a firm, the higher the incentives to innovate, – always for a hub and under conditions (a) – (c) for a spoke. Further, parts 1., 2. and 4. of the proposition imply that for a given number of

[28] Both figures are produced using the following parameter values: $\gamma = 7$, $b = 1$, $\bar{n} = 10$, $\tau = 2$; a and δ fulfill Assumptions 1 and 2. In addition, for Figure 5, we set $\psi = \varphi = 0$ and for Figure 6, $n_h = 6$, $n_s = 2$.

direct trade partners, the larger the number of competitors in these markets, the lower the incentives for innovation. In fact, the latter suggests that the "pure" competition effect of trade on R&D, considered when only the number of firm's competitors increases but the size of the market remains unchanged, is negative: two-links-away trade partners dampen R&D of a firm.[29] Still, the impact of direct trade partners on R&D tends to be positive. That is, the positive scale effect of trade between any directly linked countries is strong enough to outweigh the negative competition effect. This is always true for hubs, and in a range of specified cases (a) – (c) for spokes.[30]

To grasp the intuition for the role of asymmetry described in Proposition 1, recall that in this model the key determinant of firms' incentives to innovate is the size of its overall market. For a hub, a larger asymmetry is associated with a larger number of directly accessible markets (higher n_h) and lower number of competitors at least in some of these markets (lower n_s). For a spoke, the implications of asymmetry are the opposite. Moreover, for any given n_h and n_s, the larger the share of spokes among direct trade partners of a country (hub or spoke), the lower the foreign competition. As a result, the aggregate demand of a firm in a hub is larger when asymmetry is large, and particularly large if the share of spokes among its direct trade partners is high. For a spoke, the aggregate demand tends to be lower in highly asymmetric networks, and particularly low if the share of spokes among its direct trade partners is small. This variation in aggregate demand is translated into a difference in R&D incentives, so that increasing asymmetry amplifies R&D of a hub but reduces R&D of a spoke.

It remains to understand conditions (a) – (c) which ensure that an increase in n_s promotes spokes' R&D investments. Recall that the specification of a hub-and-spoke trade network in this model is such that an increase in the number of a spoke's direct trade partners n_s is associated with an increase in the number of *both* types of partners – hubs and spokes. Then since the market of a hub is "small" – smaller than the market of a spoke, an increase in the spoke's foreign market share due to an access to new markets may actually be smaller than a decrease in its domestic market share due to increased competition. As a result, a positive scale effect of an increase in n_s on spoke's market size and R&D may be dominated by a negative competition effect.[31] Conditions (a) – (c) ensure that this would not be the case by restraining the competition effect of trade. Namely, an increase in n_s is certain to expand spoke's market size and R&D if either (a) the trade costs of firms are sufficiently high to restrict the amount of exports from new trade partners, or (b) hubs represent only a minor share of direct trade partners of a spoke, or alternatively, (c) the number n_s of competitors in a spoke's market is comparable to n_h, so that the loss in the domestic market share of a spoke does not exceed the gain in a new hub's market.

[29] This also means that the negative market-share component of the competition effect outweighs its positive markups component.

[30] In Supplementary Appendix (Appendix C) we further investigate the impact of direct and two-links-away trade partners on R&D of a firm in a *generic* network under the assumption of *small local effects*. Consistent with the result of Proposition 1 for hub-and-spoke networks, we find that in a generic network, new direct trade partners (mostly) increase R&D of a firm, and this effect is stronger, the smaller the number of competitors in the new markets.

[31] This outcome seems to be rather rare though. For example, by calculating the model for the star network under various parameter assumptions, we find that initiating trade with the hub decreases R&D of a spoke only when the number of competitors in the hub's market (other spokes) is above 100.

The comparative statics results of Proposition 1 can be employed to rank R&D efforts of firms in different types of hub-and-spoke networks. For example, with respect to hub-and-spoke networks of Table 1, Corollary 1 describes the comparisons that can be made. It uses the notation x^*_{hi} for R&D of a hub and x^*_{si} for R&D of spoke in Type i of the hub-and-spoke structure, where $i \in 1:5$.

Corollary 1. *Consider Types 1–5 of the hub-and-spoke trade network. Suppose that (i) n_h is the same across all types, (ii) n_s is the same across all types where $n_s > 1$ (Types 3, 4 and 5), and (iii) ψ is the same across all types where $\psi > 0$ (Types 2 and 4). Then firms' equilibrium R&D efforts in Types 1–5 of the hub-and-spoke structure can be ranked as follows:*

$$x^*_{h1} > x^*_{h3} > x^*_{h4}, \quad x^*_{h1} > x^*_{h2} > x^*_{h4}, \quad \textit{and} \quad x^*_{s5} > x^*_{s3} > x^*_{s1}, \quad x^*_{s4} > x^*_{s2}.$$

The intuition for these comparisons is simple. As before, a ranking of firms' R&D efforts is a result of an analogous ranking of firms' aggregate market sizes. Note that amongst all hub-and-spoke networks, a star (Type 1 network) is the one where the hub enjoys the lowest competition in its foreign markets. Therefore, for a fixed number of a hub's direct trade partners, the hub of a star has the largest aggregate market size. As the number of rival firms in a hub's foreign markets increases, the aggregate market size of the hub declines. This is the case when either the number of a spoke's direct trade partners, n_s, increases (Type 3 network), or the share of hubs among direct trade partners, ψ, grows (Type 2 network), or both changes in n_s and ψ happen simultaneously (Type 4 network). Furthermore, the larger the increase in n_s and ψ, the smaller the size of a hub's aggregate market.

For spokes, the situation is reversed. In the star (Type 1 network) each spoke has an access to a single foreign market. Therefore, given a fixed number of a hub's direct trade partners, a spoke's market in the star is smaller than in other hub-and-spoke trade networks. As the number of direct trade partners of a spoke, n_s, increases (Type 3 network), the market of a spoke expands. It expands even further if the share of spokes among direct trade partners, φ , grows (Type 5 network). Moreover, the larger the increase in n_s and φ, the larger the aggregate market size of a spoke.

To further examine the effects of asymmetry in the trade network, in the next section we compare our results for a hub-and-spoke network with those for a symmetric network, where all countries have the same number of direct trade partners. This will also allow us to demonstrate that in any hub-and-spoke trade network R&D of a hub is strictly larger than R&D of a spoke, as the former is larger and the latter is smaller than R&D of a firm in the symmetric network. Intuitively, a symmetric network can be regarded as a "special case" of a hub-and-spoke structure, where $n_s = n_h$, and all firms exert the same R&D effort. Then as the asymmetry is "introduced" into such a network through an increase in n_h or a decrease in n_s, hubs do more and spokes do less R&D than a representative firm in the symmetric system.

This, together with the results of Proposition 1 emphasizes the importance of taking into account the asymmetry of the trade network in determining R&D decisions of firms. Moreover, interpreting our findings in application to regional/preferential trade, we conclude that allowing for regional trade agreements in the WTO leads to a situation where firms in hub economies do more R&D than firms in spoke countries, and the more asymmetric the trade relations become, the larger the gap between R&D investments. In addition, the expansion of the regional trade network through a conclusion of new agreements (creation of new links or new links and nodes) augments R&D of some but reduces R&D of some other countries. The effect of a new trade agreement on R&D tends to be positive for the immediately involved countries but negative for their existing trade partners.

5.2 Symmetric trade networks. Comparison with asymmetric networks

To better understand the effects of asymmetry, let us now consider a class of symmetric networks and then compare R&D of firms in this setting with R&D of hubs and spokes in asymmetric networks. In contrast to hub-and-spoke structures, in symmetric, or *regular* trade networks, each country has the same number n of direct trade partners. n is sometimes called the *degree* of a symmetric network. When $n = 1$, the network consists of one or more simple bilateral agreements, and when $n = N - 1$, the network is *complete*, so that every country has an agreement with every other country.

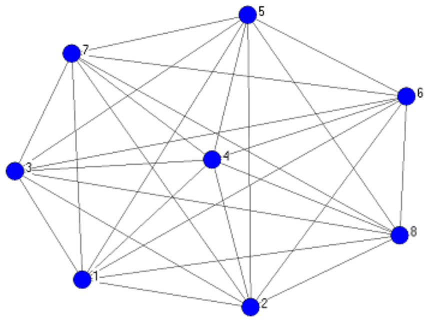


Figure 1: Complete network with 8 countries

The case of a complete network can be thought of as representing multilateral trade between countries. In general, multilateral trade is characterized by two fundamental principles of the GATT: "reciprocity" and the "most favoured nation" principle of non-discrimination. The reciprocity principle implies that each country agrees to reduce its trade barriers in return for a reciprocal reduction by another; the non-discrimination principle states that all countries in the multilateral trade system should be treated equally, that is, tariffs imposed by a country on imports from *any* other country in the system must be the same. In our model, when all countries are linked with each other and tariffs on all trade links are the same, – for example, zero, as in case of *free* trade agreements, – the non-discrimination principle applies. And the reciprocity principle is obviously fulfilled, too, in its extreme form since tariffs are reduced to the same level not only in trade within each pair of countries but also across pairs.

In the symmetric network, all countries are identical, and each firm exerts the same R&D effort in the symmetric equilibrium. It can be found from a single first-order condition (cf. (10)):

$$\left[-\frac{(n+1)^3}{(n+2)^2} + \delta b\right] x + 2n\frac{n+1}{(n+2)^2}x + n\frac{(n+1)(n-1)}{(n+2)^2}x = \frac{(n+1)^2}{(n+2)^2}(a-\gamma) + \frac{n+1}{(n+2)^2}(-2\tau n + \tau n),$$

which leads to:

$$x^*(n) = \frac{a - \gamma - \frac{n}{n+1}\tau}{-1 + \delta b\big(1 + \frac{1}{n+1}\big)^2}. \tag{5}$$

We study the functional dependence of this equilibrium R&D effort on n, and obtain that the larger the number of a country's direct trade partners, the stronger the incentives of a firm to invest in R&D. Moreover, this turns out to be the case even despite the fact that the equilibrium profit of a firm declines as the number of direct trade partners increases. Both of these comparative statics results are stated in Proposition 2, and Figure 7 provides an illustration:[32]

Proposition 2 (**symmetric network**). *Consider symmetric networks of degree n, where $n < \bar{n}$ for some $\bar{n} \geq 1$. Suppose that demand in each country is sufficiently large (Assumption 1) and R&D is sufficiently costly (Assumption 2) for all $n < \bar{n}$. Then firm's equilibrium R&D effort x^* is monotonically increasing in n, while firm's profit π is monotonically decreasing in n for any $n < \bar{n}$.*

Intuitively, as the symmetric network of trade agreements expands – due to an access of a new country or a formation of new agreements between the existing countries, – the reduction in the domestic and foreign market shares suffered by each firm is exactly compensated by the participation in the new market. That is, the negative market share effect associated with an increased competition exactly offsets the positive scale effect associated with an access to a new market. As a result, opening to trade with a new country affects R&D of each firm only through the remaining component of the competition effect – the reduction in price markups. Then since this markups reduction increases firm's aggregate demand, it also increases R&D, so that in equilibrium, R&D of each firm is increasing in the number of its direct trade partners.[33] On the other hand, the profit of each firm declines as a result of increasing R&D expenditures and decreasing prices.[34]

One interpretation of Proposition 2, when applied to the case of a complete network, is that the expansion of the multilateral trade system promotes R&D in every country. It also suggests that the R&D investment of a firm in the multilateral agreement with $n \geq 2$ is higher than R&D of a firm in the bilateral agreement ($n = 1$), which in turn is higher than R&D in autarky ($n = 0$). Furthermore, as a consequence of the above, also the aggregate level of R&D in the multilateral trade system increases in the size of the system and exceeds the aggregate R&D of the same number of countries that are either involved in simple bilateral agreements or do not trade with each other at all.

Now, bringing the results of our equilibrium analysis in this and the preceding section together, we can compare R&D incentives of firms in symmetric and hub-and-spoke trade networks. Under the same demand and cost parameter restrictions as employed in our analysis above, the following

[32] The equilibrium profit of a firm in the symmetric network is given by equation (12) in the Appendix. Figure 7 is produced using the same parameter values as for Figures 5 and 6: $\gamma = 7$, $b = 1$, $\bar{n} = 10$, and $\tau = 2$; a and δ fulfill Assumptions 1 and 2.

[33] Note that in contrast to this case of a symmetric network, in the asymmetric, hub-and-spoke structure, the interaction of scale and competition effects of trade is not so straightforward, and the negative, market-share effect of competition is generally *not* offset by the positive scale effect.

[34] Figure 7 also demonstrates that the rates of increase in R&D and decrease in profit are both declining in n. This is implied by the fact that the markup-reducing effect of trade in the symmetric network becomes weaker as the network expands. In fact, it is easy to show that the price in each market is a decreasing and convex function of n, as demonstrated by Figure 8 in the Appendix.

proposition states an important result. It claims that in any hub-and-spoke network, R&D of a hub is larger than R&D of a spoke. Moreover, given the same number n_h of direct trade partners, R&D of a hub, x_h^*, is larger than R&D of a firm in the symmetric network, $x^*(n_h)$, while for spokes, the opposite is true: given the same number n_s of direct trade partners, R&D of a spoke, x_s^*, is smaller than R&D of a firm in the symmetric network, $x^*(n_s)$.

Proposition 3 (**comparison of symmetric and hub-and-spoke networks**). *Consider a hub-and-spoke network defined by parameters* $0 \leq \psi, \varphi < 1$ *and* $1 \leq n_s < n_h \leq \bar{n}$ *for some* $\bar{n} > 1$*, and any symmetric trade networks of degree* n_s *and* n_h*. Suppose that demand in each country is sufficiently large (Assumption 1) and R&D is sufficiently costly (Assumption 2) for all* $n_s < n_h \leq \bar{n}$*. Then there exists* $\Delta > 0$ *such that for any* $\delta \geq \Delta$*, R&D of firms in hub-and-spoke and symmetric networks can be ranked as follows:*

$$x_h^* > x^*(n_h) > x^*(n_s) > x_s^*.$$

The proof of this proposition is based on the idea that a symmetric network of degree n_h can be regarded as a hub-and-spoke network "composed only of hubs", that is, where n_s in the hub-and-spoke network has increased to become equal to n_h. Similarly, a symmetric network of degree n_s can be regarded as a hub-and-spoke network "composed only of spokes", where n_h has decreased to become equal to n_s.[35] Then inequalities $x_h^* > x^*(n_h)$ and $x^*(n_s) > x_s^*$ follow immediately from parts 1 and 2 of Proposition 1 stating that R&D of a hub, x_h^*, is decreasing in n_s and R&D of a spoke, x_s^*, is decreasing in n_h. The last, "connecting" inequality $x^*(n_h) > x^*(n_s)$ is implied by Proposition 2, according to which R&D of a firm in the symmetric network is increasing in its degree.

As before, an intuitive explanation for the obtained comparison of firms' R&D levels has to do with the comparison of their market sizes. Indeed, for any number of direct trade partners, the aggregate market size of a hub in a hub-and-spoke network is larger than that of a firm in the symmetric network, and the opposite is true for a spoke. A simple calculus provides the key insight. While in a symmetric network of degree n_h, the number of firm's competitors is n_h in *each* of its foreign markets, in a hub-and-spoke network, the number of hub's competitors is n_h only in ψn_h of its foreign markets, while in the other markets it is less than n_h. Symmetrically, for a spoke, the number of its competitors in the foreign markets is larger than for a firm in a symmetric network of degree n_s. Therefore, for the same number of direct trade partners, benefits of innovation for a hub are higher and for a spoke lower than for a country in the symmetric network.

Our next proposition combines the results of Proposition 3 and Corollary 1 and provides the ranking of equilibrium R&D efforts of firms in different types of hub-and-spoke networks (see Table 1) and in the symmetric networks.

Proposition 4 (**extended comparison**). *Consider Types 1–5 of the hub-and-spoke trade network. Suppose that (i)* n_h *is the same across all types, (ii)* n_s *is the same across all types where* $n_s > 1$

[35] When $n_s = n_h$, ψ and φ become meaningless and drop from the analysis of hub-and-spoke networks. Furthermore, in the proof we note that such transformation from a hub-and-spoke to symmetric network may require adding or deleting some nodes, but as R&D decisions of firms are fully determined by parameters n_s, n_h, ψ and φ and *not* by the overall network size, the total number of nodes in the network is irrelevant.

