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A Utility-Based Model of Sales with Informative Advertising

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Abstract

This paper presents a generalised framework to understand mixed-strategy sales behaviour with informative advertising. By introducing competition in the utility space into a clearinghouse sales model, we offer a highly tractable framework that can i) provide a novel welfare analysis of intra-personal price discrimination in sales markets, ii) characterise sales in a range of new contexts including complex market settings and situations where firms conduct sales with two-part tariffs or non-price variables such as package size, and iii) synthesise past research and highlight its key forces and assumptions.

Keywords: Sales; Price Dispersion; Advertising; Clearinghouse; Utility Space; Intra-personal; Price Discrimination; Two-Part Tariffs; Bonus Packs; Package Size

JEL Codes: L13; D43; M37; D83

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1 Introduction

The evidence for the existence of price dispersion in the form of temporary price reductions or ‘sales’ is overwhelming (as reviewed by Baye et al 2006). Sales activity is estimated to account for 20-50% of retail price variation (Nakamura and Steinsson 2008, Hosken and Reiffen 2004) and 38% (25%) of all packaged consumer good purchases in the US (UK) (Steenkamp et al 2005). Consequently, sales are an active area of research across many disciplines including industrial organization, marketing, and macroeconomics.¹

One major strand of this literature has made much progress in understanding sales in the form of mixed strategy pricing equilibria that stem from either heterogeneity in consumers’ information or loyalty, and/or the costs of informative advertising (e.g. Varian 1980, Burdett and Judd 1983, Stahl 1989, Janssen and Moraga-Gonzalez 2004; Robert and Stahl 1993, Baye and Morgan 2001).² However, for tractability, this literature has focused upon a limited range of market settings in two important ways. First, models typically only consider single-product firms and most also assume unit demand. This restricts the examination of more realistic scenarios involving multi-product firms or downward-sloping demand. Second, the literature has almost exclusively studied sales in the form of single linear prices. This limits its ability to analyse firms’ increasing usage of non-price sales, such as package size offers with ‘X% Free’. More substantially, it also prevents any consideration of intra-personal price discrimination in sales markets. Such discrimination remains unstudied despite the prevalence of sales using two-part tariffs in markets such as energy, mobile phones, telecommunication/broadband services, and sports clubs.

To help analyse sales in these wider contexts, we present a new and simple framework that offers several contributions. First, and most substantively, we exploit the tractability of the framework to provide a novel characterisation and welfare analysis of intra-personal price

¹For related reviews, see Baye et al (2006) and Raju (1995). Sales research in macroeconomics is less developed; see Guimaraes and Sheedy (2011) for a recent example.

²Other strands of the literature explain sales as resulting from i) clearance sales (e.g. Lazear 1986), ii) dynamic pricing incentives when consumers vary in inventory costs (e.g. Sobel 1984, Hong et al 2002), iii) imperfect brand consumer loyalty (e.g. Raju et al 1990) and iv) behavioral biases, such as reference dependence (e.g. Zhou 2011).

discrimination in sales markets. Second, by using the framework’s generality, we examine equilibrium sales and informative advertising behaviour in a range of new or understudied contexts, including complex market settings and situations where firms non-price variables such as package size. Finally, we show how the framework is capable of synthesising a wide variety of equilibria from the existing literature, while highlighting common forces and key assumptions.

In more detail, the paper extends Armstrong and Vickers’ (2001) seminal model of competition in the utility space into a mixed strategy sales context by introducing utility competition into a version of the ‘clearinghouse’ sales model (Baye et al 2004 and 2006). In the original clearinghouse sales model, each firm sells a single homogeneous product, while consumers are potentially split into ‘loyal consumers’ that only buy from a designated firm, and ‘shopper consumers’ who exhibit no such loyalty. Firms then choose their price and whether to inform consumers of this price for some potential advertising cost. In equilibrium, as consistent with sales behaviour, each firm randomises between selecting a high price without advertising, and advertising a lower stochastic price drawn from a common support.³ By introducing utility competition into this context, we make use of Armstrong and Vickers’ dual approach to think of firms as competing directly in their provision of ‘value for money’ or utility, u , with an associated profit function, $\pi(u)$, that captures the maximum profit a firm can make per consumer for a given utility offer. With very little increase in computational costs, this approach offers us a high level of generality and tractability by allowing the characterisation an over-arching sales equilibrium for a general form of utility, u , and profit function, $\pi(u)$, where firms mix between offering a low level of utility while not advertising, and advertising a higher stochastic utility offer. This equilibrium can then apply across a wide range of demand, product, cost, and pricing conditions, while also permitting firms to

³In the ‘gatekeeper’ version of the clearinghouse model (Baye and Morgan 2001), firms decide whether to inform consumers by paying a listing fee at a clearinghouse, whilst consumers choose whether to pay a subscription fee to access the listed information, where both fees are endogenously chosen by the clearinghouse or ‘gatekeeper’. As the role of such a gatekeeper is not the focus of our study, we build upon the ‘advertising’ version of the clearinghouse model instead (Baye et al 2004 and 2006), where firms can communicate to consumers directly for an exogenous informative advertising cost and all consumers receive adverts without charge.

compete in multiple dimensions in ways that would otherwise be very difficult to model.

After presenting the framework in Sections 2 and 3, Section 4 offers a number of initial contributions by examining some particular specifications of utility and profits. First, it demonstrates how the framework can nest many equilibria from the existing literature, such as Varian (1980) and Simester (1997), as well as Baye et al (2004 and 2006), while also extending them to allow for more complex market conditions including multiple products, downward-sloping demand and/or positive advertising costs. Second, it shows how the utility framework can enable the analysis of sales behavior in contexts where firms do not use linear prices. As fully detailed and referenced later, the literature has had little to say about such sales behaviours despite their obvious real-world applicability. In one setting, we offer an original characterisation of sales when firms employ two-part tariffs. In equilibrium, firms price each product at marginal cost but randomise their fixed fee. In another setting, we analyse sales behaviour where prices remain constant but firms randomise a non-price variable as consistent with package size sales, such as ‘X% Free’ offers, and a range of other value-increasing sales activities, such as the temporary inclusion of free additional items.

However, after outlining some brief comparative statics in Section 5, Section 6 presents the paper’s main results regarding the welfare analysis of intra-personal price discrimination in sales markets. Typically, intra-personal price discrimination involves firms selling units of output at different prices to the same consumer despite no corresponding difference in marginal cost. Within their utility framework, Armstrong and Vickers (2001) broadly view such discrimination as any increase in pricing flexibility such that firms can earn higher profits per consumer through a change in the profit function, $\pi(u)$. Among other examples, this can include the move from linear pricing to two-part tariffs, the removal of uniform pricing constraints across products, or the relaxation of a ban on loss-leaders. Under their pure-strategy setting, Armstrong and Vickers then show that such discrimination tends to increase consumer welfare in competitive markets, while also providing a sufficient condition for discrimination to reduce consumer welfare in less competitive markets.

While these seminal results have proved very instructive, they cannot be applied to the commonly observed setting of mixed-strategy sales competition. Extending their results to this different context introduces new technical challenges and generates new effects for

discrimination. For instance, under pure-strategies, a change in $\pi(u)$ only affects equilibrium at the margin, while under mixed strategies, a change in $\pi(u)$ affects equilibrium at each level of u within the equilibrium sales support in a way that can provide firms with conflicting incentives. Despite these substantial differences, Sections 6.1 and 6.2 first show that our sales framework can provide some results and conditions that offer a striking similarity to Armstrong and Vickers. In particular, we are able to present some identical results for the competitive limit, and an equivalent sufficient condition for discrimination to lower utility under the restriction that advertising costs are zero, such that sales derive solely from the existence of loyal consumers. Even under this restriction, we consider the similarity in results to be notable given the large differences between the two models. However, when advertising costs are allowed to be positive some further new discrimination effects become active in our framework. For this more general sales setting, we then provide another sufficient condition that differs to the original condition of Armstrong and Vickers.⁴ To further understand the effects driving this difference, Section 6.3 explores an example setting in the context of a transition from linear prices towards two-part tariffs where discrimination improves utility in our sales framework, yet reduces it under pure-strategies. Finally, Section 6.4 briefly considers how our price discrimination results can also be applied to other factors that affect the profit function, such as production costs. Typical results in pure-strategy models suggest that a cost reduction will only affect consumer surplus if it reduces costs at the relevant level of output for equilibrium (at the margin).⁵ However, in a sales context, our results imply, quite generally, that consumers will benefit from a cost reduction even if the reduction only applies to lower ‘infra-marginal’ levels of output.

To end our analysis, before Section 8 concludes, Section 7 uses the framework’s ability to synthesise past research to highlight and extend the key assumptions within the existing literature. In particular, we discuss a range of assumptions and extensions in regard to different levels of consumer visit/search costs, multi-stop shopping, heterogeneous preferences, endogenous advertising costs and alternative advertising technologies.

⁴Results that differ to Armstrong and Vickers’ original findings have also been noted by Johnson (2014) who considers a different pure-strategy setting with boundedly rational consumers.

⁵See Weyl and Fabinger (2013) for a summary and significant extension of the standard theory for cost-pass through and incidence.

Related Literature: Due to the nature of our paper, it will be preferable to discuss most of the relevant literature as the analysis progresses. However, in this subsection, we now provide some more details on how our framework relates to i) Armstrong and Vickers' (2001) original utility analysis, ii) some previous sales papers that also make reference to utility, iii) some pure-strategy clearinghouse models, and iv) the wider advertising literature.

First, Armstrong and Vickers (2001) introduce the concept of competition in the utility space and embed this approach into a range of discrete-choice settings where consumers are fully informed and value a firm's utility offer with the addition of some idiosyncratic noise. They then use their framework to establish a variety of results about pure-strategy market equilibria and study a number of issues related to competitive price discrimination. In contrast, while we borrow their utility space set-up and study some related discrimination issues, we embed the utility approach into a qualitatively different (clearinghouse) model with advertising, where i) consumers are initially uninformed about firm's utility offers, and ii) consumers have identical preferences, such that the equilibrium necessarily involves mixing in utility for most parameter values.

Second, several past sales papers have already made reference to competing in utility rather than prices (Simester 1997, Hosken and Reiffen 2007, Wildenbeest 2011, Dubovik and Janssen 2012 and Lach and Moraga-Gonzalez 2012). However, unlike our framework, these papers only use utility as a means of computing sales equilibria in very specific market settings, and do not use the associated profit function, $\pi(u)$, to explore any general results, implications or wider questions.⁶

Third, we note that our framework, and clearinghouse models more generally, relate to the wider literature on informative advertising in markets with homogeneous products (e.g. Butters 1977, Robert and Stahl 1993, Stahl 1994 and Janssen and Non 2008). This is later discussed in the context of advertising technologies in Section 7.

Finally, a small literature considers the 'gatekeeper' version of the clearinghouse frame-

⁶In more detail, Wildenbeest (2011) and Lach and Moraga-Gonzalez (2012) each use utility to solve a non-clearinghouse sales model for their empirical studies in a single product-unit demand-linear pricing context, while Hosken and Reiffen (2007) consider a non-clearinghouse dynamic price sales setting where firms sell one storable and one non-storable good. Simester (1997) and Dubovik and Janssen (2012) are detailed later in the paper.

work with horizontally differentiated products (Galeotti and Moraga-Gonzalez 2009, Moraga-Gonzalez and Wildenbeest 2012). As a consequence, these papers exhibit pure-strategy price equilibria and therefore do not share our mixed-strategy focus.

The paper now proceeds as outlined above. All omitted proofs are contained in Appendix A.

