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Measuring mispricing in experimental markets

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# Measuring mispricing in experimental markets

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## Abstract

Mispricing (the difference between prices and their underlying fundamental values) is an important characteristic of markets. The literature on the topic consists of many different measures. This state of affairs is unsatisfactory, since different measures may produce different results. Stöckl et al. (2010) partially address this problem by proposing (among other things) that measures of mispricing be independent of certain nominal variables: the number of dividend payments and the absolute level of fundamental values. Their conditions rule out all previous measures used in the literature and leads them to propose new measures in response. This paper proposes that mispricing measures be independent of an additional variable: the unit of account. This condition rules out the measures proposed by Stöckl et al. (2010) and serves as the basis for a new measure of market mispricing, the *Geometric Average Deviation (GAD)*. The unit of account condition is relevant to many market settings, and thus calls into question the findings of previous research based on other measures that fail to satisfy this condition. An application illustrates the potential impact of this new measure on previous experimental results.

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# 1 Introduction

Market mispricing refers to the extent to which prices deviate from a certain reference level, and it forms an important part of the analysis of experimental asset markets (Palan 2013). The literature on the topic, which goes back to at least Smith et al. (1993), consists of many different measures (Stöckl et al. 2010). This state of affairs is unsatisfactory, since different measures may produce different results. Ideally, a set of theoretically-motivated conditions would be agreed upon that identify a single measure of mispricing.

The topic of identifying a unique measure of mispricing is closely related to the formation of an ideal price index (Fisher 1922), index numbers (Diewert 1979) and functional equations (Eichhorn 1978). Stöckl et al. (2010; SHK) go some way to addressing the problem. Among other things, they propose that a measure of mispricing be independent of certain nominal variables (number of dividend payments and absolute level of fundamental values). Second, they show that no measure previously used in the literature satisfies this condition, and suggest two new measures that do satisfy their conditions as alternatives.

The SHK independence conditions are only important when comparing across settings in which the associated parameters are not constant. However, independence with respect to nominal values is in general a desirable trait for mispricing measures to have. As the following section shows, the SHK measures are not independent of one particular nominal variable: the *unit of account*. Since a unit of account is always implicit when averaging prices, this problem in fact occurs quite generally, and thus may have an effect on previous research.

The unit of account, also referred to as the *numeraire*, refers to the asset (or combination of assets) in which all relative values are expressed, where a relative value is any value that expresses a ratio of two quantities. The relative value of an asset is expressed as the quantity of the unit of account that may be exchanged for a single unit of the asset. Quite naturally, for each unit of account, each relative value may have a different numerical representation. For example, the relative values 2 EUR/\$ and 0.5 \$/EUR are numerically different, even though they represent the same rate of exchange. The choice of unit of account (and hence representation) is arbitrary, and does not affect the implied rate of exchange between the assets. However, the arithmetic mean of such values is sensitive to the unit of account, which in turn has implications for the SHK measures and other arithmetic-mean based measures.

A numerical example illustrates the problem. Suppose dollars and Euros are traded over two time periods. Initially, dollars are used as the unit of account, and the exchange rates are 2 \$/EUR and 0.5 \$/EUR. The arithmetic mean of these values suggests an average exchange rate of 1.25 \$/EUR, or equivalently that a single Euro is worth more than a single dollar. If instead *Euros* are used as the unit of account, the exchange rates are 0.5 EUR/\$ and 2 EUR/\$ and the arithmetic mean (1.25 EUR/\$, or equivalently 0.8 \$/EUR) suggests instead that *dollars* are more valuable than Euros. The two implied averages, 1.25 EUR/\$ and 0.8 EUR/\$, are quite different, even though they both describe the same situation.

As a consequence, this paper proposes independence from the choice of numeraire as an additional condition for mispricing measures. Under this condition, recommendations are made about how prices should be averaged. This forms the basis for a new measure of mispricing, the *Geometric Average Deviation* (*GAD*). The condition of numeraire independence, even in combination with the original SHK conditions, still does not generate a unique measure of mispricing, but it does at least further reduce the set of such measures. In addition, the related issue of the interval length is examined, which leads to a recommendation that intervals be as small as possible.

The rest of the paper is structured as follows. Section 2 examines market mispricing in detail and makes recommendations about how a measure of mispricing be calculated under the condition of numeraire independence. In many cases where it is necessary to take an average, an appropriate solution is shown to be replacing the arithmetic mean with its geometric counterpart. Section 3 illustrates with an application and Section 4 concludes.