(Types 3, 4 and 5), and (iii) ψ is the same across all types where $\psi > 0$ (Types 2 and 4). Then firms' equilibrium R&D efforts in Types 1–5 of the hub-and-spoke network and in the symmetric network can be ranked as follows:[36]

$$x_{h1}^* > x_{h3}^* > x_{h4}^* > x^*(n_h) > x^*(n_s) > x_{s5}^* > x_{s3}^* > x_{s1}^*$$

and

$$x_{h1}^* > x_{h2}^* > x_{h4}^* > x^*(n_h) > x^*(n_s) > x_{s4}^* > x_{s2}^*.$$

The first sequence of inequalities is demonstrated with Figure 2 below. Figure 9 in the Appendix provides an analogous illustration for the second sequence.

Thus, R&D investments of firms differ substantially across symmetric and asymmetric trade networks and across hub and spoke countries within an asymmetric network. The highest R&D incentives exist for a hub, especially for a hub in the star (Type 1 network), whereas for a spoke in the star R&D incentives are the lowest. As the number of direct trade partners of a spoke and/or the share of spokes (hubs) among direct trade partners of each spoke (hub) increase, the levels of R&D investment of hubs and spokes draw closer together. They coincide at the level of R&D investment of a firm in the symmetric network, which therefore, takes an average position – lower than R&D of a hub but higher than R&D of a spoke.

Finally, in addition to the comparison of firms' individual R&D investments, we compare the aggregate R&D of the same total number of countries in the complete trade network and in the simple star. While the latter network can be thought of as representing a multilateral trade system, the former is a trade arrangement where one country, the hub, has several bilateral agreements with spoke countries. We find that even though R&D of the hub in a star is higher than R&D of a single country in the multilateral system, the aggregate R&D in the star is *lower* than in the multilateral agreement. This observation is demonstrated with Figure 10 in the Appendix.

5.3 Welfare analysis

To complement the analysis of the effects of asymmetry in the trade network on firms' R&D, this section compares the *welfare* implications of trade within a symmetric network and within a star, the simplest type of the hub-and-spoke structure. A higher degree of asymmetry in the star network is associated with a larger number of spokes, that is, hub's direct trade partners, while the number of spokes' direct trade partners is fixed at one. In what follows, the social welfare of country i, W_i, is defined as the sum of firm i's profit, π_i, and consumer surplus, CS_i, where

$$CS_i = \frac{1}{2b}\left(a - p_i\right)^2 \tag{6}$$

due to linearity of the demand function in (1).

We find that in both types of trade network, symmetric and a star, the level of welfare and the impact on welfare of a new trade agreement depends on the value of trade costs τ and, in a certain

[36]The last inequality in both sequences requires, in addition, that at least one of three conditions (a) – (c) of Proposition 1 holds, under which x_s^* is increasing in n_s.

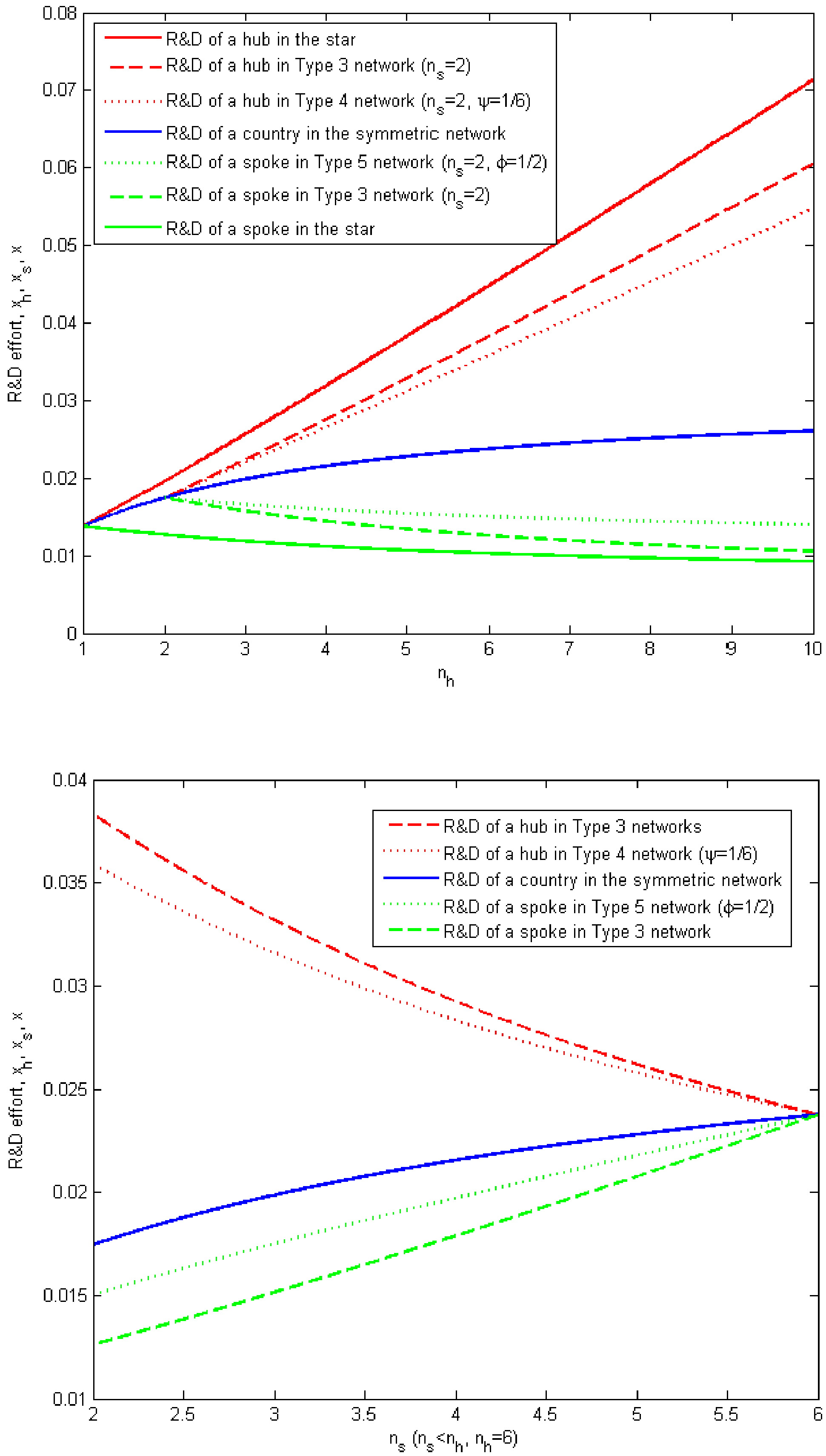


Figure 2: Equilibrium R&D efforts in the symmetric and hub-and-spoke trade networks as a function of n_h (upper panel) and as a function of n_s (lower panel).

range of trade costs, on the number of country's existing trade partners. Starting from the case of an asymmetric, star network, our first proposition states that when the trade costs are relatively low (below a certain threshold $\underline{\tau}$), the welfare of the hub economy increases in the number of its direct trade partners, n, while the welfare of spokes declines. That is, a higher degree of asymmetry in the star network widens the welfare gap between the hub and the spokes. On the other hand, at high trade costs (above another, larger threshold $\overline{\tau}$), the welfare of *both*, the hub and the spokes, decreases in the number of hub's trade partners, and at medium trade costs (between the two thresholds), the impact of a new trade partner depends on the number of hub's existing trade partners.

Proposition 5 (welfare in a star). *Consider a star network, where the number of hub's direct trade partners $n_h < \bar{n}$ for some $\bar{n} \geq 1$. Suppose that demand in each country is sufficiently large (Assumption 1) and R&D is sufficiently costly (Assumption 2) for all $n_h < \bar{n}$. Then for any $n_h < \bar{n}$, there exists $\Delta > 0$ such that for any $\delta \geq \Delta$, the social welfare of a spoke economy is monotonically decreasing in n_h (at all $\tau \geq 0$). Furthermore, as soon as an additional restriction $a > \gamma\left(1 + \frac{\bar{n_h}(2n_h^3+12n_h^2+51n_h+16)}{9(n_h+2)}\right)$ holds, there exist $\widetilde{\Delta} > 0$ and two thresholds of trade costs $0 < \underline{\tau} \leq \overline{\tau}$, such that for any $\delta \geq \widetilde{\Delta}$, the following statements are fulfilled:*

1. *the social welfare of the hub economy is monotonically increasing in n_h for any $\tau < \underline{\tau}$ and it is monotonically decreasing in n_h for any $\tau > \overline{\tau}$;*

2. *for $\tau \in [\underline{\tau}, \overline{\tau}]$, there exists $\widetilde{n}(\tau)$, $1 \leq \widetilde{n}(\tau) \leq \bar{n}$, such that the social welfare of the hub is monotonically increasing in n_h for any $n_h \leq \widetilde{n}(\tau)$ and it is monotonically decreasing in n_h for $n_h > \widetilde{n}(\tau)$.*

Such a dependence of social welfare of the hub on trade costs τ and, at medium values of τ, on the status quo number of hub's trade partners can be explained as follows. At low trade costs (below $\underline{\tau}$), an access to a new, spoke market is cheap and leads to a large expansion of the overall production at the hub firm. Then even though the price in the hub's own market declines due to an increased competition, the profit of the hub firm increases. Moreover, a lower price in the hub market benefits the consumers. As a result, at low trade costs, the welfare of the hub improves due to an increase in both, firm's profit and consumer surplus. As trade costs grow, exploitation of the spoke markets by the hub becomes more expensive, and a larger number of trade partners is associated with lower profits for the hub firm. In fact, when trade costs are high enough, the profit reduction associated with an extra trade partner is larger than the value of a simultaneous increase in the consumer surplus, so that the welfare of the hub declines. At first, this negative impact of a new trade partner shows only in large star networks, as the rate of increase in consumer surplus in such networks is particularly low (see Figure 12). However, when trade costs become very high (above $\overline{\tau}$), benefits from trade for the hub are minor in star networks of any size, and a new trade agreement unequivocally reduces hub's welfare. By contrast, in a spoke economy, welfare is decreasing in n_h at *all* levels of trade costs. While almost invariable price in a spoke market keeps the consumer surplus constant, the profit of a spoke firm at larger n_h is smaller as a result of a lower price and reduced market share in the hub, spoke's only trade partner.

In a symmetric trade network, the welfare changes associated with a larger number of direct trade partners are similar to those at the hub in a star. At low trade costs, the social welfare of each country is increasing in the number of its direct trade partners, n, and it is decreasing in n at high trade costs. At medium trade costs, the impact of a new trade agreement on welfare depends on the number of country's existing trade partners: at low levels of country's overall trade exposure, the impact of a new trade agreement on the welfare is positive but it turns negative as the number of country's trade partners exceeds a certain threshold.

However, differently from the case of a star network, in a symmetric trade network, trade with a new partner at low and medium trade costs improves country's welfare due to an increase in consumer surplus only, as firm's profits decline even when trade costs are low. The latter is explained by the fact that a new trade agreement in a symmetric network leads to a much smaller expansion of a firm's aggregate market as compared to the boost in aggregate demand of the hub in a star. Therefore, firm's overall production in the symmetric network increases only a little when its number of accessible markets grows, while the price at each market is slashed by tighter competition.[37] Then, as the level of trade costs increases, the decline in profits associated with new foreign markets becomes even stronger and at some point, it outweighs the increase in consumer surplus. Just as in a star network, first, this negative impact of a new trade partner on welfare shows only in trade networks with a large number of existing trade partners, where the rate of increase in consumer surplus is particularly low (see Figure 12). But at sufficiently high trade costs, a new trade agreement reduces social welfare of a country irrespective of its current degree of trade exposure. Formally, the welfare implications of trade in a symmetric network are stated by Proposition 6:

Proposition 6 (welfare in a symmetric network). *Consider a symmetric network of degree n, where $n < \bar{n}$ for some $\bar{n} \geq 1$. Suppose that demand in each country is sufficiently large (Assumption 1) and R&D is sufficiently costly (Assumption 2) for all $n < \bar{n}$. Then for any $n < \bar{n}$, the consumer surplus of a country in a symmetric trade network of degree n is monotonically increasing in n. Furthermore, as soon as an additional restriction $a > \gamma(1 + \bar{n} + 2\bar{n}/(n + 2))$ holds, there exist $\Delta > 0$ and two thresholds of trade costs $0 < \underline{\tau} \leq \overline{\tau}$, such that for any $\delta \geq \Delta$, the following statements are fulfilled:*

1. *the social welfare of a country is monotonically increasing in n for any $\tau < \underline{\tau}$ and it is monotonically decreasing in n for any $\tau > \overline{\tau}$;*

2. *for $\tau \in [\underline{\tau}, \overline{\tau}]$, there exists $\widetilde{n}(\tau)$, $1 \leq \widetilde{n}(\tau) \leq \bar{n}$, such that the social welfare of a country is monotonically increasing in n for any $n \leq \widetilde{n}(\tau)$ and it is monotonically decreasing in n for $n > \widetilde{n}(\tau)$.*

[37]Formally the negative impact of an increase in n on firm's profit has been established by Proposition 2.

Proposition 6 is illustrated with Figure 3. Other figures showing the dependence on n of countries' profits, consumer surplus and welfare in both symmetric and star networks are presented in the Appendix (Figures 12 – 14). Notice that the level of unit trade costs at which the social welfare of a country in a symmetric network or of the hub in a star decreases in n is "very high" relative to other costs and prices in each market. This implies that at low and medium trade costs, – such as in case of zero tariffs in free trade agreements and custom unions, – the expansion of a symmetric trade network improves the welfare of every country, and similarly, the expansion of the star increases the welfare of the hub.

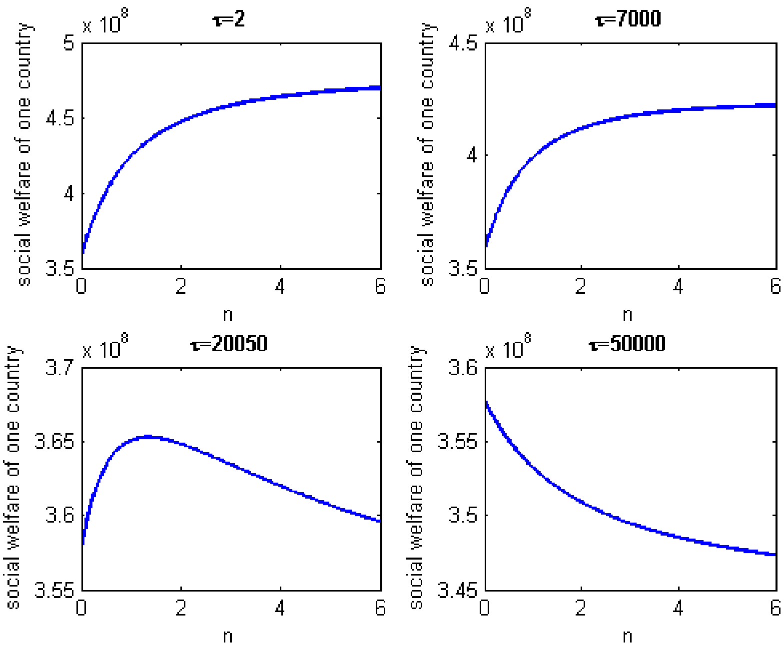


Figure 3: Social welfare of a country in a symmetric trade network of degree n.

Furthermore, the results of our numerical analysis imply that for any number n_h of direct trade partners and any trade costs, the social welfare of the hub in a star is higher than the welfare of a country in a symmetric network (Figure 14). On the other hand, the welfare of a spoke in the star is lower than the welfare of both the hub and a country in the symmetric network at least when trade costs are not too high.[38] The difference between the welfare of the hub and a country in the symmetric network is driven by a large disparity in firms' profits: while the consumer surplus of the hub and of a country in the symmetric network are essentially the same, the profit of the hub firm is much higher than the profit of a firm in the symmetric network. For a spoke in the star, the lowest level of welfare is mainly due to the lowest consumer surplus. At the aggregate level, the results point at the benefits of trade in the symmetric network: for the same total number of countries, the aggregate welfare in the symmetric, complete network exceeds the welfare in the star, with a larger difference obtained at low trade costs (Figure 15).

[38] These findings find support in earlier studies. For example, Baldwin (2004), Kowalczyk and Wonnacott (1992), Deltas et al. (2006), Lloyd and Maclaren (2004), and De Benedictis et al. (2005) find that the welfare and income levels of spokes are lower than those of hubs and of countries in the complete network.

For trade policy, the findings derived in this section imply that asymmetry in trade relations between countries benefits the hub but harms the spoke economies. In particular, in the regional trade system where one (hub) economy has several bilaterals with other countries (spokes), the welfare of the hub is larger than the welfare of a spoke, and the expansion of such trade system increases this welfare gap, at least as long as the trade costs are not too high. On the other hand, the expansion of the multilateral trade system (the complete network) improves the welfare of every country and also offers higher aggregate welfare than a regional, star system, even though each country's welfare is lower than that of the hub in the star.

