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Abstract

The well-known double marginalization problem understates the inefficiencies arising from vertical relations in consumer search markets where consumers are uninformed about the wholesale prices charged by manufacturers to retailers. Consumer search provides a monopoly manufacturer with an additional incentive to increase its price, worsening the double marginalization problem and lowering the manufacturer's profits. Nevertheless, manufacturers in more competitive wholesale markets may not have an incentive to reveal their prices to consumers. We show that retail prices decrease in search cost, and so both industry profits and consumer surplus increase in search cost. *JEL*: D40; D83; L13.
Keywords: Vertical Relations, Consumer Search, Double Marginalization.

In markets where consumers do not know retail prices and need to invest time and resources to get to know them, they often also do not know retailers' marginal cost. Even though consumers are not directly interested in retailers' costs, these costs are indicative for whether consumers should buy at the current shop or continue to search for the next offer. This is particularly true when there is a significant common component in retailers' costs. For the same observed price at a particular retailer, if the common cost is low, the retailer's margin is high and it is more beneficial to check the price at the next retailer as he is likely to maintain a lower margin and hence to charge a lower price.

This paper studies consumer search in markets where retailers have a common marginal cost that is set by a manufacturer. As a reference point, we consider the (unrealistic) case where consumers are fully informed about the wholesale price that retailers must pay the

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manufacturer and show how the familiar double marginalization problem arises. We then consider the more realistic setting where consumers are uninformed about retailers' common marginal cost. In this case, consumers cannot condition their search rule (reservation price) on the wholesale price. We show that this results in a more inelastic demand curve for the manufacturer, and hence, in even higher wholesale and retail prices.

The worsening of the double marginalization problem can be understood as follows. Consumers' optimal search strategy in the form of a reservation price is based on a *conjectured* level of the wholesale price. If the manufacturer chooses a wholesale price that is higher than the conjectured level, consumers do not adjust their reservation price. As a result, retailers are squeezed: they face a higher cost, but cannot fully adjust their prices upwards as they would if consumers knew that their marginal cost is higher. The retail price adjustment to an increase in the wholesale price is therefore lower than in the case where consumers can observe the wholesale price. For the upstream manufacturer this means that his demand is less sensitive to price changes and, therefore, it has an incentive to charge higher prices.[1]

The phenomenon discussed here may arise in markets with the following three features: (*i*) in the retail market consumers search (and retailers thus have some market power), (*ii*) the upstream market is also characterized by some market power and (*iii*) consumers do not observe the wholesale price that is paid by retailers to manufacturer(s). There are many vertical markets that share these three features, e.g. consumer electronics markets (computers, cameras, TVs, refrigerators, etc.), supermarkets, the automobile market, but also the financial industry with financial intermediaries as retailers. In all these markets, there is a limited set of manufacturers and retailers, consumers search for better prices and consumers are (typically) not aware of the wholesale price.[2]

While these markets share the above features, they also differ in other respects, such as, whether products are homogeneous or differentiated, the types of contracts that are used between retailers and manufacturers, and the nature of competition at each level of the product channel. In the main body of this paper we focus on a homogeneous products market with one manufacturer who sells its product to two retailers using a linear pricing scheme.[3] Both retailers buy the manufacturer's product at the same wholesale price and sell to consumers in a retail market that has downward sloping demand. As Stahl (1989) is the standard sequential search model for homogeneous goods, we extend this model by adding a vertical industry structure and analyze the role of searching consumers who are not aware of the wholesale price. Stahl's model has two types of consumers. Shoppers know all prices and buy at the lowest price available in the market. Non-shoppers are uninformed about prices. They search sequentially, follow a reservation price strategy, and must pay a search cost for each additional price they choose to observe.

In the case where the wholesale price is unobserved by consumers, non-shoppers cannot

[1]When the manufacturer's actions are unobservable, the problem is one of lack of commitment. As Bagwell (1995) has shown in a study of first-mover advantage, a player's ability to commit is equivalent to the observability of his actions. Clearly, observability of the manufacturer's actions is not given in our model. The inability to commit is the result of this information disparity.

[2]That consumer search is important in these markets is exemplified by many studies (see, for example, **?** on electricity markets, Lin and Wildenbeest (2013) on Medigap insurance markets, Tappata (2009) and Chandra and Tappata (2011) on gasoline markets) and Baye, Morgan and Scholten (2004) on online markets).

[3]The mechanism we uncover holds in many other markets and the next section presents a reduced-form model making this point. In Section 4 we consider oligopoly wholesale markets. Extensions dealing with oligopoly retail markets, two-part tariffs and differentiated product markets as in Wolinsky (1986) are provided in the Appendix D.

condition their reservation price on the wholesale price. Instead, they form beliefs about the wholesale price, and in equilibrium, these beliefs are correct. For a high search cost, the equilibrium is exactly the same regardless of whether the wholesale price is observed or not.[4] The crucial difference with the observed case is when the search cost is relatively low. Provided that the reservation price equilibrium exists when consumers do not observe the wholesale price,[5] we show that the wholesale price is *higher* than when consumers observe the wholesale price. This effect is strongest when the search cost is very low (as in that case the reservation price is always close to the conjectured level of retailers' cost and retailers are maximally squeezed). When search cost approaches zero, the wholesale price may be substantially higher than the monopoly price of a vertically integrated firm. When, in addition, the proportion of shoppers buying at the lowest retail price is low, the incentives to increase the wholesale price can be so strong that a reservation price equilibrium fails to exist. We show that equilibrium existence is guaranteed when the proportion of shoppers is large.

For linear demand, we show that the comparative static results with regard to search cost crucially depend on whether or not consumers observe the wholesale price. When consumers observe the wholesale price, the expected retail price increases when the search cost of non-shoppers increases from an initially low level. Thus, paradoxically, when they do not know the wholesale price, consumers are better off with a higher search cost. The underlying reason is as follows. Conditional on a given wholesale price, a higher search cost leads to higher retail margins and thus higher retail prices. This decreases the manufacturer's demand and softens the incentive to charge high prices. For linear demand, this effect is so strong that the resulting retail price is a decreasing function of search cost, even though retail margins increase. This leads to another interesting effect: total industry profits are increasing in search cost. This is because by taming the manufacturer's incentive to charge a high price, a higher search cost leads to retail prices closer to those that would be charged by an integrated vertical firm, and thus to higher industry profits. If consumers observe the wholesale price, the expected retail price is an increasing function of search cost (as one would typically expect), but the wholesale price is not monotone in search cost. First, at initially low levels of search cost, the wholesale price is decreasing in search cost, as the manufacturer accommodates higher retail margins by lowering its price. Afterwards, the wholesale price starts to increase as retail margins become inelastic towards the wholesale price.

Having discussed many (negative) implications of the wholesale price being unobserved by consumers, and since the economic forces we discuss may operate in so many different markets, one may wonder why manufacturers and retailers choose not to reveal information regarding the wholesale contractual arrangement to consumers. We see two reasons. First, as we show in Section 4, the incentives for the manufacturer(s) to announce wholesale prices depend on the market power they have. If instead of having a monopoly manufacturer (as we consider in the main body of the paper), there is sufficiently strong competition upstream, manufacturers are better off in a world where consumers do not observe wholesale prices. In this case, manufacturers have a strong incentive to conceal the wholesale arrangement from consumers in order to increase prices. Second, in many markets, it may simply be too difficult to announce wholesale arrangements in a credible and understandable way to consumers, as these relationships are usually governed by

[4]As explained below, when search cost is high, non-shoppers' search behavior does not affect the equilibrium.

[5]The equilibrium may not exist. See below for more details.

complex contracts.[6] Moreover, retailers have an incentive to make consumers believe that their margins are actually lower than they really are, as this makes consumers believe that there is no point in continuing to search. Manufacturers, on the other hand, have an interest in making consumers believe that retail margins are high, that it thus makes sense for consumers to continue to search, as this will lower retail margins and increases manufacturer's demand. Thus, both retailers and manufacturers have an incentive to conceal the details of the wholesale contract, and these incentives go in opposite directions.[7]

Most of the consumer search literature assumes that consumers know retail costs so that they know the distribution of retailers' (price) offers on the next search (see, e.g., Stahl (1989), Wolinsky (1986) and other papers in the consumer search literature). Several authors have incorporated cost uncertainty into the search literature by assuming that retailers' cost follows some random process (Benabou and Gertner (1993), Dana (1994) and Fishman (1996), and more recently, Tappata (2009), Chandra and Tappata (2011) and Janssen, Pichler and Weidenholzer (2011)). This paper is the first to introduce vertical industry structure into a consumer search model and to consider the implications of consumers not observing the wholesale arrangement.

The issues we touch upon in this paper are related to recent literature on recommended retail prices (see, e.g., Buehler and Gartner (2013) and Lubensky (2014)). Lubensky suggests that by recommending retail prices, manufacturers provide information to consumers about what level of retail prices to expect in an environment where the manufacturer's marginal cost is random and only known to the manufacturer. In our framework, when informed about the wholesale price, consumers have a better notion of how large retail margins are and this benefits all market participants by reducing the manufacturer's perverse incentive to increase its price. In Buehler and Gartner (2013), consumers are not strategic and the recommended retail price is used by the manufacturer to communicate demand and cost information to the retailer.

Our paper also provides an alternative perspective on the issue of whether it should be mandatory for intermediaries, for example in the financial sector, to disclose their margins. Recent policy discussions in, for instance, the EU and the USA have led to legislation mandating intermediaries to reveal this information.[8] Our paper argues that this legislation may actually benefit most market participants as it provides (i) consumers with a better benchmark on what prices to expect in the market, and (ii) manufacturers with a reduced incentive to squeeze intermediaries, leading overall price levels and quantities to be closer to the efficient outcomes. In a recent paper, Inderst and Ottaviani (2012) come to an opposite conclusion. Their framework focuses on the information an intermediary has on how well a product matches the current state of the economy and how competing manufacturers incentivize the intermediary to recommend their products to consumers. Our paper abstracts from these issues as it deals with a homogeneous product and instead focusses on the effect of observing retail margins on the search behavior of consumers and manufacturer's incentives to price its products.

The remainder of this paper is organized as follows. Section I offers a general, abstract formulation of the key features of our model and explains when these lead to higher

[6]By complex contracts we do not necessarily refer to non-linear pricing arrangements such as two-part tariffs. Rather, we refer to provisions for warranties, returns, delivery, and so on, that affect retailers' marginal cost but are hard to understand for consumers.

[7]It is also worth pointing out that after the wholesale price has been set, consumers do not have an incentive to spend time and resources identifying the wholesale price, as in equilibrium they have correct expectations.

[8]For details, see the references given in Footnote 4 of Inderst and Ottaviani (2012).

wholesale prices. Section II introduces, analyzes and compares the two versions of our consumer search model. In Section II we derive further results and illustrate our main general results for the case of linear demand. Section IV shows that manufacturers will not necessarily benefit from revealing the wholesale price if there is upstream competition. Section V concludes. Proofs are provided in the Appendix.

1 A General Model

The aim of this section is to illustrate the main argument using simple microeconomic tools. In so doing, we also aim to show that the argument is more general than the search model presented in this paper, and we highlight the main ingredients that are necessary for the argument to hold.

Consider a manufacturer that produces a good. The manufacturer charges a unit wholesale price w to retailers who sell the good to consumers. Consumers may or may not observe the wholesale price w. Demand for the manufacturer, denoted by Q, depends on w through some optimal behavior by retailers. We allow Q also to depend on the belief consumers hold about the wholesale price, denoted by w^e. In the next section, we will explicitly model consumer and retail behavior, and show why in search markets Q depends on w^e.

We write manufacturer's profit as

$$\pi = w \cdot Q(w, w^e). \tag{1}$$

The manufacturer's profit maximization problem depends on whether or not consumers observe w. If they do, then $w^e = w$, and the manufacturer chooses w, anticipating that consumers observe its choice. If w is not observed, then consumers' belief cannot change with a change in w and in equilibrium we have to impose that the belief w^e is correct.

Denote the optimal wholesale price in case consumers do not observe it (the unobserved case) by w^u, and the optimal wholesale price in case consumers do observe it (the observed case) by w^o. Note that regardless of whether w^u or w^o is higher, as long as $w^o \neq w^u$, equilibrium profit is higher in the observed case than in the unobserved case:

$$w^o \cdot Q(w^o, w^o) > w^u \cdot Q(w^u, w^u). \tag{2}$$

This is because in equilibrium $w^e = w$, and so equilibrium profit always equals $w \cdot Q(w, w)$. Any outcome in the unobserved case can be implemented in the observed case by choosing the same wholesale price. So, the manufacturer cannot be worse off in the observable case as it maximizes $w \cdot Q(w, w)$ directly.

In many search models, including the Stahl (1989) model we use in this paper, $Q(w, w^e)$ is a decreasing function of w^e.[9] This is because if consumers believe that the manufacturer has charged a higher price than it actually has, then consumers accept higher prices, and thus retail prices are higher. This results in lower sales by the manufacturer. We will use this fact to show that $w^o < w^u$. Note that if this were not the case and $w^o > w^u$ we would have

$$w^o \cdot Q(w^o, w^u) \geq w^o \cdot Q(w^o, w^o) \tag{3}$$

[9]Guo (2013) presents a behavioral model where buyers care about the seller's cost as they have a preference for a fair division of the surplus. In that model, demand also depends on expectations concerning retailers' cost, but unlike in our model, the higher the expected cost, the higher the demand. As a referee pointed out, there are many other settings where demand depends on expectations, such as in reputation models of quality provision.

as manufacturer's profits are a decreasing function of w^e. Combining inequalities (2) and (3) leads to a contradiction (that w^u is not the optimal choice in the unobserved case), and thus to the conclusion that it has to be the case that $w^o < w^u$.

This simple model reveals several requirements that are necessary for the wholesale price to be higher in the unobserved case. First, demand has to depend on w^e as otherwise $w^u = w^o$. Second, retailers have to have some market power. If not, retail prices are equal to w, and so the manufacturer's profit is independent of w^e. Third, there has to be market power upstream.[10] As we show below, a standard sequential search model with a vertical structure imposed upon it, has all these features.

2 Retail Search Markets with a Monopoly Manufacturer

This section introduces a wholesale level in the search model developed by Stahl (1989). We introduce and compare two versions of the model, depending on whether consumers do or do not observe the wholesale price. Henceforth, we refer to these two cases as the observed and unobserved case.

We modify Stahl's model in two dimensions. First, for analytical tractability we consider duopoly, and, second, we explicitly consider the retailers' marginal cost, whereas Stahl normalizes it to zero. So, we consider a homogenous goods market where a manufacturer chooses a price w for each unit it sells to retailers. For simplicity, we abstract away from the issue of how the cost of the manufacturer is determined and set this cost equal to 0.[11] Retailers take the wholesale price (their marginal cost) w as given and compete in prices. The distribution of retail prices charged by the retailers is denoted by $F(p)$ (with density $f(p)$). Each firm's objective is to maximize profits, taking the prices charged by other firms and the consumers' behavior as given.