## 2 Theory

Consider a market for two assets,  $A$  and  $B$ , that consists of a set of  $N$  observations. For the moment, only assume that observation  $i \in 1, \dots, N$  is composed of a price  $p_i$  and a fundamental value  $v_i$ , both expressed in units of  $A$  per unit of asset  $B$ . Prices indicate the *implied, subjective* market valuation of the two assets, whereas fundamentals  $v_i$  denote the ratio of the *actual, objective* values of holding the two assets<sup>1</sup>. Mispricing may be defined in the two following ways:

**Definition 1** *Absolute mispricing*: on average over time, how far prices for an asset *differ from* its fundamental value.

**Definition 2** *Overpricing*: on average over time, how far prices for an asset *are higher than* its fundamental value.

The two concepts are similar, yet distinct. The first measures only the magnitude of mispricing, while the second also includes a direction component. The discussion here will be restricted to measures of overpricing, however the results will extend to the first definition of mispricing as well.

It is important to recall that relative values, such as prices and fundamentals, are values that represent a ratio of two quantities. As such, they may be expressed in any arbitrary unit of account, or numeraire. Let  $y(X)$  indicate the representation of relative value  $y \in (p, v)$  in units of the numeraire  $X \in (A, B)$ . Prices and fundamentals are *defined* in units of  $A$ , therefore  $y(A) = y$  and  $y(B) = 1/y$ .

Let  $M_X = M(p(X), v(X))$  denote the measure of overpricing given the choice of numeraire  $X$ . Since the numeraire determines the form in which the

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<sup>1</sup>The actual value of holding an asset is the discounted sum of all its future payoffs. Naturally, if payoffs are stochastic, payoffs should be adjusted for risk preferences.

inputs are presented, it is natural that the interpretation of  $M_X$  also depends on  $X$ . In particular, if the inputs are presented using the numeraire  $X$ , then they represent relative prices and values of the non-numeraire asset,  $X'$ . This implies that mispricing  $M_X$  is also a measure of the overpricing of the non-numeraire asset.

Overpricing is a relative concept, and therefore overpricing in one asset implies a certain amount of underpricing in the other. Given that  $M_X$  measures overpricing in the non-numeraire asset, let  $M_X^{-1}$  denote the implied overpricing in the numeraire asset  $X$  itself. For example, if overpricing is proportional to fundamentals and centered at zero ( $p = v \Rightarrow M_X = 0$ ), then  $M_X^{-1}$  takes the form:

$$M_X^{-1} = \frac{1}{M_X + 1} - 1. \quad (1)$$

For example, in a typical asset market setting, the two assets might be cash (the numeraire) and shares. In this case, shares being overpriced by 50% of fundamental value ( $M_{cash} = 1/2$ ) implies that cash is underpriced by 33% ( $M_{cash}^{-1} = 1/(0.5 + 1) - 1 = -1/3$ ).

This means that there are two ways of calculating the overpricing of an asset  $X$ : 1) overpricing using the other asset,  $X'$ , as numeraire ( $M_{X'}$ ), and 2) *implied* overpricing when  $X$  itself is used as numeraire ( $M_X^{-1}$ ). *Numeraire independence* requires that the two methods be equivalent:

$$M_{X'} = M_X^{-1}. \quad (2)$$

For mispricing measures that satisfy (1), this simplifies further to:

$$M(1/p, 1/v) = \frac{1}{M(p, v) + 1} - 1 \quad (3)$$

As long as observations are assigned weights  $w_i$ ,  $i = 1, \dots, N$  that are not affected by the choice of numeraire, then one overpricing measure that satisfies (3) is the weighted geometric average of prices relative to fundamentals<sup>2</sup>:

$$WGM_A = \prod_i \left( \frac{p_i}{v_i} \right)^{w_i / \sum w_j} - 1. \quad (4)$$

The type of weights used depends on the type of observation under consideration. Three types of data that commonly arise in experimental asset markets are 1) indices, 2) transactions, and 3) order books.

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<sup>2</sup>(4) is only unique under additional assumptions on  $M_N$  (see Aczél (1990) and Roberts (1990)). Non-positive prices and values are unlikely to arise in practice, yet even when they do the geometric mean is not necessarily undefined (Habib 2012).

