WORKING PAPERS

Decentralized Redistribution In a Laboratory Federation

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March 2014

Working Paper No: 1402

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

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Abstract

Fiscal federalism is often hailed as an innovation procedure: successful policy experiments in one jurisdiction will, via imitation, spread through the entire system, leading to overall better policy performance. We show that such hopes set in laboratory federalism may be ill-founded. For a standard framework of decentralized redistribution in a common labor market with mobile transfer recipients imitation-with-experimentation will lead to a complete breakdown of the welfare state: zero transfers.

Keywords: Laboratory federalism, Mobility, Redistribution.
JEL Classification: H77, H75, C73.

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1 Introduction

In addition to various other benefits, fiscal federalism is often credited as a laboratory or discovery procedure for good public policies (Oates, 1999; Kollman et al., 2000; Salmon, 1987). In a federation, jurisdictions can experiment with new and innovative policies. When failing, these policies can be discarded without great damage to the entire system; when successful, however, they can and will be copied by other jurisdictions and spread across the system. This Hayekian process of imitating successful examples will, so it is hoped, converge towards efficient policy outcomes. The notion of laboratory federalism is thought to be particularly suitable for social welfare policies (Inman and Rubinfeld, 1997).[1]

This paper casts doubts on the universal validity of the idea that such imitative processes are always of beneficial nature in the context of fiscal federalism. In a model where redistribution from rich to poor is decentralized, we show that mimicking best-performing policies can imply a total breakdown of decentralized redistribution.

We arrive at this result in the classical framework of decentralized redistribution by Wildasin (1991, 1994): governments in a multi-jurisdictional, economically integrated area want to redistribute income from immobile rich to poor workers. The poor are costlessly mobile between jurisdictions and supply labor wherever they reside. In a migration equilibrium, their living standard will be equalized across jurisdictions. Governments pay transfers to the poor which are financed by taxes on the rich. In this and related frameworks, the literature routinely studies one-shot fiscal games, predicting the Nash equilibrium to be the outcome.

Here we differ: in our paper, governments engage in repeated interaction over time. Rather than playing best-response strategies, they observe which policies performed best in other jurisdictions in the past and then adopt them (imitate-the-best behavior). Policy innovation occurs via occasional experimentation; when these experiments turn out to be successful in the sense that the experimenting jurisdiction fares better than the non-experimenting ones, the new policy will be mimicked – and otherwise discarded.

The imitation-cum-experimentation dynamics captures many aspects of laboratory federalism. It implies learning from others, the dissemination of policies that perform well in relative terms, and the possibility of innovation. Moreover, unlike playing best-replies, the imitation dynamics does not require that governments fully understand the mechanics of the federation and its economic structure; only observing policies and payoffs is relevant. This corresponds to the idea that the laboratory feature of federal systems is

[1] The European Union endorsed this laboratory idea in its so-called Open Method of Coordination (OMC), an iterative mode of governance that is based on policy experiments and the mimicking of best practices (see e.g. Borrás and Jacobsson, 2004).

particularly appealing when knowledge about the functioning of the economy is limited.

For the classical decentralized redistribution game à la Wildasin (1991), we show that this imitative process leads to zero transfers (no redistribution at all). A basic intuition, detailed in Section 3, is that any jurisdiction that experiments by cutting back transfers always ends up with a relative advantage over others. This is due to the fact that in this model the consumption level of the poor in a migration equilibrium is equalized across jurisdictions and, thus, features as a public good. Reducing transfers lowers the deviant's contribution to this public good, the level of the public good goes down in all jurisdictions alike. The resulting relative advantage then triggers an imitation wave lowering transfers in all other jurisdictions. On the other hand, starting at a situation with zero transfers everywhere, any deviating jurisdiction that starts paying some transfers to its poor will end up worse off, relative to the others. This kind of experiment will never be followed by imitation. In a dynamic setting with imitate-the-best behavior, these properties drive the system towards ever lower transfers to the poor, resulting eventually in a complete breakdown of redistribution. The dynamics of laboratory federalism has, thus, fatal consequences for the public provision of consumption to the poor in this setting.

Some empirical evidence (mainly from the U.S.) indeed indicates that mimicking behavior is present in the area of welfare policy (Revelli, 2001, 2006; Saavedra 2000; or Brueckner, 2000). It mainly leads to a "race to the bottom": governments, out of fear to become welfare magnets, underbid each other in the transfers to the poor. While these findings cannot count as proof of the "laboratory failure" possibility outlined here, they nevertheless indicate that imitation need not promote efficient policies.

