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The Double Diamond Paradox^{*}

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Abstract

We study vertical relations in markets with consumer *and* retailer search. Retailers search to learn manufacturers' prices. We obtain three important new results. First, we explain why empirical distributions of retail prices are bimodal, with a regular price and a sales price. Second, under competitive conditions (many retailers or small consumer search cost) social welfare is significantly smaller than in the double marginalization outcome. Manufacturers' regular price is significantly above the monopoly price squeezing retailers' markups and providing an alternative explanation for incomplete cost pass-through. Finally, by randomizing to induce active consumer search, manufacturers can increase their profits.

JEL Classification: D40; D83; L13

Keywords: Vertical Relations, Retailer Search, Consumer Search, Diamond Paradox, Double Marginalization

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1 Introduction

For markets to be truly competitive, consumers must be able to compare price offers of different firms (Stigler, 1961). Indeed, it has been established, both theoretically and empirically, that the Law of One Price fails to hold whenever (some) consumers make their purchasing decisions before observing all relevant prices. One source of price dispersion concerns the temporal variation in prices at a given firm, generally referred to as *sales*. Sales are ubiquitous in consumer markets and represent a large share of the observed price variation.¹ Accordingly, a large empirical literature has identified a number of regularities in the temporal distribution of prices. The distribution of retail prices is double-peaked, with two prices being charged most of the time (see Hosken and Reiffen (2004) and Pesendorfer (2002)) and the typical time-series is *V-shaped* with a high price for a number of periods followed by a discrete, short-lived drop which we identify as sales. The bold curve in Figure 1 provides an example of such a pattern.² In contrast, the equilibrium price distribution generated in the theoretical literature is continuous and monotone.³

It is not difficult to see that theoretical models that only consider the retail market have difficulties explaining the empirically documented bimodal price distributions. If all consumers only search once, then firms have monopoly power and the Diamond Paradox emerges where firms set the monopoly price; see, for example, Diamond (1971) and Burdett and Judd (1983).⁴ On the other hand, if all consumers sample at least two prices, firms will price at marginal cost and again the Law of One Price will hold. As long as some consumers find search costly, this cannot be part of an equilibrium (see, e.g., Stahl, 1989). In markets where some consumers search once and others search twice or more, the equilibrium price distribution is continuous, as any hole in its support would yield incentives for rivals to undercut. Even in markets with product differentiation

¹See Nakamura and Steinsson (2008).

²This is the time series of prices reported in the Dominick's Database for one of the most popular Miller Beer Products.

³Varian (1980) and Stahl (1989), and a large subsequent literature, analyze static models where consumers differ in their information about other firms' prices and show that random prices can arise in equilibrium. Pesendorfer (2002) presents a model of durable good consumption whereby consumers differ in their willingness to pay and firms optimally vary prices over time in order to price discriminate among consumers (see also Sobel, 1984).

⁴Diamond (1971) showed that, if all consumers have a positive, yet (arbitrary) small, search cost, firms will set monopoly prices in equilibrium. Indeed, if consumers expect any price below the monopoly price to be charged, but instead observe a slightly higher price, they do not have an incentive to continue to search. Thus, no price lower than the monopoly price can be part of the equilibrium price distribution.

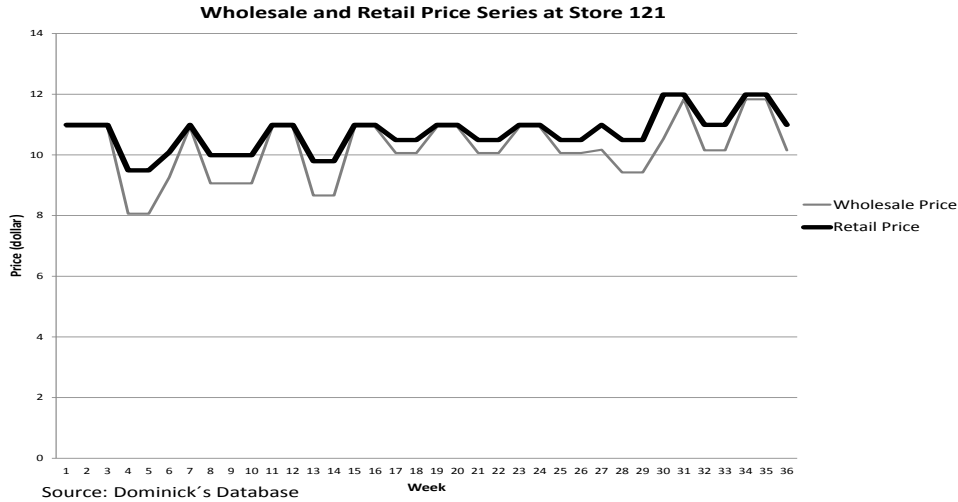


Figure 1: Typical Retail and Wholesale Price Series.

as in [Wolinsky \(1986\)](#) or [Anderson and Renault \(1999\)](#), bimodal price dispersion will not emerge as a robust phenomenon as firms need to be exactly indifferent between charging two prices.

We argue that bimodal price distributions may be explained as the natural outcome of the vertical interaction between manufacturers and retailers. Indeed, we provide evidence that price variation at the retail level may be induced by manufacturers: sales are associated with lower wholesale prices. While this pattern has so far not been documented in the literature, it is consistent with the price series in the well-known Dominick’s database. A typical example is presented by the grey curve in Figure 1.⁵ Note that in that Figure retail sales coincide with (or are induced by) contemporaneous reductions in wholesale prices. Note also that retail markups are decreasing in wholesale prices, consistent with incomplete cost pass-through.⁶

To explain bimodal retail and wholesale price distributions, our baseline model has a simple vertical industry structure. Manufacturers sell their product to retailers and retailers resell the product to final consumers. The key feature of the model is that both final consumers *and* retailers engage in sequential search and have to pay a search cost to

⁵See Section 5 for a somewhat more systematic statistical analysis.

⁶Many industries are characterized by incomplete cost pass-through ([Weyl and Fabinger, 2013](#)), i.e. higher wholesale prices induce lower margins at the retail level. For instance, using real exchange rates as a source of wholesale price variation, [Campa and Goldberg \(2005\)](#) find that the typical product has a cost pass-through elasticity smaller than one. Our model endogenously generates incomplete cost pass-through, regardless of the shape of demand.

acquire new information. Final consumers search among retailers, while retailers search among manufacturers. To emphasize that our explanation is not based on search cost heterogeneity, the baseline model has all consumers facing the same consumer search cost and all retailers having identical retailer search cost. We consider homogeneous goods markets so that in the absence of search costs, competition would force prices to be equal to marginal cost.

Retailer and consumer search are important in many real-world markets. While there is a large theoretical and empirical literature documenting the importance of search costs in retail markets, this is the first paper dealing with search in wholesale markets. Even though one may argue that retailers are professional traders with substantial knowledge of market conditions, search frictions in the wholesale market may be substantial if the relevant prices of *alternative* suppliers are hard to identify.⁷

As our baseline model is a simple Diamond model with a vertical industry structure, one may expect the interaction between manufacturers and retailers to be characterized by the classic double marginalization outcome, where retailers charge the retail monopoly price given the wholesale price that is set by manufacturers, and that given this retail behaviour, manufacturers set their optimal monopoly price as well. Since there is no price dispersion, neither consumers nor retailers have incentives to search. Indeed, we show that double marginalization constitutes an equilibrium for some parameter values, but - more importantly- it is *not* an equilibrium for other parameter values. In particular, under competitive conditions (a relatively large number of retailers or small consumers' search cost) a manufacturer is able to increase its profit by deviating to a price that is equal to the consumers' reservation price, thereby appropriating the retailer's margin. If there are many retailers and the retail market is competitive, the squeezed retailer does not have an incentive to continue to search and will pass-on this higher price to consumers. For low levels of consumer search cost, the (small) reduction in manufacturers' demand is outweighed by the large increase in

⁷Retailers often sign complex contracts with manufacturers, specifying delivery conditions, packages, buy-back policies, branding, and product positioning so that the relevant price may be hard to know ex-ante. In this sense, retailers may be subject to obfuscation practices by manufacturers (see [Ellison and Wolitzky, 2012](#)).

It may be argued that retailers often have long-term contracts with their current suppliers and that it is (almost) costless for them to continue their relations with these suppliers. This is perfectly consistent with our paper. In search theoretic terms, this would imply that the first search is for free. What we argue in this paper is that to understand the content of the contracts between retailers and their suppliers we need to understand the cost retailers need to make to be supplied by *alternative* suppliers. Both search and switching costs are relevant in this regard and we will discuss the similarities and differences between the effects of these costs.

margin.

Under the competitive conditions where double marginalization is not an equilibrium outcome, manufacturers randomize over two prices: the double marginalization price and a higher price where retailers are squeezed. Because retailers incur a search cost to visit the next manufacturer, they also buy and resell at the highest of these two prices. In a competitive market where retail profits are low, a small retail search cost is sufficient to deter retailers from continuing to search. Retail price strategies are deterministic, with a higher price response at the higher wholesale price. The resulting bimodal retail price distribution is, therefore, induced by manufacturers. This result obtains for small, but strictly positive, consumer search cost so that the retailers' price reaction is limited by consumers' reservation price.

The equilibria that exhibit bimodal price distributions are also interesting from a welfare point of view. As the lowest price over which manufacturers randomize is the double marginalization wholesale price, these equilibria are welfare-inferior to the Double Marginalization Equilibrium. To highlight the idea that search frictions at both levels of the retail chain significantly strengthen the Diamond Paradox, we refer to these equilibria as Double Diamond Equilibria. Interestingly, when the consumer search cost approaches zero, Double Diamond Equilibria exist for all values of the other parameters and they converge to the manufacturer choosing the double marginalization wholesale price with arbitrary small probability and setting the consumer reservation price with probability close to one. Thus, what is typically considered essential for competition, namely the combination of many retailers and small search costs, is disastrous for social welfare in vertically related industries where both retailers and consumers have a small search cost. Numerically, we show that the difference in outcomes between Double Diamond Equilibria and the Double Marginalization Equilibrium can be quantitatively substantial. For example, for linear demand, we show that the total surplus generated in these equilibria can be in the order of 40% lower than the total surplus generated in the Double Marginalization Equilibrium (which is already considered to be low), whereas consumer surplus is more than 60% lower! These results suggest that retailer search has important implications that have so far been unexplored.

Together, the Double Diamond Equilibria and the Double Marginalization Equilibrium span the whole parameter space. In these equilibria, consumers and retailers follow standard reservation price strategies. These equilibria are the main focus of our analysis. For certain demand functions, there exists, however, another type of equilibrium where the manufacturers make more profits than in any of the equilibria

mentioned before and where consumers follow a non-reservation price strategy. These equilibria may exist because of the possibility of consumer learning. There exists an important search literature, initiated by [Bénabou and Gertner \(1993\)](#), where consumers learn about the underlying cost structure of firms while observing prices (see also [Dana \(1994\)](#), [Fishman \(1996\)](#), and more recently [Tappata \(2009\)](#) and [Janssen et al. \(2011\)](#)). In these papers, uncertainty about firms' cost is exogenously imposed. In contrast, our paper provides a foundation for retail cost uncertainty through randomized equilibrium pricing of manufacturers in the wholesale market.