2 Model

Let there be a finite number of symmetric firms, $i = 1, \dots, n$ with $n \geq 2$, and a unit mass of risk-neutral consumers. Building on the notation of Armstrong and Vickers (2001) ('AV' from this point forward), suppose firm i chooses to provide a utility offer (net of associated payments), $u_i \in \mathfrak{R}$. However, in contrast to AV, let consumers have identical preferences in the sense that all consumers value firm i 's offering at exactly u_i . For any given firm with utility offer, u , define the resulting maximum profit per consumer as $\pi(u)$. The source of utility and the associated profit function will depend upon demand, product, cost and other factors, but to maintain generality, we are deliberately agnostic about their exact specification until we consider some applications later in the paper. However, as a simple illustrative example, let each firm sell a single good with marginal cost c and suppose consumers have unit demands with a willingness to pay of V . If firm i sets a price p_i , firm i 's utility offer is simply the difference between the product value and its price, $u_i = V - p_i$, while its profits per consumer derive from the total available surplus from the transaction minus what it gives away as utility, $\pi(u_i) = V - c - u_i$. Other later examples will include multiple products and downward-sloping demand, or the possibility of two-part tariffs. However, with some further minor assumptions about the form of $\pi(u)$ as detailed below, we always maintain AV's assumption that $\pi(u)$ is independent of the number of consumers served.

In further contrast to AV, consumers are initially uninformed about firms' utility offers. Each firm can choose whether or not to inform consumers of its offer through informative advertising under the following assumptions. First, as consistent with all previous clearing-house models, any advertisement is assumed to reach the whole consumer population. Second, following the 'advertising' version of the clearinghouse model (Baye et al 2004, 2006) we as-

sume that the advertising cost is exogenous, as consistent with the existence of a competitive advertising industry with constant returns to scale. Third, for presentational convenience, advertising costs are assumed to be strictly positive, $A > 0$. Given that our equilibrium will be continuous in A , we later consider costless advertising in a set-up akin to Varian (1980) by taking the limit, $A \rightarrow 0$. Now, by defining firm i 's advertising decision as $a_i \in \{0, 1\}$, and further denoting the chosen utility offers and advertising decisions of all firms $j \neq i$ as the respective $(n - 1)$ -dimensional vectors u_{-i} and a_{-i} , we then describe the total proportion of consumers that buy from firm i as $q(u_i, u_{-i}; a_i, a_{-i})$, together with firm i 's total profits, $\Pi_i = \pi(u_i)q(u_i, u_{-i}; a_i, a_{-i}) - a_i A$.

After having received any advert(s), each consumer must ‘visit’ an advertising firm if they wish to purchase from that firm. However, each consumer can still learn a non-advertising firm’s utility offer and potentially purchase from that firm if the consumer ‘searches’ the firm. While we later provide an extension to a more general setting, the main analysis makes the following strong assumption to simplify exposition: the costs of any first visit (with an advert) or search (without an advert) are zero, $s_1 = 0$, while the costs of making any second search/visit to another firm are prohibitively expensive, $s_k = \infty$ for $k \geq 2$. Hence, consumers can only take one of three actions; make a visit to an advertising firm to buy from its known utility offer, search a non-advertising firm to discover its utility offer and potentially buy, or exit the market.⁷

Consumers are potentially decomposed into two types, $t \in \{L, S\}$, with proportions, θ and $(1 - \theta)$ respectively, where $\theta \geq 0$. ‘Loyal’ consumers, $t = L$, only ever consider buying from their designated ‘local’ firm. Regardless of firms’ advertising decisions, these consumers simply visit/search their firm and buy according to their underlying demand function if the firm offers non-negative utility. Each firm has a symmetric share of loyal consumers, θ/n . The remaining ‘shopper’ consumers, $t = S$, have no such loyalty. Instead, they compare any advertised offers with the utility they expect at any non-advertising firms, and then visit/search the firm with the highest expected utility offer and buy according to their demand function if the firm offers non-negative utility. In the event of a tie where the shoppers are

⁷Specifically, Section 7 shows how these assumptions can be relaxed to allow for any $s_1 \geq 0$ and $s_k > 0$ under a remaining assumption of one-stop shopping such that consumers can only ever buy from one firm.

indifferent between some set of firms, we assume that the shoppers simply randomise between the tied firms with equal probability.⁸

We analyse the following game. In Stage 1, each firm simultaneously chooses how much utility to offer, $u_i \in \mathfrak{R}$, and its advertising strategy, $a_i \in \{0, 1\}$. To allow for mixed strategies, define $\alpha_i \in [0, 1]$ as firm i 's probability of advertising, together with $F_i^A(u)$ and $F_i^N(u)$ as firm i 's utility distribution when advertising and not advertising, respectively. However, as later demonstrated, the advertised utility support, $[\underline{u}^A, \bar{u}^A]$, and the non-advertised utility support, $[\underline{u}^N, \bar{u}^N]$, never overlap in equilibrium as $\underline{u}^A > \bar{u}^N$. Therefore, to ease exposition we later refer to firm i 's utility distribution *unconditional on advertising*, $F_i(u)$, on $[\underline{u}, \bar{u}]$, where $F(\bar{u}^N) = 1 - \alpha$. In Stage 2, consumers observe any advertisements and then make their visit and purchase decisions in accordance with the strategies outlined above.

We consider symmetric equilibria where all players hold correct beliefs, and where, given all other players' strategies, firms select their utility and advertising strategy optimally, and consumers have no incentive to change their visit and purchase decisions.

Finally, we assume i) that $\pi(u)$ is continuously differentiable and strictly quasi-concave in u with a unique maximiser at $u^m \geq 0$, and ii) that each firm's monopoly profits per consumer are strictly positive, $\pi(u^m) > 0$.⁹ Further, by quasi-concavity, $\pi(u)$ is strictly decreasing in $u > u^m$, and denote \hat{u} as the unique maximum utility offer that each firm can make while breaking even, such that $\pi(\hat{u}) < 0$ for all $u > \hat{u}$. It then follows that $\pi(u^m) > \pi(\hat{u}) = 0$ and

⁸This tie-breaking rule contrasts to the previous clearinghouse literature where instead shoppers are assumed to only trade with advertising firms in the case where the lowest advertised price is the same as that expected at non-advertising firms. This is correctly justified in the 'gatekeeper' version of the model (Baye and Morgan 2001) where shoppers optimally visit the clearinghouse first to observe listed firms prices before then deciding whether or not to incur a positive cost to visit a non-advertising firm. However, our random tie-breaking rule seems more natural in the 'advertising' version of the model because shoppers receive all adverts before making any search/visit decisions and so remain indifferent between any firms with the same expected price. Further, the use of this rule serves to simplify exposition while generating no difference in the equilibrium other than changing the advertised utility support from $[u^m, \bar{u}]$ to $(u^m, \bar{u}]$.

⁹For i), note that consumer's optimal behavior is contained in $\pi(u)$. Thus, for example, in the unit demand example above, $\pi(u) = 0$ for $u < 0$, and so profits are maximized at $u^m = 0$. For ii), as later formalised, note that $\pi(u^m) > 0$ is a necessary condition for sales behaviour because firms would never be willing to select any other $u \neq u^m$ if $\pi(u^m) = 0$.

$$\hat{u} > u^m \geq 0.$$

3 Equilibrium Analysis

To derive the game's symmetric equilibria, we proceed in a series of steps. First, note that any firm that chooses not to advertise will optimally set the monopoly utility level, u^m , with probability one, such that $\underline{u}^N = \bar{u}^N = u^m$. This follows from the logic of the previous literature. Specifically, under our simplifying assumption that shoppers can only visit/search one firm, it follows that i) each firm has monopoly power over its θ/n share of loyal consumers, and ii) if any shoppers choose to search a firm with expectations at or above u^m , then that firm would prefer to offer u^m as consumers cannot purchase elsewhere. Hence, the only unadvertised offer that can be consistent with expectations in equilibrium is u^m .

Next, under our random tie-breaking rule, a firm's advertising decision has no impact on consumer behavior in the event of a tie and yet has positive cost, $A > 0$. It then follows that advertising an offer of u^m will be strictly dominated by selecting u^m and not advertising. Hence, firms will only ever advertise with an offer $u > u^m$. This implies that $\underline{u}^A > \bar{u}^N = u^m$ and so from this point forward we can refer to firms' utility distributions unconditional on advertising, $F(u)$ on $[u^m, \bar{u}]$, as indicated earlier.

Now, by setting $u = u^m$ and not advertising, any given firm i will never be able to win the shoppers outright. Instead, it can only possibly trade with the shoppers if all the other $(n-1)$ firms $j \neq i$ also choose not to advertise, which occurs with probability, $(1-\alpha)^{n-1}$. In that case, the shoppers expect all firms to offer $u = u^m$ and so they search a firm at random such that firm i obtains an expected share, $(1-\theta)/n$. Consequently, when combined with the profits from firm i 's loyal consumers, firm i can always guarantee the following profits by not advertising for any given advertising probability, α ,

$$\pi(u^m) \left[\frac{\theta}{n} + (1-\alpha)^{n-1} \frac{(1-\theta)}{n} \right] \tag{1}$$

As formalised below, the unique symmetric equilibrium of the game will take one of two forms, depending upon the level of advertising costs. Given $A > 0$, there can be no equilibrium with $\alpha = 1$ as a firm would always prefer to deviate to avoid the advertising

cost. However, one form of equilibrium can exist where no firm advertises if advertising costs are sufficiently large, while for all lower advertising costs, another form of equilibrium exists where all firms advertise with a positive interior probability, $\alpha \in (0, 1)$.

We first consider the latter form of equilibrium. Suppose (as later verified) that $\alpha \in (0, 1)$. It is then straightforward to show that no equilibrium exists with pure utility strategies, and that instead, when any firm advertises, it mixes by selecting a utility from a common interval, $(u^m, \bar{u}]$. As the proof follows standard results, we simply state the following.

Lemma 1. *In the symmetric mixed strategy form of equilibrium, whenever a firm advertises, it randomises its utility offer from a common interval $(u^m, \bar{u}]$ without gaps or point masses.*

For the firms to be willing to mix over advertising in this way, we first require the profits from not advertising with offer $u = u^m$, (1), to equal the profits from advertising an offer slightly higher than u^m , where for a cost of A firm i wins all the $(1 - \theta)$ shoppers with the probability that all other firms do not advertise, $(1 - \alpha)^{n-1}$. Hence, we require

$$\pi(u^m) \left[\frac{\theta}{n} + (1 - \alpha)^{n-1} \frac{(1 - \theta)}{n} \right] = \pi(u^m) \left[\frac{\theta}{n} + (1 - \alpha)^{n-1} (1 - \theta) \right] - A \quad (2)$$

From this, one can find the equilibrium advertising probability

$$\alpha = 1 - \left(\frac{\frac{n}{n-1} A}{(1 - \theta) \pi(u^m)} \right)^{\frac{1}{n-1}} \quad (3)$$

For $\alpha \in (0, 1)$, it must be that each firm earns the equilibrium profits, $\bar{\pi}$, regardless of whether or not it advertises. To obtain an expression for such equilibrium profits, we can then substitute α from (3) into the expected profits from not advertising in (1) to give

$$\bar{\pi} = \frac{\theta}{n} \pi(u^m) + \frac{A}{n-1}. \quad (4)$$

Next, we derive the equilibrium utility distribution, $F(u)$. By advertising any offer within the equilibrium support $u \in (u^m, \bar{u}]$, firm i will gain expected profits of

$$\pi(u) \left[\frac{\theta}{n} + (1 - \theta) F(u)^{n-1} \right] - A. \quad (5)$$

This follows as firm i will always collect profits from its θ/n loyalists, and also wins the profits of the $(1 - \theta)$ shoppers with the probability that no other firm advertises a higher utility offer, $F(u)^{n-1}$. For each firm to be indifferent over the support, we then require these expected profits to be equal to the equilibrium profits, $\bar{\pi}$ in (4), for each u . As there are no mass points within the support, this requires the following for all $u \in (u^m, \bar{u}]$

$$\bar{\pi} \equiv \frac{\theta}{n}\pi(u^m) + \frac{A}{n-1} = \pi(u) \left[\frac{\theta}{n} + (1 - \theta)F(u)^{n-1} \right] - A \quad (6)$$

The equilibrium utility distribution can then be derived as

$$F(u) = \left(\frac{\bar{\pi} + A - \frac{\theta}{n}\pi(u)}{(1 - \theta)\pi(u)} \right)^{\frac{1}{n-1}} = \left(\frac{\frac{\theta}{n}[\pi(u^m) - \pi(u)] + \frac{n}{n-1}A}{(1 - \theta)\pi(u)} \right)^{\frac{1}{n-1}} \quad (7)$$

As required, the probability that a firm selects u^m is equal to the probability it does not advertise, $F(u^m) = 1 - \alpha$. This is illustrated in Figure 1 below for some example parameters.