Table 1: Market data for numerical example

Period	$p_i$	$v_i$
1	1 E/S	1 E/S
2	2 E/S	1 E/S

Table 2: Overpricing of shares (S) relative to Euros (E) for numerical example

Measure	Numeraire	Overpricing
$AM_E$	Euros	+50.0%
$AM_S^{-1}$	shares	+33.3%
$GM_E = GM_S^{-1}$	Euros, shares	+41.4%

## 2.1 Price indices

Index data arise when averaging over time periods in a market setting. Indices imply that all observation be treated equally, therefore in this case the weights are  $w_1 = \dots = w_N = 1/N$ . (4) simplifies to:

$$GM_A(p, v) = \prod_i \left( \frac{p_i}{v_i} \right)^{1/N} - 1. \quad (5)$$

In contrast, the measure suggested by SHK, *Relative Deviation (RD)*, uses a ratio of arithmetic means:

$$AM_A(p, v) = \frac{\sum_i p_i}{\sum_i v_i} - 1. \quad (6)$$

Both of these functions measure deviations proportional to fundamentals and are centered at zero, therefore the conversion function (1) applies in both cases. However, while the geometric mean satisfies numeraire independence given by (3), it is easily verified that the ratio of arithmetic means does not.

Consider a typical experimental asset market environment where shares (S) are traded for Euros (E). Suppose that the market consists of the two periods given in Table 1. With Euros as the unit of account,  $AM_E(p, v) = (0 + 1)/2 = 0.5$ , which says that shares are overpriced by 50%. If instead shares are used as the unit of account, then  $AM_S(1/p, 1/v) = (0 - 0.5)/2 = -0.25$  (Euros are underpriced by 25%), or equivalently shares are overpriced by 33% ( $AM_S^{-1} = 0.33$ ). These calculations are summarized in the first two rows of Table 2.

The table shows that overpricing implied by (6), and hence *RD*, is sensitive to the choice of numeraire. The conclusions depend on the arbitrary choice of unit of account and the representation of prices and fundamentals. This is perhaps not surprising, since *AM* implies a linear average of relative values,

while the associated conversion function (1) includes non-linear transformations. The sensitivity of the arithmetic mean of prices to the choice of unit of account has been known for some time (Jevons 1863), and makes it unsuitable not only in the current context, but also many others (for example, general equilibrium modelling (Flemming et al. 1977), exchange rates (Brodsky 1982), psychology (Aczél and Saaty 1983), and measuring technical performance (Flemming and Wallace 1986)).

The final row of Table 2 shows the value of the geometric mean (5) when applied to the previous example. It implies overpricing of 41.4%, regardless of which numeraire is chosen. Since this measure also satisfies the conditions proposed by *SHK*, there is no tradeoff to averaging indices using (5) instead of (6).

**Recommendation 1** Measure overpricing of price indices using (5).

## 2.2 Transactions

In a double auction, transactions occur at various times in the market. It may be necessary to average over the transaction prices themselves. Suppose transaction  $i$  consists of amounts  $a_i$  and  $b_i$  of the two assets, and occurs relative to a fundamental value  $v_i$  expressed in terms of units of  $A$  per unit of  $B$ . Using the same units, the implicit price of the transaction is  $p_i = a_i/b_i$ .

Before turning to the weighted geometric mean, it is useful to consider alternative measures. For transactions, *RD* uses a measure of the form:

$$AM_A = \frac{\sum_i a_i}{\sum_i b_i v_i}. \quad (7)$$

This measure takes a ratio of the weighted average of prices and fundamentals, using the weights  $w_i = b_i$ . When  $B$  is used as numeraire, the *implied* overpricing of asset  $B$  is:

$$\begin{aligned} AM_B^{-1} &= \frac{\sum_i a_i/v_i}{\sum_i b_i} \\ &\neq AM_A, \end{aligned}$$

which means that *AM* is not, in general, independent of the choice of numeraire. The one exception to this rule is when all transactions have the same fundamental value ( $v_1 = \dots = v_N$ ).

Therefore, in the general case, a different measure is required if numeraire independence is to be satisfied. Returning to the weighted geometric mean (4), a natural assumption to make about the weights  $w_i$  is that they are constructed as a sum of the exchanged quantities,  $a_i$  and  $b_i$ . Since the quantities represent units of different assets, they must be converted to a common unit of account before they may be aggregated. If conversions are made using the fundamental value  $v_i$ , then the weights have the form:

$$w_i = (a_i/v_i + b_i)v_i^\alpha, \quad (8)$$

where  $\alpha$  is a free parameter that implicitly determines the unit of account. A further restriction is that the weights be invariant to a re-labelling of the assets. Re-labelling the assets switches the arbitrary order of  $A$  and  $B$ ; in practical terms, it reverses the positions of  $a_i$  and  $b_i$ , and inverts the fundamental value  $v_i$ . For a general function  $f$ , independence with respect to asset re-labelling therefore implies:

$$f(a_i, b_i, v_i) = f(b_i, a_i, 1/v_i). \quad (9)$$

Applied to (8), this identifies  $\alpha = 0.5$ , and gives the unique weight:

$$w_i = a_i v_i^{-0.5} + b_i v_i^{0.5}. \quad (10)$$

This weight has intuitive appeal because the implied unit of account ( $A^{0.5}B^{0.5}$ ) is a combination of the two individual units  $A$  and  $B$ , and the weight itself satisfies:

$$w(a_1 + a_2, b_1 + b_2, \bar{v}) = w(a_1, b_1, \bar{v}) + w(a_2, b_2, \bar{v}).$$

