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Firm entry and exit, investment irreversibility, and business cycle dynamics

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Firm entry and exit, investment irreversibility, and business cycle dynamics*

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Abstract

This paper studies the role of firms' entry and exit for business cycle dynamics in an environment, where physical capital is partially sunk. Extending a heterogeneous-firm model à la Hopenhayn (1992) by aggregate productivity shocks and partially irreversible investment yields substantial endogenous amplification and propagation. A positive aggregate productivity shock increases the number of entrants and their initial investment levels, because the expected entry value outweighs the implicit sunk cost associated with investment irreversibility. The endogenous propagation of an exogenous stimulus arises via a built-in selection device, as the production growth of new businesses over their life-cycle exceeds the decay due to exits of the least productive firms.

JEL classification: D92, E22, E37, L11

Keywords: firm entry and exit, partial investment irreversibility, real business cycle

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1 Introduction

The Great Recession in the U.S. economy was accompanied by a low level of entry and excessive exits of firms. Using the Business Dynamics Statistics (2013a) data, I observe a 23% decrease in the number of firm births and a 14% increase in firm deaths over the period 2007 − 2009. This documented decline in the number of firms most likely exacerbated the severity of the recession and contributed to the subsequent slow recovery, as adjustment along firms' extensive margin is crucial for the overall productivity growth and net job creation[1]. However, these cyclical patterns are by no means specific to the most recent economic downturn. A number of studies have documented, across different countries and for various time periods, similar business cycle movements for various measures of firms' entry and exit[2].

Given the observed co-movements between the mass of businesses and aggregate economic activity, it is natural to investigate the extent to which firms' entry and exit help explain macroeconomic fluctuations. New businesses contribute to aggregate dynamics not only by their initial investment, but also by acquiring physical capital in later stages of their life-cycle. As it is difficult to detach and liquidate the stock of physical capital already in place, firms view a constant share of their investment as effectively sunk. Sunk costs dampen both the amount of entry and post-entry investment activity, and can lead to postponing the decision to exit by affecting the option value of staying in business. These interactions of firm entry and exit with partial investment irreversibility are essential for understanding the aggregate effects of the extensive margin adjustment by firms[3].

This paper aims to assess the role of firms' turnover for business cycle dynamics in an environment where investment in physical capital is partially irreversible. I qualitatively and quantitatively evaluate the contribution of firm turnover for the magnitude and persistence of the model's response to aggregate productivity shocks. Towards this end, I augment the heterogeneous-firms framework à la Hopenhayn (1992) by aggregate productivity shocks, endogenous entry and exit decisions, and partial irreversibility of a

[1]See Foster et al. (2001), Foster et al. (2008), and Haltiwanger et al. (2010), Sedláček and Sterk (2014).

[2]Portier (1995) reports pro-cyclicality of net business formation for French data, Sedláček and Sterk (2014) observe pro-cyclical dynamics of entrants' job creation in the U.S. private sector, Devereux et al. (1996) documents counter-cyclicality of a number of business failures in the U.S. economy, and Campbell (1998) present evidence on counter-cyclicality of plant exit rate in the U.S. manufacturing.

[3]The sunk nature of the physical capital is an empirically well-established fact, see e.g. the evidence presented by Ramey and Shapiro (2001).

firm's investment decision. Firms differ in their individual productivity levels, and decide whether to remain operational or exit, thereby recovering a fraction of their capital's value. The model is solved numerically and calibrated to match empirical moments of U.S. private sector data.

This paper contributes to the macro literature by offering a novel framework, distinguished by partial investment irreversibility, for assessing the role of firm entry and exit in business cycle dynamics. By focusing on the sunk nature of capital, I present the characteristic that affects the responses of business' turnover to productivity shocks together with the subsequent impact on the economic aggregates. The model calibration that emphasizes a realistic empirical fit of the firm-size distribution allows me to quantitatively capture the cross-firm differences in optimal investment and exit policies and the consequences thereof. This is arguably an important part of any quantitative analysis conducted using a heterogeneous-firm model.

The analysis reveals that accounting for endogenous entry and exit of firms in a business cycle model with investment irreversibility substantially increases the magnitude and propagation of the response of aggregate variables to exogenous disturbances. A positive aggregate productivity shock increases the number of entrants as well as their individual investment levels. This is due to the fact that higher productivity yields an expected entry value that outweighs the implicit sunk cost associated with investment irreversibility. As a result, exiting businesses are replaced not only by a greater number of new firms, but also by firms that invest more on average. Moreover, as the productivity growth of surviving firms exceeds the decay due to exits of the least productive firms, the output level of this new cohort increases in the subsequent periods. This built-in selection device enhances the endogenous propagation of economic activity.

The remainder of this paper is structured as follows. Section 2 contains a brief overview of the related literature. Section 3 presents the setup of the model economy and an analytical characterization of plants' steady-state optimal policies. Section 4 describes the calibration procedure and parameter values. Section 5 overviews the numerical solution of the model in settings both with and without the shocks to the level of aggregate productivity. Section 6 concludes.

2 A brief literature review

This work is connected to several strands of the literature. First and foremost, it is related to papers that analyze a role of firm entry and exit in the business cycle framework. Examples of relevant contributions are Lee and Mukoyama (2012), Sedláček (2014), and in particular Samaniego (2008), Clementi and Palazzo (2013) and Clementi et al. (2014).

Focusing on the transition dynamics of an economy after a permanent shock to aggregate productivity, Samaniego (2008) shows irresponsiveness of plant entry and exit, and subsequently no effect of turnover on aggregate dynamics. However, as Lee and Mukoyama (2012) point out, this result is driven by the assumed modeling of the entry process, in particular a form of entry costs, which are tightly linked to the wage rate. This specification generates pro-cyclical entry costs, which dampen response of entrants. Moreover, in this model there is no initial investment decision or scrap value of capital, which are key to my mechanism.

The papers by Clementi and Palazzo (2013) and Clementi et al. (2014) feature a model with a firm-level capital stock subject to a combination of convex and non-convex (fixed) adjustment costs, where the former paper feature a partial equilibrium analysis, while the latter addresses a dynamics in the general equilibrium. These works show that endogenous firm entry and exit propagate effects of productivity shocks on aggregate variables. However, these analyses don't incorporate investment irreversibility. Moreover, the entry process is modeled differently, as there is a fixed mass of potential entrants that invest after a realization of their individual productivities. In my model, entrants are ex-ante identical and they have to invest before drawing their initial productivity.

In a slightly different spirit, there are several papers that assess the business cycle implications of entry and exit in a framework with imperfect competition. The most recent contributions are Jaimovich and Floetotto (2008), Colciago and Etro (2010), Ottaviano (2011) and Bilbiie et al. (2012). Unlike my work, they focus on variations in markup as a possible transmission mechanism. Entry and exit cause a change in the number of firms, which has an effect on price markup because of supply- (tighter competition implies lower markup) or demand-side considerations (greater variety increases the elasticity of substitution). In my model, there is perfect competition, as I focus on the technology-side determinants of entry and exit. Moreover, with the exception of Ottaviano (2011), these papers feature homogeneous firms, so there is no endogenous exit decision, only the exogenously generated business failures.

The second related field of literature concerns papers that assess implications of non-convex capital adjustment costs for the aggregate dynamics in frameworks with firm heterogeneity. Examples are Veracierto (2002), Khan and Thomas (2008) and Bachmann et al. (2013). The paper by Veracierto is particularly closely related, as the author evaluates the implications of investment irreversibility for business cycle dynamics, reaching as a main result its quantitative irrelevance. However, his model differs importantly in assumptions on plant entry and exit. Both start-up and quit processes are purely exogenous. There is no role for initial investment or a scrap value of capital. Moreover, the exit hazard is the same for firms with different productivities.

3 The model

My model features an infinite time horizon with a discrete time index t. There are two types of agents present, namely households and firms. Households own plants, supply labor and consume produced goods. Establishments, on the other hand, own capital, hire labor from the households, and use these two factors to produce output. The output goods can be used either for consumption or investment, which are perfect substitutes. Plants are heterogeneous along two dimensions: the level of idiosyncratic productivity, which persistently evolves over time, and the stock of capital. The latter can be augmented by investment that is (partially) irreversible. Moreover, given their current state the production units consider whether to remain in the market or exit. Plants can be divided into two groups, namely entrants and incumbents. Entrants have to make their initial investment before becoming operational. Consequently they are relabeled as incumbents after their first period in the market.

At this point I present the assumed sequence of establishments' actions. At the beginning of each period, incumbent plants first observe shocks to both the aggregate and the idiosyncratic component of their productivity. Afterwards they hire labor and produce output while paying wages and the fixed cost. As a next step, establishments decide whether to remain in business or exit and receive the scrap value of their capital stock. Moreover, a fraction of establishments is forced to exit exogenously in order to generate failures among large production units consistent with the empirical evidence. Finally, surviving production units adjust their capital stock by investing subject to irreversibility constraint and adjustment cost.

The sequence of actions for entrants is analogous to that of incumbents with the exception that their initial investment is undertaken at the end of the previous time period. This is because when a plant decides to enter the market, it has to purchase its initial capital stock, which is done on the capital markets operating at the end of the period. After paying this setup cost, an entrant observes the aggregate state and receives its starting level of productivity. Subsequently it engages in hiring labor and producing output, and decides whether to exit or continue its operation. Finally, an entrant is relabeled as an incumbent from the second period of its existence.

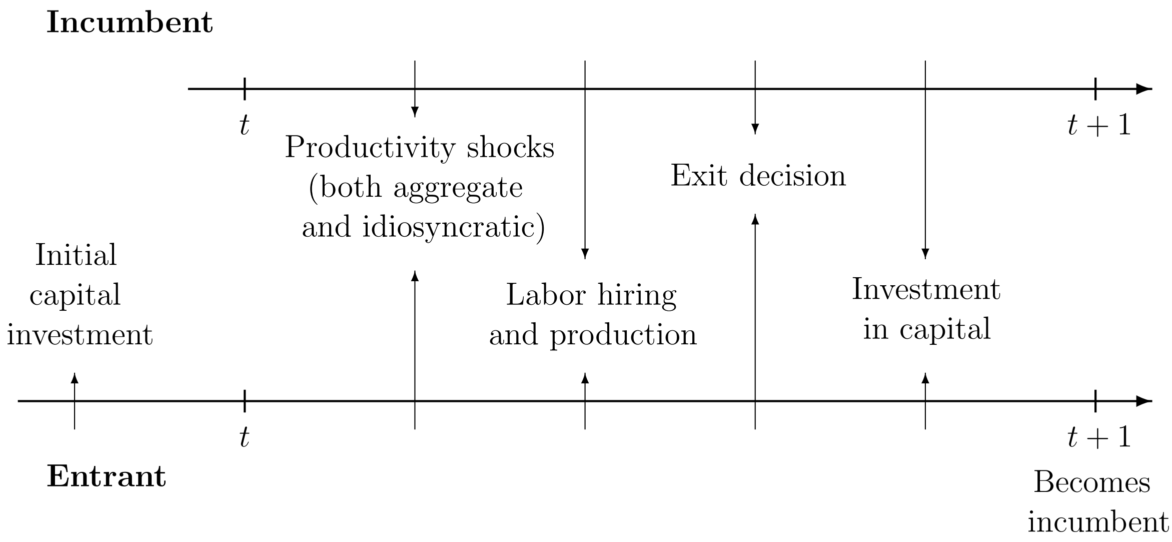


Figure 1: Timing of actions

The timing of events for continuing as well as for newly entering plants is graphically represented in Figure 1. There are several economic interpretations underlying these timing assumptions. Once they observe their current productivity, plants can adjust their capital stock only after they have produced output, as is standard in the business cycle models with physical capital. An exit decision is taken after the production process to reinforce the idea of investment sunkness: an establishment with an unfavorable draw of productivity has to engage in production one last time before exiting the market. In turn, potential entrants have to acquire their initial investment before their starting productivity is revealed. This feature, common to most dynamic models of industry, captures uncertainty connected with the entry of new plants.