It is important to observe that our results can also be re-interpreted in terms of retailers' switching cost. The difference between search and switching costs is that the first relates to an informational friction, while the second does not (see [Wilson, 2012](#)). One may argue that in some environments, retailers have a long-term relation with their supplier and do not find it easy to switch to a different supplier. As we go along, we will discuss how these results can be understood in terms of this alternative interpretation.

Notice also that our results do not depend on whether or not the first search is free (provided the first search is not too costly, prohibiting consumers or retailers to enter the market). Thus, long-term relationships between retailers and their suppliers can be accommodated in our analysis.⁸ Still, search costs are important to understand the content of long-term contracts in such markets, as they determine retailers' outside options.

As far as we know, there is no literature on search by retailers. There is a small, recent literature dealing with consumer search in markets with an explicit vertical structure. In [Janssen and Shelegia \(forthcoming\)](#) and [Janssen and Shelegia \(2015\)](#), all retailers acquire their product from one monopolistic manufacturer, while in [Lubensky \(2013\)](#), a single manufacturer deals exclusively with a large number of retailers who face a competitive fringe of independent sellers. There is no retail search in these vertical models.

Our paper also has interesting implications for empirical studies assessing the classical question of the effect of retail concentration on prices (see, e.g., [Berger and Hannan \(1989\)](#) and [Bikker and Haaf \(2002\)](#) for the banking industry, or [Cotterill \(1986\)](#) for the food industry). In our model, retail margins are lower in Double Diamond Equi-

⁸If the first visit is costly (see [Janssen et al., 2005](#)), then there exists an upper bound on the search cost such that our results hold, as the surplus of consumers and retailers should be positive for the market not to break down. Since most of our important results hold true for small search costs, costly first search is inconsequential.

libria and these equilibria typically exist when there are many retailers in the market. However, this does not imply that competitive retail markets create higher welfare. Manufacturers find it easier to squeeze retailers when retail markets are competitive, resulting in much higher wholesale prices and higher retail prices than in concentrated retail markets.

The rest of the paper is organized as follows. The next section presents the model. Section 3 presents some general characterization results that all equilibria have to satisfy. In particular, we show that the double marginalization equilibrium has the lowest wholesale and retail price in any possible equilibrium. Section 4 shows the conditions that are together necessary and sufficient for a Double Marginalization Equilibrium to exist. Section 5 focuses on Double Diamond Equilibria. It characterizes these equilibria and determines necessary and sufficient conditions for these equilibria to exist. This Section also verifies in some detail the empirical implications of these equilibria and discusses the welfare implications. Section 6 deals with the non-reservation price equilibrium in which manufacturers benefit from active consumer search, while Section 7 treats search cost heterogeneity. Section 8 concludes.

2 The Basic Model

To focus on search frictions in a vertical industry structure, consider a standard market for a homogenous product, supplied by $m \geq 2$ manufacturers. Our arguments do not depend on the manufacturers' cost structure, so we normalize cost to be equal to 0. Manufacturer i (*she*) sells the product for a price w_i per unit to $n \geq 2$ retailers, and each retailer (*he*) j resells the product to consumers at a retail price p_j at no additional cost. There is a unit mass of consumers (often referred to as *they*) and each individual consumer has a demand function $D(p)$ so that they derive a surplus of $S(\tilde{p}) = \int_{\tilde{p}}^{\infty} D(p)dp$ when buying at price \tilde{p} .⁹

The novel feature of our model is that both retailers and consumers engage in costly sequential search in order to discover the relevant prices. Retailers (consumers) have to pay a positive search cost s_R (respectively s_C) for any additional price quotation (either w for retailers or p for consumers) they observe. As we want to understand the different roles played by retailers' and consumers' search costs, we allow $s_R \neq s_C$.

⁹We assume throughout a vertical structure of independent firms signing linear pricing contracts. We believe that this is the natural starting point for any analysis of retailer search. For a discussion of this assumption see Section 8.

For simplicity, we follow most of the search literature and assume that the first price quotation is free, but most of our results continue to hold if the first search is costly. When retailers decide to stop their search and pay a wholesale price w' , their marginal cost is w' for each unit sold. When consumers stop searching, they decide to buy at the lowest price p they have observed and buy $D(p)$ units. For most of the paper, we assume that all individuals in a given layer share the same search cost. In Section 7 we show that most of our main results continue to hold with heterogeneous search costs at both levels.

A retailer's search strategy is characterized by a reservation price ρ_R : at any price $w \leq \rho_R$, the retailer buys, otherwise he continues to search. For the first search, each retailer either visits a manufacturer at random or, in the case of long-term relationships, he visits his current supplier. Reservation price strategies are optimal for the retailer since he does not update his beliefs about the distribution of prices posted by the remaining manufacturers, following any history of observed prices (see, e.g., [Kohn and Shavell, 1974](#)).¹⁰ Consumers face a slightly different problem, since they may learn about the distribution of manufacturer prices from the observed history of retail prices. For the bulk of this paper, however, we consider equilibria in which consumers' optimal search behaviour is also characterized by a reservation price strategy. In such a case, we denote this reservation price by ρ_C : at any price $p \leq \rho_C$, consumers buy, otherwise they continue to search. For the first search, an equal share of consumers visits each retailer. In Section 6 we provide an equilibrium where consumers do not follow a reservation price strategy. The determination of ρ_R and ρ_C depends on the equilibrium features, and we discuss the details in different sections.

The timing of the interaction is as follows. Manufacturers first set their wholesale prices. Retailers then search among manufacturers according to an optimal search strategy. When all retailers have stopped searching, taking their wholesale price as given, they simultaneously set retail prices without knowing wholesale prices of others. Consumers then engage in sequential search among the given retail prices.

Given some wholesale price w , we define the profit-maximizing price for a monopolist retailer as

$$p^m(w) = \arg \max_{p \geq 0} (p - w)D(p). \quad (1)$$

We refer to $p^m(w)$ as the retail monopoly price. Given the retailer's optimal price

¹⁰See [Rothschild \(1974\)](#) for the observation on the (non)optimality of the reservation price rule when the search environment is (not) stable.

$p^m(w)$, a single manufacturer would choose the wholesale monopoly price defined by

$$w^m = \arg \max_{w \geq 0} wD(p^m(w)). \quad (2)$$

We shall assume throughout that both maximization problems are well-defined. In particular, we assume that manufacturers' and retailers' (monopolistic) profit functions are single-peaked.¹¹ We refer to w^m as the wholesale monopoly price and $p^m(w^m)$ as the double marginalization (DM) retail price. Together they induce the DM outcome.

Following the vertical contracting literature (see, e.g., McAfee and Schwartz, 1994) we consider symmetric Perfect Bayesian equilibria satisfying a natural extension of passive beliefs to our environment. Namely, we assume that whenever a given player observes an off-the-equilibrium path action, she puts positive probability on *minimal deviation paths* only. For each information set, the minimal deviation path specifies a set of actions for every other player that induces that information set and such that the number of players whose action does not belong to the support of their equilibrium strategy profile is minimal. Notice that passive beliefs satisfy the minimal deviation property.

Definition 1. *A symmetric perfect Bayesian reservation price equilibrium (RPE) is a wholesale price w^* , a retail search and pricing strategy $(\rho_R^*, p^*(w))$ and a consumer search strategy ρ_C^* such that in every information set, a player's strategy is optimal given the strategies of the other players and beliefs that put zero probability on action profiles that do not belong to a minimal deviation path.¹²*

If consumers visit a retailer who has posted an unexpected retail price, they are not sure which firm has actually deviated from the equilibrium strategy: the retailer they have visited or the manufacturer from which the retailer they have visited visits had procured the product. Out-of-equilibrium beliefs are therefore of some importance to determine the optimal search behaviour of the consumer. If they believe that the retailer has deviated to another retail price, while all manufacturers followed their equilibrium price strategies, then they do not need to update their beliefs about the

¹¹Standard assumptions in $D(p)$ would suffice. For instance, if the elasticity changes smoothly and monotonically with the price, both problems are well-behaved.

¹²A definition like this is often implicitly used in the consumer search literature. Formally, if one does not require reservation prices to be optimal off the equilibrium path, marginal cost pricing may be considered equilibrium behaviour if consumers' reservation prices are equal to marginal cost. By requiring retail and consumer strategies to be optimal for all beliefs that are consistent with a minimal deviation path, the above definition rules out situations like this.

other retailers when deciding whether to continue their search. If, on the other hand, consumers believe that a manufacturer has deviated to set a different wholesale price, then they have to take into account that other retailers may have visited the same deviating manufacturer, bought at the same wholesale price, and sell at the same out-of-equilibrium retail price.

3 General Characterization Results

We first provide a general characterization of some of the properties that all equilibria in this model have to satisfy. This allows us to reduce the space of possible strategy profiles to consider. Our first result shows that in any symmetric equilibrium the firms' prices are not smaller than the DM prices.

Proposition 2. *In any symmetric equilibrium, wholesale and retail prices set by manufacturers and retailers are such that $w^* \geq w^m$ and $p^*(w^*) \geq p^m(w^m)$.*

This is an important result.¹³ It shows that when both consumers and retailers have a positive search cost, we cannot expect competition in the market to result in social welfare levels that are better than those resulting from a market served by a monopolist at both the wholesale and the retail level. Thus, it constitutes a relatively straightforward extension of the Diamond Paradox.

In addition, we show that in any equilibrium, the wholesale monopoly price is always set with some positive probability and if other prices are also set with positive probability, there has to be a non-negligible gap between the wholesale monopoly price and the rest of the support.

Proposition 3. *In any symmetric equilibrium, manufacturers set w^m with some strictly positive probability. In addition, there exists some $\varepsilon > 0$ such that the open set of prices $(w^m, w^m + \varepsilon)$ does not belong to the support of the wholesale price distribution.*

The first part of the Proposition is the combination of two basic observations. First, in any equilibrium the retail price is decreasing in the wholesale price. It follows that if a manufacturer chooses the lowest price in the support of her distribution, she guarantees that both retailers and consumers buy without further search. Second, of all prices that can be on the equilibrium path according to Proposition 1, the wholesale monopoly

¹³With minor changes in the proof, we can extend Proposition 1 to hold for any (potentially non-symmetric) equilibrium.

price gives the manufacturer the highest profits if retailers and consumers buy outright. Thus, if the wholesale monopoly price was not set with positive probability on the equilibrium path, it would be optimal to deviate to it.

For the second part, notice that for any wholesale price sufficiently close to the wholesale monopoly price, retailers are able to charge the retailer monopoly price and, given positive search costs, still sell to all incoming consumers. Since, by definition, this yields lower profits to the manufacturer, such prices are never part of the equilibrium price distribution.