On the demand side of the market, we have a continuum of risk-neutral consumers with identical preferences. A fraction $\lambda \in (0, 1)$ of consumers, the *shoppers*, have zero search cost. These consumers sample all prices and buy at the lowest price. The remaining fraction of $1 - \lambda$, the *non-shoppers*, incur no cost for the first search but incur a positive search cost s for each subsequent search.[12] If a consumer buys at price p she demands $D(p)$. In our general analysis we assume that the demand function is thrice continuously differentiable, and in addition satisfies the following regularity properties: (i) $-\infty < D'(p) < 0$, (ii) for some finite P, $D(p) = 0$ for all $p \geq P$ and $D(p) > 0$ otherwise, (iii) for every $w < p$, $\pi_r(p; w) \equiv (p - w)D(p)$ is strictly concave and maximized at $p^m(w)$, the retail monopoly price, and (iv) for all given w, $(p - w)\pi_r'(p)/\pi_r(p)^2$ is decreasing in p for

[10] Even though in the model of this section the manufacturer is a monopolist, as long as $\frac{\partial Q(w,w_e)}{\partial w} > -\infty$, the same qualitative results would hold even if we considered competing manufacturers. We do not consider competing manufacturers for expositional simplicity.

[11] To focus on the new insights derived from studying the vertical relation between retailers and manufacturers in a search environment, we assume all market participants know that the manufacturer's cost equals 0. Alternatively, the manufacturer's cost could be set by a third party or could be uncertain. However, this would create additional complexity that may obscure the results.

[12] Our analysis for low search cost is unaffected when consumers also have to incur a cost for the first search. For higher search cost, the analysis would become more complicated (see, Janssen, Moraga-González and Wildenbeest (2005) for an analysis of the participation decision of non-shoppers in the Stahl model where the first search is not free). Janssen and Parakhonyak (2014) show that the Stahl equilibrium is unaffected by costly recall.

all $p \in (w, p^m(w)]$.[13] We also assume $D(p)$ is such that the relevant manufacturer's profit functions are strictly quasiconcave. We will formulate this assumption more precisely once we derive these profit functions. In the next section, we consider the special case of linear demand with $D(p) = 1 - p$ so that $p^m(w) = \frac{1+w}{2}$ and $P = 1$.

The timing is as follows. First, the manufacturer chooses w, which is observed by retailers and, (only) in the observed case, also by consumers. Then, given w, each of the retailers i sets price p_i. Finally, consumers engage in optimal sequential search given the equilibrium distribution of retail prices, not knowing the actual prices set by individual retailers.

When consumers do not observe the wholesale price, the retail market cannot be analyzed as a separate subgame for a given w. To accommodate this asymmetric information feature we use Perfect Bayesian Equilibrium (PBE) as the solution concept, focusing in both the observed and unobserved case on equilibria where buyers use reservation price strategies. In the unobserved case, non-shoppers are not informed about w so that the reservation price ρ is independent of the wholesale price. Non-shoppers buy at a retail price $p \leq \rho$ and continue to search otherwise. The reservation price is based on beliefs about w, and in equilibrium, these beliefs are correct. In the observed case, the non-shoppers search strategy is a reservation price strategy $\rho(w)$ that depends on w.

As retailers know the wholesale price in both models, their pricing is dependent on the wholesale price. PBE imposes the requirement that retailers respond optimally to any w, not only the equilibrium wholesale price. Because of the presence of a fraction of shoppers, there is price dispersion at the retail level and we denote by $\underline{p}(w)$ and $\bar{p}(w)$ the lower- and upper- bound of the equilibrium price support. Any price outside the interval $[\underline{p}(w), \bar{p}(w)]$, is an out-of-equilibrium price and consumers have to form beliefs about who has deviated if such a retail price is observed. We define out-of-equilibrium beliefs in the respective subsections below.

Formally, the equilibrium is defined as follows.

Definition 1 *A PBE where buyers use reservation price strategies is characterized by a price w^* set by the manufacturer, a distribution of retail prices $F(p; w)$, one for each w, and a non-shoppers' reservation price strategy such that given the strategies of the other players*

1. *the manufacturer chooses w to maximize its expected profit;*

2. *each retailer uses a price strategy $F(p; w)$ with support $[\underline{p}(w), \bar{p}(w)]$, that maximizes its expected profits;*

3. *non-shoppers' reservation price strategy is such that they search optimally given their beliefs about w and $F(p)$; shoppers observe all prices and buy at the lowest retail price. Beliefs are updated using Bayes' Rule when possible.*

[13]These are standard assumptions in this literature. For example, the last condition (accounting for the fact that retailers' marginal cost is w) is used by Stahl (1989) to prove that the reservation price is uniquely defined. Stahl (1989) shows that the condition is satisfied for all concave demand functions and also for the demand function of the form $D(p) = (1 - p)^\beta$, with $\beta \in (0, 1)$.

2.1 Retailers' Cost Is Observed

When consumers observe the price set by the manufacturer, our model simply adds a price setting stage to the sequential search model of Stahl (1989). To characterize the equilibrium, it is useful to first characterize the behavior of retailers and consumers for given w. This behavior is by now fairly standard. We first consider the upper bound of the retail price distribution. Clearly, the upper bound $\bar{p}(w)$ may never exceed the monopoly price $p^m(w)$. We define the non-shoppers' reservation price $\rho(w)$, as the solution[14] to

$$ECS(w, \rho(w)) \equiv \int_{\underline{p}(w)}^{\rho(w)} D(p)F(p)\, dp = s. \tag{4}$$

If the solution to (4) does not exist, then we set $\rho(w) = P$.[15] For $\rho(w) \leq p^m(w)$, $ECS(w, \rho(w))$ is the expected benefit from searching the second retailer when a non-shopper encounters a price equal to $\rho(w)$ at her first visit. Given such a reservation price, all non-shoppers who encounter prices below $\rho(w)$ do not search, whereas those who observe prices above $\rho(w)$ do search. Thus, the upper bound for the equilibrium retail price distribution is: $\bar{p}(w) = \min(\rho(w), p^m(w))$. For relatively low search cost, $\rho(w)$ will be the upper bound, while for relatively high search cost it will be $p^m(w)$.

Because consumers can observe whether or not the manufacturer has deviated, if they observe a price $p \notin [\underline{p}(w), \bar{p}(w)]$ the should believe that the retailer they have visited has deviated . While in this case they are free to change their belief about the price of the other retailer, if the reservation price is the upper bound of the price distribution, it is common to impose *passive* beliefs (in the sense of McAfee and Schwartz (1994)), so that upon observing a deviation consumers do not change their belief about the other retailer's price. For price deviations just above the reservation price, these beliefs are essential for having a reservation price equilibrium. In case the retail monopoly price is the upper bound of the price distribution, beliefs about deviations above the upper bound are unimportant because retailers will never charge prices above the retail monopoly price. Note, however, that for this case Stahl (1989) sets the reservation price equal to infinity (and thus implicitly assumes consumers always buy from the retailer they visit) if there does not exist a $\rho(w) \leq p^m(w)$ that is the solution to (4). Our definition of $\rho(w)$ is different in this regard (see Footnote 15), and it will become important in the next section where w is unobserved.

Proposition 1 below characterizes the equilibrium retail price distribution.

Proposition 1 *For $\lambda \in (0, 1)$, the equilibrium price distribution for the subgame starting with w is given by*

$$F(p; \bar{p}(w)) = \frac{1+\lambda}{2\lambda} - \frac{(1-\lambda)\pi_r(\bar{p}(w))}{2\lambda\pi_r(p)} \tag{5}$$

with a density

$$f(p) = \frac{(1-\lambda)\pi_r(\bar{p}(w))\pi_r'(p)}{2\lambda\pi_r(p)^2} \tag{6}$$

[14]The regularity conditions on demand guarantee that $ECS(w, \rho(w))$ is monotone in $\rho(w)$, so if the solution to (4) exists, it is unique.

[15]This definition of the reservation price differs somewhat from Stahl's definition. Our and Stahl's definitions coincide when $\rho(w) \leq p^m(w)$. For $\rho(w) > p^m(w)$ Stahl sets $\rho(w) = P$ while we still use the root of (4) even though $ECS(w, \rho(w))$ no longer represents the expected benefit from further search. We nevertheless use this definition in order to avoid discontinuity of $\rho(w)$ in w at $\rho(w) = p^m(w)$. In what follows, we never interpret $\rho(w)$ as a reservation price when $\rho(w) > p^m(w)$. This also applies to profit functions containing $\rho(w)$.

and support $[\underline{p}(w), \bar{p}(w)]$ *where* $\bar{p}(w) = \min(\rho(w), p^m(w))$*,* $\rho(w)$ *is the solution to (4) and* $\underline{p}(w)$ *is the solution to:*

$$(1+\lambda)\pi_r(\underline{p}(w)) = (1-\lambda)\pi_r(\bar{p}(w)). \tag{7}$$

The proposition characterizes the retail price distribution for a given wholesale price w that is observed by consumers. For a given w, retailers do not set prices that are higher than the consumer reservation price and the retail monopoly price. The lower bound of the price distribution is such that retailers are indifferent between selling to all the shoppers and half of the non-shoppers at that price and selling to only half of the non-shoppers at the upper bound of the price distribution.

This completes the description of retail behavior for a given w. Now we turn to the optimal behavior of the manufacturer. One can easily verify that by increasing w, the manufacturer shifts retailers' price distribution to the right, regardless of whether $p^m(w)$ or $\rho(w)$ is the upper bound, so that manufacturer's expected demand is a decreasing function of w. For a given w, the manufacturer's expected profit is given by:

$$\pi(w, \bar{p}(w)) = \left((1-\lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) f(p)\, dp + 2\lambda \int_{\underline{p}(w)}^{\bar{p}(w)} D(p) f(p)(1 - F(p))\, dp \right) w.$$

The first integral is the expected demand from the non-shoppers; the second integral is expected demand from the shoppers who buy at the lowest retail price.

Using (5) and (6) this profit can be written as

$$\pi(w, \bar{p}(w)) = \left(\frac{(1-\lambda)^2}{2\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} \frac{D(p)\pi_r(\bar{p}(w))^2 \pi_r'(p)}{\pi_r(p)^3} dp \right) w \tag{8}$$

Henceforth, when we write $\pi(w, x)$ we refer to (8) where x is substituted for $\bar{p}(w)$ everywhere, while $\underline{p}(w)$ solves (7) where again x is substituted for $\bar{p}(w)$.[16]

In order to simplify the analysis, for the remainder of the paper we assume when the wholesale price is observed

Assumption 1 $D(p)$ *is such that for any* s *and* λ*,* $\pi(w, \bar{p}(w))$ *is strictly quasiconcave.*[17]

As $\rho(w)$ and $p^m(w)$ are continuous functions, so is $\pi(w, \bar{p}(w))$. We show in the proof of Theorem 1 that at $\rho(w) = p^m(w)$ the derivative of $\pi(w, \bar{p}(w))$ is the same regardless of which upper bound is taken.[18] In addition, the manufacturer's profit is equal to 0 at $w = 0$ and at $w = P$. It follows that there is a unique optimal value of $w \in (0, P)$, denoted by w^*, where w^* solves

$$\frac{\partial \pi(w, \bar{p}(w))}{\partial w} = 0. \tag{9}$$

[16] For $x > p^m(w)$ the function $\pi(w, x)$ is well defined, but it is no longer the manufacturer's expected profit (see Footnote 15).

[17] This implies that $\pi(w, p^m(w))$ is also quasiconcave. In Proposition 5, we show that in case demand is linear $\pi(w, p^m(w))$ is concave in w. Because $\rho(w)$ is only implicitly defined by (7), it is very difficult to analytically show that $\pi(w, \bar{p}(w))$ is quasiconcave in w for commonly employed demand functions. Numerical analysis for linear and constant elasticity demand functions reveals that it is. For unit demand, there is no drawback for the manufacturer to push prices up.

[18] The main intuition for this fact is that when the upper bound is the monopoly price, the density of the retail price distribution at that point is 0 (see also equation 6). In that case, the manufacturer's profit is not affected by marginal differences in the upper bound of the retail price distributions.

Whether $\bar{p}(w) = \rho(w)$ or $\bar{p}(w) = p^m(w)$ in addition to (9) determines the equilibrium wholesale price depends on s and λ and this issue is resolved in Section 2.3. In either case, as $w^* > 0$ and at the retail level $\underline{p}(w^*) > w^*$, both the manufacturer and retailers are able to charge a margin above their respective marginal cost. Thus, although retailers randomize over prices, a familiar double-marginalization problem arises in this model with an observed wholesale price.

2.2 Retailers' Cost Is Unobserved

Retailers' behavior in the unobserved case is very similar to what was described in the previous subsection, the main difference is that now the upper bound is given by $\bar{p}(w) = \min(\rho^*, p^m(w))$. Since we impose $\rho^* = \rho(w^*)$ in equilibrium, the difference between the case where w is observed and where it is unobserved is fairly subtle. In the unobserved case considered here, for every $w \leq \rho^*$ expected profit of the manufacturer is given by $\pi(w, \rho^*)$ while in the observed case it is given by $\pi(w, \rho(w))$. Note also that Assumption 1 does not imply that $\pi(w, \rho^*)$ is quasi-concave, and in fact it may well not be (leading to possible non-existence of a reservation price equilibrium). The manufacturer's profit function is given by (8), with the difference that now $\bar{p}(w) = \min(\rho^*, p^m(w))$.

As with the observed case, depending on parameters, one of two types of equilibria prevails. One type is where $\bar{p}(w^*) = p^m(w^*) < \rho^*$, and like in the observed case for relatively high search costs, the manufacturer's price w^* solves (9) for $\bar{p}(w) = p^m(w)$. The other type is where $\rho^* < p^m(w^*)$. This case is different from before as now an equilibrium is described as the intersection of two "reaction curves" in a simultaneous move game, one describing how the reservation price ρ^* depends on w^* and the other how the optimal wholesale price w^* depends on ρ^*. Therefore, if equilibrium were to exist for relatively low s, the wholesale price w^* and non-shoppers' reservation price ρ^* should simultaneously satisfy

$$w^* = \arg\max_w \pi(w, \rho^*) \text{ and } \rho^* = \rho(w^*). \tag{10}$$

In the unobserved case, out-of-equilibrium beliefs have to be treated with care. If $\bar{p}(w) = \rho^* < p^m(w)$ any price above ρ^* has to be interpreted by consumers as a deviation by the retailer, and *not* by the manufacturer, and thus we have that consumers believe the wholesale price to be w^* after observing such a price. Such a deviation is thus punished by further search. The belief that it is the retailer, and not the manufacturer, that has deviated is, as in the observed case, necessary for a reservation price equilibrium. It can be interpreted as consumers having passive beliefs. Otherwise, if consumers believe that it is the manufacturer that has deviated to a wholesale price $w > \rho^*$ and that therefore all retailers set a price $p = w > \rho^*$, then uninformed consumers will not want to incur the search cost to find out the other retail price. But this would be inconsistent with a reservation price equilibrium because retailers would have an incentive to deviate and choose prices above the reservation price. Note that if the manufacturer deviated to a wholesale price $w < \rho^*$ and the retailers did *not* deviate from their equilibrium strategy, retail prices would not be higher than the reservation price ρ^* and consumers would not know that the manufacturer had deviated.[19]

[19]Thus, if consumers would like to attribute a price observation above the reservation price to the manufacturer only, then they have to assume that the manufacturer has set the wholesale price $w > \rho^*$. Such beliefs can be rationalized further and are consistent with *the logic* of the D1 refinement that considers which firm has most incentives to deviate (Cho and Sobel (1990)). The D1 criterion is, however, developed for different types of games than ours. The informal argument for the linear case goes as follows.