3.1 The production side

There is a single final good produced, which can be consumed by households or used by plant as capital investment. Production by an establishment at time t is given by the production function

$$y_t = f(z_t, k_{t-1}, n_t) \equiv A_t z_t k_{t-1}^{\alpha} n_t^{\eta}, \tag{1}$$

where $\alpha, \eta \in (0,1); \alpha+\eta < 1$. I assume decreasing returns to scale in order to have production units choose finite optimal levels of production factors capital k_{t-1} (predetermined) and labor n_t. The terms A_t and z_t denote the aggregate and the idiosyncratic productivity of a plant respectively. The development of the logarithm of these productivities over time can be represented by the following AR(1) processes

$$\begin{aligned} \ln A_t &= \rho_A \ln A_{t-1} + \epsilon_t^A, \\ \ln z_t &= \rho_z \ln z_{t-1} + \epsilon_t^z, \end{aligned} \tag{2}$$

with $\rho_A, \rho_z \in (0,1)$, $\epsilon_t^A \sim \mathcal{N}(0, \sigma_A^2)$ and $\epsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$. Notice that the aggregate and idiosyncratic productivity processes are mutually independent. Let me denote by $f^A(A_{t+1}|A_t)$ and $f^z(z_{t+1}|z_t)$ the density functions of A_{t+1} and z_{t+1} conditional on A_t and z_t, respectively.

The first production factor is the capital stock, which is owned directly by plants. It can be adjusted by investment and is subject to depreciation over time. The corresponding law of motion is

$$k_t = (1 - \delta^K) k_{t-1} + i_t, \tag{3}$$

where $\delta^K \in (0,1)$ is the depreciation rate and i_t represents gross investment. I assume that establishments face a friction of partial irreversibility, in particular they are able to recover only a fraction κ of a remaining value of capital in the case of disinvestment. In addition, my model features also a quadratic cost of capital adjustment in order to smooth a plant's investment behavior over its life-cycle.

The assumption of frictions to capital reallocation implies that the capital stock k_t becomes a plant's individual state variable in period t in addition to the idiosyncratic productivity z_t. The distribution of establishments over the underlying state space $\mathbf{S} = \mathbb{R}_+ \times \mathbb{R}_+$ at the end of period t is characterized by a measure $\bar{\mu}_t(z_t, k_t)$ defined on the Borel algebra $\mathcal{S}$ on $\mathbf{S}$. However, for convenience I define also a measure $\mu_t(z_t, k_{t-1})$ representing the distribution of establishments after realization of the productivity shocks. This measure together with the aggregate productivity constitute the aggregate state of the economy $S_t := (A_t, \mu_t)$.

The second factor of production, labor, can be adjusted at the plant level without any costs. In each period, plants hire workers from the households in a frictionless labor market and pay them the wage rate w, which adjusts such that the labor market clears.

At the same time, there is an additional cost associated with the production process that a business has to pay each period: a fixed cost measured in output units and denoted by c^f. It captures the overhead expenditures of production faced by an operating plant. This cost is necessary to obtain endogenous exit decision of production units, as it forces the least productive establishments to quit rather than to cut their production level to zero.

After characterizing its technology, I can represent a plant's decision process as a dynamic programming problem. Following Khan and Thomas (2008), I denote the marginal utility of household's consumption by p_t and use it as a price used by establishments to value current output. As a result, a plant's decision problem is expressed in utility terms rather than in physical units.

Using this convention, I now describe the recursive problem of profit maximization of a production unit. First, the value function $V^0(z_t, k_{t-1}; S_t)$ of a plant with capital stock k_{t-1} and a productivity level z_t under the aggregate state S_t is given as follows:

$$V^0(z_t, k_{t-1}; S_t) = p(S_t)\, \tilde{\pi}(z_t, k_{t-1}; S_t) + V^{EX}(z_t, k_{t-1}; S_t). \tag{4}$$

Here $\tilde{\pi}$ denotes plant's contemporaneous profit net of the investment flows and any associated adjustment costs,

$$\tilde{\pi}(z_t, k_{t-1}; S_t) = \max_n \left\{ A_t z_t k_{t-1}^{\alpha} n^{\eta} - w(S_t) n - c^f \right\},$$

and V^{EX} depicts a plant's value at the moment when it considers whether to exit or remain operational. This decision is characterized by the following equation

$$V^{EX}(z_t, k_{t-1}; S_t) = \max \left\{ p(S_t)\, V^X,\ \delta\, p(S_t)\, V^X + (1-\delta) V^{INV}(z_t, k_{t-1}; S_t) \right\}, \tag{5}$$

where V^X represents the value of exit (characterized below) and δ is the probability of an exogenous exit. In equation (5), V^{INV} captures a plant's value prior to its investment decision,

$$V^{INV}(z_t, k_{t-1}; S_t) = \max_{k_t} \Big\{ -p(S_t)\, C^K(k_t, k_{t-1}) \ + \ \beta \mathbb{E}\big[V^0(z_{t+1}, k_t; S_{t+1})|z_t, S_t\big] \Big\}, \quad (6)$$

where $C^K(k_t, k_{t-1})$ is a function that summarizes capital adjustment. The household's discount factor $\beta \in (0, 1)$ is used by the establishments to value future cash flows. Note that the standard stochastic discount factor $\tilde{\beta} = \beta u_c(c_{t+1}, l_{t+1})/u_c(c_t, l_t)$ is implicitly present in the price $p(S_t)$, which is assumed to equal current marginal utility of consumption. Furthermore, the expectations operator in equation (6) relates to next period's realizations of both aggregate and idiosyncratic productivities, conditional on their current values. Let me conclude by denoting $k^*(z_t, k_{t-1}; S_t)$ the plant's optimal choice of the next period stock of physical capital k_t subject to its current individual state (z_t, k_{t-1}) and the aggregate state S_t.

3.2 A plant's decision margins

Next, I characterize the behavior of establishment. Specifically, I describe in detail the three decision margins that are present in the model: investment in the future capital stock, exit decision, and entry together with the initial capital investment.

3.2.1 Capital adjustment

As represented by equation (6), plants choose their future level of a capital stock in order to maximize their market value. There $C^K(k_t, k_{t-1})$ is a function of costs associated with the capital investment (or possibly disinvestment) $k_t - (1 - \delta^K)k_{t-1}$. This functional form incorporates both the adjustment costs C_A and the investment itself (including the irreversibility constraint) C_I, such that $C^K = C_A + C_I$. The underlying functions have the following forms:

$$C_A(k_t, k_{t-1}) = \gamma \frac{(k_t - (1 - \delta^K)k_{t-1})^2}{k_{t-1}},$$

$$C_I(k_t, k_{t-1}) = \begin{cases} \kappa(k_t - (1 - \delta^K)k_{t-1}), & \text{if } (k_t - (1 - \delta^K)k_{t-1}) < 0, \\ (k_t - (1 - \delta^K)k_{t-1}), & \text{otherwise.} \end{cases}$$

Note that a parameter value $\kappa = 1$ implies full reversibility, whereas a zero value of κ indicates complete irreversibility.

3.2.2 Exit

In each period, after it observes current realizations of productivity and produces output, each plant considers whether to remain in the market or to exit by comparing the expected value from remaining active with the outside option from exiting, V^X. As I treat the scrap value of capital as part of the irreversibility friction, the value of exit depends on the current stock of plant's capital $(1-\delta^K)k_{t-1}$. This dependence can be represented by a function $V^X(k_{t-1})$ with the following properties:

$$V^X(0) = 0, \quad \forall k_1 < k_2: \; V^X(k_1) \leq V^X(k_2), \quad \text{and} \quad V^X(k) \leq k.$$

The economic interpretation is straightforward. A production unit without capital cannot recover any value after exiting, a rise in the capital stock does not decrease scrap value, which cannot be higher than the actual value of capital itself. In the numerical solution of my model, I assume that an exiting plant receives a fraction κ of its current capital stock net of depreciation, i.e., $V^X(k) = \kappa(1-\delta^K)k$.

The notion of the exit value allows to rewrite the value function V^{EX} in (5) as follows

$$V^{EX}(z_t, k_{t-1}; S_t) = \max\Big\{p(S_t)V^X(k_{t-1}), \delta V^X(k_{t-1}) + (1-\delta)V^{INV}(z_t, k_{t-1}; S_t)\Big\}. \tag{7}$$

Note that in addition to plants exiting endogenously, a fraction δ of surviving units is forced to exit irrespective of their current state. This modeling feature is introduced to generate exit also among large establishments, which is consistent with the empirical evidence.

An establishment with individual state (z_t, k_{t-1}) exits whenever the expected future profits, including the optimal level of investment in capital, are lower than the recoverable scrap value:

$$\max_{k_t}\Big\{-p(S_t)C^K(k_t, k_{t-1}) \; + \; \beta\mathbb{E}\Big[V(z_{t+1}, k_t; S_{t+1}|z_t, S_t)\Big]\Big\} < p(S_t)V^X(k_{t-1}). \tag{8}$$

Following this and using expression (6), I characterize the continuation decision of a plant by an indicator function $\chi(z_t, k_{t-1}; A_t, \mu_t)$ as follows:

$$\chi(z_t, k_{t-1}; A_t, \mu_t) = \begin{cases} 1 & \text{if } V^{INV}(z_t, k_{t-1}; S_t) \geq V^X(k_{t-1}), \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

3.2.3 Entry

In each period there is an unbounded mass of potential entrants. Those who decide to enter in period t have first to acquire the initial capital stock k^E_{t-1} at the end of period t. Consequently the entering production units observe the aggregate productivity shock and draw their starting productivity z^E_t from a common distribution $g(z)$ independent of their initial investment. I assume that $g(z)$ is log-normal, i.e. $\log z^E_t \sim \mathcal{N}(\bar{z}^E, \sigma^2_E)$. With their productivity revealed, they next hire labor and produce just like the incumbent plants. Finally, they consider whether to remain operational and pursue business activity or exit from the market.

The level of initial investment is selected such that the entrants maximize their expected value of future profits conditional on the current aggregate conditions:

$$k^E_{t-1}(S_{t-1}) = \arg\max_k \left\{ \beta\, \mathbb{E}\left[V^0(z_t, k; S_t | S_{t-1}) \right] - p(S_{t-1})\, k \right\}. \tag{10}$$

Since all entrants draw their starting productivity level from the same distribution and face equal aggregate conditions, they acquire identical levels of capital k^E_{t-1}[4].

At the same time, I assume the free-entry condition to hold in equilibrium. This means that a mass of entrants M_{t-1} adjusts such that the expected value of entry is zero:

$$\beta\, \mathbb{E}\left[V^0(z_t, k^E_{t-1}(S_{t-1}); S_t) | S_{t-1} \right] = p(S_{t-1})\, k^E_{t-1}(S_{t-1}) \tag{11}$$

This is ensured by the general equilibrium effect of wages. A positive value of entry, i.e. the expected future cash flows exceeding the entry investment, motivates more establishments to enter. This increases the (expected) demand for labor (in the next period) and through the labor market clearing causes upward pressure on the wage rate. Consequently, profits (and thus also the expected value of entry) fall. This mechanism is at

[4]Note that a level of capital selected by the entering plants is always finite due to the decreasing returns to scale technology.

work until the expected value of entry is zero.

My modeling feature of initial investment choice is similar to the approach by Poschke (2010), where the entrants also make their investment decision prior to the productivity draw in the stationary environment. However, in his paper entrants choose a level of expected lifetime productivity, which cannot be changed later in the plant's life cycle, whereas in my model the individual capital stock can be adjusted in every period. Moreover, Poschke (2010) assumes that this initial choice affects the distribution of initial productivity draws, while I assume mutual independence between them.