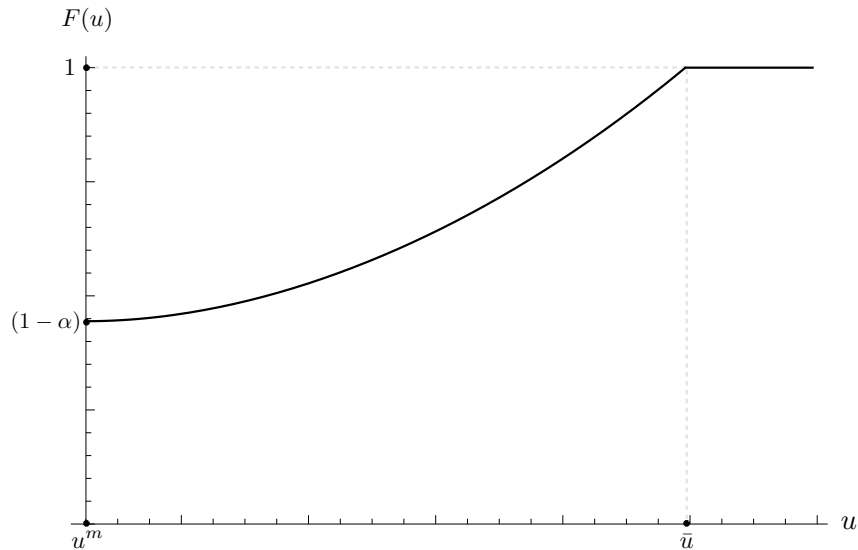


Figure 1: Example equilibrium utility distribution, $F(u)$ on $[u^m, \bar{u}]$

To complete the derivation, it remains to establish the upper utility bound, \bar{u} . Recall that there is no point mass at \bar{u} , and so a firm that advertises \bar{u} will definitely win all the $(1 - \theta)$ shoppers in addition to its θ/n loyalists. Hence, in equilibrium, it must be that

$$\bar{\pi} = \pi(\bar{u}) \left[\frac{\theta}{n} + (1 - \theta) \right] - A. \quad (8)$$

After expanding and rearranging, this gives an expression for \bar{u} in terms of $\pi(\bar{u})$,

$$\frac{\left(\frac{\theta}{n}\right)\pi(u^m) + \left(\frac{n}{n-1}\right)A}{\frac{\theta}{n} + (1 - \theta)} = \pi(\bar{u}) \quad (9)$$

The existence of a unique solution for \bar{u} is guaranteed if A is sufficiently low as the LHS of (9) is strictly increasing in \bar{u} and A . In equilibrium, we require $\bar{u} > u^m$, and so this implies an upper bound on A which can be found by substituting u^m for \bar{u} in (9) and rearranging:

$$A < \frac{n-1}{n}(1 - \theta)\pi(u^m) \quad (10)$$

If this condition is not met, such a mixed strategy form of equilibrium cannot exist. Instead, the alternative form of equilibrium exists where the firms engage in a pure-strategy equilibrium with no advertising and $u = u^m$. There, each firm earns $\pi(u^m)/n$ and advertising costs are sufficiently high to discourage any firm from advertising a slightly higher utility to win all the shoppers and collect $\pi(u^m)\left[\frac{\theta}{n} + (1 - \theta)\right] - A$.

We are now able to fully characterise the symmetric equilibria:

Proposition 1. *For any given $A > 0$, the unique symmetric equilibrium takes one of two forms, depending upon the level of advertising costs:*

1. *If $A \in (0, \frac{n-1}{n}(1 - \theta)\pi(u^m))$, each firm offers $u = u^m$ and does not advertise with probability $(1 - \alpha) \in (0, 1)$ according to (3), and advertises an offer u from the interval $(u^m, \bar{u}]$ according to (7) with probability α , where \bar{u} solves (9).*
2. *If $A \geq \frac{n-1}{n}(1 - \theta)\pi(u^m)$, each firm offers $u = u^m$ and never advertises.*

From this point forward, the paper will focus on the more interesting ‘sales’ equilibrium by assuming $A < \frac{n-1}{n}(1 - \theta)\pi(u^m)$.

4 Example Applications

This section provides a number of example applications by specifying the exact determinants of each firm's utility offer, u , and the associated profits per consumer, $\pi(u)$. As well as helping to build some foundation results for our later analysis of price discrimination, the findings from this section demonstrate how our framework can i) synthesise and extend many models from the existing literature, and ii) offer characterisations of sales practices in new contexts.

For a given form of u and $\pi(u)$, one can think of firms as facing a two-step decision. As already analysed in Proposition 1, firms first need to decide what level of utility to offer. However, firms may then need to decide *how* to provide this level of utility because, in many cases, firms can provide a given level of utility in a variety of different ways by, say, discounting different products. In equilibrium, firms will prefer the method of utility provision that is most profitable.

With this in mind, we first consider the standard setting where sales are in the form of linear prices and illustrate a range of cases under both unit demand and downward sloping demand. We then examine some new examples where firms engage in sales with variables other than linear prices. In all cases, we assume that consumer preferences are quasi-linear in income. All detailed technical derivations are postponed to Appendix B.

4.1 Linear Price Applications

Suppose each firm sells $K \geq 1$ products, indexed by $k = 1, \dots, K$. Let each firm have a marginal cost of producing product k equal to $c_k \geq 0$, as denoted by the vector $\mathbf{c} = \{c_1, \dots, c_K\}$, and assume there are no fixed costs. Further suppose that firm i selects individual prices for each of its K products, with p_{ik} for product k , as summarised by the vector $\mathbf{p}_i = \{p_{i1}, \dots, p_{iK}\}$.

4.1.1 Unit Demand

Under unit demand, let the maximum willingness to pay for product k equal V_k and assume that $V_k > c_k$. This is without loss of generality because products with $V_k \leq c_k$ would not be sold. Using our notation above, it then follows that $u_i = \sum_{k=1}^K (V_k - p_{ik})$ and

$\pi(u_i) = \sum_{k=1}^K (V_k - c_k) - u_i$. Further, given unit demands, the monopoly price of each product k will equal $p_{ik}^m = V_k$ and so $u^m = 0$ and $\pi(u^m) = \sum_{k=1}^K (V_k - c_k)$.

Without further specification, one cannot determine unique individual product prices for utility levels $u' > u^m$ when firms sell $K > 1$ products as any set of prices such that $u = u'$ will provide the same level of profits. Hence, when $K > 1$, one can only determine the aggregate price at firm i , $\sum_{k=1}^K p_{ik} = \sum_{k=1}^K V_k - u_i$, which we denote as $p^*(u_i)$.

By substituting these notations into the expressions derived in Proposition 1, this particular application of our framework can then provide a simple K -product synthesis of some key papers within the previous literature. First, it nests the (popularised) version of Varian (1980) when $A \rightarrow 0$.¹⁰ Second, it nests a version of the ‘advertising’ version of the clearinghouse model (Baye et al 2004, 2006).

4.1.2 Downward-Sloping Demand

Now suppose that each consumer has downward-sloping demand functions for the K products and that these are potentially interdependent across products, as denoted by the demand vector $\mathbf{q}(\mathbf{p}_i) = \{q_1(\mathbf{p}_i), \dots, q_K(\mathbf{p}_i)\}$. For given demand and price vectors, the utility offer available at firm i is then defined as $u_i = S(\mathbf{p}_i)$, where $S(\mathbf{p}_i)$ denotes the surplus available at firm i where $\partial S(\mathbf{p}_i)/\partial p_i = -q_i$.

In terms of prices, each firm’s profits per consumer equal $\pi(\mathbf{p}) = \mathbf{q}(\mathbf{p})'(\mathbf{p} - \mathbf{c})$. In a monopoly case, a firm’s prices equal $\mathbf{p}^m = \operatorname{argmax}_{\mathbf{p}} \pi(\mathbf{p})$ with an implied monopoly utility offer, $u^m = S(\mathbf{p}^m)$, and associated monopoly profits per consumer of $\pi(u^m) \equiv \pi(\mathbf{p}^m)$. More generally, to provide a given utility offer $u' > u^m$, unlike the unit demand case, a firm will now have a unique set of profit maximising individual product prices (under suitable demand assumptions). Indeed, under suitable regularity conditions, there can exist a unique price vector that maximises profits subject to offering $u = u'$ such that $\mathbf{p}^*(u) = \operatorname{argmax}_{\mathbf{p}} \pi(\mathbf{p})$ subject to $S(\mathbf{p}) = u$, with resulting profits per consumer, $\pi(u) \equiv \pi(\mathbf{p}^*(u))$.¹¹

¹⁰The popularised version of the Varian model as summarised by Baye et al (2006) abstracts from some complications within the original model such as costs that depend on the number of consumers served.

¹¹This constrained pricing decision can be thought of as a Ramsey problem. Individual prices can be hard to fully characterise, but with additional restrictions, firms can be shown to optimally reduce prices on products that are more price-elastic and complementary to other products. See Bliss (1988), Simester (1997)

By substituting the above notations into the expressions derived in Proposition 1, this application can nest, and provide generalisations of, some key papers within the previous literature. First, it generalises a version of the ‘advertising’ version of the clearinghouse model (Baye et al 2004, 2006) to allow for multiple products with downward-sloping, inter-related demand. Second, it nests a version of Simester’s (1997) analysis of multi-product sales with zero advertising costs, when $A \rightarrow 0$, and then generalises it to allow for any level of advertising costs $A > 0$.¹²

4.2 New Applications

Contrary to much of the existing literature, sales behavior often extends beyond the use of linear prices. By making some different specifications to utility and firms’ profits, this subsection now demonstrates how our framework can characterise such wider sales behavior.

4.2.1 Sales with Two-Part Tariffs

Suppose firms employ two-part tariffs and conduct sales, as consistent with the markets for energy, mobile phones, telecommunication and broadband services among many others. Little is known about such sales but to see how our framework can provide a characterisation, let us build on the last section where each firm sells K products with downward sloping demand functions. Let firm i now set a K -dimensional vector of marginal prices (per unit of consumption), \mathbf{p}_i , together with a single fixed fee, $f_i \geq 0$. In terms of \mathbf{p}_i and f_i , firm i ’s per consumer profits then equal $\pi(\mathbf{p}_i, f_i) = \mathbf{q}(\mathbf{p}_i)'(\mathbf{p}_i - \mathbf{c}) + f_i$. By now denoting $S(\mathbf{p}_i)$ as a consumer’s surplus at firm i *gross of firm i ’s fixed fee* it then follows that $u_i = S(\mathbf{p}_i) - f_i$. To generate any given utility, u' , firm i will choose \mathbf{p}_i and f_i to maximise $\pi(\mathbf{p}_i, f_i)$ subject to $S(\mathbf{p}_i) - f_i = u'$. This allows us to state the following.

and AV for more discussion.

¹²Simester’s model also allows loyal and shopper consumers to have different demand functions and so our claims above also require the additional restriction of a common demand function. However, Section 7 later discusses how our framework can be extended to allow for such demand heterogeneity.

Lemma 2. *To provide a given level of utility, $u = u'$, firm i will set each marginal price equal to its marginal cost, $\mathbf{p}_i = \mathbf{c}$, and select a fixed fee, $f_i = S(\mathbf{c}) - u'$. This implies that $\pi(u') = S(\mathbf{c}) - u'$, $u^m = 0$ and $\pi(u^m) = S(\mathbf{c})$.*

This result follows standard logic. To provide a utility, u' , most profitably, firm i should maximise surplus by using marginal cost pricing and then extract all such surplus apart from u' through the manipulation of its fixed fee. In the monopoly case, firm i can use its fixed fee to extract all of the surplus because the consumer only has a zero outside option.