Lastly, notice that if we interpret trade costs as tariffs, then the social welfare of country i can be defined alternatively as

$$W_i = \pi_i + CS_i + T_i,$$

where $T_i = \tau \sum_{k \in N_i} y_{ki}$ is the total trade tariffs collected by country i. Given such definition of welfare, we find numerically that the social welfare of each country within a symmetric trade network and of the hub in a star is monotonically increasing in the number of their direct trade partners, n_h, for *any* value of trade costs. The social welfare of a spoke country is decreasing in n_h, just as in case of the original welfare formulation. These findings are illustrated with Figure 16.

6 Conclusions and policy implications

This paper develops a model of intra-industry trade with firm-level productivity improvements via R&D. Trade relations between countries are represented by a network, and the two large classes of networks considered are hub-and-spoke and symmetric networks. In this framework we study the effects of asymmetry in the number of countries' trade partners on firms' innovation incentives and countries' welfare.

We find that the trade network asymmetry does play an important role. First, with respect to innovation, R&D investment of a hub is always larger than R&D of a spoke, and R&D of a firm in the symmetric network is lower than in a hub but higher than in a spoke even if the number of trade partners of a hub (resp., spoke) is the same as the number of trade partners of a country in the symmetric network. Furthermore, the larger the degree of asymmetry, associated with an increase in the number of direct trade partners of a hub or a decrease in the number of direct trade partners of a spoke (or both), the larger the disparity in innovation efforts of hubs and spokes.

Intuitively the role of asymmetry in a trade network for R&D decisions of firms can be understood in light of its effects on firms' aggregate demand. Indeed, the aggregate demand of a firm determines its incentives to innovate, as larger demand amplifies the returns but does not affect the cost of R&D. Thus, any change in the degree of trade network asymmetry that increases firm's aggregate demand, also increases its R&D incentives, and vice versa. For a hub, a larger degree of asymmetry in the hub-and-spoke networks is associated with larger aggregate demand, as the number of directly accessible markets is larger and/or the number of competitors at least in some of these markets is lower. On the contrary, for a spoke, a larger degree of asymmetry implies smaller aggregate demand. A symmetric network, on the other hand, can be regarded as a "special case" of a hub-and-spoke structure, where the number of direct trade partners of a hub and a spoke are the same. The aggregate demand of a firm in such network is in-between those of a hub and a spoke, where the latter obtain when asymmetry is "introduced" through an increase or decrease in the number of direct trade partners of some countries. This difference in the aggregate demand of hubs, spokes and firms in the symmetric network translates into a difference in their R&D incentives. As a result, the largest R&D incentives exist for a hub in the hub-and-spoke network, while for a spoke, the R&D incentives are the lowest. For a firm in the symmetric network, R&D is lower than in a hub but higher than in a spoke.

Furthermore, concerning the welfare implications of trade in the symmetric network and in the simple *star* network, we find numerically that for the same number of trade partners, the welfare of the hub in a star is larger than the welfare of a country in the symmetric network, while the welfare of a spoke in the star is the lowest. We also find that when the trade costs are not too high, an increasing degree of asymmetry in the star network (due to an expansion in the number of hub's foreign markets) enhances the welfare of the hub but lowers the welfare of spokes. By contrast, in the symmetric network, a larger number of countries' trade partners improves the welfare of every country.

The findings of the paper suggest some policy implications. First, interpreting the hub-and-spoke network as a trade network that emerges in the process of regional/preferential trade liberalization, and the symmetric *complete* network as a result of multilateral trade liberalization, we conclude that R&D and welfare gains of regionalism versus those of multilateralism depend heavily on the relative number of regional trade agreements signed by countries. If a country signs a relatively large number of regional trade agreements and thus becomes a hub in the trade network, then its R&D/productivity and social welfare are higher than R&D/productivity and welfare of a country in the multilateral trade system. On the other hand, a country that is involved in a relatively small number of regional trade agreements derives lower R&D and welfare gains than a country in the multilateral system. This implies that from the perspective of a small economy, which can only become a spoke in the regional trade system, the prospects for productivity improvements and welfare gains are generally better in the multilateral trade system than in the regional system. In contrast, for hubs the regional alternative is more attractive, and thus, allowing regional trade agreements in the WTO may inhibit the pursuit of multilateral free trade by "powerful" hub economies. Furthermore, the finding that opening to trade with new countries in the symmetric trade network promotes R&D and welfare of every country (at least as long as trade costs are low) implies that an expansion of the WTO or unification of several plurilateral blocks benefit all member states. Differently, in asymmetric regional trade systems an expansion of the system tends to increase R&D and welfare of the countries that are immediately involved in a new trade agreement but suppresses R&D and welfare of their existing trade partners.

At last, considering the results of the paper in application to the inter-regional trade within one country, we obtain that regions where businesses are better connected to markets in other parts of the country outperform their more isolated neighbours, both in terms of welfare and firm productivity. This points towards the importance of regional trade and business integration and provides new insights into the reasons for large productivity and wealth gaps between regions in many countries.

A number of extensions remain for further research. First, note that in the current setting all countries are regarded as absolutely identical in all but their position in the trade network. This allows isolating the effects of trade network asymmetry and countries' involvement in trade. Endogenizing the trade network in such setting is bound to produce symmetric or almost symmetric network structures, as demonstrated by the network formation models of Goyal and Joshi (2006) and Mauleon et al.(2010). It would, therefore, be interesting to consider endogenous trade network formation in a model where countries are *a priori* different in their market size and/or firms' marginal production costs. Such approach is likely to generate trade network asymmetries observed in the real world, where countries that are larger and/or more productive are more involved in the international trade. Second, an important direction for further research is testing the results of the paper empirically. To the best of our knowledge, the impact on R&D and productivity of asymmetries in the network of international trade and the issue of potential variations in the impact on R&D and productivity of different types of trade liberalization (multilateral versus regional) have not been addressed in the empirical research. Our theoretical findings suggest the relevance of this research direction. More generally, they emphasize the importance of taking into account the asymmetry and degree of asymmetry in countries' trade exposure in any empirical investigation of the impact of trade on innovation, productivity and countries' welfare.

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7 Appendix

Appendix A: Proofs[39]

Solving the two-stage game (section 4)

The game is solved by backward induction. Consider each stage in turn.

Stage 2: output choice At the second stage, each firm $i \in 1:N$ chooses a profit maximizing vector of production quantities $\{y_{ij}\}_{j\in N_i\cup\{i\}}$, given the outputs of the rivals and R&D efforts $\{x_i\}_{i\in 1:N}$. The profit of firm i is equal to the sum of revenues collected at the domestic and foreign markets minus the costs of overall production, R&D and transportation:

$$\begin{aligned}\pi_i &= \sum_{j\in N_i\cup\{i\}}\left(a - by_{ij} - b\sum_{k\in N_j\cup\{j\},k\neq i} y_{kj}\right)y_{ij} - (\gamma - x_i)y_i - \delta x_i^2 - \tau\sum_{j\in N_i} y_{ij} = \\ &= \sum_{j\in N_i\cup\{i\}}\left(-by_{ij}^2 - b\sum_{k\in N_j\cup\{j\},k\neq i} y_{kj}y_{ij}\right) + (a-\gamma+x_i)y_i - \delta x_i^2 - \tau\sum_{j\in N_i} y_{ij}. \qquad (7)\end{aligned}$$

Note that function π_i is additively separable and linear-quadratic in output levels $\{y_{ij}\}_{j\in N_i\cup\{i\}}$ of firm i. Moreover, it's strictly concave in own production quantities since $b > 0$. Then the solution of the system of linear first-order conditions delivers maximum to the profit function in (7) and determines the equilibrium of the second stage as long as this solution is interior, that is, $y_{ij} > 0$ for any i and any $j \in N_i \cup \{i\}$. The unique solution of the system is given by the following vector of production quantities of every firm i:

$$y_{ii} = \frac{1}{b(|N_i|+2)}\left(a - \gamma + (|N_i|+1)x_i - \sum_{j\in N_i} x_j + |N_i|\tau\right), \qquad (8)$$

$$y_{ij} = \frac{1}{b(|N_j|+2)}\left(a - \gamma + (|N_j|+1)x_i - \sum_{k\in N_j\cup\{j\},k\neq i} x_k - 2\tau\right), \ \forall j \in N_i. \qquad (9)$$

All of these quantities are strictly positive and hence, form the Nash-Cournot equilibrium of the second stage, as soon as the demand in each market is sufficiently high relative to costs of production and exporting. This is guaranteed by the assumption that the demand parameter a is sufficiently large relative to the cost parameters γ and τ:[40]

Assumption 1 $\quad a > \gamma\left(1 + \max_{i\in 1:N}|N_i|\right) + 2\tau.$

[39] Due to computational complexity, proofs of some propositions in this section are presented schematically. More details are available from the author.

[40] Note that under Assumption 1 and condition that $0 \le x_i \le \gamma$,

$$y_{ij} \ge \frac{1}{b(|N_j|+2)}(a - \gamma - |N_j|\gamma - 2\tau) = \frac{1}{b(|N_j|+2)}(a - (\gamma + \gamma|N_j| + 2\tau)) > 0 \quad \forall i \in 1:N \quad \forall j \in N_i \cup \{i\}.$$

Moreover, $x_k \le \gamma$ ensures that $\sum_{k\in N_j\cup\{j\}} y_{kj} < a/b$ (see demand specification in (1)):

$$\sum_{k\in N_j\cup\{j\}} y_{kj} = \frac{|N_j|+1}{b(|N_j|+2)}(a-\gamma) + \frac{1}{b(|N_j|+2)}\sum_{k\in N_j\cup\{j\}} x_k - \frac{|N_j|}{b(|N_j|+2)}\tau \le \frac{|N_j|+1}{b(|N_j|+2)}a - \frac{|N_j|}{b(|N_j|+2)}\tau < \frac{a}{b}.$$

Moreover, as we demonstrate below, when Assumption 1 holds, firms also have an incentive to invest a strictly positive amount in R&D.

Two observations are in order. First, the equilibrium output of firm i produced for any country $j \in N_i \cup \{i\}$ is increasing in firm's own R&D effort and decreasing in R&D efforts of i's rivals in market j. That is, the higher the equilibrium R&D of firm i and the lower the equilibrium R&D of every other firm $k \in N_j \cup \{j\}$, $k \neq i$, competing with i in market j, the higher the share of market j gained by i. Second, the presence of non-negative trade costs τ often gives firm i a competitive advantage over its rivals on the domestic market ($y_{ii} \geq y_{ji}$ for $j \in N_i$), and it also implies that i's production for the domestic market is at least as large as its production for foreign markets ($y_{ii} \geq y_{ij}$).[41]

Stage 1: R&D choice At the stage preceding the production decisions, firms choose their R&D efforts. Incorporating the second-stage solution (8)– (9) into firm i's profit function (7), we obtain a function of R&D efforts of i and its direct and two-links away trade partners:

$$\pi_i = \left[\frac{1}{b}\sum_{j \in N_i \cup \{i\}} \frac{(|N_j|+1)^2}{(|N_j|+2)^2} - \delta\right] x_i^2 + \frac{2}{b}\left[(a-\gamma-2\tau)\sum_{j \in N_i} \frac{|N_j|+1}{(|N_j|+2)^2} + (a-\gamma+|N_i|\tau)\frac{|N_i|+1}{(|N_i|+2)^2}\right] x_i$$

$$-\frac{2}{b}\sum_{j \in N_i}\left[\frac{|N_i|+1}{(|N_i|+2)^2} + \frac{|N_j|+1}{(|N_j|+2)^2}\right] x_i x_j - \frac{2}{b}\sum_{j \in N_i}\sum_{k \in N_j, k \neq i} \frac{|N_j|+1}{(|N_j|+2)^2} x_i x_k + f(\{x_k\}_{k \in N_i \cup N_i^2}),$$

where $f(\{x_k\}_{k \in N_i \cup N_i^2})$ is a term independent of x_i.[42]

This profit function is linear-quadratic in firm i's own R&D effort x_i. It is strictly concave in x_i for sufficiently high values of parameter δ ($\delta > \frac{1}{b}\sum_{j \in N_i \cup \{i\}} \frac{(|N_j|+1)^2}{(|N_j|+2)^2}$), that is, when the cost of R&D is sufficiently high. In this case the vector of equilibrium R&D efforts of all firms can be found as a solution of the system of linear first-order conditions, provided that this solution is interior, that is, $0 < x_i < \gamma$:

$$\left[-\frac{1}{b}\sum_{j \in N_i \cup \{i\}} \frac{(|N_j|+1)^2}{(|N_j|+2)^2} + \delta\right] x_i + \frac{1}{b}\sum_{j \in N_i}\left[\frac{|N_i|+1}{(|N_i|+2)^2} + \frac{|N_j|+1}{(|N_j|+2)^2}\right] x_j + \frac{1}{b}\sum_{j \in N_i}\sum_{k \in N_j, k \neq i} \frac{|N_j|+1}{(|N_j|+2)^2} x_k$$

$$= \frac{1}{b}(a-\gamma-2\tau)\sum_{j \in N_i} \frac{|N_j|+1}{(|N_j|+2)^2} + \frac{1}{b}(a-\gamma+|N_i|\tau)\frac{|N_i|+1}{(|N_i|+2)^2}, \quad i \in 1:N. \tag{10}$$

The solution is unique whenever this system is non-singular, which is the case for most arbitrary network structures. However, for the types of networks that will be in focus of our analysis later on, multiple solutions are possible as either $|N_i| = |N_j|$ for all $i \neq j$ (in a symmetric network), or $|N_i|$ takes just one of two values, high or low, for any i (in a hub-and-spoke network). In these cases, we think it is reasonable to focus on symmetric equilibria, such that in a symmetric network all firms

[41] To be more precise, as soon as $x_i = x_j$ for some $j \in N_i$, the equilibrium level of production for market i of firm i is at least as high as that of firm j. Similarly, if for some $j \in N_i$ $|N_i| = |N_j|$ and $\sum_{k \in N_i, k \neq j} x_k = \sum_{k' \in N_j, k' \neq i} x_{k'}$, the equilibrium level of production of firm i for the domestic market is at least as high as its production for market j.

[42] Simple algebra confirms that

$$f(\{x_k\}_{k \in N_i \cup N_i^2}) = \frac{1}{b}\sum_{j \in N_i} \frac{1}{(|N_j|+2)^2}\left(a-\gamma-2\tau-\sum_{k \in N_j \cup \{j\}, k \neq i} x_k\right)^2 + \frac{1}{b}\frac{1}{(|N_i|+2)^2}\left(a-\gamma+|N_i|\tau-\sum_{j \in N_i} x_j\right)^2.$$

exert the same R&D effort, and in a hub-and-spoke network, R&D efforts of all hub firms are the same, and R&D efforts of all spoke firms are the same.

Irrespective of the trade network structure, let $\{x_i^*\}_{i\in 1:N}$ denote a solution of system (10). Below we describe the conditions on demand and cost parameters which guarantee that $0 < x_i^* < \gamma$ for any i, so that the vector of solutions $\{x_i^*\}_{i\in 1:N}$ defines the equilibrium R&D efforts. First, observe that since all coefficients multiplying the R&D efforts of firms on the left-hand side of (10) are positive (provided that the profit function is concave in x_i), the best-response function of firm i is decreasing in R&D efforts of its direct and two-links-away trade partners. This means that the efforts of firm i and its direct and two-links-away trade partners are *strategic substitutes* from i's perspective. Intuitively, by exerting higher R&D efforts, firm i's rivals capture larger market shares, and this reduces returns to R&D for firm i. Then conditions that are sufficient for equilibrium R&D efforts to be positive and to not exceed γ require that the best-response effort of a firm is greater than zero even when the efforts of its rivals are equal to γ, and that it remains below γ even when the efforts of its rivals are zero. The latter is provided by Assumption 1, while the former is ensured by

Assumption 2 $\quad \delta > \frac{1}{\gamma b}\max\limits_{i\in N}\left[\sum\limits_{j\in N_i}\frac{|N_j|+1}{(|N_j|+2)^2}\left(\gamma|N_j|+a-2\tau\right)+\frac{|N_i|+1}{(|N_i|+2)^2}(\gamma|N_i|+a+|N_i|\tau)\right].$

That is, as soon as the demand for the good in each market is sufficiently large (Assumption 1), the returns to R&D turn out to be high enough relative to costs to induce strictly positive R&D investments in all countries. On the other hand, if R&D is sufficiently costly (Assumption 2), the amount of R&D investments will not exceed the upper threshold of γ. In fact, Assumption 2 is also sufficient for the concavity of the profit function as it is stronger than $\delta > \frac{1}{b}\sum_{j\in N_i\cup\{i\}}\frac{(|N_j|+1)^2}{(|N_j|+2)^2}$, at least as soon as Assumption 1 holds. Thus, both assumptions together imply that the system of the first-order conditions (10) determines the equilibrium R&D efforts of firms, which are strictly positive and smaller than γ. Below we will also show that under Assumptions 1 and 2, the equilibrium profits of all firms are strictly positive.

