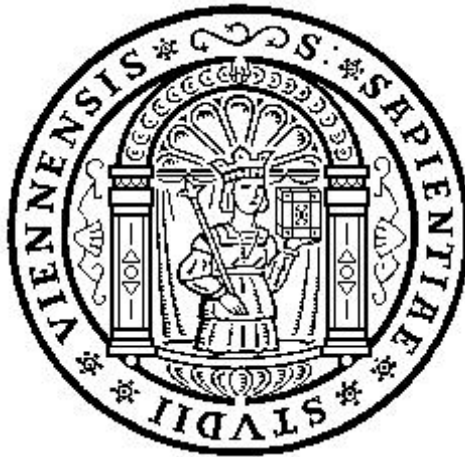


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Maarten Janssen
Sandro Shelegia
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Beliefs and Consumer Search*

Maarten Janssen[†] and Sandro Shelegia[‡]

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Abstract

When consumers search sequentially for prices and product matches, their beliefs of what they will encounter at the next firm are important in deciding whether or not to continue to search. In search environments where retailers have a common cost that is not known to consumers and is either the outcome of a random process or strategically set by an upstream firm, it is natural for consumers to have symmetric beliefs. We show that market outcomes under symmetric beliefs are quantitatively and qualitatively different from outcomes when consumers hold passive beliefs. Market prices are higher with symmetric beliefs (and can be as high as the joint profit maximizing prices), and are non-monotonic in the search cost. Moreover, price rigidities arise endogenously as retailers are not willing to charge prices above consumers' reservation utility. These phenomena become exacerbated in a vertical relations environment.

JEL Classification: D40; D83; L13

Keywords: Vertical Relations, Consumer Search, Double Marginalization, Product Differentiation, Price Rigidities

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[†]Department of Economics, University of Vienna and State University Higher School of Economics. Email: maarten.janssen@univie.ac.at

[‡]Department of Economics, University of Vienna. Email: sandro.shelegia@univie.ac.at

1 Introduction

In consumer search markets, the market power of firms depends on consumers' willingness to search. When at a firm, consumers compare the benefits of buying now with the expected benefits of continuing to search. These expected benefits of search crucially depend on the price consumers believe a next firm charges. Consumers will form beliefs differently depending on the environment they are in and the amount of information (about the environment) they possess. Resulting beliefs determine how consumers react to price changes, and thus how profitable price changes are. If consumers are pessimistic about whether the next search will yield a good offer, they are more likely to accept the current offer, giving firms incentives to set higher prices. This basic insight is important in any search market, but - as we will argue in this paper - it is particularly important in markets where consumers face uncertainty concerning factors that influence market prices. If, for example, consumers know that prices may depend on firms' common marginal cost, but are uninformed about this cost, then the price the consumer observes at the current firm is likely to be interpreted as a signal of the common cost level and, thus (presuming prices positively depend on cost), of the expected benefits of search. In a similar way, if retailers' common marginal cost is determined by a manufacturer who strategically determines the wholesale price (as an important component in the retailers' cost) and consumers do not observe the wholesale price, then beliefs about retail prices that are not yet observed will be influenced by the prices consumers observe at the retailer they are currently visiting.

When market power depends on consumer beliefs, it is important to understand their impact of beliefs and what type of beliefs consumers may have. The consumer search literature has, by and large, side-stepped the issue of consumer beliefs and implicitly or explicitly assumed that consumers hold *passive* beliefs, i.e., no matter what they observe at a particular firm, consumers believe that all other firms will stick to their equilibrium strategies.¹ We have argued above that there are natural environments where one would expect consumers to have other types of beliefs, such as *symmetric* beliefs,² where consumers believe that firms whose prices they have not yet observed charge the same price as the firm they are currently visiting. Going one step further, one may argue that even if consumers fully know the firms' environment, consumers may hold symmetric beliefs, i.e., there is no particular reason why consumers should hold passive beliefs even in standard models of consumer search.

¹This is the case in both large strands of the sequential search literature. For heterogeneous goods models that build on Wolinsky (1986), passive beliefs are the norm (e.g. see Anderson and Renault (1999), Armstrong, Vickers and Zhou (2009), and Bar-Isaac, Caruana and Cuñat (2011) among many others). The same is true for homogenous goods models that build on Stahl (1989), but there passive beliefs are necessary for equilibrium to exist (see also Footnote 4).

²See, for instance, Hart et al. (1990), McAfee and Schwartz (1994) and more recently Pagnozzi and Piccolo (2012).

This paper shows that symmetric beliefs lead to equilibrium predictions that are qualitatively and quantitatively significantly different from the traditional analysis that assumes passive beliefs. In particular, we provide new explanations for such diverse phenomena as price rigidities, sales and seemingly collusive outcomes. We do so in three interconnected models. We start our analysis by introducing symmetric beliefs in the standard model by Wolinsky (1986). We then consider the two adaptations of this model that were alluded to in the first paragraph: one where there is asymmetric information about firms' common cost and one where this cost is determined by an upstream manufacturer. To explain how these phenomena arise under symmetric beliefs in these different environments, it is important to understand how the Wolinsky model relates to the Diamond Paradox (Diamond (1971)). Diamond showed that with homogenous goods, for any positive search cost, there will be no search beyond the first firm and all firms charge the monopoly price. As discussed in Anderson and Renault (1999), Wolinsky solved the paradox by introducing product differentiation, giving some consumers incentives to search. However, when the search cost is sufficiently high, Wolinsky's solution fails because even consumers with very bad utility draws are not willing to pay the search cost. As a result, when the search cost exceeds a certain threshold, search stops, firms set the monopoly price, and demand drops discontinuously. The existence of such a search cost threshold, and the associated demand drop is also new to the literature.

An immediate implication of symmetric beliefs in the Wolinsky (1986) model is that, whenever consumers search in equilibrium, prices are higher with symmetric beliefs. The reason is that with symmetric beliefs consumers are less willing to search after observing a price that is higher than the equilibrium price, so a switch from passive to symmetric beliefs increases prices. A less immediate change, but one that is even more important in the two applications we consider, is that with symmetric beliefs the equilibrium price is non-monotonic in search cost. For relatively small search costs, the price is increasing in search cost (as it is under passive beliefs). Once the search cost is sufficiently high, however, firms find themselves in a situation where, conditional on consumers searching, they would like to charge prices in excess of the consumers' reservation utility, but when they do so, consumers do not want to search. The latter increases firms' demand elasticity and induces them to reduce prices to the reservation utility. This creates an interval of intermediate search costs where firms charge prices equal to the reservation utility. As the reservation utility is decreasing in search cost, prices are also decreasing.³ Overall, price is hump-shaped in search cost. Interestingly, at the maximum, both firms set the price that a monopolist selling both goods would set. Thus, with symmetric beliefs and intermediate

³This inverse relationship between search cost and prices is unlike the earlier contributions with the same conclusion. In Janssen, Moraga-González and Wildenbeest (2005), the search cost changes the composition of heterogeneous consumers and may result in lower prices at higher search cost. In Zhou (2014) consumers search for multiple products. In this environment, products are search complements and if the search cost increases, firms may compete more intensely to prevent consumers searching further.

search costs a fully collusive outcome results even though firms act non-cooperatively!

Another way to look at the same equilibrium is to fix the search cost, and see how equilibrium changes as the marginal cost increases. This perspective is important for both of our applications where marginal cost plays the key role. With passive beliefs, price is always increasing in marginal cost. With symmetric beliefs, there is an intermediate level of cost where price equals the reservation utility, and because the reservation utility does not depend on marginal cost, neither does price. So with symmetric beliefs the price is first increasing, then constant, and then increasing again. This creates a new search theoretic explanation for price rigidities: firms will not increase prices above consumers' reservation utility.

We next adapt the Wolinsky model to an environment where firms' common marginal cost is random, known to firms but not to consumers.⁴ This is relevant in a variety of product markets where the cost of inputs fluctuates, such as products where precious metals and fossil fuels are used. In a separating equilibrium of this model, consumers have to infer marginal cost from the prices they observe, resulting in symmetric beliefs. Once we focus on symmetric beliefs, several conclusions follow immediately from our prior analysis. First, for any sufficiently low realization of marginal cost, prices are higher in the model with uncertain cost than with known cost. Second, for some cost realizations price is equal to reservation utility, and so independent of cost. This allows us to provide an alternative explanation for (i) price stickiness (see Sheshinski and Weiss (1977) and Akerlof and Yellen (1985); Akerlof and L. Yellen (1985) for seminal contributions), and (ii) for the observation that retailers often follow a pricing strategy that is characterized by a regular price and periodic price reductions. It is important to note that our search theoretic explanation of price rigidity does not depend on some exogenously assumed menu cost, nor does it assume that costs are somehow sticky (as in Cabral and Fishman (2012)): cost fluctuates, but for a (large) range of cost realizations, retailers do not change their price.⁵ Our explanation of irregular sales is, unlike the existing literature, in terms of a pure strategy equilibrium where a firm's price is a function of its fluctuating cost, but for a range of high cost realizations firms charge the same 'regular' price.

Our second application is a vertical relations model where retailers' common cost is set by an upstream monopolist and the cost is not observed by consumers. In this environment consumers may have passive or symmetric beliefs as the final retail price

⁴In the context of an alternative, homogeneous goods search model by Stahl (1989), Dana (1994), and more recently, Tappata (2009), Chandra and Tappata (2011) and Janssen, Pichler and Weidenholzer (2011) have analyzed consequences of asymmetric information about firms' common marginal cost. Janssen and Shelegia (2014) analyze a vertical relations model where retailers' common cost is determined by an upstream firm and is unobserved by consumers. These models use passive beliefs, however, as a reservation price equilibrium does not exist under symmetric beliefs in homogeneous goods search models.

⁵Cabral and Fishman offer a search theoretic explanation for why "output prices are stickier than input costs". In our model, costs are not sticky at all, but prices are.

a consumer observes is the product of the wholesale price set by the manufacturer and the way the retailer reacts to that price. Under passive beliefs, the consumer blames an individual retailer for setting an unexpected price and believes that the other firms (the manufacturer and the second retailer) have not deviated. Under symmetric beliefs, the consumer blames the manufacturer for the fact that the consumer has observed an unexpected price and believes that retailers react in the same way to the deviation by the manufacturer.

Under passive beliefs, the equilibrium structure is as follows. For low search cost, the manufacturer sets a wholesale price such that retail prices are lower than the reservation utility. For higher search costs, the manufacturer's optimal price is such that retail prices will be higher than the reservation utility and consumers do not search. Thus, for high search cost there is no active search and a classic double-marginalization outcome results, with the retail prices being equal to the retail monopoly price (given the wholesale price) and the manufacturer setting his optimal price (given this retail behavior).

Also in the vertical relations model, symmetric beliefs create a difference in that there may exist a large intermediate region of search costs where both wholesale and retail prices are decreasing in search cost. All the features that we have emphasized above, such as price rigidities, non-monotonicity of retail prices in search cost, sales-type pricing behavior and retail prices being higher than monopoly prices (here: double marginalization prices), may also occur in the vertical relations model. The result on price rigidities holds, even if the search cost is very small. In a specific example, we show that the vertical relations model may exacerbate all these phenomena as the manufacturer deliberately sets a wholesale price so as to have retail prices equal to the reservation utility in order to prevent a drop in market demand. To the best of our knowledge, there is no other literature where a firm proactively seeks to prevent the Diamond Paradox from arising. Under symmetric beliefs, the manufacturer is able to manipulate consumers' beliefs and, thus, their search behavior.

The rest of the paper is organized as follows. In Section 2 we set up the Wolinsky model, and show how a firm's demand depends on consumer beliefs. In Section 3 we analyze firm pricing with passive and symmetric beliefs. In Section 4 we introduce marginal cost uncertainty into the Wolinsky model and solve for equilibrium using our results from previous sections. In Section 5, we introduce an upstream monopolist who determines the retailers' marginal cost in the Wolinsky model. We show that by acting non-cooperatively the Diamond Paradox can be avoided by the manufacturer. The final section concludes. Proofs are contained in Appendix A. Subsequent appendices deal with possible alternative formulations of the model.