If, however, $\bar{p}(w) = p^m(w) < \rho^*$, then the retailers have no incentive to deviate to a price above $p^m(w)$ unless the manufacturer has also deviated. In this case it is natural to assume that upon observing a deviation above the upper bound consumers believe the manufacturer has deviated. In particular, we assume that consumers will believe the manufacturer has chosen w such that the observed price corresponds to the monopoly price given w, i.e., $p = p^m(w)$.[20] This assumption on out-of-equilibrium beliefs is implied by "symmetric" beliefs (i.e. for any out-of-equilibrium retail price observed, consumer believe that the other retailer charges the same price). If we were to use passive beliefs also in this case, then the equilibrium would not exist for some parameter values. The reason is that the manufacturer may deviate to w such that $p^m(w) > \rho^*$, where the reservation price is computed according to consumers' prior belief that the upper bound is $p^m(w)$. Since ρ^* does not change with w, retailers may be squeezed resulting in a profitable deviation for the manufacturer. Under our assumption on out-of-equilibrium beliefs, retailers are able to charge prices above the reservation price and up to the retail monopoly price, eliminating profitable deviations for the manufacturer. Thus, the asymmetry in assumptions concerning out-of-equilibrium beliefs is not driven by equilibrium selection arguments, but by the necessity to preserve the existence of equilibrium whenever possible. Note that, as described in the previous subsection, beliefs in our model are equivalent those implicitly assumed by Stahl (1989).

Having defined the most favorable beliefs for the equilibrium existence, we nevertheless note that Assumption 1 does not guarantee that (10) has a solution or that the solution is unique. The next subsection characterizes equilibria in the observed and unobserved cases, and analyzes the existence of equilibrium in the unobserved case.

2.3 Characterization and Comparison

In order to facilitate the comparison of equilibrium behavior in the two cases, define $w^m = \arg\max \pi(w, p^m(w))$. By Assumption 1, w^m is unique and solves (9) for $\bar{p}(w) = p^m(w)$. Note that in general w^m is not equal to the vertically integrated monopoly price or the wholesale price in the standard double-marginalization model with one retailer. As we show in Section 3, w^m equals the vertically integrated monopoly price for a class of demand functions that includes linear demand as a special case.

It becomes evident from the definition of w^m that if $p^m(w)$ is the upper bound of the retail price distribution for all w, then there is no difference between the observed and unobserved cases and w^m is the manufacturer's equilibrium choice. The intuition is simple: the only difference between the observed and unobserved cases is in the determination of the upper bound of the retail price distribution and whether this upper bound depends on conjectured or actual wholesale price. When the upper bound is given by

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Suppose that after observing a deviation price $p > \rho^*$ a non-shopper buys with probability $q(p)$. Clearly, if $q(p)$ is sufficiently close to 1, a retailer has an incentive to deviate. Then consider the case where only the wholesale firm deviates. The only way a deviation by the upstream firm alone could account for retail prices above the reservation price is $w > \rho^*$ resulting in a profit of $w(1-w)$. Given that, later on, we show the equilibrium wholesale price to be always larger than 0.5 in the linear demand case (and the equilibrium reservation price even higher than that), this is never optimal for the wholesale firm (whatever the consumers do). As there are values of $q(p)$ such that a retail firm has an incentive to deviate, whereas there are no values of $q(p)$ such that the wholesale firm has an incentive to deviate, D1 requires non-shoppers to consider it infinitely more likely that the individual retailer has deviated.

[20] The precise specification of these beliefs is not important. What is important is that, when the manufacturer adjusts its prices upwards, the retail monopoly price remains the upper bound of the retail price distribution.

the retailers' behavior and *not* determined by non-shoppers' search behavior, the upper bound is determined in the same way in both cases. As this happens when the search cost is high, it is clear that for sufficiently high search cost the equilibrium outcomes of the two models coincide.

For relatively low search costs in the observed case, define w^o as the solution to $\frac{\partial \pi(w,\rho(w))}{\partial w} = 0$, which is obtained by substituting $\bar{p}(w) = \rho(w)$ into equation (9).[21] By Assumption 1, the solution exists and is unique. Further, for the unobserved case, let w^u be a solution, if it exists, to $\frac{\partial \pi(w,\rho^u)}{\partial w} = 0$, which is (9) for $\bar{p}(w) = \rho^u$, where ρ^u itself solves $\rho^u = \rho(w^u)$. This system may have multiple solutions. Abusing notation, we use w^u to refer to the set of all solutions.[22] Note that neither uniqueness nor existence of w^u is guaranteed by Assumption 1, but it can be verified that if equilibrium exists for linear demand, it is unique. Because Theorem 1 that follows does not depend on uniqueness, we choose not to impose further assumptions on $D(p)$.

Finally, for every λ define $\bar{s}_\lambda$ as the search cost such that $\rho(w^m) = p^m(w^m)$, i.e., $\bar{s}_\lambda$ is such that the non-shoppers' reservation price equals the retail monopoly price in case the manufacturer sets its price equal to w^m. As $w^m < p^m(w^m) < P$ and the reservation price is an increasing function of s up to P starting from w^m (when $s = 0$), it is clear that $\bar{s}_\lambda$ is uniquely defined.

The next theorem states our first main result. For search cost values lower than the critical threshold value $\bar{s}_\lambda$, if in the unobserved case a PBE where buyers use reservation price strategies exists, then the manufacturer chooses a higher wholesale price than in the equilibrium of the observed case. As, in equilibrium, consumers correctly anticipate the manufacturer's behavior, the reservation price will be higher in the unobserved case implying that the distribution of retail prices in the unobserved case first-order stochastically dominates the distribution of retail prices in the observed case. For search cost values higher than $\bar{s}_\lambda$ the manufacturer's behavior is independent of the search behavior of consumers and the equilibria in the two cases coincide.

Theorem 1 *(i) If $s \geq \bar{s}_\lambda$, then equilibrium always exists and the equilibrium wholesale price in both the observed and unobserved cases equals w^m; (ii) If $s < \bar{s}_\lambda$, then the equilibrium wholesale price in the observed case equals w^o and, if equilibrium exists, any equilibrium wholesale price w^u in the unobserved case satisfies $w^o < w^u$.*

The technical part of the proof, dealing with the threshold value $\bar{s}_\lambda$, is in the Appendix. For the substantive part arguing that $w^o < w^u$ when $s < \bar{s}_\lambda$, note that in the previous section, we have argued that the main difference between the two cases is the term $\frac{\partial Q}{\partial w^e}$. In our model, w^e affects demand through ρ. The next lemma shows that ρ positively depends on w^e. The reason is that if w^e increases, consumers expect to get a worse deal if they continue to search the next retailer and are therefore more willing to buy now.

Lemma 1 *In the unobserved case, the reservation price is an increasing function of the non-shoppers' belief about the wholesale price: $\rho'(w^e) > 0$.*

For the effect of the reservation price ρ on demand, the following lemma states that manufacturer's demand is a decreasing function of ρ and in the proof we argue that the retail price distributions first-order stochastically dominate each other as ρ increases.

[21]Note that w^o may be such that $\rho(w^o) > p^m(w^o)$, in which case $f(p)$ as defined in (6) is negative.

[22]As with w^o, w^u may be such that $\rho(w^u) > p^m(w^u)$, in which case $f(p)$ is negative for the range of w where $\rho(w) > p^m(w)$.

Lemma 2 *For a given w, the manufacturer's demand and profit are decreasing functions of ρ for $w \leq \rho < p^m(w)$.*

Taken together, the two lemmas imply that manufacturer's demand decreases as w^e increases and thus, in terms of the previous section, $\frac{\partial Q}{\partial w^e} < 0$.

Figure 1 illustrates the difference between the two cases for the same increase in w. In the observed case the manufacturer demand falls more because $\rho(w)$ also increases, whereas in the unobserved case ρ is constant because it depends on consumers' belief about w, and not on w itself. Thus, with an increase in w the retail price distribution shifts more to the right if w is observed than when it is not observed.

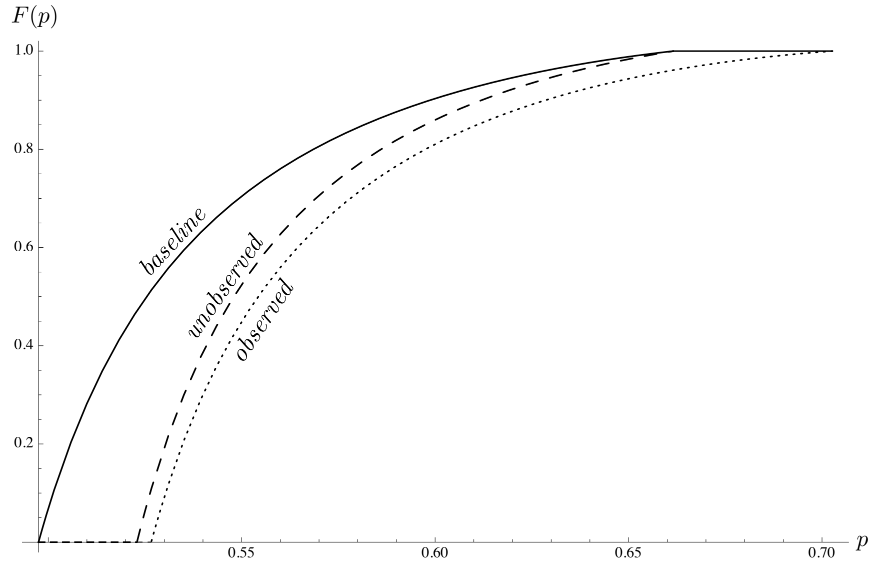

Figure 1: Cumulative distribution of downstream prices for $D(p) = 1 - p$, $w = 0.45$ and $\rho = \rho(0.45) = 0.6615$ (solid), $w = 0.48$ and $\rho = \rho(0.48) = 0.7026$ (dotted), and $w = 0.48$ and $\rho = \rho(0.45) = 0.6615$ (dashed).

Since w^o maximizes the manufacturer's profits for correct beliefs, and in the unobserved case beliefs are correct in equilibrium, the manufacturer prefers that its price is observed by consumers. A higher wholesale price leads to higher retail prices and so consumers are also worse off in the unobserved case. Retail profits depend on w in a complex way. When w, their marginal cost, increases, their profits fall, but the reservation price also increases, which given that the reservation price is below p^m, increases their profits. One can show that for sufficiently high λ, or s sufficiently close to, but lower than, $\bar{s}_\lambda$, retailers are also worse off in the unobserved case.[23]

Proposition 2 *The manufacturer and consumers are worse off in the unobserved case than in the observed case. There exist $\tilde{\lambda}$ and $\tilde{s}$ such that retailers are also worse off for $\tilde{s} < s < \bar{s}_\lambda$ or $\tilde{\lambda} < \lambda < 1$.*

Given that $w^u > w^o$ for $s < \bar{s}_\lambda$, one may wonder how large the difference is when s is very low. In particular, it seems natural that as $s \to 0$ the equilibrium outcomes

[23]These conditions are sufficient, but not necessary. We have been unable to find examples where retailers are better off in the unobserved case and believe, but were unable to show, that retailers are always worse off in the unobserved case regardless of λ or s.

of the two models should converge. Our second main result compares the observed and unobserved case by considering the limiting results of the two models when the search cost s vanishes. Surprisingly, we find that the difference between the equilibrium outcomes of the two models remains for prlowop s.

Theorem 2 *If $s \to 0$, then*
(i) w^o approaches the solution to $D(w)+wD'(w)=0$ (the integrated monopoly price),
(ii) w^u approaches the solution to $D(w)+\lambda wD'(w)=0$, if a reservation price equilibrium exists.

In the observed case, the limiting result for search cost approaching zero can be easily understood. As the reservation price has to converge to the retailers' cost there is (almost) Bertrand competition at the retail level with retail prices close to marginal cost. As this cost is known to consumers, they effectively demand $D(w)$ and therefore, the manufacturer's profit function is simply $D(w)w$, which is maximized at the integrated monopoly price.

In the unobserved case, the limiting result is very different. To see this, note that the vertically integrated monopoly price is the solution to $D(w)+wD'(w)=0$ and that for any $\lambda < 1$, the result says that the manufacturer charges a strictly higher price. Although the formal proof in the appendix requires taking limits in a more subtle way (as the reservation price depends on s), some intuition for the result can be obtained as follows. When non-shoppers do not observe retailers' cost and search cost is low, non-shoppers' reservation price is close to the conjectured wholesale price w^e. Moreover, as shoppers anyway observe all prices and as retailers know their cost, retailers choose retail prices close to their marginal cost. Thus, when s approaches zero, the behavior of the manufacturer can be understood as if it is maximizing

$$w\left(\lambda D(w)+(1-\lambda)D(w^e)\right),$$

where shoppers react to the real wholesale price and non-shoppers react to the expected wholesale price. Maximizing this expression with respect to w and imposing $w^e = w$ gives

$$D(w)+\lambda wD'(w)=0. \tag{11}$$

Thus, when $s \to 0$, the manufacturer sets a price that is higher than the optimal price of a vertically integrated monopolist.

We have consistently made statements (e.g. Theorems 1 and 2) conditional on the existence of an equilibrium. Theorem 2 helps to identify the cause of the possible non-existence of a reservation price equilibrium. When λ is sufficiently low and s approaches zero, an equilibrium where consumers follow a reservation price strategy and the manufacturer plays a pure strategy does not exist. This is because the equilibrium candidate wholesale price w^u converges to P, which implies that the manufacturer would want to deviate and charge a lower price. Intuitively, when λ is sufficiently low, the manufacturer's demand is almost inelastic, so if equilibrium is to exist, the manufacturer has to charge a very high price. But then the manufacturer can deviate to a lower price, e.g. w^m, and make more profit. Such a lower price cannot itself be part of an equilibrium, because for the corresponding reservation price the manufacturer wants to deviate to a higher price due to inelastic demand. Mathematically, the reason is that for low s and

λ, $\arg\max_w \pi(w, \rho)$ is discontinuous in ρ and there is no solution to the system (10) of equilibrium equations.[24]

Proposition 3 *For $s \to 0$ there exists λ sufficiently lowlow such that no equilibrium exists where consumers follow a reservation price strategy and the manufacturer plays a pure strategy.*

If a reservation price equilibrium does not exist, the question is what type of equilibrium does exist. If in equilibrium the manufacturer follows a pure strategy, then the equilibrium must be of a reservation price nature as consumers must correctly anticipate the wholesale price. Without uncertainty, the optimal search behavior is characterized by a reservation price strategy. It thus follows that if a reservation price strategy does not exist, the manufacturer has to randomize. In that case there can be learning along the equilibrium path and the optimal search strategy may not have the reservation price property (see, e.g., Rothschild (1974)). In the linear demand case that we analyze in the next section, we numerically determine the size of this non-existence region.

Proposition 3 raises the issue of whether we can guarantee that an equilibrium does exist for other parameter values. The next proposition answers in the affirmative for high λ (equilibrium also exists for high s, but that is less interesting given that the equilibrium outcomes of the two models coincide, as shown in Section 2.1).

Proposition 4 *When the wholesale price is unobserved by consumers, for every $s > 0$ there is a reservation price equilibrium for sufficiently high λ.*

3 Linear Demand

In this section we illustrate the quantitative significance of our results for the linear demand function $D(p) = 1 - p$.[25] We also derive a new result, namely, that when search costs are relatively high, the wholesale price is equal to the monopoly price of a vertically integrated monopolist. Using the latter, we show that the wholesale price is decreasing in search cost in the unobserved case.