Derivation of the profit function in section 7.

The profit function in (7) can be written as:

$$\begin{aligned}\pi_i &= 2x_i^*\Bigg[\frac{1}{b}(a-\gamma-2\tau)\sum_{j\in N_i}\frac{|N_j|+1}{(|N_j|+2)^2}+\frac{1}{b}(a-\gamma+|N_i|\tau)\frac{|N_i|+1}{(|N_i|+2)^2}-\\ &- \frac{1}{b}\sum_{j\in N_i}\left[\frac{|N_i|+1}{(|N_i|+2)^2}+\frac{|N_j|+1}{(|N_j|+2)^2}\right]x_j^*-\frac{1}{b}\sum_{j\in N_i}\sum_{k\in N_j,k\neq i}\frac{|N_j|+1}{(|N_j|+2)^2}x_k^*\Bigg]-\\ &- \left[-\frac{1}{b}\sum_{j\in N_i\cup\{i\}}\frac{(|N_j|+1)^2}{(|N_j|+2)^2}+\delta\right]x_i^{*2}+f(\{x_k\}_{k\in N_i\cup N_i^2}).\end{aligned}$$

By the first-order conditions (10), this reduces to:

$$\begin{aligned}
\pi_i &= 2\Bigg[-\frac{1}{b}\sum_{j\in N_i\cup\{i\}}\frac{(|N_j|+1)^2}{(|N_j|+2)^2}+\delta\Bigg]x_i^{*2}-\Bigg[-\frac{1}{b}\sum_{j\in N_i\cup\{i\}}\frac{(|N_j|+1)^2}{(|N_j|+2)^2}+\delta\Bigg]x_i^{*2}+\\
&+ f(\{x_k\}_{k\in N_i\cup N_i^2}) = \Bigg[-\frac{1}{b}\sum_{j\in N_i\cup\{i\}}\frac{(|N_j|+1)^2}{(|N_j|+2)^2}+\delta\Bigg]x_i^{*2}+\\
&+ \frac{1}{b}\sum_{j\in N_i}\frac{1}{(|N_j|+2)^2}\Bigg(a-\gamma-2\tau-\sum_{k\in N_j\cup\{j\},k\neq i}x_k^*\Bigg)^2+\\
&+ \frac{1}{b}\frac{1}{(|N_i|+2)^2}\Bigg(a-\gamma+|N_i|\tau-\sum_{j\in N_i}x_j^*\Bigg)^2. \qquad (11)
\end{aligned}$$

It is easy to see that this expression is strictly positive whenever Assumptions 1 and 2 hold.

In a symmetric network, where $|N_i| = |N_j| = n$ for all $i \neq j$, the profit function in (11) becomes:

$$\begin{aligned}
\pi &= \left(-\frac{1}{b}\frac{(n+1)^3}{(n+2)^2}+\delta\right)\left(\frac{a-\gamma-\frac{n}{n+1}\tau}{-1+\delta b(1+\frac{1}{n+1})^2}\right)^2+\frac{n}{b(n+2)^2}\left(a-\gamma-2\tau-n\frac{a-\gamma-\frac{n}{n+1}\tau}{-1+\delta b(1+\frac{1}{n+1})^2}\right)^2+\\
&+ \frac{1}{b(n+2)^2}\left(a-\gamma-n\left(\frac{a-\gamma-\frac{n}{n+1}\tau}{-1+\delta b\big(1+\frac{1}{n+1}\big)^2}-\tau\right)\right)^2. \qquad (12)
\end{aligned}$$

Proof of Proposition 1. First, we observe that the right-hand side of the inequality in Assumption 1 is monotonically increasing in n_h. Therefore, Assumption 1 holds for all $n_s < n_h \leq \bar{n}$ as soon as it holds for $n_h = \bar{n}$. Similarly, Assumption 2 holds for all $n_s < n_h \leq \bar{n}$ as soon as a stricter condition is fulfilled. Thus, the following two conditions guarantee that both assumptions hold for all $n_s < n_h \leq \bar{n}$:

$$\begin{aligned}
a &> \gamma(1+\bar{n})+2\tau, \qquad (13)\\
\delta &\geq \frac{1}{\gamma b}\left[\frac{\bar{n}+1}{(\bar{n}+2)^2}\bar{n}(\gamma\bar{n}+a-2\tau)+\frac{2}{9}((\gamma\bar{n}+a)(\bar{n}+1)-\tau\bar{n})\right]. \qquad (14)
\end{aligned}$$

The R&D efforts of a hub and a spoke in the symmetric equilibrium are given by the solution to the system of the first-order conditions (10), which in case of the hub-and-spoke network reduces to just two equations:

$$\begin{aligned}
\Bigg(&- (1-\psi)n_h\frac{(n_s+1)^2}{(n_s+2)^2}-(n_h\psi+1)\frac{(n_h+1)^2}{(n_h+2)^2}+\delta b+\psi n_h\frac{2(n_h+1)}{(n_h+2)^2}+ \qquad (15)\\
&+ (1-\psi)n_h\frac{n_s+1}{(n_s+2)^2}((1-\varphi)n_s-1)+\psi n_h\frac{n_h+1}{(n_h+2)^2}(\psi n_h-1)\Bigg)\cdot x_h+\\
&+ (1-\psi)n_h\left(\frac{n_h+1}{(n_h+2)^2}+\frac{n_s+1}{(n_s+2)^2}+\frac{n_h+1}{(n_h+2)^2}n_h\psi+\frac{n_s+1}{(n_s+2)^2}\varphi n_s\right)\cdot x_s=\\
&= (a-\gamma-2\tau)\left((1-\psi)n_h\frac{n_s+1}{(n_s+2)^2}+\psi n_h\frac{n_h+1}{(n_h+2)^2}\right)+\frac{n_h+1}{(n_h+2)^2}(a-\gamma+n_h\tau),
\end{aligned}$$

$$\begin{aligned}
\Bigg(&- (1-\varphi)n_s\frac{(n_h+1)^2}{(n_h+2)^2} - (\varphi n_s+1)\frac{(n_s+1)^2}{(n_s+2)^2} + \delta b + \varphi n_s\frac{2(n_s+1)}{(n_s+2)^2} + \\
&+ (1-\varphi)n_s\frac{n_h+1}{(n_h+2)^2}((1-\psi)n_h-1) + \varphi n_s\frac{n_s+1}{(n_s+2)^2}(\varphi n_s-1)\Bigg)\cdot x_s + \\
&+ (1-\varphi)n_s\left(\frac{n_h+1}{(n_h+2)^2} + \frac{n_s+1}{(n_s+2)^2} + \frac{n_h+1}{(n_h+2)^2}n_h\psi + \frac{n_s+1}{(n_s+2)^2}\varphi n_s\right)\cdot x_h = \\
&= (a-\gamma-2\tau)\left((1-\varphi)n_s\frac{n_h+1}{(n_h+2)^2} + \varphi n_s\frac{n_s+1}{(n_s+2)^2}\right) + \frac{n_s+1}{(n_s+2)^2}(a-\gamma+n_s\tau)
\end{aligned} \tag{16}$$

The unique solution of this system is:

$$x_s^* = \frac{\begin{gathered}\Big((a-\gamma-2\tau)(1-\varphi)n_s(n_h+1)(n_s+2)^2+ \\ +(n_s+1)(n_h+2)^2\big[(a-\gamma-2\tau)\varphi n_s + a-\gamma+n_s\tau\big]\Big)\cdot \\ \cdot\Big((n_h+2)^2\big[b\delta(n_s+2)^2 - n_h(1-\psi)(n_s+1)(2+\varphi n_s)\big]- \\ -(n_h\psi+1)(n_h+1)(n_h(1-\psi)+1)(n_s+2)^2\Big)- \\ -(1-\varphi)n_s\Big((n_h\psi+1)(n_h+1)(n_s+2)^2 + (\varphi n_s+1)(n_s+1)(n_h+2)^2\Big)\cdot \\ \cdot\Big((a-\gamma-2\tau)\big[n_h\psi(n_h+1)\,(n_s+2)^2 + (n_h-n_h\psi)(n_s+1)\,(n_h+2)^2\,\big]+ \\ +(n_h+1)\,(n_s+2)^2\,(a-\gamma+n_h\tau)\Big)\end{gathered}}{\begin{gathered}\Big((n_h+2)^2\big[b\delta(n_s+2)^2 - n_h(1-\psi)(n_s+1)(2+\varphi n_s)\big]- \\ -(n_h\psi+1)(n_h+1)(n_h(1-\psi)+1)(n_s+2)^2\Big)\cdot \\ \cdot\Big(-(1-\varphi)n_s(n_s+2)^2(n_h\psi+2)(n_h+1) + (n_h+2)^2\delta b(n_s+2)^2- \\ -(n_h+2)^2\big[(\varphi n_s+1)(n_s+1)^2 - n_s\varphi(n_s+1)(1+\varphi n_s)\big]\Big)- \\ -(1-\varphi)(1-\psi)n_s n_h\Big((n_h+1)(n_s+2)^2(1+n_h\psi) + (n_s+1)(n_h+2)^2(1+\varphi n_s)\Big)^2\end{gathered}} \tag{17}$$

$$x_h^* = \frac{\begin{gathered}(a-\gamma-2\tau)\big(n_h\psi(n_h+1)(n_s+2)^2 + (n_h-n_h\psi)(n_s+1)(n_h+2)^2\big)+ \\ +(n_h+1)(n_s+2)^2(a-\gamma+n_h\tau) - x_s^*(n_h-n_h\psi)\big((n_s+1)(n_h+2)^2+ \\ +(n_h+1)(n_s+2)^2 + \varphi n_s(n_s+1)(n_h+2)^2 + n_h\psi(n_h+1)(n_s+2)^2\big)\end{gathered}}{\begin{gathered}b\delta(n_h+2)^2(n_s+2)^2 - (n_h\psi+1)(n_h+1)^2(n_s+2)^2- \\ -(n_h-n_h\psi)(n_s+1)^2(n_h+2)^2 + n_h\psi(2n_h+2)(n_s+2)^2+ \\ +((1-\varphi)n_s-1)(n_h-n_h\psi)(n_s+1)(n_h+2)^2 + n_h\psi(n_h\psi-1)(n_h+1)(n_s+2)^2\end{gathered}} \tag{18}$$

Taking a derivative of x_h^* and x_s^* with respect to each of the parameters n_s, n_h, φ and ψ, we obtain a ratio, where the denominator is unambiguously positive while the sign of the numerator is determined by the sign of a cubic polynomial in δ. As soon as δ is sufficiently large – greater than the largest real root of the polynomial, the sign of the polynomial is defined by the sign of the coefficient at the highest degree.

Thus, to simplify calculations, we assume that δ is large enough ($\delta > \Delta$) and focus on the sign of the polynomial's coefficient at δ^3. We obtain that under the parameter restriction (13), partial derivatives $\frac{\partial x_s^*}{\partial n_h}$, $\frac{\partial x_h^*}{\partial n_s}$, and $\frac{\partial x_h^*}{\partial \psi}$ are negative and the derivatives $\frac{\partial x_h^*}{\partial n_h}$ and $\frac{\partial x_s^*}{\partial \varphi}$ are positive. Regarding the derivative $\frac{\partial x_s^*}{\partial n_s}$, this derivative is positive if and only if the following inequality holds:

$$(a-\gamma-2\tau)(1-\varphi)\cdot A+(a-\gamma-2\tau)\cdot B+\tau\cdot C>(a-\gamma-2\tau)(1-\varphi)\cdot D-(a-\gamma-2\tau)\varphi\cdot E, \tag{19}$$

where

$$\begin{aligned}
A &= n_s^6n_h^4+7n_s^6n_h^3+18n_s^6n_h^2+20n_s^6n_h+8n_s^6+12n_s^5n_h^4+84n_s^5n_h^3+216n_s^5n_h^2+\\
&\quad +240n_s^5n_h+96n_s^5,\\
B &= -30n_s^4n_h^4\varphi+50n_s^4n_h^4-300n_s^4n_h^3\varphi+380n_s^4n_h^3-840n_s^4n_h^2\varphi+1000n_s^4n_h^2-\\
&\quad -960n_s^4n_h\varphi+1120n_s^4n_h-384n_s^4\varphi+448n_s^4+40n_s^3n_h^4\varphi+100n_s^3n_h^4-\\
&\quad -320n_s^3n_h^3\varphi+880n_s^3n_h^3-1280n_s^3n_h^2\varphi+2400n_s^3n_h^2-1600n_s^3n_h\varphi+2720n_s^3n_h-\\
&\quad -640n_s^3\varphi+1088n_s^3+240n_s^2n_h^4\varphi+120n_s^2n_h^4+240n_s^2n_h^3\varphi+1200n_s^2n_h^3-\\
&\quad -480n_s^2n_h^2\varphi+3360n_s^2n_h^2-960n_s^2n_h\varphi+3840n_s^2n_h-384n_s^2\varphi+1536n_s^2+288n_sn_h^4\varphi\\
&\quad +112n_sn_h^4+576n_sn_h^3\varphi+1024n_sn_h^3+384n_sn_h^2\varphi+2816n_sn_h^2+3200n_sn_h+1280n_s+\\
&\quad +16n_h^5\varphi+96n_h^4\varphi+64n_h^4+192n_h^3\varphi+448n_h^3+128n_h^2\varphi+1152n_h^2+1280n_h+512,\\
C &= 160n_h^4+1024n_s+1280n_h+768n_s^2+256n_s^3+32n_s^4+1280n_h^2+640n_h^3+512+\\
&\quad +16n_h^5+1929n_s^2n_h^2+960n_s^2n_h^3+640n_s^3n_h^2+320n_s^3n_h^3+80n_s^4n_h^2+24n_s^2n_h^5+\\
&\quad +80n_s^3n_h^4+40n_s^4n_h^3+1286n_sn_h^3+8n_s^3n_h^5+10n_s^4n_h^4+n_s^4n_h^5+2560n_sn_h+\\
&\quad +2560n_sn_h^2+1920n_s^2n_h+640n_s^3n_h+80n_s^4n_h+32n_sn_h^5+320n_sn_h^4+240n_s^2n_h^4,
\end{aligned}$$

$$\begin{aligned}
D &= n_s^4n_h^5+6n_s^3n_h^5+12n_s^2n_h^5+8n_sn_h^5,\\
E &= 2n_s^4n_h^5+14n_s^3n_h^5+36n_s^2n_h^5+40n_sn_h^5.
\end{aligned}$$

Notice that all of these expressions, A, B, C, D, and E, are positive, so that the left-hand side of (19) is positive, while the sign of the right-hand side is determined by the relative values of $(1-\varphi)\cdot D$ and $\varphi\cdot E$.

It is easy to see that $2D<E$, so that for $\varphi\geq 1/3$, $(1-\varphi)\cdot D<\varphi\cdot E$, and the right-hand side of (19) is negative. This establishes condition (b) of the proposition.

Next, observe that $C>D$. Then as soon as $\tau\geq(a-\gamma-2\tau)(1-\varphi)$, inequality (19) holds. This justifies condition (a).

Finally, condition (c) follows from the series of inequalities. First, when $n_h\leq n_s^2$,

$$A>n_s^4n_h^5+12n_s^3n_h^5+7n_s^2n_h^5+84n_sn_h^5. \tag{20}$$

Secondly, since $n_s<n_h$,

$$n_s^4n_h^5+12n_s^3n_h^5+7n_s^2n_h^5+84n_sn_h^5>n_s^4n_h^5+6n_s^3n_h^5+13n_s^2n_h^5+84n_sn_h^5>D. \tag{21}$$

Combining (20) and (21), we obtain that $A>D$, so that inequality (19) is satisfied. □

Proof of Proposition 2. Notice that in case of a symmetric network of degree n, the right-hand side of inequality in Assumption 1 is an increasing function of n and also the right-hand side of inequality in Assumption 2 is an increasing function of n, *provided* that Assumption 1 holds. Therefore, for Assumptions 1 and 2 to be fulfilled for all $n < \bar{n}$, it is enough to ensure that these assumptions hold for $n = \bar{n}$. The resulting restrictions are

$$a > \gamma(1+\bar{n}) + 2\tau, \tag{22}$$

$$\delta \geq \frac{1}{\gamma b}\frac{(\bar{n}+1)}{(\bar{n}+2)^2}((\gamma\bar{n}+a)(\bar{n}+1) - \tau\bar{n}). \tag{23}$$

The proof of Proposition 2 is then established in two steps.

