WORKING PAPERS

A Dynamic Politico-Economic Model of Intergenerational Contracts

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March 2013

Working Paper No: 1304

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

A Dynamic Politico-Economic Model of Intergenerational Contracts*

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March 8th, 2013

Abstract

This paper proposes a dynamic politico-economic theory of intergenerational contracts, whose driving force is the intergenerational conflict over government spending. Embedding a repeated probabilistic voting setup in a standard OLG model with human capital accumulation, we find that the empowerment of elderly constituencies is key in order to enforce productive policies. The paper characterizes the Markov-perfect equilibrium of the voting game, as well as the welfare properties. The main results are: (i) the existence of a Markov-perfect equilibrium which attains a growth-enhancing intergenerational contract does not require pre-commitment through the establishment of long-lasting institutions; (ii) the political sustainability of the intergenerational contract relies solely on the politico-economic fundamentals that are payoff-relevant for future constituents; (iii) the implementation of pork-barrel transfers does not necessarily crowd out productive public investment; and, (iv) the greater the degree of intergenerational conflicts over the government spending, the lower the inefficiency.

JEL Classification: D72, E62, H23, H30, H53

Keywords: Efficient allocation, human capital, intergenerational transfers, Markov-perfect equilibrium, repeated voting.

*We are indebted to Graziella Bertocchi and Michele Boldrin for their advice and mentorship. Marco Bassetto, Roland Benabou, Alejandro Cunat, Vincenzo Denicolò, Daniel Garcia, Bård Harstad, John Hassler, David K. Levine, Anirban Mitra, Nicola Pavoni, Gilles Saint Paul, Karl Schlag, Paolo Siconolfi, Gerhard Sorger, Kjetil Storesletten, and Fabrizio Zilibotti provided valuable comments. We also thank seminar participants at 2011 NBER Summer Institute Meeting in Income Distribution and Macroeconomics, 2011 SED Annual Meeting in Ghent, 2010 NSF/NBER/CEME Conference in Mathematical Economics and General Equilibrium Theory in New York, XV Workshop on Macroeconomic Dynamics in Vigo, 2^{nd} Conference on Recent Development in Macroeconomics in Mannheim, 24^{th} ESPE Conference in Essen and 8^{th} Workshop on Macroeconomic Theory in Pavia, as well as the seminar participants at the Bank of Italy, ETH in Zurich and the Universities of Bologna, Modena, Milan, Napoli, Oslo and Vienna for the useful discussions. All errors are our own.

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Why should I care about future generations? What have they done for me?" (Groucho Marx)

1 Introduction

The sustainability of intergenerational redistributive welfare programs is a crucial issue in the current political debate: the intergenerational disagreement over the allocation of public resources turns out to be a battleground, pitting young against old and taxpayers against recipients, especially when balanced budget conditions are required to be met. For this reason, it becomes critical to explore (i) the conditions under which intergenerational transfers can be implemented and (ii) why the welfare system developed so far has become a stable institution of the modern society.

Two major stylized observations that pertain to demographic and political considerations motivate this study. On the one hand, the shift of the age balance toward the elderly alters the economic and institutional nature underlying the enforcement of redistributive welfare programs. Focussing on the U.S., the median age of the population declined steadily in the period from 1955 to 1970 as an echo effect of the baby boom, and began to increase in the 1970s. At the same time, the fraction of the elderly - people aged 65+ - of the population steadily increased, whereas the fraction of youth - people under the age of 25 - of the population steadily decreased.[1] The simultaneous variation in both the median age (Figure 1) and the relative cohort size (Figure 2) had an impact both on the financial solvency of the public system - since the share of recipients increased, while the share of contributors decreased - and on the outcome of the voting competition - as population aged, so did the voters.

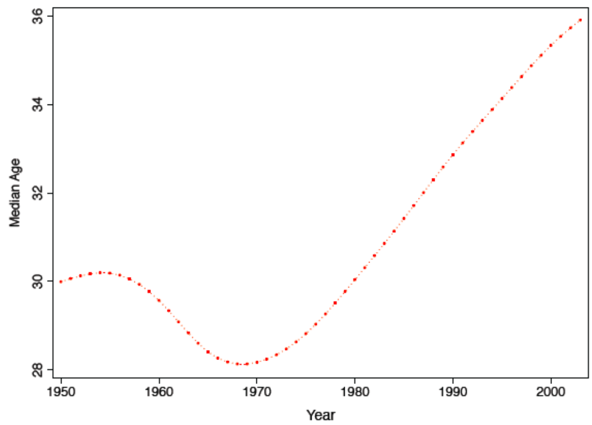

Fig. 1: Median Age of the Population in the U.S.

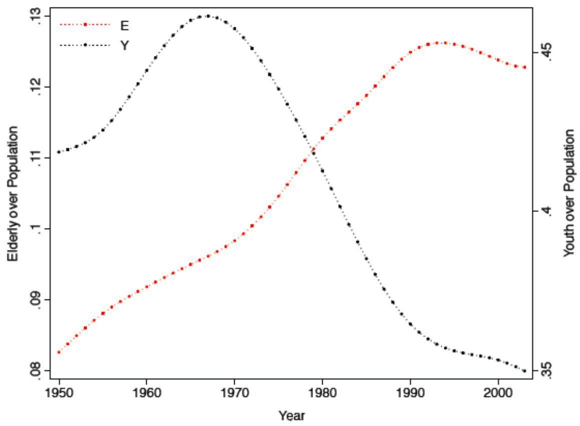

Fig. 2: Fraction of Elderly and Youth of the Population in the U.S.

On the other hand, the political power imbalance among different cohorts does not just rely on demographic considerations. As has been widely documented by political scientists, single-mindedness is a crucial ingredient in the political success of an interest group or voting block. According to Mulligan and Sala-i-Martin (1996), while the young citizens disperse their political interests among different and often contrasting issues, their older counterparts are likely to target fewer programs while making their voting decisions, such as social security and medicare. Accounting only for demographics and keeping political influence per beneficiary as constant, one would expect in the U.S. in the period from 1980 to 2003 the share of GNP devoted to government programs for the elderly to increase in a commensurate fashion with their population share (a fraction of 1.089); analogously, the share of the youth public spending per GNP to decrease in tandem with the fall of the population share under the age of 25 (a

[1] Source: U.N. (2011).

fraction of 0.85). On the contrary, Figures 3 and 4 show how, on the one hand, the public spending for the elderly (including social security and health care) has grown by a factor of 1.196, and, on the other hand, the public spending for the youth (including education) has also grown by a factor of 1.06.[2]

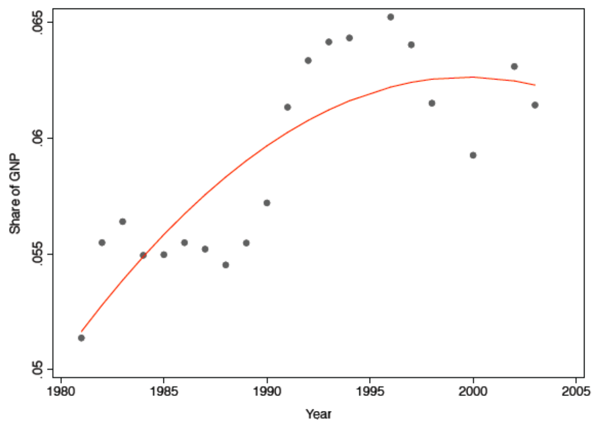

Fig. 3: Government Spending (Pension and Health Care) per GNP in the U.S.

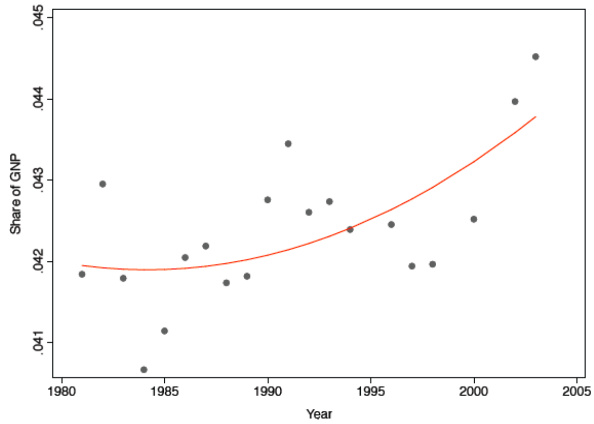

Fig. 4: Government Spending (Education) per GNP in the U.S.