Technically, our paper borrows from the literature on imitative learning in games (see e.g. Alós-Ferrer and Ania, 2005) and the notion of finite-population evolutionarily stable strategies (Schaffer, 1988). The application of these ideas to fiscal federalism is rather new. Wagener (2013) is the first one to point out the relevance of relative-payoff considerations in the context of tax competition and shows in a game with taxable mobile capital that they result in long-run efficiency losses beyond those in a Nash equilibrium. In a related paper, Ania and Wagener (2013) consider an alternative model of income redistribution with perfect mobility in an economically integrated area. There, small changes in redistribution levels are assumed to trigger drastic migration flows and consumption levels only equalize in the symmetric case in which all jurisdictions set the same transfers. Jurisdictions are assumed to have a utilitarian welfare function that values income transfers to any group of poor residents, no matter how large this group is, while at the same time keeping a certain standard of living for the rich with immobile taxable assets. In that setting, the same kind of imitation and experimentation behavior as in the present paper allows for coordination on positive redistribution levels. In particular, it is shown that jurisdictions coordinate in the long run on the set of transfer policies that would be sustainable even if half of the jurisdictions would decide to lower their transfers. Taken together, these papers illustrate how evolutionary learning models of imitation may contribute to a more detailed understanding of laboratory federalism.

The rest of the paper is organized as follows: Section 2 presents the standard model of decentralized redistribution due to Wildasin (1991). Section 3 reviews the relevant notions of evolutionary stability and identifies zero transfers as the unique evolutionary equilibrium of the static game. Section 4 presents the imitation-and-experimentation dynamics and argues that zero transfers are indeed the only possible long-run outcome of this dynamics. Section 5 concludes.

2 The decentralized redistribution game

2.1 Framework

Our framework of redistribution from rich to poor with mobility of the poor is a symmetric version of Wildasin (1991, 1994). In an economically integrated area there is a finite number $N > 1$ of identical jurisdictions. Each jurisdiction $i \in \{1, \ldots, N\}$ is inhabited by one (representative) rich household who is immobile and owns the fixed factors. The poor are workers who are mobile at zero costs within the economic area. Their total number in the economy, $N \cdot \bar{\ell}$, is exogenously fixed. By ℓ_i we denote the number of poor locating in jurisdiction i.

Each worker inelastically supplies one unit of labor wherever he resides. Production in each jurisdiction follows a Ricardian technology $f(\ell)$ with $f'(\ell) > 0 > f''(\ell)$ for all $\ell \geq 0$. The fixed factors are already embodied in f. To ensure that every jurisdiction is populated with some workers in all scenarios below, we assume that $f(0) = 0$ and $f'(0) \to \infty$. Workers in i are paid their marginal product $f'(\ell_i)$ plus a subsidy s_i (net of any taxes), such that their net income and consumption equal

$$c_i = f'(\ell_i) + s_i. \tag{1}$$

The rich in jurisdiction i consumes the residual income at his location:

$$y_i = f(\ell_i) - [f'(\ell_i) + s_i] \cdot \ell_i. \tag{2}$$

At subsidies $\mathbf{s} = (s_1, \ldots, s_N)$, a migration equilibrium is a distribution $(\ell_1(\mathbf{s}), \ldots, \ell_N(\mathbf{s}))$ of workers across jurisdictions such that there is full employment and workers' consumption levels are equalized across jurisdictions, i.e.:

$$\sum_{i=1}^{N} \ell_i(\mathbf{s}) \quad = \quad N \cdot \bar{\ell} \tag{3}$$

$$f'(\ell_i(\mathbf{s})) + s_i = f'(\ell_j(\mathbf{s})) + s_j \quad =: \quad c(\mathbf{s}) \quad \text{for all } i, j \in 1, \ldots, N. \tag{4}$$

From (3) and (4), jurisdictions that pay equal transfers will attract equally many mobile poor. Note that migration flows are invariant to permutations of the subsidies in other jurisdictions; i.e. $\ell_i(\mathbf{s}) = \ell(s_i; \mathbf{s}_{-i})$ for all i and any order of the elements in the vector $\mathbf{s}_{-i}$, denoting the subsidies of all jurisdictions except i. As a consequence, also workers' consumption level in a migration equilibrium, $c(\mathbf{s})$ is invariant to permutations of subsidies across jurisdictions.