First, we show that the equilibrium R&D effort x^* in (5) is monotonically increasing in n. Consider a derivative of x^* with respect to n:

$$\begin{aligned}\frac{\partial x^*}{\partial n} &= \frac{-\frac{\tau}{(n+1)^2}\big(-1+\delta b\big(1+\frac{1}{n+1}\big)^2\big) + 2\delta b\big(1+\frac{1}{n+1}\big)\frac{1}{(n+1)^2}\big(a-\gamma-\frac{n}{n+1}\tau\big)}{\big(-1+\delta b\big(1+\frac{1}{n+1}\big)^2\big)^2} = \\ &= \frac{\frac{1}{(n+1)^2}\Big(\tau + \delta b\big(1+\frac{1}{n+1}\big)\big(-\tau\big(1+\frac{1}{n+1}\big) + 2\big(a-\gamma-\frac{n}{n+1}\tau\big)\big)\Big)}{\big(-1+\delta b(1+\frac{1}{n+1})^2\big)^2}.\end{aligned}$$

The sign of this derivative is positive as soon as

$$2\big(a-\gamma-\frac{n}{n+1}\tau\big) > \tau\big(1+\frac{1}{n+1}\big),$$

which holds due to the restriction on a in (22).

Next, we prove that the profit function, π, in 12 is monotonically decreasing in n. Given the complexity of algebraic expressions, below we present only a sketch of the argument.

Taking a derivative of π with respect to n, we obtain an expression equal to the product of the ratio $\frac{1}{b(2n-4b\delta+n^2-4bn\delta-bn^2\delta+1)^3}$ and the quadratic polynomial of τ. The ratio is negative for any $n \geq 1$ due to the restriction on δ in (23), and the polynomial is positive for any $n \geq 1$ as soon as both (22) and (23) hold. The latter is established via two steps.

- First, from (23) it follows that the coefficient of the polynomial at the quadratic term τ^2 is negative for any $n \geq 1$, while the constant term is positive. Hence, the graph of the quadratic function is a parabola with downward-directed branches and two real roots – one positive and one negative.

- Since the unit trade cost τ is non-negative and does not exceed $\frac{1}{2}(a-\gamma)$ due to (22), the value of the polynomial is positive for all $\tau \in [0, \frac{1}{2}(a-\gamma))$, as soon as it is positive at $\tau = \frac{1}{2}(a-\gamma)$. One can show that this is indeed the case, provided that (22) and (23) hold.

Thus, for all $n \geq 1$ and any parameters satisfying (22) and (23), the derivative of π with respect to n is negative. □

Proof of Proposition 3. Consider an arbitrary hub-and-spoke network defined by parameters $0 \leq \psi, \varphi < 1$ and $1 \leq n_s < n_h$. Then observe that a symmetric network of degree n_h can be obtained from this hub-and-spoke network by increasing the number of spokes' direct trade partners from n_s to n_h. Similarly, a symmetric network of degree n_s can be obtained from the hub-and-spoke network by decreasing the number of hubs' direct trade partners from n_h to n_s.[43] Note that such transformation from a hub-and-spoke to symmetric network may require adding or deleting some nodes in the network, but as R&D decisions of firms are fully determined by parameters n_s, n_h, ψ and φ and *not* by the overall network size, the total number of nodes is irrelevant. Then inequalities $x_h^* > x^*(n_h)$ and $x^*(n_s) > x_s^*$ follow from parts 1 and 2 of Proposition 1 stating that R&D of a hub, x_h^*, is decreasing in n_s and R&D of a spoke, x_s^*, is decreasing in n_h. The last remaining inequality $x^*(n_h) > x^*(n_s)$ follows from Proposition 2.

□

Proof of Proposition 4. Consider the first series of inequalities in Proposition 4:

$$x_{h1}^* > x_{h3}^* > x_{h4}^* > x^*(n_h) > x^*(n_s) > x_{s5}^* > x_{s3}^* > x_{s1}^*.$$

Each of these inequalities follows either from Proposition 1 or from Proposition 2. Indeed, $x_{h1}^* > x_{h3}^*$ and $x_{h4}^* > x^*(n_h)$ since x_h^* is decreasing in n_s, and $x_{h3}^* > x_{h4}^*$ since x_h^* is decreasing in ψ. Similarly for spokes, $x^*(n_s) > x_{s5}^*$ since x_s^* is decreasing in n_h, $x_{s5}^* > x_{s3}^*$ since x_s^* is increasing in φ, and $x_{s3}^* > x_{s1}^*$ since x_s^* is increasing in n_s. Finally, the intermediate inequality, $x^*(n) > x^*(m)$, is the immediate implication of Proposition 2.

Likewise, in the second series of inequalities,

$$x_{h1}^* > x_{h2}^* > x_{h4}^* > x^*(n_h) > x^*(n_s) > x_{s4}^* > x_{s2}^*,$$

the first inequality $x_{h1}^* > x_{h2}^*$ is implied by the fact that x_h^* is decreasing in ψ, while the second and the third, $x_{h2}^* > x_{h4}^* > x^*(n_h)$, follow from the fact that x_h^* is decreasing in n_s. With regard to spokes, $x^*(n_s) > x_{s4}^*$ since x_s^* is decreasing in n_h, and $x_{s4}^* > x_{s2}^*$ since x_s^* is increasing in n_s. The intermediate inequality, $x^*(n_h) > x^*(n_s)$, is again the result of Proposition 2. □

Proof of Proposition 5. As in the proof of Proposition 1, we employ the parametric restrictions (13) and (14), which ensure that Assumptions 1 and 2 hold for any $n_h < \bar{n}$. The proof of Proposition 5 is then established in two steps.

First, we show that the social welfare of a spoke in the star is monotonically decreasing in n. The social welfare of a spoke, W_s, is given by the sum of the firm's profit, π_s, and consumer surplus, CS_s, where

$$\begin{aligned}
\pi_s &= -by_{ss}^2 - by_{hs}y_{ss} - by_{sh}^2 - b(n_h-1)y_{sh}^2 - by_{hh}y_{sh} + \\
&\quad +(a-\gamma+x_s^*)(y_{ss}+y_{hs}) - \delta(x_s^*)^2 - \tau y_{sh}, \\
CS_s &= \frac{1}{2b}(a-p_i)^2 = \frac{b}{2}(y_{ss}+y_{hs})^2.
\end{aligned}$$

as implied by (11), (6) and the definition of the demand function in (1). Here y_{ss} and y_{hh} denote the domestic market production of a spoke and hub firm, respectively, and y_{hs}, y_{sh} denote the

[43] When $n_s = n_h$, ψ and φ become meaningless and drop from the analysis of hub-and-spoke networks.

production of the hub for each spoke's market and the production of a spoke firm for the hub's market, respectively. Using (8) and (9) for the equilibrium production levels and (17), (18) for the equilibrium R&D efforts of the hub and a spoke in a star,[44] W_s can be written as a function of n_h and parameters only. Then taking a derivative of W_s with respect to n_h, we obtain a product of the ratio

$$-\frac{3}{b\big(27b^2(n_h+2)^3\delta^2+(-12bn_h^4-111bn_h^3-378bn_h^2-537bn_h-258b)\delta+ \\ +(4n_h^4+48n_h^3+159n_h^2+170n_h+51)\big)^3} \tag{24}$$

and a polynomial of the second degree in τ. To evaluate the sign of this product, we first observe that the ratio is negative for any $n_h \geq 1$ due to the restrictions (13) and (14). We then evaluate the sign of the polynomial, focusing on the sign of its coefficients. Each coefficient is itself a polynomial of the sixth degree in δ and thus, when δ is large enough – greater than the largest real root of this polynomial, – the sign of a coefficient is determined by the sign of the term at δ^6. Focusing on this term, we find that for sufficiently large δ and any $n_h \geq 1$, the coefficient at the quadratic term and the constant term in the polynomial of τ are positive, while the coefficient at the linear term is negative. Taking into account the negative sign of the ratio in front of the polynomial, we then conclude that the derivative of W_s with respect to n_h can be graphically represented by a parabola with downward-directed branches, crossing the axis $\tau = 0$ in its negative part and reaching the extremum at a negative part of the τ-scale. This means that for $\tau \geq 0$ the value of the derivative is always negative. That is, the welfare in a spoke economy is decreasing in n_h for all trade costs τ.

Next, the effects of an increase in n_h on the welfare of the hub can be established using Lemma 1 stated below. They obtain as a corollary of Lemma 1 if we define the lower threshold $\underline{\tau} = \min_{n_h\in[1,\bar{n}]} \widetilde{\tau}(n_h)$ and the upper threshold $\overline{\tau} = \max_{n_h\in[1,\bar{n}]} \widetilde{\tau}(n_h)$, where $\widetilde{\tau}(n_h)$ is characterized by Lemma 1.

Lemma 1. *Suppose that inequalities* (13) *and* (14) *and an additional restriction* $a > \gamma\left(1+\frac{\bar{n}(2n_h^3+12n_h^2+51n_h+16)}{9(n_h+2)}\right)$ *hold. Then there exist* $\widetilde{\Delta} > 0$ *such that for any* $\delta \geq \widetilde{\Delta}$*, the following statements are fulfilled:*

1. *for any* $n_h \in [1,\bar{n}]$ *there exists a threshold of trade costs* $\widetilde{\tau}(n_h)$*,* $0 < \widetilde{\tau}(n_h) < \widehat{\tau}$*, where* $\widehat{\tau} = \frac{a-\gamma(1+\bar{n})}{2}$ *is the largest value of trade costs at which* (13) *holds, such that the derivative of the social welfare function with respect to* n_h *is positive for any* $\tau < \widetilde{\tau}(n_h)$ *and it is negative for any* $\tau > \widetilde{\tau}(n_h)$*;*
2. *the threshold* $\widetilde{\tau}(n_h)$ *is decreasing in* n_h*.*

Proof of Lemma 1. 1. The social welfare, W_h, of the hub in a star is given by the sum of the firm's

[44] (17) and (18) should be considered under the parameter values $\psi = \varphi = 0$ and $n_s = 1$, to address the case of a star network.

profit, π_h, and consumer surplus, CS_h, where

$$\begin{aligned}\pi_h &= -by_{hh}^2 - bn_h y_{sh} y_{hh} + n_h(-by_{hs}^2 by_{ss} y_{hs}) + \\ &\quad +(a-\gamma+x_h^*)(y_{hh}+n_h y_{sh}) - \delta(x_h^*)^2 - \tau n_h y_{hs}, \\ CS_h &= \frac{1}{2b}(a-p_i)^2 = \frac{b}{2}(y_{hh}+n_h y_{sh})^2,\end{aligned}$$

as follows from (11), (6) and the demand function in (1). Using (8) and (9) for the equilibrium production levels and (17), (18) for the equilibrium R&D efforts of the hub and a spoke in a star, W_h can be expressed in terms of n_h and parameters only.

Taking a derivative of W_h with respect to n_h, we obtain a product of the ratio

$$\frac{3}{\begin{array}{c} b\big(27b^2(n_h+2)^3\delta^2 + (-12bn_h^4 - 111bn_h^3 - 378bn_h^2 - 537bn_h - 258b)\delta + \\ +(4n_h^4 + 48n_h^3 + 159n_h^2 + 170n_h + 51)\big)^3 \end{array}}$$

and the quadratic polynomial of τ. The ratio turns out to be the same as in (24) but without a minus sign in front of it, so that it is positive for any $n_h \geq 1$ due to (13) and (14). On the other hand, the value of the quadratic polynomial is positive for any given $n_h \geq 1$ when τ is below a certain threshold $\widetilde{\tau}(n_h)$ and negative when τ is above that threshold, provided that the parameter restrictions of the Lemma hold.

The latter is established in two steps.

- The coefficients of the polynomial are themselves polynomials of the sixth degree in δ. In each of these polynomials we focus on the sign of the term at δ^6, the highest degree of δ, since for sufficiently large δ the sign of this term determines the sign of the whole polynomial (in δ), that is, the sign of the corresponding coefficient in the polynomial of τ. Following this approach, we find that for sufficiently large δ and any $n_h \geq 1$ the coefficient at the quadratic term and the constant term in the polynomial of τ are positive. The coefficient at the linear term is negative, provided that (13) holds. Thus, the graph of the polynomial is a parabola with upward-directed branches, crossing the axis $\tau = 0$ in its positive part and reaching the extremum at a positive value of τ. Furthermore, two roots of the polynomial are real-valued and positive functions of n_h.

  From these observations it follows that the smaller root of the polynomial is a point on a scale of τ, $\widetilde{\tau}(n_h)$, where the polynomial changes its sign from positive to negative. That is, it is such a value $\widetilde{\tau}(n_h)$ of trade costs that the derivative of W_h with respect to n_h is positive for any τ between 0 and $\widetilde{\tau}(n_h)$ and negative for any τ between $\widetilde{\tau}(n_h)$ and the second, larger root of the polynomial. It remains to show that the larger root of the polynomial is actually greater than the upper bound of trade costs $\widehat{\tau} = \frac{a-\gamma(1+\bar{n})}{2}$ defined by (13), so that the derivative of the hub's welfare is negative for *any* possible $\tau \in (\widetilde{\tau}(n_h), \widehat{\tau})$.
- We prove that the larger root of the polynomial is greater than $\widehat{\tau}$ by showing that the sign of the polynomial at $\tau = \widehat{\tau}$ is negative.

Evaluating the polynomial at $\tau = \widehat{\tau}$, we obtain another polynomial of the sixth degree in δ. For sufficiently high δ – higher than the largest real root of that polynomial, only the sign of the coefficient at the highest degree of δ, δ^6, matters. This coefficient is equal to

$$\frac{729}{2} b^6 \bar{n}\gamma (n_h+2)^6 \left((a-\gamma)(-18-9n_h) + \bar{n}\gamma(2n_h^3 + 12n_h^2 + 51n_h + 16) \right)$$

Due to (13), a is at least as large as $\gamma(1+\bar{n})$. In fact, if $a > \gamma\left(1 + \frac{\bar{n}(2n_h^3+12n_h^2+51n_h+16)}{9(n_h+2)}\right)$ (an additional restriction on a in Lemma 1), then this coefficient is negative for any $n_h \geq 1$. Hence, as soon as δ is large enough and the additional restriction on a holds, the value of the polynomial at $\tau = \widehat{\tau}$ is negative.

2. To prove that $\widetilde{\tau}(n_h)$ is decreasing in n_h, consider a derivative of $\widetilde{\tau}(n)$ with respect to n_h. We find that it is given by the product of a ratio, which is negative under condition (13), and a polynomial of δ. For δ greater than the largest real root of this polynomial, the sign of the function is determined by the sign of the term at the highest degree of δ. We focus on that term and find that for sufficiently large δ and any $n_h \geq 1$, the highest-degree term is positive. So, at least for sufficiently large δ, the derivative of $\widetilde{\tau}(n)$ with respect to n_h is negative for any $n_h \geq 1$, which implies that $\widetilde{\tau}(n_h)$ is monotonically decreasing in n_h.

This completes the (schematic) proof of Lemma 1 and Proposition 5.