2 The Wolinsky Model and Preliminaries

The retail side of the models we study follows Wolinsky (1986). There are two retailers, 1 and 2, who can acquire some input at a common cost c . The retailers transform the input into a final differentiated good, using a one for one technology. There is a unit mass of consumers per retailer. Utility to a consumer from buying the good at retailer i is v_i . This utility is drawn from the distribution function $G(v)$, which is the same for both retailers and defined over the (possibly infinite) interval $[\underline{v}, \bar{v}]$. As is standard in the literature, we require that $1 - G(v)$ is logconcave, but in the vertical model, we sometimes restrict ourselves to $G(v)$ being uniformly distributed. The a consumer's valuation of the product of retailer 1 is independent of his valuation of the product of retailer 2. Each consumer visits one of the retailers at random and finds out v_i and p_i . After observing the match value v_i and the price p_i consumers decide whether or not to visit the second firm. If they do so, they make their purchase at the best available surplus $v_i - p_i$, provided that it exceeds zero, the outside option. We assume that the first visit is free.⁶ The second visit is costly, and the cost is denoted by s . If after visiting the first retailer, consumers decide to search the second firm, they can always go back to the first retailer at no additional cost (free recall).

The above set-up is common. We now explicitly specify the role of beliefs in the model, and derive retailers' demand. Assume retailer i sets price p_i and retailer j sets price p_j . Consider consumers who make their first visit at retailer i , where they find out p_i and v_i . These consumers do not know p_j , and they will have to form some belief about it, denoted by p_j^e . We write $p_j^e(p_i)$ as in general, p_j^e may depend on p_i . The same is true for consumers who visit firm j first and have to form beliefs about p_i , denoted by $p_i^e(p_j)$. The consumer search literature implicitly or explicitly uses "passive" beliefs, implying that p_j^e is independent of p_i (and p_i^e is independent of p_j). The vertical contracting literature (see, e.g., Hart et al. (1990) and McAfee and Schwartz (1994)) also consider "symmetric" beliefs, which in our setting can be described as $p_j^e = p_i$: consumers who first visit firm i believe that firm j charges the same price as i . Many other beliefs are possible, but we shall concentrate on passive and symmetric beliefs for two reasons. First, both have been used in the literature, are well understood and are likely to be used by consumers. Second, once we understand how the standard model changes with symmetric beliefs, it will be relatively straightforward to extrapolate how the equilibrium predictions change with other beliefs.

Consider a consumer who visits firm i . As is well known, a rational consumer will use a reservation utility strategy, where she searches if the utility drawn is below a certain threshold, and stops if the utility exceeds this threshold. This threshold depends on the utility realization, firm i 's price and the belief about firm j 's price. Before finding it,

⁶See Appendix C for the solution of our model when the first search is costly.

we determine the reservation utility w , a utility level at which a consumer is indifferent between searching the second firm, or accepting w , assuming that both firms charge equal prices. The reservation utility w is the solution to

$$\int_w^{\bar{v}} (v - w) f(v) dv = s, \quad (1)$$

if the solution exists, and is equal to $w = \underline{v}$ if it does not.

The threshold utility level is then simply computed as w plus the (expected) price difference. Formally, a consumer who draws v_i and p_i will search the other firm if

$$v_i < w + (p_i - p_j^e).$$

This condition guarantees that the consumer prefers to search rather than to accept the current offer. In order to guarantee that consumers search we need that their reservation utility exceeds the price expected at the other firm, $w > p_j^e$, or else the expected benefit from search is negative and no consumer searches beyond the first firm.

Given the optimal search behavior above, retailer i 's demand is given by

$$\begin{aligned} Q_i = & (1 - G(w - p_j^e + p_i)) + \int_{p_i}^{w - p_j^e + p_i} G(p_j - p_i + v) g(v) dv \\ & + G(w - p_i^e + p_j)(1 - G(w - p_i^e + p_i)) + \int_{p_i}^{w - p_i^e + p_i} G(p_j - p_i + v) g(v) dv. \end{aligned} \quad (2)$$

The first term is the demand from consumers who visit retailer i and buy outright because their utility draw is smaller than the threshold $w - p_j^e + p_i$. The first integral is the demand from those consumers who first visit retailer i , decide to visit retailer j , but come back and buy from retailer i . The third term is the demand from consumers who first visit retailer j , draw a value v_j that is smaller than the threshold value $w - p_i^e + p_j$, decide to visit retailer i , encounter a value v_i that is higher than $w - p_i^e + p_i$ and buy at retailer i . Finally, the second integral is similar to the third term, but accounts for those consumers who first visit retailer j , decide to search retailer i , also draw a relatively low value at retailer i , but still buy at retailer i as that price/product offer is considered to be better than that offered by retailer j .

This expression is different from the standard expression in the literature following Wolinsky (1986) in several respects. It allows retailer i , whose demand is being computed, to set a price p_i , which may differ from all of the following: (i) the price of its rival, p_j , (ii) the belief consumers who visit firm 2 hold about its price, p_i^e , and (iii) the belief consumers who visit it hold about firm 2's price, p_j^e . When looking for the equilibrium with passive beliefs, the Wolinsky literature sets all p_j, p_j^e and p_i^e equal to the candidate equilibrium price p^* . As our aim is to study symmetric beliefs, we must allow p_j^e to

potentially change with p_i . Furthermore, we will study two models where firms' common marginal cost is known to them, but unknown to consumers. Because of this, our demand equation has to allow for the possibility that firm i is aware that its price differs from retailer j 's price (because retailer i is planning to deviate), which in turn differs from the belief consumers who visit it hold about retailer i 's price, and also consumers who visit retailer i have beliefs about the price of retailer j that are different from all the previous prices.

The demand expression for firm i is illustrated in Figure 1. Area A corresponds to the first term in (2) which refers to consumers who arrive at firm i first, draw v_i above $w - p_i + p_j^e$ and buy outright. Area B corresponds to consumers who arrive at i first but continue to search because their utility draw v_i is smaller than $w - p_i + p_j^e$ and then come back to buy because their utility draw at firm j is even worse (the second term in (2)). Areas C and D correspond to consumers who first visit firm j , draw a utility level v_j that is smaller than $w - p_j + p_i^e$, continue to search firm i and purchase there as their utility at firm i is higher than p_i and higher than the utility at firm j . Note that consumer beliefs affect who searches, but not whether consumers buy or not.

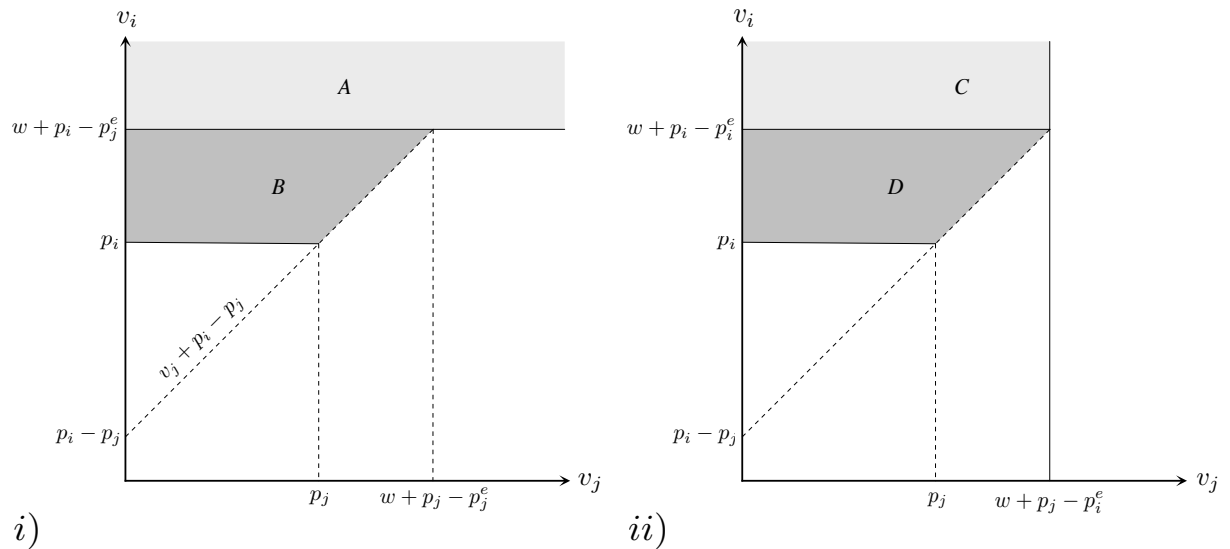


Figure 1: Figures i) and ii) depict consumers who buy from firm i after having made a first visit to firm i and firm j , respectively.

3 Equilibrium Behavior

In this section, we characterize the Perfect Bayesian Equilibrium (PBE) of the Wolinsky model for both passive and symmetric beliefs. This is the key ingredient for our subsequent analysis. The analysis with passive beliefs is mostly well known, except for the part where we solve the model for large search cost (or alternatively, for large marginal cost). We show that once search cost exceeds a certain threshold, demand drops discontinuously.

The analysis with symmetric beliefs is new, and also features a drop in demand. This drop-in-demand feature of the model is important when we consider vertical relations, because under symmetric beliefs a manufacturer has the incentive and the ability to avoid such a drop.

To be precise about the role of beliefs in the Wolinsky model, we formally define the equilibrium notion below. Beliefs about the price to be observed at the next search may depend on the price observed at the first search, so consumers' expected utility of searching another retailer may depend on the price observed at the first firm, and we write $w(p_1)$.

Definition 1 *A symmetric perfect Bayesian equilibrium (PBE) of the Wolinsky model is a price p^* (the same for both firms) and a reservation utility $w(p_1)$ such that*

1. *Each firm i chooses $p_i = p^*$ to maximize its expected profit given the price of the other firm and consumers' reservation utility;*
2. *Consumers follow an optimal search rule given their beliefs and the match value v_i and price p_i they observe at firm i ;*
3. *Consumers' beliefs are updated using Bayes' Rule when possible, i.e., whenever they observe $p_1 = p^*$ at their first search, they believe that the other firm has also set a price p^* . Out-of-equilibrium beliefs p_2^e of the price set by the firm that is not yet visited are either*
 - i. *(passive beliefs) such that $p_2^e = p^*$ if $p_1 \neq p^*$, or*
 - ii. *(symmetric beliefs) such that $p_2^e = p_1$ if $p_1 \neq p^*$.*

It is important to understand why we focus on the duopoly case. First, focusing on duopoly allows us to simplify the analysis considerably. In particular, with two retailers, a consumer has to form a belief once (prior to the first search), and so her belief-formation is relatively simple. This is not true for three or more retailers, where it may be the case that, after having searched two firms, a consumer observes two different prices and needs to re-form her belief in order to decide whether to search or not. Second, regardless of how subsequent beliefs are formed, the issues we uncover in the duopoly model are still relevant because they concern the first search. In fact, in virtue of the fact consumers are more, and often much more, likely to search once rather than two or more times, what our analysis in this paper is of first-order relevance even in oligopoly models.

It is intractable to provide a proof that the equilibrium exists for a general match function $G(\cdot)$. As far as we know none of the papers in the literature based on the Wolinsky model provides such a general existence proof. The existence proof is trivial, however, when $G(\cdot)$ is uniform, where it is also easy to see that the symmetric equilibrium is

unique. Moreover, numerical simulations show that the existence and the uniqueness of the symmetric equilibrium are also guaranteed for other frequently used distribution functions, such as the normal and logistic distribution. The propositions below are formulated assuming that the equilibrium exists.

3.1 Passive Beliefs

In order to find the symmetric equilibrium of the Wolinsky model with passive beliefs, we use demand from (2) and adapt it to passive beliefs. Let p^* denote the equilibrium price where it does not exceed w . Then from the perspective of retailer i who intends to deviate to $p_i \neq p^*$, retailer j charges the equilibrium price, and consumers who first visit retailer i or retailer j also expect the other retailer to charge the equilibrium price, no matter what price they observe at their first visit. Therefore, we set $p_j = p_j^e = p_i^e = p^*$.

When the equilibrium price satisfies $w \geq p^*$, expected demand for retailer i simplifies to:

$$Q_i = (1 - G(w - p^* + p_i))(1 + G(w)) + 2 \int_{p_i}^{w - p^* + p_i} G(p^* - p_i + v)g(v) dv.$$

The first-order condition for firm 1's profit maximization, along with the equilibrium condition $p_i = p^*$ gives the following pricing rule:

$$p^* = \hat{p} \equiv c + \frac{1 - G(p^*)^2}{2 \int_{p^*}^w g(v)^2 dv + 2G(p^*)g(p^*) + (1 - G(w))g(w)}, \quad (3)$$

where \hat{p} is defined as the equilibrium price under passive beliefs for low search cost. Note that \hat{p} is an increasing function of c . As shown by Anderson and Renault (1999), \hat{p} is also increasing in s when $1 - G(v)$ is logconcave.