2.2 Policy objectives

With respect to political preferences, we follow Wildasin (1991) and assume that each jurisdiction cares for its social welfare that depends on the consumption levels of the rich (y_i) and a representative poor (c):

$$U_i = U(y_i, c). \tag{5}$$

We assume the function U is strictly quasi-concave and has strictly positive partial derivatives $U_y := \partial U(y_i, c)/\partial y > 0$ and $U_c := \partial U(y_i, c)/\partial c > 0$ for all (y_i, c). Given a migration equilibrium at subsidies $\mathbf{s}$, the payoffs of jurisdiction i can be expressed as follows:

$$\begin{aligned} \pi_i(\mathbf{s}) &= U\left(f(\ell_i(\mathbf{s})) - [f'(\ell_i(\mathbf{s})) + s_i]\ell_i(\mathbf{s}), c(\mathbf{s})\right) \\ &= U\left(f(\ell_i(\mathbf{s})) - c(\mathbf{s})\ell_i(\mathbf{s}), c(\mathbf{s})\right). \end{aligned} \tag{6}$$

Jurisdictions choose subsidies out of a common strategy set of subsidies $S \subseteq [0, \bar{s}]$ where $\bar{s} < \infty$ and subsidies $\mathbf{s}$ affect the allocation of the poor across jurisdictions $(\ell_1(\mathbf{s}), \ldots, \ell_N(\mathbf{s}))$.

It is important to note that the payoff function is symmetric, since migration flows and, thus, the social welfare of any jurisdiction are invariant to permutations of the strategies of others; i.e., we can write the payoffs to any jurisdiction i, π_i, as a function of own subsidy s_i and the vector $\mathbf{s}_{-i}$ of the subsidies in other jurisdictions or any permutation thereof:

$$\pi_i(\mathbf{s}) = \pi(s_i; \mathbf{s}_{-i}) = U\left(f(\ell(s_i; \mathbf{s}_{-i})) - c(s_i; \mathbf{s}_{-i})\ell(s_i; \mathbf{s}_{-i}), c(s_i; \mathbf{s}_{-i})\right). \tag{7}$$

2.3 Comparative statics of the migration equilibrium

The response of $(\ell_1(\mathbf{s}), \ldots, \ell_N(\mathbf{s}))$ and $c(\mathbf{s})$ to changes in any of the subsidies s_i can be obtained by totally differentiating (3) and (4) with respect to s_i.[2] Specifically, for $i, j = 1, \ldots, N$ and $i \neq j$,

$$
\begin{aligned}
\frac{\partial c(\mathbf{s})}{\partial s_i} &= \frac{1/f''(\ell_i)}{\sum_{j=1}^{N} 1/f''(\ell_j)} > 0, \\
\frac{\partial \ell_i(\mathbf{s})}{\partial s_i} &= -\frac{1}{f''(\ell_i)} \cdot \left(1 - \frac{1/f''(\ell_i)}{\sum_{j=1}^{N} 1/f''(\ell_j)}\right) > 0, \\
\frac{\partial \ell_j(\mathbf{s})}{\partial s_i} &= \frac{1}{f''(\ell_j)} \cdot \frac{1/f''(\ell_i)}{\sum_{j=1}^{N} 1/f''(\ell_j)} < 0.
\end{aligned}
$$

Thus, higher transfers offered by any jurisdiction lead to an inflow of mobile workers into that jurisdiction, to an outflow from every other jurisdiction, and to an economy-wide increase in workers' consumption.

In what follows, situations of particular interest are those where m jurisdictions (with $1 \leq m \leq N-1$) each grant a certain subsidy (s^m, say), while the remaining $N-m$ jurisdictions all set another one (s^{N-m}, say). We denote such a subsidy vector by

$$
(\mathbf{s}^m, \mathbf{s}^{N-m}) = (\underbrace{s^m, \ldots, s^m}_{m}, \underbrace{s^{N-m}, \ldots, s^{N-m}}_{N-m})
$$