4 Double Marginalization Equilibrium

In our model, Double Marginalization (DM) is the natural counterpart to the monopoly outcome that arises in Diamond's seminal paper. In the strategy profile that leads to the DM outcome, (i) each manufacturer sets the wholesale monopoly price, (ii) each retailer visits one single manufacturer, buys at that wholesale price, and then sets the retail monopoly price, and (iii) consumers visit one single retailer and buy at that retail price. More formally, we have the following. Each manufacturer chooses w^m . Retailers' behaviour as a function of the wholesale price w is given by

$$\sigma_R^r(w) = \begin{cases} \text{buy and set } p^m(w) & \text{if } w \leq \rho_R \text{ and } p^m(w) \leq \rho_C, \\ \text{buy and set } \rho_C & \text{if } w \leq \rho_R \text{ and } p^m(w) > \rho_C, \\ \text{buy and set } w & \text{if } \rho_C < w < \rho_R, \\ \text{continue to search} & \text{if } w > \rho_R, \end{cases} \quad (3)$$

while consumers' optimal search rule $\sigma_C^r(p)$ as a function of the retail price p is given by

$$\sigma_C^r(p) = \begin{cases} \text{buy at } p & \text{if } p \leq \rho_C, \\ \text{continue to search} & \text{if } p > \rho_C. \end{cases} \quad (4)$$

If this strategy profile constitutes an equilibrium, there is no active search and no price dispersion. Retailers are effective monopolists in their incoming demand and, therefore, optimize by setting the monopoly retail price $p^m(w)$.

Definition 4. *A symmetric (pure) strategy profile σ^{DM} is a double marginalization (DM) strategy profile if $\sigma^{DM} = (w^*, \sigma_R^r(w), \sigma_C^r(p))$ is such that each manufacturer sets*

$w^* = w^m$ as in (2), each retailer follows the strategy $\sigma_R^r(w)$ as in (3), and consumers follow the search rule $\sigma_C^r(p)$ as defined by (4). If σ^{DM} is an equilibrium, then we call it a Double Marginalization Equilibrium (DME).

Given the construction of these strategies, it is clear that the only profitable deviation may come from a manufacturer who charges a higher wholesale price at the expense of a lower retailer margin. To derive necessary and sufficient conditions for this deviation not to be profitable, we still have to specify the reservation prices ρ_R and ρ_C . As we mentioned in Section 2, retailers' reservation price is relatively easy to characterize. We can compute the wholesale price $\rho_R > w^m$ such that retailers are indifferent between buying the product at that price and continuing to search for a lower (equilibrium) price w^m that they expect from other manufacturers. Given that, after observing ρ_R a retailer would set a retail price equal to $\min\{\rho_C, p^m(\rho_R)\}$. ρ_R is implicitly determined by

$$(\min\{\rho_C, p^m(\rho_R)\} - \rho_R) \frac{D(\rho_C)}{n} = (p^m(w^m) - w^m) \frac{D(p^m(w^m))}{n} - s_R,^{14}$$

where because of the first random search, consumer demand is evenly split among all retailers. The above equation is rewritten as

$$(p^m(w^m) - w^m)D(p^m(w^m)) - (\min\{\rho_C, p^m(\rho_R)\} - \rho_R)D(\rho_C) = ns_R.$$

The characterization of the consumers' reservation price is more complicated because $\rho_C > p^m(w^m)$. If consumers observe the reservation price (whatever its precise value), then they know that either this retailer has deviated or the manufacturer has deviated that has sold the product to the retailer he has visited. Notice that both interpretations of the deviation are consistent with the minimal deviation property on beliefs as discussed in Section 2.

Assume for now that consumers *blame* the manufacturer for a possible deviation from the equilibrium retail price. In this case, if a consumer continues to search, there is a chance of $\frac{1}{m}$ that the next retailer has bought the product from the same

¹⁴If we were to consider retailer switching cost instead of search cost, the same equation would apply as there is no uncertainty for retailers as regards to what price to expect on the next search in the DM strategy profile.

manufacturer. The consumer reservation price ρ_C is then given by

$$\int_{\rho_C}^{\infty} D(p)dp = \frac{1}{m} \int_{\rho_C}^{\infty} D(p)dp + \frac{m-1}{m} \int_{p^m(w^m)}^{\infty} D(p)dp - s_C,$$

that is,

$$\frac{m-1}{m} \int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \quad (5)$$

We have two reasons to focus on the case in which consumers blame a manufacturer for a possible deviation. First, our main results suggest that retailer search brings about equilibrium outcomes that may be significantly worse than the DM outcome. We do not want this argument to depend on specific choices of out-of-equilibrium beliefs. Later in this section, we show that the most critical deviation to consider is the manufacturer choosing a wholesale price w equal to the consumer reservation price ρ_C , thereby fully squeezing the retail margins. This deviation is more profitable if the reservation price is low, which happens if consumers blame retailers.¹⁵ In other words, if the double marginalization equilibrium is *not* an equilibrium if consumers blame the manufacturer, it is certainly not an equilibrium if consumers blame the retailer. Second, by assuming consumers blame the manufacturer, the next Section provides a clear characterization of all reservation price equilibria. In Section 6, we consider out-of-equilibrium beliefs where consumers blame retailers and show that these beliefs are necessary for other (non-reservation price) equilibria to exist.

We are now in the position to provide a characterization of the conditions under which the Double Marginalization strategy profile is an equilibrium. As argued above, it suffices to focus on potential deviations by manufacturers. Moreover, it suffices to verify whether manufacturers can profitably deviate to a wholesale price that induces retailers to buy and charge the consumers' reservation price (whenever retailers react to a deviation by setting the retail monopoly price, the manufacturer's profit is always lower than in the candidate equilibrium, by definition of the wholesale monopoly price). There are two cases to consider. First, suppose s_C is so large that $\rho_C D(\rho_C) \leq w^m D(p^m(w^m))$.¹⁶

¹⁵If consumers believe the retailer has deviated to an unexpectedly high retail price, then they should expect that all other retailers will set the equilibrium retail price $p^m(w^m)$. Therefore, in this case, the reservation price is given by

$$\int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \quad (6)$$

Comparing expressions (5) and (6), it is clear that if consumers blame retailers, their reservation price is lower than if they blame manufacturers.

¹⁶Note that ρ_C is increasing in s_C and that because of the single peakedness assumption, $\rho_C D(\rho_C)$

It is clear that in this case no deviation is profitable. Whether or not retailers buy from the manufacturer that deviated, the profit the deviating manufacturer makes is smaller than the equilibrium profit $w^m D(p^m(w^m))$. Formally, we can define \bar{w} as the lowest wholesale price such that if retailers buy and react by selling themselves at \bar{w} and consumers react by buying as well, the manufacturer is indifferent between setting this price and the wholesale monopoly price, i.e.,

$$\bar{w}D(\bar{w}) = w^m D(p^m(w^m)). \quad (7)$$

Whenever consumers observe the retail price \bar{w} , their pay-off of buying (resp. continue to search) is given by $\int_{\bar{w}}^{\infty} D(p)dp$ (resp. $\frac{1}{m} \int_{\bar{w}}^{\infty} D(p)dp + (1 - \frac{1}{m}) \int_{p^m(w^m)}^{\infty} D(p)dp - s_C$). Thus, we can define a threshold $\bar{s}_C(m)$ by

$$\bar{s}_C(m) = \frac{m-1}{m} \int_{p^m(w^m)}^{\bar{w}} D(p)dp \quad (8)$$

at which the consumer is indifferent between continuing to search after observing a price \bar{w} and buying. Thus, for $s_C \geq \bar{s}_C(m)$, the manufacturer's deviation is not profitable, even if retailers and consumers react by buying at this price. In that case the DME exists, while for smaller search costs, we have to inquire into the retailers' decision problem.

Second, suppose then $\rho_C D(\rho_C) > w^m D(p^m(w^m))$, or equivalently that $s_C < \bar{s}_C(m)$. In this case if the retailer still buys at ρ_C (even if fully squeezed), which requires that the retailer search cost is larger than the profit she would make if she continues to search, i.e., $s_R > \frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m)) \equiv \bar{s}_R(n)$, it is clearly optimal for the manufacturer to deviate and the DME does not exist. But even if $s_R < \frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m))$, it is still profitable to deviate if, and only if, $\rho_R D(\rho_C) > w^m D(p^m(w^m))$, where ρ_R is defined by

$$(\rho_C - \rho_R) \frac{D(\rho_C)}{n} = (p^m(w^m) - w^m) \frac{D(p^m(w^m))}{n} - s_R. \quad (9)$$

From (9) it follows that

$$\rho_R D(\rho_C) = \rho_C D(\rho_C) - (p^m(w^m) - w^m)D(p^m(w^m)) + ns_R,$$

is decreasing in s_C for all $\rho_C > p^m$.

and so a manufacturer's gain from deviating is positive if

$$\rho_R D(\rho_C) - w^m D(p^m(w^m)) = \rho_C D(\rho_C) - p^m(w^m) D(p^m(w^m)) + n s_R > 0.$$

As the RHS is strictly increasing in s_R and is positive at $\bar{s}_R(n)$, there is a threshold retailer search cost level $0 < s_R^*(m, n, s_C) < \bar{s}_R(n)$ defined by

$$s_R^*(m, n, s_C) = \frac{1}{n} \left(p^m(w^m) D(p^m(w^m)) - \rho_C D(\rho_C) \right), \quad (10)$$

such that for any $s_R \in (s_R^*(m, n, s_C), \bar{s}_R(n))$ partial squeezing of retailers is optimal and a double marginalization equilibrium does not exist. If, however, $s < s_R^*(m, n, s_C)$, the deviation is not profitable and the DM strategy profile constitutes an equilibrium.

From the above, we can characterize the existence of the DME as follows.

Proposition 5. *For any given m and n the Double Marginalization strategy profile is an equilibrium if and only if s_C and s_R are such that (i) $s > \bar{s}_C(m)$ as defined in (8) or (ii) $s < \bar{s}_C(m)$ and $s_R > s_R^*(m, n, s_C)$ as defined in (10).*

Note that the threshold value $s_R^*(m, n, s_C)$ is decreasing in n and increasing in s_C (via ρ_C). Importantly, the threshold value approaches 0 when n becomes very large or when s_C is close to 0. In this case, it is profitable for the manufacturer to deviate for almost all retailer search cost levels. In addition, $\bar{s}_C(m)$ is increasing in m . Thus, in markets that are thought of as being competitive because the search frictions are small and there are many retailers and manufacturers, DM is not an equilibrium.

Figure 2 illustrates Proposition 5. The region to the right of the thick black line indicates the parameter region where the DM strategy profile is an equilibrium. Above the curve $s < s_R^*(m, n, s_C)$ and to the left of $\bar{s}_C(m)$, the strategy profile is not an equilibrium because manufacturers are better off by partially or fully squeezing retailers.

To provide some quantitative illustration of our findings so far, we briefly consider the special case of a linear demand function $D(p) = 1 - p$. As is well-known, the wholesale and retail monopoly prices are equal to $w^m = \frac{1}{2}$ and $p^m(w^m) = \frac{1+w^m}{2} = \frac{3}{4}$, respectively. Accordingly, the profit of manufacturers (resp. retailers) is given by $\frac{1}{8}$ (resp. $\frac{1}{16}$). Social welfare and consumer surplus are given by $\frac{7}{32}$ and $\frac{1}{32}$.