The equilibrium price is derived under the assumption that all consumers search, which in this setting requires that the benefit from search, equal to $w - \hat{p}$, is non-negative. For high enough s , however, the solution to (3) exceeds w . This is because w is a decreasing function of s , and so there always exists a sufficiently large s such that $\hat{p} > w$. Each firm is then a single-good monopolist that faces demand $1 - G(p)$. The resulting profit $(p - c)(1 - G(p))$ is maximized at the monopoly price, denoted by p^M , that solves

$$p^M = c + \frac{1 - G(p^M)}{g(p^M)}. \quad (4)$$

The threshold search cost level s_2 (s_1 will be defined shortly) is such that $w = \hat{p}$ at which point (3) and (4) coincide. Therefore, s_2 solves

$$\int_{p^M}^{\bar{v}} (v - p^M)g(v) dv = s_2. \quad (5)$$

Proposition 1 *Under passive beliefs, the equilibrium price p^* of the Wolinsky model is*

equal to \hat{p} for $s \leq s_2$, and is equal to p^M for $s > s_2$.

Figure 2 shows that the equilibrium price is continuous in search cost s and strictly increasing for $s \leq s_2$ and constant for $s > s_2$. To intuitively understand why for passive beliefs \hat{p} is equal to p^M when $\hat{p} = w$ it is useful to consider in Figure 1 a marginal upward deviation from $p_i = \hat{p}$ when \hat{p} is close to w and $p_j^e = \hat{p}$. The first thing to note is that for \hat{p} close to w and $p_j^e = \hat{p}$, the areas B and D are negligibly small. Moreover, in areas B and D, the number of consumers firm i loses by increasing its price is (almost) proportional to the size of these areas. The second thing to note is that a monopolist considers the value of the demand he loses by increasing price relative to the increased revenue he makes over all consumers that continue to buy. As valuations for product i and j are independent of each other and the losses are proportional to the size for all areas A-D, a firm that under passive beliefs marginally increases its price for \hat{p} close to w and $p_j^e = \hat{p}$ has identical considerations as a monopolist (who only considers area A). More intuitively, as s increases and w approaches \hat{p} , almost no consumers search and compare prices. Thus, each firm is only concerned with whether or not consumers accept their price, which is exactly the consideration of a monopolist.

Note that for s higher than the threshold value s_2 , equilibrium demand falls discontinuously. For $s > s_2$, consumers only buy when their match value at the first firm exceeds \hat{p} , so that total demand equals $1 - G(p)$. For lower s , consumers will search on and total demand equals $1 - G(p)^2$, so the equilibrium demand (and thus retailers' profits) is discontinuous at $s = s_2$. Figure 4 shows the equilibrium level of total demand as a function of search cost.

One may wonder why retailers cannot prevent demand from decreasing discontinuously at $s = s_2$ by collectively abstaining from increasing their prices above w ? The reason is that the demand drop is a pure externality - demand at firm i drops because consumers who first visit firm j do not search, but firm i has no way to induce those consumers to search. In other words, when $s > s_2$, even though both firms would prefer to collectively charge $p_1 = p_2 = w$, individually they have an incentive to increase their price to a level above w .

The equilibrium for $s > s_2$, where firms charge the monopoly price and no consumer searches beyond the first firm, closely resembles the Diamond Paradox, but in the current setting, unlike Diamond's, the first search is free and so the market does not fully break down (see Appendix C for the version of our model with costly first search).

3.2 Symmetric Beliefs

Having solved the model for passive beliefs, we now proceed to constructing a PBE with symmetric beliefs. Let \tilde{p} denote the equilibrium price under symmetric beliefs when it

does not exceed w . In order to find the expected demand for retailer i at $p_i \neq \tilde{p}$, we note that with symmetric beliefs $p_i^e = p_j = \tilde{p}$ if only firm i deviates. This follows from the supposition that retailer j does not deviate from the equilibrium price \tilde{p} and hence consumers who visit it first expect firm i to also stick to its equilibrium strategy. Unlike the model with passive beliefs, $p_j^e \neq \tilde{p}$ but rather $p_j^e = p_i$. Thus, under symmetric beliefs expected demand for retailer i charging p_i , is given by

$$Q_i = G(w - \tilde{p} + p_i) + \int_{p_i}^w G(\tilde{p} - p_i + v)g(v)dv + \int_{p_i}^{w-\tilde{p}+p_i} G(\tilde{p} - p_i + v)g(v)dv. \quad (6)$$

For small s , the equilibrium price is then given by

$$\tilde{p} = c + \frac{1 - G(\tilde{p})^2}{2 \int_{\tilde{p}}^w g(v)^2 dv + 2G(\tilde{p})g(\tilde{p})}. \quad (7)$$

Recall that (7) is derived under the assumption that $\tilde{p} < w$. As w is decreasing in s , for large enough s we have $\tilde{p} = w$ and

$$p = c + \frac{1 - G(p)^2}{2g(p)G(p)}.$$

This price turns out to be the profit-maximizing price for a monopolist selling both products. Such a monopolist would set $p_1 = p_2 = p$ and maximize $(1 - G(p)^2)(p - c)$, which is maximized at p^{JM} that solves

$$p^{JM} = c + \frac{1 - G(p^{JM})^2}{2g(p^{JM})G(p^{JM})}. \quad (8)$$

The joint profit maximizing price p^{JM} is always higher than the single-product monopoly price p^M because joint profit maximization takes into account demand externalities between the two products. Therefore, for a range of search costs, the two firms set prices that are higher than the single-good monopoly price and as high as the price of a multi-good monopolist. It is surprising that a ‘‘collusive’’ outcome is achieved via consumer beliefs, while firms act independently, given these beliefs.

To explain this joint profit maximizing result, it is useful to return to Figure 1 and consider a marginal upward deviation from $p_i = \hat{p}$ when \hat{p} is close to w and $p_j^e = p_i$ (and $p_i^e = \hat{p}$). With symmetric beliefs, firm i does not trigger additional searches (the lower bound of area A is w which is independent of p_i). Thus, an increase in p_i only results in the shrinking of area B, which when this area is very small is roughly equivalent to losing only those consumers whose valuations for i are smaller than p_i , and at the same time whose valuations for j are even lower. Thus firm i only loses marginal consumers who prefer product i to j . This is exactly how a joint profit maximizing monopolist acts - when increasing p_i it only loses consumers whose valuation for good i is higher than for

good j : other consumers simply switch to the other good. Thus, for sufficiently large s , firms act as if they are jointly maximizing profits.

The highest search cost for which \tilde{p} is the equilibrium price is such that $w = p^{JM}$. Denote this level of search cost by s_1 . It solves

$$\int_{p^{JM}}^{\bar{v}} (v - p^{JM})g(v) dv = s_1. \quad (9)$$

Then for $s \leq s_1$, the equilibrium price is given by \tilde{p} . As $p^{JM} > p^M$, it is clear that $s_1 < s_2$.

Now consider equilibrium prices for $s > s_1$. Note that in this case $w < \tilde{p}$. This means that whatever the symmetric equilibrium price, it cannot be lower than w because in that case each firm wants to deviate to a higher price. Can it be the case that firms charge equilibrium prices above w ? In this case, consumers do not search and firm 1's demand is the monopoly demand $1 - G(p)$ and profits are maximized at p^M . Depending on whether w is lower or higher than p^M there are two cases. First, if $s_1 < s \leq s_2$ (and $w \geq p^M$), firms will never want to charge a price higher than w as this reduces profits even further. So the only possible candidate for the equilibrium price is $p_1 = p_2 = w$. For this interval of search cost the price is decreasing in s . Second, if $s > s_2$ (and $w < p^M$), firms cannot stop themselves from deviating to prices higher than w for the same reasons as explained under passive beliefs, even though they suffer from a discrete drop in demand. Thus, the equilibrium price is p^M and consumers stop searching.

The following proposition summarizes.

Proposition 2 *Under symmetric beliefs, the retailers' equilibrium price for the Wolinsky model is equal to \tilde{p} for $s < s_1$, is equal to w for $s_1 \leq s \leq s_2$, and is equal to p^M for $s_2 < s$.*

Figure 2 illustrates how the equilibrium price varies with search cost in the Wolinsky model with symmetric beliefs (it also shows the price for passive beliefs for comparison). For low search cost the price first rises in s , until the joint monopoly price, and then falls until it is equal to the monopoly price. For higher s , the equilibrium price is constant and equal to the monopoly price.⁷

It is easy to establish that for $s \in (0, s_2)$, the equilibrium price under symmetric beliefs, is strictly higher than under passive beliefs and non-monotonic in s . Furthermore, for $s \in (s_1, s_2)$, where price is decreasing in s , it does not even depend on firms' marginal cost c , and is fully determined by the reservation utility w . Figure 4 also shows how

⁷Unlike the same phenomenon in Janssen et al. (2005), in our model the non-monotonicity of the equilibrium price in search cost does not depend on whether or not the first search is costly. With costly first visits, the monopoly pricing for $s > s_2$ would disappear as the market breaks down (as in the Diamond Paradox), but the part of the equilibrium construction where $s_1 < s < s_2$ would not be affected.

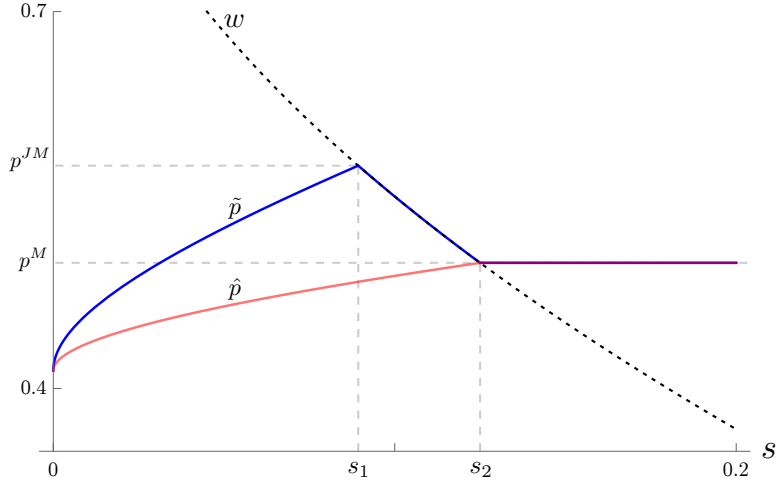


Figure 2: Price in the Wolinsky model with passive (light red) and symmetric (dark blue) beliefs as a function of s for $G(\cdot) \sim U(0, 1)$ and $c = 0$.

equilibrium total demand depends on the search cost under symmetric beliefs. It clearly shows the drop in demand when search cost increases beyond s_2 .

Having characterized the equilibrium in terms of search cost s , we now turn to the characterization in terms of c . For this purpose, fix a search cost s , and consider what happens for various levels of c . Let $p^{JM}(c)$ and $p^M(c)$ be defined as before, where c in brackets highlights the dependence on c . It is clear that $p^{JM}(c)$ and $p^M(c)$ are increasing in c , and reach \bar{v} when c is sufficiently high, at least in the limit. Then, for any given s , define c_1 and c_2 as solutions to $p^{JM}(c_1) = w$ and $p^M(c_2) = w$, respectively.⁸ These thresholds can be defined in terms of w from (4) and (8):

$$c_1 = w - \frac{1 - G(w)^2}{2g(w)G(w)}, \quad (10)$$

$$c_2 = w - \frac{1 - G(w)}{g(w)}. \quad (11)$$

For any c we have $p^{JM}(c) > p^M(c)$ so that $c_1 < c_2$. We can now state a counterpart to Proposition 2 in terms of c .

Proposition 3 *Under symmetric beliefs, the Wolinsky model has a unique equilibrium where the downstream price is \tilde{p} for $c < c_1$, is equal to w for $c_1 \leq c \leq c_2$, and is equal to p^M for $c > c_2$.*

Note that the first two regions of marginal cost in the Proposition may be empty for sufficiently high s if we impose the natural restriction that $c \geq 0$. When s is sufficiently low, so that $c_1 > 0$, we have the following comparative statics with respect to c . When

⁸Note that both values can be negative when w is sufficiently small. We keep this in mind in what follows.