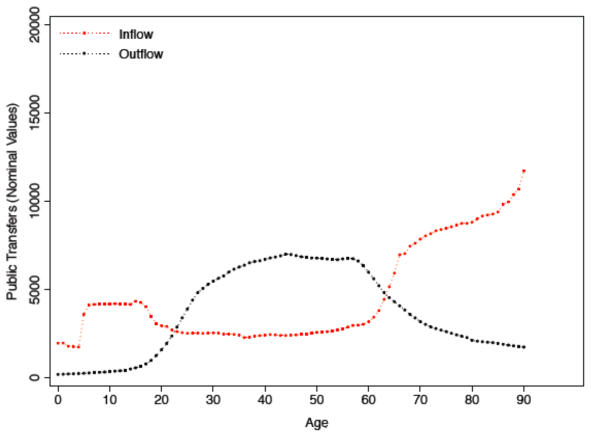

Fig. 5: Per-capita Fiscal Pattern in the U.S. (1980)

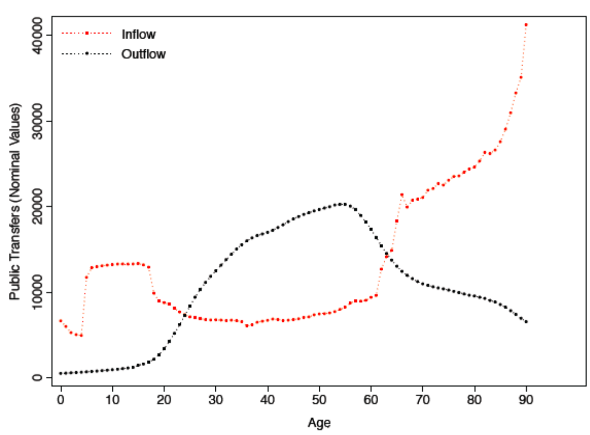

Fig. 6: Per-capita Fiscal Pattern in the U.S. (2003)

In this paper, we show that an increase in the political clout of elderly voters can explain this counter-intuitive simultaneous co-movement of age-targeted public spending and the difference from what is expected due to shifts in the demographic structure alone. Furthermore, by adopting a tractable dynamic politico-economic model in a general equilibrium framework, we provide a micro-foundation of the cross-country and historically stable intergenerational contract, which entails both the young and the old enjoying benefits (in the form of backward and forward transfers, respectively) and the working-age population paying taxes.[3] Figure 5 and 6 show how the age-targeted fiscal policy pattern has evolved between 1980 and 2003 in the U.S.

The analysis is based on a standard overlapping generation economy populated by ideologically heterogeneous agents, living up to three periods. At the young age individuals acquire skills, whereas when they are adults they offer inelastic labor and partially save their proceeds; finally, when they become old they retire. From their adult age onwards, agents exert their voting right. The presence of

[2] Source: Lee, Donehower, and Miller (2011).

[3] Throughout the paper we adopt the notion of forward and backward intergenerational transfers as introduced by Rangel (2003). The former are youth-age-targeted transfers that generate a cost for the current generation and a benefit for the future one, being crucial for future productivity. By contrast, the latter are old-age-targeted transfers, generating a cost for the current generation and a benefit for the past one.

a political system is justified by the need to finance the provision of public spending under incomplete credit markets, which otherwise would preclude the young from accumulating human capital. To infer the role of incomplete contracting, let us consider the case with no transaction costs. In this scenario, an intergenerational form of the Coase Theorem would arise: children would commit themselves to compensating their parents for the cost of the investment, and parents would have an incentive to finance them. However, in many organizations such contracts are not allowed, for example because of the absence of collateral for the youth.[4] Furthermore, even with complete markets, private contracts can at most comply with short term projects, failing to internalize all the technological spillover whose impact outranges the individual lifespan and, in turn, to achieve long-term growth.[5] Therefore, it becomes relevant to assess the conditions under which non-market institutions are able to enforce efficient intergenerational agreements.

With this aim, we consider constituents who envision a government of office-seeking but short-lived representatives in a majoritarian probabilistic voting environment. The politicians compete by proposing a fiscal bundle of intergenerational transfers and taxes subject to an intra-period balanced budget.[6] The politicians' objective is to win the largest number of votes among the currently living voters, with no concern for the well-being of unborn generations. We have intentionally made each generation completely selfish in order to focus on the implications of having short-term mandate politicians. Furthermore, we abstract away from commitment considerations in the following sense: no government can bind the policies of its successors, regardless of whether the successor belongs to the same party or not. Hence, promises made during an electoral campaign are not credible unless they are optimal ex post, i.e. when the party is in power. We embody the "minor causes should have minor effects" principle to implement differentiable Markov-perfect politico-economic equilibria and to identify the fundamental and robust forces that shape the size of the welfare state and, therefore, growth. Specifically, we model policy decisions as the outcome of a non-cooperative dynamic Stackelberg game: in each period, the government selects a fiscal platform, internalizing through the evolution of state variables the effect on subsequent policy decisions and, in turn, on the welfare of the current young generation. Such policies are also the equilibrium outcome of a finite horizon game when the time goes to infinity. The equilibrium refinement we adopt rules out equilibria in which the current political outcome depends directly on the past outcome, as in reputation equilibria.[7] This seems appropriate in our setup, where periods are very long (around 30 years) and political competition takes place among different agents at each point in time.

Our characterization is quite flexible for a large class of preferences and technology and, despite the model's simple structure, generates several interesting results, consistent with the stylized observations described above.

A simple model with only human capital helps to articulate the main argument, which we resume

[4] For a recent discussion on the missing credit markets to finance human capital, see Kehoe and Levine (2000).

[5] From a normative perspective, Boldrin and Montes (2005) have formalized public education and pay-as-you-go system as two parts of an intergenerational contract where social security is the return from the investment into the human capital of the next-period generation. In presence of credit market constraints, the authors have shown how an interconnected intergenerational transfers scheme can replicate the allocation achieved by complete markets. Relaxing the definition of optimality by explicitly considering the positive spillover generated by productive investments, Docquier, Paddison, and Pastieau (2007) have pointed out that there are no justifications to providing backward benefits, when the dynastic welfare weight is sufficiently high.

[6] To single out the impact of political institutions on intergenerational transfers and to highlight the asynchronous timing of public exchange we assume away the provision of public goods - a key element in the political economy of fiscal policy. See Bassetto (2008) for a treatment of the role of public good provision in an OLG environment.

[7] Previous literature has focused on reputational mechanisms to justify the provision of productive investment. Although, trigger strategies may be analitically convenient, they lead to multiplicity of equilibria. Furthermore, they require coordination among agents and costly enforcement of a punishment technology, which may not work when agents are not patient enough. Finally, they are not robust to refinement such as backward induction in a finite horizon economy when time tends to infinity. See among others Rangel (2003).

here: the existence of a Markov-perfect equilibrium which attain a growth-enhancing intergenerational contract does not require pre-commitment through the establishment of institutions that necessarily outlive the current government and bind future decision-makers. In a probabilistic voting framework, the parents always have a claim over the children's productivity, so as to say that they partially own their production. Therefore, selfish adults can have incentives to invest in forward transfers in order to raise their children's labor productivity and, by exerting their influence, partially grab the bigger stake as a political rent when old. To recap: when the relative political clout of the elderly is large enough, the two age-targeted programs can self-sustain. Clearly, the redistributive scheme works only if the cost of providing forward transfers is at least compensated by the expected benefit of receiving backward transfers when old. For plausible parameter values, the model foretells an inverse-U shaped relationship between the political influence of the elderly voters and the provision of productive transfers and, in turn, the rate of economic growth. This result can partially explain the gap of the age-targeted transfers with respect to the pure variation due to demographics, as documented above.