3.3 Cross-sectional distribution

Finally I present an expression for a distribution of entrants E_t and a transition law for the distribution of all establishments $\mu_{t+1} = \Gamma(A_{t+1}, \mu_t)$. Recall that the entering plants first acquire start-up capital and subsequently receive their initial productivity. This leads to the following identity:

$$\forall (Z_t, K_{t-1}) \in \mathcal{S}: \; E_t(Z_t, K_{t-1}; S_{t-1}) = M_{t-1}(S_{t-1}) \int_{Z_t} \mathbb{1}_{\{k^E_{t-1}(S_{t-1}) \in K_{t-1}\}} dg(z), \qquad (12)$$

where $\mathbb{1}$ represents the indicator function.

Regarding the incumbent plants, they consider whether to exit before the investment decisions and realization of shocks. Recall that $k^*(z_t, k_{t-1})$ denotes the optimal choice of capital for a production unit in the state (z_t, k_{t-1}). The distribution of establishments μ_t evolve according to the following law of motion:

$$\begin{aligned} \forall (Z_{t+1}, K_t) \in \mathcal{S} : \mu_{t+1}(Z_{t+1}, K_t) \; &= \; E_{t+1}(Z_{t+1}, K_t; S_t) + \\ + (1-\delta) \int_{\mathbb{R}^+} \int_{Z_{t+1}} \int_{\mathbf{S}} \mathbb{1}_{\{z_t > \tilde{z}_t(k_{t-1})\}} \, &\mathbb{1}_{\{k^*(z_t, k_{t-1}) \in K_t\}} \, d\mu_t(z_t, k_{t-1}) \, f^Z(z_{t+1}|z_t) \, f^A(A_{t+1}|A_t). \end{aligned} \qquad (13)$$

3.4 The household side

There is a continuum of infinitely-lived identical households of unit measure that consume output, supply labor, and own the production units, whose profits they claim. The households' objective is choose consumption c and labor n^h to maximize the expected life-time stream of utility

$$\max_{\{c_t, n_t^h\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^h) \tag{14}$$

subject to the budget constraint

$$\forall t: \; c_t = w_t n_t^h + \Pi_t,$$

where $\beta \in (0,1)$ is the time-invariant discount factor and Π_t stands for plants' profits, which can be specified explicitly in the following way:

$$\begin{aligned}
\Pi_t = & \int_{\mathbf{S}} \tilde{\pi}(z_t, k_{t-1}; S_t)\, d\mu(z_t, k_{t-1}) - M(S_t)\, k^E(S_t) - \\
& - (1-\delta) \int_{\mathbf{S}} C^K \Big(k^*(z_t, k_{t-1}; S_t), k \Big) \chi(z_t, k_{t-1}; S_t)\, d\mu(z_t, k_{t-1}) + \\
& + \int_{\mathbf{S}} \kappa\, (1-\delta^K)\, k_{t-1} \Big(1 - (1-\delta)\chi(z_t, k_{t-1}; S_t) \Big)\, d\mu(z_t, k_{t-1}).
\end{aligned}$$

The three lines capture respectively the cash flows from production and entry investment, those associated with investment carried out by the surviving plants, and those from a capital scrapping of the exiting establishments.

The optimization procedure yields the following first-order conditions

$$\begin{aligned}
c_t: \; & u_c(c_t, n_t^h) = \lambda_t, \\
l_t: \; & u_n(c_t, n_t^h) = -\lambda_t w_t, ,
\end{aligned} \tag{15}$$

where λ_t is a Lagrange multiplier associated with the budget constraint in time t. These two conditions together with the assumption $u_c(c_t, n_t^h) = p_t$ yield

$$w_t(A_t, \mu_t) = -\frac{u_n(c_t, n_t^h)}{u_c(c_t, n_t^h)} = -\frac{u_n(c_t, n_t^h)}{p_t(A_t, \mu_t)}.$$

Finally, I assume the utility function to be of the form $u(c_t, n_t^h) = \log c_t - \theta n_t^h$, which results from the assumption of indivisible labor following Hansen (1985) and Rogerson (1988). This allows me to rewrite conditions (15) as follows:

$$p_t = \frac{1}{c_t}, \qquad w_t = \frac{\theta}{p_t}. \tag{16}$$

3.5 Recursive equilibrium

A recursive competitive equilibrium of my model is defined as a set consisting of functions

$$(p, w, V^0, k^*, N^D, \chi, C, N^H, k^E, M, \Gamma)$$

that satisfy the following conditions:

- Plants maximize their profits: Taking $p(A, \mu)$, $w(A, \mu)$ and $\Gamma(A, \mu)$ as given, plants with individual state (z, k) follow optimal policies in investment $k^*(z, k; A, \mu)$, labor demand $N^D(z, k; A, \mu)$ and exit $\chi(z, k; A, \mu)$ such that $V^0(z, k; A, \mu)$ solves (4).
- Households maximize their utility: Taking $p(A, \mu)$ and $w(A, \mu)$ as given, households choose optimal consumption $C(A, \mu)$ and labor supply $N^H(A, \mu)$ to satisfy (16).
- Entry is optimal: Taking $p(A, \mu)$, $w(A, \mu)$ and $\Gamma(A, \mu)$ as given, entrants choose initial investment $k^E(A, \mu)$ to satisfy (10) and their mass adjusts $M(A, \mu)$ such that (11) holds.
- The goods market clears: Price $p(A, \mu)$ adjusts such that household's consumption $C(A, \mu)$ is equal to the aggregate output minus the fixed cost and the entry investment (first row of (17)), minus the cost of investment of surviving incumbents (second row), and plus the disinvestment of exiting production units (third row):

$$\begin{aligned} C(A, \mu) = & \int_{\mathbf{S}} \left\{ f\Big(z, k, N^D(z, k; A, \mu); A, \mu\Big) - c^f \right\} d\mu(z, k) - M(A, \mu)\, k^E(A, \mu) - \\ & - (1-\delta) \int_{\mathbf{S}} C^K\Big(k^*(z, k; A, \mu), k\Big) \chi(z, k; A, \mu)\, d\mu(z, k) + \\ & + \int_{\mathbf{S}} \kappa\, (1-\delta^K)\, k\, \Big(1 - (1-\delta)\omega(z, k; A, \mu)\Big)\, d\mu(z, k). \end{aligned} \tag{17}$$

- The labor market clears: $w(A, \mu)$ adjusts such that the aggregate labor demand equals the labor supply:

$$\int_{\mathbb{S}} N^D(z, k; A, \mu) d\mu(z, k) = N^H(A, \mu), \tag{18}$$

- Dynamics are consistent: The transition law $\Gamma(A, \mu)$ for the distribution μ is compatible with the mass of entrants $M(A,\mu)$ and the optimal policies $K(z,k;A,\mu)$, $\omega(z,k;A,\mu)$ and $k^E(A,\mu)$ following (13).

3.6 Characterization of optimal exit policy

To conclude the model's outline, I characterize a plant's optimal policies with respect to its individual state (z,k) in the steady-state economy. Studying the stationary equilibrium is a simplification with respect to the full model, which incorporates the dynamics arising in the presence of shocks to aggregate productivity. Yet, I argue that it is instructive to first inspect the characteristics of the stationary environment, as many results can be derived analytically unlike in the framework with aggregate dynamics. Moreover, the firms' optimal exit policies in the steady state can provide useful insights into the economic mechanisms that are at work in the dynamic model. To keep the exposition smooth, all the formal statements and proofs, including the proof of existence and uniqueness of the value function, are presented in Appendix D.

I take equations (4) - (6) as a starting point and rewrite them as in the stationary equilibrium. This implies an independence of a plant's decision problem with respect to the aggregate state S_t. At the same time, I'm able to normalize the final goods price p_t to one, since it does not change over time and thus does not affect a plant's optimal policies. As a result, the Bellman equation of a plant with a capital stock k and a productivity level z has the following form:

$$V^0(z,k) = \tilde{\pi}(z,k) + \max\Big\{V^X(k), \delta V^X(k) + (1-\delta)\max_{k'}\Big\{-C^K(k',k) + \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\}\Big\}, \tag{19}$$

where $V^X(k)$ represents the exit value $V^X(k) = \kappa(1-\delta^k)k$, $\tilde{\pi}(z,k)$ denotes a plant's contemporaneous profit net of the capital adjustment,

$$\tilde{\pi}(z,k) = \max_n \Big\{zk^\alpha n^\eta - wn - c^f\Big\}, \tag{20}$$

and C^K characterizes any costs related to the adjustment of a plant's stock of physical capital

$$C^K(k',k) = \begin{cases} \gamma\frac{(k'-(1-\delta^K)k)^2}{k} + \kappa(k'-(1-\delta^K)k), & \text{if } (k'-(1-\delta^K)k) < 0, \\ \gamma\frac{(k'-(1-\delta^K)k)^2}{k} + (k'-(1-\delta^K)k), & \text{otherwise.} \end{cases} \tag{21}$$

As a main analytical result, it can be shown that for each level of a plant's capital stock k there exists a unique productivity threshold $\tilde{z}(k) > 0$, which determines whether the establishment exits or remains operational. Specifically, if a plant with a stock of physical capital k draws a productivity level z lower than $\tilde{z}(k)$, then its optimal policy is to quit production. On the other hand, a high-enough productivity shock increases the value of an establishment above the scrap value of its capital, thus making exiting suboptimal. This pattern is consistent with the empirical evidence showing that less productive plants are more likely to exit (see a survey in Bartelsman and Doms (2000)).

4 Calibration

Prior to presenting the model's numerical solution, I have to decide on the parameter values, which are calibrated in correspondence with the selected statistics of the annual data for the U.S. private business sector. To make the exposition smooth, I present a detailed description of data sources and construction of the targeted statistics in Appendix A.

The parameters in my model can be divided into two groups. The first group contains those that can be calibrated based on the literature or direct empirical evidence. The output elasticity with respect to labor η is set to match the labor share of national income of 0.60, and the discount factor β is set in a way that the real annual interest rate equals 4%. Consequently, the depreciation rate δ_K is set to match the annual value 6.02%, which is calculated as the average fraction of depreciated capital based on the Bureau of Economic Analysis (2013) dataset (BEA). At the same time, the fraction of production units exiting for exogenous reasons, δ, is pinned down at the value 0.7% by the exit rate of the largest establishments as calculated based on the Business Dynamics Statistics (2013a) dataset (BDS). Finally, the parameter values governing the stochastic process for the aggregate TFP (persistence ρ_A and volatility σ_A) are estimated from the Solow residual using the Bureau of Labor Statistics (2014) dataset (BLS) for the employment level, and the BEA data for the capital stock and output.

The second group of parameters is calibrated such that the selected moments implied by the model economy are consistent with corresponding data statistics. It is important to bear in mind that these parameters have interacting effects on the targeted statistics, therefore I have to calibrate them jointly. First, I target the household labor disutility parameter θ to imply a labor force participation rate equal to 63.9% taken from the BLS dataset. Next, the value of the output elasticity with respect to capital α targets the capital-to-output ratio at value 1.76 following the data provided by BEA. For the parameter $\bar{z}^E$ governing the mean of entrants' productivity distribution, it is reasonable to target a ratio of the average size of entering plants to the average size of exits at a value 1.07 obtained from the BDS dataset. Consequently, the standard deviation of entrants' productivity distribution σ_E matches the average 19.75% of entering units that exit during the first year of their existence (BDS). At the same time, a fixed cost of production c^f serves as normalization, therefore I use it to pin down the steady state mass of firms at unit value.

As a final step, I comment on the calibration of parameters that characterize the stochastic process of the idiosyncratic TFP (persistence ρ_z and volatility σ_z), and frictions to capital reallocation (the capital quadratic adjustment cost function parameter γ and a degree of irreversibility κ). The literature typically targets selected moments of the investment rate distribution[5] documented by Cooper and Haltiwanger (2006) (see e.g. Khan and Thomas (2008) or Khan and Thomas (2013)). These statistics characterize behavior of a sample of large manufacturing plants that survived throughout the sample period (17 years). Subsequently, a comparable sample of production units from the model economy needs to be generated in order to target data moments consistently. In a model with entry and exit, this arguably excludes smaller establishments from the identification process, as they are more prone to exit (and also entry), and puts too much of an emphasis on the large plants, which are typically less subject to turnover risk. However, my focus is on the interaction between the equilibrium distribution of establishments and the optimal entry and exit policies, which are most pronounced in the lower part of the plant size distribution. Therefore, the aforementioned approach is not suitable for the calibration of my model given the pursued research question.