□

□

Proof of Proposition 6. As we explained in the beginning of the proof of Proposition 6, Assumptions 1 and 2 hold for all $n < \bar{n}$ as soon as they hold for $n = \bar{n}$. Under the resulting restrictions, (22) and (23), the proof of Proposition 6 is established in two steps. First, we show that the consumer surplus of a country within a symmetric trade network of degree n is monotonically increasing in n. According to (6) and the definition of the demand function in (1), the consumer surplus of country i in the symmetric trade network is given by:

$$CS_i = \frac{1}{2b}(a-p_i)^2 = \frac{b}{2}(y_{ii} + ny_{ji})^2, \tag{25}$$

where y_{ji} is the equilibrium production of any trade partner j of country i for i's market. Using equations (8) and (9) for the production levels and (5) for the R&D effort of a country in the symmetric network, (25) can be written as

$$\begin{aligned} CS_i = CS = \;&= \frac{b}{2}\Big(\frac{1}{b(n+2)}\big(a-\gamma+\frac{a-\gamma-\frac{n}{n+1}\tau}{-1+\delta b\big(1+\frac{1}{n+1}\big)^2}+n\tau\big)+ \\ &+ \frac{n}{b(n+2)}\big(a-\gamma+\frac{a-\gamma-\frac{n}{n+1}\tau}{-1+\delta b\big(1+\frac{1}{n+1}\big)^2}-2\tau\big)\Big)^2 \\ &= \frac{1}{2}b\delta^2(n+2)^2\frac{(\gamma-a+n\gamma+n\tau-an)^2}{(2n-4b\delta+n^2-4bn\delta-bn^2\delta+1)^2}. \end{aligned}$$

Taking a derivative of CS with respect to n, we obtain:

$$\begin{aligned}\frac{\partial CS}{\partial n} &= \frac{(\gamma - a + n\gamma + n\tau - na)^2}{(2n - 4\delta b + n^2 - 4n\delta b - n^2\delta b + 1)^2}\Big(\frac{1}{2}\delta^2 b(n+2)(n+4) + \\ &+ \frac{\delta^2 b(n+2)^2(4\delta b - 2n + 2n\delta b - 2)}{(n+1)^2 - \delta b(n+2)^2} + \frac{\delta^2 b(n+2)^2(\gamma + \tau - a)}{\gamma - a + n\gamma + n\tau - na}\Big).\end{aligned}$$

The sum of the first two terms in large brackets is positive due to the fact that $\delta b > 1$, which follows from inequalities (22) and (23). The last, third term is positive, too, as an immediate consequence of (22). Therefore, the whole expression for the derivative of CS with respect to n is positive.

Next, the effects of an increase in n on country's welfare can be established as a corollary of Lemma 2 stated below. The corollary obtains if we define the lower threshold $\underline{\tau} = \min_{n\in[1,\bar{n}]}\widetilde{\tau}(n)$ and the upper threshold $\overline{\tau} = \max_{n\in[1,\bar{n}]}\widetilde{\tau}(n)$, where $\widetilde{\tau}(n)$ is characterized by Lemma 2.

Lemma 2. *Suppose that inequalities* (22) *and* (23) *and an additional restriction* $a > \gamma(1 + \bar{n} + 2\bar{n}/(n+2))$ *hold. Then there exist* $\Delta > 0$ *such that for any* $\delta \geq \Delta$, *the following statements are fulfilled*:

1. *for any* $n \in [1, \bar{n}]$ *there exists a threshold of trade costs* $\widetilde{\tau}(n)$, $0 < \widetilde{\tau}(n) < \widehat{\tau}$, *where* $\widehat{\tau} = \frac{a-\gamma(1+\bar{n})}{2}$ *is the largest value of trade costs at which* (22) *holds, such that the derivative of the social welfare function with respect to* n *is positive for any* $\tau < \widetilde{\tau}(n)$ *and negative for any* $\tau > \widetilde{\tau}(n)$*;*

2. *the threshold* $\widetilde{\tau}(n)$ *is decreasing in* n.

Proof of Lemma 2. 1. The social welfare, W, of a country in the symmetric network is given by the sum of the firm's profit, π, in (12) and consumer surplus, CS, calculated above. Taking the derivative of W with respect to n, we obtain the expression represented by the product of the ratio $-\frac{1}{b(2n-4b\delta+n^2-4bn\delta-bn^2\delta+1)^3}$ and the quadratic polynomial of τ. The ratio is positive for any $n \geq 1$ due to the restriction on δ in (23). Thus, the sign of the derivative is determined by the quadratic polynomial of τ. The value of this polynomial for any given $n \geq 1$ turns out to be positive when τ is below a certain threshold $\widetilde{\tau}(n)$ and negative when τ is above that threshold, provided that the parameter restrictions of the Lemma hold. The latter is established in two steps.

- First, we find that due to the restrictions (22) and (23), the coefficient of the polynomial at the quadratic term τ^2 and the constant term are positive, while the coefficient at the linear term τ is negative for any given $n \geq 1$. This means that the graph of the polynomial is a parabola with upward-directed branches, crossing the axis $\tau = 0$ in the positive part and reaching the extremum at a positive value of τ. Moreover, two real-valued and positive roots of the polynomial exist, each being a function of n.

  Given these observations, the smaller root of the polynomial is a point on a scale of τ at which the polynomial changes its sign from positive to negative. In other words, it is such a value $\widetilde{\tau}(n)$ of trade costs that the derivative of the social welfare with respect to n is positive for any τ between 0 and $\widetilde{\tau}(n)$ and negative for any τ between $\widetilde{\tau}(n)$ and the second, larger root of the polynomial. It remains to show that the larger root of the polynomial is actually greater than the upper bound of trade costs $\widehat{\tau} = \frac{a-\gamma(1+\bar{n})}{2}$ defined by inequality (22), so that the derivative of the social welfare is negative for *any* $\tau \in (\widetilde{\tau}(n), \widehat{\tau})$.

- To prove that the larger root of the polynomial is greater than $\widehat{\tau}$, we show that the sign of the polynomial at $\tau = \widehat{\tau}$ is negative.

  Evaluating the polynomial at $\tau = \widehat{\tau}$, we obtain a cubic polynomial in δ. Then as soon as δ is high enough – higher than the largest real root of the cubic polynomial, the sign of the polynomial is determined by the sign of the coefficient at the highest degree of δ. To simplify things, we assume that δ is indeed sufficiently high and focus on the sign of the coefficient at δ^3. This coefficient is equal to

  $$\frac{1}{2}\bar{n}\gamma(n+2)^3\left(-a(2+n)+\gamma(2+4\bar{n}+n+\bar{n}n)\right).$$

  It is easy to see that the sign of this coefficient is negative for any $n \geq 1$ as soon as a satisfies an additional restriction (apart from (22)), $a > \gamma(1+\bar{n}+2\bar{n}/(n+2))$. Thus, if δ is large enough and the additional restriction on a holds, the value of the polynomial at $\tau = \widehat{\tau}$ is negative.

2. To prove that the threshold $\widetilde{\tau}(n)$ is decreasing in n, consider a derivative of $\widetilde{\tau}(n)$ with respect to n. The derivative is given by the product of the fraction

$$\frac{1}{2}\delta b \frac{\gamma - a}{\begin{gathered}(-2n^4\delta^3b^3 + 8n^4\delta^2b^2 - 5n4\delta b + n^4 - 20n^3\delta^3b^3 + 44n^3\delta^2b^2 - 24n^3\delta b + \\ +4n^3 - 72n^2\delta^3b^3 + 102n^2\delta^2b^2 - 45n^2\delta b + 6n^2 - 112n\delta^3b^3 + \\ +116n\delta^2b^2 - 38n\delta b + 4n - 64\delta^3b^3 + 48\delta^2b^2 - 12\delta b + 1)^2\end{gathered}},$$

which is negative due to (22), and the sum of two terms. The first term is a polynomial of the fifth degree in δ and the second is a product of a positive ratio and a polynomial of the sixth degree in δ. As soon as δ is greater than any of the largest real roots of the two polynomials, the sign of each term is defined by the sign of the coefficient at the highest degree. Both coefficients are positive: $4b^5(n+2)^6 > 0$ and $4b^6(n+2)^8(n+3) > 0$. So, at least for sufficiently large δ, the derivative of $\widetilde{\tau}(n)$ is negative for any $n \geq 1$.

This concludes the (schematic) proof of Lemma 2 and Proposition 6.

□

Appendix B: Figures

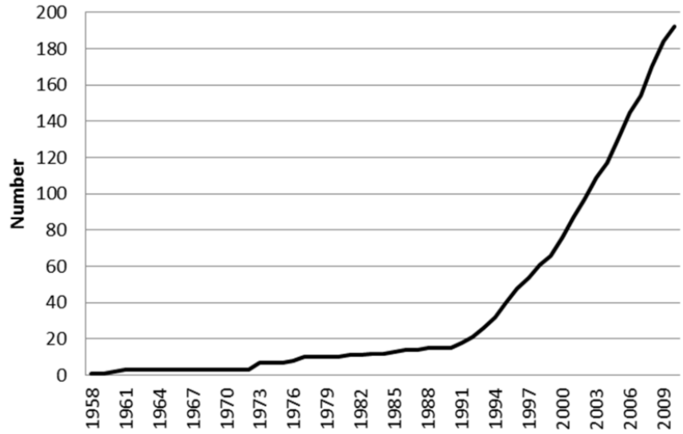

Figure 4: Evolution of the number of RTAs notified to the GATT/WTO during the period and in force (consolidated figures). Source: www.econstor.eu

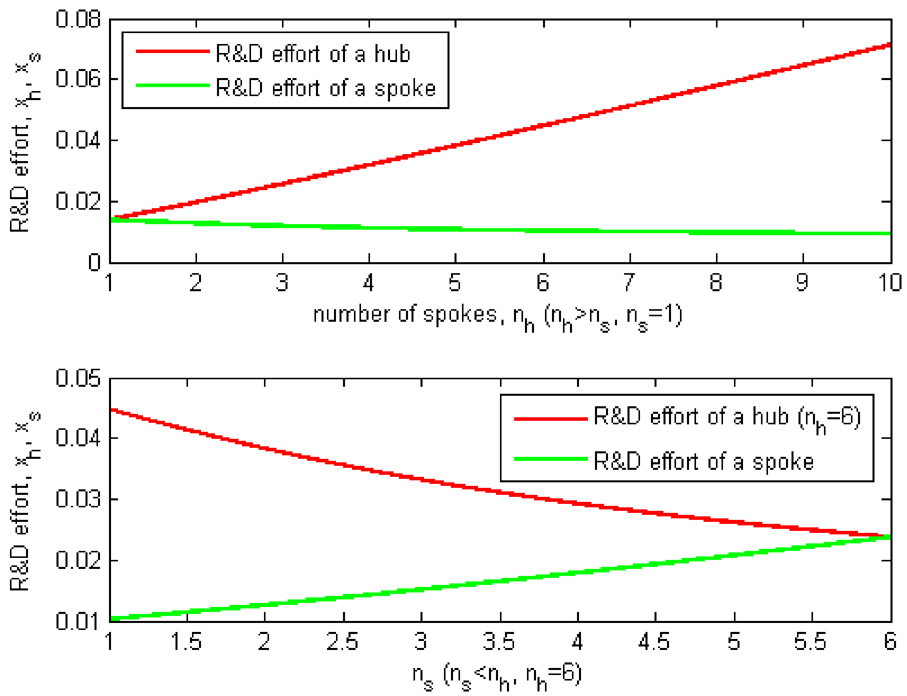

Figure 5: Equilibrium R&D efforts in the hub-and-spoke trade network as a function of n_h (upper panel) and as a function of n_s (lower panel).

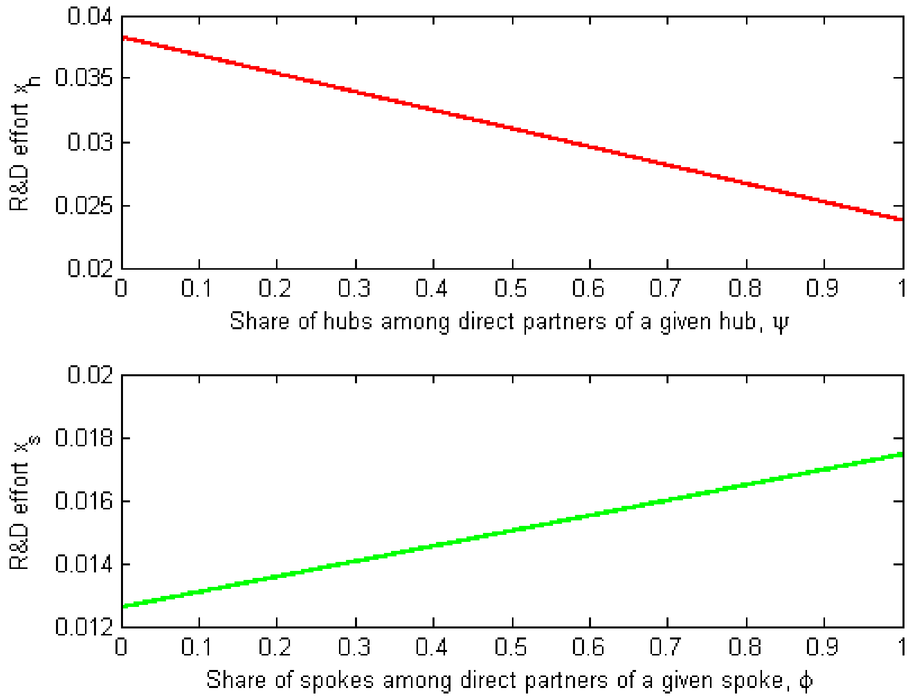

Figure 6: Equilibrium R&D efforts in the hub-and-spoke trade network as a function of ψ (upper panel) and as a function of φ (lower panel).

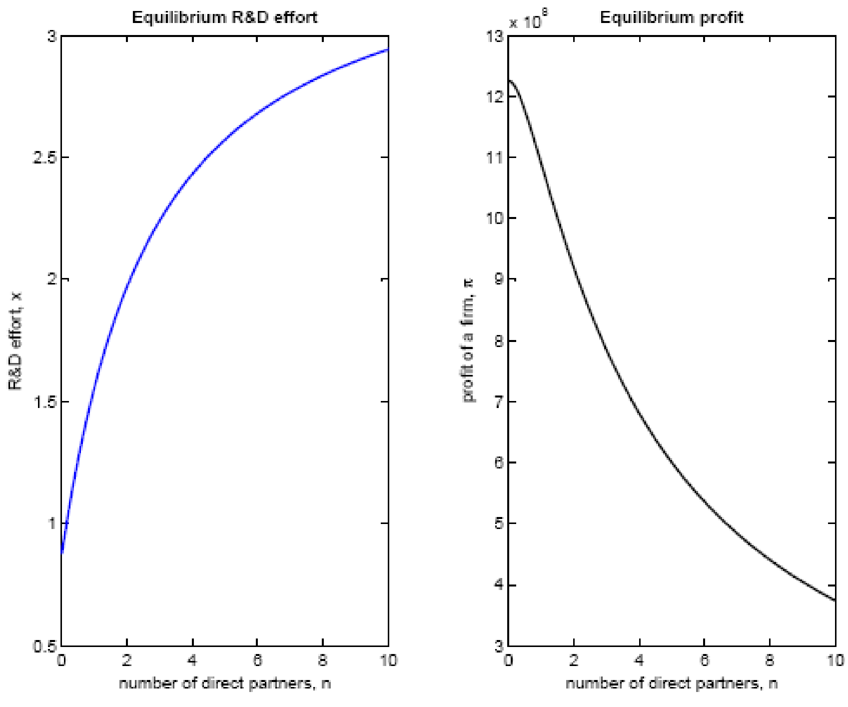

Figure 7: Equilibrium R&D effort and profit of a country in the symmetric network of degree n.

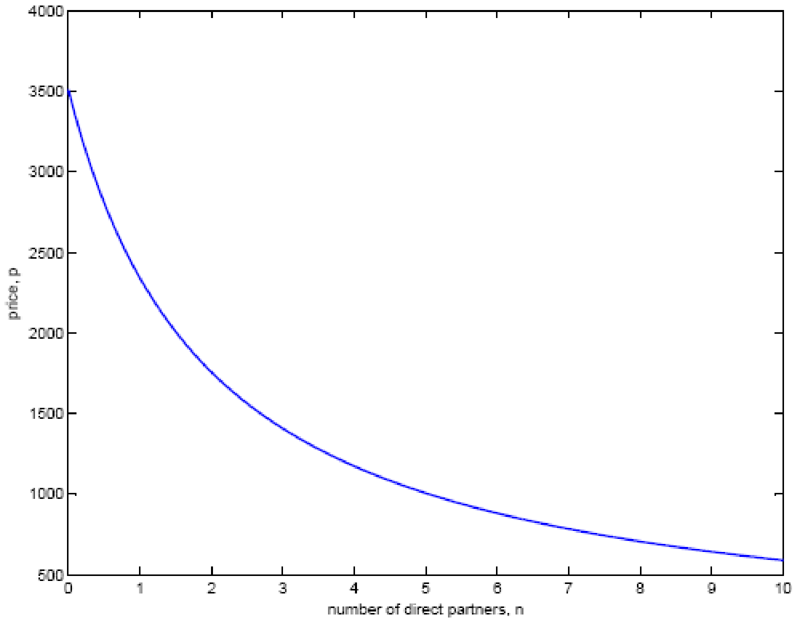

Figure 8: Price on a market in the symmetric network of degree n.