This means that if a transaction is split into separate pieces, the sum of the weights of the individual pieces will still equal the weight of the original transaction.

**Recommendation 2** Measure overpricing of a set of transactions by (4) and (10).

### 2.3 Order book

An order book consists of a set of potential transactions that individuals are willing to engage in. Each offer  $i$  implies a certain proportional deviation  $d_i$  of prices from fundamentals. If  $A$  is the numeraire, then  $d_i = a_i/b_i v_i$ .

In a typical asset market environment, where cash (the numeraire) is traded for shares, bids (asks) are orders in which an individual has offered to purchase (sell) shares in exchange for cash. For rational traders, the market-clearing deviation at any point in time lies between the lowest bid and highest ask deviations. The largest bid deviation (lowest ask deviation) represents a minimum bound  $d_{min}$  (maximum bound  $d_{max}$ ) on the market-clearing deviation from fundamental value. These bounds may be averaged to arrive at a point estimate of overpricing at any point in time. Since these bounds are indices, they are weighted equally ( $w_1 = w_2 = 1/2$ ) and the overpricing measure is a special case of (5):

$$OBM_A = d_{min}^{1/2} d_{max}^{1/2}. \quad (11)$$



**Recommendation 3** Measure order book overpricing by (11).

The order book uses the average of an interval to measure overpricing. For this reason, it is less precise than the measure based on transactions. Therefore, when possible, it is preferable to use transaction prices.

## 2.4 Summary

Given a market composed of transactions  $t = (a_t, b_t, v_t)$  that occur over  $N > 0$  intervals of equal length, Recommendations 1-3 suggest a new measure of overpricing, the *Geometric Average Deviation (GAD)*:

$$GAD = \prod_i^N d_i^{1/N} - 1 \quad (12)$$

where

$$d_i = \begin{cases} \prod_t^{T_i} \left( \frac{a_{i,t}}{b_{i,t}v_{i,t}} \right)^{w_{i,t}/\sum_s^{T_i} w_{i,s}}, & \text{if } T_i > 0 \\ d_{i,min}^{1/2} d_{i,max}^{1/2}, & \text{if } T_i = 0. \end{cases}$$

$$w_{i,t} = a_{i,t}v_{i,t}^{-1/2} + b_{i,t}v_{i,t}^{1/2}.$$

An intermediate measure of overpricing  $d_i$  is calculated for each interval  $i$ . For intervals that contain at least one transaction ( $T_i > 0$ ), the interval measure is the weighted geometric average of transaction deviations. When an interval contains no transactions ( $T_i = 0$ ), the geometric average of the deviation tunnel bounds is used. Intervals with no transactions and no bounded deviation tunnel are omitted. Overall overpricing of asset  $B$  is the geometric average of the interval deviation measures, and the implied overpricing for asset  $A$  is given by  $GAD^{-1} = 1/(GAD + 1) - 1$ .

## 2.5 Interval length

Typically, markets are composed of transactions that occur over time, and the choice of interval length determines how transactions are grouped before being aggregated into indices. This has an important affect on any overpricing measure with this structure, regardless of whether or not it satisfies numeraire independence. *RD* restricts the interval length to values such that the fundamental value within every interval is constant. However, there is no theoretical justification for this restriction. In general, the issue of optimal interval length remains an open question, and it is important to think about 1) the implications of different interval lengths, and 2) whether or not there are any reasons for choosing one value over the others.

First, consider the impact of different interval lengths. Conceptually, longer intervals give more weight to larger transactions (transactions in which large

quantities of the assets are exchanged), whereas smaller intervals tend to treat all transactions more equally. Limiting cases arise as the interval length becomes very large or very small. In the first case, the entire market eventually becomes a single interval, and all transactions are averaged together based on their individual weights. Since there is only one interval, no subsequent averaging is required. On the other hand, as the interval approaches zero, the probability that an interval contains more than one transaction also approaches zero. In the limit no interval contains multiple transactions, the intermediate averaging step is eliminated, and all transactions are weighted equally in the final measure. This suggests that interval length may be interpreted as the extent to which the relative weight of transactions is taken into account.