In order to address the above concerns, I use parameters ρ_z and σ_z to directly discipline the establishment size distribution observed in the stationary equilibrium of my model in accordance with the empirical evidence. More precisely, I split the model-implied size distribution into nine disjunct bins separated by the percentiles corresponding to the size categories presented in the data, and compare their respective employment shares.

[5] E.g. a cross-sectional mean and standard deviation of investment rates, their average serial correlation together with fractions of plants undertaking positive, negative, or negligible investment.

Details on the data, which are publicly available from the Business Dynamics Statistics (2013b) dataset, are presented in Appendix A.3. As the number of size categories exceeds that of free parameters, my calibration minimizes a weighted sum of squared differences between the empirical and model employment shares[6]. Next, recall that the convex costs of capital adjustment have been introduced to smoothen firm's growth over its life-cycle. Therefore it is appropriate to use a ratio of average firm sizes of the second year cohort to the first year cohort 1.2 in order to pin down the parameter value for γ . Last but not least, in order to discipline the magnitude of plant entry and exit, I use the investment irreversibility parameter κ to align the model-implied turnover rate defined as the sum of entry and exit rates with its empirical counterpart 0.215.

Table 1: Parameter values of the benchmark calibration

η	β	δ_K	δ	ρ_A	σ_A			
0.60	0.96	0.0602	0.007	0.889	0.0145			
θ	α	$\bar{z}^E$	σ_E	c^f	ρ_z	σ_z	γ	κ
1.373	0.205	0	0.115	0.210	0.92	0.175	0.09	0.95

The baseline calibration of parameters is presented in Table 1. First, note that the production function exhibits returns-to-scale at value 0.805, well within the range of empirical estimates. Next, the model-implied degree of reversibility has a value 0.95, which is a reasonable one with respect to numbers considered by the literature[7]. Finally, my calibration of the transition law of idiosyncratic productivity implies a process that is more persistent and volatile compared to the related contributions[8]. This is a consequence of my calibration strategy, namely of an emphasis on the establishment size distribution.

Table 2 contains a comparison of the values of targeted variables generated by the model with their empirical counterparts. Subsequently, Table 3 provides an overview of the fit of establishment size distribution. It can be observed that the model-implied distribution tracks it empirical counterpart very closely especially at the bottom part,

[6]The weights used are the normalized inverted standard deviations of time series observed for respective size class percentiles.

[7]For a loss in a value of displaced physical capital relative to its acquisition price and net of depreciation (i.e. equivalent to $1-\kappa$ in my model), Ramey and Shapiro (2001) observe on average $40+\%$ for closing plants in the aerospace industry, Bloom (2007) estimates a value of 33.9%, and Khan and Thomas (2013) obtain 4.6% via calibration. As Khan and Thomas (2013) note, this parameter value is sensitive to the model specification, and therefore a proper measure of whether it is reasonable should be a model-implied behavior of individual production units rather than a magnitude of the raw number.

[8]E.g. Khan and Thomas (2008) use $\rho_z = 0.859$ and $\sigma_z = 0.022$, and Clementi et al. (2014) feature $\rho_z = 0.653$ and $\sigma_z = 0.138$, while Clementi and Palazzo (2013) have $\rho_z = 0.55$ and $\sigma_z = 0.22$.

Table 2: Comparison of targeted empirical statistics with the model

Name of statistic	U.S.data	the model
(i) Labor force participation rate	0.639	0.639
(ii) Capital to output ratio	1.76	1.77
(iii) Ratio of entrants-to-exits size	1.06	0.80
(iv) Fraction of exiting entrants	0.198	0.198
(v) Ratio of 2^{nd} year to 1^{st} year firm size	1.214	1.269
(vi) Turnover rate	0.215	0.217

Notes: All statistics are calculated as annual averages over the underlying samples. Sources: (i) own calculations using the BLS dataset, time period 1960-2013; (ii) own calculations using the BEA dataset, time period 1960-2013; (iii) - (vi) own calculations using the BDS dataset, time period 1979-2013. Detailed description available in Appendix A.

whereas a fit of population shares at the top percentiles is somewhat worse. As I discuss in Appendix A.3, this is due to the assumption of normality for the underlying shock process. Nevertheless, I argue that this does not bias my analysis in a quantitatively significant way, because the economic mechanisms of interest, namely interactions between production units' entry and exit decisions with an irreversible nature of physical capital, mainly concern the smallest production units.

Table 3: Comparison of empirical establishment size distribution with the model

Size class	Employment shares	Population shares		Size class	Employment shares	Population shares	
		Data	Model			Data	Model
1−4	0.0663	0.4928	0.4840	100−249	0.1523	0.0170	0.0225
5−9	0.0883	0.2236	0.2166	250−499	0.0890	0.0044	0.0069
10−19	0.1123	0.1387	0.1379	500 − 999	0.0687	0.0017	0.0034
20−49	0.1647	0.0903	0.0915	1000+	0.1317	0.0010	0.0025
50−99	0.1264	0.0305	0.0346				

Notes: Data statistics are calculated as the annual averages over the underlying sample from the BDS dataset, time period 1979-2013. Detailed description available in Appendix A.3. Model-implied plant population shares are calculated for the corresponding cumulative employment shares, and sizes of the establishments do not match exactly to the respective empirical size classes.

5 Numerical results

5.1 Stationary equilibrium

I start the numerical analysis by characterizing the solution of the model's stationary equilibrium. The numerical algorithm that solves the steady-state of the model economy is described in detail in Appendix B.1. Even though this paper addresses the aggregate dynamics, it is instructive to first inspect the characteristics of the stationary environment. In particular, the steady-state distribution of firms and their optimal entry, exit and investment policies can provide useful insights into the economic mechanisms that are at work in the dynamic model.

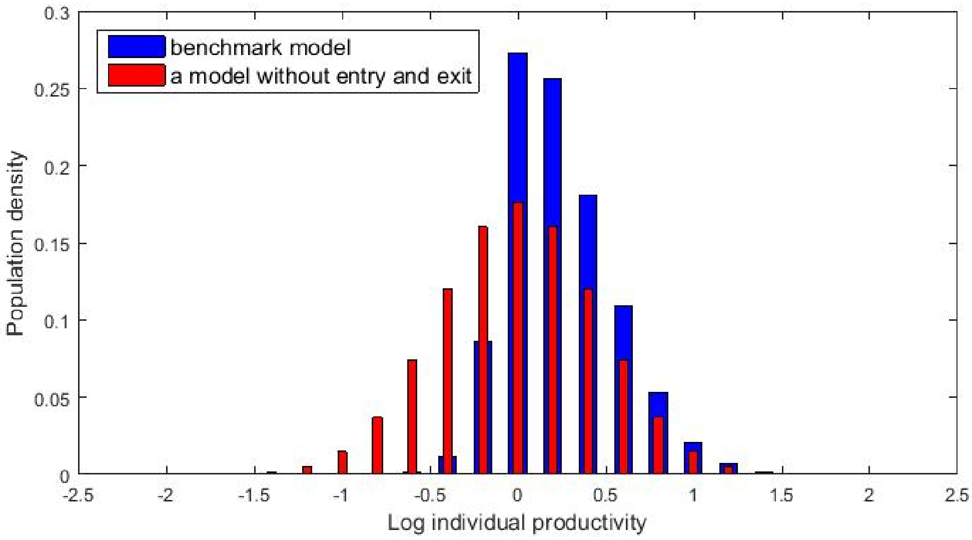


Figure 2: Effect of entry and exit on productivity distribution

Notes: The figure captures the steady-state distribution of plants over their individual productivity levels in the benchmark model economy (blue) and in the framework without the establishment entry and exit (red).

First, I present the distribution of establishments across their individual productivity levels and confront it against its equivalent in the steady-state of a model without firm entry and exit[9]. The comparison is presented in Figure 2. It is apparent that the benchmark model features businesses with productivity levels that are higher on average and less dispersed relative to the framework without plant turnover. This difference is caused by two channels. The first one is a standard selection mechanism, where the least productive establishments choose to exit, thus allowing for a reallocation of resources toward the more productive plants. The second channel operates through the entry of new production units, whose productivity is less dispersed in comparison to that of the exiting establishments and thus contribute to the observed discrepancy. All in all, these results indicate that introducing the extensive margin of reallocation into the framework

[9]Details on a model without firm turnover together with a corresponding numerical solution are presented in Appendix C.

with firm heterogeneity shifts the mass of establishments towards the higher productivity levels relative to the case without entry and exit.

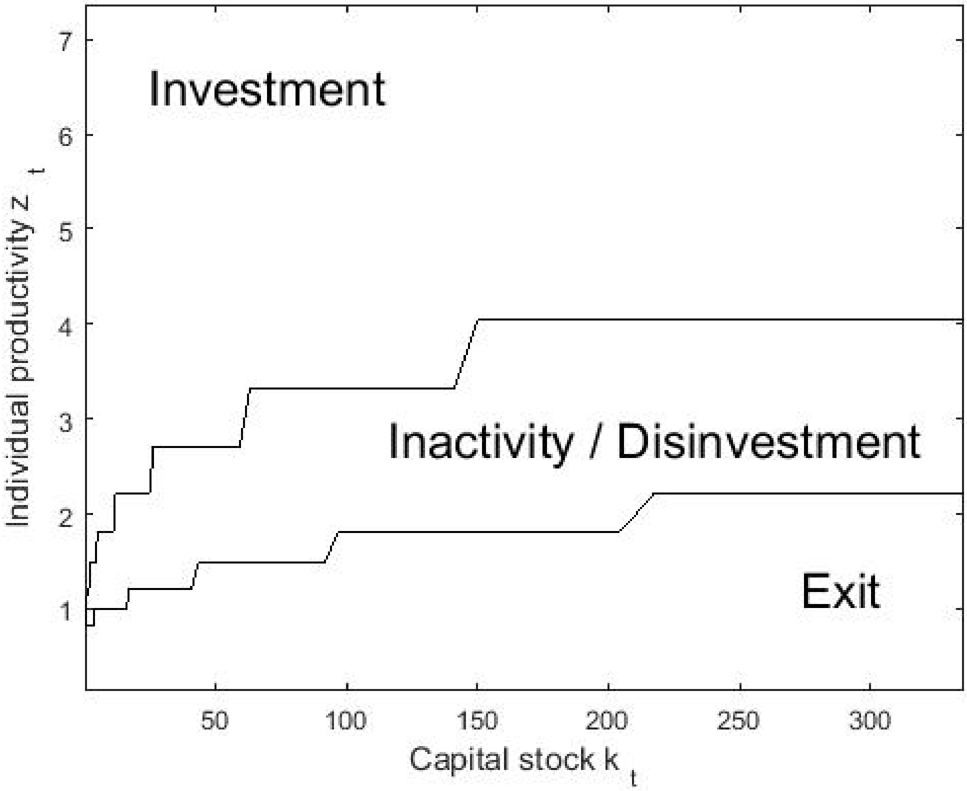


Figure 3: Plant optimal policy over the state space

Notes: The figure represents plants' optimal investment and exit policies over the space of their current individual states in the stationary equilibrium.