Having established these results, one can now apply Proposition 1 to show how our framework can provide a theoretical characterization of firms' optimal promotional behavior under two-part tariffs. Equilibrium sales behaviour involves marginal cost pricing at all times, with firms mixing between not advertising with a high 'regular' fixed fee, $f = S(\mathbf{c})$, and advertising a stochastic lower fixed fee.

Existing work on such sales behaviour is very limited. Theoretically, we know of only Hendel et al (2014) who find that sales behaviour emerges when firms use non-linear pricing in a dynamic context when consumers are able to store units for future consumption. Among other results, they find that sales are more relevant for larger units. In contrast, our sales behaviour derives from consumer loyalty and/or the costs of informative advertising. Empirically, there is very little formal evidence on two-part tariff sales, but our results seem anecdotally consistent with some examples. For instance, at the time of writing, many of the major UK suppliers of broadband, landline and TV services provide sales on their packages by offering reduced monthly fees but unchanged prices for charged telephone calls.¹³ More formally, Giulietti et al (forthcoming) cannot reject the hypothesis that firms are playing mixed strategies in regard to the implied 'final bill' for the average consumer within the British electricity market where suppliers often employ two-part tariffs. However, the literature appears silent over how firms actually vary their marginal price and fixed fee elements in a sales context. Future work on two-part tariff sales competition would appear valuable.

¹³See virginmedia.com, bt.com, and talktalk.com; accessed 18/07/14.

4.2.2 Non-Price Sales

Other common forms of sales that do not use linear prices involve firms holding prices constant but engaging in sales by using some non-price variable. One such type of sale involves the use of ‘bonus packs’ where firms provide temporary extensions to package size with no additional price increase, as consistent with ‘X% Free’ offers. These types of sales are becoming increasingly popular (Chen et al 2012). Alternatively, another form of sale involves a temporary increase in product quality or ‘value’. One example is the inclusion of free items with products, commonly known as ‘premiums’, such as toys in breakfast cereals or free drinks at restaurants. Again, this type of sale is increasing in popularity (Palazon and Delgado-Ballester 2009). Other value-increasing sales could include offering temporary consumer finance arrangements (e.g. “Buy now, pay later”), or use prize draws, competitions or charity donations within products.

To briefly illustrate how our framework can provide a useful characterisation of such non-price sales, we focus on a stylised example with single products and unit demand. In line with the motivation above and as further discussed below, suppose that the price of each firm’s product is fixed with $p_i = p > 0$, in the sense that no firm is willing or able to use it as its sales variable. Instead, as consistent with firms varying their package size or product value (either directly or through an upstream manufacturer), now let each firm choose some strategic variable z from the interval $[\underline{z}, \bar{z}]$, where z influences both the consumers’ willingness to pay for its product, $V(z)$, and its marginal (per unit) cost, $c(z)$. Consumer utility and profits per consumer can then be denoted as $u(z) = V(z) - p$ and $\pi(z) = p - c(z)$. As detailed in Appendix B, after making some suitable regularity assumptions, one can then use Proposition 1 to show how our framework can provide a characterisation of non-price sales. As consistent with practice, this shows that equilibrium sales behaviour will involve firms mixing between not advertising with a minimum ‘regular’ package size/product value, and advertising a stochastic offer with an increased package size or product value.

Little is known about equilibrium non-price sales within the existing literature. Research within marketing has taken different directions by i) explaining the existence of non-price sales by arguing that the over-use of price sales can lead to a weaker brand image and a reduced perception of product quality (e.g. Grewal et al 1998 and Darke and Chung 2005),

and ii) empirically analysing their efficacy relative to price discounts (e.g. see Hardesty and Bearden 2003 and Chen et al 2012 for bonus packs, and Palazon and Delgado-Ballester 2009 for premiums). Within economics, there is a small theoretical literature that exhibits firms mixing in quality, as consistent with quality sales. However, most of these models also exhibit mixing in prices in a way that is inconsistent with the non-price sales phenomena detailed above (e.g. Armstrong and Chen 2009, Chioveanu 2012 and Dubovik and Janssen 2012.) However, one notable exception relates to the recent independent work by Armstrong (2014). Within a broad study on the role of ‘savvy’ consumers, he presents an equilibrium that exhibits all firms selecting a single price while mixing in quality. While he does not choose to make the connection, this equilibrium could also be interpreted in terms of non-price sales. Further related theoretical and empirical work seems highly warranted. In particular, a detailed documentation of firms’ actual non-price sales and an empirical test of our theoretical predictions would seem most useful.

5 Comparative Statics

To build some further foundation results for our later analysis of price discrimination, this section briefly provides a number of comparative statics. These results apply across a general range of market settings including our previous example applications.

Importantly, in both this and the next section, we will often consider the effect of some variable on equilibrium utility offers. Such effects may differ between loyal consumers (who receive the expected utility offer at a given firm) and shopper consumers (who receive the expected maximum utility offer across the market). However, to ensure that the direction of any effect is the same for both consumer groups, it is sufficient to examine the effects on the equilibrium utility distribution in terms of first-order stochastic dominance (FOSD). While not technically necessary, it is useful to define this with explicit reference to u^m to aid later results:

Definition 1. *An ‘improvement’ (or ‘reduction’) in utility offers occurs when there is i) a weak increase (decrease) in u^m , and ii) a weak decrease (increase) in $F(u)$ for all $u \in [u^m, \bar{u}]$, and a strict decrease (increase) in $F(u)$ for some $u \in [u^m, \bar{u}]$.*

Now consider the marginal effects from an increase in either of the two sources of sales activity; advertising costs, A , or the proportion of loyal consumers, θ .

Proposition 2. *A marginal increase in advertising costs, A , or the proportion of loyal consumers, θ , leads to a lower advertising probability, α , higher equilibrium profits, $\bar{\pi}$, and reduced utility offers.*

These results echo past findings in the literature but substantially extend their generality within our broader framework. Intuitively, higher advertising costs or a larger proportion of loyal consumers both discourage firms from advertising higher utility offers. This softens competition, raises firms' equilibrium profits, and reduces consumer utility.

The next related result considers the 'competitive limit' equilibrium where firms are unconstrained by advertising costs and where consumers show no firm loyalty, with $A, \theta \rightarrow 0$.

Proposition 3. *When advertising costs and the proportion of loyal consumers both tend to zero, with $A, \theta \rightarrow 0$, the equilibrium advertising probability, $\alpha \rightarrow 1$, equilibrium profits, $\bar{\pi} \rightarrow 0$, the utility distribution, $F(u) \rightarrow 0$ for all $u \in (u^m, \bar{u})$, and the upper utility bound tends to the break-even utility level, $\bar{u} \rightarrow \hat{u}$, such that utility offers tend to the break-even utility level, $u \rightarrow \hat{u}$.*

When the two sources of sales activity vanish to zero, firms converge to a competitive pure-strategy equilibrium at the break-even level of utility, \hat{u} . Intuitively, when $A \rightarrow 0$, firms must advertise with a probability tending to one in order to maintain indifference between advertising and not advertising as shown in (2). Hence, consumers are close to being fully informed in equilibrium. Further, when $\theta \rightarrow 0$, no consumers are loyal. Together, these facts imply that the firms face some strong incentives to simply over-cut their rivals' offers until the point where utility reaches the break-even level, \hat{u} , with $\pi(\hat{u}) = 0$.

Finally, at this stage, we choose not to report any further comparative statics in regard to a change in the profit function, $\pi(u)$, or the number of firms, n . The effects from a change in the profit function are relatively complex and context dependent and so we defer these to the next section where we consider a price discrimination setting. However, as shown, the section's results can be interpreted more widely in the context of other factors related to the

profit function, such as production costs. The effects from a change in the number of firms are not typically reported in the clearinghouse literature as one must further specify (perhaps arbitrarily) how the change also affects advertising costs, market size, and the relationship between market size on advertising costs. Hence, we omit such results here, but briefly return to discuss them further in the context of endogenous advertising costs in Section 7.

6 Intra-Personal Price Discrimination in Sales Markets

We are now ready to use the flexibility of our framework to study the welfare effects of intra-personal price discrimination when firms compete in sales. Intra-personal price discrimination covers many possible strategies including the use of two-part tariffs, as studied in Section 4.2.1. Indeed, within their original pure-strategy framework, AV analyse such discrimination under a wide definition which views it as being consistent with any increase in pricing flexibility such that firms can earn higher profits per consumer. After defining a profit function under no-discrimination as $\pi_N(u)$ and a profit function under discrimination as $\pi_D(u)$, they state that such discrimination is consistent with $\pi_D(u) \geq \pi_N(u)$ where the inequality is almost always strict apart from cases such as the break-even utility \hat{u} , where the inequality can be weak to allow for the possibility that $\pi_N(u) = \pi_D(u) = 0$. Among other examples, they suggest that this definition encompasses the transition from linear pricing to two-part tariffs, the easing of uniform pricing constraints across products and the removal of a ban on loss-leaders. In addition, broader interpretations could also include the relaxation of pricing restrictions that limit the content or size of firms' product ranges.

As further detailed below, AV show that intra-personal price discrimination tends to increase consumer welfare in competitive markets. However, in less competitive markets, they present a sufficient condition for discrimination to reduce consumer surplus; $\log \pi_D(u) - \log \pi_N(u)$ decreasing in u , which implies $\frac{\pi_D(u)}{\pi_N(u)}$ decreasing in u , such that the benefits from engaging in price discrimination are higher for lower values of u (Lemma 4). Intuitively, discrimination gives firms the ability to supply higher utility while still breaking even in competitive markets, but allows firms to extract more surplus and thus give lower utility to consumers in less competitive markets.

While these seminal results have proved very instructive, they cannot be applied to the commonly observed mixed strategy setting of sales competition. As we shall see, the consideration of intra-personal price discrimination under mixed strategies introduces further technical challenges and new effects. For instance, within AV's pure-strategy setting, a change in $\pi(u)$ only affects the equilibrium level of u at the margin, while in the case of mixed strategies, a change in $\pi(u)$ affects the equilibrium at each level of u within the sales support, $[u^m, \bar{u}]$. Hence, to proceed, we find it useful to use a continuous, parameterised version of AV's definition of intra-personal price discrimination. Specifically, after indexing the per consumer profit function by a 'discrimination parameter' d , $\pi(u; d)$, we denote an improved ability for firms to conduct such discrimination by an increase in d such that i) $\pi_d(u; d) > 0$ for all $u \in [u^m, \hat{u})$, and ii) $\pi_d(u; d) \geq 0$ for $u = \hat{u}$. As equivalent to AV, this stipulates that $\pi(u; d)$ must be strictly increasing in the discrimination parameter for all utilities within the relevant support apart from the break-even utility level, \hat{u} , where instead, $\pi(u; d)$ need only be weakly increasing.

6.1 The Effects on Firms

First, we consider the effects of an increase in discrimination, d , on equilibrium profits, $\bar{\pi}$. These effects (and the later effects on consumers) are potentially complicated by the fact that a change in the ability to discriminate may also affect the monopoly level of utility, which we now denote as $u^m(d)$. Hence, from (4), we re-express $\bar{\pi} = (\theta/n)\pi(u^m(d); d) + \frac{A}{n-1}$. However, with the use of an envelope-style argument, it still follows that per consumer monopoly profits are increasing in the level of discrimination, $\pi_d(u^m(d); d) > 0$. By then noting that $\frac{\partial \bar{\pi}}{\partial d} = \frac{\theta}{n}\pi_d(u^m(d); d)$, we can state:

Proposition 4. *An increase in discrimination raises equilibrium profits if and only if there is positive proportion of loyal consumers, $\theta > 0$.*

Firms only benefit from discrimination if there is positive proportion of loyal consumers. This immediately highlights a difference between the two sources of sales activity relating to loyal consumers, θ , and advertising costs, A . If $\theta > 0$, then equilibrium profits have a component deriving from the monopoly custom of loyal consumers, $(\theta/n)\pi(u^m(d); d)$, and so

profits rise in discrimination. If, instead, $\theta = 0$, such that sales activity derives solely from the existence of advertising costs, then we know from (1) that equilibrium profits only derive from the probability that no firm advertises, $\frac{\pi(u^m(d);d)}{n}(1-\alpha)^{n-1}$. As we know, once the advertising probability is endogenised in equilibrium, (3), these profits reduce to $\bar{\pi} = A/(n-1)$ as indicated above. Hence, in this case, an increase in discrimination has no effect because any change in the profit function is neutralised through a corresponding change in the endogenous advertising probability.