Interestingly, the efficient equilibrium can be enforced only because of the establishment of short-term agreements between constituents and representatives. If politicians were committed at each time, then they would finance productive spending and redistribute today by promising expropriation of next-period adults. Thus, the commitment solution would fail in achieving long-term growth.

We also deal with a more realistic environment in which agents accumulate physical capital. Here, the conflict between age-cohorts arises not only because of different life spans, but also from the difference in ownership of productive factors as well as the source of income. In this case, short-lived politicians take into account through the evolution of physical capital, that future representatives will compensate the fiscal cost of current adults by paying backward transfers in their old age. Thus, the expected policy response of future politicians reduces the current cost of transferring resources to the elderly. In equilibrium, we show how the implementation of backward transfers does not crowd out productive public investment, which is in contrast to the common view.[8] However, unlike the case with only human capital, efficient growth is not always guaranteed. Indeed, the politico-economic outcome can exhibit a growth rate that is substantially below the efficient rate of the social planner, who attaches independent decaying weights to all future generations. Furthermore, for the same Pareto weights, the gross return to capital generally fall below the efficient one. These two simultaneous sources of distortion in the capital accumulation (i.e. underaccumulation of human capital and overaccumulation of physical capital) also imply that the Cass sufficient condition (1972) for dynamic efficiency fails in general to be met. The intuition for this qualitative result is that the benefits of growth largely spread over onto subsequent generations, who are not required to appropriately reward the previous generations for their sacrifices. Due to the lack of commitment, each generation can solely extract some political rents from growth through its influence on subsequent policy decisions. The governments exploit this mechanism to the largest possible extent as compatible with their political agenda.

The degree of inefficiencies caused by intergenerational conflicts is quantified in the analytical examples, where we evaluate how distant the solution is from the Pareto-frontier for any arbitrary set of weights. The inefficiency is shown to be lower, the larger the degree of substitutability among factors of production, i.e. the greater the degree of the intergenerational conflicts over public resources due

[8] Recent political economy models of growth (see, for instance, Alesina and Rodrick, 1994; Persson and Tabellini, 1994; Azzimonti, 2011) have suggested that the political disagreement over the composition of public expenditure and uncertainty may lead to extensive redistribution, which has a depressive effect on growth. As long as parties compete to detain power via democratic process, politicians tend to be endogenously short-sighted. As a result, the economy experiences under-investment of productive assets. Our framework yields a different perspective. As long as redistribution is crucial to reach social consensus for growth-oriented policies, intergenerational conflicts over the redistribution of public resources may enhance growth and improve welfare.

to the difference in the ownership of productive assets. It vanishes when the elasticity of substitution among productive factors tends to be infinite. Technically, it comes from the fact that the lower the technological substitutability among the factors of production, the larger the strategic political power exerted by the adults compared to the elderly and the weaker the strategic complementarity channel between forward and backward transfers, which is the unique mechanism to enforce an efficient allocation. This effect appears only in overlapping generation economies with long-lasting productive spending and has not been derived in other papers in the literature. Furthermore, this provides new fundamentals to the theory which recognizes the link between the elasticity of substitution in the technology and the economic growth (Klump and De La Grandville, 2000).

Our theoretical framework applies to a variety of settings that essentially involve two fundamental aspects: (i) the nature of short-term agreements among politicians and current living voters, i.e. the absence of commitment, and (ii) the prospect of follow-up intergenerational contracts, which serves as a discipline device to implement current policies. Clearly, there are many applications that fit into this setting. A prominent example is the government decision of how much to invest on forward productive transfers such as environmental preservation or R&D and education. These programs entail a transfer to future generations that is financed through taxes paid by current generations and whose benefits are long-lasting. In our political economy of intergenerational exchange, the backward intergenerational transfer plays the role of a pay-as-you-go social insurance program such as social security or medicare.

This paper augments the literature of dynamic politico-economic models with overlapping generations that incorporates forward-looking decision makers in a multidimensional policy space (Krusell, Quadrini and Rios-Rull, 1997). Previous studies have introduced altruism (Tabellini, 1991, Kaganovich and Zilcha 1999), public good provision (Bassetto, 2008, Hassler, Storesletten and Zilibotti, 2007, and Song, Storesletten and Zilibotti, 2012), or reputational mechanisms (Bellettini and Berti Ceroni, 1999, and Rangel, 2003) to justify the emergence of an intergenerational contract. While recognizing the theoretical relevance of these channels, we emphasize the role played purely by political institutions and strategic incentives: each adult generation's willingness to transfer wealth to the old and the young depends on its beliefs that the same thing will happen in the subsequent time period, which are enforced by established institutions, like democratic voting.

Adopting a perspective similar to ours and in the spirit of Samuelson (1958), recent works (Grossman and Helpman, 1998, Azariadis and Galasso, 2002, Forni, 2005, and Gonzales-Eiras and Niepelt, 2008) have developed models of social security in a repeated voting environment. However, unlike our work, most of these findings suffer in equilibrium of indeterminacy. Moreover, they have focused only on backward transfers, whereas our theory recognizes the fundamental link between productive and redistributive public spending. Closer to our work is Gonzales-Eiras and Niepelt (2011). However, they have limited their study to the analysis of demographic ageing on long-term growth. None of the existing papers - to the best of our knowledge - have provided an implicit characterization of the general equilibrium politico-economic outcome and have posited implications in terms of welfare analysis like we have in this present paper.[9]

The rest of the paper is organized as follows. Section 2 describes the model environment. Section 3 derives the equilibrium characterization. Section 4 presents the social planner allocation and provides the criterion for the welfare comparison. Section 5 presents two analytical examples. Finally, Section 6 concludes. All proofs are contained in the Appendix.

[9] Battaglini and Coate (2007) have explored how pork-barrel spending affects the overall size of government and distorts investment in public capital good. They have focused the analysis on the efficiency of the steady state level of taxation and allocation of tax revenues. Unlike us, they have explored an environment where infinite-living agents take policy decisions by legislative bargaining.

2 The Model

Consider an overlapping generation (OLG) economy inhabited by an infinite number of ideologically heterogeneous agents, living up to three periods: young, adult and old. $i \in \{a, o\}$ labels the adult and elderly cohorts, respectively. Agents of different age differ in their wealth holdings. Time is discrete, indexed by t, and runs from zero to infinity. Population grows at a constant rate $\nu - 1$, thus the mass of the adult generation born at time $t-1$ and living at time t is equal to $N_t^{t-1} = \nu^t N_0$. At each time two short-lived office-seeking parties, denoted by $\iota_t \in \{\mathcal{L}_t, \mathcal{R}_t\}$, compete by proposing a political platform in order to maximize the probability of winning election.

2.1 Households

An agent j born at time $t-1$ and living at time t evaluates consumption and ideology according to the following expected and additive intertemporal (non altruistic) utility function:

$$u\left(c_{\iota_t}^a\right) + \varsigma_{j,\iota_t}^a + \beta E_{\iota_t}\left[u\left(c_{\iota_{t+1}}^o\right) + \varsigma_{j,\iota_{t+1}}^o\right] \tag{1}$$

where $\beta \in (0,1)$ is the individual discount factor and $E_{\iota_t}[\cdot]$ is the expectation operator conditioned on the political platform implemented by the current incumbent party. The random variable ς_{j,ι_t}^i summarizes the utility derived by agent j belonging to cohort i at time t from political factors that are orthogonal to consumption. It embeds two components: an idiosyncratic ideological bias, σ_j^i, and an aggregate ideological bias, η. Thus,

$$\varsigma_{j,\iota_t}^i = \left(\sigma_j^i + \eta\right)\mathcal{I}_{\iota_t}$$

where $\mathcal{I}_{\iota_t}$ is an indicator function such that $\mathcal{I}_{\mathcal{R}_t} = 1$ and $\mathcal{I}_{\mathcal{L}_t} = 0$.