Secondly, I illustrate plants' optimal decision rules with respect to investment and exit. This exercise complements a theoretical analysis of the state space characterization of the optimal policies as presented in Section 3.6. In Figure 3 I display the individual state space split into the disjunct regions, which differ from each other by the policy that is optimal for given states. More specifically, a firm can choose either to exit or to remain operational. In the latter case, three different adjustments of firm's physical capital stock can take place, namely an additional purchase (investment), a sale (disinvestment), or inactivity associated with capital depreciation. The patterns observed in the figure are intuitive. In particular, least productive establishments leave the market for any level of physical capital, whereas those with higher levels of productivity remain in operation and invest. Moreover, the optimal policy changes from the disinvestment through the inactivity up to the investment as productivity increases, conditional on the stock of capital.

What is a little counterintuitive is the fact that the exit hazard is not monotonic in the level of physical capital conditional on productivity. More precisely, there is a range of productivities where firms with the highest capital stock decide to exit, whereas the ones with somewhat lower size remain active[10]. This feature is due to the presence of capital adjustment frictions, as the firms with an excessive stock of capital and not sufficient productivity find it optimal to exit rather than to downsize and incur convex costs.

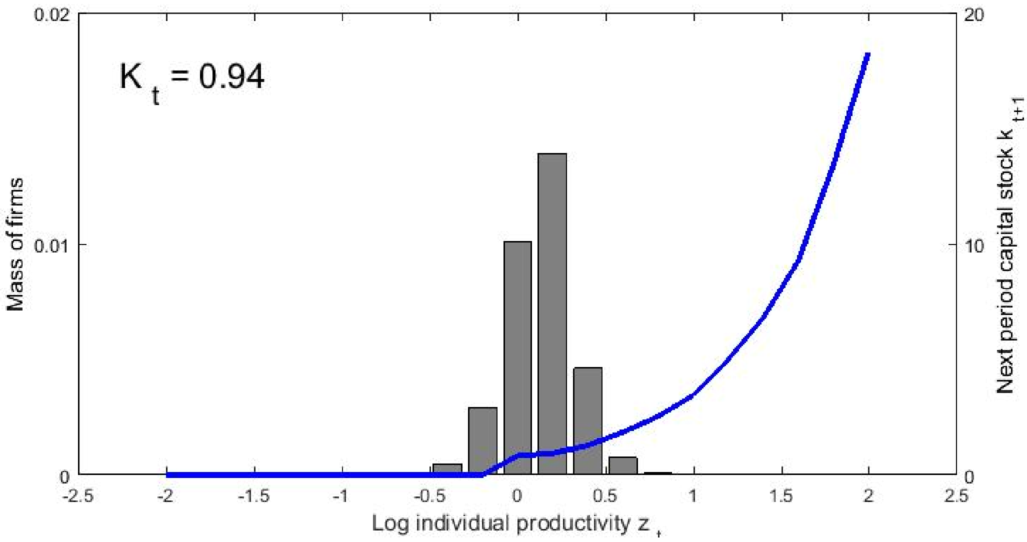


Figure 4: Distribution and optimal policies of firms with a median stock of capital

Notes: The figure represents the distribution of plants over their individual productivity levels (gray bars) and the establishments' optimal exit and investment policies (blue line) in the stationary equilibrium at the median level of current physical capital stock.

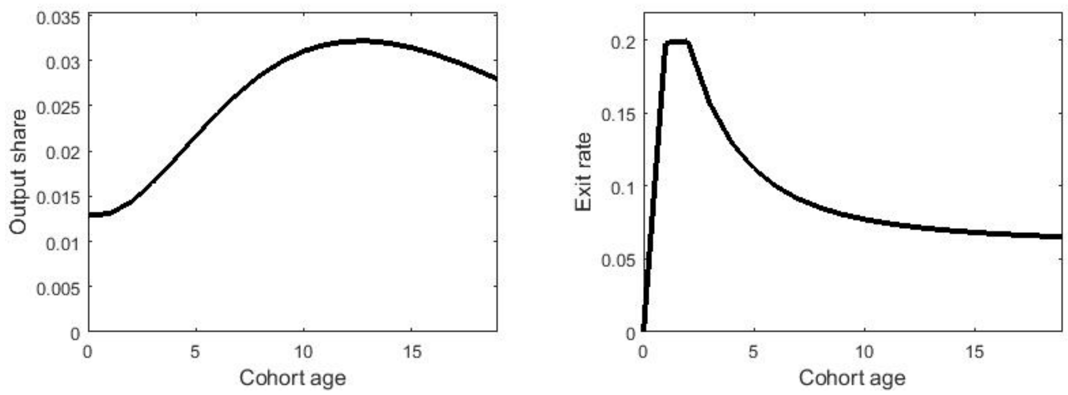
Next, I focus more directly on the intuition for a quantitative significance of the economic mechanisms that are my primary interest. As discussed in the introduction, what lies at the heart of my business cycle analysis of firms' entry and exit is the concentration of business units (both entrants and incumbents) along the exit and investment decisions. This is important as an occasional synchronization of actions taken at a firm's level can lead to large changes in the aggregate outcomes. Figure 4 displays the equilibrium mass of firms and optimal policies for firms that currently possess the median stock of physical capital. It can be seen that the lion's share of the businesses is concentrated at productivity levels just about the exit threshold. This indicates that in the case of a rightward shift of the optimal policy curve, as happens under a negative shock to the common productiv-

[10]This observation is present also in alternative models with firm entry, exit and heterogeneity in the physical capital, e.g. Clementi et al. (2014).

ity component, the number of exiting businesses would increase disproportionately, thus amplifying the drop in economic activity. On the other hand, the leftward shift would decrease the number of exits only moderately. However, a relatively large mass of firms would increase their stock of physical capital, which boosts the aggregate investment.

These observations carry potentially important implications for the business cycle dynamics. A relatively high mass of businesses switching into the exit state in the case of negative shock worsens the economic downturn. On the other hand, comparably substantial measure of firms increases their investment in the case of positive productivity shock. These movements have a potential to amplify the dynamics driven by fluctuations in aggregate productivity.

However, it is important to notice that the patterns observed in the stationary equilibrium, or even in the model environment with aggregate fluctuations under partial equilibrium, need not fully translate into the dynamics in the full model. As observed e.g. in Khan and Thomas (2008), price movements in the general equilibrium analysis can quantitatively mitigate a nonlinear behavior observed in settings where joint supply and demand side considerations are neglected. In any case, the above observation show that my model has the potential to thoroughly inquire the research question at hand.

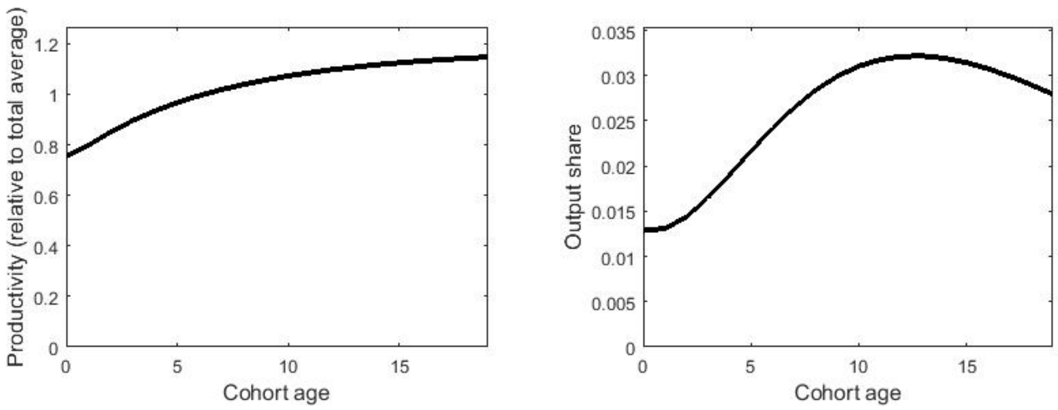


(a) Population share (b) Exit rate

Figure 5: Population and exit patterns of firm cohorts

Notes: The figure on the left hand side represents the age-profile of the total plant population share constituted by the establishments from a given age cohort. The figure on the right hand side depicts the age-profile of the exit rate of the plants from a given age cohort. Both patterns are observed in the stationary equilibrium.

Finally, I overview plants' dynamic behavior over their life-cycles. Figure 5a depicts a share of a cohort's firm mass of a total business population and figure 5b characterizes respective exit rates. It can be observed that a firm population of a given cohort decreases over time, as in each year some of the business units quit production. However, the rate of this outflow, i.e. the exit rate, is decreasing, as the older businesses are on average more productive and thus less likely to find the exit decision optimal. This is captured by a growth of a mean cohort productivity, which is displayed relative to the economy-wide average in figure 6a. Although initially low, a cohort's mean level of productivity eventually grows above the total average.



(a) Relative productivity (b) Output share

Figure 6: Production and productivity patterns of firm cohorts

Notes: The figure on the left hand side represents the age-profile of the mean idiosyncratic productivity of the plants from a given age cohort relative to the economy-wide average. The figure on the right hand side contains the ratio of the output produced by the establishments in individual age cohorts to the aggregate output depicted over the cohorts' life cycle. Both patterns are observed in the stationary equilibrium.

These observations imply that a cohort's contribution to the aggregate economic activity is a non-monotonic function of its age. Two forces ar work are a decrease in a mass of cohort firms and an increase in their productivity. Figure 6b captures a cohort's share of total output observed over its age. It is apparent that the productivity growth channel prevails in the early stage of a cohort's life, as the output share grows until ultimately peeking at 3.2% at age of 14 years. Afterwards the mass decrease channel begins to dominate and the output share falls, gradually approaching value zero.

The observed patterns clearly point to a propagating effect of firm entry on the business cycle dynamics. The above-average number of entering firms in case of a positive productivity shock translates into even higher increase in the economic activity after several years, when a contribution of a given cohort is most pronounced. On the other hand, a drop in the common productivity level leads to *the missing generation effect*, where a loss due to the absent entrants becomes especially evident once the cohort's output is the highest. To summarize, the model exhibits a life-cycle dynamics consistent with the empirical evidence, thus proving its suitability to address research questions concerning firm dynamics.

5.2 The model with aggregate fluctuations

Now I turn to the outcomes of the model in the environment where the aggregate total factor productivity is subject to the shock. In what follows, I first report the standard business cycle statistics for the benchmark model. Next, I use the impulse-response functions to compare dynamic behavior of my framework with the model without firm's extensive margin of adjustment. Finally, I observe the dynamic patterns of the firms' distribution and optimal policies, thus identifying the important channels for the aggregate effect of entry and exit. The numerical algorithm is described in detail in Appendix B.2.

As a first step, I characterize the equilibrium of the benchmark model using the statistics regularly used in the real business cycle literature. In Table 4, I provide an overview of standard deviations of selected aggregate variables expressed relative to the standard deviation of the economy's output. Note that all variables have been logged and detrended using the HP-filter prior to calculating the statistics. The variability of variables generated by my model is roughly similar to the empirical evidence and the outcomes of the standard RBC model (cf. Table 3 in King and Rebelo (2000)), with the patterns of investment and consumption being less aligned with the data. Most precisely, the volatility of aggregate investment is more than eight times that of output, which is too high, while the respective value for consumption is relatively low. At the same time, a volatility of labor just below that of output resembles the data well.

Table 5 displays the aggregates' contemporaneous correlations with output. Here, the patterns are more similar to the empirical evidence. In particular, both investment and labor exhibit a high degree of procyclicality. The correlation of output with aggregate consumption is somewhat lower, yet still significantly high. On the other hand, the aggregate stock of physical capital exhibit negative correlation with the business cycle, contrary to the data.

Next, I present the impulse responses of aggregates to both positive and negative tem-

Table 4: Standard deviations of variables relative to output

	output	investment	consumption	employment	capital
Data[1]	(0.0144)	3.043	0.777	0.677	0.338
Benchmark Model[2]	(0.0144)	8.930	0.423	0.701	0.409
No Entry and Exit[3]	(0.0125)	5.187	0.515	0.504	0.344

Notes: All time series are logged and detrended by the HP-filter with the smoothing parameter of 6.25. Sources: 1 - own calculations using annual data for the US private sector from the Bureau of Economic Analysis (2013) and Bureau of Labor Statistics (2014) datasets, time period 1960-2013, detailed description available in Appendix A; 2 - outcomes of the benchmark model; 3 - outcomes of the model without establishment entry and exit.