To see for which parameter values this is an equilibrium, we need to compute the consumer and retailer reservation prices, ρ_C and ρ_R , given by (5) and (9), and calculate $\bar{s}_C(m)$ and $s_R^*(m, n, s_C)$. By (5), we derive the consumer reservation price to be

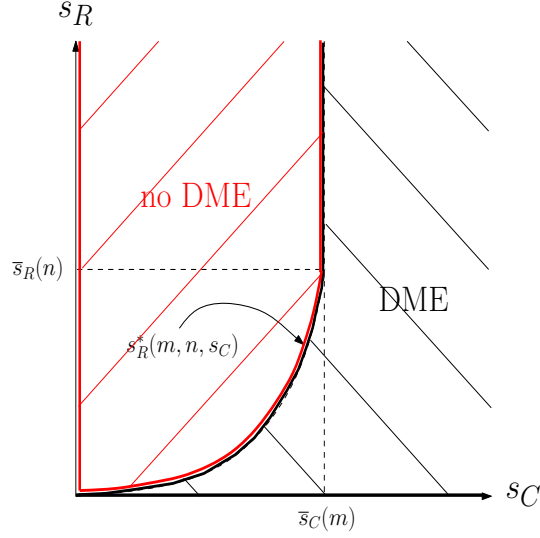


Figure 2: The region where the DME does (not) exist for a given pair (m, n) .

$\rho_C(s_C) = 1 - \frac{1}{4}\sqrt{\frac{m-1-32ms_C}{m-1}}$. It follows that $\bar{s}_C(m) = \frac{4\sqrt{2-5}}{32} \frac{m-1}{m}$. Given $\rho_C(s_C)$, using (9), we derive the retailer reservation price to be $\rho_R(s_R, s_C) = \rho_C(s_C) - \frac{1-16ns_R}{4} \sqrt{\frac{m-1}{m-1-32ms_C}}$, while $s_R^*(m, n, s_C) = \frac{1}{4n} \left(1 - \frac{8ms_C}{m-1} - \frac{1}{4} \sqrt{1 - \frac{32ms_C}{m-1}} \right)$. It can be easily shown that $s_R^*(m, n, s_C)$ is strictly increasing and convex in s_C and strictly decreasing in both m and n . Proposition 5 shows that the DME exists, if and only if, $s_R \leq s_R^*(m, n, s_C)$ or $s_C \geq \bar{s}_C(m)$. In Figure 2 we show this region for $m = n = 2$. In this case $\bar{s}_C(2) = \frac{4\sqrt{2-5}}{64}$, which is approximately 33% of the consumer surplus generated in the DME. Thus, the DM strategy profile is *not* an equilibrium for a large set of relevant parameter values.

5 Double Diamond Equilibria

We now show that manufacturers may partially or fully squeeze retailers *along the equilibrium path* by setting a wholesale price that is larger than the wholesale monopoly price with positive probability. This will result in retail prices that are higher than the DM price and in welfare that is smaller than the welfare generated in the DME. Interestingly, these equilibria exist whenever the DME does not, i.e., when the consumer search cost is small and the retailer search cost is not too small (depending on how many retailers there are in the market). We call these equilibria Double Diamond Equilibria (DDE) because the combination of search costs for consumers and retailers renders market outcomes that are significantly worse than those in the Diamond model.

Moreover, the discontinuity as search costs become negligible is even more severe here than in the original Diamond model. Since manufacturers randomize *in equilibrium* between two prices, consumers' reservation price no longer depends on off-the-equilibrium path beliefs, but is directly pinned down by equilibrium strategies.

The simplest equilibrium generating market outcomes that are worse than the DM outcome has manufacturers randomizing over two wholesale prices: the wholesale monopoly price w^m defined by (2) is set with probability γ and a higher wholesale price $w^{dd} > w^m$ is set with probability $1 - \gamma$. So, manufacturers adopt a mixed strategy given by

$$\sigma_M = \begin{cases} w^m & \text{w.p. } \gamma \\ w^{dd} & \text{w.p. } 1 - \gamma. \end{cases} \quad (11)$$

We know that retailers will react to w^m by setting the retail monopoly price p^m . We also know that in any equilibrium where consumers choose reservation price strategies, it must be the case that retailers choose $p(w) = \min\{p^m(w), \rho_C\}$ (if $w \leq \min\{\rho_R, \rho_C\}$). As the manufacturer has to be indifferent between choosing w^m and w^{dd} (and w^m is the unique maximizer of $wD(p^m(w))$), it therefore has to be that in a Double Diamond equilibrium, retailers react to w^{dd} by choosing ρ_C . Thus, we may have two types of Double Diamond Equilibria: one where with positive probability manufacturers fully squeeze retailers and set $w^{dd} = \rho_C$ and one where they partially squeeze retailers and set $w^{dd} = \rho_R < \rho_C$ with positive probability.

Formally, we define a Double Diamond strategy profile as follows.

Definition 6. *A symmetric (pure) strategy profile σ^{DD} is a Double Diamond (DD) strategy profile if $\sigma^{DD} = (\sigma_M, \sigma_R^r(w), \sigma_C^r(p))$ is such that each manufacturer sets a strategy as in (11), each retailer follows the strategy $\sigma_R^r(w)$ as in (3), and consumers follow the search rule $\sigma_C^r(p)$ as in (4). If σ^{DD} is an equilibrium, then we call it a Double Diamond Equilibrium (DDE).*

We first inquire into the conditions for existence of DDE whereby retailers are fully squeezed. In this case, consumers' reservation price ρ_C has to be equal to \bar{w} as defined in (7). Given that manufacturers now explicitly randomize, ρ_C is defined as follows. Consumers who visit a retailer charging a retail price ρ_C are indifferent between buying and continuing to search if, and only if,

$$\int_{\rho_C}^{\infty} D(p)dp = \frac{1}{m} \int_{\rho_C}^{\infty} D(p)dp + (1 - \frac{1}{m}) \left(\gamma \int_{p^m(w^m)}^{\infty} D(p)dp + (1 - \gamma) \int_{\rho_C}^{\infty} D(p)dp \right) - s_C.$$

To understand the RHS of this expression note that if a consumer continues to search for a lower retail price, there are two cases to consider: (i) with probability $\frac{1}{m}$, another retailer has visited the same manufacturer setting the high wholesale price ρ_C (with the retail price ρ_C as the best response), and (ii) when each other retailer visits one of the other manufacturers, because of manufacturers' randomized pricing, the retailer sets $p^m(w^m)$ with probability γ and ρ_C with probability $1 - \gamma$. Thus, after observing the retail price ρ_C , consumers are indifferent if, and only if,

$$\frac{m-1}{m} \int_{p^m(w^m)}^{\rho_C} D(p) dp = \frac{s_C}{\gamma}. \quad (12)$$

As $\rho_C = \bar{w}$, this equation determines γ , the probability manufacturers choose the wholesale monopoly price, i.e.,

$$\gamma = \frac{s_C}{\frac{m-1}{m} \int_{p^m(w^m)}^{\bar{w}} D(p) dp}.$$

Note that this expression is linear in s_C and, as will be important later, that γ approaches 0 when s_C approaches 0. Note that if $s_C < \bar{s}_C(m)$ as defined in the previous section, this equation will always define γ in such a way that it is smaller than 1.

We next consider the retailer strategy. If a retailer visits a manufacturer with a wholesale price of ρ_C , the best he can do is to buy and sell at ρ_C if

$$\gamma(p^m(w^m) - w^m) \frac{D(p^m(w^m))}{n} \leq s_R. \quad (17)$$

This equation is easily understood: the chance of observing a wholesale price of w^m on the next search equals γ and the expected pay-off that is obtained should be smaller than the search cost.

Thus, a DDE with full squeezing of retailers exists if $s_C < \bar{s}_C(m)$ and

$$\frac{s_C(p^m(w^m) - w^m) D(p^m(w^m))}{\frac{(m-1)n}{m} \int_{p^m(w^m)}^{\bar{w}} D(p) dp} \leq s_R. \quad (13)$$

We have the following result.

Proposition 7. *For any given m and n , there exists a DDE where retailers are fully*

¹⁷If the retailer cost is a switching cost rather than a search cost, and retailers know all manufacturer prices, this condition needs to be modified to $(p^m(w^m) - w^m) \frac{D(p^m(w^m))}{n} \leq s_R$.

squeezed with strictly positive probability if, and only if, $s_C < \bar{s}_C(m)$ and s_R is such that (13) holds.

The lower bound on s_R such that retailers do not want to continue to search goes to 0 if s_C approaches 0 or if the number of retailers n is large. The reason why retailers do not want to continue to search after observing ρ_C if s_C is close to 0 (and s_R is also small, but larger than the threshold value defined by the LHS of (13)) is that the probability with which manufacturers charge the monopoly wholesale price w^m is also close to 0 so that the chance of getting a lower wholesale price and making a profit on the next search is arbitrarily small.

Note that in a full squeezing DDE, γ ranges between 0 and 1, depending on whether s_C is close to 0 or close to $\frac{m-1}{m} \int_{p^m(w^m)}^{\bar{w}} D(p) dp$. For larger s_C values in this range, we have an equilibrium with a "regular" high price and an irregular lower "sales price" that is induced by the manufacturer, very much like what we have observed in Figure 1. At the sales price, retailers' margins are positive (and are in fact equal to the monopoly level given the wholesale price), while they are 0 at the regular price. For low s_C values in this range, we have that the low price is charged most of the time and becomes the regular price.

We next consider the possibility that the manufacturers partially squeeze retailers in a DDE. In this case, it is clear that the following conditions should be satisfied. First, manufacturers should be indifferent between charging w^m and ρ_R :

$$\rho_R D(\rho_C) = w^m D(p^m(w^m)). \quad (14)$$

As the RHS is constant, it follows from the single-peakedness of $pD(p)$ and the fact that the manufacturer has to be indifferent between choosing w^m and $\rho_R < \rho_C$ that consumers' reservation price under partial squeezing has to be smaller than under full squeezing.

Second, after observing ρ_C , consumers should be indifferent between buying and continuing to search:

$$\frac{m-1}{m} \int_{p^m(w^m)}^{\rho_C} D(p) dp = \frac{s_C}{\gamma}.$$

This condition is similar to what we saw before in the full squeezing DDE, the only difference being that ρ_C is now determined differently. Finally, at ρ_R retailers should be indifferent between buying and continuing to search for the lower wholesale price w^m :

$$(\rho_C - \rho_R) \frac{D(\rho_C)}{n} = \frac{\gamma}{n} (p^m(w^m) - w^m) D(p^m(w^m)) + \frac{1-\gamma}{n} (\rho_C - \rho_R) D(\rho_C) - s_R,$$

which reduces to

$$(p^m(w^m) D(p^m(w^m)) - \rho_C D(\rho_C)) = \frac{n s_R}{\gamma}. \quad (15)$$

To inquire when such a partial squeezing DDE exists, consider for any given m and n all pairs (s_C, s_R) such that (10) is satisfied. In the previous section, we have argued that these (s_C, s_R) pairs define the boundary of the region where the DME exists. These (s_C, s_R) pairs satisfying (10) can, however, also be re-interpreted as partial squeezing DDE where $\gamma = 1$. From equation (14), (15) and (12) it is then clear, however, that if $(\rho_R, \rho_C, 1)$ is a solution for the parameters (m, n, s_C, s_R) , then for any $\gamma \in (0, 1)$ it should be the case that (ρ_R, ρ_C, γ) is a solution for the parameters (m, n, s'_C, s'_R) , where $s'_C = \gamma s_C$ and $s'_R = \gamma s_R$. This fact is used in the proof of the next proposition to argue that for small values of s_C and s_R , the partial squeezing DDE exists in the region in between the DME and the full squeezing DDE. Thus, at least one of these types of equilibria always exists. In addition, the next Proposition argues that, for a given set of parameters, the equilibrium is unique in the class of reservation price equilibria where both retailers and consumers follow reservation price strategies.