For $m = 1$ or $m = N-1$ this corresponds to the case of a single mutant in the presence of an otherwise homogeneous situation. Given any $\mathbf{s} = (\mathbf{s}^m, \mathbf{s}^{N-m})$ where i is one of the jurisdictions setting $s_i = s^m$ and j is one of the jurisdictions setting $s_j = s^{N-m}$ we denote by

$$
\ell^m(s^m, s^{N-m}) = \ell_i(\mathbf{s}^m, \mathbf{s}^{N-m}) \quad \text{and} \quad \ell^{N-m}(s^m, s^{N-m}) = \ell_j(\mathbf{s}^m, \mathbf{s}^{N-m})
$$

the number of poor in any of the jurisdictions in each group. Then, a migration equilibrium at $(\mathbf{s}^m, \mathbf{s}^{N-m})$ is characterized by:

$$
\begin{aligned}
m \cdot \ell^m(s^m, s^{N-m}) + (N-m) \cdot \ell^{N-m}(s^m, s^{N-m}) &= N \cdot \bar{\ell} \\
f'(\ell^m(s^m, s^{N-m})) + s^m = f'(\ell^{N-m}(s^m, s^{N-m})) + s^{N-m} &= \hat{c}(s^m, s^{N-m})
\end{aligned}
$$

where $\hat{c}(s^m, s^{N-m}) := c(\mathbf{s}^m, \mathbf{s}^{N-m})$, as defined in (4). Suppose that *all* m jurisdictions simultaneously deviate from s^m. The change in the migration equilibrium can be obtained

[2] See Wildasin (1991) p. 761 for details. Note that (1) implicitly defines $\ell_i = \ell_i(c_i - s_i) = (f')^{-1}(c_i - s_i)$ and, thus, $\ell_i'(c_i - s_i) = 1/f''(\ell_i)$.

by totally differentiating the previous two equations:[3]

$$\frac{\partial \hat{c}}{\partial s^m} = \frac{f''(\ell^{N-m})}{\frac{N-m}{m} f''(\ell^m) + f''(\ell^{N-m})} > 0, \tag{8}$$

$$\frac{\partial \ell^m}{\partial s^m} = -\frac{1}{f''(\ell^m) + \frac{m}{N-m} f''(\ell^{N-m})} > 0, \tag{9}$$

$$\frac{\partial \ell^{N-m}}{\partial s^m} = \frac{1}{\frac{N-m}{m} f''(\ell^m) + f''(\ell^{N-m})} < 0. \tag{10}$$

3 Evolutionary stability

The present section reviews the relevant notion of evolutionary stability and it applies this idea of equilibrium to the game defined in the previous section. We will distinguish different levels of evolutionary robustness. A finite-population evolutionarily stable strategy (ESS) is one that is robust against all possible mutations that come one at a time: single mutants deviating from an ESS fare worse than non-mutants. Much stronger, a globally stable ESS (GSS) is robust against mutations *independently* of the number of mutants. Together, ESS and GSS constitute a stability check against any competing strategy coming in at small or large scale.

3.1 Evolutionarily and globally stable strategies

Consider a subsidy vector of type $\mathbf{s} = (\mathbf{s}^m, \mathbf{s}^{N-m})$ for $1 \leq m \leq N-1$. We say that a subsidy $s^{N-m} = s^*$ is (weakly) m-stable if

$$\pi(s^*; \underbrace{s, \ldots, s}_{m}, \underbrace{s^*, \ldots, s^*}_{N-m-1}) \geq \pi(s; \underbrace{s, \ldots, s}_{m-1}, \underbrace{s^*, \ldots, s^*}_{N-m})$$

for all $s^{N-m} = s$. The special case of a weakly 1-stable strategy is called a finite-population evolutionarily stable strategy (ESS); it satisfies

$$\pi(s^*; s, s^*, \ldots, s^*) \geq \pi(s; s^*, \ldots, s^*) \quad \text{for all } s \in S.$$

If all jurisdictions set an ESS s^*, a unilateral deviator, choosing s, will never be better off than any of the non-deviators who stick to s^*. The notion of m-stability extends this idea

[3]Starting with a uniform subsidy in all jurisdictions, the change in the migration equilibrium if only one of them slightly deviates is captured by the case $\ell^m = \ell^{N-m} = \bar{\ell}$ and $m = 1$. Specifically, we obtain

$$\frac{\partial c(\mathbf{s})}{\partial s_i} = \frac{1}{N} > 0, \; \frac{\partial \ell_i(\mathbf{s})}{\partial s_i} = -\frac{N-1}{Nf''\left(\bar{\ell}\right)} > 0, \text{ and } \frac{\partial \ell_j(\mathbf{s})}{\partial s_i} = \frac{1}{Nf''\left(\bar{\ell}\right)} < 0 \; \forall i, j = 1, \ldots, N, \; i \neq j.$$

Of course, these expressions can also be obtained for the general comparative statics given earlier.

to groups of players: if all other jurisdictions set subsidies s^*, then a joint deviation of m jurisdictions to any alternative strategy s leaves all deviators (weakly) worse off than any of those jurisdictions that stuck to s^*.