Table 5: Contemporaneous correlations with output

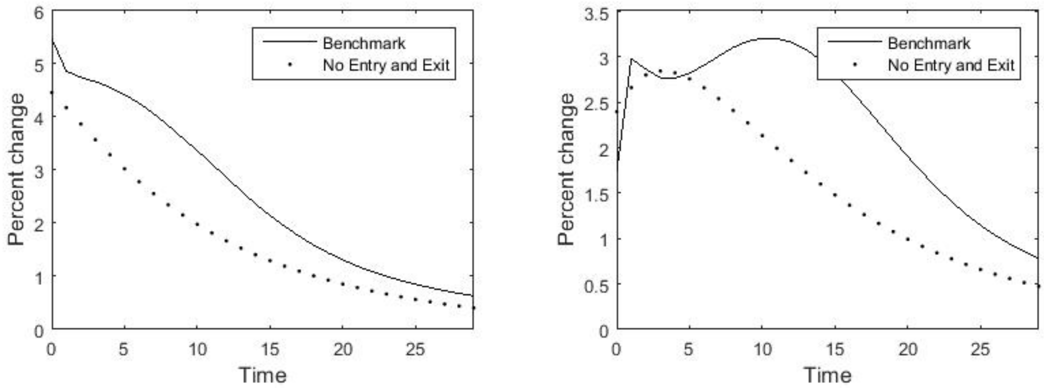
	investment	consumption	employment	capital
Data	0.878	0.959	0.853	0.352
Benchmark Model	0.764	0.812	0.936	−0.061
No Entry and Exit	0.982	0.982	0.981	−0.309

Notes: See above the description of Table 4.

porary shock to the level of aggregate productivity and compare them with their equivalent counterparts in the framework without plant entry and exit. First, figures 7a and 7b display respectively the responses of aggregate output and consumption to a positive temporary shock to aggregate productivity. The size of the shock is set to one standard deviation.

It is apparent that aggregate output exhibits more amplified and pronounced response in the model economy with plants' entry and exit relative to the one without it. First, the response on impact in the benchmark model is 1 percentage point higher than the response in the alternative, which means an important amplifying role by plants' entry and exit on the aggregate production. Second, the level of production remains above it's steady state value well after the shock arrival. This is consistent with the life-cycle patterns of the plants' cohorts as outlined in Section 5.1, where it is demonstrated that the highest share of a cohort's output on the aggregate production is at the plants' age of around 14 years. This maturing process of the above-average number of the new entrants provides for the propagating effect of the plants' turnover.

Let me compare also the responses of aggregate consumption in the respective economies as presented in Figure 7b. Here it is clear that the difference between the responses in respective models is quite small at the early periods after the shock hits, but it becomes more pronounced later. I argue that this is due to the consumption smoothing behavior of



(a) Output (b) Consumption

Figure 7: Responses of the aggregates to a positive TFP shock

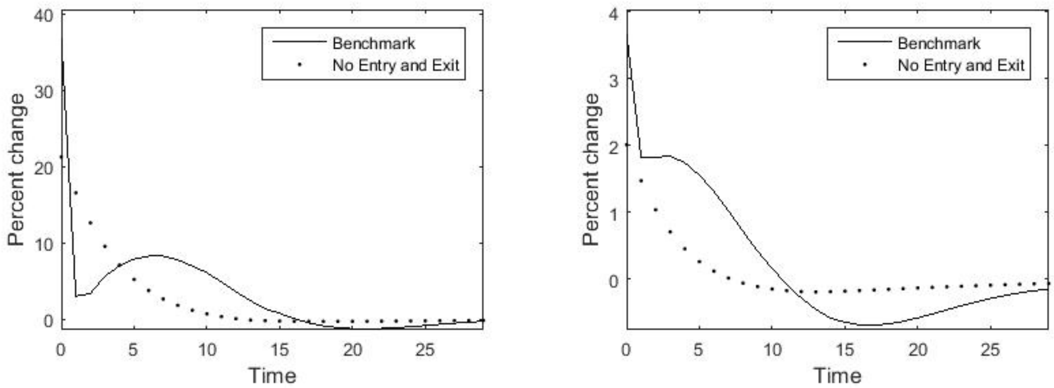
Notes: The impulse-responses are calculated as follows: Starting from the stationary equilibrium, the aggregate TFP is increased one standard deviation above its steady-state level and subsequently is allowed to return back following the underlying AR(1) process. The figure on the left hand side depicts the responses of aggregate output. The figure on the right hand side captures the responses of aggregate consumption.

the representative household. Once a shock hits the economy, growing output leads to an increase in consumption. However, the corresponding decrease in the contemporaneous marginal utility of consumption prompts household to postpone some the consumption for the future.

As a next step, Figures 8a and 8b present the responses of aggregate investment and employment level to a positive temporary shock to aggregate productivity. It can be observed that the investment level exhibits the strongest response, increasing by more than 35% on impact relative to its steady-state value. This is in striking contrast with a model without plant entry and exit, where the response is much smaller, around 21%. The decomposition reveals that the majority of this difference is driven by the additional initial investment of new entrants.

6 Conclusions

This paper analyzes the role of firms' entry and exit for business cycle dynamics. It emphasizes the contribution of investment irreversibility, as it affects businesses' decisions to start or shut down production. My analysis reveals that allowing for endogenous extensive margins of firms' adjustment substantially increases the magnitude and propagation of the aggregate variables' response to exogenous productivity shocks. An increase in the



(a) Investment

(b) Employment

Figure 8: Responses of the aggregates to a negative TFP shock

Notes: The impulse-responses are calculated as follows: Starting from the stationary equilibrium, the aggregate TFP is decreased one standard deviation below its steady-state level and subsequently is allowed to return back following the underlying AR(1) process. The figure on the left hand side depicts the responses of aggregate investment. The figure on the right hand side captures the responses of aggregate employment.

level of total factor productivity leads to an increase in the aggregate level of output, investment, and employment. The responses in the environment with firms' entry and exit are higher compared to a situation where the extensive margin of firms' adjustment is absent. These differences are persistent, as they propagate long in time after the shock has hit.

An important feature of my work is the interaction between the extensive margin of firms' adjustment and the partial investment irreversibility. These two characteristics jointly determine the dynamic pattern of the firm-size distribution, which drives the main results of my work. However, frictions to adjusting labor also determine a firm's decision whether to enter, grow or exit. Firm dynamics are likely to be affected not only by the costs of adjusting labor *per se*, but also by possible interactions between frictions to capital and labor adjustment. My model currently features fully flexible labor markets only. I consider it a promising avenue for future research to analyze firm turnover in an environment featuring adjustment costs to both capital and labor. This is likely to further improve our understanding of the role of business turnover for the aggregate economic activity.

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A Data

In the following part, I describe in detail data sources and a construction of statistics that are used both to discipline corresponding model-generated moments during the calibration process, and to assess the quantitative patterns of the model economy. I use the annual data for the U.S. private business sector with the time horizon explicitly described for each data source individually.

A.1 Capital stock and production

I start with the data on the aggregate level of output, capital stock, investment and depreciation. This information is provided in the sections GDP & Personal Income (GDPPI) and Fixed Assets (FA) of the U.S. Economic Accounts database of the Bureau of Economic Analysis (2013) over the time period 1947-2013. I construct respective aggregate time-series as follows:

(i) Output for the business sector, which I denote by Y_t, can be obtained from the statistic *real gross value added* in GDPPI Table 1.3.6, line 2.

(ii) For the aggregate capital stock, I use data on nonresidential fixed assets and do not consider residential structures or consumption durables. This decision is driven by my focus that lies on frictions to capital reallocation related to firm operation, where the latter arguably do not play a significant role. As a results I use *the current-cost net stock of private nonresidential fixed assets* from FA Table 4.1, line 1. As these are expressed in nominal terms, I adjust them using *the chain-type quantity indexes* from FA Table 4.2, line 1, to receive the time-series of aggregate capital stock K_t.

(iii) The data on depreciation can be observed in *the current-cost depreciation of private nonresidential fixed assets* from FA Table 4.4, line 1. Again, this time-series needs to be expressed in real value using *the chain-type quantity indexes* from FA Table 4.5, line 1, which yields the time-series of aggregate level of depreciation D_t.

(iv) Finally, the data on nonresidential investment taken from *the investment in private nonresidential fixed assets* from FA Table 4.7, line 1 are converted to real investment I_t using *the chain-type quantity indexes* from FA Table 4.8, line 1.

These time-series are in turn used to construct statistics for the calibration procedure. The capital-to-output ratio is defined as K_t/Y_t, and the value targeted in the calibration is a sample average over the period $1972 - 2013$. Similarly, I construct the aggregate investment rate and the depreciation rate as sample averages of the respective ratios I_t/K_{t-1} and D_t/K_{t-1}.

As a following step, I comment on my source of evidence on the labor force participation rate. The underlying dataset is the Labor Force Statistics of the Current Population Survey available from Bureau of Labor Statistics (2014), which presents the *civilian labor force participation rate* at monthly frequency starting from 1948. I calculate the targeted value of this statistics (64.9%) as an average over the period $1972 - 2013$.

A.2 Establishment demography

Next, I describe the data characterizing establishment demography, i.e. entry, growth and exit of production units. The relevant information is available in the dataset of Business Dynamics Statistics (2013a), which provides data on the number of establishments, their entry/exit activity and job flows. This evidence can be also decomposed into categories according to their age and size characteristics. The sample covers the period 1979-2012 at annual frequency[11].

Let me denote the number of production units of age a in year t by $M_{a,t}$, the employment level of this cohort by $N_{a,t}$, the number of plants exiting in year t by M_t^{EX} and their total employment level by N_t^{EX}. Given this notation, I can observe, for a given year t, entry and exit rates respectively defined as $EN_t = M_{0,t}/\sum_a M_{a,t}$ and $EX_t = M_t^{EX}/\sum_a M_{a,t}$, and target average turnover rate at value 0.215, which is calculated as a sum of sample averages of entry and exit rates. Subsequently, I can derive the average establishment size as $\bar{N}_t = \sum_a N_{a,t}/\sum_a M_{a,t}$, the average size of entering unit as $\bar{N}_{0,t} = N_{0,t}/M_{0,t}$, and the average size of exiting plant as $\bar{N}_t^{EX} = N_t^{EX}/M_t^{EX}$. Consequently, a ratio of the average size of entering establishments relative to the average firm is given by $\bar{N}_{0,t}/\bar{N}_t$, and a ratio of the average size of exiting firms relative to the average plant size can be expressed as $\bar{N}_t^{EX}/\bar{N}_t$. The value of statistics targeted in the calibration—0.517 for entry and 0.486 for exit—are calculated as sample averages of these ratios. Next calibration target is the ratio of average firm sizes of the second year cohort to the first year cohort, which has value 1.214 and is calculated as $(N_{1,t}/M_{1,t})/(N_{0,t}/M_{0,t})$.

[11] The entire available sample spans 1977-2012, however, I drop the first two years following the concern by Moscarini and Postel-Vinay (2012) regarding the measurement error in the first two observations.

The final BDS-based data moment targeted in the calibration is a fraction of entering firms that exit during their first year of existence at value 19.75%. This statistic is calculated as a sample mean of year-specific exit rates of entrants defined as $1 - M_{1,t+1}/M_{0,t}$.