Proposition 8. *For any given set of parameter values m, n, s_C and s_R there exists a unique RPE that is either of the DM or the DD type.*

In the Introduction, we made some observations about the empirical relevance of Double Diamond Equilibria (DDEs) and about their welfare implications. We will now further detail these points.

Empirical Relevance

The price distribution emerging in a DDE provides a micro-foundation for several regularities of real-world consumer markets. First, the distribution of wholesale prices is bimodal, with a high (regular) price and a lower (sale) price. This pattern has been documented for a number of retail markets and is a salient feature of the Dominick's database (see [Midrigan, 2011](#)).

Second, DDEs exhibit a positive correlation between wholesale and retail prices, since retail price dispersion is due to manufacturers' incentives to randomize over two

different prices. This pattern of contemporaneous correlation has not been documented before. We follow the methodology in [Midrigan \(2011\)](#) and construct a series of regular prices for each of the products sold in one of the supermarkets in Dominick’s Database. A price is deemed regular for a given week if it is the modal price in a time-window around that date. Our model predicts that in a cross-section of products, conditional on the regular price, a higher realized cost (which proxies for wholesale price) induces a higher realized retail price. We then run a simple log-log regression on prices on costs, controlling for the regular price:

$$\log p_{i,t} = \alpha + \beta \log p_{i,t}^R + \gamma \log c_{i,t} + \epsilon_{i,t}. \quad (16)$$

The results are shown in Table 1. As predicted by the model, cost increases are correlated with higher realized prices.¹⁸

Table 1: Estimation Results : Posted Prices, Regular Prices and Wholesale Prices

Variable	Coefficient	(Std. Err.)
ln regular price	0.901***	(0.000)
ln cost	0.096***	(0.000)
Intercept	0.014***	(0.000)

Third, our model predicts an incomplete pass-through from wholesale to final prices, independently of the shape of the demand curve. This prediction is also confirmed by our data. In particular, we run a simple fixed-effect regression of observed prices on costs:

$$p_{i,t} = \alpha + \beta c_{i,t} + \delta_t + \mu_i + \epsilon_{i,t} \quad (17)$$

The estimated coefficient is $0.515 < 1$ ($p < 0.001$) (see Table 2). Therefore, an increase in wholesale prices reduces observed markups.

Welfare

DDEs yield very inefficient outcomes, and this inefficiency is largest in *competitive* conditions. Indeed, DDEs exist when s_C is small and n is relatively large (or s_R is

¹⁸We also run separate regressions for all 29 product categories in the sample and the estimated elasticities range from 0.03 in toothpastes to 0.34 in frozen dinners. Similar results are obtained from a set of category-specific Probit regressions and Linear Probability fixed effects on the probability of sales given costs.

Table 2: Estimation Results : Pass-Through

Variable	Coefficient	(Std. Err.)
cost	0.515	(0.001)
date	0.001	(0.000)
Intercept	1.588	(0.002)
Product Fixed Effects	(Y)	

not too small). Further, as consumer search cost decreases, by (12), the equilibrium value of γ also decreases. In particular, when s_C goes to zero, γ converges to 0 and so the randomized market prices converge to $w = p = \rho_C$. Competitive forces, far from improving equilibrium outcomes, lead to equilibria with *lower* social welfare in markets with search frictions in *both layers* of the product chain. In particular, the social welfare loss becomes the largest when the consumer search cost approaches zero.

How much social welfare in the DDEs is lower than that in the DME depends on the shape of the demand function, the number of firms, and the level of search costs. We now illustrate the size of the effects by considering linear demand $D(p) = 1 - p$.

From the above, it follows that a full squeezing DDE exists if $s_C < \bar{s}_C(m) = \frac{4\sqrt{2}-5}{32} \frac{m-1}{m}$ and $s_R > \frac{2m}{(4\sqrt{2}-5)(m-1)n} s_C$. From (7) it follows that in this equilibrium $\rho_C = \frac{2+\sqrt{2}}{4}$ and $\gamma = \frac{s_C}{\frac{m}{m-1} \int_p^{\rho_C} D(p) dp} = \frac{32}{4\sqrt{2}-5} \frac{m-1}{m} s_C$. The region where this equilibrium exists is also depicted in Figure 3. Total surplus and consumer surplus are decreasing in s_C . When s_C approaches 0, total and consumer surplus are at their lowest level and equal to approximately 0.135 and 0.01, respectively, which is around 38% and 66% lower than the corresponding figures for the double marginalization outcome, which is already known to generate low welfare levels. The remaining area in Figure 3 is where the partial squeezing DDE exists.

Figure 4 shows how total surplus and consumer surplus vary with changes in s_C and s_R for a given level of the other search cost. In the left panel we show the dependence on s_C for $s_R = 0.016$ that is such that the equilibrium gradually changes from a DDE with full squeezing to one with partial squeezing and, finally for large enough values of s_C , to the DME. The right panel of Figure 4 shows a similar picture for the impact of s_R for a value of s_C equal to 0.005. Notice that the impact of search cost on welfare is large and that the two search costs have opposite effects: welfare is the lowest for large values of s_R and small values of s_C .

The panels clearly show the discontinuity of total surplus and consumer surplus at a search cost equal to 0 and how the interaction between retail and consumer search

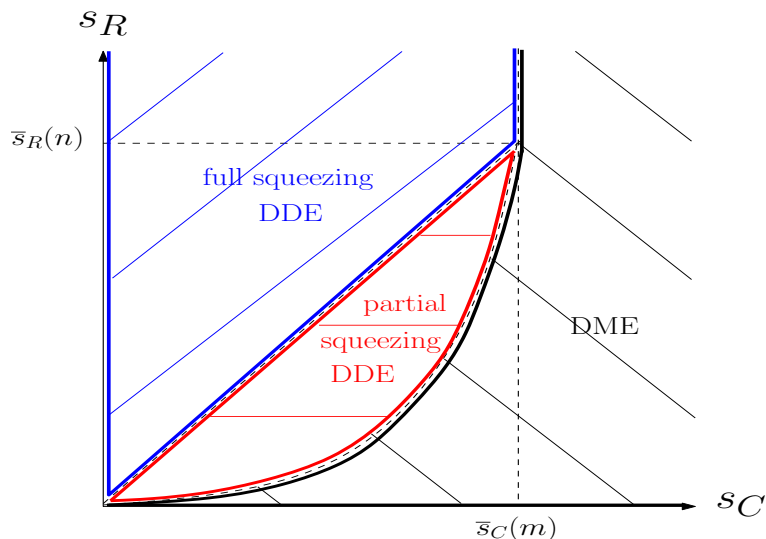


Figure 3: Equilibrium for Different Search Costs.

severely affects this discontinuity. If both retailer and consumer search costs are equal to 0, we have marginal cost pricing and total surplus and consumer surplus being both equal to 0.5. With retail search costs being equal to 0 and consumer search costs approaching 0, we have the Diamond Paradox with monopoly pricing resulting in total surplus and consumer surplus being equal to 0.375 and 0.125, respectively. With both search costs approaching 0 at a ratio such that the full squeezing DDE prevails, we have a limiting total surplus and consumer surplus of 0.135 and 0.01.

To sum up, in our model, seemingly competitive conditions yield equilibria with large manufacturer's markups and substantial welfare losses. This theoretical prediction is consistent with recent evidence on the Chilean supermarket industry (see [Noton and Elberg \(2015\)](#)). Using directly estimated manufacturer's costs, they find substantial markups for *all* coffee suppliers of big-box supermarkets.¹⁹

6 Non-Reservation Price Equilibria

Until now, we have considered reservation price equilibria (RPEs) where prices are such that retailers and consumers immediately buy at the first price they observe. There is no active search in these equilibria and the manufacturers' profits are always equal to the profit in the DM outcome. We will now show that for certain demand functions

¹⁹The bulk of the literature *infers* manufacturer's costs from equilibrium pricing conditions. See, in particular, [Villas-Boas \(2009\)](#).

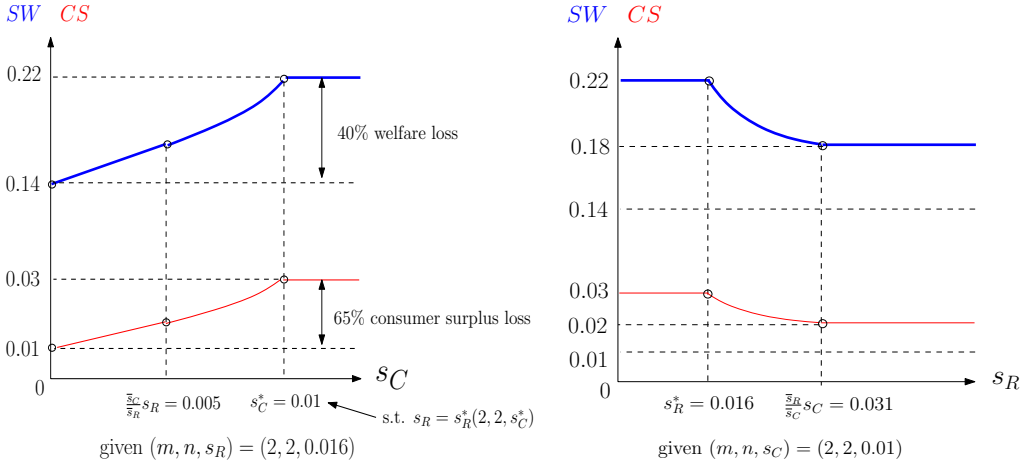


Figure 4: Welfare and Consumer Surplus as a Function of Search Costs.

there exists another type of equilibrium with active search among consumers. This equilibrium is interesting for two reasons. First, by randomizing between the wholesale monopoly price and a higher price at which consumers continue to search with probability α , manufacturers make more profits than in a DME where they choose a pure strategy (or any other reservation price equilibrium). Manufacturers randomize in such a way that consumers have an incentive to search and by doing so, they increase the number of consumers who buy from them at the wholesale monopoly price. Second, the non-reservation price equilibrium Pareto-dominates the DDE.

We illustrate the possibility of this type of equilibrium by building on the simple structure of the full squeezing DDE where the manufacturer randomizes between w^m and a price $\hat{w} > w^m$ with probabilities γ and $(1 - \gamma)$, respectively, and the retailers buy at wholesale price $w \leq \hat{w}$ and react by setting the retail monopoly price $p^m(w^m)$ and \hat{w} .²⁰ Since the different search paths that can arise under an arbitrary number of retailers yield complicated expressions, we concentrate on the simplest case with two retailers, *i.e.*, $n = 2$. The main difference with the analysis so far is that consumers choose a non-reservation price strategy (cf., Janssen et al., 2014) where they buy at prices $p \leq \rho_C$, randomize between continuing to search and buying with probabilities α and $(1 - \alpha)$ after observing $\hat{w} > \rho_C$ and continue to search at all other prices.