6.2 The Effects on Consumers

To begin, we consider the effects at the competitive benchmark where advertising costs and consumer loyalty vanish to zero, with $A, \theta \rightarrow 0$. Here, from Proposition 3, we know that firms advertise an offer equal to the break-even utility, \hat{u} , and earn zero equilibrium profits. Therefore, regardless of any effects on $u^m(d)$, discrimination has no effect on firm profits but weakly improves consumer utility because $\partial \hat{u} / \partial d$ is non-negative from the definition of \hat{u} and from our assumption, $\pi_d(\hat{u}; d) \geq 0$. Thus, we can state the following

Proposition 5. *When $A, \theta \rightarrow 0$, an increase in discrimination leads to a (weak) improvement in utility offers.*

This echoes AV's findings where they consider some competitive limit results within their pure-strategy framework. In our framework, when $A, \theta \rightarrow 0$, the firms refrain from conducting sales. Hence, the two frameworks become quite similar - the firms offer a break-even utility in the limit and so an improvement in the ability to profitably offer utility results in a weak improvement in consumer welfare under either framework.

We now move away from this simple case to consider the more substantial setting of less competitive sales markets where there are positive levels of advertising costs, $A > 0$, and/or consumer loyalty, $\theta > 0$. To begin, we initially leave aside any potential effects from discrimination on u^m , and focus on the effects on the utility distribution, $F(u)$, for $u \in (u^m, \bar{u}]$. To build some intuition, it is useful to recall the indifference equation in (6) which required equilibrium profits to equal expected profits for all $u \in (u^m, \bar{u}]$:

$$\bar{\pi} \equiv \frac{\theta}{n}\pi(u^m(d); d) + \frac{A}{n-1} = \pi(u; d) \left[\frac{\theta}{n} + (1-\theta)F(u)^{n-1} \right] - A \quad (11)$$

Here, we can see that an increase in discrimination can provide conflicting effects. On the one hand, from the LHS, it can raise the equilibrium profits which derive from offering u^m to the loyal consumers and not advertising, $\pi(u^m(d); d)$. Hence, an increase in discrimination may make firms more inclined to select lower utilities and refrain from advertising. On the other hand, from the RHS, it can also raise the expected profits from advertising higher utility levels to win the shoppers, related to $\pi(u; d)$, and so it may prompt firms to select higher utilities and advertise more.

By differentiating the resulting equilibrium expression for $F(u)$ from (7), these two effects combine, to give

$$\frac{\partial F(u)}{\partial d} = \frac{F(u)^{2-n}}{n-1} \cdot \frac{\frac{\theta}{n} \left[\pi(u; d) \frac{\partial \pi(u^m(d); d)}{\partial d} - \pi(u^m(d); d) \frac{\partial \pi(u; d)}{\partial d} \right] - \frac{n}{n-1} A \frac{\partial \pi(u; d)}{\partial d}}{(1-\theta)\pi(u; d)^2}. \quad (12)$$

With this foundation, we now consider some different cases before presenting a more general result. First, we consider the case where where $A \rightarrow 0$ but $\theta > 0$ such that sales derive solely from the presence of loyal consumers. In this case, despite the existence of mixed strategies, we can provide a result that is highly connected to AV's original pure-strategy findings. From (12), it now follows that $\partial F(u)/\partial d$ will be strictly positive (negative) for all $u \in (u^m, \bar{u}]$ when $\frac{\pi_d(u^m(d); d)}{\pi(u^m(d); d)}$ is larger (smaller) than $\frac{\pi_d(u; d)}{\pi(u; d)}$ for all $u \in (u^m, \bar{u}]$. Intuitively, this condition dictates whether or not the effect from an increase in discrimination on equilibrium profits dominates the effect on expected profits and so determines whether or not firms must select 'higher' or 'lower' utilities within $F(u)$ to maintain the indifference condition, (11). However, to consider the effects from discrimination in terms of FOSD, one must also consider the effects on u^m .¹⁴ As verified in the associated proof, we know that $u_d^m < (>)0$ is necessarily implied under the condition $\frac{\pi_d(u^m(d); d)}{\pi(u^m(d); d)} > (<) \frac{\pi_d(u; d)}{\pi(u; d)}$. Hence, we can state:

¹⁴Note that we do not need to consider the effects on the other bound, \bar{u} , because it always moves in the same direction as the rest of the distribution. Namely, if (12) is negative (positive) for all u , then \bar{u} increases (decreases) in d .

Proposition 6. *Let A be sufficiently small and $\theta > 0$. Suppose $\frac{\pi_d(u;d)}{\pi(u;d)}$ is decreasing (increasing) in u . Then an increase in discrimination leads to a reduction (improvement) in utility offers.*

The utility reduction part of this result states that if the percentage change in profit due to discrimination is larger for smaller utility levels, then an increase in discrimination will reduce utility offers in the sense of FOSD. When translated into discrete changes in discrimination levels, this decreasing $\frac{\pi_d(u;d)}{\pi(u;d)}$ condition is simply an equivalent continuous version of AV's original condition for utility reduction, which implied $\frac{\pi_D(u)}{\pi_N(u)}$ decreasing in u . The reverse utility improvement part of Proposition 6 also has an exact parallel in AV's framework. While they chose not to state this result, it is trivial to use their own proof to demonstrate that when $\frac{\pi_D(u)}{\pi_N(u)}$ is increasing in u , equilibrium utility improves. Hence, for this special case with $A \rightarrow 0$, Proposition 6 provides an exact parallel of AV's findings in a sales context. This similarity is striking given the large differences in contexts and effects between the two frameworks.

However, our mixed strategy framework also exhibits another channel that further differs to AV and which derives from the existence of advertising costs. To understand this effect, we now consider the special case where $\theta = 0$ and $A > 0$ such that sales derive solely from advertising costs. Here, we know from Proposition 4 that an increase in d will have no effect on equilibrium profits. However, from (11), we know that it will still raise the expected profits from advertising higher utilities and so firms will be encouraged to make more generous offers in equilibrium. Specifically, from (12), it is clear that $\partial F(u)/\partial d$ must be negative for all $u \in (u^m, \bar{u}]$ regardless of the form of the profit function. However, for a FOSD improvement, we still require that the monopoly utility is non-decreasing, $u_d^m(d) \geq 0$ which may not always be implied. Hence, we can now state

Proposition 7. *Let θ be sufficiently small and $A > 0$. Suppose $u_d^m \geq 0$. Then an increase in discrimination leads to an improvement in utility offers.*

With these findings in place, we are now ready to provide a general result for the case with both $\theta > 0$ and $A > 0$. From the expression for $F(u)$ in (7), or from $\partial F(u)/\partial d$ in (12), we can see that $\partial F(u)/\partial d$ will be negative for all $u \in (u^m, \bar{u}]$ if d generates an increase in

profits that is relatively uniform across the range of u , so that $\pi_d(u; d)$ is ‘approximately’ the same for all u . Further, we can see that this beneficial effect on utility is strengthened when an increase in discrimination increases profits proportionally more for higher utilities such that $\pi_d(u; d) > \pi_d(u^m(d); d)$ for $u > u^m$. As these restrictions on the profit function also ensure that the monopoly utility is non-decreasing in d , utility offers will necessarily improve in terms of FOSD. Hence,

Proposition 8. *Let $\theta > 0$ and $A > 0$, and suppose $\pi_d(u; d)$ is (weakly) increasing in u , $\pi_{ud}(u; d) \geq 0$. Then an increase in discrimination leads to an improvement in utility offers.*

Proposition 8 presents our most general result about the effects of intra-personal discrimination in sales markets. It states that utility offers will improve under a condition which implies that an increase in discrimination raises profits at higher utility levels by a weakly higher amount than at lower utility levels. As readily seen from (12), this condition ensures that firms offer higher utility levels because we also know that $\pi(u^m; d) > \pi(u; d)$ for all $u > u^m$ and that $A > 0$. Intuitively, if discrimination makes higher utility levels more profitable than lower ones, then firms will tend to provide more generous offers in the market. However, a reverse result in terms of utility reductions need not be true. As illustrated by Proposition 7, there are situations where discrimination can induce firms to offer higher utility even when $\pi_{ud}(u; d) < 0$.

6.3 A Counter-Example to AV with Restricted Two-Part Tariffs

To further highlight the differences between the effects of discrimination in a sales context relative to the pure-strategy case, we now examine an example setting where an increase in discrimination yields higher utility in our framework but lower utility in AV’s framework.

In constructing such an example, we exploit the fact that when θ is small, utilities are improved in our framework when $u_d^m \geq 0$. However, in AV, utilities are reduced under the (equivalent) condition that $\frac{\pi_d(u; d)}{\pi(u; d)}$ is decreasing in u , which further implies that $u_d^m \leq 0$ because discrimination increases profits proportionally more for lower utilities, so that u^m can never increase. Hence, by combining these two inequalities, we can see that any such example where utility improves in our framework but reduces in AV’s, must satisfy $u_d^m = 0$.

To explore this possibility further, we examine discrimination in the form of a transition from linear pricing towards two-part tariffs in a setting that involves what we term as ‘restricted two-part tariffs’. In line with the notation used in Section 4 to study two part tariffs, suppose that firms charge a unit price p and fixed fee f for a single product with demand $q(p)$, such that $u = S(p) - f$. Further, let marginal costs be normalised to zero, $c = 0$, and denote the standard monopoly linear price as p^m . As a conceptual tool, we now suppose that there is some restriction on the level of the fixed fee, such that firms must set $f \leq d$. An increase in d is therefore consistent with an increased ability to conduct intra-personal price discrimination as previously assumed. Suppose d is always binding with $d < S(0)$.¹⁵ After developing some base results as detailed in the associated proof, one can state:

Lemma 3. *Under restricted two-part tariffs, the monopoly level of utility, u^m , equals zero when $d \geq \bar{d}$, and is positive when $d < \bar{d}$, where $\bar{d} = S(p^m)$.*

Intuitively, when the restriction on the fixed fee is sufficiently slack, $d \geq \bar{d}$, the firm can still extract all associated consumer surplus, $S(p)$, such that $u^m = 0$, by setting the highest permitted level of fixed fee, $f = d$, together with some positive price, $p > 0$. However, when the restriction on the fixed fee is tighter, with $d < \bar{d}$, the firm is unable to extract all surplus even if it raises its price to the monopoly level, p^m , as $f < S(p^m) = \bar{d}$. Now, when $d \geq \bar{d}$ such that $u^m = 0$, and $\theta \rightarrow 0$, we can apply Proposition 7 to state:

Proposition 9. *Under restricted two-part tariffs when $d \geq \bar{d}$, $A > 0$ and θ is sufficiently small, an increase in discrimination leads to an improvement in utility offers as $u_d^m = 0$ despite the fact that $\frac{\pi_d(u;d)}{\pi(u;d)}$ may be decreasing in u .*

As detailed in the footnote below, after a suitable specification of the demand function, such as the linear case, one can show the following two conditions can hold: i) our own condition for a utility improvement ($u_d^m = 0$ from Proposition 7), and ii) the equivalent AV pure-strategy condition for a utility reduction ($\frac{\pi_d(u;d)}{\pi(u;d)}$ decreasing in u from our Proposition 6).¹⁶ Hence, the mixed-strategy and pure-strategy results diverge. Intuitively, this difference

¹⁵From Section 4.2.1, when there are no restrictions, we know that for a given u , profit is maximised by setting $p = c = 0$ and $f = S(0) - u$. Therefore, $d < S(0)$ will bind for u sufficiently close to 0.

¹⁶AV’s condition that $\log \pi_D(u) - \log \pi_N(u)$ is decreasing in u , or $\frac{\pi_D(u)}{\pi_N(u)}$ is decreasing in u , is equivalent to

derives from our previous logic. As $\frac{\pi_d(u;d)}{\pi(u;d)}$ is decreasing in u , we should expect an increase in discrimination to reduce utility under pure strategies. However, under mixed strategies, as $\theta \rightarrow 0$, an increase in discrimination has no effect on equilibrium profits yet still raises the expected profits from advertising higher utilities beyond u^m . Therefore, as inconsistent with AV, firms face increased incentives to offer higher utility levels.