A strategy is called a weakly *globally stable strategy (GSS)* if it is m-stable for *every* m with $1 \leq m \leq N-1$. A GSS resists any type of experiments, be they of small fractions of jurisdictions ($m=1$, ESS) or larger coalitions of mutants. The imitation-cum-experimentation dynamics by which we characterize laboratory federalism below will lead to stochastically stable states; global stability will imply such stochastic stability.

As an aside that will facilitate the interpretation below, recall that a finite-population ESS corresponds to a symmetric Nash equilibrium of a game with *relative* payoffs (Schaffer 1988). I.e., s^* is an ESS if and only if

$$s^* = \arg\max_{s \in S} \left[\pi(s; s^*, \ldots, s^*) - \pi(s^*; s, s^*, \ldots, s^*)\right]. \tag{11}$$

3.2 ESS and GSS for decentralized redistribution

We study global stability and ESS of the decentralized redistribution game applying a direct technique inspired by Tanaka (2000). For $\mathbf{s} = (\mathbf{s}^m, \mathbf{s}^{N-m})$, as defined above, denote by

$$\varphi(s^m, s^{N-m}) := \pi(s^m; \underbrace{s^m, \ldots, s^m}_{m-1}, \underbrace{s^{N-m}, \ldots, s^{N-m}}_{N-m}) - \pi(s^{N-m}; \underbrace{s^m, \ldots, s^m}_{m}, \underbrace{s^{N-m}, \ldots, s^{N-m}}_{N-m-1})$$

the payoff differential between a jurisdiction that sets s^m and one that sets s^{N-m} when a total of m jurisdictions choose s^m and the rest s^{N-m}. Then s^* is m-stable if s^* solves the following problem:

$$\max_{s^m} \varphi(s^m, s^{N-m}), \quad \text{given } s^{N-m} = s^*. \tag{12}$$

An m-stable strategy s^* maximizes the difference in payoffs for any of m mutants and any of $N-m$ non-mutants who choose strategy $s^{N-m} = s^*$. Moreover, it can be interpreted as a strategy where $N-m$ mutants to s^* from any symmetric profile where all players choose some s^m would earn higher payoffs.

Proposition 1 *In the decentralized redistribution game, zero transfers ($s^* = 0$) are uniquely m-stable for all $m = 1, \ldots, N-1$. Hence, they are an ESS and also a GSS.*

Proof: Fix m with $1 \leq m \leq N-1$ and consider $\mathbf{s} = (\mathbf{s}^m, \mathbf{s}^{N-m})$. Substituting the expressions for a migration equilibrium at such $\mathbf{s}$ into (12) we get

$$\begin{aligned}\varphi(s^m, s^{N-m}) \;&=\; U\left(f(\ell^m(s^m, s^{N-m})) - \hat{c}(s^m, s^{N-m})\ell^m(s^m, s^{N-m}), \hat{c}(s^m, s^{N-m})\right) \\ &\quad -U\left(f(\ell^{N-m}(s^m, s^{N-m})) - \hat{c}(s^m, s^{N-m})\ell^{N-m}(s^m, s^{N-m}), \hat{c}(s^m, s^{N-m})\right).\end{aligned}$$

To maximize this expression, we partially differentiate φ with respect to s^m and obtain:[4]

$$\begin{aligned}\frac{\partial \varphi}{\partial s^m} &= U_y^m \cdot \left[(f'(\ell^m) - \hat{c}) \cdot \frac{\partial \ell^m}{\partial s^m} - \ell^m \cdot \frac{\partial \hat{c}}{\partial s^m} \right] + U_c^m \cdot \frac{\partial \hat{c}}{\partial s^m} \\ &\quad -U_y^{N-m} \cdot \left[\left(f'(\ell^{N-m}) - \hat{c} \right) \cdot \frac{\partial \ell^{N-m}}{\partial s^m} - \ell^{N-m} \cdot \frac{\partial \hat{c}}{\partial s^m} \right] - U_c^{N-m} \cdot \frac{\partial \hat{c}}{\partial s^m}.\end{aligned}$$