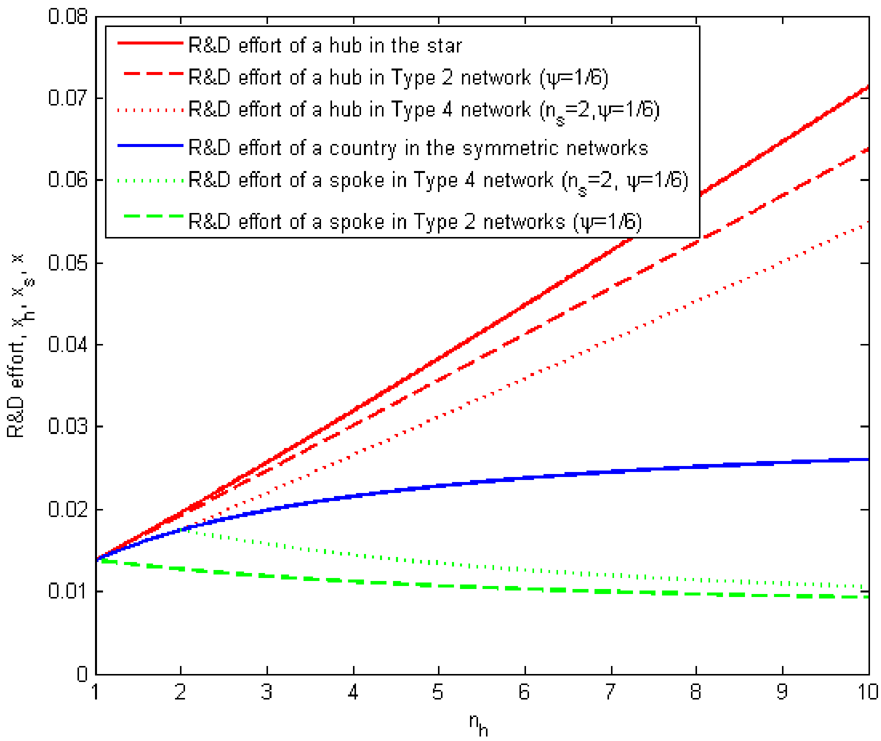

Figure 9: Equilibrium R&D efforts in the symmetric and hub-and-spoke trade networks as a function of n_h.

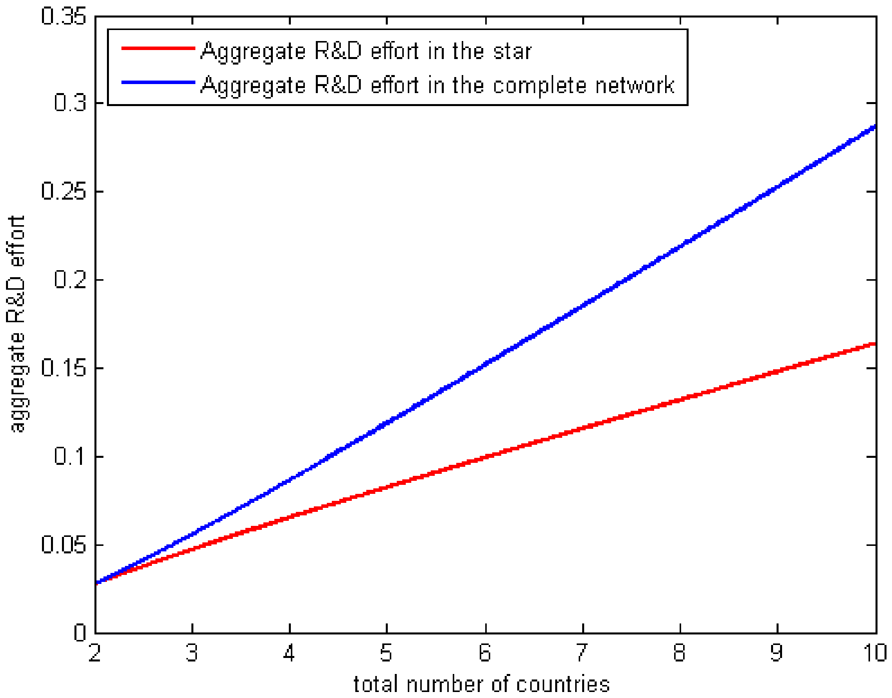

Figure 10: Aggregate equilibrium R&D efforts in the star and in the complete trade network.

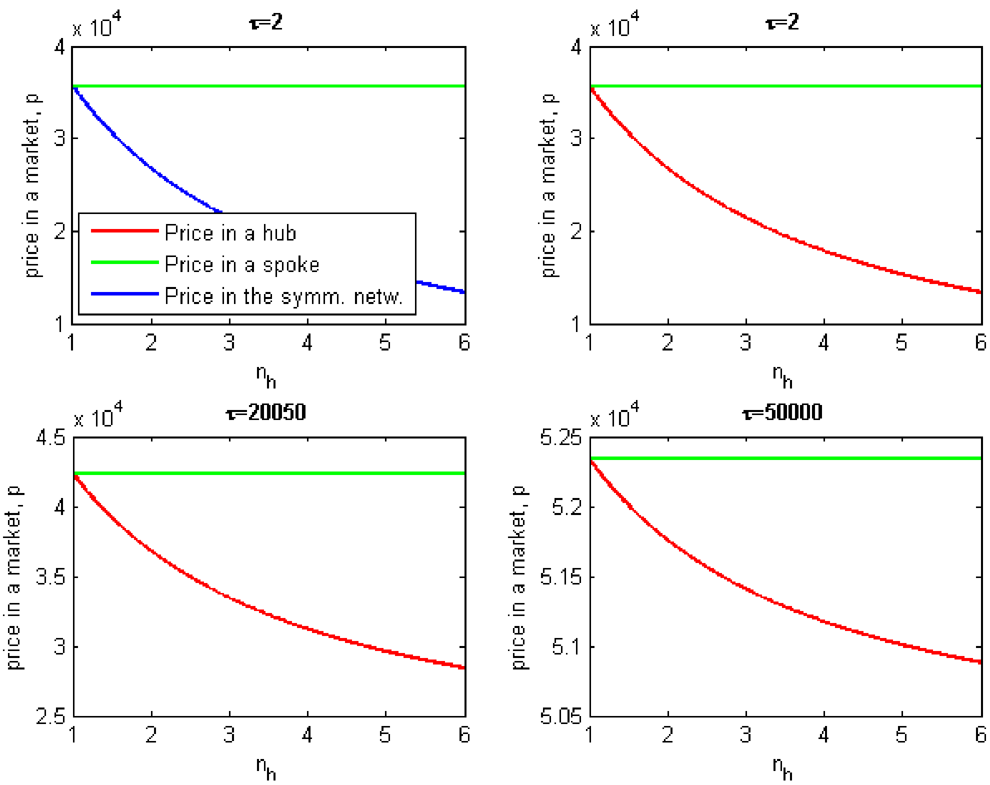

Figure 11: Price in the star and in the symmetric network of degree n_h. Price in the hub of a star and in a country within the symmetric network coincide.

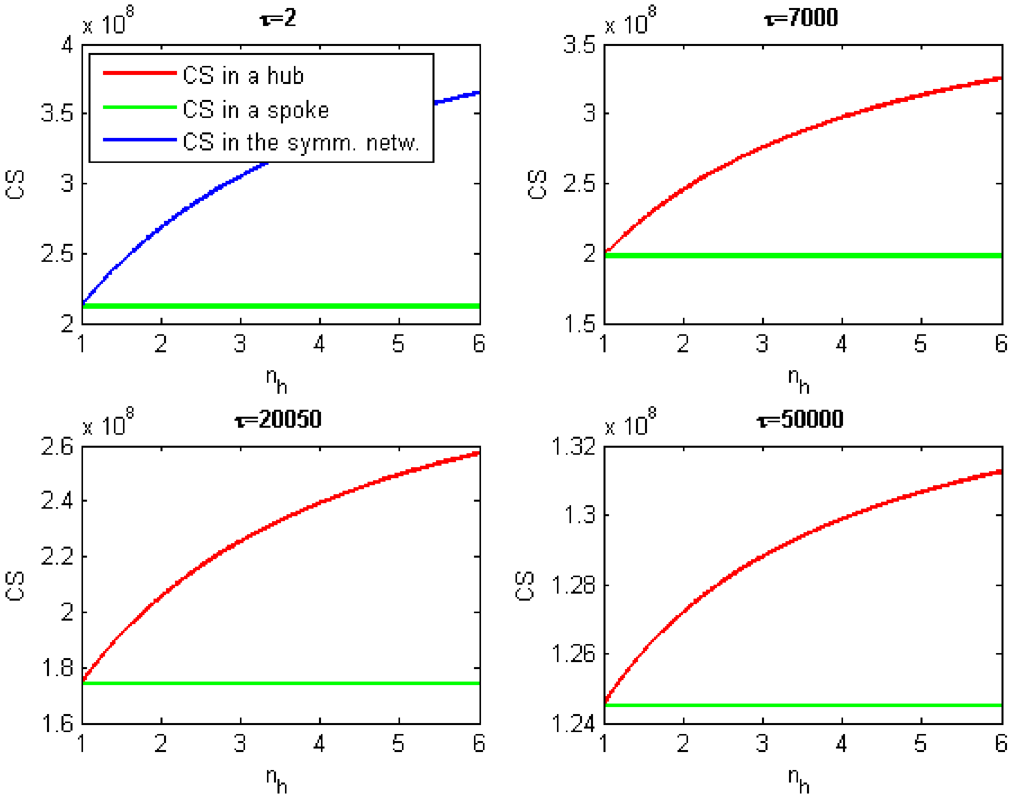

Figure 12: Consumer surplus of a country in the star and in the symmetric network of degree n_h. Consumer surplus in the hub of a star and in a country within the symmetric network coincide.

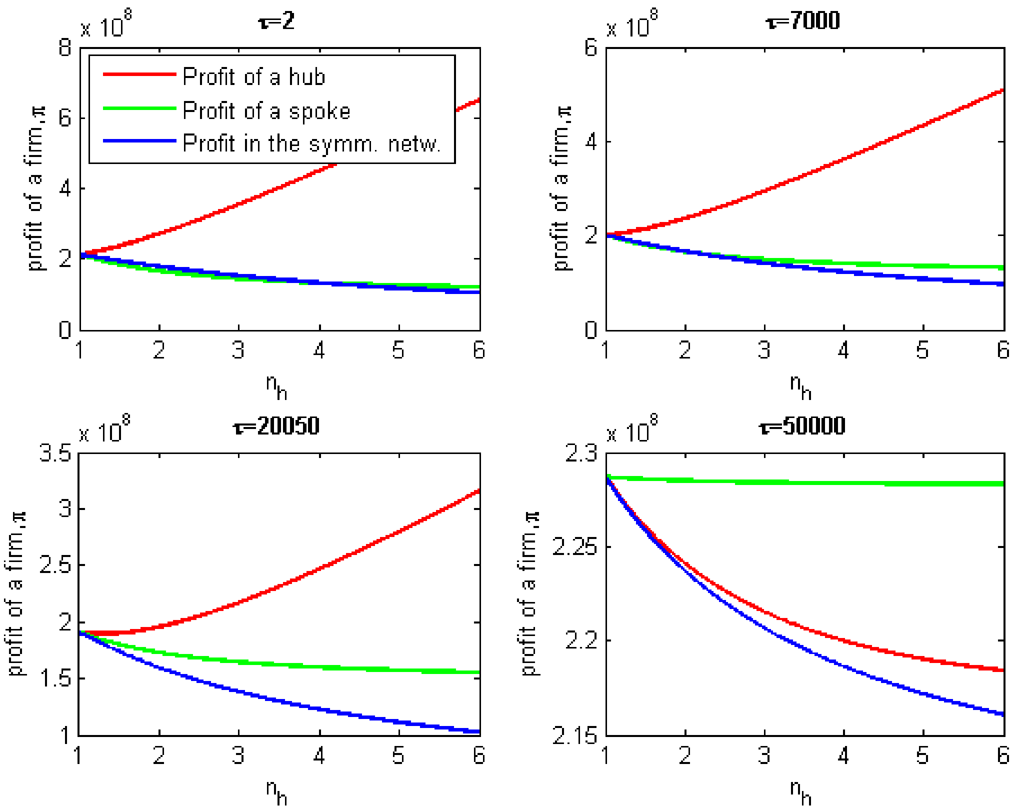

Figure 13: Firm's profit in the star and in the symmetric network of degree n_h.

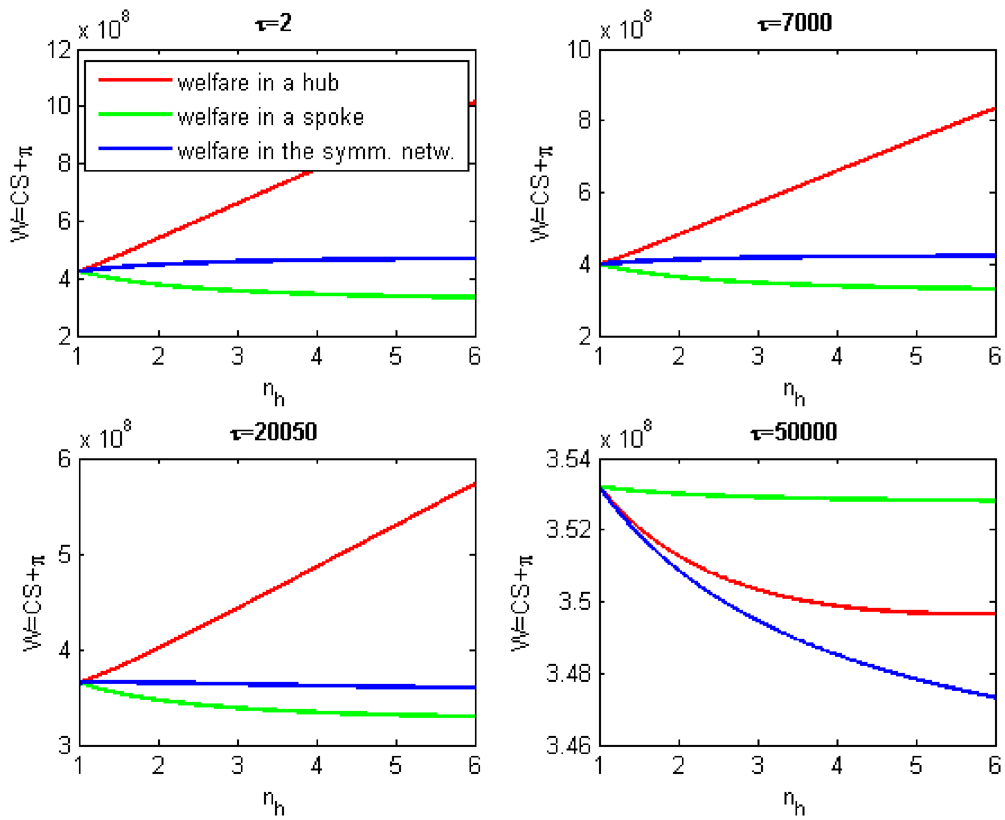

Figure 14: Social welfare of a country in the star and in the symmetric network of degree n_h.

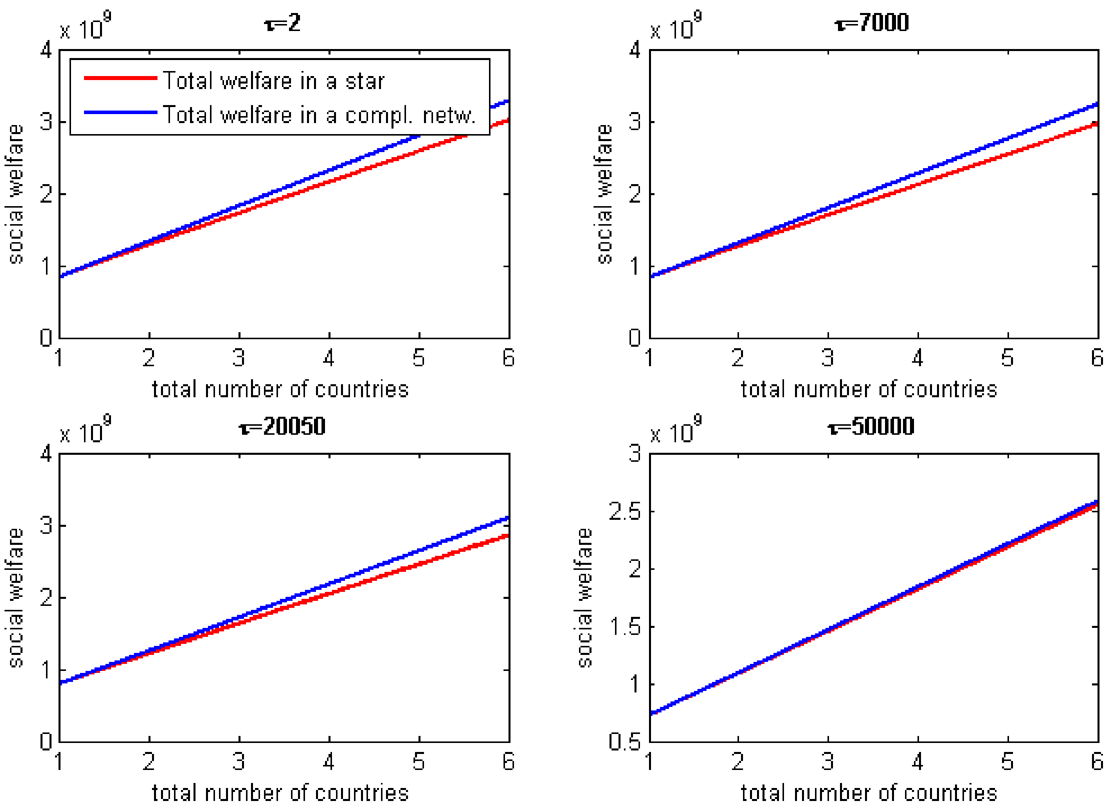

Figure 15: Aggregate welfare in the star and in the complete network.