6.4 Other Interpretations: Production Costs

Finally, given the flexibility of the formulation in the previous subsections, one can easily consider different interpretations of the discrimination parameter, d , and its effect on firm profits, $\pi(u; d)$. Here, we briefly explore some implications for the case where d relates to a reduction in firms' production costs.

In more detail, let consumers have some weakly decreasing demand function, $q(p)$, to allow for both downward-sloping and unit demand. Further, in contrast to our previous applications, suppose that firms' costs may be nonlinear. In particular, we denote the total costs per-consumer as $C(q, d)$ where q is the level of output purchased by the consumer and where d is a cost parameter. This function can be relatively general so long as it is differentiable in both q and d , increasing in q , and such that $C(0, d) = 0$. Via an increase in d , we then consider the effects of any 'relevant' reduction in costs, where such a reduction is defined *at any interval within the active range of output*, such that $C_d(q, d) \leq 0$ for all q and $C_d(q, d) < 0$ for some q , including all $q \in [0, \varepsilon]$ where $\varepsilon > 0$. This definition ensures that the cost reduction occurs at either the relevant level of output for an (pure-strategy) equilibrium 'at the margin' and/or lower 'infra-marginal' levels of output. We can then state the following.

$\frac{\pi_D(u) - \pi_N(u)}{\pi_N(u)}$ decreasing in u . In a continuous discrimination context, the equivalent counterpart is then $\frac{\pi_d(u;d)}{\pi(u;d)}$ decreasing in u . Under restricted two-part tariffs, $\pi_D(u) = S(0) - u$ and $\pi_N(u) = S^{-1}(d+u)q(S^{-1}(d+u)) + d$ for $u < S(0) + d$ and $\pi_D(u) = \pi_N(u) = S(0) - u$ otherwise. $\frac{\pi_D(u)}{\pi_N(u)}$ is more than 1 for $u < S(0) + d$, and 1 for $u \geq S(0) + d$. So $\frac{\pi_D(u)}{\pi_N(u)}$ is decreasing in u at least for some u , and decreasing for all u when $\frac{1}{1 - \varepsilon(S^{-1}(d+u))} > \frac{\pi_D(u)}{\pi_N(u)}$, where $\varepsilon(p)$ is the price elasticity of demand ($\varepsilon(p) < 1$ for all $d > \bar{d}$). Thus, for $q(p)$ that satisfies the condition on elasticity, e.g. $q(p) = 1 - p$, AV's condition holds. Further, our own condition from Proposition 7 also holds when $d \geq \bar{d}$ as $u_d^m = 0$.

Proposition 10. *When firms use linear prices or two-part tariffs, any ‘relevant’ reduction (increase) in per-consumer costs always results in improved (reduced) utility offers.*

This offers an interesting difference to typical results in pure-strategy models where a cost reduction will only affect consumer surplus if it reduces costs at the relevant level of output for equilibrium (at the margin). In contrast, in a sales context, our result suggests, quite generally, that (both loyal and shopper) consumers will benefit from a cost improvement even if the cost reduction only applies to lower ‘infra-marginal’ levels of output. Intuitively, from (11), any relevant reduction in costs will raise both the incentive to select lower utilities and the incentive to advertise higher utility levels as both $\pi(u^m(d); d)$ and $\pi(u; d)$ will increase. However, when d is interpreted as a cost parameter, it is always the case that $\pi_{ud}(u; d) \geq 0$ and so utility must improve from Proposition 8. To see why $\pi_{ud}(u; d) \geq 0$, note that the effects of a cost reduction on profits, $\pi_d(u; d)$, must be weakly larger for higher utility levels because higher utility levels involve weakly higher output levels and so cover a weakly larger interval of possible cost changes.

7 Robustness and Extensions

While our framework adds significant generality to the existing literature, this final section now considers some remaining limitations and offers a variety of extensions. Due to our framework’s ability to synthesise much previous research, this section can also be used as a guide to the key assumptions within the wider literature. We proceed by discussing the following issues: i) costly first visits/searches, ii) non-prohibitive costs of further visits/searches, iii) multi-stop shopping, iv) heterogeneous preferences, v) endogenous advertising costs, and vi) alternative advertising technologies.

7.1 Costly First Visits/Searches

In the main framework, we assumed that consumers could make a first visit/search for free, with $s_1 = 0$. This ensured that consumers were willing to visit/search a firm even if they expected $u^m = 0$, as in the case of unit demand or two-part tariffs. However, we now show that sales equilibria remain in our model when $s_1 > 0$.

First, suppose $0 < s_1 \leq u^m$. Here, consumers receive positive utility under monopoly, $u^m > 0$, as in the case of downward sloping demand and linear price. Then, as long as the monopoly utility offsets s_1 with $0 < s_1 \leq u^m$, then consumers will still be willing to search/visit and the equilibrium will remain unchanged.

Second, and more generally, now let $s_1 > u^m$. Here, consumers will not be willing to search a firm in equilibrium due to a familiar hold-up problem akin to the Diamond Paradox (1971): if a firm does not commit to a sufficiently high offer by advertising $u \geq s_1$ then both loyal and shopper consumers should rationally expect $u = u^m$ and refuse to search such that the firm earns zero profits. Therefore, now consider a (modified) sales equilibrium where each firm advertises with probability one and selects an advertised utility $u \geq s_1$ from some $F(u)$. Here, a firm can guarantee equilibrium profits, $\bar{\pi} = \pi(s_1)[\theta/n] - A$ by advertising $u = s_1$. To ensure that these profits are positive we assume that i) the proportion of loyal is positive, $\theta > 0$, and ii) first visit/search costs and advertising costs are not too large, such that $\pi(s_1) > [An/\theta]$. One can then characterise a symmetric sales equilibrium with a well-behaved $F(u)$ on $[s_1, \bar{u}]$, where $s_1 < \bar{u}$, by using some similar steps to those used for Proposition 1.¹⁷ In essence, the equilibrium follows a structure related to Varian's (1980) price sales model with the addition of a fixed cost $A > 0$.

7.2 Non-Prohibitive Costs of Further Visits/Searches

To ease exposition, the main analysis made a simplifying assumption that consumers could only visit/search a single firm by ensuring that the costs of visiting/searching any further firms were prohibitively expensive, with $s_k = \infty$ for all $k \geq 2$. We now demonstrate that our sales equilibrium remains under the much weaker assumption that $s_k > 0$ all $k \geq 2$, subject to a persistent assumption of one-stop shopping, such that a consumer cannot buy from more than one firm.

To see this, first consider the main model where $s_1=0$, but now allow $s_k > 0$ all $k \geq 2$. We know that the behaviour of the loyal consumers will remain unchanged as they will only ever consider buying from their local firm. Therefore, to demonstrate that our equilibrium

¹⁷Briefly, i) $F(u)$ can be constructed from $\bar{\pi} = \pi(u)[(\theta/n) + (1 - \theta)F(u)^{n-1}] - A$, and ii) \bar{u} follows from $\bar{\pi} = [(\theta/n) + (1 - \theta)]\pi(\bar{u}) - A$.

remains we need to show that shopper consumers will endogenously refrain from making a second visit/search in equilibrium. Initially suppose that firms keep playing their original equilibrium strategies. Then a shopper that receives $h \in [0, n]$ adverts will either i) visit the advertised firm with the highest utility, u^* , if $h \geq 1$, or ii) search a firm at random to discover an offer, u^m , if $h = 0$. In either case, given the assumption of one-stop shopping, the gains from any second search/visit will always be strictly negative if $s_2 > 0$ because any other advertising firm will necessarily have $u < u^*$ and all non-advertising firms will have $u^m < u^*$. Now suppose that firms can deviate from their original equilibrium strategies. To see that the logic still holds, note that only the behaviour of non-advertising firms is relevant and that such firms are unable to influence second search/visit decisions due to their inability to communicate or commit to any $u < u^m$. Hence, firms' advertising and utility incentives remain unchanged and the original equilibrium still applies.

Finally, consider the case in the previous subsection where the first search/visit is costly, with $s_1 > 0$. When $0 < s_1 \leq u^m$, the original equilibrium remains and our argument from above still applies. Alternatively, when $s_1 > u^m$, firms advertise with probability one and so shoppers are fully informed such that the costs of further visits/searches become irrelevant.

7.3 Multi-Stop Shopping

The main framework assumed that consumers engage in one-stop shopping in the sense that consumers can only purchase from one firm. The extension in the previous subsection also maintained this assumption to allow for different visit/search costs. One-stop shopping may be reasonable in some markets, such as supermarkets and restaurants, and is often used within the wider literature on price discrimination (e.g. AV, and the models reviewed in Stole 2007). However, in practice, one-stop shopping can be a strong assumption within some markets when firms sell multiple products because consumers may be willing and able to buy different goods from different suppliers in order to 'cherry-pick' the best discounts (Fox and Hoch 2005). It remains a significant challenge to allow for multi-stop shopping within our framework and so this appears to be a key assumption within our paper and within the wider clearinghouse literature. Indeed, very few papers on sales (or even consumer search, more generally) consider any such possibility (e.g. McAfee 1995, Shelegia 2012, Rhodes 2013

and Zhou 2014). Future work in this direction is of clear importance.

7.4 Heterogeneous Preferences

The original model assumed that consumers had identical preferences such that all consumers valued a given offer at exactly u . However, our model can be extended to allow for heterogeneous preferences across the two consumer groups. In particular, we can allow loyal consumers and shoppers to value a given offer at u_L and u_S respectively. For clarity, we focus on the case where $u_L \geq u_S$ for all offers, as consistent with loyal consumers having a weakly higher willingness to pay as well as a higher level of firm loyalty. However, the reverse case can also be easily considered.

Now suppose that each firm chooses the level of some sales variable, $r \in [-\infty, \infty]$, and define the utility offer for any given r for type $t \in \{L, S\}$ as $u_t(r)$, where utilities are increasing in r for both types, $u'_t(r) > 0$ for all t , and where $u_L(r) \geq u_S(r)$ for all r . Hence, the variable r can be thought of as (the negative of) price, but alternatively, it could be a non-price variable such as pack size. Further, allow the per-consumer profit obtained from a given level r to vary by consumer type, with $\pi_t(u_t(r))$. This captures the potential impact on firms' profits from the difference in consumer groups' preferences, as well as any potential differences in the costs of supplying each group.

Under suitable regularity conditions to ensure that the monopoly level of r , r^m , is uniquely defined, and that $E(\Pi(r^m)) > 0$ and $u_S(r^m) \geq 0$ such that equilibrium profits are positive and no consumer group is excluded, one can apply a straightforward modification of Proposition 1 to establish the following.

Proposition 11. *If $A \in (0, \frac{n-1}{n}(1-\theta)\pi_S(u_S(r^m)))$, there exists a symmetric sales equilibrium where each firm selects $r = r^m$ and does not advertise with probability $(1-\alpha) \in (0, 1)$, and advertises with a selection, r , from the interval $(r^m, \bar{r}]$ according to $F(r)$ as defined implicitly by (15) with probability α , where*

$$r^m = \operatorname{argmax}_r \frac{\theta}{n} \pi_L(u_L(r)) + \frac{A}{n-1} \cdot \frac{\pi_S(u_S(r))}{\pi_S(u_S(r^m))} \quad (13)$$

$$\alpha = 1 - \left(\frac{\frac{n}{n-1}A}{(1-\theta)\pi_S(u_S(r^m))} \right)^{\frac{1}{n-1}} \quad (14)$$

$$F(u_s(r)) = \left(\frac{\bar{\pi} + A - \frac{\theta}{n}\pi_L(u_L(r))}{(1-\theta)\pi_S(u_S(r))} \right)^{\frac{1}{n-1}} \quad (15)$$

$$\bar{\pi} = \frac{\theta}{n}\pi_L(u_L(r^m)) + \frac{A}{n-1} \quad (16)$$

$$\bar{r} \text{ solves } \bar{\pi} = \pi_L(u_L(\bar{r}))[\theta/n] + \pi_S(u_S(\bar{r}))(1-\theta) - A \quad (17)$$

Compared to the main framework, the resulting mixed strategy equilibrium now balances the differing profits available from the two consumer groups. As noted earlier, Simester (1997) also allows for similar form of between-group heterogeneity in a simplified price setting. In particular, our equilibrium can provide the basis to nest his as a special case when $A \rightarrow 0$, firms sell multiple products with linear prices, and the costs of supplying the two groups are equal.