We start by providing, in Figure 2, a numerical analysis of when a reservation price does and does not exist in case demand is linear. The upward-sloping line separating the region where the equilibrium upper bound of the retailers' price distribution is given by the reservation price and the region where it is given by the retailers' monopoly price represents $\bar{s}_\lambda$ as defined in the previous section. In the case of linear demand one can find an explicit solution for $\bar{s}_\lambda$.[26] To the left of $\bar{s}_\lambda$ the reservation price is binding and around the vertical axis, the manufacturer's optimal price approaches $w^* = 1/(1 + \lambda)$, which is the solution to (11) for linear demand if a reservation price equilibrium exists. For s approaching 0, one can implicitly define the critical value of λ^* such that equilibrium exists if, and only if $\lambda > \lambda^*$. It turns out that λ^* solves

$$\tfrac{2(1-\lambda(1+\lambda))}{\lambda} - \tfrac{(1-\lambda)^2}{\lambda^2} \ln\left(\tfrac{1+\lambda}{1-\lambda}\right) = 0,$$

[24]The non-existence is not related to the discreteness of the search cost distribution (zero for shoppers and $s > 0$ for non-shoppers). If we assumed a continuous search cost distribution that is close to the discrete distribution used here, the same issue would arise.

[25]The results can be generalized to demand functions of the form $D(p) = (1 - \alpha p)^\beta$, where $\alpha, \beta > 0$. This class includes both concave and convex functions.

[26]The detailed calculation is based on expressions in the proof of Proposition 5 resulting in $\bar{s}_\lambda = \frac{1}{16}\sqrt{\frac{1+\lambda}{2\lambda}} + \frac{1}{32} - \frac{1-\lambda}{32\lambda} \ln \frac{1}{1-\sqrt{\frac{2\lambda}{1+\lambda}}}$.

which gives $\lambda^* \approx 0.47$. To the right of $\bar{s}_\lambda$ curve, s is relatively high and the reservation price is not binding.

We next characterize the manufacturer's equilibrium behavior for $s > \bar{s}_\lambda$. It turns out that the manufacturer charges the monopoly price of a vertically integrated monopolist, i.e., $w^* = 1/2$.

Proposition 5 *Suppose $D(p) = 1 - p$. If $s \geq \bar{s}_\lambda$, the manufacturer's profit takes the form $g(\lambda)w(1-w)$, for some function $g(\lambda)$. Thus, the manufacturer's profit is concave and $w^m = 1/2$.*

The proof exploits the special relationship between the upper and lower bounds of the price distribution for linear demand and, as noted in the previous section, the statement does not readily generalize beyond the class of demand functions described in Footnote 25.

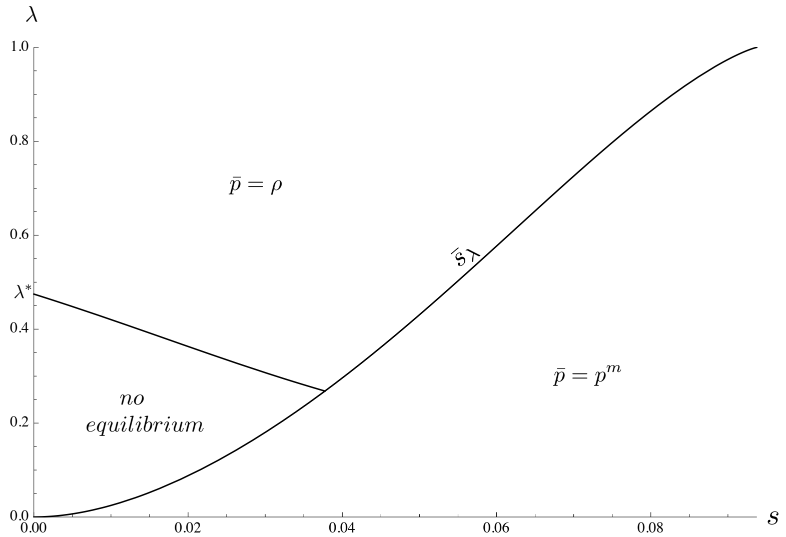

Figure 2: Different equilibrium cases depending on s and λ. Triangular area is where there is no equilibrium in the unobserved case.

We next explore the size of the price-raising effect due to the non-observability of the wholesale price and inquire into the comparative statics analysis of changes in s. Figure 3 illustrates how much the wholesale price is higher in the unobserved case for different parameter values. When $\lambda = 1/2$, Figure 3 confirms that for high search cost s, the wholesale price w equals 1/2, while for lower values of s the wholesale price is higher in the unobserved case. Note that in the observed case the wholesale price is U-shaped. The downward-sloping part is easily understood. When search cost increases, retailers gain more market power and are able to increase their margins for a given cost. The manufacturer internalizes this effect and charges a lower wholesale price. The upward-sloping part is more subtle. As s gets closer to $\bar{s}_\lambda$, the upper bound of the retail price distribution approaches $p^m(w)$ and becomes less responsive to w. Moreover, the probability that retail prices are close to the retail monopoly price becomes low. As s increases, the optimal wholesale price is increasing in s because the manufacturer's demand is less responsive to w. The same effects are also present in the unobserved case, but there the wholesale price starts from a much higher level when s is close to zero. The same convex-shaped relationship between w and s holds, however, but wholesale prices are decreasing throughout.

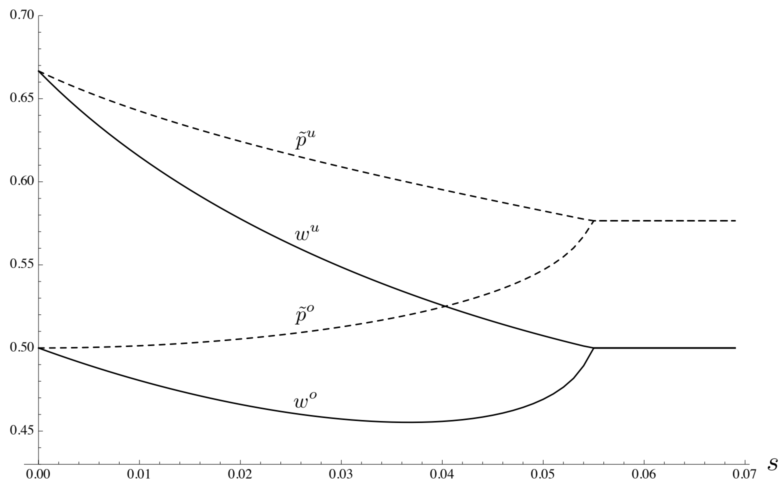

Figure 3: Wholesale prices (solid lines) and average weighted retail prices (dashed) for the two cases as functions of s for $\lambda = \frac{1}{2}$.

Figure 3 also depicts $\tilde{p} = \lambda E \min(p_1, p_2) + (1 - \lambda)Ep$ for both cases. This is the average price a randomly chosen consumer pays. Figure 3 confirms the limiting results when s becomes very low (Theorem 2). It shows that the price raising effect due to unobservability can be quite substantial with the wholesale and retail price being up to 33 percent higher in the unobserved case. Identifying the normal double marginalization problem with the extent to which $\tilde{p}$ is higher than the vertically integrated monopoly price of $\frac{1}{2}$, the figure also shows that the worsening of the double marginalization problem by the consumers' inability to observe wholesale prices may outweigh the standard double marginalization problem.

Figure 3 clearly shows the different effect of search cost s on $\tilde{p}$. When the wholesale price is observed, we have the standard effect in the consumer search literature that expected prices are increasing in the search cost. However, in the unobserved case, $\tilde{p}$ is decreasing in s. The reason for this unusual relationship is that there are two countervailing forces here. First, when the search cost increases, retailers have more market power and increase their margins for a given cost. Second, the problem arising due to consumers' inability to observe wholesale prices becomes less severe with higher search cost as retailers are able to pass on wholesale price increases to consumers. The manufacturer internalizes this effect and charges significantly lower prices in response (see the steep decrease in the w^u curve in Figure 3). As the second effect dominates for linear demand, the total effect is that retail prices are decreasing in s. This is the content of the next proposition. It compares retail price $\frac{1}{1+\lambda}$ at $s = 0$ with the expected price for high s. In addition, the proposition compares manufacturer's and retailers' profits for high and low search cost.

Proposition 6 *Suppose $D(p) = 1 - p$. If a reservation price equilibrium exists in the unobserved case, then $\tilde{p}$ is higher, manufacturer's profit is higher, and total industry profits are lower if $s \to 0$ than if $s > \bar{s}_\lambda$.*

This result offers an alternative perspective on the role of price comparison websites' that effectively reduce search costs. Websites that are believed to help consumers get better deals may in the end lead to higher prices, unless in addition to retail prices they also inform consumers about wholesale prices.[27] Proposition 6 also points to the fact that

[27]While the latter is uncommon, some websites such as www.edmunds.com try to provide this infor-

the manufacturer and the retailers have opposite interests, with the former preferring low search cost and the latter preferring high search cost.

4 Upstream Competition

In this section, we point out that with upstream competition, manufacturers may not have an incentive to truthfully reveal the wholesale arrangement. We also show that wholesale and retail prices continue to be higher in the unobserved case, resulting in lower welfare. To illustrate that the manufacturers' incentive to reveal information depends on the severity of upstream competition, we model upstream competition in such a way that it does not affect the analysis of the retail market. To do so, we have one manufacturer serving all retailers, but different manufacturers competing for the right to serve the market. The choice of the monopoly manufacturer depends on the wholesale prices that are offered by the different manufacturers, but also on other factors that are beyond their control. With N manufacturers, we introduce a probability $\gamma(w_i, w_{-i})$ that firm i wins the competition and serves the retail market at price w_i if other firms charge w_{-i}. The function γ can take different forms. If $\gamma(w_i, w_{-i}) = 1/N$ for all (w_i, w_{-i}), then effectively there is no price competition upstream. There is Bertrand competition in the upstream market if $\gamma(w_i, w_{-i})$ is such that $\gamma(w_i, w_{-i}) = 1/N$ if all firms choose the same wholesale price and $\gamma(w_i, w_{-i}) = 0$ if $w_i > \min w_{-i}$ and $\gamma(w_i, w_{-i}) = 1$ if $w_i < \min w_{-i}$. The severity of upstream competition is measured by the derivative of $\gamma(w_i, w_{-i})$ with respect to w_i evaluated at the symmetric point where $w_i = w_j$ for all i, j.[28]

One way to think of this modeling approach is that all retailers have to pay a fixed "conversion" cost ε_i in order to customize output of manufacturer i into retail output. If manufacturers do not know this cost, and assuming that all retailers adopt the same manufacturer's product, retailers will adopt manufacturer i's product if $(1-\lambda)\pi_r(\bar{p}(w_i)) - \varepsilon_i > (1-\lambda)\pi_r(\bar{p}(w_j)) - \varepsilon_j$ for all $j \neq i$. This can then be rewritten as $\gamma(w_i, w_{-i}) = \Pi_{j\neq i}\Phi((1-\lambda)(\pi^r(\bar{p}(w_i)) - \pi^r(w_j)))$, where Φ is the symmetric distribution of the difference between the epsilons, with $\Phi(0) = 1/2$.

Depending on whether or not consumers observe the wholesale price, we then model upstream competition by having manufacturers choose w_i by maximizing $\gamma(w_i, w_{-i})\pi(w_i, \rho(w_i))$ or $\gamma(w_i, w_{-i})\pi(w_i, \rho^*)$, for the observed and unobserved cases, respectively.

In both cases, in a symmetric equilibrium the first-order condition for firm i is

$$\frac{1}{N}\frac{\partial \pi(w_i, \bar{p}(w_i))}{\partial w_i} + \frac{\partial \gamma(w_i, w_{-i})}{\partial w_i}\pi(w_i, \bar{p}(w_i)) = 0. \tag{12}$$

If the probability γ is unaffected by w_i, in which case $\frac{\partial \gamma(w_i, w_{-i})}{\partial w_i} = 0$, then all manufacturers will choose the same price as the upstream monopolist would choose. If, on the other hand, $\frac{\partial \gamma(w_i, w_{-i})}{\partial w_i}$ is negative, then wholesale prices will be lower than in the monopoly model.

In this modified model, it is still the case that wholesale prices are higher in the unobserved case than in the observed case. In the monopoly model, this means that

mation for cars by announcing the retailers' invoice prices. However, as retailers receive holdbacks from manufacturers when selling cars and these holdbacks are unknown, the relevant marginal cost remains unknown to consumers.

[28] The advantage of this modeling approach is that the probability $(1-\gamma(w_i, w_{-i}))$ of losing i's monopoly puts downward pressure on w_i, as competition should do, but the model is kept tractable by preserving the assumption of a monopoly supply to retailers.

the manufacturer always prefers to be in the observed case, as in the unobserved case it ends up distorting its price choice upward. Under upstream competition, however, if the equilibrium wholesale price falls enough because of competitive pressure, manufacturers will prefer to be in the unobserved case where prices are higher.

Thus, under upstream competition, manufacturers may not have an incentive to reveal information to consumers about the price they set as the unobservability of their price relaxes price competition. Apart from the fact that it may simply be too difficult to credibly convey information about wholesale prices, upstream competition may be another reason why in real-world markets, firms do not reveal their wholesale price arrangements.

5 Discussion and Conclusion

This paper has investigated the implications of vertical industry structure in markets where consumer search is prevalent. The most important result is that when the wholesale price is not observed by consumers, manufacturer's demand is less elastic (as consumers' reservation price is not affected by the actual choice of the wholesale price) giving the manufacturer an incentive to increase its price, squeezing the retailers' profits. In equilibrium, the actual choice of the wholesale price has to be correctly anticipated by consumers, but this can only arise at levels of the wholesale price that are far higher than the price a vertically integrated monopolist would set. In the Appendix we show that the argument we present in this paper is robust to several extensions, such as retail oligopoly markets (Appendix B) and two-part tariffs (Appendix C). We also show that the effects hold true in search markets with product differentiation based on Wolinsky (1986) (Appendix D).

The paper identifies several policies a monopolistic manufacturer may pursue to alleviate the problem associated with unobservable wholesale prices. If it can credibly do so, the manufacturer may want to provide information about the wholesale price as this will act as a commitment not to increase prices. The manufacturer may also want to subsidize search cost as this will reduce retail margins. In Janssen and Shelegia (2014) we also show that the double marginalization is weakened, but not eliminated, by choosing two-part tariffs. The manufacturer will never want to price discriminate between retailers as its profits will be lower than when it charges all retailers the same price. Finally, the manufacturer does not have an incentive to eliminate competition between retailers as this will increase retail margins, thereby reducing manufacturer's demand and profits.

Overall, the paper draws attention to the importance of taking into account the incentives of manufacturers when studying retail markets where consumer search is important. Given the findings in this paper, we expect that other findings in the consumer search literature may be refined once the upstream perspective is taken into account. In addition, the paper raises the question whether other topics in the study of vertical market structures (such as exclusive dealing, retail price maintenance, etc.) should be reconsidered for markets with consumer search. Finally, the result on the non-existence of reservation price equilibria in this paper redirects the theoretical literature on consumer search to consider non-reservation price equilibria.