A.3 Establishment size distribution

Finally, my calibration procedure aims to discipline a size distribution of establishments generated by the model in a way that is consistent with the data. The evidence for this purpose can be again found in the dataset of Business Dynamics Statistics (2013a), where counts and employment levels of production units are shown for distinct size categories. In table 6 I present respective shares of both statistics across the different size categories averaged over the entire sample period. For example, 49.28% share of establishments with an employment level 1 to 4 workers is an average share of production units with at most 4 employees over the years $1979 - 2012$.

size class	$1-4$	$5-9$	$10-19$	$20-49$	$50-99$
establishments	0.4928	0.2236	0.1387	0.0903	0.0305
employment	0.0663	0.0883	0.1123	0.1647	0.1264
size class	$100-249$	$250-499$	$500-999$	$1000+$	
establishments	0.0170	0.0044	0.0017	0.0010	
employment	0.1526	0.0890	0.0687	0.1317	

Table 6: Shares of production units for different size classes (averages over 1979-2012)

With respect to the calibration procedure, note that the employment level of a firm in my model is not a discrete variable. Therefore I use data to calculate cumulative distributions in both statistics and compare these values against their equivalents from the model. Specifically, I order all the firms in my model economy according to their size, then evaluate cumulative shares in both their number and labor force, and finally compare establishment shares for the employment percentiles corresponding to the available empirical observations, i.e. at 6.63^{th} percentile, 15.46^{th} percentile, etc.

Note that my assumption of normally distributed idiosyncratic productivity shock processes implies that it is not possible to match long top tail of the empirical size distribution for the reasonable parameter values. However, I believe this does not effect my results significantly, as the main focus is on the distribution of plants along the optimal entry and exit policies, and these are arguably most relevant for the establishments at the bottom of the size distribution.

B Numerical algorithm

In this part I outline a numerical routine that solves my model. It follows closely a computational approach used by Chang and Kim (2007) in a model with heterogeneous households, incomplete asset market and indivisible labor supply, while also incorporating correcting comments on the former by Takahashi (2014). First, I describe an algorithm for the stationary equilibrium, which is used for calibration of parameters and as a starting point for the analysis of economy with aggregate shocks. Next, I present a solving procedure for model economy with fluctuations in aggregate productivity.

B.1 Stationary equilibrium

1. As a starting point, I initialize the grid points for an individual firm's capital stock k and idiosyncratic productivity z.

   - The discretization of capital $\{k^1, \ldots, k^{N_k}\}$ is equidistant in logs and its range is extended sufficiently to so that in equilibrium firms don't reach upper and lower boundary. Subsequently, a number of grid points N_k is set so that depreciation spans one grid point[12]. In my benchmark calibration this amounts to $N_k = 160$ grid points between $k^1 = 0.213$ and $k^{160} = 3989.03$.

   - On the other hand, the stochastic process for z_t is approximated on a finite number $N_z = 21$ of grid points $\{z^j\}_{j=1}^{N_z}$, where the values of the grid points and the transition probabilities $\pi_z(z'|z)$ are calculated using algorithm by Rouwenhorst as presented by Kopecky and Suen (2010).

2. Before calculating value function and optimal policies, I guess a value for the wage rate w. This variable will be later adjusted in order to pin down the free entry condition. At this stage I can remain agnostic about a value of the price p, because given its multiplicative nature it doesn't affect optimal policies and solely rescales the value function in the steady-state[13].

3. I compute the steady state value function and firms' optimal policies on each grid point. I first initialize the value function and then update it by iterating in time using

[12]This means that a depreciation of the capital stock at a grid point k^j yields a value of capital k^{j-1}.
[13]Notice that this is no longer the case once I introduce aggregate uncertainty.

firm optimal investment and exit policies and exogenous transition for productivity until convergence is achieved. To increase speed of the computation, firms are allowed to re-optimize their policy functions only on each hundredth time-iteration. I check that this simplification doesn't effect stationary equilibrium outcomes.

4. As a next step I use the steady-state value function and the distribution of entrants' productivity to calculate the optimal level of initial investment by new firms and consequently evaluate the expected value of entry. If this is different from 0 up to a given level of precision (10^{-10}), i.e. the free-entry condition doesn't hold, I adjust the wage rate w accordingly[14] and the algorithm returns to step 3.

5. Using the optimal policies for investment and exit together with the exogenous transition probabilities for individual productivity, I construct the equilibrium transition matrix $T_{(z,k)}$ that characterizes transitions between the individual states (z, k).

6. Next I compute the equilibrium distribution of firms. First, the *probability* distribution μ^* and the entry rate M are obtained by solving a fixed-point problem given by $\mu^* = T'_{(z,k)}\mu^* + M\mu^E$, where μ^E is the probability distribution of entrants over the state space. Consequently I adjust the mass of firms which rescales the distribution such that the aggregate labor demand matches labor supply given by the targeted labor participation rate.

7. Finally, goods market clearing condition (17) yields a level of aggregate consumption and thus also the equilibrium price p, which is assumed to equal the household's marginal utility of consumption. This together with the equilibrium wage rate w provides the household labor disutility parameter θ using the labor market clearing condition (16).

B.2 Equilibrium with aggregate fluctuations

In an economy with aggregate fluctuations, the distribution of firms μ_t becomes a state variable in an individual firm's problem. As this is an infinitely dimensional object, calculation of the equilibrium becomes numerically unfeasible. Therefore, following the method developed by Krusell and Smith (1998), I calculate the approximate equilibrium

[14] In case of a strictly positive (negative) expected value of entry I increase (decrease) the wage rate w.

by assuming that agents track only first moments of this distribution and use them to forecast both current and future prices. Notice that a prediction of the contemporary prices is necessary because they are determined by the aggregate variables, which in turn depend on current actions of the agents.

The parametric law of motion for the aggregate capital stock K_t is assumed to take a form

$$\log K_{t+1} = b_0 + b_1 \log K_t + b_2 \log A_t, \tag{22}$$

and the current equilibrium price p_t is forecasted by the rule

$$\log p_t = d_0 + d_1 \log K_t + d_2 \log A_t. \tag{23}$$

Note that the first moment of the second individual state variable, the idiosyncratic productivity, is constant by the law of large numbers and thus is included in the constant terms b_0 and d_0. Moreover, I don't need to forecast the current wage rate w_t, because in equilibrium it is unambiguously determined by the price p_t through the households' first order condition (16).

Using this approximation, the algorithm goes as follows:

1. In a first step, I initialize grids for the state variables: individual firm's capital stock k, idiosyncratic productivity z, aggregate capital stock K and aggregate productivity A. The grids for k and z are identical to those for the calculation of stationary equilibrium.

   - A level of aggregate capital K spans $N_K = 11$ grid points that are equally spaced on interval $[0.75K^*, 1.25K^*]$, where K^* is the aggregate capital stock in the steady state. During the numerical simulations I make sure that lower and upper bound of this interval are never reached.

   - On the other hand, the stochastic process for the aggregate productivity A_t is approximated in a way similar to that of the idiosyncratic productivity process. In particular, there is a finite number $N_A = 7$ of grids equally spaced in logs with the transition probabilities $\pi_A(A'|A)$ calculated by the Rouwenhorst's algorithm.

2. Before the calculation of the equilibrium value function and optimal policies, I ini-

tialize the parameter values for the law of motion (22) and the price forecasting function (23).

3. Next, I compute the value function and firms' optimal policies for all the grid points. This step is similar to step 3 in the algorithm solving for the equilibrium of the model without aggregate shocks. The difference is in the fact that firms' optimization problem during the iterations in time is conditional on the assumed law of motion and forecast function.

4. In the following step I simulate a behavior of the model economy over a large number of periods (5500) using the distribution of firms from the stationary equilibrium as a starting point. Note that I cannot use firms' optimal policies derived in the value-function-iteration step, but I need to solve for equilibrium in each step of simulation. However, I use the value function calculated in step 3 to compute the expected continuation value, which is used in a firm's optimization problem in each step of the simulation[15] Each time period consists of the following sequence of steps:

   (a) At the beginning I observe the distribution of firms, a realization of shock to the aggregate productivity, and the current level of the aggregate capital stock.

   (b) Secondly, I use the transition law for the aggregate capital (22) to calculate the next period aggregate stock of physical capital as perceived by agents in an economy.

   (c) Next I use the value function obtained in step 3 to calculate the expected continuation value of firms at each individual state. I use linear interpolation over both the next period aggregate capital and the expected realization of aggregate productivity to allow for values not on the grid points.

   (d) Given the expected continuation value, I solve the entry problem to obtain a level of optimal entry investment while adjusting the equilibrium price to satisfy the free entry condition. This immediately determines the equilibrium wage rate through the households' first order condition.

   (e) Consequently, I solve a firm's problem under the *current* equilibrium prices using the expected continuation value derived in step 4c, obtaining firms' optimal

[15] See Section II in Takahashi (2014) for a detailed discussion.

investment and exit policies. Again, I use linear interpolation so that the firms can choose a level of physical capital not on the grid points. Subsequently, the firms' optimal policies allow me to calculate the current values of aggregate variables (output, investment).

(f) As a following step, I calculate a mass of entrants that is consistent with the goods market clearing.

(g) Finally, I update the distribution of firms using a current period optimal investment and exit policies, a transition dynamics of idiosyncratic productivity, together with an addition of entrants.

5. After the simulation procedure, I estimate parameters in (22) and (23) by the OLS on the simulated data, where I discard first 500 observations to eliminate the effect of initial conditions. If the estimated values are close to the previous ones, the equilibrium law of motion and forecast function have been found. Otherwise I update the coefficients with the new estimates and return to step 3.

As a last point, I present the coefficient values in the approximating functions. The approximate law of motion for the aggregate capital stock K_t has form

$$\log K_{t+1} = 0.1385 + 0.8513 \log K_t + 0.3636 \log A_t,$$

and the current equilibrium price p_t is forecasted by the rule

$$\log p_t = 0.4514 + -0.5507 \log K_t + -0.5789 \log A_t.$$

The accuracy here is somewhat lower compared to the model without entry and exit, in particular R^2 is respectively 0.9857 and 0.9995. This indicates that some additional distribution moment should be added to improve performance. A prime candidate is the aggregate level of idiosyncratic productivity, which changes due to entry and exit of firms. This extension will be done as the next step of my analysis.

C The model without entry and exit

In order to make any statements on an importance of firm entry and exit for the business cycle dynamics, I need to construct a model without firm turnover, which serves as a reference point. In order to construct such an environment, I assume away entry and exit decisions of firms from my benchmark model. In addition, the parameters governing exogenous exit δ and fixed costs c^f are set equal to zero. This yields a fixed mass of firms, whose value is fixed to its full-model equivalent observed in the stationary equilibrium.

With respect to the numerical solution, it is very similar to the algorithm used for the benchmark model and described in detail in part B. However, there are certain differences that require clarification. First, in solving for the steady state equilibrium the iteration over the wage targets clearing of the labor market instead of the free-entry condition (see point 4 in section B.1). Second, in the solution of the model with aggregate fluctuations, each step of a simulation requires iterating over the equilibrium price in order to obtain the goods market clearing. This is fairly more computationally demanding relative to the model with firm entry and exit, as there the market clearing in the simulations is ensured by adjustment of a mass of entering firms (see point 4f in section B.2).

I conclude with the specific values of coefficients in the approximating functions. In both cases, the accuracy is very high with R^2 well above 0.9999. The approximate law of motion for the aggregate capital stock K_t has form

$$\log K_{t+1} = 0.1905 + 0.7794 \log K_t + 0.4324 \log A_t,$$

and the current equilibrium price p_t is forecasted by the rule

$$\log p_t = 0.0668 + -0.3165 \log K_t + -0.7467 \log A_t.$$

D Analytical results and proofs

Now I formally state and prove the results that underlie my characterization of a plant's optimal policies over the state space in the stationary equilibrium as presented in section 3.6. First, I provide several auxiliary results regarding the properties of individual components of the Bellman equation. Second, I prove existence and uniqueness of the value function in the stationary equilibrium. Finally, I characterize plants' optimal policies with respect to the individual state space.