Assumption 1 (Ideology) *The i.i.d. random variables σ_j^i and η are uniformly distributed over the interval $\left[-\frac{1}{2\boldsymbol{\sigma}^{\mathbf{i}}}, \frac{1}{2\boldsymbol{\sigma}^{\mathbf{i}}}\right]$ and $\left[-\frac{1}{2\boldsymbol{\eta}}, \frac{1}{2\boldsymbol{\eta}}\right]$, respectively.*

In each period, voters form opinions about a candidate's position (for example, religious issues, constitutional law, civil rights, and so on) and personal characteristics (for example, honesty, leadership, trustworthiness, and so on). The idiosyncratic ideological bias, σ_j^i, is drawn from cohort specific distributions. Thus, individuals belonging to the same cohort may vote differently. The aggregate ideological bias, η, measures the average popularity of candidates from party $\mathcal{R}_t$ relative to those from party $\mathcal{L}_t$. Then, individuals belonging to different cohorts may support the same party. Both shocks are i.i.d. over time, hence candidate specific. A zero value of bias indicates neutrality in terms of voters' ideology, while positive values indicate that agent j prefers the candidate belonging to party $\mathcal{R}_t$ over his opponent.

Assumption 2 (Utility) *The function $u : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, strictly increasing and concave, and satisfies the usual Inada condition, i.e. $\lim_{c \to 0} u_c(\cdot) = \infty$ and $\lim_{c \to \infty} u_c(\cdot) = 0$.*

$c_{\iota_t}^a$ denotes the consumption at time t when adult and $c_{\iota_{t+1}}^o$ represents the consumption at time $t+1$ when old. In the first period of life (i.e. childhood), individuals do not consume. When young, agents spend all their time endowment in acquiring skills if productive forward transfers, f_{ι_t}, are publicly provided without having access to private credit markets. At the adult age, individuals inelastically supply labor and use their gross income, $w_t h_t$, for consumption and saving, s_t. When old, agents retire and consume their savings capitalized at a gross rate of return, R_{t+1}. Depending upon the political setting, the adults and the old pay different amounts of lump-sum taxes (possibly negative) denoted by z_{ι_t} and b_{ι_t}, respectively. Thus, the individual budget constrains for the adult and elderly agents are:

$$c^a_{\iota_t} + s_t \leq w_t h_t - z_{\iota_t} \tag{2}$$

$$c^o_{\iota_{t+1}} \leq R_{t+1} s_t + b_{\iota_{t+1}} \tag{3}$$

The present value of after-tax lifetime income of an agent born at time $t-1$ is given by $I_t \equiv I^a_t + I^o_{t+1}$, where $I^a_t \equiv w_t h_t - z_{\iota_t}$ and $I^o_{t+1} \equiv \frac{b_{\iota_{t+1}}}{R_{t+1}}$.

At the initial time $t = 0$, an exogenous human capital endowment $h_0 > 0$ characterizes the economy. The budget constraint of each adult is equal to $c^a_{\iota_0} = w_0 h_0 - z_{\iota_0} - s_0$. At the same time, elderly agents are endowed with an exogenous amount of physical capital, $k_0 > 0$. Their individual budget constraint is $c^o_{\iota_0} = R_0 k_0 + b_{\iota_0}$.

2.2 Technology

At each time t, the economy produces a single homogenous private good, Y_t, combining physical capital, K_t, and aggregate human capital or effective labor supply, H_t, according to the following technology:

$$y_t = \Theta(k_t, h_t) \tag{4}$$

where $Y_t \equiv y_t N^{t-1}_t$, $K_t \equiv k_t N^{t-1}_t$, and $H_t \equiv h_t N^{t-1}_t$.

Assumption 3 (Production) *The function $\Theta : \mathbb{R}^2_+ \to \mathbb{R}_+$ exhibits a constant return to scale and it is strictly monotonic increasing and weakly concave in each of the two inputs with $\Theta(0, h_t)$, $\Theta(k_t, 0) \geq 0$ and $\Theta_{k_t h_t}$, $\Theta_{h_t k_t} \geq 0$.*

Under Assumption 3 it follows that $y_t = h_t \Theta\left(\frac{k_t}{h_t}, 1\right) \equiv h_t \vartheta\left(\tilde{k}_t\right)$ and, in turn, $\tilde{y}_t = \vartheta\left(\tilde{k}_t\right)$, where $\tilde{y}_t \equiv \frac{y_t}{h_t}$ and $\tilde{k}_t \equiv \frac{k_t}{h_t}$ refer to the per-efficiency units of final good and physical capital, respectively. The inverse demand for factor prices are $\Theta_{k_t} = \vartheta_{\tilde{k}_t}\left(\tilde{k}_t\right)$ and $\Theta_{h_t} = \vartheta\left(\tilde{k}_t\right) - \tilde{k}_t \vartheta_{\tilde{k}_t}\left(\tilde{k}_t\right)$. Let us denote by ζ the elasticity of factor substitution. Thus, $\Theta_{k_t h_t} = \Theta_{h_t k_t} = \frac{1}{\zeta} \frac{\vartheta_{\tilde{k}_t}\left(\tilde{k}_t\right)}{h_t} \frac{\left(\vartheta\left(\tilde{k}_t\right) - \tilde{k}_t \vartheta_{\tilde{k}_t}\left(\tilde{k}_t\right)\right)}{\vartheta\left(\tilde{k}_t\right)}$ and $\Theta_{k_t k_t} = -\frac{1}{\zeta} \frac{\vartheta_{\tilde{k}_t}\left(\tilde{k}_t\right)}{k_t} \frac{\left(\vartheta\left(\tilde{k}_t\right) - \tilde{k}_t \vartheta_{\tilde{k}_t}\left(\tilde{k}_t\right)\right)}{\vartheta\left(\tilde{k}_t\right)}$. Physical capital depreciates fully after one period. The human capital of an adult born at time t is produced according to a technology, which combines productive forward transfers, f_{ι_t}, and parental human capital, h_t, as complement factors:

$$h_{t+1} = \Phi(f_{\iota_t}, h_t) \tag{5}$$

Assumption 4 (Human Capital) *The function $\Phi : \mathbb{R}^2_+ \to \mathbb{R}_+$ exhibits a constant return to scale and it is strictly monotonic increasing and strictly concave in each of the two inputs with $\Phi(0, h_t)$, $\Phi(f_{\iota_t}, 0) \geq 0$ and $\Phi_{f_{\iota_t} h_t}$, $\Phi_{h_t f_{\iota_t}} > 0$.*

Under Assumption 4, a higher level of knowledge attained by one generation makes it less costly for the next one to achieve the same level (human capital spillover). It follows that $h_{t+1} = h_t \Phi\left(\frac{f_{\iota_t}}{h_t}, 1\right) \equiv h_t \varphi\left(\tilde{f}_{\iota_t}\right)$, where $\tilde{f}_{\iota_t} \equiv \frac{f_{\iota_t}}{h_t}$ denotes the per-efficiency units of productive transfers. The human capital growth rate is equal to $\frac{h_{t+1}}{h_t} = \varphi\left(\tilde{f}_{\iota_t}\right)$. The marginal impact of forward transfers and parental human capital on the human capital production are $\Phi_{f_{\iota_t}} = \varphi_{\tilde{f}_{\iota_t}}\left(\tilde{f}_{\iota_t}\right)$ and $\Phi_{h_t} = \varphi\left(\tilde{f}_{\iota_t}\right) - \tilde{f}_{\iota_t}\varphi_{\tilde{f}_{\iota_t}}\left(\tilde{f}_{\iota_t}\right)$, respectively.[10]

2.3 Fiscal Constitution

In each period, short-lived governments, democratically elected by their constituents, use fiscal authority to transfer income among different age-groups. The transfers serve simultaneously the political scope of the elected representatives and the economic needs of their constituents. We assume that politicians are prevented from borrowing, and the public balance must hold in every period. It implies:

$$N_t^t f_{\iota_t} + N_t^{t-1} z_{\iota_t} + N_t^{t-2} b_{\iota_t} = 0 \tag{6}$$

where the collection $\{f_{\iota_t}, z_{\iota_t}, b_{\iota_t}\}$ represents the age-targeted fiscal bundle. When the middle-aged are the net payers and the other generations are the net receivers, then youth-age-targeted transfers are tagged as *forward transfers*, since they aim to benefit future generations, whereas old-age-targeted transfers are defined as *backward transfers*. At each time t, Eq. (6) reduces the multidimensionality of the political platform to a bi-dimensional plan $p_{\iota_t} \equiv \{z(b_{\iota_t}, f_{\iota_t}), b_{\iota_t}, f_{\iota_t}\}$ where the fiscal feasibility conditions require that $b_{\iota_t} \in \left[\underline{b}_t, \bar{b}_t\right]$ and $f_{\iota_t} \in \left[0, \bar{f}_t\right]$ with $\underline{b}_t \equiv -\nu R_t k_t$, $\bar{b}_t \equiv \nu w_t h_t$, and $\bar{f}_t \equiv \frac{1}{\nu}\left(w_t h_t - \frac{b_t}{\nu}\right)$.