In considering which (if any) interval length is optimal, it helps to return to the definition of overpricing. In particular, it refers to overpricing as a measurement over time. This emphasizes the temporal aspect of overpricing, and is consistent with choosing the interval length to be as short as possible, given the dataset at hand:

**Recommendation 4** Make the interval length as small as possible.

This recommendation for interval length is not specific to  $GAD$ , and applies equally to other measures of overpricing, regardless of whether or not they satisfy numeraire independence.

### 3 Application

This section illustrates in practice how conclusions based on other measures (such as  $RD$ ) may change when applying a measure that satisfies independence of numeraire, such as  $GAD$ . Stöckl et al. (2014) conduct an experiment in which shares and cash are traded for one another. Table 3 reports the following overpricing measures (expressed as implied overpricing of shares) averaged over the five treatments of the study:

1.  $RD(\text{shares})$ :  $RD$  using cash as numeraire (column 1),
2.  $RD(\text{cash})$ :  $RD$  using shares as numeraire (column 2),
3.  $GAD_{150}$ :  $GAD$  using original intervals of 150 seconds (column 3), and
4.  $GAD_1$ :  $GAD$  using the preferred interval length of one second (column 4).

The first column is  $RD$  as it is reported in Stöckl et al. (2014), and the final column presents the preferred measure of overpricing,  $GAD_1$ . The intermediate columns break down the difference between the two. The first two columns illustrate the sensitivity of  $RD$  to the choice of numeraire. The  $GAD_{150}$  measure compared to the  $RD$  measures isolates the effect of removing this sensitivity by changing from an arithmetic to a geometric mean. Finally, the effect of moving

Table 3: Average overpricing per treatment, reported as implied overpricing of shares (%)

Treatment	$RD(shares)$	$RD(cash)$	$GAD_{150}$	$GAD_1$
$T1$	39.8	15.2	35.8	2.6
$T2$	1.3	1.3	1.3	0.1
$T3$	-15.2	-13.7	-14.5	-1.6
$T4$	2.8	3.3	3.0	0.3
$T5$	6.0	6.6	6.3	1.1

from an intermediate interval length to the preferred minimum length of 1 is given by the difference between  $GAD_{150}$  and  $GAD_1$ . Values are reported averaged over all markets in a treatment (individual market measures are available in the Appendix).

For this dataset, the treatment averages are such that the two  $RD$  measures always indicate the same direction of overpricing, with  $GAD_{150}$  always falling somewhere between the two  $RD$  measures. However, the data in the appendix for individual markets (see Table 5) show that these patterns need not hold in general. Reducing the interval length decreases the impact of large deviations, and therefore measured overpricing in column 4 is an order of magnitude lower than in columns 1-3.

The main interest is the significance of the treatment differences given in Table 4. Each row reports the  $p$ -value resulting from a Mann-Whitney  $U$ -test that a pair of treatments are similar. The difference between certain treatments is so extreme that the result is unaffected by the choice of measure (row 5, for example). However, in many cases the treatment comparison is sensitive to the choice of measure. For example, in row 1 the difference is significant ( $p = 0.004$ ) using  $RD(shares)$ , insignificant using  $RD(cash)$  ( $p = 0.485$ ), and marginally significant using  $GAD_{150}$  and  $GAD_1$  ( $p = 0.093$ ). Thus the conclusions based on the usual RD (shares) measure (column 1) can be significantly affected by moving to measures that are not sensitive to the choice of unit of account (columns 3 and 4).

In order to understand where these differences come from, it helps to look at an individual market. Figures 1-3 show data from market 1 of  $T1$ , in which the four measures differ substantially in their estimates of overpricing:  $RD(shares) = 16.7\%$ ,  $RD(Euros) = -13.7\%$ ,  $GAD_{150} = 2.9\%$  and  $GAD_1 = 0.3\%$ , respectively. Figure 1 displays prices and fundamentals using cash as the numeraire, and based on this information it appears that shares are significantly overpriced in the middle of the market, followed by mild under-pricing towards the end.