As a starting point I take the Bellman equation outlined in the expression (19). Specifically, the optimization problem of a plant with a capital stock k and a productivity level z has the following form:

$$V^0(z,k) = \tilde{\pi}(z,k) + \max\Big\{V^X(k), \max_{k'}\Big\{-C^K(k',k) \ + \ \beta\mathbb{E}\big[V^0(z',k')|z\big]\Big\}\Big\}, \qquad (24)$$

where $V^X(k)$ represents the exit value $V^X(k) = \kappa(1-\delta^k)k$, $\tilde{\pi}(z,k)$ denotes a plant's contemporaneous profit net of the capital adjustment as given in equation (20), and C^K characterizes any costs related to adjustment of a plant's stock of physical capital and is explicitly stated in equation (21).

Now I derive certain properties of profit and costs functions with respect to the state variables. The proofs are a trivial algebraic exercise and therefore I skip them to keep the exposition smooth.

Lemma 1. *$\tilde{\pi}(z,k)$ is increasing and concave in k, and increasing and convex in z.*

In later derivations, it is convenient to use the explicit analytical expression for $\tilde{\pi}(z,k)$:

$$\tilde{\pi}(z,k) = \left(\eta^{\frac{\eta}{1-\eta}} - \eta^{\frac{1}{1-\eta}}\right) w^{\frac{-\eta}{1-\eta}} z^{\frac{1}{1-\eta}} k^{\frac{\alpha}{1-\eta}} - c^f. \qquad (25)$$

Lemma 2. *(i) Consider any $k' \in \mathbb{R}^+$. If $(1-\delta^K)\gamma \le \kappa$, then $-C^K(k',k)$ is strictly increasing in k for $k \in \mathbb{R}^+$. Otherwise, $-C^K(k',k)$ is strictly increasing in k for $k \in (0, \frac{k'}{\sqrt{(1-\delta^K)(1-\delta^K-\frac{\kappa}{\gamma})}})$, and strictly decreasing in k for $k \in (\frac{k'}{\sqrt{(1-\delta^K)(1-\delta^K-\frac{\kappa}{\gamma})}}, \infty)$.*
(ii) Consider any $k \in \mathbb{R}^+$. Then $-C^K(k',k)$ is negative for $k' \in (0, (1-\delta^K-\frac{\kappa}{\gamma})k) \cup ((1-\delta^K)k, \infty)$, positive for $k' \in ((1-\delta^K-\frac{\kappa}{\gamma})k, (1-\delta^K)k)$, strictly increasing in k' on $(0, (1-\delta^K-\frac{\kappa}{2\gamma})k)$, and strictly decreasing in k' on $((1-\delta^K-\frac{\kappa}{2\gamma})k, \infty)$.

Now I move to the second part of this section, where I prove that a plant's problem, as it is outlined in the Bellman equation (24), has a solution, which is unique. Subsequently, I prove several properties of this value function, which allow me to later to characterize a plant's optimal policies over the state space.

Proposition 1. *Consider the recursive optimization problem of a plant given by (24). Then there is a unique value function $V^0(z,k)$ that solves it.*

Proof. First, I reformulate the Bellman equation in a way consistent with the propositions derived in Stokey et al. (1989). Let me introduce a binary variable x, which explicitly represents a plant's exit/survival decision. Specifically, $x = 1$ means that an establishment has decided to remain operational, whereas $x = 0$ indicates exit. Then the equation (24) can be equivalently rewritten as follows:

$$\begin{aligned}\tilde{V}^0(z,(k,1)) = \max_{(k',x)\in\mathbb{R}^+\times\{0,1\}} \Big\{ & \tilde{\pi}(z,k) \ + \ (1-x+x\delta)V^X(k) - \\ & - \ x(1-\delta)C^K(k',k) \ + \ x(1-\delta)\beta\mathbb{E}\big[\tilde{V}^0(z',(k',x))|z\big]\Big\}, \\ \tilde{V}^0(z,(k,0)) = 0. & \end{aligned} \tag{26}$$

Now I can apply Theorem 9.12 from Stokey et al. (1989) to prove existence and uniqueness of $\tilde{V}^0(z,(k,x))$ (and thus also of $V^0(z,k)$). Assumption 9.1 holds trivially. Assumption 9.2 is ensured by the finite expected value of z and the outside option provided by a plant's exit. Conditions (a) and (b) are also ensured by the finite expected value of z together with the decreasing returns-to-scale production function in capital (see Lemma 1) and the convex capital adjustment costs. Under these conditions, Theorem 9.12 from Stokey et al. (1989) implies the existence of a unique value function $\tilde{V}^0(z,(k,x))$ solving equation (26), which in turn provides a unique solution $V^0(z,k)$ to the equation (24). □

After establishing the existence and uniqueness of the value function, I prove its several properties, which will be useful for the characterization of a plant's optimal policies. In the proofs I follow the approach taken by Dixit (1997). First, the value function V^0 is shown to be a fixed point of a contraction mapping $\mathcal{M} : \mathcal{V} \to \mathcal{V}$ defined on a complete functional space of continuous functions $\mathcal{V}$ as follows:

$$\mathcal{M}V(z,k) = \max_{k'}\Big\{\tilde{\pi}(z,k)+\max\Big\{V^X(k),\delta V^X(k)-(1-\delta)C^K(k',k)+(1-\delta)\beta\mathbb{E}\big[V(z',k')|z\big]\Big\}\Big\}.$$

Secondly, I demonstrate that $\mathcal{M}$ maps a complete subspace $\mathcal{V}_1 \subseteq \mathcal{V}$ onto itself, which implies via the contraction-mapping theorem that the fixed point lies in the subspace $\mathcal{V}_1$. However, V^0 is the unique fixed point in $\mathcal{V}$ and therefore it has to be $V^0 \in \mathcal{V}_1$.

Proposition 2. *$V^0(z,k)$ is continuous and it is increasing in z. Moreover, if $(1-\delta^K)\gamma \leq \kappa$, then $V^0(z,k)$ is increasing in k.*

Proof. Continuity is a trivial consequence of V^0 being the fixed point of $\mathcal{M}$ on $\mathcal{V}$ as outlined above, since $\mathcal{V}$ is the space of continuous functions. Moreover, as $\tilde{\pi}(z,k)$ is strictly increasing in z, $\mathcal{M}$ maps a closed subspace $\mathcal{V}_1$ of functions increasing in z into itself, which yields desired result. Finally, consider monotonicity with respect to k. Both $\tilde{\pi}(z,k)$ and $V^X(k)$ are strictly increasing in k. At the same time, $(1-\delta^K)\gamma \leq \kappa$ is sufficient for $-C^K(k',k)$ to be increasing in k, therefore I obtain for $k_1 < k_2$

$$\max_{k'} \Big\{ -C^K(k',k_2) \ + \ \beta \mathbb{E}\big[V^0(z',k')|z\big]\Big\} \geq -C^K(k_1^*,k_2) \ + \ \beta \mathbb{E}\big[V^0(z',k_1^*)|z\big] > \\ > -C^K(k_1^*,k_1) \ + \ \beta \mathbb{E}\big[V^0(z',k_1^*)|z\big],$$

where $k_1^* = \arg\max_{k'} \Big\{ -C^K(k',k_1) \ + \ \beta \mathbb{E}\big[V^0(z',k')|z\big]\Big\}$. Repeating the argument about $\mathcal{M}$ mapping a subspace $\mathcal{V}_1$ of functions increasing in k into itself concludes the proof. □

A strict monotonicity of $V^0(z,k)$ with respect to z and k allows me to prove two corollaries, which will be useful later on.

Corollary 1. *Function* $W(z,k) := \max_{k'} \Big\{ -C^K(k',k) \ + \ \beta \mathbb{E}\big[V^0(z',k')|z\big]\Big\}$ *is strictly increasing in* z.

Proof. Consider k, $z_1 < z_2$ and denote $k_1^* := \arg\max_{k'} \Big\{ -C^K(k',k) \ + \ \beta \mathbb{E}\big[V^0(z',k')|z_1\big]\Big\}$. Observe that the assumed process (AR(1) in logs) of individual productivity is monotone (see Exercise 12.11 in Stokey et al. (1989)) and thus $z_1 < z_2$ imply an inequality $\mathbb{E}\big[V^0(z',k')|z_1\big] < \mathbb{E}\big[V^0(z',k')|z_2\big]$. Using this, simple algebraic steps yield $W(z_2,k) \geq -C^K(k_1^*,k) \ + \ \beta \mathbb{E}\big[V^0(k_1^*,k')|z_2\big] > -C^K(k_1^*,k) \ + \ \beta \mathbb{E}\big[V^0(k_1^*,k')|z_1\big] = W(z_1,k)$. □

Corollary 2. *If* $(1-\delta^K)\gamma \leq \kappa$, *then function* $\mathbb{E}\big[V^0(z',k)|z\big]$ *is strictly increasing in* k.

Proof. Consider z and $k_1 < k_2$. Proposition 2 yields $V^0(z',k_2) - V^0(z',k_1) > 0, \ \forall z'$. However, the mean value of a strictly positive function is necessarily positive. □

Finally, I present and prove the characterization of a plant's optimal policies with respect to its individual state. Consistently with the notation introduced in Section 3, $\chi(z,k)$ represents the exit/survival decision of a plant with the productivity level z and the current capital k as defined in equation (9). Specifically, $\chi(z,k) = 1$ if an establishment decides to remain operational, whereas $\chi(z,k) = 0$ in case of exit. Additionally, $k^*(z,k)$ denotes the optimal choice of the next period capital stock.

I start with the proof of existence of the exit cut-off level of productivity as a function of the current capital stock. More precisely, for each plant there is a well-defined threshold in the productivity, such that a shock that decreases the TFP level below this boundary induces a production unit to quit operation.

Proposition 3. *For any k there exists a unique $\tilde{z}(k) > 0$ such that:*
(i) $\max_{k'} \Big\{ -C^K(k', k) \ + \ \beta \mathbb{E}\big[V^0(z', k') | \tilde{z}(k)\big] \Big\} = \kappa(1 - \delta^k)k$;
(ii) $\forall z < \tilde{z}(k) \ \chi(z, k) = 0$ *and* $\forall z > \tilde{z}(k) \ \chi(z, k) = 1$.

Proof. Consider function $W(z, k)$ as defined in Corollary 1. I proceed by showing that: (a) $\lim_{z \to +\infty} W(z, k) = +\infty$; and (b) $\lim_{z \to 0^-} W(z, k) < \kappa(1 - \delta^k)k$. These properties together with the result from Corollary 1 and a continuity of $W(z, k)$ with respect to z yield the existence and uniqueness of the intersection point $\tilde{z}(k) > 0$ as well as the optimality of the exit (survival) for the productivity levels below (above) it.
(a) First, observe that $W(z, k) \geq \beta \mathbb{E}\big[V^0(z', (1 - \delta^K)k) | z\big]$. Following equation (25), $\lim_{z \to +\infty} \tilde{\pi}(z, k) = +\infty$, which implies $\lim_{z \to +\infty} V^0(z, (1 - \delta^K)k) = +\infty$. Subsequently, the persistent Markov structure of the productivity process z yields $\lim_{z \to +\infty} \mathbb{E}\big[V^0(z', (1 - \delta^K)k) | z\big] = +\infty$, which finally implies $\lim_{z \to +\infty} W(z, k) = +\infty$.
(b) Observe that $\lim_{z \to 0} \tilde{\pi}(z, k) = -c^f$ from equation (25), and therefore $\lim_{z \to 0} \mathbb{E}\big[\tilde{\pi}(z', k) | z\big] < 0$. Then for every level of the capital stock $\bar{k}$ I can choose the productivity level $\bar{z}$ low enough, so that (starting from the individual state $(\bar{z}, \bar{k})$) conditional on survival a plant generates a negative stream of contemporaneous profits for sufficiently long time unless the unit always disinvests. However, eventually there is no capital to sell and therefore negative profits prevail. Moreover, disinvestment is associated with an additional convex cost of adjustment relative to exiting. As a result, it is not optimal for a plant to remain operational at the very first decision point, thus $\lim_{z \to 0^-} W(z, k) < \kappa(1 - \delta^k)k$. □