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Appendix A: Proofs

PROOF OF THEOREM 1:

We first show that if $s = \bar{s}_\lambda$, then $w^o = w^u = w^m$. The expected quantity sold by the manufacturer is given by

$$Q(w) = \left(\frac{(1-\lambda)^2}{2\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} \frac{\pi_r(\bar{p}(w))^2 \pi_r'(p)}{(p-w)^3 D^2(p)} dp \right).$$

As $\rho(w^m) = p^m(w^m)$ at $s = \bar{s}_\lambda$ demand is the same in all three relevant cases if the manufacturer charges w^m: (i) when the upper bound is the retail monopoly price, (ii) when the upper bound is the reservation price conditional on w^m (as in the observed case) and (iii) when the upper bound is the reservation price conditional on the non-shoppers' belief about the wholesale price being w^m (as in the unobserved case). Thus, to determine whether the first-order condition for profit maximization yields the same w^m in all three cases, we only need to evaluate the derivate of the manufacturer's demand with respect to w and show that it is equal in all three cases.

As $\frac{\partial \bar{p}(w)}{\partial w} = 0$ at $\bar{p}(w) = p^m(w)$, this derivative equals $\frac{(1-\lambda)^2}{2\lambda}$ times

$$\int_{\underline{p}(w)}^{\bar{p}(w)} \left[\frac{\partial \pi_r(\bar{p}(w))^2}{\partial w} \frac{\pi_r'(p)}{(p-w)^3 D^2(p)} + \pi_r(\bar{p}(w))^2 \frac{\partial \frac{\pi_r'(p)}{(p-w)^3 D^2(p)}}{\partial w} \right] dp$$
$$- \frac{\pi_r(\bar{p}(w))^2 \pi_r'(\underline{p}(w))}{(\underline{p}(w)-w)^3 D^2(\underline{p}(w))} \frac{\partial \underline{p}(w)}{\partial w}.$$

As

$$\frac{\partial \pi_r(\bar{p}(w))}{\partial w} = \frac{\partial (\bar{p}(w)-w) D(\bar{p}(w))}{\partial w}$$
$$= \left[(\bar{p}(w)-w) \frac{\partial D(\bar{p}(w))}{\partial w} + D(\bar{p}(w)) \right] \frac{\partial \bar{p}(w)}{\partial w} - D(\bar{p}(w)) = -D(\bar{p}(w)),$$

the derivative of the manufacturer's demand with respect to his own decision variable w at $w = w^m$ when $s = \bar{s}_\lambda$ can only differ between these three cases if $\frac{\partial \underline{p}(w)}{\partial w}$ differs between these cases. However, in all three cases the relation between the upper and lower bound of the retail price distribution is given by $(1-\lambda)\pi_r(\bar{p}(w)) = (1+\lambda)\pi_r(\underline{p}(w))$. Taking the total differential and using the fact that at $s = \bar{s}_\lambda$, $\rho(w) = \rho = p^m(w)$ we have that in all three cases

$$\frac{\partial \underline{p}(w)}{\partial w} = \frac{(1+\lambda)D(\underline{p}(w)) - (1-\lambda)D(\bar{p}(w))}{(1+\lambda)\pi_r'(\underline{p}(w))}.$$

This completes the proof that if $s = \bar{s}_\lambda$, then $w^o = w^u = w^m$.

We next prove that in the observed case, when $s = \overline{s}_\lambda$, the manufacturer sets its price equal to w^m in equilibrium. Assume the opposite. Then there should exist w' such that $\pi(w') > \pi(w^m)$. Given the definition of w^m, it cannot be that $\rho(w') \geq p^m(w')$. So it has to be the case that $\rho(w') < \rho(w^m)$. Then, the first order condition

$$\left.\frac{\partial \pi(w, \rho(w))}{\partial w}\right|_{w=w'} = 0$$

should hold. This is the condition that defines w^o, but as shown above, when $s = \overline{s}_\lambda$, this condition coincides with the condition that defines w^m. Since w^m is unique, we conclude that $w' = w^m$, a contradiction.

Now consider the observed case when $s > \overline{s}_\lambda$. If the manufacturer charges w^m, then its profit is the same as for $s = \overline{s}_\lambda$ because the upper bound on retail prices is $p^m(w^m) < \rho(w^m)$. Since for any given w, $\pi(w, \rho(w))$ is a decreasing function of s, no $w \neq w^m$ can increase manufacturer's profits. Thus w^m is the equilibrium price for all $s > \overline{s}_\lambda$.

Next consider $s < \overline{s}_\lambda$. In this case $\rho(w^m) < p^m(w^m)$. At the w that maximizes manufacturer's profits, $\rho(w) \geq p^m(w)$ cannot hold, or otherwise the manufacturer can earn higher profits by charging w^m (in this case the profit will be at least as high as $\pi(w^m, p^m(w^m))$ and thus higher than profit if w is charged). Thus, at the new maximizer $\rho(w) < p^m(w)$ holds and so the manufacturer's profit is maximized at w^o.

This proves that in the observed case for $s \geq \overline{s}_\lambda$ the manufacturer's profit is maximized at w^m, and for $s < \overline{s}_\lambda$ it is maximized at w^o.

Now we prove that in the unobserved case, when $s = \overline{s}_\lambda$, the manufacturer also sets his price equal to w^m. Assume the opposite, so that some $w' \neq w^m$ is charged in equilibrium. If it is the case that $\rho(w') < p^m(w')$, then the necessary condition for the manufacturer's profit maximization implies that $w' = w^u$, which in turn leads to a contradiction because, as proven above, $w^u = w^m$. Now consider $\rho(w') > p^m(w')$. If the manufacturer charges w^m instead of w', its profit will be at least as high as $\pi(w^m, p^m(w^m))$, and by the definition of w^m, the profit will be higher than at w'. Thus w' cannot be charged in equilibrium. Finally, if w' is such that $\rho(w') = p^m(w')$, by deviating down the manufacturer will ensure that the upper bound is the monopoly price, and by deviating up it will ensure that the upper bound is $\rho^* = \rho(w')$. Since when the monopoly price is equal to the reservation price, derivatives are equal on both sides (as proven above), the derivative of profit on both sides has to be zero, and thus $w' = w^m$, a contradiction. Thus, it has to be the case that the manufacturer charges w^m, and this is the equilibrium wholesale price because the manufacturer has no profitable deviation.

Now consider the unobserved case where equilibrium exists. Take $s > \overline{s}_\lambda$. Then w^m is the equilibrium price. First note that w^m is an equilibrium becuase for any w manufacturer's profit is as in the observed case (consumers always blame retailers for deviations, thus $\bar{p}(w) = p^m(w)$). Therefore, w^m has to maximize manufacturer's profits. There cannot be a more profitable price $w' \neq w^m$. If $\rho(w') \geq p^m(w')$, contradiction follows from the definition of w^m. If $\rho(w') < p^m(w')$, then by deviating to w^m the manufacturer can guarantee itself profit of at least $\pi(w^m, p^m(w^m))$ (because the upper bound will be at most $p^m(w^m)$ after the deviation). Given that in the observed case w^m maximizes profits (proven above), and that in the unobserved case beliefs are correct in equilibrium, it has to be the case that $\pi(w^m, p^m(w^m)) > \pi(w', \rho(w'))$, thus we get a contradiction. For $s < \bar{s}_\lambda$, $\rho(w^m) < p^m(w^m)$. At the w that maximizes manufacturer's profits, $\rho \geq p^m(w)$ cannot hold, or otherwise the manufacturer can earn higher profits by charging w^m, in which case the profit will be higher than $\pi(w, p^m(w^m))$ and thus higher than $\pi(w, \rho(w))$.

Thus, for the equilibrium wholesale price $\rho < p^m(w)$ holds and so w^u will be the wholesale price. So, for all $s \geq \bar{s}_\lambda$, in both the observed and unobserved cases the equilibrium exists, and the wholesale price equals w^m. If $s < \bar{s}_\lambda$, equilibrium wholesale price in the observed case is given by w^o, and in the unobserved case, if equilibrium exists, it is given by w^u.

PROOF OF LEMMA 1:

Note first that (4) is defined for consumer's conjectured wholesale price w^e. Thus the condition for the reservation price is $ECS(\rho, w^e) = s$. From this, we have

$$\frac{\partial ECS(\rho, w^e)}{\partial \rho}\frac{\partial \rho}{\partial w^e} + \frac{\partial ECS(\rho, w^e)}{\partial w^e} = 0.$$

As expected consumer surplus decreases with w^e when ρ is held constant, $\rho'(w^e) > 0$ immediately follows if we can show that $\frac{\partial ECS(\rho,w^e)}{\partial \rho} > 0$. Using (5) and realising that here $\overline{p}(w) = \rho$, we have

$$\frac{\partial F(p;\rho)}{\partial \rho} = -\frac{1-\lambda}{2\lambda}\left(\frac{\pi_r'(\rho)}{\pi_r(p)}\right)$$

and

$$\frac{\partial F(p)}{\partial p} = \frac{1-\lambda}{2\lambda}\left(\frac{\pi_r(\rho)\pi_r'(p)}{\pi_r(p)^2}\right)$$

we can write

$$\frac{D(p)\frac{\partial F(p)}{\partial \rho}}{D(\rho)\frac{\partial F(p)}{\partial p}} = -\frac{D(p)\frac{\pi_r'(\rho)}{\pi_r(p)}}{D(\rho)\frac{\pi_r(\rho)\pi_r'(p)}{\pi_r(p)^2}} = -\frac{(\rho - w)\pi_r'(\rho)/\pi_r^2(\rho)}{(p-w)\pi_r'(p)/\pi_r(p)^2},$$

which, given the assumption that $(p-w)\pi_r'(p)/\pi_r(p)^2$ is increasing in p, lies in the interval $[-1, 0)$. Thus, we can rewrite

$$\frac{\partial ECS(\rho,w^e)}{\partial \rho} = \frac{\int_{\underline{p}}^{\rho} D(p)F(p)dp}{\partial \rho} = D(\rho) + \int_{\underline{p}}^{\rho} D(p)\frac{\partial F(p)}{\partial \rho}dp$$

$$> D(\rho) - \int_{\underline{p}}^{\rho} D(\rho)\frac{\partial F(p)}{\partial p}dp = D(\rho)\left(1 - \int_{\underline{p}}^{\rho}\frac{\partial F(p)}{\partial p}dp\right) = D(\rho)(1 - F(\rho)) = 0.$$

PROOF OF LEMMA 2:

Take the derivative of $F(p)$ from (5) for $\bar{p}(w) = \rho$ with respect to ρ. This gives

$$-\frac{(1-\lambda)\pi_r'(\rho)}{2\lambda\pi_r(p)}.$$

This expression is negative given that $\rho < p^m(w)$ and $\pi_r'(\rho) > 0$. So for a higher ρ the distribution of retail prices first-order stochastically dominates the one for a lower ρ. Given that retail demand is downward-sloping, for a given w, manufacturer's profit is a decreasing function of ρ.

PROOF OF PROPOSITION 2:

The first part is trivial. Since $w^u > w^o$, and beliefs are correct in equilibrium, manufacturer's profits are lower in the unobserved case. As for consumers, the price distribution in the unobserved case first-order stochastically dominates the distribution in the observed case. This means that consumers are worse of in the unobserved case.

In order to prove that retail profits are lower in the unobserved than in the observed case, it is sufficient to show that retail profits are everywhere decreasing in w. Since retail profits are equal to

$$(1-\lambda)\pi_r(\rho(w))$$

we need to show that this expression is decreasing in w. This is equivalent to

$$\rho'(w) < \frac{D(\rho(w))}{\pi_r'(\rho(w))}.$$

Using the definition of $\rho(w)$ we get

$$\begin{aligned}
\rho'(w) &= \frac{-\int_{\underline{p}(w)}^{\rho(w)} D(p)\frac{\partial F(p)}{\partial w}\,dp}{D(\rho(w)) + \int_{\underline{p}(w)}^{\rho(w)} D(p)\frac{\partial F(p)}{\partial \rho}\,dp} = \frac{-\int_{\underline{p}(w)}^{\rho(w)} D(p)\frac{(p-\rho(w))(1-\lambda)D(\rho(w))}{2\lambda(p-w)^2D(p)}dp}{D(\rho(w)) + \int_{\underline{p}(w)}^{\rho(w)} D(p)\frac{-(1-\lambda)\pi_r'(\rho(w))}{2\lambda(p-w)D(p)}dp} \\
&= \frac{\frac{(1-\lambda)D(\rho(w))}{2\lambda}\int_{\underline{p}(w)}^{\rho(w)}\frac{(\rho(w)-p)}{(p-w)^2}dp}{D(\rho(w)) - \frac{(1-\lambda)\pi_r'(\rho(w))}{2\lambda}\int_{\underline{p}(w)}^{\rho(w)}\frac{1}{(p-w)}\,dp}.
\end{aligned}$$

So the sufficient condition is

$$\begin{aligned}
\frac{\frac{(1-\lambda)D(\rho(w))}{2\lambda}\int_{\underline{p}(w)}^{\rho(w)}\frac{(\rho(w)-p)}{(p-w)^2}dp}{D(\rho(w)) - \frac{(1-\lambda)\pi_r'(\rho(w))}{2\lambda}\int_{\underline{p}(w)}^{\rho(w)}\frac{1}{(p-w)}\,dp} &< \frac{D(\rho(w))}{\pi_r'(\rho(w))} \iff \\
\pi_r'(\rho(w))\frac{(1-\lambda)}{2\lambda}\int_{\underline{p}(w)}^{\rho(w)}\left(\frac{\rho(w)-w}{(p-w)^2}\right)dp &< D(\rho(w)) \iff \\
\left(D(\rho(w)) + (\rho(w)-w)D'(\rho(w))\right)\frac{(1-\lambda)(\rho(w)-\underline{p}(w))}{2\lambda(\underline{p}(w)-w)} &< D(\rho(w)).
\end{aligned}$$

This condition is satisfied for s sufficiently close to $\bar{s}$ because as s approaches $\bar{s}$, $\rho(w)$ approaches $p^m(w)$ and so the LHS approaches 0, while the RHS is bound away from zero. Rewriting $\frac{(\rho(w)-\underline{p}(w))}{(\underline{p}(w)-w)}$ as $\frac{(\rho(w)-w)}{(\underline{p}(w)-w)} - 1$ and then as $\frac{(1+\lambda)D(\underline{p}(w))}{(1-\lambda)D(\rho(w))} - 1 = \frac{(1+\lambda)D(\underline{p}(w))-(1-\lambda)D(\rho(w))}{(1-\lambda)D(\rho(w))}$ gives the other sufficient condition. For λ sufficiently close to 1, $\frac{(1-\lambda)(\rho(w)-\underline{p}(w))}{2\lambda(\underline{p}(w)-w)}$ converges to 1, and so the condition is satisfied due to the negativity of $D'(\rho(w))$.