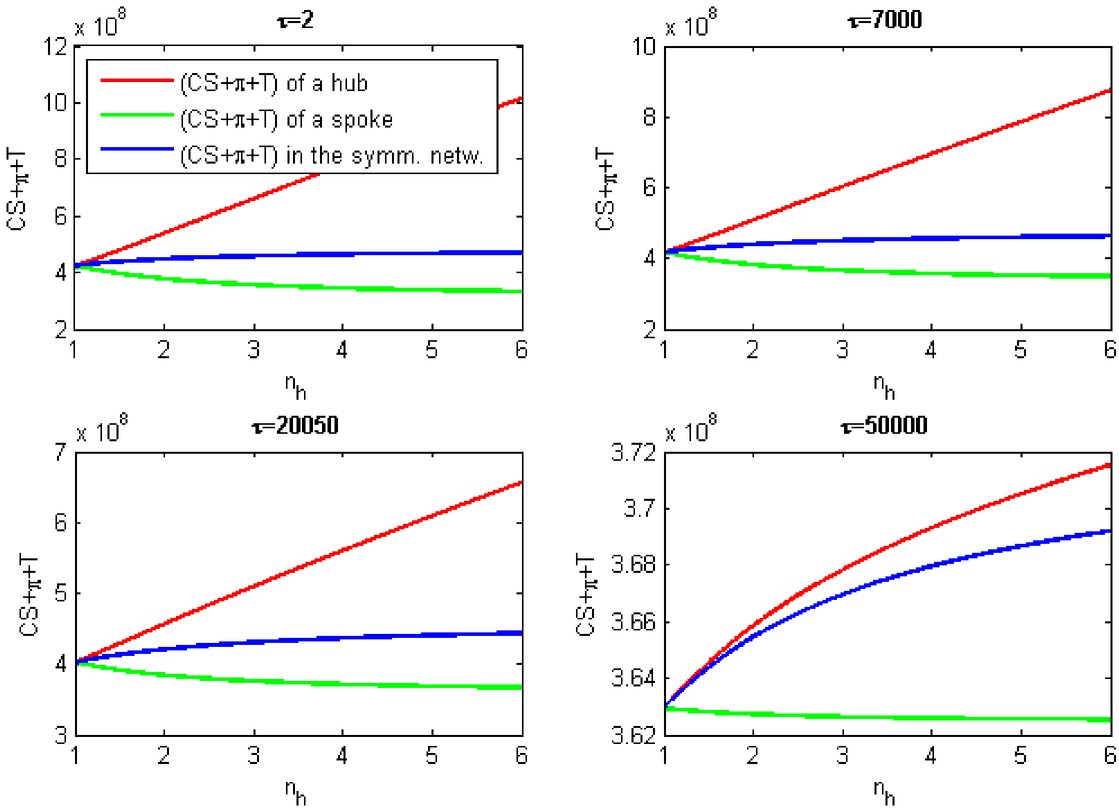

Figure 16: Social welfare with trade tariffs of a country in the star and in the symmetric network of degree n_h.

Supplementary Appendix for "R&D in Trade Networks: The Role of Asymmetry" (Not intended for publication)

January 15, 2016

Appendix A: Equilibrium R&D efforts in arbitrary network. The case of small local effects

Consider an arbitrary trade network where the magnitude of *local effects*, that is, the effects of strategic interaction between firms in a two-links-away neighbourhood of each other, is small in a well-defined sense. Below we show that even though a generic analytical solution for firms' equilibrium R&D decisions is not feasible in this case, an assumption of small local effects allows deriving the ranking of these R&D decisions in accordance with firms' simple network characteristics, such as the number of their direct and two-links-away trade partners. The results turn out to be broadly consistent with our findings in the analysis of hub-and-spoke trade networks. They indicate that among two otherwise identical firms the one with a smaller number of two-links-away trade partners and a larger number of direct trade partners (of sufficiently large market size) does more R&D. That is, two-links-away trade partners and the competition effect of trade associated with them undermine firm's incentives to innovate, while direct trade partners tend to encourage innovation, at least as long as their market size is not too small. To arrive at these conclusions, we employ the asymptotic approach proposed by Bloch and Quérou (2008).[1]

Recall that firms' equilibrium R&D decisions are a solution of the system of linear first-order conditions (10), which can be written as:

$$\begin{aligned}
\delta x_i \quad &- \quad \frac{1}{b}\left[\sum_{j\in N_i\cup\{i\}} \frac{(|N_j|+1)^2}{(|N_j|+2)^2}x_i - \sum_{j\in N_i}\left(\frac{|N_i|+1}{(|N_i|+2)^2}+\frac{|N_j|+1}{(|N_j|+2)^2}\right)x_j - \right.\\
&- \left.\sum_{j\in N_i}\sum_{k\in N_j,k\neq i}\frac{|N_j|+1}{(|N_j|+2)^2}x_k\right] = \\
&= \quad \frac{1}{b}(a-\gamma-2\tau)\sum_{j\in N_i}\frac{|N_j|+1}{(|N_j|+2)^2}+\frac{1}{b}(a-\gamma+|N_i|\tau)\frac{|N_i|+1}{(|N_i|+2)^2}, \qquad i\in 1:N.
\end{aligned}$$

In the matrix form this has a simple representation:

$$\left(\delta\mathbf{I}-\frac{1}{b}\mathbf{B}\right)\cdot\mathbf{x}=\frac{1}{b}\tilde{\mathbf{u}},$$

[1] As emphasized in Bloch and Quérou (2008), at least two arguments can defend the usefulness of studying network effects whose magnitude is small. First, given an arbitrary network structure, where the matrix of interactions is complex, this may be the only way to evaluate the equilibrium R&D decisions. Secondly, the insights obtained for small local effects by continuity also hold as the magnitude of externalities increases.

or

$$(\mathbf{I} - \lambda\mathbf{B}) \cdot \mathbf{x} = \lambda\tilde{\mathbf{u}}, \tag{1}$$

where $\lambda = \frac{1}{b\delta}$, $\mathbf{x}$ is a vector of firms' equilibrium R&D decisions, $\mathbf{I}$ is the identity matrix, and $\lambda\mathbf{B}$ is the matrix of local effects. The norm of matrix $\lambda\mathbf{B}$ captures the magnitude of local effects, and in what follows, we will focus on the solution of (1) under the assumption that this magnitude is small.

First, following Bloch and Quérou (2008), let us define a vector sequence $\mathbf{f} = (\mathbf{c}^1, \mathbf{c}^2, \ldots, \mathbf{c}^m, \ldots)$, where each vector $\mathbf{c}^m$ is given by:

$$\mathbf{c}^m = \lambda^m \tilde{\mathbf{u}}\mathbf{B}^{m-1},$$

so that the first terms of this sequence are $\mathbf{c}^1 = \lambda\tilde{\mathbf{u}}$, $\mathbf{c}^2 = \lambda^2\tilde{\mathbf{u}}\mathbf{B}$, and $\mathbf{c}^3 = \lambda^3\tilde{\mathbf{u}}\mathbf{B}^2$.

Now, using sequence $\mathbf{f}$, we will show that equilibrium R&D efforts of firms, though not observed explicitly, can be compared by means of comparing firms' network characteristics. This follows from Proposition 1 which states that under the assumption of small local effects, the ranking of firms' equilibrium R&D decisions (solution of (1)) is approximately equivalent to the lexicographic ordering of the components of $\mathbf{f}$. And the components of $\mathbf{f}$ can, in turn, be interpreted in terms of firms' network characteristics.

Proposition 1. *Consider a system of linear equations* (1)*. Suppose that $\|\lambda B\|$ is sufficiently small,* [2] *so that for a given $0 < \bar{\varepsilon} < 1$, $\|\lambda B\| \leq \frac{\bar{\varepsilon}}{N}$. Then system* (1) *has a unique solution $\mathbf{x}^*$, and there exists $K > 1$ such that for any $i, j \in 1 : n$, $i \neq j$,*

$$|x_i^* - x_j^* - (c_i^M - c_j^M)| \leq \lambda \cdot \frac{\bar{\varepsilon}^{K+1}}{1 - \bar{\varepsilon}} \cdot 2\|\tilde{\mathbf{u}}\|,$$

where $(c^M)_i$ and $(c^M)_j$ are the first unequal elements of the sequences $\mathbf{f}_i = (\mathbf{c}_i^1, \mathbf{c}_i^2, \ldots, \mathbf{c}_i^m, \ldots)$ and $\mathbf{f}_j = (\mathbf{c}_j^1, \mathbf{c}_j^2, \ldots, \mathbf{c}_j^m, \ldots)$, i. e., $\mathbf{c}_i^M \neq \mathbf{c}_j^M$ and $\mathbf{c}_i^m = \mathbf{c}_j^m$ for all $m < M$.

Proof. The proof is based on the proof of Lemma 2.3 and Lemma 7.1 in Bloch and Quérou (2008). Consider the system of linear equations (1). Since $\|\lambda\mathbf{B}\| \leq \frac{\bar{\varepsilon}}{N} < \frac{1}{N}$, Lemma 7.1 in Bloch and Quérou (2008) states that (1) possesses a unique solution and

$$\|\mathbf{x}^* - \lambda\tilde{\mathbf{u}} \cdot \sum_{k=0}^{K} \lambda^k \mathbf{B}^k\| \leq \frac{n^{K+1}\|\lambda\mathbf{B}\|^{K+1}\lambda\|\tilde{\mathbf{u}}\|}{1 - N\|\lambda\mathbf{B}\|} \leq \frac{\lambda\bar{\varepsilon}^{K+1}\|\tilde{\mathbf{u}}\|}{1 - \bar{\varepsilon}}.$$

Observe that $\mathbf{c}^m$ is defined so that

$$\lambda\tilde{\mathbf{u}} \sum_{k=0}^{K} \lambda^k \mathbf{B}^k = \sum_{m=1}^{K+1} \mathbf{c}^m.$$

So,

$$\|\mathbf{x}^* - \sum_{m=1}^{K+1} \mathbf{c}^m\| \leq \frac{\lambda\bar{\varepsilon}^{K+1}\|\tilde{\mathbf{u}}\|}{1 - \bar{\varepsilon}}.$$

By definition of the l_∞ vector norm, this means that $\forall i \in 1 : N$

$$|\mathbf{x}_i^* - \sum_{m=1}^{K+1} \mathbf{c}_i^m| \leq \frac{\lambda\bar{\varepsilon}^{K+1}\|\tilde{\mathbf{u}}\|}{1 - \bar{\varepsilon}}. \tag{2}$$

[2] As in Bloch and Quérou (2008), we use the l_∞ vector norm defined by $\|\mathbf{A}\| = \max\limits_{i,j}|a_{ij}|$.

Consider a pair (i, j) of players and let M be the first element of the sequences $\mathbf{f}_i$, $\mathbf{f}_j$ such that $\mathbf{c}_i^M \neq \mathbf{c}_j^M$. Applying (2) to i and j, we obtain:

$$|\mathbf{x}_i^* - \mathbf{x}_j^* - (\mathbf{c}_i^M - \mathbf{c}_j^M)| \leq 2 \cdot \frac{\lambda \bar{\varepsilon}^{K+1} \|\tilde{\mathbf{u}}\|}{1 - \bar{\varepsilon}}$$

This concludes the proof. □

Thus, if the upper bound for the magnitude of local effects is close to zero, then

$$x_i^* > x_j^* \text{ if and only if } \mathbf{f}_i \succ \mathbf{f}_j,$$

where $\mathbf{f}_i \succ \mathbf{f}_j$ means that $\mathbf{f}_i$ lexicographically dominates $\mathbf{f}_j$. That is, in order to compare the equilibrium R&D efforts of different firms, one can restrict attention to the first-order term of sequence $\mathbf{f}$, $\mathbf{c}^1$, or if the i-th and j-th elements of the first order term are equal, to the second-order term, $\mathbf{c}^2$, etc. Then the task of ranking firms' equilibrium R&D choices reduces to ranking firms' network characteristics.

Indeed, consider a pair of firms (i, i'), $i, i' \in 1 : N$, such that $\tilde{\mathbf{u}}_i \neq \tilde{\mathbf{u}}_{i'}$. Then by Proposition 1, if

$$\|\lambda B\| \leq \frac{\bar{\varepsilon}}{N}$$

for some $0 < \bar{\varepsilon} < 1$, then the difference between x_i^* and $x_{i'}^*$ can be approximated by the difference between $\tilde{\mathbf{u}}_i$ and $\tilde{\mathbf{u}}_{i'}$ such that the measurement error does not exceed $\lambda \cdot \frac{\bar{\varepsilon}^{K+1}}{1-\bar{\varepsilon}} \cdot 2\|\tilde{\mathbf{u}}\|$, where

$$\tilde{u}_i = (a - \gamma - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \gamma + |N_i|\tau) \frac{|N_i| + 1}{(|N_i| + 2)^2}$$

and

$$\|\tilde{\mathbf{u}}\| = \max_i (a - \gamma - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \gamma + |N_i|\tau) \frac{|N_i| + 1}{(|N_i| + 2)^2}.$$

This means that when local effects are small, the R&D effort of firm i is at least as high as the R&D effort of firm i' if and only if

$$\begin{aligned} (a - \gamma - 2\tau) \sum_{j \in N_i} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \gamma + |N_i|\tau) \frac{|N_i| + 1}{(|N_i| + 2)^2} \geq \\ \geq (a - \gamma - 2\tau) \sum_{j \in N_{i'}} \frac{|N_j| + 1}{(|N_j| + 2)^2} + (a - \gamma + |N_{i'}|\tau) \frac{|N_{i'}| + 1}{(|N_{i'}| + 2)^2}. \end{aligned} \tag{3}$$

This suggests two observations. First, if the number of direct trade partners of both firms is the same, i.e., $|N_i| = |N_{i'}|$, then inequality (3) holds, and thus $x_i^* \geq x_{i'}^*$, if and only if the number of two-links-away trade partners of firm i' is at least as large as the number of two-links-away trade partners of firm i. In this sense, the equilibrium R&D effort of a firm is increasing in the number of its two-links-away trade partners. Second, by the same logic, the equilibrium R&D effort of a firm is increasing in the number of its direct trade partners, at least as soon as the new trade partner j' does not have too many of its own direct trade partners. The latter is implied by the observation that all else being equal, a larger number of direct trade partners of i, $|N_i| = |N_{i'}| + 1$ turns inequality (3) into

$$\frac{|N_{i'}|+2}{(|N_{i'}|+3)^2}(a-\gamma+(|N_{i'}|+1)\tau) - \frac{|N_{i'}|+1}{(|N_{i'}|+2)^2}(a-\gamma+|N_{i'}|\tau) + (a-\gamma-2\tau)\frac{|N_{j'}|+1}{(|N_{j'}|+2)^2} \geq 0,$$

which holds when $|N_{j'}|$ is sufficiently small.

That is, an additional direct trade partner of i increases i's incentives to innovate as soon as the number of i's competitors in the market of this trade partner is sufficiently low. The two observations together imply that two-links-away trade partners of a firm reduce its incentives for innovation, while direct trade partners increase these incentives, at least when the actual market size of a new trade partner is large enough. Thus, consistent with our conclusions for the hub-and-spoke trade network, the competition effect of trade is negative but the positive scale effect, associated with access to a new market, often outweighs the negative competition effect.

Bloch F., and N. Quérou. 2008. "Pricing in networks." No. hal-00356356.