$c < c_1$ price equals $\tilde{p}(c)$, which is increasing in c . For $c \in [c_1, c_2]$ price equals w , which is independent of c . The flat part stems from the fact that for $c \in [c_1, c_2]$, the search cost s is in the interval $[s_1, s_2]$. Finally, when $c > c_2$, price equals $p^M(c)$ and is again increasing in c . See Figure 3 for an illustration. For comparison, the figure also shows equilibrium prices under passive beliefs. Figure 5 shows how equilibrium total demand depends on the firm's marginal cost under symmetric beliefs. It clearly shows the drop in demand from $1 - G(w)^2$ to $1 - G(w)$ when the search cost increases beyond c_2 . The figure also displays the dependence under passive beliefs.

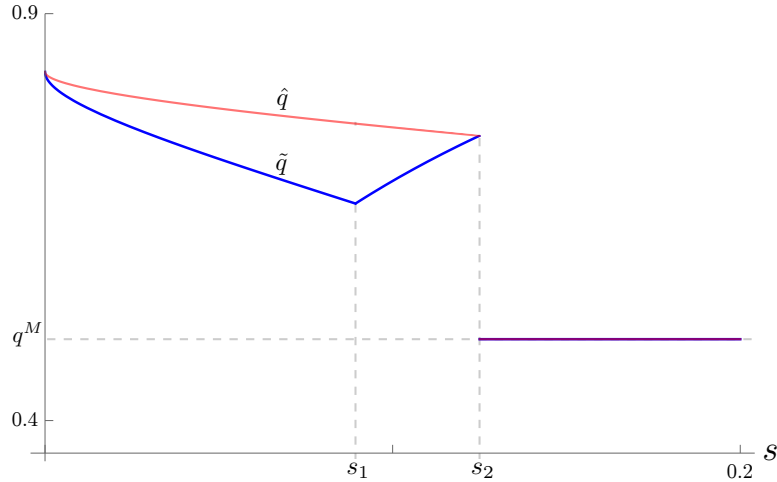


Figure 3: The equilibrium demand in the Wolinsky model with symmetric (dark blue) and passive (light red) beliefs as a function of s for $G(\cdot) \sim U(0, 1)$.

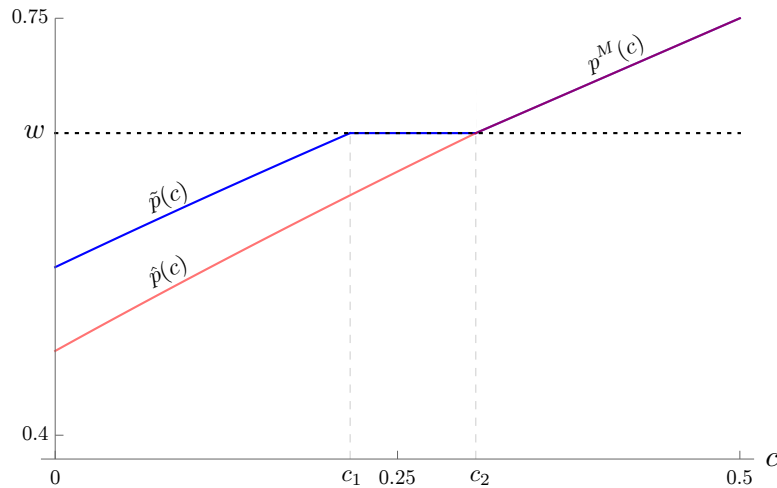


Figure 4: The equilibrium price in the Wolinsky model with symmetric (dark blue) and passive (light red) beliefs as a function of c for $G(\cdot) \sim U(0, 1)$.

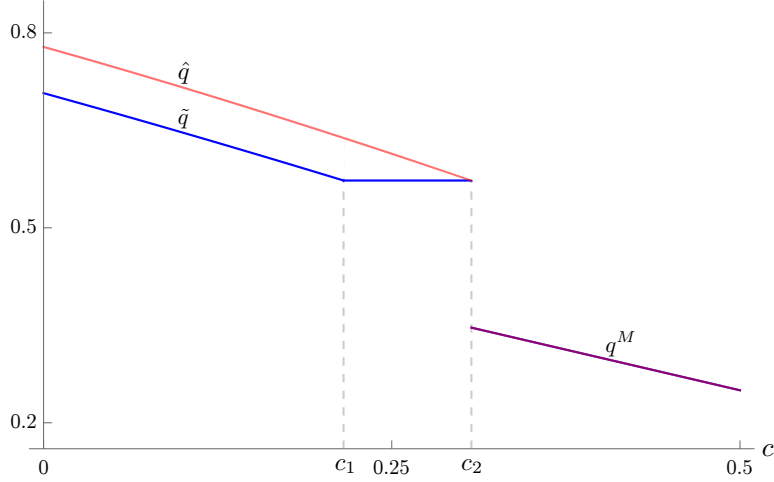


Figure 5: The equilibrium demand in the Wolinsky model with symmetric (blue) and passive (red) beliefs as a function of c for $G(\cdot) \sim U(0, 1)$.

4 Asymmetric Information about Common Cost

In this section, we study the environment where firms have a common cost that is known to them, but unknown (and uncertain) to consumers.⁹ Let firms' common marginal cost be drawn from a continuous distribution $F(\cdot)$ with support $[\underline{c}, \bar{c}]$. Firms observe the cost before posting prices, but consumers do not. We concentrate on symmetric equilibria of this model. We first define P as the reach of prices in equilibrium. Thus P is the set of prices that are observed along the equilibrium path. Because equilibrium is symmetric, if a consumer arrives at firm i and observes $p_i \in P$, she has to believe that firm j also charges p . That means that symmetric beliefs are implied on the equilibrium path and for all prices in P beliefs have to be symmetric! For all out-of-equilibrium $p_i \notin P$, consumers may have different beliefs. For simplicity, however, we concentrate on equilibria where beliefs are also symmetric for prices $p_i \notin P$. In this asymmetric information model where retailers' cost is randomly determined, retailers' (symmetric) strategy is given by a function $p(c)$, whereas the search strategy is characterized by $w^*(p_1)$. Formally, a symmetric equilibrium for this model is then given as follows.

Definition 2 *A symmetric perfect Bayesian equilibrium in continuous strategies of the search model with asymmetric information about cost is a continuous pricing strategy $p^*(c)$ with reach P and a reservation utility $w^*(p_1)$ such that*

1. *for every cost realization c , each firm i chooses $p_i(c) = p^*(c)$ to maximize its expected profit given the pricing strategy $p^*(c)$ of the other firm and consumers' reservation utility $w(p_1)$;*

⁹The effect of common cost uncertainty has been studied in the context of the Stahl (1989) model of sequential price search with homogeneous goods (see, e.g., Dana (1994), Tappata (2009) and Janssen et al. (2011)), but not in the differentiated goods sequential search model based on Wolinsky (1986), where the equilibrium price follows a pure strategy.

2. Consumers follow an optimal search rule given their beliefs and the match value v_i and the price p_i they observe at firm i ;
3. Consumers' beliefs are updated using Bayes' Rule when possible, i.e., whenever they observe $p_1 \in P$ at their first search, they believe that the other firm has also set a price $p_2^e = p_1$. Out-of-equilibrium beliefs p_2^e of the price set by the firm that is not yet visited are symmetric, i.e., $p_2^e = p_1$ if $p_1 \notin P$.

Given that beliefs are symmetric, the model can be easily solved by using Figure 4 and noting that for any realization of c , the equilibrium price is given by the blue curve. Then, depending on the relationship between c_1 and c_2 on the one hand, and \underline{c} and \bar{c} on the other, we have one of the following equilibria:

Proposition 4 *For any given s , depending on the range of cost uncertainty $[\underline{c}, \bar{c}]$ the equilibrium of the asymmetric information model takes one of the following forms:*

- (a) *If $\bar{c} \leq c_1$, the equilibrium is fully separating and the equilibrium price is $\tilde{p}(c)$ for cost realization c .*
- (b) *If $\underline{c} < c_1 < \bar{c}$, the equilibrium is partially separating and the equilibrium price is $\tilde{p}(c)$ for $c < c_1$ and w for $c_2 \geq c \geq c_1$. If, in addition, $\bar{c} > c_2$, then for $c > c_2$, the equilibrium price is $p^M(c)$.*
- (c) *If $c_1 \leq \underline{c} < \bar{c} \leq c_2$, the equilibrium is pooling and the equilibrium price is w for all c ;*
- (d) *If $\underline{c} < c_2 < \bar{c}$, the equilibrium is partially separating and the equilibrium prices is w for $c_1 \leq c \leq c_2$, and $p^M(c)$ for $c_2 < c$. If, in addition, $\underline{c} < c_1$, the equilibrium price is $\tilde{p}(c)$ for $c < c_1$.*
- (e) *If $\underline{c} \geq c_2$ the equilibrium is fully separating and the equilibrium price is $p^M(c)$.*

While the Proposition has many cases, each of them can be understood by placing the interval $[\underline{c}, \bar{c}]$ into Figure 4. As the blue curve has increasing and flat parts, the equilibrium can be fully separating, fully pooling or partially separating, depending on where \underline{c} and \bar{c} fall relative to c_1 and c_2 . For instance, if the exogenous cost uncertainty is small (\underline{c} is close to \bar{c}), the equilibrium is likely to be fully separating or fully pooling, whereas if the range of cost uncertainty is large, it will be partially separating.

It is interesting to relate some aspects of this equilibrium characterization to two independent literatures: the literature on sticky prices and the literature on sales. Starting with the seminal contributions by Sheshinski and Weiss (1977), Akerlof and Yellen (1985a,b) and Mankiw (1985), there is a large literature that explains why firms do not

adjust prices following cost shocks assuming an exogenous cost of price adjustment, the menu cost. Our model generates price stickiness without assuming menu cost. Instead, prices are sticky because, for a range of marginal costs, firms find it optimal to set prices equal to the consumer’s reservation utility that is independent of marginal cost. Sherman and Weiss (2014) empirically find that retailers in the Shuk Mahane Yehuda market in Jerusalem do not react to cost changes. They explain this finding with a dynamic model where consumers are not informed about the cost and do not adjust their expectations about prices charged by other retailers. In this world, if retailers were to increase their prices consumers would walk away to the next store. Compared to their paper, we have a static model with product differentiation and a continuum of cost states. Cabral and Fishman (2012) have also proposed a search theoretic foundation for price stickiness. Their framework relies, however, on some stickiness in retailers’ cost and they show that consumer search may lead to retail prices that are even stickier. Our results in Proposition 4 do not rely in any way on stickiness of retailers’ cost. The result that prices are fully rigid does depend, however, on the assumption that all consumers have identical search costs. If search costs are heterogeneous (see, also, Moraga, Sandor and Wildenbeest (2014)), but the search cost distribution is concentrated around a certain value, prices will be “almost” sticky, and we would obtain a search theoretic explanation for incomplete cost pass-through (Weyl and Fabinger (2013)).

Following the seminal article by Varian (1980), the literature on sales provides an explanation for the common observation that retailers often follow a pricing strategy for their products that is characterized by a regular price most of the time and a sales price at random moments in time, where the discount given on the regular price is also subject to large variations (see, e.g., Narasimhan (1988)). This literature explains this phenomenon by an asymmetric mixed strategy equilibrium with a mass point for the regular price and a continuous price distribution for the sales prices. From the viewpoint of the consumers (or the empirical economist who does not have data on cost, but only on retail prices) the equilibrium under b) in the Proposition can also be viewed as an equilibrium with sales. For many cost levels, firms’ (regular) price is independent of cost, but when cost becomes sufficiently low, the retailers give a discount. When cost is unobserved by the empirical economist, the firm acts as if it is giving a random price promotion, but in fact (in our model) the sale arises as the realization of a pure strategy that is symmetric across firms. Empirically, Pesendorfer (2002) finds evidence for the fact that a firm has a regular price and then provides discounts at irregular points in time. Note, however, that in line with (d) in Proposition 4, a firm may also have a regular price with irregular price movements going up and down. This is the pricing pattern that is documented in Hosken and Reiffen (2004).