To understand the nature of non-reservation price equilibrium (NRPE) in the present

²⁰One could also build other non-reservation price equilibria, for example where retailers are partially squeezed, but our construction below is probably the simplest type of non-reservation price equilibrium. The purpose here is not to provide a complete characterization of all equilibria that may exist, but rather to illustrate that there may well be other types of equilibria than the one we focussed on so far.

context, it is important to return to the fact that the reservation price may depend on out-of-equilibrium beliefs of the consumers regarding who they held responsible for a deviation. In Section 4 we have argued that if consumers blame retailers then the reservation price can be low. To get a NRPE, it has to be the case that the reservation price is off-the-equilibrium path and that consumers blame the retailer for the deviation.²¹ Moreover, in the present context, consumers can also hold beliefs about the wholesale price the deviating retailer has obtained. In particular, it is more likely that the retailer has deviated to a price $p \in (p^m, \hat{w})$ after having bought at a wholesale price of w^m , rather than after a wholesale price of \hat{w} as in the latter case the retailer would make a loss by selling at a price below cost. Thus, the consumer believes that the deviating retailer bought at a wholesale price of w^m so that the pay-off of continuing to search equals

$$\frac{1}{m} \int_{p^m(w^m)}^{\infty} D(p)dp + \frac{m-1}{m} \left[(1-\gamma) \int_{\rho_C}^{\infty} D(p)dp + \gamma \int_{p^m(w^m)}^{\infty} D(p)dp \right] - s_C.$$

which (importantly) is larger than the continuation pay-off if they had searched after observing \hat{w} .²² Thus, the consumer reservation price ρ_C such that the consumer buys at all $p \leq \rho_C$ equals

$$\frac{1 + (m-1)\gamma}{m} \int_{p^m(w^m)}^{\rho_C} D(p)dp = s_C. \quad (18)$$

To make this into an equilibrium, we need three conditions to hold along the equilibrium path. First, the manufacturer should be indifferent between charging the two different prices. If she charges w^m , she will get the following demand. With probability $(\frac{1}{m})^2$ both retailers will visit her and all consumers will buy. With probability $2(\frac{1}{m})(1 - \frac{1}{m})$ exactly one of the retailers will visit her and then a fraction of $\frac{1}{2}(1 + \alpha(1 - \gamma))$ of all consumers will buy from this retailer. If she charges \hat{w} , the only difference is when exactly one retailer is visiting her, in which case a fraction of $\frac{1}{2}(1 - \alpha\gamma)$ of all consumers will buy from this retailer. Thus, the indifference condition

²¹The first statement follows from the fact that ρ_C cannot be equal to $p^m(w^m)$, while if it were equal to \hat{w} both manufacturers and retailers would have an incentive to slightly undercut. The second statement follows from the fact that in this case the reservation price would be implicitly defined by $\frac{2s_C}{\gamma} = \int_{p^m(w^m)}^{\rho_C} D(p)dp$ so that in fact $\rho_C = \hat{w}$ and the previous argument cuts in.

²²One could go one step further and say that it is more likely that the retailer has deviated to $\hat{w} - \varepsilon$ after having observed a manufacturer price of w^m , then after a manufacturer price of \hat{w} —as in the latter case it makes a loss.

for the manufacturer is

$$(m + (m - 1)\alpha(1 - \gamma))w^m D(p^m(w^m)) = (m - (m - 1)\alpha\gamma)\widehat{w}D(\widehat{w}). \quad (19)$$

The second condition requires that at the high price \widehat{w} consumers are indifferent between searching and buying. As before, the condition is given by

$$\frac{m - 1}{m} \int_{p^m(w^m)}^{\widehat{w}} D(p) dp = \frac{s_C}{\gamma}.$$

The third condition is that retailers should not continue to search after observing \widehat{w} . If a retailer deviates and continues to search, then there is a probability γ that he will find a lower price at one of the other manufacturers. In that case, he sells to all consumers who visit him first. In addition, due to active consumer search, he can sell to some other consumers who visit the other retailer first, depending on the manufacturer the other retailer visited first. Basically, there are three kinds of manufacturers to be considered: (i) the same manufacturer that he visited first before continuing to search (which happens with probability $\frac{1}{m}$); (ii) the same manufacturer that he visited after continuing to search (which happens with probability $\frac{1}{m}$); (iii) one of the other manufacturers (which happens with probability $1 - \frac{2}{m}$). Since some consumers who first visited the other retailer continue to search and visit him in cases (i) and (iii) but not in case (ii), continuing to search gives him the pay-off of

$$\gamma \left(\frac{1}{m} \frac{1 + \alpha}{2} + \frac{1}{m} \frac{1}{2} + \frac{m - 2}{m} \left[\frac{\gamma}{2} + (1 - \gamma) \frac{1 + \alpha}{2} \right] \right) (p^m(w^m) - w^m) D(p^m(w^m)).$$

As they get a profit of 0 if they buy, the retailers' condition not to continue to search reduces to:

$$\frac{\gamma}{2m} (m + \alpha [1 + (m - 2)(1 - \gamma)]) (p^m(w^m) - w^m) D(p^m(w^m)) < s_R. \quad (20)$$

These are the conditions that should hold on the equilibrium path when the manufacturer chooses w^m or \widehat{w} . To see when it is not optimal for a manufacturer to deviate, we have first to develop the optimal strategy of retailers given such a deviation, and then to consider the optimal deviation profit of a manufacturer. The analysis around these considerations is provided in the Appendix, where we show that for some demand functions, most notably for linear demand, all conditions for a NRPE to exist

are satisfied.

Proposition 9. *In a vertical industry structure with two retailers, there exists parameter values m, s_C and s_R and a demand function $D(p)$ such that a NRPE exists. Manufacturers make more profit than in any RPE. If a NRPE exists, then generically, there is a continuum of them.*

As only two conditions have to hold with equality and there are three "free parameters" that are determined endogenously, namely α, γ and \hat{w} , it is clear that if there exists an equilibrium for given parameter values such that all inequalities hold strictly, then there must also be another equilibrium in the neighbourhood of these parameter values. That is why generically, there is a continuum of non-reservation price equilibria (NRPEs) if there exists at least one.

We can show the following relationship between a NRPE and a fully squeezing DDE.

Proposition 10. *If a NRPE exists, it coexists with and is Pareto-superior to the full squeezing DDE.*

The intuition why the NRPE is Pareto-superior to the fully squeezing DDE is the following. In the NRPE, manufacturers induce consumers to actively search by randomizing wholesale prices. Active search increases the expected profits of manufacturers. In addition, manufacturers set the wholesale monopoly price with a higher probability than in the fully squeezing DDE, i.e., denoting γ^{non} (resp. γ^{DD}) the probability with which the wholesale monopoly price is set in the two respective equilibria, we have $\gamma^{non} > \gamma^{DD}$. Thus, since retailers are fully squeezed at a high wholesale price in both equilibria, the higher probability to get the wholesale monopoly price gives them higher expected profits. Furthermore, since retailers are more likely to set the retail monopoly price, consumers also receive a higher surplus. Note that Proposition 10 can be extended to the general case of multiple manufacturers and multiple retailers.

To illustrate these two Propositions, a NRPE exists for the linear demand function $D(p) = 1 - p$ and parameter values $(s_C, m) = (0.009, 10)$ and $s_R \geq 0.016$. For these parameters, there is an equilibrium where $(\gamma, \alpha, \hat{w}) \approx (0.5, 0.022, 0.85)$ and where $\rho_C = 0.828 (< \bar{w} \approx 0.85)$. For these parameter values a fully squeezing DDE also exists. The parameter region under which the NRPE exists given $m = 10$ is depicted in Figure 5.

Table 3 summarizes the expected profit for the manufacturers and the retailers and consumer surplus for both NRPE and full squeezing DDE in comparison to the corresponding values in the DM outcome. Each value is expressed as a percentage

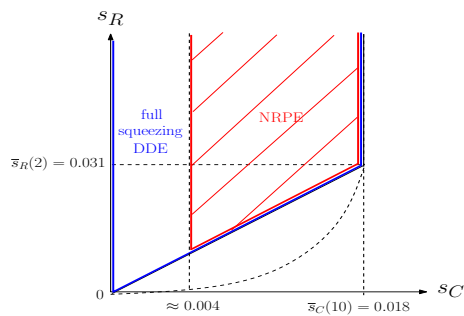


Figure 5: An Approximated Parameter Region of the NRPEs given $m = 10$.

of the corresponding value in the DM outcome. Note that per Proposition 6, if a NRPE exists, there is a continuum of them, which exists in a neighbourhood of the full squeezing DDE. It is easily verified that all NRPE constitute a Pareto improvement over the full squeezing DDE.

Table 3: Welfare Comparison across Equilibria.

	Full Squeezing DDE	NRPE
Manufacturer's Profits	100%	(100%,102.1%]
Retailers' Profits	48.7%	(48.7%,52.9%]
Consumer Surplus	66.3%	(66.3%,70.2%]
Social Welfare	80.5%	(80.5%,83.2%]

7 Search Cost Heterogeneity

So far, we have assumed that all consumers have the same search cost and also that all retailers have the same search cost. In this extension, we consider to what extent our results can be generalized to allow for search cost heterogeneity at both levels. It is not our intention to provide a full analysis under search cost heterogeneity, as it is clear that this is beyond the scope of this paper. Rather, we want to demonstrate that the two types of equilibria (DME and DDE) we have discussed in this paper continue to exist if there is some search cost heterogeneity.

For the DME to exist, search costs (s_R, s_C) have to be large enough so that both manufacturers and retailers set their respective DM prices and make profits equal to $\frac{1}{m}w^m D(p^m(w^m))$ and $\frac{1}{n}(p^m(w^m) - w^m)D(p^m(w^m))$, respectively. Now, if any consumer i faces a sufficiently high search cost ($s_{C_i} \geq \bar{s}_C(m)$), then regardless of the search cost distribution of retailers, DM remains an equilibrium. On the other hand, whenever

some consumers have a search cost s_C smaller than $\bar{s}_C(m)$, the DME may continue to exist, but the analysis is somewhat more subtle and it is clear that whether or not the DME exists depends on the search cost distribution of both consumers and retailers.