7.5 Endogenous Advertising Fee

Our framework assumed advertising costs were exogenous as consistent with a competitive advertising industry with constant returns to scale. However, advertising costs can be made endogenous. For instance, suppose that any advertising firm must advertise through a monopoly advertising company which has marginal operating costs that are normalised to zero. The advertising company then selects its advertising fee, A , in order to maximize its profits, $\Pi_A = n\alpha \cdot A$. With the use of (3) this equals

$$\Pi_A = nA \left(1 - \left(\frac{\frac{n}{n-1}A}{(1-\theta)\pi(u^m)} \right)^{\frac{1}{n-1}} \right). \quad (18)$$

Such profits are strictly concave and maximized with an advertising fee,

$$A^* = \left(\frac{n-1}{n} \right)^n \pi(u^m)(1-\theta). \quad (19)$$

The advertising company's chosen level of fee necessarily ensures the existence of the mixed strategy equilibrium by satisfying (10). Moreover, by inserting A^* into the necessary original equilibrium equations, one can derive a set of expressions to describe the new equilibrium with endogenous advertising fees. Among other results, this equilibrium involves an advertising probability, $\alpha = \frac{1}{n}$, a total advertising activity that remains fixed as part of equilibrium, $n\alpha = 1$, and advertising company profits, $\Pi_A = A^*$.

One can now re-consider the effects of an increase in the proportion of loyal consumers. With exogenous A , Proposition (5) showed that this reduced the advertising probability and raised firm profits. Here, an increase in θ now reduces A^* in a way that leaves the advertising probability unchanged. Consequently, this reduces the advertising company's profits, Π_A , but still ensures that firms' equilibrium profits rise. As before, it can also still be shown that consumers suffer from reduced utility offers.

Now consider the effects of an increase in the number of firms. As explained in Section 5, results are generally hard to state here without further (arbitrary) specification of the additional impacts on market size and advertising costs. However, with the use of our endogenous advertising costs, we can now offer the following limit results when $n \rightarrow \infty$ under the assumption of a fixed market size. Here, the advertising fee converges to $A^* \rightarrow \pi(u^m)(1-\theta)$. However, while the aggregate advertising activity remains fixed at one, the probability of an individual firm winning the shoppers now vanishes to zero such that advertising becomes a near-dominated strategy with $\alpha \rightarrow 0$. Consequently, utility offers tend to u^m . At an industry level, the firms' aggregate profits, $n\bar{\pi}$, then tend to $\theta\pi(u^m)$, while the advertising company extracts all the remaining profits that are associated with the shoppers, $\Pi_A \rightarrow \pi(u^m)(1-\theta)$. However, at the individual level, each firm's share of loyal consumers, θ/n , tends to zero and so each firm's equilibrium profits, $\bar{\pi}$, converge to zero. We note that the documented effects on the advertising fee and the total advertising activity differ to existing results in Baye and Morgan (2001) because there, the gatekeeper optimally responds to an increase in n by extracting more rent through consumers' subscription fees rather than firms' advertising

fees.

7.6 Alternative Advertising Technologies

In line with the standard clearinghouse literature, our framework assumed that all consumers received each advertisement such that advertising had a perfect ‘reach’. In the gatekeeper version of the clearinghouse model (Baye and Morgan 2001), this assumption simply implies that all listed firms are observed by consumers that visit the clearinghouse. However, in our framework and within previous advertising versions of the clearinghouse model (Baye et al 2004, 2006), this assumption may seem less reasonable because, in practice, advertising may only reach a fraction of consumers, $R < 1$. This possibility adds substantial complexity to the model. When a consumer does not receive an advert from a firm, the consumer can no longer assume that the firm did not advertise or infer that $u = u^m$ as it could be that $u > u^m$ and the consumer simply did not receive the firm’s advert. In fact, contrary to the one visit/search feature of our framework, after receiving a best known offer that is sufficiently close to u^m , a shopper consumer will now strictly prefer to make a further search to a non-advertising firm because such a firm will have an expected utility offer larger than u^m . Thus our equilibrium does not apply.

Imperfect advertising reach is often considered in the wider, non-clearinghouse literature on informative advertising (e.g. Butters 1977, Robert and Stahl 1993, Stahl 1994 and Janssen and Non 2008). Of particular relevance is Robert and Stahl’s (1993) consumer search model of pricing and advertising, where they let firms choose the reach of their advertising for any given advertised price, $R(p)$, for an advertising cost that is increasing in reach, $A(R)$. After deriving consumers’ optimal reservation search rules and other substantial equilibrium details, they show that firms constrain their prices in equilibrium to ensure that consumers do not find it optimal to make any second search, as consistent with the simplifying assumption in our framework. However, the formal completion of such a substantial extension within our model would take us away from our clearinghouse set-up and remains well beyond the scope of the current paper.

8 Conclusions

By extending Armstrong and Vickers' (2001) seminal model of competition in the utility space into a mixed strategy setting, this paper has provided a simple and tractable framework to analyse sales and advertising behavior. The framework can i) provide a novel analysis of intra-personal price discrimination in sales markets, ii) characterise sales in a range of new contexts including complex market settings and situations where firms conduct sales with two-part tariffs or non-price variables such as package size, and iii) synthesise past research and highlight its key forces and assumptions.

We believe the flexibility of our framework will facilitate the examination of further research questions in the future. For instance, market asymmetries typically present large technical challenges for standard sales models and are therefore currently ill-understood despite their obvious importance. However, our framework may permit a tractable analysis of their effects, as the authors are currently exploring in a separate paper.

Finally, we would also like to further highlight some directions for empirical research. The ability of our model to consider sales in more realistic settings beyond single product markets with unit demand should help empirical work study sales and advertising behaviour in more detail. In addition, despite the common existence of sales in markets with two-part tariffs, and the increasing usage of non-price sales, such as those using bonus packs or temporary increases in product value, there appears to be remarkably little empirical documentation of such activities. Such further work, together with a test of our theoretical characterisation of these forms of sales, would appear fruitful.

Appendix A - Proofs of Propositions

Proof of Proposition 1: By building on the steps in the text it remains to formally demonstrate for each of the two parts of the Proposition that i) the proposed symmetric equilibrium exists and ii) that no other symmetric equilibria can exist. We consider each part in turn.

Part 1: For existence, first note that it is indeed true that $\alpha \in (0, 1)$ as $\alpha = 1 -$

$\left(\frac{nA}{(n-1)(1-\theta)\pi(u^m)}\right)^{\frac{1}{n-1}}$ is less than one given $A < \frac{n-1}{n}(1-\theta)\pi(u^m)$ and greater than zero given $A > 0$. Second, note that the advertised utility support $(u^m, \bar{u}]$ is well-defined because i) $u^m > \bar{u}$ and ii) $\bar{u} < \hat{u}$ such that the upper bound is less than the break-even utility level. The first inequality i) follows from the derivation in the text. The second inequality ii) follows as $\pi'(u) < 0$ for $u > u^m$ and the profits from selecting $u = \bar{u}$ equal $\bar{\pi} = \frac{\theta}{n}\pi(u^m) + \frac{A}{n-1} > 0 = \pi(\hat{u})$. Third, the equilibrium distribution $F(u)$ is also well-defined as it can be verified from the text that i) $F(u^m) = \alpha$ and ii) $F(\bar{u}) = 1$, while it is also true that iii) $F'(u) > 0$ for $u \in (u^m, \bar{u}]$. This follows from (7) as $F'(u) = \frac{1}{n-1}Z^{(\frac{1}{n-1})-1} \cdot \frac{\partial Z}{\partial u}$ where $Z = \left(\frac{\bar{\pi} + A - \frac{\theta}{n}\pi(u)}{(1-\theta)\pi(u)}\right) > 0$ such that $\frac{\partial Z}{\partial u} = -\frac{\pi'(u)(\bar{\pi} + A)}{(1-\theta)\pi(u)^2} > 0$ given $\pi'(u) < 0$ for $u > u^m$. Finally, a firm can do no better than price according to $F(u)$ when advertising as i) $u = u^m$ is uniquely optimal if the firm does not advertise, ii) it remains dominated to price outside the support, $(u^m, \bar{u}]$, when advertising, and iii) each firm's equilibrium profits for all u within the advertised support are equal to $\bar{\pi}$ and so it remains a best response to use $F(u)$ when the other $n - 1$ firms also use $F(u)$. For uniqueness, note that the proposed equilibrium with $\alpha \in (0, 1)$ is uniquely defined and so we only need to rule out other potential symmetric equilibria with i) $\alpha = 0$ or ii) $\alpha = 1$. For i), the low advertising cost threshold rules out $\alpha = 0$ as advertising an offer above u^m is a best response if all other firms do not advertise. For ii), no equilibrium with $\alpha = 1$ can exist given $A > 0$ because such equilibrium has to have mixed strategies with no point masses, as per usual logic. The lower bound on such mixed strategies is at least u^m , and hence equilibrium profit is at most $\frac{\theta}{n}\pi(u^m) - A$, which is dominated by no advertising and setting u^m .

Part 2: If no firm advertises and all firms select u^m , then shoppers are allocated according to the random tie-breaking rule. From the text, we know that no firm would want to deviate to any other $u \neq u^m$ when not advertising. However, we also know that no firm would want to deviate to any other $u \neq u^m$ when advertising if $A \geq \frac{n-1}{n}(1-\theta)\pi(u^m)$, because even advertising slightly above u^m , at $u^m + \varepsilon$, to win all the shoppers for sure would not be profitable. To see this, note that such a deviation would yield at most $\frac{\theta}{n}\pi(u^m + \varepsilon) + (1-\theta)\pi(u^m + \varepsilon) - A$ as compared to an equilibrium profit of $\frac{\theta}{n}\pi(u^m) + \frac{(1-\theta)}{n}\pi(u^m)$. The gain from such a deviation is then $(1-\theta)\pi(u^m + \varepsilon) - A - \frac{(1-\theta)}{n}\pi(u^m)$ which is negative for all $\varepsilon > 0$ given our condition on A . Thus, this equilibrium exists. To then show that no other symmetric equilibrium exists we just need to consider a possible equilibrium where all firms

advertise. This is not possible because if all firms advertise, they would have to do so in the fashion described in Part 1, and such an equilibrium cannot be constructed because A exceeds the threshold. *Q.E.D*

Proof of Lemma 2: To derive the first part of the result, we need to maximise $\pi(\mathbf{p}_i, f_i) = \mathbf{q}(\mathbf{p}_i)'(\mathbf{p}_i - \mathbf{c}) + f_i$ subject to $S(\mathbf{p}_i) - f_i = u'$. By substituting for f_i in the objective function, this reduces to the maximisation of $\mathbf{q}(\mathbf{p}_i)'(\mathbf{p}_i - \mathbf{c}) + S(\mathbf{p}_i) - u'$. The resulting FOC implies that \mathbf{p}_i should be chosen to maximize total welfare, as consistent with marginal cost pricing, $\mathbf{p}_i = \mathbf{c}$. By substituting this back into the constraint, the optimal fixed fee can then be derived as $f_i = S(\mathbf{c}) - u'$. Firm i 's optimal profits can then be written in terms of u by substituting the optimal marginal prices and fixed fee to give $\pi(u') = S(\mathbf{c}) - u'$. The monopoly case then follows immediately as $u^m = 0$. *Q.E.D*

Proof of Proposition 2: First, consider the results for the effects on α and $\bar{\pi}$. These follow immediately from differentiating (3) and (4) with respect to A and θ . Second, for the utility offer results, we now show that marginal changes in A and θ lead to reductions in utility in the sense of FOSD. From Definition 1, as a change in A or θ leaves u^m unchanged, this only requires $F(u)$ to be increasing in A and θ for all $u \in [u^m, \bar{u}]$. This follows easily from differentiating $F(u)$ in (7). *Q.E.D*