Definition 1 (Intergenerational Contract) *An intergenerational contract is a mutual political agreement, which simultaneously enforces both backward and forward transfers.*

3 Maximization and Equilibrium

We characterize the equilibrium of the economy as a subgame perfect politico-economic equilibrium. Within each time period, the sequence of moves is as follows:

i. A new generation of young people is born;

ii. Before the realization of the ideological shocks among voters, parties democratically compete proposing their political platforms;

iii. After the realization of the ideological shocks, agents vote for their preferred candidates;

iv. Agents save and firms hire workers and rent capital;

v. The older generation dies; the young and adult generations age and become adult and old, respectively.

Within a given period, the sequential politico-economic game can be viewed as Stackelberg and it is solved by backward induction. This procedure is the standard fixed-point problem, which nests two interdependent parts. Firstly, the adults determine the individual level of savings and firms produce the homogenous final good given the law of motion for policies (competitive economic equilibrium). Secondly, short-lived office-seeking politicians promise voters an age-targeted fiscal bundle in order to maximize the probability of winning the election (politico-economic equilibrium). The final fixed-point problem is to ensure that the law of motion for policies underlying the competitive economic equilibrium is consistent with the political selection.

[10] From now on, we will abstract from the functional arguments. For example, $\vartheta_t \equiv \vartheta\left(\tilde{k}_t\right)$ and $\varphi_t \equiv \varphi\left(\tilde{f}_{\iota_t}\right)$.

3.1 Competitive Economic Equilibrium

In a competitive economic equilibrium, each individual j in the adult-age chooses her lifetime consumption taking factor prices and fiscal policies as given. Maximizing Eq. (1) subject to the individual budget constraints, Eqs. (2) and (3), and the feasibility constraints $c^a_{\iota_t} \geq 0$, $c^o_{\iota_{t+1}} \geq 0$, and $s_t \geq 0$, the following first order condition yields:

$$u_{c^a_{\iota_t}} \geq \beta E_{\iota_t}\left[R_{t+1} u_{c^o_{\iota_{t+1}}}\right] \tag{7}$$

In terms of life-cycle after-tax income $s_t = \max\left\{0, E_{\iota_t}\left[\gamma\left(R_{t+1}\right) I^a_t - \left(1-\gamma\left(R_{t+1}\right)\right) I^o_{t+1}\right]\right\}$ where, for any separable additive utility function, $\gamma(\cdot)$ and its complement to one represent the derivative of the saving function with respect to the per period net income. Under full depreciation, the level of saving completely determines the dynamics of physical capital.

Firms produce in a perfectly competitive environment, and in equilibrium, they choose the level of physical capital and the per-efficiency units of labor so as to maximize profits, i.e. $\max\limits_{k_t,h_t}[\Theta\left(k_t,h_t\right) - w_t h_t - R_t k_t]$. Firms optimality and markets clearing imply that factor prices are given by the marginal productivity of each factor:

$$w_t = \Theta_{h_t} \tag{8}$$

$$R_t = \Theta_{k_t} \tag{9}$$

Definition 2 (Competitive Economic Equilibrium) *Given the initial conditions $\{k_0, h_0\}$ and the sequence of policies $\{p_{\iota_t}\}^{\infty}_{t=0}$, a competitive economic equilibrium is defined as a sequence of allocations $\left\{c^a_{\iota_t}, c^o_{\iota_t}, k_{t+1}, h_{t+1}\right\}^{\infty}_{t=0}$ and factor prices $\{w_t, R_t\}^{\infty}_{t=0}$ such that for all $t \geq 0$:*

i. The allocation solves the maximization problem of the adults, i.e. Eq. (7) is satisfied;

ii. The factor prices are consistent with the profit maximization of firms, i.e. Eqs. (8) and (9) are satisfied;

iii. The market for the private good clears, i.e. $c^a_{\iota_t} + \frac{c^o_{\iota_t}}{\nu} + \nu f_{\iota_t} + \nu k_{t+1} = \Theta\left(k_t, h_t\right)$;

iv. The market for physical capital clears, i.e.

$$\nu k_{t+1} = \max\left\{0, \mathrm{E}_{\iota_t}\left[\gamma\left(\Theta_{k_{t+1}}\right) I^a_t - \left(1-\gamma\left(\Theta_{k_{t+1}}\right)\right) I^o_{t+1}\right]\right\} \tag{10}$$

Under the balanced budget constraint, Eq. (6), plugging Eqs. (8), (9), and (10), into Eqs. (2) and (3), yields the following individual consumption levels:

$$c^a_{\iota_t} = \mathcal{C}^a_{\iota_t}\left(k_t, h_t, k_{t+1}, b_{\iota_t}, f_{\iota_t}\right) \equiv \Theta_{h_t} h_t - \nu f_{\iota_t} - \frac{b_{\iota_t}}{\nu} - \nu k_{t+1} \tag{11}$$

$$c^o_{\iota_{t+1}} = \mathcal{C}^o_{\iota_{t+1}}\left(k_{t+1}, h_{t+1}, b_{\iota_{t+1}}\right) \equiv \nu \Theta_{k_{t+1}} k_{t+1} + b_{\iota_{t+1}} \tag{12}$$

Hence, conditional on the current realization of the ideological bias, the indirect utility of any individual belonging to the adult and elderly cohorts are respectively equal to:

$$\mathcal{W}^a_{\iota_t} \equiv u\left(\mathcal{C}^a_{\iota_t}\left(k_t, h_t, k_{t+1}, b_{\iota_t}, f_{\iota_t}\right)\right) + \beta E_{\iota_t}\left[u\left(\mathcal{C}^o_{\iota_{t+1}}\left(k_{t+1}, h_{t+1}, b_{\iota_{t+1}}\right)\right)\right] \tag{13}$$

and

$$\mathcal{W}^o_{\iota_t} \equiv u\left(\mathcal{C}^o_{\iota_t}\left(k_t, h_t, b_{\iota_t}\right)\right) \tag{14}$$

When taxation and public spending are precluded, the *autarky* indirect utility yields $\underline{\mathcal{W}}_t \equiv \max_{s_t}\{u(c_t^a) + \beta u(c_{t+1}^o) \mid I_t = w_t h_0\}$.