Consider then the possibility of a DDE existing under search cost heterogeneity. Fix a pair (s'_R, s'_C) such that a DDE exists if all retailers and consumers had the same search cost. Consider then a search cost distribution such that all consumers have a search cost larger than s'_C and all retailers have a search cost larger than s'_R . Given prices, these search cost distributions translate into distributions of reservation prices ρ_C and ρ_R respectively. Denote these distributions by $F_C(\rho_C)$ and $F_R(\rho_R)$ with lower bounds of the support given by ρ'_C and ρ'_R respectively (corresponding to s'_C and s'_R). In a partial squeezing DDE whereby all retailers, resp. all consumers, have the same search cost s'_R , resp. s'_C , manufacturers make a profit equal to $\rho'_R \frac{D(\rho'_C)}{m}$, while retailers' profit equals $(\rho'_C - \rho'_R) \frac{D(\rho'_C)}{n}$ if they buy at ρ'_R and sell at ρ'_C . A full squeezing DDE is a special case where $\rho'_C = \rho'_R$. We shall now inquire whether under search cost heterogeneity it is possible that we have an identical equilibrium outcome with manufacturers and retailers setting the same prices ρ'_R and ρ'_C and all retailers and consumers buying immediately. First, it is clear that consumers will not change their behaviour and will keep on buying immediately as their search costs are not smaller than s'_C . Second, retailers will also continue buying, but they may set a different price. For a given wholesale price ρ'_R , their profit $\pi_R(p)$ of setting a price p different from ρ'_C (given that their competitors choose ρ'_C or $p^m(w^m)$) is $\pi_R(p) = (1 - F_C(p))(p - \rho'_R) \frac{D(p)}{n}$, with $F_C(p) = 0$ for $p < \rho'_C$. To see this, note that if a retailer sets a price $p < \rho'_C$ all consumers will continue to buy, while if he sets a price $p > \rho'_C$ only a fraction $1 - F_C(p)$ will buy, namely those consumers who have a reservation price $\rho_C > p$. It is then clear that it is certainly not optimal to set a price smaller than ρ'_C , while it is *not* optimal to increase the price above ρ'_C if $\partial\pi_R(p)/\partial p < 0$, or in other words, if

$$(1 - F_C(p)) \left[(p - \rho'_R) \frac{\partial D(p)}{\partial p} + D(p) \right] - f_C(p)(p - \rho'_R)D(p) < 0 \quad \text{for } p \geq \rho'_C.$$

If $\rho'_C = \rho'_R$, i.e., if we are in a full squeezing DDE, then this condition should also hold at $p = \rho'_C$. However, this condition is never met at $p = \rho'_R = \rho'_C$. Thus, any full-squeezing DDE cannot exist under consumer search cost heterogeneity as retailers will have an incentive to deviate and raise prices above the wholesale price ρ'_R . In a partial squeezing DDE, however, the condition can be fulfilled if the hazard rate of the

consumer reservation price distribution satisfies

$$\frac{f(p)}{1 - F(p)} > \frac{(p - \rho'_R) \frac{\partial D(p)}{\partial p} + D(p)}{(p - \rho'_R) D(p)}.$$

It is clear that this condition is more easily satisfied the larger the retailers' margin.

Finally, we need to check whether manufacturers want to deviate. For given strategies of retailers, consumers and other manufacturers, a manufacturer's profit $\pi_M(w)$ of setting a price w different from ρ'_R (given that their competitors choose ρ'_R or w^m) equals $\pi_M(w) = (1 - F_R(w))w \frac{D(\rho'_C)}{m}$, with $F_R(w) = 0$ for $w < \rho'_R$. The interpretation is similar to what we had above for the retailer. Thus, it is clearly not optimal to set a price smaller than ρ'_R , while it is not optimal to increase the price above ρ'_R if $\partial \pi_M(w) / \partial w < 0$, or in other words, if

$$(1 - F_R(w)) - f_R(w)w < 0,$$

for all $w > \rho'_R$, which is a condition on the hazard rate of the retailers' reservation price distribution.

Thus, we conclude that both the DME and the DDE where retailers are partially squeezed do not depend on all retailers and all consumers having one and the same search cost. These equilibria also exist when the search cost heterogeneity satisfies some distributional requirements that are characterized above in terms of distributional requirements on the respective reservation prices.

8 Discussion and Conclusion

This is the first paper to consider costly retailer search. We show that in conjunction with costly consumer search, retailer search has important implications for the functioning of markets. From a welfare perspective, our main result is that markets with small, but positive consumer search cost will produce outcomes that are not better, and often much worse, than the double marginalization outcome. This is especially true when there are many manufacturers and many retailers. Interestingly, the lower bound on retailer search cost for which this is true converges to zero, when the consumer search cost approaches zero or when the number of retailers becomes large. That is, circumstances that we often think of as favouring competitive outcomes may actually

generate very low welfare levels. This is true even if there is some heterogeneity in consumer search cost. The discontinuity that is at the heart of the Diamond Paradox is considerably strengthened when we consider the interaction between manufacturers and retailers in search markets.

The welfare results of our paper are surprising and disturbing. As already mentioned, the direct evidence on the size of manufacturer's margins in vertically related industries is very scarce. However, the recent paper by (Noton and Elberg, 2015) suggests that markups for producers of fairly homogenous goods are substantial. Further, notice that even if we had evidence suggesting that prices are close to marginal cost at the retail level, manufacturer margins may be substantial. Indeed, retail margins in the Double Diamond Equilibria are lower than those in the Double Marginalization Equilibrium, but the welfare generated by the Double Diamond Equilibria is much lower.

From an empirical point of view, our paper generates a new and interesting way to explain retail price distributions. In a given store, the distribution of prices for a given product tends to be concentrated around two price levels: a regular price and a sales price. Existing models of price dispersion typically fail to generate this bimodal nature of the (retail) price distribution. Our model also suggests that it is manufacturers who induce sales by offering discounts to retailers. While there is surprisingly little empirical research on the interaction between retail and wholesale prices, we have provided some suggestive evidence that these decisions are very much related. These results draw attention to the importance of studying price fluctuations at both levels simultaneously and call for a more ambitious empirical study of pricing along the vertical product chain with search frictions.

Our paper also points in the direction of taking search frictions seriously when doing horizontal merger analysis at both the retail and wholesale level. More concentration at the retail level potentially gives retailers more incentive to continue to search for better price offers from manufacturers. Markets with relatively low consumer search costs tend to end up less likely in a Double Diamond Equilibrium if the number of retailers is small. Also, the number of manufacturers may affect consumer reservation prices, as with fewer manufacturers it is more likely that alternative retailers will be supplied by the same manufacturer, making it less beneficial for consumers to search and, therefore, not optimal for manufacturers to deviate. It should be noted, though, that our results are derived in a model where it is the manufacturer that has the power to propose the terms of trade with retailers. It would be interesting to reverse this

power and to investigate how our conclusions change when retailers have the power to determine wholesale conditions (as is certainly true in some markets).

We have focused on a simple vertical industry with linear pricing contracts between independent firms. It is not difficult to see that if the first search is costly, allowing for two-part tariffs will result in a market break down as the manufacturers cannot commit to a contract where retailers make a profit. In a sense, this would strengthen our result of the inefficiency of the market, but one could also argue that sequential search models are not well-suited to studying non-linear contracts.²³ Allowing for vertical integration provides firms with incentives to charge simple monopoly prices at the retail level. The reason not to consider vertical integration is that there are many markets where retailers carry products of many different manufacturers, creating barriers for vertical integration. When retailers carry many different products, multi-product search is important, and it would be interesting to combine the analysis of [Rhodes \(forthcoming\)](#) and [Zhou \(2014\)](#) with ours.

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²³If the first search is costless, two-part tariffs would give rise to double marginalization. However, this would create high risks for the company that is supposed to pay the high fixed fee. These companies may not be willing to bear these risks in a world where there is high uncertainty about demand.

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A Appendix: Proofs

Proof of Proposition 2

Consider an arbitrary symmetric (possibly mixed) equilibrium strategy profile of manufacturers, retailers, and consumers. Let \underline{w} be the lower bound of the wholesale price distribution. Without loss of generality, we assume that given any symmetric strategies of manufacturers and retailers, there exists a maximum price ρ_C such that for any retail price that is lower than ρ_C , consumers do not continue to search and buy at that price.

First of all, we show that if retailers observe the minimum wholesale price $\underline{w} \leq \rho_C$, it is optimal for retailers to set the optimal retail price $p^m(\underline{w})$ because consumers would not search at that price. This corresponds to Lemma 11 below. Next, given Lemma 11, we show that if $\underline{w} < w^m$, manufacturers have an incentive to unilaterally deviate from \underline{w} to a slightly higher wholesale price. This corresponds to Lemma 12 below. Thus, Lemmas 11 and 12 imply that $w \geq w^m$ (resp. $p \geq p^m(w^m)$) holds in any equilibrium strategy profile.

Lemma 11. *Assume that manufacturers adopt a symmetric strategy whereby they put positive probability on a price $\underline{w} \in (0, w^m)$. If a retailer visits a manufacturer setting \underline{w} , it is optimal for the retailer to buy the product and set the retail price at $p^m(\underline{w})$.*

Proof. First of all notice that in such a strategy profile, $\rho_C > p(\underline{w})$, since consumers should be compensated for their search costs. Therefore, for any price in $(p(\underline{w}), \rho_C)$ retailers' profits are just proportional to per-consumer profits. Since per-consumer profits are single-peaked, if $p(\underline{w}) < p^m(\underline{w})$, increasing the price leads to higher profits. On the other hand, any $p(\underline{w}) > p^m(\underline{w})$ is dominated. Thus, $p(\underline{w}) = p^m(\underline{w})$. \square

Lemma 12. *In any equilibrium, $\underline{w} \geq w^m$.*

Proof. Assume to the contrary that there exists a symmetric equilibrium such that the lower bound of the wholesale prices in the strategy of manufacturers is $\underline{w} \in [0, w^m)$. Notice that whenever retailers visit a manufacturer setting the minimum wholesale price \underline{w} , they buy the product from the manufacturer and set the retail price $p^m(\underline{w}) < \rho_C$ to consumers,²⁴ and then consumers visiting the retailers buy the product.

Suppose that a manufacturer deviates from \underline{w} to a slightly higher wholesale price w that is close enough to \underline{w} . Since w is sufficiently close to \underline{w} , $p^m(w) < \rho_C$ holds. Since $\underline{w} < w < w^m$ and $p^m(\underline{w}) < p^m(w) < \rho_C$ hold, by single-peakedness of the profit function, it follows that $\underline{w}D(p^m(\underline{w})) < wD(p^m(w))$. Thus, the deviating manufacturer can slightly increase the profit by setting a slightly higher wholesale price w .

Since we can repeat the above argument until the minimum wholesale price \underline{w} reaches the wholesale monopoly price w^m , it implies that manufacturers never set a wholesale price $\underline{w} \in [0, w^m)$. \square

Proof of Proposition 3

By Proposition 2, we do not need to consider a case where a manufacturer sets some wholesale price $w < w^m$ with some probability.

Assume to the contrary that there exists a symmetric equilibrium such that the lower bound of the wholesale prices in the strategy of manufacturers is $\underline{w} > w^m$. When a manufacturer sets \underline{w} , a visiting retailer buys the product at \underline{w} and sells the product to consumers at $p^m(\underline{w})$ due to Lemma 11, and then a visiting consumer buys the product at $p^m(\underline{w})$ because of $p^m(\underline{w}) < \rho_C$. Given this, each manufacturer has an incentive to deviate to the wholesale monopoly price w^m for which visiting retailers buy and sell at the retail monopoly price $p^m(w^m)$, and then visiting consumers buy at $p^m(w^m)$. This gives the deviating manufacturer the profit given by $w^mD(p^m(w^m))$ that is larger than

²⁴The inequality follows because $p^m(\underline{w})$ is the minimum retail price and consumers have a positive search cost.

the profit under \underline{w} given by $\underline{w}D(p^m(\underline{w}))$ due to single-peakedness of the profit function. Thus, the deviation is profitable, a contradiction.