Before we proceed to the comparative statics of this equilibrium, note that there is no equilibrium where beliefs are passive everywhere. This is because with passive beliefs,

price is increasing in c for all c (see the red curve in Figure 4). Therefore the interval $[\underline{c}, \bar{c}]$ maps into some interval $P = [p(\underline{c}), p(\bar{c})]$, but for all $p \in P$, beliefs have to be symmetric, which contradicts the initial supposition that beliefs are passive. While there may exist equilibria where beliefs are symmetric for $p \in P$, and passive for $p \notin P$, we shall not pursue these further.

Comparative statics

The comparative statics of this model with respect to search cost depends on which of the five cases described in Proposition 4 applies. There are two fully separating cases, (a) and (e). In (a), the distribution of prices shifts to the right as s increases, while in (e), prices are independent of s .

In all partially separating and pooling equilibria, for a range of marginal cost values, price equals w , and because w is decreasing in s , so is the price for any corresponding c . As an example, consider case (b) where $\underline{c} < c_1 < \bar{c}$, which has a frequently observed structure of a ‘regular’ price w for cost realizations $c \geq c_1$ and random discounts for cost realizations $c < c_1$ where the price is $\tilde{p}(c) < w$. In this case, prices are distributed continuously on the interval $[\tilde{p}(\underline{c}), w)$ with a CDF given by $F(\tilde{p}^{-1}(p))$, where $\tilde{p}^{-1}(\cdot)$ is the inverse of $\tilde{p}(c)$, and there is a mass point equal to $1 - F(c_1)$ at w . Now consider a marginal increase in s . It shifts the distribution of prices below w to the right, as per increase in $\tilde{p}(c)$ for every c , but w shifts to the left, while the mass at w increases. Thus, an increase in search cost leads to a lower and more frequently charged regular price (w), but shallower and less frequent discounts. Whether the average price increases or decreases is ambiguous, and depends on $G(\cdot)$ and $F(\cdot)$.

Another way to think about comparative statics is to fix a narrow interval $[\underline{c}, \bar{c}]$ (in the sense that all five cases will appear for some s), and consider what happens as s changes. When s is low, equilibrium falls into case (a) and as s increases, the price increases. As s increases further, equilibrium transits from (a) to (e), where the price becomes independent of s . In (b) and (d) average price may be falling or increasing, but in (c) where the price equals w it is falling.

5 Vertical Industry Structure

We now turn to the environment where retailers’ cost is (partially) determined by a manufacturer and consider the impact of consumer beliefs on prices and upstream and downstream profits. As consumers do not know the retail prices (and have to pay a search cost to observe them), it is realistic to also assume that consumers do not observe the wholesale price.¹⁰ Note that in this setting the final retail price consumers observe

¹⁰Janssen and Shelegia (2014) focus on the comparison of markets where consumers observe the wholesale price and ones where they do not observe the wholesale price in the context of the Stahl (1989)

is the end result of the wholesale price set by the manufacturer and the retail pricing strategy (a retail price for any given wholesale price) chosen by the retailer the consumer visits. After observing an out-of-equilibrium retail price, consumers may hold any beliefs regarding who has deviated from the equilibrium path. For example, they may hold passive beliefs and reason that the retailer they visited has deviated, leaving beliefs about other firms' (the other retailer and the manufacturer) pricing unchanged. Alternatively, they may hold symmetric beliefs and think that the retailer they visited has not deviated at all, but that the manufacturer charged a different wholesale price expecting the other retailer to react in the same way to the manufacturer's deviation. As we focus on the role of consumer beliefs, we compare the resulting equilibria under both belief systems.

Consider a monopolist manufacturer selling a homogeneous good at a price m to both retailers.¹¹ Retailers have to spend t to transform the homogenous input into the differentiated output they sell, so that each retailer's marginal cost is given by $c = m + t$. We can interpret t as the marginal cost of transformation or a unit sales tax, or the sum of the two. The reason we introduce t is to be able to discuss the extent to which our results on price rigidity hold true in a vertical industry structure.¹² For a given cost, consumer behavior is described in Section 2 if this cost were known to consumers.

The equilibrium notion we employ in the vertical relation models is formally defined as follows.

Definition 3 *For a given t , the marginal cost of transformation, a symmetric perfect Bayesian equilibrium of the search model with vertical relations is a wholesale price m^* , a retail pricing strategy $p^*(c)$, with $c = m + t$, and a reservation utility $w(p)$ such that*

1. *the manufacturer chooses m so as to maximize its profit given the pricing strategy $p^*(c)$ of the retailers and consumers' reservation utility $w(p)$;*
2. *each retailer i chooses $p_i(c) = p^*(c)$ to maximize its expected profit given the pricing strategy $p^*(c)$ of the other retailer and consumers' reservation utility $w(p)$;*
3. *Consumers follow an optimal search rule given their beliefs and the match value v_i and the price p_i they observe at firm i ;*

sequential search model. We focus here on the role of consumer beliefs where consumers do not observe the wholesale price. In the earlier version of this paper we also considered the vertical relations model where the wholesale price is observed by consumers, and confirmed Janssen and Shelegia's finding that, holding beliefs the same, prices are higher when the wholesale price is unobserved.

¹¹See Appendix B showing that the same equilibrium can be sustained for retailers having symmetric beliefs when the manufacturer can discriminate between retailers and gives each retailer a private offer in terms of an individualized unit price m_i to retailer i .

¹²Without t , we cannot assess how the industry reacts to changes in cost as the retailer cost would be equal to the wholesale price and this price is endogenously determined. Alternatively, one could introduce a separate cost for the manufacturer.

4. Consumers' beliefs are updated using Bayes' Rule when possible, i.e., whenever they observe $p_1 = p^*(m^* + t)$ at their first search, they believe that the other firm has also set a price $p_2^e = p^*(m^* + t)$. Out-of-equilibrium beliefs p_2^e of the price set by the firm that is not yet visited are either

- i. (passive beliefs) such that $p_j^e = p^*(m^* + t)$ if $p_i \neq p^*(m^* + t)$, or
- ii. (symmetric beliefs) such that $p_j^e = p_i$ if $p_i \neq p^*(m^* + t)$.

5.1 Passive Beliefs

We first consider the vertical relations model with passive beliefs. This implies consumers blame individual retailers for price deviations. When beliefs are passive (and the manufacturer's actions are unobserved by consumers) consumers search without being able to infer the action chosen by the manufacturer.

Before analyzing the model, it is useful to define the classic double marginalization prices m^M and $p^M(m + t)$, where the latter is simply the monopoly price for a given marginal cost, and the former is the optimal upstream monopoly price given that downstream demand is given by $(1 - G(p))$ and the retail price is the monopoly price for marginal cost $m + t$. In order to simplify the analysis, we shall assume that $G(\cdot)$ is such that m^M is uniquely defined (see the proof of Proposition 5 for the condition that $G(\cdot)$ needs to satisfy).

We first solve for the retail competition phase (which given that m is unobserved by consumers is not a separate subgame), where the manufacturer charges some price m , consumers believe that both retailers are charging p^* , retailer 1 expects retailer 2 to charge p_2 and is considering whether to charge p_1 . Then, using (2) where we note that $p_1^e = p_2^e = p^*$, the symmetric price \hat{p} as a function of $c = m + t$ and the belief p^* is given by

$$\hat{p}(c) = c + \frac{1 - G(p(c))^2}{2 \int_{p(c)}^{p(c)-p^*+w} g(v)^2 dv + (1 - G(p(c) - p^* + w))g(p(c) - p^* + w) + 2G(p(c))g(p(c))}. \quad (12)$$

The manufacturer is now facing a market demand that is given by $Q = 1 - G(\hat{p}(c))^2$ where in $\hat{p}(c)$, under passive beliefs and consumers not observing m , the belief p^* is simply the equilibrium price that does not depend on the actual choice of m . Using the expression for $p(c)$, taking the FOC of the manufacturer's profit maximization problem one can implicitly determine the manufacturer's optimal price \hat{m} by imposing that $c = \hat{m} + t$ and $\hat{p} = p(\hat{m} + t)$ to be equal to

$$\hat{m} = \frac{1 - G(\hat{p}(\hat{m} + t))^2}{2g(\hat{p}(\hat{m} + t))G(\hat{p}(\hat{m} + t))\frac{\partial p(c)}{\partial c}}, \quad (13)$$

where $\frac{\partial p(c)}{\partial c}$ is defined implicitly from (12).

Under passive beliefs, the manufacturer's equilibrium price m^{pas} equals \hat{m} and the retailers' equilibrium price p^{pas} equals $\hat{p} = p(\hat{m} + t)$ if $\hat{p}(\hat{m} + t) \leq w$, which is the case if s is sufficiently low. In what follows, we shall assume that $G(\cdot)$ is such that \hat{m} is unique. For sufficiently high s , however, this condition does not hold. Denote by \underline{s}^{pas} the level of search cost such that $\hat{p}(\hat{m} + t) = w$. If $s > \underline{s}^{pas}$, then if consumers would search, the equilibrium prices are such that $\hat{p}(\hat{m} + t) > w$ and so consumers do not want to search. As the wholesale price is unobserved by consumers and consumers hold passive beliefs, no firm can prevent search from breaking down. Thus, for $s > \underline{s}^{pas}$ there can be no pure strategy equilibrium where consumers search. The problem is that when consumers do not search firms charge classic double marginalization prices, but there is no guarantee that $p^M(m^M) > w$ and that consumers do not want to search. In fact, the next Proposition states that there always is an interval of search cost such that no pure strategy equilibrium exists.

Proposition 5 *When consumers hold passive beliefs, there exist \underline{s}^{pas} and \bar{s}^{pas} , with $\underline{s}^{pas} < \bar{s}^{pas}$, such that, for $s \leq \underline{s}^{pas}$ the manufacturer price is \hat{m} and the corresponding retail price equals $\hat{p}(\hat{m})$, for $s \geq \bar{s}^{pas}$ the manufacturer price is m^M , and the corresponding retail price equals $p^M(m^M)$. For $s \in (\underline{s}^{pas}, \bar{s}^{pas})$ no pure strategy equilibrium exists.*

The Proposition is illustrated in Figure 6 for the case where the match value $G(\cdot)$ is uniformly distributed. For low search cost, prices are lower than the reservation utility and retail margins increase in search cost. There is a small intermediate region of search cost where a pure strategy equilibrium does not exist. For higher values of the search cost, the double marginalization outcome materializes. The manufacturer cannot prevent a drop in market demand because consumers do not react in any way to the actual wholesale price that is chosen.

5.2 Symmetric Beliefs

In the vertical industry model, one can rationalize consumers having symmetric beliefs, by having them blame the manufacturer for deviating after observing out-of-equilibrium low prices. Symmetric beliefs are convenient in that, given the downstream strategy $p^*(c)$, for every c (or m), total market demand does not depend on what consumers believe the manufacturer charges. This is because consumers who visit firm i search according to p_i , and p_j^c which equals p_i , none of which depends on beliefs about c (or m). Accordingly, downstream market behavior solely depends on the actual retail prices that are charged.

Denote by $\pi(m)$ the manufacturer's profit function. One important consideration is that demand, and hence $\pi(m)$, is discontinuous at c_2 , where demand drops as consumers stop searching. To be able to emphasize that in the vertical industry model the threshold

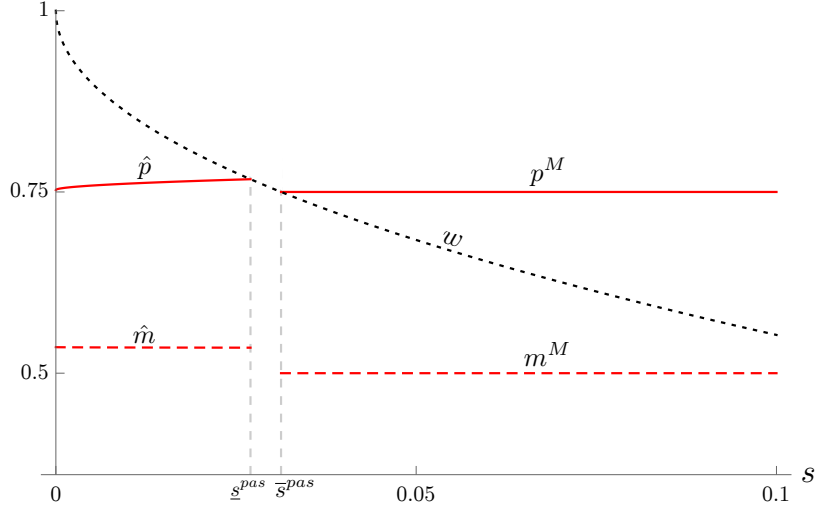


Figure 6: Wholesale (solid) and retail (dashed) prices with passive beliefs for $G(\cdot) \sim U[0, 1]$ and $t = 0$.

values c_1 and c_2 translate into critical values for the manufacturer price m , we will write $m_1 \equiv c_1 - t$ and $m_2 \equiv c_2 - t$. For $m < m_1$, the profit function is $\pi(m) = (1 - G(\tilde{p}(c)))^2 m$. For $m_1 \leq c \leq m_2$, retail prices are equal to w regardless of m , and so the manufacturer's profit is linear in m and given by $\pi(m) = (1 - G(w)^2)m$. Finally, for $m > m_2$, retail prices exceed w and are equal to $p^M(c)$, consumers do not search beyond the first retailer, and the manufacturer's profit is $(1 - G(p^M(c)))m$. Figure 7 illustrates the three regions of the profit function.