Definition 3 (Equilibrium Feasible Allocation) *An equilibrium feasible allocation is a sequence of competitive economic equilibrium allocations* $\{c_t^a, c_t^o, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$*, factor prices* $\{w_t, R_t\}_{t=0}^{\infty}$*, and policies* $\{p_{\iota_t}\}_{t=0}^{\infty}$ *that satisfies the balanced budget constraint, Eq. (6), and the fiscal feasibility conditions at each time* t*.*

3.2 Political Competition

In this section, we describe how short-lived office-seeking parties interact in the electoral competition. Public policies are chosen through a repeated voting system according to majority rule. The young have no political power.[11] To describe the behavior of politicians we consider a dynamic probabilistic voting setting adapted to an OLG environment with intergenerational transfers. The parties' objective function concerns the maximization of the probability of winning elections at each time with no ability to commit to future policies. Agents vote for party $\mathcal{R}_t$ as long as the idiosyncratic ideological bias, σ_j^i, is larger than the difference in the indirect utility achieved from voting the two alternative platforms, net of the aggregate ideological bias, η. It implies $\sigma_j^i \geq \sigma^i(k_t, h_t) \equiv \mathcal{W}_{\mathcal{L}_t}^i - \mathcal{W}_{\mathcal{R}_t}^i - \eta$. Formally, $\sigma^i(k_t, h_t)$ represents the voter in cohort i, who is indifferent between the two parties. Thus, the share of voters belonging to cohort i and supporting party $\mathcal{R}_t$ is equal to $\lambda_t^i \equiv \frac{1}{2} - \boldsymbol{\sigma}^i\left(\mathcal{W}_{\mathcal{L}_t}^i - \mathcal{W}_{\mathcal{R}_t}^i - \eta\right)$. Under majoritarian rule, party $\mathcal{R}_t$ wins the election if and only if it obtains the largest share of votes, namely if $N_t^{t-1}\lambda_t^a + N_t^{t-2}\lambda_t^o > \frac{1}{2}\left(N_t^{t-1} + N_t^{t-2}\right)$. It implies that η must be larger than the threshold level $\eta(k_t, h_t) \equiv \nu\frac{\boldsymbol{\sigma}^a}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{W}_{\mathcal{L}_t}^a - \mathcal{W}_{\mathcal{R}_t}^a\right) + \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{W}_{\mathcal{L}_t}^o - \mathcal{W}_{\mathcal{R}_t}^o\right)$. As a result, the objective function of party $\mathcal{R}_t$, i.e. $\max_{p_{\mathcal{R}_t}} \Pr(\eta \geq \eta(k_t, h_t))$, simplifies to:

$$\max_{p_{\mathcal{R}_t}} \frac{1}{2} - \boldsymbol{\eta}\eta(k_t, h_t) \tag{15}$$

Likewise for party $\mathcal{L}_t$, the $\max_{p_{\mathcal{L}_t}} \Pr(\eta \leq \eta(k_t, h_t))$ collapses to:

$$\max_{p_{\mathcal{L}_t}} \frac{1}{2} + \boldsymbol{\eta}\eta(k_t, h_t) \tag{16}$$

3.3 Politico-Economic Equilibrium

We now characterize the subgame perfect politico-economic equilibrium of the intergenerational voting game. At each time t, the implementation of a political platform generates dynamic linkages of policies across periods through the evolution of the asset variables of the economy. Fully rational agents internalize the overall dynamic effect of the current fiscal policies onto the future one, changing their strategic position over the time. In principle, the construction of policies contingent on alternative histories and enforced by reputation mechanisms allows for multiple subgame-perfect equilibria. We rule out such mechanisms and focus instead on differentiable stationary Markov policies as equilibrium refinement.[12] The payoff-relevant state variables for the political candidates are the assets held by the

[11] Our assumption matches the evidence that young agents show a much lower turnout rate at elections in comparison to the adults and the old. As Galasso and Profeta (2004) report, in some countries the elderly have a higher turnout rate at elections than the youth. In the U.S., the turnout rate among those aged 60-69 years is twice as high as among young (19-29 years).

[12] Markov perfectness implies that outcomes are history-dependent only on the fundamental state variables. The stationary part is introduced to focus on equilibrium policy rules, which are not indexed by time, i.e. the structural relation among

pivotal constituencies at each time, i.e. physical and human capital. In a probabilistic voting environment, when voters condition their strategies on those assets, the intergenerational contract is enforced and sustained even in a finite-horizon economy. Thus, the equilibrium characterized here corresponds to the limits of a finite-horizon game. The objects we are interested in are the intergenerational policy rules, $\mathcal{P}_{\iota_t} \equiv \{\mathcal{B}_{\iota_t}(k_t, h_t), \mathcal{F}_{\iota_t}(k_t, h_t)\}$, where $\mathcal{B}_{\iota_t} : \mathbb{R}^2_+ \to [\underline{b}_t, \bar{b}_t]$ and $\mathcal{F}_{\iota_t} : \mathbb{R}^2_+ \to [0, \bar{f}_t]$, and the rules governing the evolution of both physical and human capital. In a perfect forward-looking environment, the following Definition of politico-economic equilibrium is adopted.

Definition 4 (Politico-Economic Equilibrium) *Given the initial conditions $\{k_0, h_0\}$, a Markov-perfect politico-economic equilibrium is a sequence of equilibrium feasible allocation such that the collection of differentiable policy rules, $\mathcal{P}_{\iota_t} \equiv \{\mathcal{B}_{\iota_t}, \mathcal{F}_{\iota_t}\}$, satisfies the following points:*

i. *For interior savings, parties $\mathcal{R}_t$ and $\mathcal{L}_t$ implement $\check{\mathcal{B}}_{\iota_t}(k_t, h_t)$ and $\check{\mathcal{F}}_{\iota_t}(k_t, h_t)$ as the* arg max *of Eqs. (15) and (16), subject to:*

$$\Delta^k (k_{t+1}, b_{\iota_t}, f_{\iota_t}) \equiv \nu k_{t+1} - E_{\iota_t} \left[\gamma_{t+1} I^a_{\iota_t} - (1 - \gamma_{t+1}) I^o_{\iota_{t+1}}\right] = 0 \tag{17}$$

where $\gamma_{t+1} = \gamma\left(\Theta_{k_{t+1}}\left(k_{t+1}, \Phi\left(\mathcal{F}_{\iota_t}(\cdot), h_t\right)\right)\right)$, $I^a_{\iota_t} = \Theta_{h_t} h_t - \nu \mathcal{F}_{\iota_t}(\cdot) - \frac{\mathcal{B}_{\iota_t}(\cdot)}{\nu}$, and $I^o_{\iota_{t+1}} = \frac{\mathcal{B}_{\iota_{t+1}}\left(k_{t+1}, \Phi\left(\mathcal{F}_{\iota_t}(\cdot), h_t\right)\right)}{\Theta_{k_{t+1}}\left(k_{t+1}, \Phi\left(\mathcal{F}_{\iota_t}(\cdot), h_t\right)\right)}$.

ii. *The fixed point conditions hold, i.e. $\check{\mathcal{B}}_{\iota_t}(k_t, h_t) = \mathcal{B}_{\iota_t}(k_t, h_t)$ and $\check{\mathcal{F}}_{\iota_t}(k_t, h_t) = \mathcal{F}_{\iota_t}(k_t, h_t)$.*

The first equilibrium condition requires the political control variables, $\{b_{\iota_t}, f_{\iota_t}\}$, to be chosen in order to maximize the party's objective function, constrained to the capital market clearing condition, Eq. (17). Upon the existence of a Markov-perfect equilibrium, the recursive formulation of the decision rule for the private savings is as $k_{t+1} = \mathcal{K}(k_t, h_t, b_{\iota_t}, f_{\iota_t})$. Its partial derivatives $\mathcal{K}_{f_{\iota_t}} \equiv -\frac{\Delta^k_{f_{\iota_t}}}{\Delta^k_{k_{t+1}}}$ and $\mathcal{K}_{b_{\iota_t}} \equiv -\frac{\Delta^k_{b_{\iota_t}}}{\Delta^k_{k_{t+1}}}$ quantify the private sector responsiveness to any one-shot deviation of the government, when agents rationally expect future backward transfers to be set according to $\mathcal{B}_{\iota_{t+1}}(k_{t+1}, h_{t+1})$. The second condition requires that, if the equilibrium exists, it must satisfy the fixed point requirement.