PROOF OF THEOREM 2:

For a given ρ the manufacturer's profit function can be written as

$$\pi(w,\bar{p}(w)) = \left(\frac{(1-\lambda)^2(\rho-w)^2D^2(\rho)}{2\lambda}\int_{\underline{p}(w)}^{\rho}\frac{(p-w)D'(p)+D(p)}{(p-w)^3D^2(p)}dp\right)w. \tag{13}$$

The integral part can be written as

$$\int_{\underline{p}(w)}^{\rho}\frac{(p-w)D'(p)+2D(p)-D(p)}{(p-w)^3D^2(p)}dp = \frac{-1}{(p-w)^2D(p)} - \int_{\underline{p}(w)}^{\rho}\frac{1}{(p-w)^3D(p)}dp.$$

We can also write the integral part as

$$\int_{\underline{p}(w)}^{\rho}\frac{(p-w)D'(p)+D(p)}{(p-w)^3D^2(p)}dp = \int_{\underline{p}(w)}^{\rho}\frac{D'(p)}{(p-w)^2D^2(p)}dp + \int_{\underline{p}(w)}^{\rho}\frac{1}{(p-w)^3D(p)}dp.$$

As these two equations are identical, we have

$$\int_{\underline{p}(w)}^{\rho} \frac{1}{(p-w)^3 D(p)} dp = \frac{1}{2}\left[\frac{-1}{(p-w)^2 D(p)} - \int_{\underline{p}(w)}^{\rho} \frac{D^{'}(p)}{(p-w)^2 D^2(p)} dp\right]$$

so that the original demand integral can be written as

$$\int_{\underline{p}(w)}^{\rho} \frac{(p-w)D^{'}(p) + D(p)}{(p-w)^3 D^2(p)} dp = \frac{1}{2}\left[\frac{-1}{(p-w)^2 D(p)} + \int_{\underline{p}(w)}^{\rho} \frac{D^{'}(p)}{(p-w)^2 D^2(p)} dp\right].$$

As ρ is independent of w, maximizing manufacturer's profit is equal to maximizing

$$\begin{aligned} & w(\rho-w)^2\left[\frac{-1}{(\rho-w)^2 D(\rho)} + \frac{1}{(\underline{p}-w)^2 D(\underline{p})} + \int_{\underline{p}(w)}^{\rho} \frac{D^{'}(p)}{(p-w)^2 D^2(p)} dp\right] \\ = \quad & \frac{-w}{D(\rho)} + \frac{w(\rho-w)^2}{(\underline{p}-w)^2 D(\underline{p})} + \int_{\underline{p}(w)}^{\rho} \frac{w(\rho-w)^2 D^{'}(p)}{(p-w)^2 D^2(p)} dp, \end{aligned}$$

which using (16) can be written as

$$\frac{-w}{D(\rho)} + \frac{(1+\lambda)^2 w D(\underline{p})}{(1-\lambda)^2 D^2(\rho)} + \int_{\underline{p}(w)}^{\rho} \frac{w(\rho-w)^2 D^{'}(p)}{(p-w)^2 D^2(p)} dp.$$

Taking the derivative with respect to w yields

$$\begin{aligned} & \frac{-1}{D(\rho)} + \frac{(1+\lambda)^2 D(\underline{p})}{(1-\lambda)^2 D^2(\rho)} + \frac{(1+\lambda)^2 w D^{'}(\underline{p})}{(1-\lambda)^2 D^2(\rho)} \frac{\partial \underline{p}}{\partial w} - \frac{w(\rho-w)^2 D^{'}(\underline{p})}{(\underline{p}-w)^2 D^2(\underline{p})} \frac{\partial \underline{p}}{\partial w} \\ & \qquad + \int_{\underline{p}(w)}^{\rho} \frac{D^{'}(p)}{D^2(p)}\left[\frac{(\rho-w)^2}{(p-w)^2} + \frac{2(\rho-w)(\rho-p)w}{(p-w)^3}\right] dp \qquad (14) \\ = & \frac{-1}{D(\rho)} + \frac{(1+\lambda)^2 D(\underline{p})}{(1-\lambda)^2 D^2(\rho)} + \int_{\underline{p}(w)}^{\rho} \frac{D^{'}(p)}{D^2(p)}\left[\frac{(\rho-w)^2}{(p-w)^2} + \frac{2(\rho-w)(\rho-p)w}{(p-w)^3}\right] dp. \end{aligned}$$

The integral part can be written as

$$\int_{\underline{p}(w)}^{\rho} \frac{D^{'}(p)}{D^2(p)}\left[\frac{(\rho-w)^2}{(p-w)^2} + \frac{2(\rho-w)^2 w}{(p-w)^3} - \frac{2(\rho-w)w}{(p-w)^2}\right] dp.$$

As (i) $\frac{D^{'}(p)}{D^2(p)}$ is finite in a neighborhood of w, (ii) for s close to 0, ρ approaches $\underline{p}$, and (iii) from equation (7) it follows that

$$\frac{\underline{p}-w}{\rho-w} = \frac{(1-\lambda)D(\rho)}{(1+\lambda)D(\underline{p})}$$

so that

$$\lim_{s\to 0} \frac{\underline{p}-w}{\rho-w} = \frac{1-\lambda}{1+\lambda},$$

for s close to 0 the whole expression (14) is close to

$$\frac{1}{D(w)}\left[-1 + \left(\frac{1+\lambda}{1-\lambda}\right)^2\right] + \frac{D^{'}(w)}{D^2(w)}\left[w + \left(\frac{1+\lambda}{1-\lambda}\right)^2 w - 2\frac{1+\lambda}{1-\lambda} w\right].$$

This reduces to

$$\frac{4\lambda}{(1-\lambda)^2 D(w)}\left[1+\lambda\frac{wD^{'}(w)}{D(w)}\right],$$

which is equal to zero if, and only if, $1+\lambda\frac{wD^{'}(w)}{D(w)}=0$.

PROOF OF PROPOSITION 3:

From Theorem 2 we know that in the limit where $s\to 0$, a necessary condition on the wholesale price is to solve $D(w)+\lambda wD^{'}(w)=0$. Using the regularity conditions on $D(p)$, as λ approaches zero, w approaches P and so the manufacturer's profit would approach zero. However, it is clear that the manufacturer can earn strictly positive profit by setting a price close to the integrated monopoly price, in which case retailers use the monopoly price as the upper bound, and thus for λ sufficiently close to 0 the manufacturer has an incentive to deviate to a (much) lower price. Therefore, no equilibrium exists when $s\to 0$ and λ is sufficiently low.

PROOF OF PROPOSITION 4:

From Theorem 1 we know that for any given λ it is the case that $\overline{p}=\rho$ for all $s<\overline{s}_\lambda$. As we know that an equilibrium exists for $s\geq\overline{s}_\lambda$ we focus on $s<\overline{s}_\lambda$. Denote by $w^*(\rho)$ the set of optimal wholesale prices given a reservation price ρ. This set is not empty because of the continuity of $\pi(w,\rho)$ on a compact support $[0,P]$. Also, denote by $\rho^*(w)$, the reservation price for a given wholesale price w and given retailers' optimal behavior. An equilibrium exists, if there is a pair $\{w,\rho\}$ such that $\rho=\rho^*(w)$ and $w\in w^*(\rho)$.

Note that for $\lambda=1$ we have that the manufacturer's profit equals $D(w)w$ for any ρ, and by Assumption 1, this is strictly concave and maximized at $p^m(0)$. It is also the case that $\lim_{\lambda\to 1}\pi(w,\rho)=D(w)w$ and thus, by continuity, that $\lim_{\lambda\to 1}w^*(\rho)=p^m(0)$. For all $s<\overline{s}_\lambda$ we have that $\rho^*(p^m(0))<p^m(p^m(0))<P$ and as $\lim_{\lambda\to 1}\rho^*(w)=w+s$ and $\rho^*(w)$ is a continuous function for all λ, we have that for λ values close to 1, we can find $\varepsilon_\lambda>0$ such that $\rho^*(w)<p^m(0)+s$ for all $w<p^m(0)-\varepsilon_\lambda$ and $\rho^*(w)>p^m(0)+s$ for all $w>p^m(0)+\varepsilon_\lambda$. Thus, for λ values close to 1 equilibrium may fail to exist only if there is a discontinuity in $w^*(\rho)$. To prove that there is no such discontinuity, we prove that $\pi(w,\rho)$ is strictly concave for high enough λ so that the set $w^*(\rho)$ has only one element.

The proof that $\pi(w,\rho)$ is strictly concave for high enough λ essentially shows that the second-order derivative is strictly lower than 0. In the proof of Theorem 2 we have derived that the derivative of the manufacturer's profit can be written as

$$\frac{\partial\pi}{\partial w}=\frac{-(1-\lambda)^2}{D(\rho)}+\frac{(1+\lambda)^2 D(\underline{p})}{D^2(\rho)}+(1-\lambda)^2\int_{\underline{p}(w)}^{\rho}\frac{D^{'}(p)}{D^2(p)}\left[\frac{(\rho-w)^2}{(p-w)^2}+\frac{2w(\rho-w)(\rho-p)}{(p-w)^3}\right]dp$$

so that the second-order derivative is equal to

$$\frac{\partial^2\pi}{\partial w^2}=\frac{(1+\lambda)^2 D^{'}(\underline{p})}{D^2(\rho)}\frac{\partial\underline{p}}{\partial w}-(1+\lambda)^2\frac{D^{'}(\underline{p})}{D^2(\rho)}\left[1+\frac{2(\rho-\underline{p})w}{(\underline{p}-w)(\rho-w)}\right]\frac{\partial\underline{p}}{\partial w}$$
$$+(1-\lambda)^2\int_{\underline{p}(w)}^{\rho}\frac{D^{'}(p)}{D^2(p)}\left[\frac{4(\rho-w)(\rho-p)}{(p-w)^3}-\frac{2(\rho-p)w}{(p-w)^3}+\frac{6(\rho-w)(\rho-p)w}{(p-w)^4}\right]dp=$$
$$-\frac{2(1+\lambda)^2(\rho-\underline{p})wD^{'}(\underline{p})}{(\underline{p}-w)(\rho-w)D^2(\rho)}\frac{\partial\underline{p}}{\partial w}+(1-\lambda)^2\int_{\underline{p}(w)}^{\rho}\frac{D^{'}(p)}{D^2(p)}\left[\frac{2(2\rho p-3pw+\rho w)(\rho-w)}{(p-w)^4}\right]dp.$$

We develop first the integral part of this expression by using integration by parts:

$$\int_{\underline{p}(w)}^{\rho}\frac{D^{'}(p)}{D^2(p)}\left[\frac{2(2\rho p-3pw+\rho w)(\rho-w)}{(p-w)^4}\right]dp=-\frac{D^{'}(\rho)}{D^2(\rho)}\frac{2\rho(\rho-\rho)^2}{(\rho-w)^3}+\frac{D^{'}(\underline{p})}{D^2(\underline{p})}\frac{2\underline{p}(\rho-\underline{p})^2}{(\underline{p}-w)^3}$$
$$-\int_{\underline{p}(w)}^{\rho}\frac{-2\left(D^{'}(p)\right)^2+D(p)D^{''}(p)}{D^3(p)}\left[\frac{2p(\rho-p)^2}{(p-w)^3}\right]dp.$$

Using again integration by parts on the integral part, a second time yields

$$\int_{\underline{p}(w)}^{\rho} \frac{-2\left(D'(p)\right)^2+D(p)D''(p)}{D^3(p)} \left[\frac{2p(\rho-p)^2}{(p-w)^3}\right] dp = 2\Big(\rho - \frac{w(\rho-w)^2}{2(\rho-w)^2} - \frac{(\rho-3w)(\rho-w)}{\rho-w}$$
$$+(-2\rho+3w)\ln(\rho-w)\Big)\frac{-2\left(D'(\rho)\right)^2+D(\rho)D''(\rho)}{D^3(\rho)} - 2\Big(\underline{p} - \frac{w(\rho-w)^2}{2(\underline{p}-w)^2} - \frac{(\rho-3w)(\rho-w)}{\underline{p}-w}$$
$$+(-2\rho+3w)\ln(\underline{p}-w)\Big)\frac{-2\left(D'(\underline{p})\right)^2+D(\underline{p})D''(\underline{p})}{D^3(\underline{p})}$$
$$-2\int_{\underline{p}(w)}^{\rho}\Bigg(\Big(p - \frac{w(\rho-w)^2}{2(p-w)^2} - \frac{(\rho-3w)(\rho-w)}{p-w} + (-2\rho+3w)\ln(p-w)\Big)\times$$
$$\times\frac{6\left(D'(p)\right)^3-6D(p)D'(p)D''(p)+D^2(p)D'''(p)}{D^4(p)}\Bigg)dp$$

Using the fact that $\frac{1-\lambda}{\underline{p}-w} = \frac{(1+\lambda)D(\underline{p})}{(\rho-w)D(\rho)}$ is finite for any $w<\rho$ we know that for λ close to 1, $\frac{(1-\lambda)^2}{\left(\underline{p}-w\right)^k}$ is close to 0 for $k<2$. As in the last integral the fraction with different derivatives of the demand function is finite, the integral can be approximated with terms that only involve $\frac{1}{\left(\underline{p}-w\right)^k}$ for $k<2$. Hence, the second-order derivative of the manufacturer's profit function for λ close to 1 (dropping also all other terms that are close to 0) will be approximately equal to

$$\begin{aligned} &-\frac{2(1+\lambda)^2(\rho-\underline{p})wD'(\underline{p})}{(\underline{p}-w)(\rho-w)D^2(\rho)}\frac{\partial \underline{p}}{\partial w} + \frac{D'(\underline{p})}{D^2(\underline{p})}\frac{2(1-\lambda)^2\underline{p}(\rho-\underline{p})^2}{(\underline{p}-w)^3} \\ &\quad + \frac{(1-\lambda)^2 w(\rho-w)^2}{(\underline{p}-w)^2}\frac{-2\left(D'(\underline{p})\right)^2+D(\underline{p})D''(\underline{p})}{D^3(\underline{p})}. \end{aligned} \tag{15}$$

Using $\frac{(1-\lambda)^2(\rho-w)^2}{\left(\underline{p}-w\right)^2} = \frac{(1+\lambda)^2D^2(\underline{p})}{D^2(\rho)}$, $\frac{(1+\lambda)^2}{D^2(\rho)}$ times (15) can be rewritten as

$$-\frac{2(\rho-\underline{p})wD'(\underline{p})}{(\underline{p}-w)(\rho-w)}\frac{\partial \underline{p}}{\partial w} + \frac{2\underline{p}(\rho-\underline{p})^2D'(\underline{p})}{(\underline{p}-w)(\rho-w)^2} + \frac{-2w\left(D'(\underline{p})\right)^2+wD(\underline{p})D''(\underline{p})}{D(\underline{p})}.$$

We now develop an approximation for $\frac{\partial \underline{p}}{\partial w}$. Differentiating (7) with respect to w yields

$$(1+\lambda)\left(\left(\frac{\partial \underline{p}}{\partial w}-1\right)D(\underline{p}) + (\underline{p}-w)D'(\underline{p})\frac{\partial \underline{p}}{\partial w}\right) = -(1-\lambda)D(\rho),$$

which can be written as

$$\frac{\partial \underline{p}}{\partial w} = \frac{1-\frac{(1-\lambda)}{(1+\lambda)}\frac{D(\rho)}{D(\underline{p})}}{1+(\underline{p}-w)\frac{D'(\underline{p})}{D(\underline{p})}} = \frac{D(\underline{p})(\rho-\underline{p})}{(\rho-w)\left(D(\underline{p})+(\underline{p}-w)D'(\underline{p})\right)}. \tag{16}$$

So we have that

$$\frac{(1+\lambda)^2}{D^2(\rho)}\frac{\partial^2\pi}{\partial w^2} = \frac{2(\rho-\underline{p})^2D'(\underline{p})}{(\underline{p}-w)(\rho-w)^2}\frac{(\underline{p}-w)\left(\underline{p}D'(\underline{p})+D(\underline{p})\right)}{\left(D(\underline{p})+(\underline{p}-w)D'(\underline{p})\right)} + \frac{-2w\left(D'(\underline{p})\right)^2+wD(\underline{p})D''(\underline{p})}{D(\underline{p})},$$

which for λ close to 1 is approximately equal to

$$\frac{2D'(\underline{p})\left(\underline{p}D'(\underline{p})+D(\underline{p})\right)}{D(\underline{p})}+\frac{-2w\left(D'(\underline{p})\right)^2+wD(\underline{p})D''(\underline{p})}{D(\underline{p})},$$

which for λ close to 1 is approximately equal to

$$2D'(w)+wD''(w)$$

which is the second-order derivative in case $\lambda=1$. As we assume $pD(p)$ is twice differentiabile and strictly concave, this is strictly lower than 0.