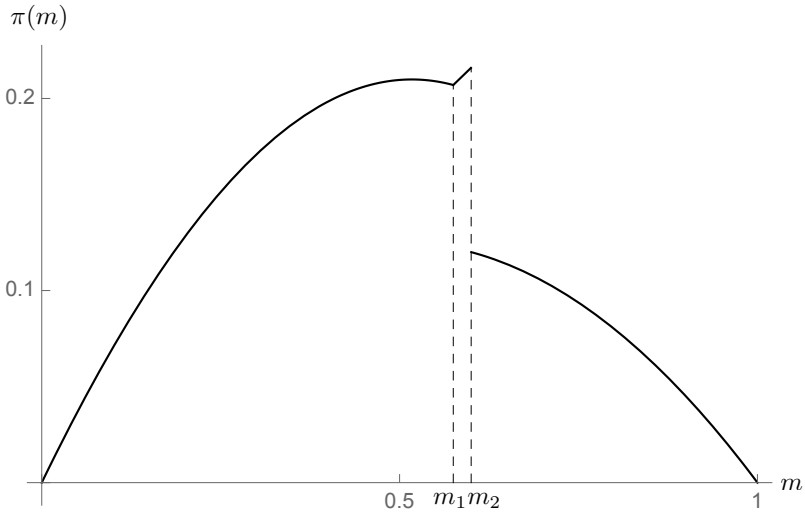


Figure 7: The manufacturer's profit for unobserved c and symmetric beliefs ($G(\cdot) \sim U[0, 1]$, $t = 0$, $s = 0.02$.)

In order to characterize the solution to the manufacturer's profit maximization problem, we make the following assumptions on $G(\cdot)$. First, let $G(\cdot)$ be such that $(1 - G(\tilde{p}(c)))^2 m$ and $(1 - G(p^M(c)))m$ are strictly concave on their domain, and let \tilde{m} and m^M denote their unique maximizers, respectively. These conditions are satisfied for commonly

used distribution functions, such as the uniform, normal and logistic distribution (as can be shown analytically for the uniform distribution, and numerically for the others).

For any given t , if search cost s is low, m_2 is close to \bar{v} , the relevant profit of the manufacturer is $\pi(m) = (1 - G(\tilde{p}(c))^2)m$, and so the manufacturer chooses $m^* = \tilde{m}$. When s is high, m_2 is close to or equal to \underline{v} , so that the relevant profit is $(1 - G(p^M(c)))m$ and $\pi(m)$ is maximized at m^M . There may be an intermediate region of s where profit is maximized at m_2 .¹³ Formally, we have

Proposition 6 *When consumers hold symmetric beliefs, there are two threshold search costs \underline{s}^{sym} and \bar{s}^{sym} , with $\underline{s}^{sym} \leq \bar{s}^{sym}$. The equilibrium outcome is such that: (i) if $s < \underline{s}^{sym}$ the manufacturer price is given by \tilde{m} and the retail price is $\tilde{p}(\tilde{m} + t)$; (ii) if $s > \bar{s}^{sym}$ the manufacturer price is m^M and the retail price is $p^M(m^M + t)$; (iii) if in addition $\bar{s}^{sym} > \underline{s}^{sym}$, then for $\underline{s}^{sym} \leq s \leq \bar{s}^{sym}$ the manufacturer price is either equal to \tilde{m} or equal to m_2 and the retail price is equal to $\tilde{p}(\tilde{m} + t)$ or $\tilde{p}(m_2 + t)$, respectively.*

While formally we cannot ascertain whether \tilde{m} or m_2 is the maximizer in the range $[\underline{s}^{sym}, \bar{s}^{sym}]$, and there may be smaller subintervals where one and then the other is the maximizer, for many match value distributions it will be the case that either \tilde{m} or m_2 is the maximizer for the whole range. Below, in Figure 8, we illustrate the above result with an example where $G(w)$ follows a uniform distribution on $[0, 1]$.

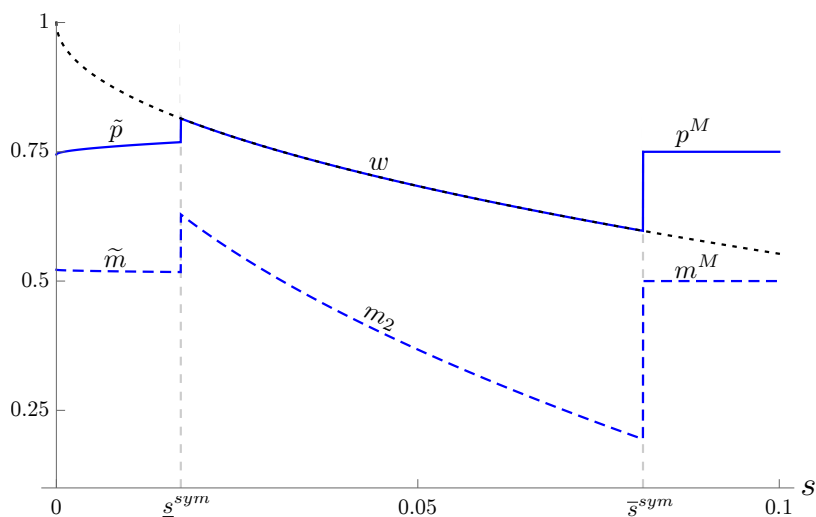


Figure 8: Wholesale (solid) and retail (dashed) prices with symmetric beliefs for $G(\cdot) \sim U(0, 1)$ and $t = 0$.

For the uniform distribution the range $[\underline{s}^{sym}, \bar{s}^{sym}]$ is reasonably large and m_2 is the maximizer over the full interval. This interval is interesting for the following reason. With symmetric beliefs, the manufacturer controls (through its implicit control over the retail price) consumer beliefs about the price that is charged by the retailer on the next

¹³Note that profit can never be maximized at $m \in [m_1, m_2)$ because profit is linearly increasing in m in that interval.

search. Moreover, the manufacturer has an interest in keeping demand high by making consumers with a low match value at a particular retailer believe that it is worthwhile to continue searching the next retailer. The manufacturer achieves this by lowering m (when s increases) so as to give retailers an incentive to continue charging ever lower values of w (as s increases). By doing so, the manufacturer proactively avoids the Diamond Paradox (with the associated drop in demand) from arising. The resulting price reduction at both the wholesale and retail level due to an increase in search costs can be quantitatively significant. For example, for the uniform distribution, the decline in wholesale price from \underline{s}^{sym} to \bar{s}^{sym} is in the order of 60% (from around 0.5 to 0.2), and the corresponding decline in retail prices is around 25% from around 0.8 to 0.6).

Figure 8 confirms that under symmetric beliefs retail prices are non-monotonic in search cost and that the retail price can be substantially higher than the double marginalization price. In a vertical industry structure, both wholesale and retail prices are non-monotonic and the fluctuations in retail prices can be quite substantial. What is new to the vertical industry model is that at the wholesale and retail level, equilibrium prices as a function of s have two discontinuous jumps. The first jump occurs because for a range of values of m , retailers set their price equal to w , which is independent of m . Thus, once it is optimal for the manufacturer to push retailers to setting the price equal to the reservation utility, there is an upward jump in the optimal wholesale price set by the manufacturer. This explains why retail prices can be higher than the double marginalization price. The second jump occurs because, with symmetric beliefs, the manufacturer attempts to prevent consumers from stopping to search the second retailer as discussed above. However, at sufficiently high values of s (sufficiently low w), the corresponding m that is needed to keep consumers searching the second retailer is so low that it is not profitable for the manufacturer to do so and he sets the double marginalization wholesale price and accepts that no consumer searches beyond the first retailer. Market demand is lower at that point, but this is compensated by a much higher price.

We finally want to come back to the issue of price rigidity and sales. Even though m is endogenized in the vertical relations model, we still can discuss cost-pass-through in this setting by investigating how firms react to changes in the transformation cost t . Intuitively, the model with vertical relations exhibits more cost rigidity because the manufacturer pushes the retailers to charge reservation prices for lower values of t , while he also delays the market breakdown by lowering the wholesale price and keeping the reservation utility pricing for higher t values.

To pursue this issue further, we provide a counterpart to Proposition 3 (and 6) where we characterize the equilibrium of the vertical relations model with symmetric beliefs for various levels of t .

Proposition 7 *When consumers hold symmetric beliefs, there are two threshold search costs \underline{t}^{sym} and \bar{t}^{sym} , with $\underline{t}^{sym} \leq \bar{t}^{sym}$, such that: (i) if $t < \underline{t}^{sym}$ the manufacturer price is*

given by \tilde{m} and the retail price is $\tilde{p}(\tilde{m} + t)$; (ii) if $t > \bar{t}^{sym}$ the manufacturer price is m^M and the retail price is $p^M(m^M + t)$; (iii) if in addition $\bar{t}^{sym} > \underline{t}^{sym}$, then for $\underline{t}^{sym} \leq t \leq \bar{t}^{sym}$ the manufacturer price is equal to m_2 and the retail price is constant and equal to $\tilde{p}(m_2 + t)$.

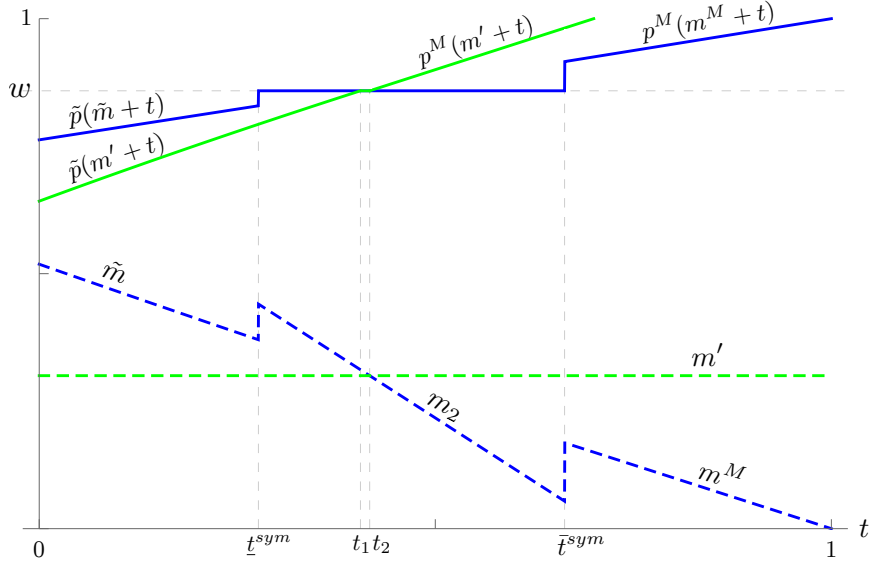


Figure 9: Equilibrium prices as a function of t in the Wolinsky model (green) and the vertical relations model (blue), both with symmetric beliefs for $G(\cdot) \sim U[0, 1]$ and $s = 0.01$.