Condition 1 (Uniqueness of Competitive Economic Equilibrium) *For $\forall\ h_t, k_t > 0$ and $k_{t+1} > 0$, if $\Delta^k (k_{t+1}, b_{\iota_t}, f_{\iota_t}) = 0$ then $\Delta^k_{k_{t+1}} > 0$.*

Condition 1 is necessary and sufficient for a unique intersection point k_{t+1} solving $\Delta^k (k_{t+1}, b_{\iota_t}, f_{\iota_t}) = 0$. This implies that the elasticity of variation of the rate of return on the savings decision must be sufficiently large at the solution, so that the substitution effect will not be dominated too much by the income effect.

By modelling the political mechanism as a probabilistic voting model à la Persson and Tabellini (2000), if a Markov-perfect politico-economic equilibrium of the intergenerational voting game exists, then at each time t, the parties propose the same political platform, i.e. $p_{\mathcal{L}_t} = p_{\mathcal{R}_t} = p_t$. In equilibrium, no candidate is able to change current policies and obtain a net gain in the number of votes. Hence, they set policies so that the marginal effect on the probability of reelection of the last unit invested is actually zero.[13] Using Eqs. (13) and (14), the equilibrium political platform maximizes a weighted sum of the adult and elderly voters' utility, given by:

$$\nu \mathcal{W}^a_t (k_t, h_t, k_{t+1}, b_t, f_t, b_{t+1}) + \phi \mathcal{W}^o_t (k_t, h_t, b_t) \tag{18}$$

payoff-relevant state variables and political controls is time invariant. The differentiable part is a convenient requirement to avoid multiplicity of equilibrium outcomes and in order to give clear positive predictions.

[13] An explicit microfoundation of the probabilistic voting game is provided in Appendix B.

The parameter $\phi \equiv \frac{\boldsymbol{\sigma^o}}{\boldsymbol{\sigma^a}} \in (0, \infty)$ denotes a synthetic measure of the ideological bias among constituencies, which captures the relative political clout of the voters belonging to the elderly cohort. In the extreme case of $\boldsymbol{\sigma^o} \to \infty$ (or alternatively $\boldsymbol{\sigma^a} \to 0$), which implies $\phi \to \infty$, the dictatorship of the old shapes the institutional process. The elderly cohort forms a single-minded ideological block, ready to compromise their partisan loyalties in return for particularistic benefits. When $\phi \to 0$ (i.e. $\boldsymbol{\sigma^o} \to 0$ or $\boldsymbol{\sigma^a} \to \infty$), the opposite holds. Namely, the adults hold the only relevant ideological position in the political competition. Finally, when $\boldsymbol{\sigma^a} = \boldsymbol{\sigma^o}$, i.e. $\phi = 1$, the densities of the ideological bias are identical across cohorts. An alternative interpretation of ϕ is in terms of effective intergenerational political power. On the one hand, it reflects the existence of formal institutions that guarantee active and passive political participation (i.e. candidacy age, electoral rules, lobby power, voting enfranchisement, and so on). On the other hand, it measures how informal institutions alter the age-cohorts' representativeness (i.e. civil society, clientelism, corruption, social norms, traditional culture, and so on).

At each time t, the political objective function, Eq. (18), has to be maximized with respect to its arguments, i.e. the pair $\{b_t, f_t\}$, subject to Eq. (17). The following system of first order conditions for interior b_t and f_t characterizes the Markov-perfect politico-economic equilibrium:

$$b_t : 0 = \phi u_{c_t^o} - u_{c_t^a} + \nu\beta u_{c_{t+1}^o} \left(\frac{d\mathcal{B}_{t+1}}{db_t} + \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t} \right) \tag{19}$$

$$f_t : 0 = -\nu u_{c_t^a} + \beta u_{c_{t+1}^o} \left(\frac{d\mathcal{B}_{t+1}}{df_t} + \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t} \right) \tag{20}$$

Eqs. (19) and (20) depict how politicians strategically manipulate the amount of future policies through their current decisions on the fiscal platform. We disentangle the strategic effects in two parts. The total derivatives $\frac{d\mathcal{B}_{t+1}}{db_t} = \mathcal{B}_{k_{t+1}}\mathcal{K}_{b_t}$ and $\frac{d\mathcal{B}_{t+1}}{df_t} = \mathcal{B}_{k_{t+1}}\mathcal{K}_{f_t} + \mathcal{B}_{h_{t+1}}\Phi_{f_t}$ capture the allocation effects generated by a variation in the current level of intergenerational transfers on the next-period amount of backward transfers through the channels of physical and human capital. The total derivatives $\frac{d\Theta_{k_{t+1}}}{db_t} = \Theta_{k_{t+1}k_{t+1}}\mathcal{K}_{b_t}$ and $\frac{d\Theta_{k_{t+1}}}{df_t} = \Theta_{k_{t+1}k_{t+1}}\mathcal{K}_{f_t} + \Theta_{k_{t+1}h_{t+1}}\Phi_{f_t}$ pin down the pecuniary externalities generated by a variation in the current level of intergenerational transfers on the next-period rate of return on capital.

According to Eq. (19), an interior solution for the backward transfers is determined by the relative political clout of the current living voters and the impact on consumption of next-period elderly voters. The first order condition features two components: the direct effect on the individual consumption of redistributing resources from tax-payers to tax-recipients and the indirect effects on the next-period consumption of the current adult. Eq. (20) yields the trade-off for the adults between the public productive investment and private savings. On the one hand, an increase in the total fiscal burden raises the opportunity cost of saving. On the other hand, the current taxpayers will be rewarded with higher next-period private consumption implied by the adjustment in both backward transfers and rental price of capital.

3.4 Human Capital

In order to highlight the main mechanism at work, let us analyze the basic setup with human capital as the sole payoff-relevant state variable. In this scenario, we abstract away the general equilibrium effect via prices and isolate the allocation channel of political competition as the main determinant for the emergence of the intergenerational contract.

The absence of physical capital destroys the dynamic strategic linkages across backward policies.

Thus, the redistributive wedge is entirely determined by the relative political power of adults and elderly voters. The equilibrium condition described in Eq. (19) reduces to:

$$\phi u_{c_t^o} - u_{c_t^a} = 0 \tag{21}$$

The consumption of both cohorts turns out to be a constant share of the total outcome. Moreover, the higher the degree of elderly single-mindedness, i.e. the larger ϕ, the lower the marginal rate of intergenerational substitution (MRIS), $\frac{u_{c_t^o}}{u_{c_t^a}}$, which measures the consumption tightness among agents. It implies a more unbalanced distribution of consumption in favor of the elderly voters, namely a deeper intergenerational inequality. At the same time, Eq. (20) collapses to:

$$-\nu u_{c_t^a} + \beta u_{c_{t+1}^o} \mathcal{B}_{h_{t+1}} \Phi_{f_t} = 0 \tag{22}$$

According to Eq. (22), the marginal cost of current taxation suffered by the adults will be offset by the marginal benefits of larger next-period consumption. Since without physical capital the elderly agents have no private wealth, then $b_t = 0$ whenever $\phi = 0$ for each t. It implies that $\mathcal{B}_{h_{t+1}} = 0$ and, from Eq. (22), no productive investment, i.e. $f_t = 0$. By contradiction, suppose $\phi > 0$ and $f_t = 0$, then from Eq. (21) $b_t > 0$ for each t. This cannot be an equilibrium, since $f_t = 0$ and, in turn, $h_{t+1} = 0$ implies $b_{t+1} = 0$.

Proposition 1 (Empowerment) *A unique intergenerational contract is enforced if and only if $\phi > 0$.*
Proof. *(See appendix).* ■

Proposition 1 highlights the main prediction: when the elderly agents are disempowered, i.e. $\phi = 0$, voters fail in supporting productive investments, although a growth-enhancing technology is at their disposal. Thereby, the economy reverts to a bad equilibrium with $f_t = 0$. On the contrary, if the old-aged constituents actively participate to the public debate, they exert their influence to extract a political rent in terms of backward transfers. Therefore, in the adult-age they support forward-oriented policies in order to grab a larger share of the next-period production. To recap: the existence of a Markov-perfect equilibrium which attains a growth-enhancing intergenerational contract does not require pre-commitment through the establishment of an institution that outlives the current government and binds future decision-makers. Rather, the empowerment of the elderly cohort acts as a credible commitment device. Their concern for future backward transfers provision is key in order to enforce the intergenerational contact, given the lack of intergenerational altruism.