For $s^* = s^m = s^{N-m}$, we get $\ell^m = \ell^{N-m} = \bar{\ell}$. Hence, utility functions and their derivatives are evaluated at identical points $\hat{c} = f'(\bar{\ell}) + s^*$ and $y = f(\bar{\ell}) - \hat{c}\bar{\ell}$. Using the comparative statics (9) and (10), the partial derivative simplifies to

$$\frac{\partial \varphi}{\partial s^m} = -s^* U_y \cdot \left[\frac{\partial \ell^m}{\partial s^m} - \frac{\partial \ell^{N-m}}{\partial s^m} \right] = -s^* \cdot \frac{U_y}{f''(\bar{\ell})} \left[-\frac{N-m}{N} - \frac{m}{N} \right] = s^* \cdot \frac{U_y}{f''(\bar{\ell})} < 0$$

for all $s^* > 0$. Hence, $s^* = 0$ is m-stable for any m. ■

Proposition 1 states that a situation without any redistribution from rich to poor ($s^* = 0$) resists any mutation, both by single jurisdictions and by groups of jurisdictions. In such a situation, workers' consumption just equals their marginal product, $c = f'(\bar{\ell})$.

The intuition behind this result can most easily be understood for the ESS (i.e., for $m = 1$). Recall that the ESS is a Nash equilibrium in a game where players care for their relative performance (see (11)). Consider a symmetric situation with positive transfers $s_i = s > 0$ for all i. A slight deviation downwards to $s' < s$ by, say, jurisdiction 1 will make the poor's consumption level c decrease (see (8)). This leads both to a direct reduction in welfare by U_c, and to an indirect increase in welfare by $U_y \bar{\ell}$, since redistribution has now become less costly for the rich. These two effects, whatever their sign when summed up, are identical for all jurisdictions – and therefore irrelevant for their relative performance. However, also the migration equilibrium changes. The mutant jurisdiction 1 will experience a drop in its poor population by, say, $\mathrm{d}\ell_1 < 0$ while all other jurisdictions will see their poor population increase by $-\mathrm{d}\ell_1/(N-1)$. The utility effect from this change is $-sU_y \mathrm{d}\ell_1 > 0$ in jurisdiction 1 and $sU_y \mathrm{d}\ell_1/(N-1) < 0$ in every other jurisdiction. Altogether, the utility increase [loss] for the mutant jurisdiction 1 is larger [smaller] than for non-deviating jurisdictions. At any uniform $\mathbf{s} = (s, \ldots, s)$ with strictly positive s every jurisdiction has an incentive to cut back its transfers in order to improve its payoff, *relative* to others. Only zero transfers ($s = 0$) everywhere can be an ESS, since downward deviations are no longer feasible. The same line of reasoning also applies when there are m mutants rather than just one: starting from any positive, uniform transfer level, cutting back transfers is always beneficial in relative terms to the jurisdictions that do so.

[4] We omit most of the arguments to simplify the expressions. Notation U_y^m is meant to indicate $\frac{\partial}{\partial y} U(f(\ell^m(s^m, s^{N-m})) - \hat{c}(s^m, s^{N-m}) \cdot \ell^m(s^m, s^{N-m}), \hat{c}(s^m, s^{N-m}))$; analogously for the other partials.

The drastic tendency to zero redistribution follows in this model from the pure public good nature of the poor's consumption level. This, in turn, is a result of the strong externalities caused by free mobility of the poor population. In a model with frictions to mobility one would always expect to see some redistribution. The main point of Proposition 1, though, would remain valid: relative performance considerations imply an erosion of redistribution as long as migration externalities are present.

4 Imitate-the-best and experimentation

So far, we have described a static economy. To capture the experimentation-with-imitation feature of a laboratory federation, suppose that decisions on transfers are made in any period over a long time horizon, say in periods $t = 0, 1, 2, \ldots$. In each period $t > 1$, all governments observe all governments' subsidies and performances (in terms of realized payoffs (7)) from the previous period. For simplicity, assume that the set S of feasible transfers to the poor is a (potentially very fine) grid $G = \{0, \delta, 2\delta, \ldots, \nu\delta\}$ with $\delta > 0$ and $\nu \in \mathbb{N}$ from which subsidides can be chosen. We require that 0 is on the grid. Consider the following imitation dynamics:

- **Imitate-the-best:** With some positive probability less than one, each government gets the opportunity to adjust its subsidy to that subsidy (or, if there are several, to one of those subsidy levels) that performed best in the previous period. If all governments chose the same subsidy in the previous period, no adjustment occurs.