It is clear from the Proposition that, depending on the distribution of match values, there may be a region of t values where retail prices are independent of the transformation cost t . We illustrate the content of the Proposition by considering $G(\cdot)$ to be uniform. Figure 9 illustrates that it may well be that the vertical relations model exhibits more price rigidity at the retail level than the model without vertical relations in the sense that there always exists an exogenous marginal cost in the Wolinsky model with symmetric beliefs such that the interval of rigidity is larger in the model with vertical relations. To make this point more precise, define for a given exogenous marginal retail cost m' , $t_i = c_i - m'$ for $i = 1, 2$. These are two threshold levels of transformation cost such that for $t < t_1$ the equilibrium price in the Wolinsky model with symmetric beliefs is given by \tilde{p} , it is w for $t \in [t_1, t_2]$ and it is p^M for $t > t_2$. In Figure 9 the green curves are the equilibrium retail (solid) and wholesale (dashed) prices in the Wolinsky model with symmetric beliefs and exogenously fixed m' . The blue curves are the equilibrium retail (solid) and wholesale (dashed) prices in the model with vertical relations and symmetric beliefs. Figure 9 shows that retail prices are rigid in the Wolinsky model with symmetric beliefs when $t \in [t_1, t_2]$, while they are rigid in the vertical relations model with symmetric beliefs for $t \in [\underline{t}^{sym}, \bar{t}^{sym}]$. As is clear, the interval of transformation cost t where the Wolinsky model exhibits price rigidity is much smaller than the corresponding interval in the vertical relations model.

Figure 9 also shows that the sales interpretation of the pricing pattern in Section 4 continues to be relevant. There is a regular price equal to w , which is charged for a wide range of cost levels t and sales prices that are charged for lower levels of t . The vertical relations model adds to this sales interpretation the point that the sales prices are substantially lower than the regular price, as prices just below w are not charged in equilibrium. This confirms the findings in Pesendorfer (2002) and Hosken and Reiffen (2004). Moreover, the figure shows that price rigidities may persist when the search cost is very small.

6 Conclusion

We see our contribution as twofold. First, we contribute to the methodology of the consumer search literature by explicitly focusing on the role of consumer beliefs on the optimal search rule and on market outcomes. We consider the (rightly) celebrated Wolinsky model and solve it (i) for symmetric, rather than passive, beliefs and (ii) for low and high search costs. We show that the market price is non-monotonic in search cost and independent of marginal cost for a range of parameter values, and that it can be as high as the joint profit maximizing price even under noncooperative behavior. We also show that when firms switch to monopoly pricing, demand drops discontinuously. Second, we show that our methodological innovation has important ramifications in two natural adaptations of the Wolinsky model (firms having random cost and firms' costs being determined by an upstream firm), where we are able to provide new search theoretic explanations for well documented phenomena, such as price rigidities and periodic sales.

The phenomenon of price rigidity, or more generally the recent discussion on cost-pass-through, focuses on the extent to which firms react to changes in their cost. Our model with homogeneous search costs generates price rigidities in a natural way as firms have an incentive to charge prices so that consumers do not continue to search the next firm. This phenomenon occurs in all of our models, but may especially be strong in the vertical industry model, where the manufacturer has an incentive and the ability to proactively resolve the Diamond Paradox by lowering its price. It is not difficult to see that a small search cost heterogeneity (see, also, Moraga-González, Sándor and Wildenbeest (2014)) retail prices will not react much to changes in marginal cost. Many industries are characterized by incomplete pass-cost-through (Weyl and Fabinger (2013)), i.e. higher wholesale prices induce lower margins at the retail level. In our models, low levels of cost pass-through arise because of retailers optimally reacting to the consumer search strategies.

Our models are purposefully simple in nature. We did not study retail oligopoly, or wholesale competition. We also did not include nonlinear prices or other arrangements (such as exclusive dealing) that are typically found in wholesale markets. We see our

results as interesting stepping stones upon which an understanding of these issues can be built. Further, we believe that our methodological contributions can be used in studies of mergers, advertising, prominence, provision of variety and other policy relevant adaptations of the Wolinsky model.

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Appendix A: Proofs

Proposition 2. Under symmetric beliefs, the Wolinsky model has a unique equilibrium where the retailers’ price is \tilde{p} for $s < s_1$, is equal to w for $s_1 \leq s \leq s_2$, and is equal to p^M for $s_2 < s$.

Proof. We prove each part in turn and start with $s < s_1$. It is easy to see that both firms charging \tilde{p} is the equilibrium because each firm maximizes its own profit given the price of the other firm, and all consumers who draw a utility realization below w search, giving rise to the profit function that is maximized by \tilde{p} . There is no other symmetric equilibrium because by assumption, (7) has a unique solution. An equilibrium where consumers do not search does not exist because then both firms would charge $p^M < p^{JM} < w$, and consumers would search.

Now take $s_1 \leq s \leq s_2$. There are two possibilities, either in equilibrium $p > w$ and consumers do not search or, $p \leq w$ and consumers search. The first is impossible because if consumers do not search, firms charge p^M , which given $s_1 \leq s \leq s_2$ is lower than w , which means that consumers will search, a contradiction. Firms cannot charge $p < w$ either, because then all consumers search and demand for each firm is given by (6). Therefore in a symmetric equilibrium p should solve (7), which is impossible for $s_1 < s \leq s_2$.

This leaves the only possible candidate for equilibrium, $p = w$. For this to be the equilibrium, consumers should search, which is assured by $p \leq w$, and each firm’s profit should be maximized at w . Recall, that at $p = w$ the profit where consumers search is strictly increasing (both firms want to charge a strictly higher price), and for p slightly above w , profit is identical to the monopoly profit (as consumers do not search and the firm faces a demand of $(1 - G(p))$). Thus w is at a kink, and is the maximizer. ■

Proposition 5. When consumers hold passive beliefs, there exist \underline{s}^{pas} and \overline{s}^{pas} , with $\underline{s}^{pas} < \overline{s}^{pas}$, such that, for $s \leq \underline{s}^{pas}$ the manufacturer price is \hat{m} and the corresponding retail price equals $\hat{p}(\hat{m})$, for $s \geq \overline{s}^{pas}$ the manufacturer price is m^M , and the corresponding retail price equals $p^M(m^M)$. For $s \in (\underline{s}^{pas}, \overline{s}^{pas})$ no pure strategy equilibrium exists.

Proof. Define \underline{s}^{pas} as a search cost such that

$$\frac{(1 - G(w)^2) ((1 - G(w))g'(w))}{2G(w)g(w)^3} = w - t - \frac{1 - G(w)}{g(w)}. \quad (14)$$

The LHS of (14) is obtained from the manufacturer's FOC given by (13) where we impose $p^* = w$ and solve for m . The RHS is obtained from the retailer's FOC, imposing again that $p = w$ and solving for m . In equilibrium, both conditions have to be satisfied, giving rise to equation (14). As \hat{m} is unique by assumption, (14) has a unique solution.

At \underline{s}^{pas} the manufacturer's profit maximization given equilibrium downstream behavior leads to a retail price exactly equal to w . For all search cost lower than \underline{s}^{pas} , the manufacturer's profit is maximized at \hat{m} and $p(\hat{m}) < w$. Now take a search cost just above \underline{s}^{pas} . For such a search cost no consumer will search. Thus if a pure strategy equilibrium is to exist, the manufacturer should charge m^M and retailers charge $p^M(m^M)$. By definition of m^M and p^M , p^M solves

$$\frac{(1 - G(p)) (pg'(p) + 2g(p))}{(1 - G(p))g'^2} = p - t - \frac{1 - G(p)}{g(p)}, \quad (15)$$

where again the LHS is obtained from the manufacturer's FOC in the classic double marginalization problem, where we solve for m^M , and the RHS is the solution for m from the monopoly pricing rule (the FOC for the retailer). By assumption, this equation also has a unique solution.

Note that the RHS is the same for both equations above (at $p = w$, the retail pricing rule is the monopoly one), and both have unique solutions, so in order to compare their roots, we shall first establish that the LHS in both crosses the RHS from above. This follows from the fact that the solutions to both equations are unique, while at (large) values of p or w such that $(1 - G(\cdot))$ is close to 0, the LHS of (14) and (15) are close to 0, whereas the RHS remains strictly positive. Then, given that the RHS of the previous two equations is the same, the solution to (15) is always smaller than the solution to (14) provided that at the solution to (14), the LHS of (15) is smaller than the LHS of (14). Investigating when the LHS of (15) is smaller than the LHS of (14), both evaluated at w , we obtain the following condition:

$$(1 - G(w))g(w)^2 (g(w)^2 + (1 - G(w))g'(w)) > 0,$$

which is always implied by the log-concavity of $1 - G(x)$, which is true by assumption. Thus, at $s = \underline{s}^{pas}$, where $p = w$, $w < p^M(m^M)$, so that at the threshold \underline{s}^{pas} , the equilibrium price $\hat{p}(\hat{m})$ equals the reservation utility, and is higher than the double marginalization price.

Given that w is decreasing in s , there exists a unique search cost, denoted by \bar{s}^{pas} , such

that $w = p^M(m^M + t)$. Thus for any $s \in (\underline{s}^{pas}, \bar{s}^{pas})$ there is no pure strategy equilibrium because if consumers search, then firms charge prices in excess of w , which implies that consumers should not search, and if consumers do not search then firms charge prices lower than w , which requires that consumers do search. For all $s \geq \bar{s}^{pas}$, consumers do not search, the manufacturer charges m^M , retailers charges $p^M(c)$, and consumers do not search. ■

Proposition 6. When consumers hold symmetric beliefs, there are two threshold search costs \underline{s}^{sym} and \bar{s}^{sym} , with $\underline{s}^{sym} \leq \bar{s}^{sym}$. The equilibrium outcome is such that: (i) if $s < \underline{s}^{sym}$ the wholesale price is given by \tilde{m} and the retail price is $\tilde{p}(\tilde{m} + t)$; (ii) if $s > \bar{s}^{sym}$ the wholesale price is m^M and the retail price is $p^M(m^M + t)$; (iii) if in addition $\bar{s}^{sym} > \underline{s}^{sym}$, then for $\underline{s}^{sym} \leq s \leq \bar{s}^{sym}$ the wholesale price is either equal to \tilde{m} or equal to m_2 and the retail price is equal to $\tilde{p}(\tilde{m} + t)$ or $\tilde{p}(m_2 + t)$, respectively.

Proof. We have that m_2 is decreasing in s , at $s = 0$ equals \bar{v} , and is zero for s that solves $w = \frac{1-G(w)^2}{2G(w)g(w)}$. Also, $\pi(m)$ is decreasing in s for all $m \in [0, m_1]$. Thus, $\max_{m \leq m_1} \pi(m)$ is decreasing in s because the function and its support are decreasing. As before, for $s = 0$ the maximizer is $\tilde{m} < m_1$. Also note that $\pi(m)$ is increasing on $[m_1, m_2]$. Therefore if the global maximum is achieved on this interval, then it is achieved at m_2 . Thus, as s increases from 0, one of the following two cases will happen first: (i) $\pi(\tilde{m}) = \pi(m_2) > (1 - G(p^M(m^M)))m^M$, or (ii) $\pi(\tilde{m}) = (1 - G(p^M(m^M)))m^M > \pi(m_2)$. One of the two cases has to occur for sufficiently high s as then $m_2 < 0$. Let us start with (i) so that $\pi(\tilde{m}) = \pi(m_2) > (1 - G(p^M(m^M)))m^M$ occurs first, and denote s where this is the case by \underline{s}^{sym} . Then for all $s < \underline{s}^{sym}$, the maximizer of $\pi(m)$ is $\tilde{m} < m_1$. For $s > \underline{s}^{sym}$ but sufficiently close to \underline{s}^{sym} , the maximizer is \tilde{m} or m_2 . Then, because $\pi(m_2)$ is strictly decreasing in s and reaches 0, and $(1 - G(p^M(m^M)))m^M$ is independent of s , there will be $\bar{s}^{sym} > \underline{s}^{sym}$, such that at \bar{s}^{sym} we have $\pi(m_2) = (1 - G(p^M(m^M)))m^M$. Once $s > \bar{s}^{sym}$ the maximizer is m^M because $\pi(\tilde{m})$ and $\pi(m_2)$ are strictly decreasing in s for $m < m_2$, while $\pi(m^M)$ is independent of s . This establishes the proof of proposition for the case where $\underline{s}^{sym} < \bar{s}^{sym}$. Now consider possibility (ii) so that $\pi(\tilde{m}) = (1 - G(p^M(m^M)))m^M > \pi(m_2)$ occurs first. Denote such a search cost by s^{sym} . For any $s < s^{sym}$ the maximizer is \tilde{m} , while because both $\pi(\tilde{m})$ and $\pi(m_2)$ are strictly decreasing in s , for $s > s^{sym}$, the maximizer is m^M . Therefore in this case $\underline{s}^{sym} = \bar{s}^{sym} = s^{sym}$. ■

Proposition 7. When consumers hold symmetric beliefs, there are two threshold search costs \underline{t}^{sym} and \bar{t}^{sym} , with $\underline{t}^{sym} \leq \bar{t}^{sym}$, such that: (i) if $t < \underline{t}^{sym}$ the wholesale price is given by \tilde{m} and the retail price is $\tilde{p}(\tilde{m} + t)$; (ii) if $t > \bar{t}^{sym}$ the wholesale price is m^M and the retail price is $p^M(m^M + t)$; (iii) if in addition $\bar{t}^{sym} > \underline{t}^{sym}$, then for $\underline{t}^{sym} \leq t \leq \bar{t}^{sym}$ the wholesale price is either equal to m_2 and the retail price is equal to $\tilde{p}(m_2 + t)$.