Figure 2 uses shares as the numeraire, and tells a different story. Now shares appear to be correctly priced during most of the market, yet significantly underpriced for a period of time near the end of the market. The two measures are not consistent with one another because the arithmetic average treats deviations

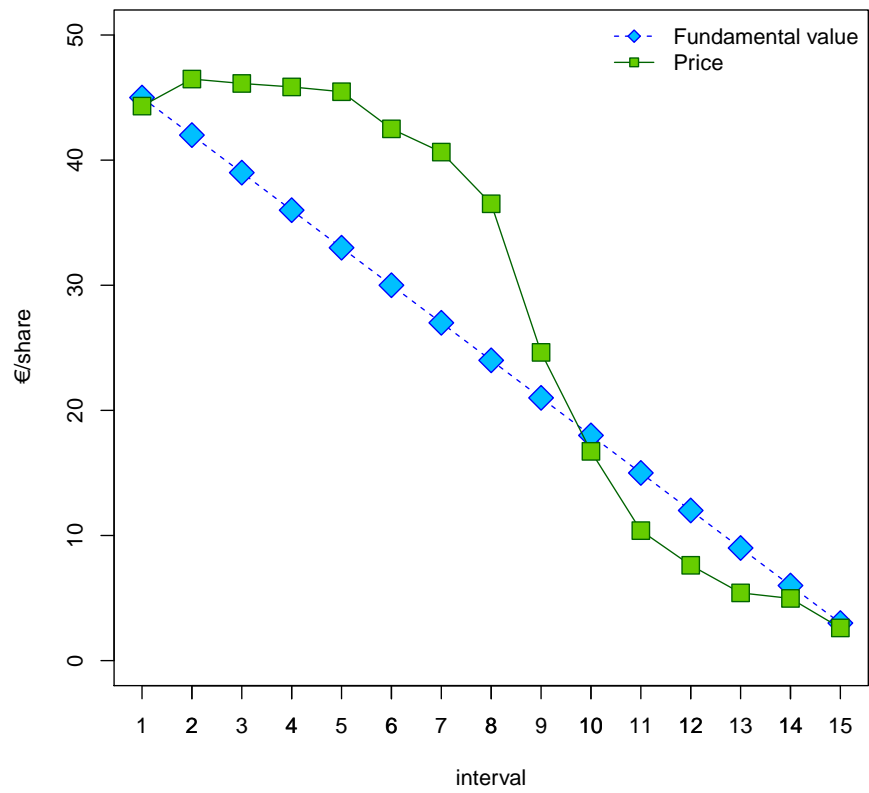


Figure 1: Prices and Fundamentals per Interval (Market 1 of  $T_1$ , numeraire = cash)

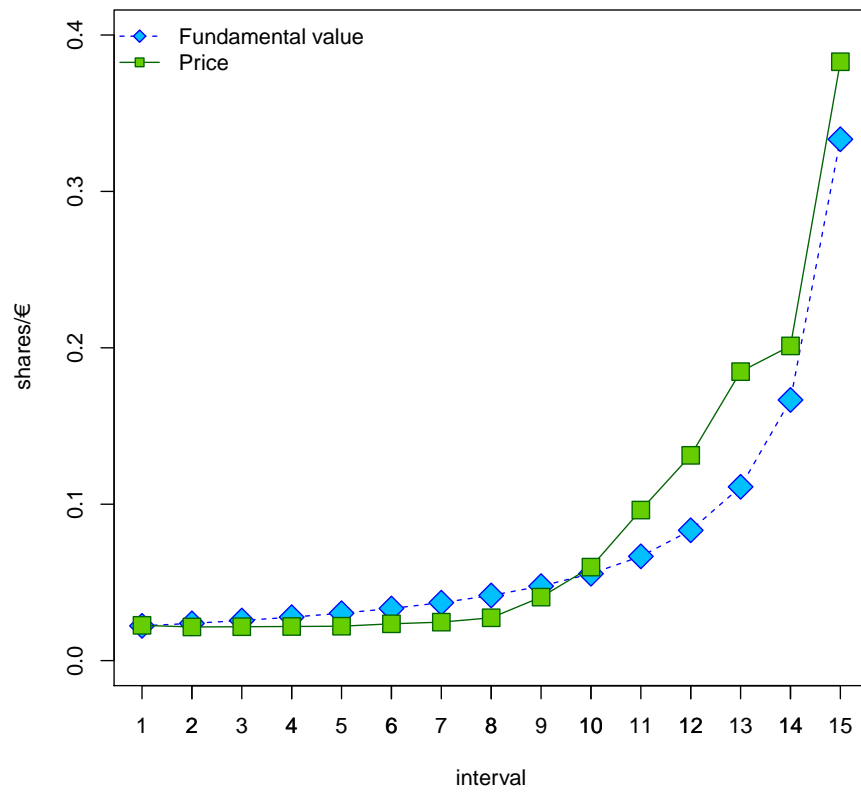


Figure 2: Prices and Fundamentals per Interval (Market 1 of  $T_1$ , numeraire = shares)