- **Experimentation:** with independent probability $\varepsilon > 0$, each government ignores the rule to imitate-the-best. Rather, it sets a subsidy in G according to some probability distribution with full support on G.

This dynamics entails important features of laboratory federalism: The imitate-the-best rule captures that governments learn from the experience of others and adopt successful policies observed elsewhere. By contrast, experimentation can generate policies that have not been adopted previously. We do not need to specify the rationale for experimentation here; it may occur due to deliberate design, error, inertia, innovation, political considerations outside the model, etc.

At each period $t = 0, 1, 2, \ldots$, the imitation dynamics with experimentation leads the economy to a (typically different) state, identified with the transfer vector $\mathbf{s} \in G^N$. The dynamics forms an (ergodic) Markov chain for transfer vectors in discrete time, indexed by the experimentation probability ε. The stationary distribution of such a stochastic process is unique and represents the frequency distribution of transfer vectors over time. We are interested in *stochastically stable states*, i.e., in subsidy vectors $\mathbf{s} \in G^N$ that are in the support of the *limit* invariant distribution of the Markov chain as ε goes to zero (see e.g. Young, 1993). These states can be interpreted as the long-run outcomes of the dynamics.

Alós-Ferrer and Ania (2005, Proposition 4) show generally that if a symmetric N-player game has a globally stable ESS s^*, then the monomorphic state $\mathbf{s}^* = (s^*, \ldots, s^*)$ is the unique long-run outcome of the imitation-cum-experimentation dynamics for that game. Using Proposition 1, this immediately implies our next result.

Proposition 2 *If decentralized redistribution is guided by imitation of best-performing policies with occasional experimentation, the long-run outcome will not entail any redistribution:* $\mathbf{s}^* = (0, \ldots, 0)$ *is the unique stochastically stable state.*

The intuition for Proposition 2 is as follows. If the dynamics is at $\mathbf{s}^* = (0, \ldots, 0)$, any mutant will receive worse payoffs than the incumbents and hence will never be imitated; this is the ESS property. However, starting from any symmetric situation $\mathbf{s} = (s, \ldots, s)$ with $s > 0$, a single mutant to $s^* = 0$ will, by the property of $N-1$-stability, attain higher payoffs than the incumbents and, hence, be imitated. This drives the dynamics towards $\mathbf{s}^* = (0, \ldots, 0)$.

As argued in Alós-Ferrer and Schlag (2009, Proposition 5), this result can indeed be considerably strengthened: a globally stable ESS is the unique long-run outcome for *any* imitation rule such that actions with maximal payoffs are imitated with some positive probability while actions with lower payoffs than the own are never copied. Hence, laboratory federalism in the model of Wildasin (1991, 1994) will lead to a complete breakdown of redistribution for quite general imitation dynamics.

5 Discussion and conclusions

The result of zero transfers as an ESS, GSS, and stochastically stable state should be contrasted with the Nash equilibrium of the decentralized redistribution game of Section 2. A strategy $s^N \in S$ constitutes a symmetric Nash equilibrium if $\pi(s^N; s^N, \ldots, s^N) \geq \pi(s; s^N, \ldots, s^N)$ for all $s \in S$. Suppose that

$$\frac{U_c\left(f(\bar{\ell}) - f'(\bar{\ell}) \cdot \bar{\ell}, f'(\bar{\ell})\right)}{U_y\left(f(\bar{\ell}) - f'(\bar{\ell}) \cdot \bar{\ell}, f'(\bar{\ell})\right)} > \bar{\ell} \tag{13}$$

holds, meaning that the motivation in a jurisdiction to redistribute towards its poor in any symmetric situation is strong enough. Then the transfer level in a symmetric Nash equilibrium is implictly defined by the following condition:[5]

$$s^N = -\frac{f''(\bar{\ell})}{N-1} \cdot \left(\frac{U_c(y(s^N), c(s^N))}{U_y(y(s^N), c(s^N))} - \bar{\ell} \right). \tag{14}$$

Given assumption (13), the subsidy level s^N determined by (14) is positive – *some* redistribution will take place in equilibrium. Zero transfers can only constitute a Nash equilibrium if redistributive concerns are weak in the sense that (13) does not hold.

The subsidy level s^N in (14) is inefficiently low: a coordinated increase in s would lead to a Pareto improvement. In addition, the subsidy level s^N strictly decreases in the number of jurisdictions, N. If N gets very large, subsidies will approach zero according to (14). Then the welfare state breaks down and the inefficiencies associated with decentralization are most pronounced (see Wildasin, 1991, or more generally Wilson, 1999).