Next, we show that manufacturers do not charge $w' = w^m + \varepsilon$ for a sufficiently small $\varepsilon > 0$. From above, we know that manufacturers set w^m with some positive probability. Given this, assume to the contrary that there exists a symmetric equilibrium such that manufacturers charge a slightly higher wholesale price $w' = w^m + \varepsilon$ with a positive probability for a sufficiently small $\varepsilon > 0$. Since w' is sufficiently close to w^m , a visiting retailer buys the product at w' without continuing to search due to search cost $s_R > 0$ and sells to consumers at $p^m(w')$ that is a little bit above $p^m(w^m)$ because consumers visiting the retailer buy the product at $p^m(w')$ without continuing to search due to search cost $s_C > 0$. From above, the manufacturer setting w' gets the profit given by $w'D(p^m(w'))$ that is smaller than the profit under w^m given by $w^mD(p^m(w^m))$. Thus, each manufacturer has an incentive to deviate to w^m , a contradiction.

Proof of Proposition 8

We first restate the Proposition in terms of admissible beliefs. As explained in the text, we shall impose the Minimum Deviation Property. See Appendix B for details.

Lemma 13. *There exists a unique RPE Outcome for any belief system satisfying the Minimal Deviation Property.*

Proof. First, in a RPE, there cannot be active search. We show this for the case of consumers, but the same argument works for retailers. For a contradiction, assume that there exists a set of equilibrium retail prices P such that consumers search at P and let \bar{p} be the maximum of those prices. Clearly, by the reservation price property, \bar{p} is also the maximum equilibrium price. Hence, demand at \bar{p} is zero. Hence, either the wholesale price is also \bar{p} or the retailer could profitably deviate to a lower price. But a wholesale price of \bar{p} cannot be optimal for a manufacturer who could charge the double marginalization price. Thus, there is no active search. We are left with two cases to consider. First, if there is price dispersion in equilibrium, both retailers' and consumers' reservation prices are pinned down uniquely in any belief system satisfying the MDP. Since consumers' demand is $D(p)$ for any $p \leq \rho_C$, the retailers' optimal price is $\max\{p^m(w), \rho_C\}$ if $w \leq \rho_R$, $p = w$ otherwise (provided that $w \leq \rho_R$ holds and retailers do not continue to search). This implies that retailers' demand is proportional to $D(p^m(w))$ for any $w < \min\{\rho_R, \rho_C\}$. Clearly, if $w < \min\{\rho_R, \rho_C\}$, then $p(w) = p^m(w)$.

Otherwise, $p(w) = \min\{\rho_R, \rho_C\}$. Second, if there is no price dispersion, Proposition 1 implies that the unique equilibrium outcome is the DME outcome. Next, notice that DM is an equilibrium outcome, if and only if, there is no profitable deviation for a manufacturer to charge a higher price than the wholesale monopoly price. Hence, if there exists a pure-strategy equilibrium there does not exist an equilibrium with price dispersion and vice versa. Hence, a RPE is unique. \square

Proof of Proposition 9

We specify strategies for all the players and then verify that they constitute an equilibrium.

Manufacturers randomize between the wholesale monopoly price w^m and a higher price \hat{w} with probabilities γ and $1 - \gamma$, respectively. Retailers buy and choose $p^m(w)$ if the manufacturer they have visited sets w such that $p^m(w) \leq \rho_C$, and choose either ρ_C or \hat{w} (to be determined below) if the manufacturer they visit sets a price w such that $p^m(w) > \rho_C$. Finally, consumers buy at any price $p \leq \rho_C$, randomize whether to buy or continue their search if they observe a price $p = \hat{w}$ (so that $1 - \alpha$ is the probability with which they buy) and always continue their search at a price $p > \hat{w}$. If the consumer has visited both manufacturers and both of them have a price $p = \hat{w}$, the consumer simply buys at the last store she visited.

We next have to make sure that deviations are not profitable. Given the retailer strategy, it is clear that if it is optimal for the manufacturer to deviate to a price $w \in (w^m, \hat{w})$ then it must be the case that retailers prefer to set a price ρ_C rather than \hat{w} , for otherwise manufacturers could obtain higher profits but setting \hat{w} themselves. Suppose that the other retailer would also choose ρ_C if she meets a manufacturer charging w . Their profit by setting ρ_C is

$$\left(\frac{1}{2} + \frac{m-1}{m} \frac{(1-\gamma)\alpha}{2} \right) (\rho_C - w) D(\rho_C).$$

If one of them chooses \hat{w} she makes a profit of

$$\left(\frac{1-\alpha}{2} + \frac{m-1}{m} \frac{(1-\gamma)\alpha}{2} \right) (\hat{w} - w) D(\hat{w}).$$

These expressions can be understood as follows. If the retailer chooses ρ_C , half of the consumers who first visit him buy immediately, while of the remaining consumers who

first visit the other retailer, only a fraction α will come back if this retailer has visited another manufacturer (which happens with probability $\frac{m-1}{m}$) and this manufacturer chooses \hat{w} (which happens with probability $1 - \gamma$). The second expression is obtained by considering that (i) half of the consumers first visit him and buy with probability $1/2$, (ii) if the other retailer has visited another manufacturer (which happens with probability $\frac{m-1}{m}$), he gets half of the consumers who continue to search if the other manufacturer sets the wholesale price \hat{w} .

Thus, the retailer is indifferent between setting ρ_C and \hat{w} (given that the other retailer chooses ρ_C) if the manufacturer chooses the price \tilde{w} that is given by

$$\tilde{w} = \frac{(m + (m - 1)(1 - \gamma)\alpha)\rho_C D(\rho_C) - (m + \alpha - (m - 1)\gamma\alpha)\hat{w}D(\hat{w})}{(m + (m - 1)(1 - \gamma)\alpha)D(\rho_C) - (m + \alpha - (m - 1)\gamma\alpha)D(\hat{w})}. \quad (21)$$

Also, it must be the case that the retailer does not want to just undercut \hat{w} after a manufacturer has chosen some w . Undercutting \hat{w} yields a profit of almost

$$\left(\frac{1 + \alpha}{2m} + \frac{m - 1}{2m}(1 - \gamma)(1 + \alpha)\right)(\hat{w} - w)D(\hat{w}).$$

This expression is understood as follows. If the retailer chooses $\hat{w} - \varepsilon$ for a sufficiently small $\varepsilon > 0$, all of the consumers who first visit him continue to search because $\hat{w} - \varepsilon$ is an unexpectedly high price above the reservation price ρ_C , but they may come back, depending on the other retailer's price.²⁵ If the other retailer has visited the same manufacturer and sets \hat{w} , which happens with probability $\frac{1}{m}$, all of the consumers who first visited him come back after continuing to search and buy, because he offers a slightly lower price than the other retailer. In addition, a fraction α of the consumers who first visit the other retailer continues to search and buy from the retailer under consideration. The same argument holds if the other retailer has visited another manufacturer.

Thus, the retail profit of undercutting \hat{w} is smaller than the retail profit of setting \hat{w} if

$$m\alpha < (m - 1)\gamma. \quad (22)$$

From above, for prices $w \leq \tilde{w}$, the sequential rationality of the retailer requires him to buy and set a price equal to ρ_C , while for wholesale prices $w > \tilde{w}$ he will react by buying and setting \hat{w} . This fully determines the retail strategy.

²⁵Notice that when observing $p \in (\rho_C, \hat{w})$ off the equilibrium path, consumers continue to search, though they buy at $p = \hat{w} (> \rho_C)$ with probability $1 - \alpha$ on the equilibrium path.

Given this strategy, the best profitable deviation for a manufacturer is to set the price \tilde{w} . At lower wholesale prices retailers will react by choosing $\min\{\rho_C, p^m(w)\}$, but it is clearly not optimal to set w such that $p^m(w) < \rho_C$, while for all w such that the retailer sets ρ_C it is best to squeeze the retailer as much as possible. Finally, if the retailer anyway reacts by choosing \hat{w} it is better to choose \hat{w} as well. Since retailers sell to the same fraction of consumers at the retail price $p^m(w^m)$, deviating to \tilde{w} is not profitable if

$$\tilde{w}D(\rho_C) < w^m D(p^m(w^m)). \quad (23)$$

These conditions characterize the equilibrium for arbitrary demand function. We now show by example that such conditions can be met. Suppose that demand is linear $D(p) = 1 - p$ and parameter values $(s_C, m) = (0.009, 10)$ and $s_R \geq 0.016$, there exists an NRPE where $(\gamma, \alpha, \hat{w}) \approx (0.5, 0.021, 0.75)$ with $\rho_C = 0.828$. Moreover, it is easily verified that if all conditions are satisfied for a certain α^* , they are satisfied for any $\alpha < \alpha^*$. This establishes the second part of the Proposition.

Proof of Proposition 10

Proof. Assume that a NRPE exists for some parameter constellation. We first show that there also exists a DDE. By (19), $\hat{w} < \rho_C^{DD}$ should hold. Given (m, s_C) , from (12) and (18) it follows that $\gamma^{non} > \gamma^{DD}$. In addition, if (21) holds we have that

$$\gamma^{DD} \frac{1}{2} (p^m(w^m) - w^m) D(p^m(w^m)) < \gamma^{non} \left(\frac{1}{2} + \frac{\alpha(1 + (m-2)(1-\gamma))}{2m} \right) (p^m(w^m) - w^m) D(p^m(w^m)). \quad (24)$$

Hence, if an NRPE exists, a DDE also exists.

Next, we show that the NRPE is Pareto-superior to the corresponding DDE. By construction, manufacturers make more profits, both at the lower price (since there is more demand) and at the higher price (by randomization). Thus, the higher price in an NRPE is lower than in the corresponding DDE. Since consumers are indifferent whether to search or not at the higher price in *both equilibria* it must be that $\gamma^{non} > \gamma^{DD}$. Hence, they are better off ex-ante. Finally, in both equilibria retailers are fully squeezed if they visit a manufacturer with the highest price and get the same margin if they observe the lower price $p^m(w^m)$. Thus, since $\gamma^{non} > \gamma^{DD}$ and their expected demand at the lower price is higher, they make higher profits. \square

B Appendix 2: Minimal Deviation Property (Not for Publication)

Let I be an information set. Let \mathcal{I} be the set of information sets and given an equilibrium σ^* , let \mathcal{I}^* be the set of on-the-equilibrium path information sets. For every $I \notin \mathcal{I}^*$, let $A(I)$ be the set of action profiles that lead to I . For each $a \in A(I)$, let $m(a)$ be the number of actions that do not belong to the support of the equilibrium strategy profile σ^* . Let $m^*(I) = \min_{a \in A(I)} m(a)$. Notice that $m^*(I) \geq 1$.

Definition 14 (Minimal Deviation Property). *A belief system β at information set I satisfies the Minimal Deviation Property (MDP) if and only if $\beta(a; I) > 0$ if and only if $m^*(I) = m(a)$ and $a \in A(I)$.*

In words, a belief system satisfies MDP if at every information set, players put positive probability on paths that lead to that information set and that contains the minimal number of deviations from equilibrium strategies.

Remark 15. *Passive beliefs satisfy MDP but the converse is not true.*

Remark 16. *In Vertical Contracting Games (e.g. McAfee and Schwartz, 1994), MDP is equivalent to Passive Beliefs. In Janssen and Shelegia (2015) MDP is consistent both with Passive and with Symmetric Beliefs (regarding retailer prices) but not with Wary Beliefs.*