Table 4: Significance of treatment differences, reported as  $p$ -value from corresponding Wilcoxon rank-sum test

Comparison	$RD(\text{shares})$	$RD(\text{cash})$	$GAD_{150}$	$GAD_1$
$T1$ vs $T2$	0.004***	0.485	0.093*	0.093*
$T1$ vs $T3$	0.002***	0.041**	0.004***	0.002***
$T1$ vs $T4$	0.026**	0.485	0.093*	0.093**
$T1$ vs $T5$	0.065*	0.818	0.093*	0.310
$T2$ vs $T3$	0.002***	0.002***	0.002***	0.002***
$T2$ vs $T4$	0.485	0.485	0.485	0.485
$T2$ vs $T5$	0.485	0.485	0.485	0.485
$T3$ vs $T4$	0.026**	0.026**	0.026**	0.093*
$T3$ vs $T5$	0.015**	0.015**	0.015**	0.015**
$T4$ vs $T5$	0.699	0.818	0.818	0.589

\*, \*\*, and \*\*\* denote statistical significance at the 10%, 5%, and 1% levels, respectively.

of prices above fundamental values the same as deviations below fundamentals. This procedure is sensitive to the choice of numeraire, which is why the general conclusions (+16.7% for RD (shares), and -13.7% for RD (Euros)) depend on which way the information is presented.

On the other hand,  $GAD$  offers an alternative interpretation (Figure 3). Geometric measures consider proportional, rather than level, deviations per interval. The graph shows relative deviations, where the scale of the vertical axis has been transformed so that deviations of the same magnitude (as measured by their proportional distance from unity) have the same length, regardless of their sign. This is a more appropriate representation of how the deviations impact the geometric mean. The market begins with significant overpricing of shares, followed by significant (albeit slightly smaller) under-pricing towards the end of the market. On aggregate, a small amount of overpricing (+2.9%) of shares occurs.

The answer to the question of on average over time, how overpriced was an asset? depends on which type of measure is used. The example in this section shows how, for the case of measures based on the arithmetic mean (such as  $RD$ ), i) individual observations, ii) treatment averages, and iii) conclusions can all be sensitive to the nominal choice of numeraire. This highlights why it is important to use a numeraire-independent measure such as  $GAD$  to capture overpricing in such circumstances.

## 4 Conclusion

Measuring market efficiency depends on how various types of prices are compared to fundamentals. Crucially, prices and fundamentals are both relative values. This study shows that due to this property, measures that rely on the

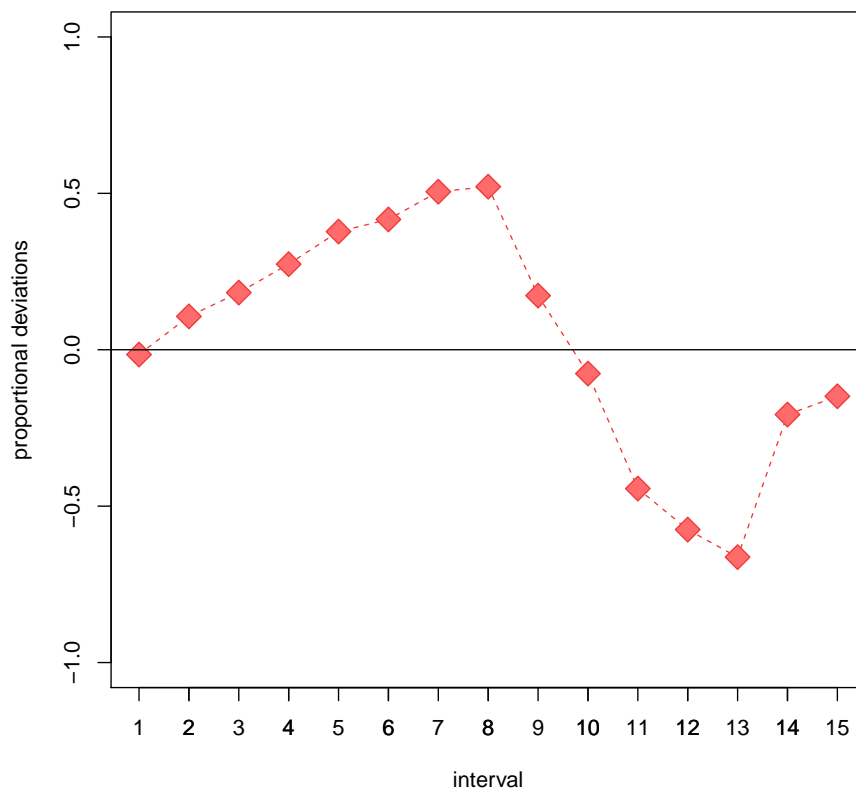


Figure 3: Normalized Deviations per Interval (Market 1 of  $T1$ )