By contrast, note that the prediction of zero subsidies in Propositions 1 and 2 holds even when redistributive motives are strong and irrespective of the number N of jurisdictions: however large or small the federation, laboratory federalism *always* yields the most competitive and most inefficient outcome that standard fiscal competition can produce.[6]

Furthermore, together with (14), Propositions 1 and 2 illustrate that imitation and experimentation do not lead to a Nash equilibrium in general. Rather, the concept of finite-population ESS is the appropriate tool to study imitation outcomes in these settings.

The main finding of our paper, the breakdown of redistribution as the long-run outcome of an imitation dynamics with experimentation in a classical decentralized redistribution game, is certainly a bleak message for those who view fiscal federalism as a discovery procedure with mutual learning and the diffusion of good policies. Yet, it should be emphasized that this paper only studies one specific (though well-acclaimed) scenario of fiscal federalism, involving strong positive fiscal externalities among jurisdictions (in fact, a public goods game). Ania and Wagener (2013) present a scenario of decentralized redistribution where imitation-and-experimentation indeed is a learning procedure that helps jurisdictions to coordinate on reasonable policies. Contrasting this and other more promising observations on laboratory federalism with the negative result of this paper indicates that our understanding of policy learning and diffusion in federations is still incomplete.

[5] See the Appendix or Wildasin (1991) for details. On existence problems of Nash equilibria in games of fiscal competition see Bayindir-Upmann and Ziad (2005).

[6] This reproduces the observation for oligopolies where the ESS in Cournot games corresponds to the Walrasian (price-taking) outcome (see Schaffer, 1988; Vega-Redondo, 1997; Alós-Ferrer and Ania, 2005). For an application to tax competition, see Wagener (2013).

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Appendix: Nash equilibrium

The first derivative of (7) with respect to s_i is given by:

$$\begin{aligned}
\frac{\partial \pi_i(\mathbf{s})}{\partial s_i} &= U_y \cdot \left[\left(f'(\ell_i(\mathbf{s})) - c(\mathbf{s})\right) \frac{\partial \ell_i(\mathbf{s})}{\partial s_i} - \ell_i(\mathbf{s}) \frac{\partial c(\mathbf{s})}{\partial s_i} \right] + U_c \cdot \frac{\partial c(\mathbf{s})}{\partial s_i} \qquad (15)\\
&= U_y \cdot \frac{s_i}{f''(\ell_i(\mathbf{s}))} \left(1 - \frac{1/f''(\ell_i)}{\sum_{j=1}^N 1/f''(\ell_j)} \right) - (U_y \cdot \ell_i(\mathbf{s}) - U_c) \cdot \frac{1/f''(\ell_i)}{\sum_{j=1}^N 1/f''(\ell_j)},
\end{aligned}$$

Suppose that $s_i = 0$ for all i. Then $\ell_i = \bar{\ell}$, $c = f'(\bar{\ell})$ and $y_i = f(\bar{\ell}) - f'(\bar{\ell}) \cdot \bar{\ell} = f(\bar{\ell}) - c \cdot \bar{\ell}$ for all i. The derivative of the payoff function becomes

$$\frac{\partial \pi_i(0, \ldots, 0)}{\partial s_i} = -\frac{1}{N} \left[U_y \left(f(\bar{\ell}) - f'(\bar{\ell}) \cdot \bar{\ell}, f'(\bar{\ell}) \right) \cdot \bar{\ell} - U_c \left(f(\bar{\ell}) - f'(\bar{\ell}) \cdot \bar{\ell}, f'(\bar{\ell}) \right) \right],$$

which is strictly positive if and only if (13) holds. In that case any jurisdiction has an incentive to subsidize its poor when all jurisdictions set $s = 0$, implying that s^N is strictly positive in a symmetric Nash equilibrium.

An interior solution for a maximum of (7) with respect to s_i must satisfy the necessary condition that (15) is equal to zero:

$$U_y \left[s_i \frac{\partial \ell_i(\mathbf{s})}{\partial s_i} + \ell_i \frac{\partial c(\mathbf{s})}{\partial s_i} \right] = U_c \frac{\partial c(\mathbf{s})}{\partial s_i}.$$

If the payoff function (7) is quasi-concave and a symmetric Nash equilibrium exists, this condition must hold for all i in equilibrium. Using the comparative statics in Footnote 3 one obtains condition (14).