Proof. For a fixed s and t the profit function of the monopolist has three regions. For $m < m_1$, profit equals $(1 - G(\tilde{p}(m + t)))^2 m$, for $m \in (m_1, m_2]$, profit equals $(1 - G(w))^2 m$, and for $m > m_2$ profit equals $(1 - G(p^M(m + t)))m$. This profit function has

a kink at m_1 , and then a discontinuous drop immediately above m_2 . It is maximized either at \tilde{m} , m_2 or m^M . Let us first consider the case where $s < \underline{s}^{sym}$ for $t = 0$. We evaluate the derivative of profits with respect to t at these three wholesale price levels. Due to the envelope theorem, $\frac{\partial(1-G(\tilde{p}(\tilde{m}+t))^2)\tilde{m}}{\partial t} = -2\tilde{m}G(\tilde{p}(\tilde{m}+t))\tilde{p}'(\tilde{m}+t)g(\tilde{p}(\tilde{m}+t))$, where using the definition of \tilde{m} we can substitute $\frac{1-G(\tilde{p}(\tilde{m}+t))^2}{2\tilde{m}G(\tilde{p}(\tilde{m}+t))g(\tilde{p}(\tilde{m}+t))}$ for $\tilde{p}'(\tilde{m}+t)$, which simplifies the relevant profit derivative to $-(1-G(\tilde{p}(\tilde{m}+t)))$. Using the same method, $\frac{\partial(1-G(p^M(m^M+t)))m^M}{\partial t} = -(1-G(p^M(m^M+t)))$. As for $\frac{\partial(1-G(w)^2)m_2}{\partial t}$, note that $m_2 = c_2 - t$, thus $\frac{\partial(1-G(w)^2)m_2}{\partial t} = -(1-G(w)^2)$. Therefore, the derivative of profit at either of these three points is negative and equal in absolute value to the demand at the respective wholesale price m . Because demand is the highest at \tilde{m} , second highest at m_2 and lowest at m^M , it follows that as t increases, the profit at \tilde{m} decreases more than the profit at m_2 , which in turn falls more than the profit at m^M .

If t is sufficiently large, and therefore m_2 sufficiently small, the maximum is achieved at m^M . If t is sufficiently small, then the maximum is achieved at \tilde{m} . As t increases from 0, the maximizer is initially \tilde{m} . Using the results above, then the maximizer will transfer to m^M immediately, or first to m_2 and then m^M . Thus, if there are three regions, \underline{t}^{sym} is t such that $(1-G(\tilde{p}(\tilde{m}+t))^2)\tilde{m} = (1-G(w)^2)m_2$ and \bar{t}^{sym} is t such that $(1-G(w)^2)m_2 = (1-G(p^M(m^M+t)))m^M$. The only alternative is that $\underline{t}^{sym} = \bar{t}^{sym}$ and $(1-G(\tilde{p}(\tilde{m}+t))^2)\tilde{m} = (1-G(p^M(m^M+t)))m^M \geq (1-G(w)^2)m_2$ at $t = \underline{t}^{sym} = \bar{t}^{sym}$.

Now consider $s \in (\underline{s}^{sym}, \bar{s}^{sym})$ for $t = 0$. The only difference to the above proof is that $\underline{t}^{sym} = 0$. Finally, consider $s > \bar{s}^{sym}$ for $t = 0$. Then the only maximizer is m^M and so $\underline{t}^{sym} = 0$ and $\bar{t}^{sym} = 0$. ■

Appendix B: Wholesale Price Discrimination

Throughout the paper, the manufacturer was not allowed to discriminate between retailers and charge them different prices. This was because we concentrate on symmetric equilibrium (it is notoriously hard to find asymmetric equilibria of the Wolinsky model, except numerically and in special cases such as uniform $G(\cdot)$), and thus allowing the manufacturer to discriminate will not create any new equilibria, but may destroy the one where he is not able to discriminate.

The aim of this section is to show that due to the complete symmetry between firms, even if the manufacturer were able to discriminate between retailers, the equilibrium without discrimination can be sustained.

Let m_i denote the price charged to retailer i . Before we proceed, we note that in any vertical relations model it is important to fix whether retailers observe prices charged to others. Following the literature (e.g. Pagnozzi and Piccolo (2012)), and given that this is fairly intuitive, we focus on the case when retailer i does not observe m_j .

For every belief that consumers hold, we seek to show that there exist out-of-equilibrium

beliefs held by retailers such that the equilibrium price without discrimination is also the equilibrium price with discrimination.

It is simple to see that if we are to sustain the equilibrium with discrimination, the retailer should hold symmetric beliefs about each other's prices. This is because without discrimination, wholesale prices are effectively symmetric, thus to emulate the same retailer behavior, their beliefs should also be symmetric. Thus assume that if in equilibrium both retailers are charged m^* , and the manufacturer charges retailer i a price $m_i \neq m^*$, then firm i believes that $m_j = m_i$.

Provided that retailers hold symmetric beliefs, in the model with discrimination retailer i will behave the same way for any m_i as she does in the model without discrimination where both are charged m_i . Let $p(m_i + t)$ denote the price retailer i charges for any price m_i it is charged by the manufacturer. This price will depend on beliefs, and in particular for passive consumer beliefs it is given by $\hat{p}(m_i + t)$, and for symmetric consumer beliefs it is given by $\tilde{p}(m_i + t)$. Let $Q_i(m_i, m_j)$ denote the demand for retailer i given the manufacturer's price vector (m_i, m_j) . Due to symmetry $Q_j(m_i, m_j) = Q_i(m_i, m_j)$, and thus we drop the subscript and write $Q(m, m')$.

The manufacturer's profit is given by

$$\Pi = Q(m_1, m_2)m_1 + Q(m_2, m_1)m_2.$$

It is easily verified that FOC with respect to m_i imposing symmetry afterwards, and FOC when imposing symmetry before taking the derivative, are the same. Given that with symmetric out-of-equilibrium beliefs retailers behave the same with or without discrimination, it is clear that the equilibrium without discrimination can be sustained when discrimination is allowed. This is because demand is completely symmetric, thus the manufacturer's profit maximizing vector (m^*, m^*) remains unchanged even if the manufacturer is allowed to charge different prices to different retailers.

Appendix C: Costly first visits

In this appendix we revisit our main results in the paper under the assumption of costly first visits. For simplicity, we assume that the first search cost is the same as the second one, s .

Wolinsky model

We start this analysis with our study of the Wolinsky model pricing from Section 3. Costly first search only affects the market when consumers expect that market prices are above the reservation utility, in which case it is not worthwhile to make the first, or for that matter second, search. As a result, for $s \leq s_2$, whether the first search is costly or not,

does not change the equilibrium. For $s > s_2$, if consumers make the first search, then our previous results apply and $p = p^M > w$, and then consumers do not wish to search. Thus in equilibrium consumers do not search at all, and firms charge prices above w , e.g. p^M . In this instance Wolinsky's solution for the Diamond Paradox does not work, and the market breaks down.

Random cost model

With these results in mind, we now turn to our model where c is random and unknown to consumers. The first and second search are qualitatively different in the following respect. Once at the first firm, consumers can infer the price of the other firm, and thus face no uncertainty about the next price. Because of this, they can simply compare their benefit from the next search to the price at the next firm, and decide whether to search further or not.

In contrast, when deciding whether to embark on the first search or not, they neither know utilities from each product, nor firms' marginal costs, and therefore the prices they should expect. In this case, we need to explicitly derive the expected benefit from the first search.

Let $p(c)$ stand for the equilibrium pricing rule. Then a consumer's expected benefit from the first search is

$$E(v - p(c) | v - p(c) > 0) = \int_{\underline{c}}^{\bar{c}} \left(\int_{p(c)}^{\bar{v}} (v - p(c))g(v)dv \right) f(c)dc.$$

Now consider the case where $\bar{c} < c_2$, such that $p(c) \leq w$. Then we can write the following

$$E(v - p(c) | v - p(c) > 0) > \int_{\underline{c}}^{\bar{c}} \left(\int_w^{\bar{v}} (v - w)g(v)dv \right) f(c)dc = \int_{\underline{c}}^{\bar{c}} sf(c)dc = s.$$

The above uses the definition of w . Thus, we have that the expected benefit from the first search is strictly higher than s , therefore consumers in this case will search the first time, and all our results apply verbatim. The intuition here is very simple. When $\bar{c} < c_2$, all prices are below the reservation utility, thus the expected price is below w and so consumers get positive expected utility from searching.

Next consider the mirror image of the previous scenario, where $\underline{c} > c_2$, so that all prices are above w . Then

$$E(v - p(c) | v - p(c) > 0) < \int_{\underline{c}}^{\bar{c}} \left(\int_w^{\bar{v}} (v - w)g(v)dv \right) f(c)dc = \int_{\underline{c}}^{\bar{c}} sf(c)dc = s.$$

and so there is a full market breakdown where no consumer searches the first time, and

firms charge $p^M(c)$.

Finally, now consider a more complicated situation where for some, but not all, cost realizations price exceeds w , which is the case for $\bar{c} > c_2 > \underline{c}$. Here it is unclear whether $E(v - p(c) | v - p(c) > 0)$ exceeds s or not, and this depends on the mass of marginal costs above c_2 , as well as distributions $F(\cdot)$ and $G(\cdot)$. Clearly, for \bar{c} sufficiently close to c_2 , consumers will search because of the strict inequality in (16). Then, we have a situation where consumers search the first time, but for some realizations of c do not search the second time, thus we have a partial Diamond Paradox, where the second search is (sometimes) discouraged, but the first search is not.

Vertical relations model

Now we turn to our analysis of vertical relations, and start with the model where consumers hold passive beliefs.

Costly first visits do not alter our analysis for sufficiently low search cost because provided that consumers are willing to search the second firm, they will also search the first one. Therefore, for $s \leq \underline{s}^{pas}$ the equilibrium described in Section 5.1 is the same regardless of whether the first visits are costly or free.

Now consider the case where search costs are so large that with costless first visits consumers do not search and retailers charge the monopoly price. This is the case when $s > \bar{s}^{pas}$. If the first visit is also costly, then consumers will not search the first firm either, and thus the market breaks down completely. In equilibrium retailers charge prices above w , e.g. $p^* = p^M$, and consumers choose not to search. The interesting point to note is that the manufacturer and retailers cannot prevent the market from breaking down. Conditional on consumers searching the first time, firms will charge retail prices above w , and thus now consumers will want to search.

Finally, for intermediate search costs, $s \in (\underline{s}^{pas}, \bar{s}^{pas})$, there is no pure strategy equilibrium when the first search is costless. This is not true, however, when the first search is costly. This is because one can sustain an equilibrium where consumers do not search by assuming that retailers charge prices above w . Consumers then rationally do not search, and retailers are indifferent between charging any price, including the one prescribed.¹⁴ The same equilibrium cannot be constructed for costless first visits because in that case, if consumers do not search, retailers charge $p^M < w$, which induces consumers to search, but if consumers do search, then retailers charge $\tilde{p} > w$, which means that consumers do not search.

Costly first visits affect the model with symmetric beliefs in the same way as they do passive beliefs. For all parameters where in equilibrium prices do not exceed w , equilibrium is unchanged. Thus for $s \leq \bar{s}^{sym}$ equilibria of the model do not depend on whether the

¹⁴This equilibrium is present for all s in the model with costly first visits, but it is more intuitive for $s \in (\underline{s}^{pas}, \bar{s}^{pas})$ because here no pure strategy equilibrium with all consumers searching can exist.

first search is costly or costless. But if $s > \bar{s}^{sym}$, then the market breaks down - consumers do not search, and retailers charge prices above w .