arithmetic mean, such as those currently used in the literature ( $RD$  and others), are sensitive to the arbitrary choice of numeraire unit. Since a numeraire is always implicit when measuring mispricing in experimental markets, this calls for a careful re-examination of previous work based on measures that do not take this into account.

In the definition of the measures discussed here, such as  $RD$  and  $GAD$ , deviations of prices above fundamentals offset negative deviations. Implicitly, these measures capture the magnitude and direction of mispricing. A different class of measures, such as the measure  $RAD$  proposed by SHK, considers only the magnitude of mispricing, regardless of its direction. The two classes of measures address fundamentally different concepts, thus both have a place in the mispricing literature. Since the two types of measures are very similar in nature, the problems associated with the arithmetic mean extend to  $RAD$ , and again the geometric mean proves to be a suitable replacement. The equivalent of  $GAD$  in the case of absolute mispricing is the *Geometric Average Absolute Deviation* ( $GAAD$ , see Appendix for definition). The only difference between the two measures is that deviations with a value less than unity are inverted in the case of  $GAAD$  - this is the equivalent of taking the absolute value for relative values.

This paper highlights the importance of recognizing the relative nature of prices in the area of experimental asset markets. However, the recommendations made here apply in general to any setting in which prices are averaged, and thus there are likely to be other areas of economics and finance where the recommendations of this study should be considered.

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## 5 Appendix

Definition of *Geometric Average Absolute Deviation (GAAD)*

$$GAAD = \prod_i^N d_i^{1/N} - 1 \quad (13)$$

where

$$d_i = \begin{cases} \prod_t^{T_i} (d'_{i,t})^{w_{i,t}/\sum_s^{T_i} w_{i,s}}, & \text{if } T_i > 0 \\ (d'_{i,min} d'_{i,max})^{1/2}, & \text{if } T_i = 0. \end{cases}$$

$$w_{i,t} = a_{i,t} v_{i,t}^{-1/2} + b_{i,t} v_{i,t}^{1/2}.$$

Deviations  $d'_{i,s}$ ,  $s \in (t, min, max)$  are the original deviations ( $d_{i,s}$ ) as defined in *GAD*, when  $d_{i,s} > 1$ . Otherwise, they are the inverse:  $d'_{i,s} = 1/d_{i,s}$  if  $d_{i,s} < 1$ .

Table 5: Mispricing per market, reported as implied overpricing of shares (in %)

Treatment	Market	$RD(\text{shares})$	$RD(\text{Euros})$	$GAD_{150}$	$GAD_1$
$T1$	1	16.7	-13.7	2.9	0.3
	2	44.4	30.0	49.8	4.2
	3	54.5	18.3	49.9	3.6
	4	0.9	-4.7	-1.4	0.0
	5	62.9	5.8	48.1	3.8
	6	59.4	55.6	65.7	3.8
$T2$	1	0.0	0.0	0.0	0.0
	2	0.6	0.6	0.6	0.1
	3	0.2	0.2	0.2	0.0
	4	5.9	5.9	5.9	0.4
	5	0.5	0.4	0.5	0.1
	6	0.7	0.7	0.7	0.0
$T3$	1	-5.6	-3.8	-4.7	-0.3
	2	-15.2	-12.5	-13.9	-1.0
	3	-0.6	-0.8	-0.7	-0.2
	4	-22.3	-21.5	-22.1	-1.6
	5	-28.0	-26.7	-27.4	-5.0
	6	-19.2	-17.2	-18.3	-1.1
$T4$	1	-6.8	-6.5	-6.7	-1.0
	2	13.7	14.1	13.9	2.3
	3	1.3	1.7	1.5	0.2
	4	8.9	9.5	9.2	1.0
	5	-12.0	-11.6	-11.8	-1.3
	6	11.7	12.4	12.1	0.8
$T5$	1	-2.9	-2.7	-2.8	-0.3
	2	18.9	19.4	19.1	2.0
	3	-7.8	-7.3	-7.5	-0.7
	4	20.1	21.6	20.8	4.3
	5	1.4	1.0	1.2	0.2
	6	6.2	7.5	6.8	1.0