PROOF OF PROPOSITION 5:

In case of linear demand the manufacturer's profits can be written as

$$\left(\tfrac{(1-\lambda)^2(1-w)^4}{32\lambda}\int_{\underline{p}(w)}^{\bar{p}(w)}\tfrac{1+w-2p}{(p-w)^3(1-p)^2}dp\right)w=$$
$$\tfrac{(1-\lambda)^2(1-w)^4w}{32\lambda(1-w)^3}\int_{\underline{p}(w)}^{\bar{p}(w)}\left(\tfrac{A}{(p-w)^3}+\tfrac{B}{(p-w)^2}+\tfrac{C}{(1-p)^2}+\tfrac{D}{p-w}+\tfrac{E}{1-p}\right)dp.$$

where $A=(1-w)^2, B=0, C=-(1-w)$ and $D=E=-1$. Thus, manufacturer's profits can be written as

$$\pi(w,\bar{p}(w))=\frac{(1-\lambda)^2}{32\lambda}(1-w)w\left[\frac{(1-w)^2}{-2(p-w)^2}-\frac{1-w}{1-p}-\ln\frac{p-w}{1-p}\right]_{\underline{p}(w)}^{p^m(w)}.$$

Using $p^m(w)=\frac{1+w}{2}$ and $1-p^m(w)=p^m(w)-w=\frac{1-w}{2}$, we have that $\underline{p}(w)=\frac{1+w-(1-w)\sqrt{\frac{2\lambda}{1+\lambda}}}{2}$. Plugging these into the profit function and simplifying, manufacturer's profits can be written as

$$\pi(w,\bar{p}(w))=g(\lambda)(1-w)w$$

for some function $g(\lambda)$. It follows that manufacturer's profits are concave in w and that the optimal choice of w equals 1/2.

PROOF OF PROPOSITION 6:

We first concentrate on the part dealing with the weighted expected retail price. This part of the proposition is true if the retail price $\frac{1}{1+\lambda}$ at $s=0$ is higher than the expected retail price for high s. The weighted average of the expected price can be written as

$$\begin{aligned}\tilde{p} &= (1-\lambda)\int_{\underline{p}(w)}^{\bar{p}(w)}pf(p)\,dp+2\lambda\int_{\underline{p}(w)}^{\bar{p}(w)}pf(p)(1-F(p))\,dp\\ &= \frac{(1-\lambda)^2\pi_r(\bar{p}(w))^2}{2\lambda}\int_{\underline{p}(w)}^{\bar{p}(w)}\frac{p\pi_r'(p)}{\pi_r(p)^3}dp,\end{aligned}$$

which for linear demand yields

$$\frac{(1-\lambda)^2\pi_r(\bar{p}(w))^2}{2\lambda}\int_{\underline{p}(w)}^{\bar{p}(w)}\frac{p(1+w-2p)}{(p-w)^3(1-p)^3}dp.$$

The expression under the integral can be written as

$$\frac{A}{(p-w)^3}+\frac{B}{(1-p)^3}+\frac{C}{(p-w)^2}+\frac{D}{(1-p)^2}+\frac{E}{p-w}+\frac{F}{1-p},$$

where $A = \frac{w}{(1-w)^2}, B = -\frac{1}{(1-w)^2}, C = E = F = \frac{1}{(1-w)^3}$, and $D = -\frac{w}{(1-w)^3}$. The integral is then given by

$$\left[\frac{-\frac{1-w}{(1-p)^2} - \frac{w(1-w)}{(p-w)^2} - \frac{2w}{1-p} - \frac{2}{p-w} + 2\ln\left(\frac{p-w}{1-p}\right)}{2(1-w)^3}\right]_{\underline{p}(w)}^{\bar{p}(w)}.$$

Using $\underline{p}(w) = \frac{1+w-(1-w)\sqrt{\frac{2\lambda}{1+\lambda}}}{2}$, $w = 1/2$ and $\bar{p}(w) = 3/4$, the weighted average price can be written as

$$\frac{-6\lambda\sqrt{2\lambda(1+\lambda)}+48\lambda-2\sqrt{2\lambda(1+\lambda)}-(1-\lambda)^2\ln\left(\frac{1-\sqrt{\frac{2\lambda}{\lambda+1}}}{1+\sqrt{\frac{2\lambda}{1+\lambda}}}\right)}{64\lambda}.$$

It is readily verified numerically that for the relevant λ, i.e., $\lambda > 0.47$, the above is lower than $\frac{1}{1+\lambda}$. Thus average weighted price for any $s > \bar{s}_\lambda$ is lower than at $s \to 0$.

We now concentrate on the part dealing with the manufacturer's profits. When $s \to 0$, manufacturer's profit is given by $\frac{1}{1+\lambda}\left(1 - \frac{1}{1+\lambda}\right) = \frac{\lambda}{(1+\lambda)^2}$. From Proposition 5, we know that for high s manufacturer's profits can be written as $g(\lambda)(1-w)w$, where $w = 1/2$ and

$$g(\lambda) = \frac{(1-\lambda)^2}{32\lambda}\left[2\left(\frac{1}{\sqrt{\frac{2\lambda}{1+\lambda}}+1} + \frac{1}{\left(1-\sqrt{\frac{2\lambda}{1+\lambda}}\right)^2} - 2\right) + \ln\left(\frac{1-\sqrt{\frac{2\lambda}{1+\lambda}}}{1+\sqrt{\frac{2\lambda}{1+\lambda}}}\right)\right].$$

Comparing this expression with $\frac{\lambda}{(1+\lambda)^2}$ one can either simply plot the two expressions to verify that $\frac{\lambda}{(1+\lambda)^2}$ is higher for all $0.47 < \lambda < 1$, or one can follow a more tedious route and verify the following points. First, $\frac{1}{4}g(\lambda)$ and $\frac{\lambda}{(1+\lambda)^2}$ are both equal to $\frac{1}{4}$ for $\lambda = 1$ and for $\lambda \approx 0.47$ the manufacturer's profit $\frac{\lambda}{(1+\lambda)^2}$ for $s \to 0$ is higher. Second, by using the substitution $x = \sqrt{\frac{2\lambda}{1+\lambda}}$, one can show that both expressions are decreasing in λ and the derivative of $\frac{\lambda}{(1+\lambda)^2}$ is lower at $\lambda = 1$ whereas it is higher at $\lambda \approx 0.47$, and for only one value of λ the derivatives are equal to each other. It follows that manufacturer's profits are higher when $s \to 0$ and $\lambda > 0.47$.

Finally, we deal with total industry profits. When $s \to 0$, industry profit equals manufacturer's profit and is thus equal to $\frac{\lambda}{(1+\lambda)^2}$. Total industry profits for high s equals $\pi(0.5, 0.75) + (1-\lambda)\frac{1}{16}$. As for $\lambda = 1$ total profit equals 1/4 and the derivative of $\frac{\lambda}{(1+\lambda)^2}$ is higher than that of the expression above for all relevant parameters ($\lambda > .47$), it follows that total profits are higher when s is high than when $s \to 0$.

Appendix B: Retail Oligopoly

In this appendix we show that the qualitative properties of the equilibria under retail duopoly extend to the case where there are more than two retail firms. The effects shown by the numerical analyses we present here are easily interpreted. Roughly speaking, there are two effects. First, the region of parameters where the knowledge consumers have on the wholesale price makes a difference becomes smaller. This in quite intuitive if we recall the result by Stahl (1989) that the reservation price is increasing in the number of firms. In the context of our model, this implies that when the number of firms is larger the upper bound of the retail price distribution is given by the retail monopoly price for smaller values of s. As this is the region where the two models coincide, the region where there is a difference between the two information scenarios becomes smaller. Second, when the search cost is small, the triple marginalization effect becomes stronger. That is, the larger the number of firms, the stronger is the effect that the equilibrium wholesale price is decreasing in s. The reason is that with a larger number of firms, the more probability mass the retail price distribution gives to higher prices and the smaller the effect of an increase in the wholesale price on the demand for the upstream firm.

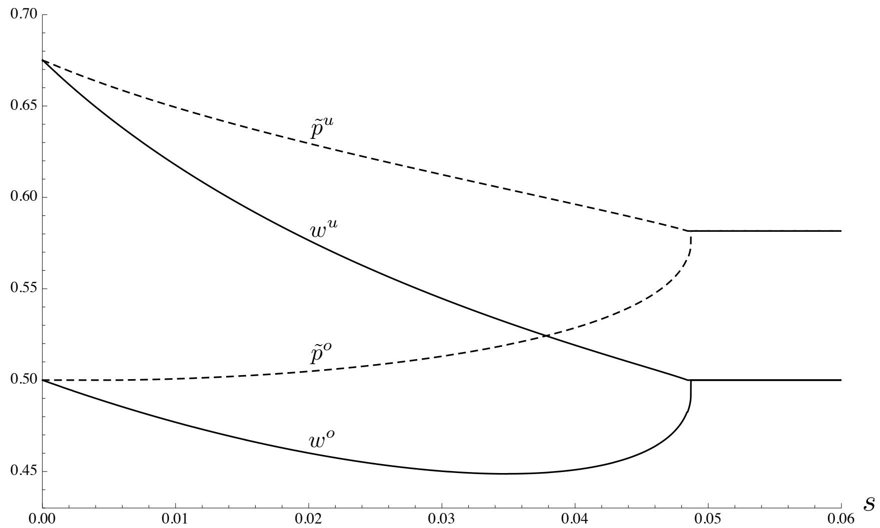

Figure 4: Upstream and weighted average downstream prices for the two models for $N = 3$.

We show these effects in two different ways. For $N = 3$, we show that Figure 4 depicting the relationship between wholesale and expected retail price as a function of s is similar to the corresponding figure for $N = 2$ (apart from the two differences noted above). Next, we also show the relationship between wholesale price and the number of firms when the search cost is small. As explained above, Figure 5 shows that this relationship is increasing.

Appendix C: Two-part tariffs

When the upstream firm uses in two-part tariffs, it charges a price per unit and a fixed fee to maximize profits. As firms are risk neutral, the upstream firm can charge a fixed fee equal to the expected profits of the retailers such that the retailers are still willing to participate. Thus, when charging an optimal two-part tariff, the upstream firm effectively maximizes total industry profits. In this case, the upstream firm has clearly less incentives to squeeze the retailers' profits with a higher unit price as it can recover these profits by charging a higher fixed fee. So, the question is whether two-part tariffs completely eliminate the problem (like they do for standard double marginalization problem) or the problem remains.

First, note that when the wholesale price arrangement is observed by consumers, the upstream firm wants to set its upstream price per unit such that the retail price distribution centers around the integrated monopoly price to maximize total industry profit. As there is price dispersion at the retail level, the upstream firm will never charge a price per unit that is equal to its marginal cost. To do so, would imply that the upper

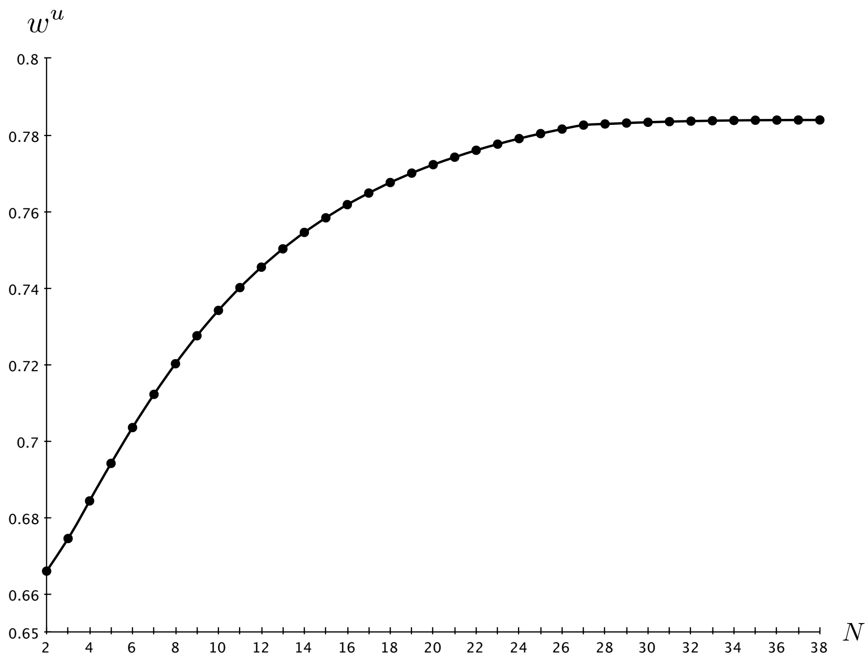

Figure 5: The upstream price for $\lambda = 0.5$ and $s = 0.0001$ as function of N.

bound of the retail price distribution is equal to the integrated monopoly price (for large search cost s) or below (for smaller values of s) so that almost surely the retail price is effectively too small to maximize profits. To have the retail price distribution center around the integrated monopoly price, the upstream firm sets a positive price per unit and a fixed fee. This is shown in Figure 6 for the case where $\lambda = 0.5$ and demand is linear and given by $D(p) = 1 - p$. Note that because of the price distribution at the retail level, the upstream firm cannot get the maximal profit of an integrated monopolist.

Next, consider the case where the wholesale price arrangement is not observed by consumers. If the upper bound of the retail price distribution is given by the retail monopoly price, the same consideration apply as above and the outcome of the two models coincide. However, when the search cost s is small, the upper bound of the retail price distribution is given by the non-shoppers' reservation price, which now does not depend on the wholesale price per unit. Thus, like in the baseline model without fixed fee, the retailers' price distribution (and thus the demand for the upstream firm) reacts less to an increase in the price per unit set by the upstream firm. thus, compared to the case where the wholesale price arrangement is observed by consumers, the upstream firm has (again) an incentive to increase the per unit wholesale price. As with two-part tariffs, the upstream firm effectively maximizes total industry profits and the downstream profits decrease with an increase in the per unit wholesale price, the incentives to increase the per unit wholesale price are dampened. The figure shows that the triple marginalization problem continues to exist, but the magnitude is much smaller.