Proof of Proposition 3: First, note that the results for α , $\bar{\pi}$, and $F(u)$ follow by taking the limits of equations (3), (4) and (7) with respect to $A, \theta, \rightarrow 0$. Second, to derive the result for \bar{u} , note from (9), that $\pi(\bar{u}) \rightarrow 0$ when $\theta \rightarrow A \rightarrow 0$ such that \bar{u} must converge to \hat{u} . *Q.E.D*

Proof of Proposition 6: From (12), when $A \rightarrow 0$, $\frac{\partial F(u)}{\partial d}$ is positive (negative) for all $u \in (u^m, \bar{u}]$ when $\frac{\pi_d(u^m(d);d)}{\pi(u^m(d);d)} > (<) \frac{\pi_d(u;d)}{\pi(u;d)}$, which holds necessarily when $\frac{\pi_d(u;d)}{\pi(u;d)}$ is decreasing (increasing) in u . Also, the fact that $\frac{\pi_d(u;d)}{\pi(u;d)}$ is decreasing (increasing) in u implies that $\pi_{ud}(u;d) < (>)0$, which itself implies that $u^m(d)$ is decreasing (increasing) in d . *Q.E.D*

Proof of Lemma 3: To begin, we derive the profit function, $\pi(u)$. For any required utility level u , a firm chooses p and f to maximise its profits $pq(p) - f$ such that $S(p) - f = u$ and $f \leq d$. For $u \geq S(0) - d$, the optimal price is $p = 0$ with fee $f = S(0) - u \leq d$ which gives the profit $\pi(u) = S(0) - u$. For $u < S(0) - d$, the optimal fee is $f = d$, such

that $u = S(p) - d$, and so the optimal price equals $p = S^{-1}(u + d)$, with resulting profit $\pi(u) = S^{-1}(u + d)q(S^{-1}(u + d)) + d$. The overall profit function can be then described by

$$\pi(u) = \begin{cases} \pi_1(u) = S^{-1}(u + d)q(S^{-1}(u + d)) + d & \text{if } u \leq S(0) - d \\ \pi_2(u) = S(0) - u & \text{if } u > S(0) - d \end{cases}$$

This profit function is maximized at some $u \leq S(0) - d$ because for all higher u it is strictly decreasing. There may be a corner solution at 0. To characterize the problem, assume that $\pi_1(u)$ is concave. Then its derivative with respect to u evaluated at $u = 0$ is given by

$$\frac{S^{-1}(d)q'(S^{-1}(d)) + q(S^{-1}(d))}{-q(S^{-1}(d))} = \frac{S^{-1}(d)q'(S^{-1}(d))}{-q(S^{-1}(d))} - 1 = \epsilon((S^{-1}(d)) - 1,$$

where $\epsilon(p) = -q'(p)(p/q)$ is the price elasticity of demand at p . Note that ϵ is decreasing in d (increasing in p) and so $\pi'(0)$ goes from being positive infinity for $d = 0$ to negative when d is sufficiently high. Thus, for sufficiently high d the maximization of $\pi(u)$ has a corner solution at $u^m = 0$. In particular, $u^m = 0$ for $d \geq \bar{d}$ where \bar{d} solves $\epsilon(S^{-1}(\bar{d})) = 1$. But from standard monopoly results, \bar{d} then has a simple interpretation such that $\bar{d} = S(p^m)$. If $d < \bar{d}$, then the monopoly utility solves $\pi'_1(u^m) = 0$ such that $u^m > 0$ and $u_d^m < 0$. Regardless of d , for all $u < S(0) - d$ we have that

$$\pi_{ud} = \frac{q(p)(pq''(p) + q'(p)) - pq'(p)^2}{q(p)^3},$$

where $p = S^{-1}(d + u)$. Given that $pq(p)$ is concave, we have $\pi_{ud} < 0$. *Q.E.D*

Proof of Proposition 10: We need to show that an increase in d always improves utility offers in the sense of FOSD. From Proposition 8, it is sufficient that $\pi_{ud}(u; d) \geq 0$. To show this is true for linear prices, we build on our previous notation, to note that $u = S(p) = \int_p^\infty q(p)$. It then follows that $p = S^{-1}(u)$. Then $\pi(u, d)$ can be expressed as $S^{-1}(u)q(S^{-1}(u)) - c(q(S^{-1}(u)), d)$. Therefore, $\pi_{du}(u, d) = \frac{q'(p)}{q(p)} \cdot c_{qd}(q(p), d)$ where we use $S^{-1}(u) = -q(p)^{-1}$. This is then non-negative as required as i) first term is negative for downward-sloping demand and zero for unit demand, and ii) the second term is simply the derivative of marginal cost with respect to d , which is non-negative by assumption. For two-part tariffs, the optimal way to provide utility with non-linear cost function is to always set per unit price p^* such that

$C_q(q(p^*), d) = q(p^*)$ and fixed fee as $f = \int_{p^*}^{\infty} q(x)dx - u$. Then, the profit function is simply $\pi(u) = p^*q(p^*) - C(q(p^*), d) - u$. Because $\pi_{ud}(u) = 0$, the result follows from Proposition 8 immediately. *Q.E.D*

Proof of Proposition 11: The proposed equilibrium implies expected profits for r^m , the lower bound of the distribution of r , as $\pi_L(u_L(r^m))[\theta/n] + \pi_S(u_S(r^m))[(1-\alpha)^{n-1}((1-\theta)/n)]$. If the firm advertises a slightly higher r then it earns $\pi_L(u_L(r^m))[\theta/n] + \pi_S(u_S(r^m))[(1-\alpha)^{n-1}((1-\theta))] - A$. Profits have to be equal for all r , including the two cases above, and so $(1-\alpha)^{n-1} = \frac{A}{\frac{n-1}{n}(1-\theta)\pi_S(u_S(r^m))}$ which gives (14). When substituted back into either profit above, this implies an equilibrium profit equal to (16). r^m should be such that the firm chooses to set it when all other firms set α according to r^m . Consider a firm that does not advertise and sets r , then its expected profit is $\pi_L(u_L(r))[\theta/n] + \frac{A}{n-1} \frac{\pi_S(u_S(r))}{\pi_S(u_S(r^m))}$. Then the level of r^m compatible with equilibrium is such that it maximizes the above, as summarised in (13)/ In order to guarantee that no consumer group is excluded, we need that $u_S(r^m) \geq 0$. $F(r)$ can be then be constructed indirectly from (15), which itself derives from $\bar{\pi} = \pi_L(u_L(r))[\theta/n] + \pi_S(u_S(r))[F(u_S(r))^{n-1}(1-\theta)] - A$. \bar{r} follows intuitively from (17). Finally, to derive the condition on A to guarantee the existence of the sales equilibrium, one can follow similar steps as Proposition 1 by requiring $\bar{r} > r^m$ and then setting $\bar{r} = r^m$ in (17) and rearranging. *Q.E.D*

Appendix B - Technical Details of Applications

Unit Demand: With the additional use of $F(p^*(u)) = 1 - F(u)$, we can derive the following equilibrium details from Proposition 1: $\bar{\pi} = \frac{\theta}{n} \sum_{k=1}^K (V_k - c_k) + \frac{A}{n-1}$, $\alpha = 1 - \left(\frac{\frac{n-1}{n}A}{(1-\theta) \sum_{k=1}^K (V_k - c_k)} \right)^{\frac{1}{n-1}}$, $F(p^*) = 1 - \left(\frac{\frac{\theta}{n} \sum_{k=1}^K (V_k - p^*) + \frac{n-1}{n}A}{(1-\theta) \sum_{k=1}^K (p^* - c_k)} \right)^{\frac{1}{n-1}}$, and $\bar{p} = p^*(u^m) = \sum_{k=1}^K V_k$. Finally, it follows that $\bar{u} = \sum_{k=1}^{K_i} (V_k - c_k) - \left(\frac{\frac{\theta}{n} \sum_{k=1}^K (V_k - c_k) + \left(\frac{n-1}{n}\right)A}{\frac{\theta}{n} + (1-\theta)} \right)$. Therefore, $\underline{p} = p^*(\bar{u}) = \sum_{k=1}^K c_k + \left(\frac{\frac{\theta}{n} \sum_{k=1}^K (V_k - c_k) + \left(\frac{n-1}{n}\right)A}{\frac{\theta}{n} + (1-\theta)} \right)$.

Downward-Sloping Demand: A detailed derivation of this equilibrium involves the following: $\bar{\pi} = (\theta/n)\pi(\mathbf{p}^m) + \frac{A}{n-1}$, $\alpha = 1 - \left(\frac{\frac{n-1}{n}A}{(1-\theta)\pi(\mathbf{p}^m)} \right)^{\frac{1}{n-1}}$, $F(u) = \frac{(\theta/n)[\pi(\mathbf{p}^m) - \pi(\mathbf{p}^*(\mathbf{u}))] + (n/(n-1))A}{(1-\theta)\pi(\mathbf{p}^*(\mathbf{u}))}$,

where $\mathbf{p}^*(\mathbf{u}) = \operatorname{argmax}_{\mathbf{p}} \pi(\mathbf{p})$ subject to $S(\mathbf{p}, \mathbf{q}^*(\mathbf{p})) = u$, $\bar{\mathbf{p}} = \mathbf{p}^*(u^m) = \mathbf{p}^m$ and $\underline{\mathbf{p}} = \mathbf{p}^*(\bar{u})$, and where \bar{u} follows from an amended version of (9). To show how this nests Baye and Morgan (2001) (with exogenous A and θ) when $K = 1$, note that now $\mathbf{p} \equiv p$ and so $F(p) = 1 - F(u)$. Hence, in equilibrium, from Proposition 1, firms earn $\bar{\pi} = (\theta/n)\pi(p^m) + \frac{A}{n-1}$ and advertise with probability $\alpha = 1 - \left(\frac{\frac{n}{n-1}A}{(1-\theta)\pi(p^m)}\right)^{\frac{1}{n-1}}$. Further, when firms advertise, they employ a price distribution conditional on advertising (as consistent with Baye and Morgan's approach), $F^A(p) \equiv \frac{1-F(u)}{\alpha}$ equal to $\frac{1}{\alpha} \left[1 - \left(\frac{(\theta/n)[\pi(p^m) - \pi(p)] + (n/(n-1))A}{(1-\theta)\pi(p)}\right)\right]$ on a support with $\underline{p} = \pi^{-1}\left(\frac{(\frac{\theta}{n})\pi(p^m) + (\frac{n}{n-1})A}{\frac{\theta}{n} + (1-\theta)}\right)$ and $\bar{p} = p^m$.

Non-Price Sales: To help characterise a simple solution, we make two assumptions that apply for all $z \in [\underline{z}, \bar{z}]$. First, assume that $V(z)$ and $c(z)$ are both strictly increasing in z , such that a higher level of z corresponds to a product with a higher unit cost but a larger package size or product value. As such, firm profits per consumer, $\pi(z) = p - c(z)$, are strictly decreasing in z , whilst consumer utility, $u(z) = V(z) - p$, is strictly increasing in z . Second, we suppose that $V(z) \geq p > c(z) \geq 0$ to ensure that profits per consumer can be positive and consumer utility is always non-negative, as then $\pi(\bar{z}) = p - c(\bar{z}) > 0$ and $u(\underline{z}) = V(\underline{z}) - p \geq 0$. Together, these assumptions ensure a unique correspondence between z and i) consumer utility, u , with $z = V^{-1}(p + u)$, and ii) firm's profit per consumer, as now $\pi(u) = p - c(V^{-1}(p + u))$. Finally, it follows that $z^m = \underline{z}$ such that $u^m = V(\underline{z}) - p$ because a monopolist would prefer the lowest cost option, \underline{z} . Then after substituting in the appropriate notation one can then apply Proposition 1 to derive the equilibrium utility selections and use $z = V^{-1}(p + u)$ to establish the firms' associated choices of z .

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