Moreover, the effect analyzed in the previous section that the triple marginalization problem is most severe for small search cost, disappears. The reason why, as explained above, under two-part tariffs the triple marginalization problem continues to exist for positive search cost is because of the price dispersion at the retail level. When the search cost becomes arbitrarily small, however, the retail price dispersion disappears and the upstream firm can extract maximal profits by setting the wholesale price per unit equal to the integrated monopoly price. Thus, under two-part tariffs the two models converge

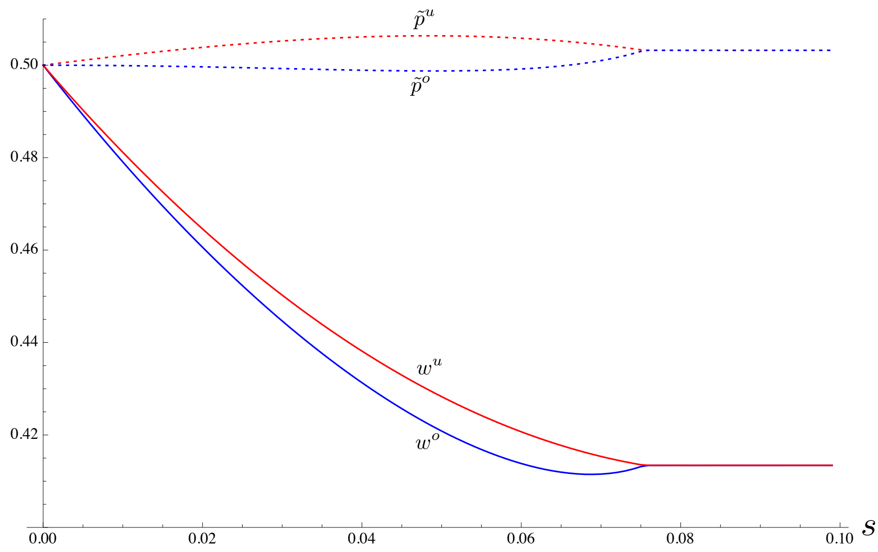

Figure 6: Two-part tariffs. Upstream prices (solid) and weighted average downstream prices (dashed) for the observed (blue) and unobserved (red) retailers' costs as functions of s for $\lambda = 0.5$.

for both large search cost and when the search cost is arbitrarily small.

Appendix D: Extension to Differentiated Goods

In this appendix we show that our results can be extended to a setting with differentiated goods. To do so, instead of the homogeneous goods model used in the body of the paper, we use a modification of Wolinsky (1986).

Assume an upstream manufacturer produces an essential input at the marginal cost of zero. He charges w for one unit to two competing retailers, 1 and 2. Retailers transform the input costlessly into the final differentiated good one for one. Valuation a consumer attaches to a good sold by retailer$i$ is v_i which is drawn according to $F(v_i)$ over the interval $[\underline{v}, \overline{v}]$. These valuations are independent between the two firms and consumers.

Each consumer costlessly visits one of the retailers with equal probability and finds out v_i and p_i. Consumer can then choose to visit the other retailer at a cost s. He can always go back to the first retailer at no additional cost. There is a unit mass of consumers per firm.

The timing is as follows: the upstream firm charges w which is observed by downstream firms (and possibly by consumers). The downstream firms charge prices, consumers visit one retailer for free, decide whether to visit the other one and make purchases if the best available $v_i - p_i$ is above zero, their outside option. We use Perfect Bayesian Equilibrium as the solution concept, and impose passive beliefs for consumers, thus upon observing a deviation by a retailer, consumers believe that this and only this retailer has deviated.

Observed upstream price

Let us solve the game backwards. At the stage when downstream firms choose prices w is known.

Assume that a consumer visits retailer 1 that charges p_1 and believes the other retailer charges p^*, the symmetric equilibrium price. As standard in this literature, define u^* as the solution to:

$$\int_{u^*}^{\bar{v}} (v - u^*) f(v)\, dv = s.$$

If such solution does not exist, then $u^* = \underline{v}$.

As is well known in the search literature, u^* is the expected utility of searching another firm including the search cost. This means that a consumer who draws v_1 will search the other firm if $v_1 < u^* - p^* + p_1$. For consumers to ever search we need that $u^* > p^*$, an assumption we make now and impose later.

Expected demand for firm 1 which charges p_1, while firm 2 charges p^* and all consumers expect the firm they have not visited first to charge p^* is given by:

$$D_1 = (1 - G(u^* - p^* + p_1))(1 + G(u^*)) + 2 \int_{p_1}^{u^* - p^* + p_1} G(p^* - p_1 + v) g(v)\, dv$$

The first order condition for firm 1's profit maximization, along with the equilibrium condition $p_1 = p^*$ gives the following price setting rule:

$$p^* = c + \frac{1 - G(p^*)^2}{g(u^*)(1 - G(u^*)) + 2\left(g(p^*)G(p^*) + \int_{p^*}^{u^*} g(v)^2\, dv\right)}. \tag{17}$$

Manufacturer's demand for equal retail prices is given by $(1 - G(p)^2)$ and so the upstream firm will set w^* that solves

$$\frac{\partial}{\partial w}(1 - G(p^*(w))^2) w = 0$$

where $p^*(w)$ is implicitly defined by (17). Assuming that the upstream profit is well behaved, and that $p^*(w^*) < u^*$, we have the solution to the model. Condition $p^*(w^*) < u^*$ will hold when s is sufficiently small. We shall assume this from now on.

Unobserved upstream price

Now we turn to the case where w is not observed by consumers. As in our baseline model with homogeneous goods, consumers cannot change their belief p^e in response to a change in w. To accommodate this, assume that consumers hold some beliefs w^e and p^e, that they do not update upon observing a deviation by a retailer they visit first (these are the passive beliefs used in the previous section).

Expected demand for retailer 1 that charges p_1, while firm 2 charges p_2 and all consumers believe that both retailers charge p^e is given by:

$$D_1 = (1 - G(u^* - p^e + p_1))(1 + G(u^* - p^e + p_2)) + 2 \int_{p_1}^{u^* - p^e + p_1} G(p_2 - p_1 + v) g(v)\, dv$$

Normally, in the last expression p_2 would be set equal to p^e. But when the manufacturer deviates and its deviation is not observed by consumers, consumers expect both firms to charge p^e and both retailers know that neither will charge p^e, so we have to allow for p_2 to differ from p^e.

To see why misplaced expectations are important note two things. First, the total demand when $p_1 = p_2 = p$ is equal to $1 - G(p)^2$ and does not depend on p^e. So the manufacturer does not *per se* care about misplaced expectations. Second, the derivative of the demand for firm 1 at around $p_1 = p_2$, which is negative, depends on p^e. To see this let us take the derivative of D_1 with respect to p_1 and set $p_1 = p_2 = p$:

$$-2g(p)G(p) - g(p - p^* + w)(1 - G(p - p^* + w)) - 2\int_p^{p-p^*+w} g(v)^2\,dv. \tag{18}$$

When beliefs are correct ($p^e = p$) previous simplifies to:

$$-2g(p)G(p) - g(w)(1 - G(w)) - 2\int_p^{w} g(v)^2\,dv \tag{19}$$

Both of these expressions are negative. Let us define $x = p - p^*$

and take the derivative of (18) with respect to x and evaluate it at $x = 0$. This derivative is given by

$$-g(w)^2 - (1 - G(w))g'(w). \tag{20}$$

If (20) is negative, then when consumers expect to see lower prices than both firms charge ($x > 0$), derivative of a firm's demand with respect to its own price is larger in absolute value, and so retailers charge lower prices. This means that $\frac{\partial Q}{\partial w^e} < 0$, and so the manufacturer charges a higher price in the unobserved case.

Note that if (20) is negative, it is the condition for log-concavity of $1 - G(w)$. This condition is implied by log-concavity of $g(w)$. It is necessary in the standard Wolinsky model to have that prices decrease in search cost (see Anderson and Renault (1999)). It is satisfied for many commonly used distributions such as uniform, normal and logistic, but for exponential (20) is zero, and therefore prices coincide in the observed and unobserved cases.

Appendix E: Price Discrimination Between Retailers

In the main body of the paper we considered environments where the upstream firm cannot price discriminate between different retailers. In this extension, we discuss the kind of considerations that apply when price discrimination is feasible and ask whether the symmetric equilibrium without price discrimination we characterized so far remains an equilibrium or not.

To consider this question, we have to distinguish two different cases, depending on whether or not rival retailers observe the wholesale price the other retailer pays. In the main part of the analysis where retailers know the upstream firm cannot price discriminate, these two cases coincide as knowing your own cost level (the wholesale price) you also know the cost of your competitor. Consider now first the case where retailers do not observe the wholesale price rival firms pay. In this case, whether or not it is optimal for the upstream firm to deviate depends on the beliefs retailers have about the type of deviation chosen by the upstream firm. Note that in this case of private information about their own wholesale price, after observing a deviation by the upstream firm, retailers have to form beliefs about the wholesale price the deviating upstream firm charges to the rival retailer, and they have to have beliefs about the rival's beliefs about their own wholesale price, and so on. There is no standard refinement notion in game theory that restricts these type of beliefs, and one can easily construct beliefs such that the retailers react in such a way that the upstream firm's deviation is not profitable. For example, if after observing a deviation, a retailer believes that the upstream firm has deviated in such a way that both retailers have the same cost and believes that the other retailer has similar beliefs, then our analysis of no price discrimination applies.[29] Thus, one can support the symmetric equilibrium we have concentrated on so far, also in case the upstream firm can price discriminate between retailers by appropriate beliefs of the retailers after a deviation of the upstream firm.

Next consider, the case where the rivals observe each others' costs. In that case we can show that local deviations are not profitable, but to show that global deviations are also unprofitable is beyond analytical tractability, and it is not clear that this case is worthwhile to pursue (as retailers may actually not observe each others' cost). What is easy to see, however, is that the upstream firm does not have an incentive to charge one of the retailers such a high price that it effectively forecloses this firm from actively selling in the market. The reason is that given that the equilibrium upstream price is larger than the vertically integrated monopoly price, the upstream firm benefits from downstream competition, and even though it is imperfect because of consumer search, it is better than not having any downstream competition.

In the rest of this subsection, we show that the upstream cannot locally increase profits by price discriminating between the two retailers who know each others' cost. To this end, let us assume that the upstream firm deviates and charges w_1 to firm 1 and $w_2 (< w_1)$ to firm 2 and that $p^m(w_i) > \rho$ $(i = 1, 2)$. The proof proceeds by showing that the upstream is always better off by charging both firms w_1 instead of charging one retailer a lower price. Given that any asymmetric pricing strategy of the upstream firm is dominated by a symmetric one, it follows that charging both firms the w^* derived in the previous sections has to be locally optimal.

If $w_1 > w_2$ and both retailers know each others' cost, both downstream firms will randomize continuously over an interval $[\underline{p}, \rho)$ and firm 1 will have a mass point at ρ. For any price $p \in [\underline{p}, \rho)$ that firm 1 charges, the pricing strategy of firm 2, denoted by F_2, should satisfy

$$(p - w_1)D(p)(1 - \lambda + 2\lambda(1 - F_2(p))) = (1 - \lambda)(\rho - w_1)D(\rho) \tag{21}$$

in order to make firm 1 indifferent between any of its prices in the support of the mixed strategy distribution. This gives

$$F_2(p) = 1 - \left(\frac{1-\lambda}{2\lambda} \frac{(\rho - w_1)D(\bar{p}) - (p - w_1)D(p)}{(p - w_1)D(p)} \right).$$

From (21), the lower limit $\underline{p}$ is implicitly defined by

$$(1 + \lambda)(\underline{p} - w_1)D(\underline{p}) = (1 - \lambda)(\rho - w_1)D(\rho). \tag{22}$$

Note that for a given ρ this implies that the lower bound of the price distribution in this asymmetric case where $w_1 > w_2$ is equal to the lower bound of the price distribution in case both firms are charged a wholesale price of w_1. Moreover, observe that $F_2(p)$ coincides with the distribution that would have been used if both downstream firms were charged w_1.

[29] This is only one set of beliefs that work. There are many other beliefs the retailers may hold, and depending on these beliefs the upstream firm either has or does not have an incentive to deviate.

For any price that firm 2 charges in the interval $[\underline{p}, \rho)$, F_1 should satisfy

$$(p - w_2)D(p)(1 - \lambda + 2\lambda(1 - F_1(p))) = (1 + \lambda)(\underline{p} - w_2)D(\underline{p}) = (1 - \lambda + 2\lambda\gamma_1)(\rho - w_2)D(\rho),$$

in order to make firm 2 indifferent between any of its prices in the support of the mixed strategy distribution. This yields

$$F_1(p) = 1 - \frac{(1 - \lambda + 2\lambda\gamma_1)(\rho - w_2)D(\rho) - (1 - \lambda)(p - w_2)D(p)}{2\lambda(p - w_2)D(p)}. \tag{23}$$

From the above, firm 1's mass point at ρ is equal to

$$\gamma_1 = 1 - F_1(\rho) = \frac{(1 - \lambda)}{2\lambda}\frac{(w_1 - w_2)(\rho - \underline{p})}{(\underline{p} - w_1)(\rho - w_2)}. \tag{24}$$

Observe that

$$\begin{aligned} & F_1(p) < F_2(p) \Longleftrightarrow \\ \Longleftrightarrow\ & (1 - \lambda + 2\lambda\gamma_1)(\rho - w_2)(p - w_1) > (1 - \lambda)(\rho - w_1)(p - w_2) \\ & \Longleftrightarrow 2\lambda\gamma_1(\rho - w_2)(p - w_1) > (1 - \lambda)(\rho - p)(w_1 - w_2), \end{aligned}$$

which, given the expression for γ_1, is certainly the case

$$\begin{aligned} (1 - \lambda)(w_1 - w_2)\frac{\rho - \underline{p}}{\underline{p} - w_1}(p - w_1) &> (1 - \lambda)(\rho - p)(w_1 - w_2) \Longleftrightarrow \\ (\rho - \underline{p})(p - w_1) &> (\rho - p)(\underline{p} - w_1) \Longleftrightarrow \\ (\rho - w_1)(p - \underline{p}) &> 0. \end{aligned}$$

This means that F_1 first-order stochastically dominates F_2.

The above implies that the upstream firm always earns larger profit when charging both retailers w_1 than when charging one retailer w_1 and the other one a lower w_2. This is true because in the asymmetric case retail prices are higher (firm 2 behaves in the same way in both cases and firm 1 charges higher prices), and hence upstream demand is lower. Moreover, one of the firms (firm 2) gets, on average, a larger proportion of this smaller demand and pays less to the upstream firm. All these factors reduce the upstream firm's profit.

So for the upstream firm any asymmetric pricing is dominated by some form of symmetric pricing and we have shown before that of all symmetric prices, charging w^* to both retailers gives the highest profit. Thus, for the upstream firm, there is no profitable local deviation from $w_1 = w_2 = w^*$.

The above analysis remains true as long as $p^m(w_2) > \rho$. However, when the upstream firm sets w_2 such that $p^m(w_2) < \rho$, retailer 2 will not charge prices up to ρ and the price distributions will be truncated at $p^m(w_2)$ with firm 2 having a mass point at $p^m(w_2)$ and firm 1 having a mass point at ρ. Thus, the above argument that the upstream firm's demand is decreasing if one decreases w_2 from an intial level of w_1 is a local argument, and does not continue to hold if w_2 is decreased sufficiently. The fact that the retailers choose mixed strategies and that we do not have clean expressions for how upstream profits depend on w (let alone how they depend on asymmetric choices of w_1 and w_2) makes it difficult to come up with a global argument. Given that it is not clear that this case of retailers knowing each others' cost level is more interesting than the case where they do not, we have decided not to develop this point further.