Proposition 2 (Single-Mindedness and Growth) $\frac{df_t}{d\phi} \geq (<)\, 0$ *if and only if* $\rho_{\mathcal{B}_{h_{t+1}},\phi} \geq (<)\, \rho_{\frac{\phi u_{c_t^o}}{u_{c_{t+1}^o}},\phi}$, *where* $\rho_{\mathcal{B}_{h_{t+1}},\phi} \equiv \mathcal{B}_{h_{t+1},\phi} \frac{\phi}{\mathcal{B}_{h_{t+1}}}$ *and* $\rho_{\frac{\phi u_{c_t^o}}{u_{c_{t+1}^o}},\phi} \equiv \frac{d}{d\phi}\left(\frac{\phi u_{c_t^o}}{u_{c_{t+1}^o}}\right)\frac{u_{c_{t+1}^o}}{u_{c_t^o}}$ *denote the elasticity with respect to ϕ of both the marginal impact of human capital on backward transfers and the marginal rate of substitution between current and next-period cohort, which is weighted for the relative political clout of the old voters.*
Proof. *(See appendix).* ■

An increase in ϕ has a two-fold impact. On the one hand, it alters the distribution of consumption in favor of the current elderly voters. A higher degree of the elderly single-mindedness positively affects the level of backward transfers and, in turn, the amount of the consumption of the old-aged agents, i.e.

$\rho_{\frac{\phi u_{c_t^o}}{u_{c_{t+1}^o}},\phi}$ increases. On the other hand, it improves the future ability of the current adult to extract the electoral rent generated by the investment in productive transfers, i.e. $\rho_{\mathcal{B}_{h_{t+1}},\phi}$ increases. Proposition 2 states that if the latter effect dominates the former, then the larger the relative political power of elderly voters, the more the investment in the growth-enhancing technology.

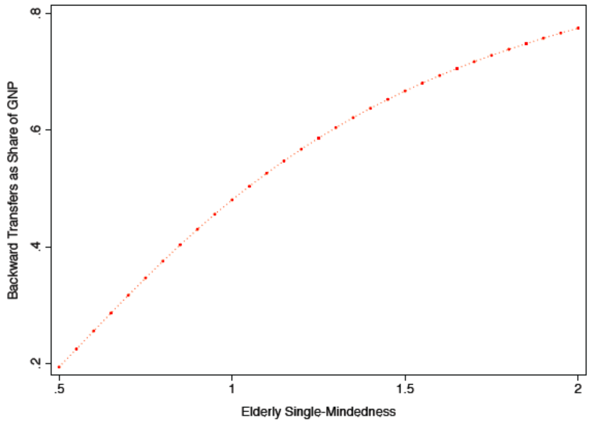

Fig. 7: Elderly Single-Mindedness and Backward Transfers

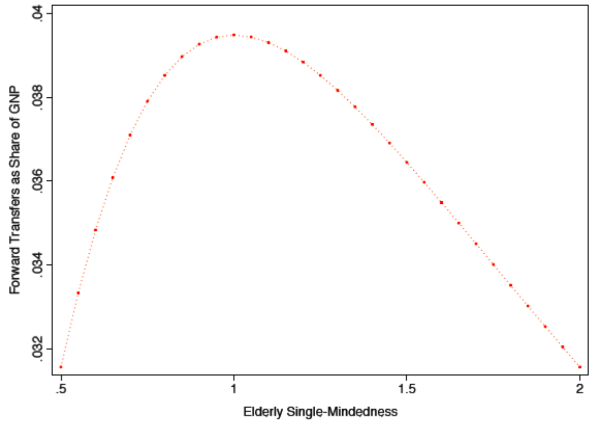

Fig. 8: Elderly Single-Mindedness and Forward Transfers

For specific parameter values, Figures 7 and 8 illustrate the properties of the equilibrium policy rules that follow from the previous discussion: (i) b_t is a non-decreasing policy as the elderly single-mindedness gets more polarized (Figure 7); (ii) when the agents are low risk adverse and, in turn, less responsive to changes in the consumption, the model foretells an inverse-U shaped relationship between the political influence of the elderly voters and the provision of forward transfers and, in turn, the rate of economic growth (Figure 8).[14] Indeed, when the elderly-single mindedness is sufficiently small, an increase in ϕ induces a variation in $\rho_{\mathcal{B}_{h_{t+1}},\phi}$, which offsets the variation in $\rho_{\frac{\phi u_{c_t^o}}{u_{c_{t+1}^o}},\phi}$. The overall effect reverses for sufficiently large elderly political clout. As a consequence, according to Proposition 2, the economy's growth rate turns out to be a non-monotone function of the elderly single-mindedness. More generally, a higher curvature of utility over consumption implies a monotone decreasing relation between the sustainable productive transfers and the relative political power of elderly voters.

3.5 Discussion

When agents condition their strategies also on the evolution of physical capital, more sophisticated strategic effects occur. The definition of property rights on the different production inputs creates divergent economic interests among cohorts, putting the intergenerational cooperation under strain. Nevertheless, we show how the core results of the human capital economy still characterize an environment with savings.

Proposition 3 (Indeterminacy and Uniqueness) *The properties of the Markov-perfect politico-economic equilibrium are as follows:*

i. *If $\phi = 0$, then there exists a continuum of undetermined differentiable policy functions;*

ii. *$\phi > 0$ breaks the equilibrium indeterminacy;*

[14] Appendix B provides the quantitative framework to replicate Figures 7 and 8.

iii. *If there exists a unique intergenerational contract then necessarily $\phi > 0$.*

Proof. *(See appendix).* ■

Proposition 3 argues that as long as $\phi > 0$, the political sustainability of the intergenerational contract does not rely on the self-fulfilling expectations of future agreements, but on the politico-economic fundamentals that are indeed payoff-relevant for future constituents. In some recent papers related to ours, dynamic complementarity between the households' savings decision and the government's policy rules gives rise to a multiplicity of self-fulfilling expectations-driven Markov-perfect equilibria: households' actions, which are based on their fiscal policy expectations, make it optimal for the government to fulfill these expectations.[15] As a main implication, economies with identical fundamentals might significantly differ in their levels of intergenerational transfers. Moreover, the set of Markov-perfect equilibria would include the optimal allocation as well as distortionary policy rules. We consider these features as strong limitations of a substantial part of the previous literature on dynamic public finance with Markov-perfect equilibrium, since such models fail both in the positive description of real world phenomena and in their normative prescriptions. One way to allow the voters to correctly form their expectations about future policies and pin down the good allocation is to introduce some degree of commitment. For example, Azariadis and Galasso (2002) propose the establishment of a fiscal constitution, which allow generations to play a veto power. However, a general critique that can be raised, is why a society, which can agree on sophisticated constitutional constraints, is not able to agree on the efficient outcome in the first place? We opt for a different solution: in a probabilistic voting framework under Assumption 2, poor agents care at the margin more about their private consumption than rich cohorts. Their votes are thus more responsive to the transfers they receive, which allows them to claim for greater redistribution. At the same time, political uncertainty in the form of idiosyncratic realization of ideological bias plays the role of a credible commitment device, which allows each generation to be at least partially rewarded for the sacrifices afforded during its life-cycle. In the spirit of the economy with only human capital, as a further implication of Proposition 3, the political process enforces a unique intergenerational contract as long as ϕ is strictly positive.

To determine the intra-temporal wedge of political redistribution, let combine Eqs. (7) and (19) as:

$$\Delta^b (b_t) \equiv \phi u_{c_t^o} - \Upsilon_{t+1} u_{c_t^a} = 0 \tag{23}$$

where $\Upsilon_{t+1} \equiv 1 - \nu^2 \frac{\frac{d\mathcal{B}_{t+1}}{db_t} + \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t}}{\frac{d\mathcal{B}_{t+1}}{df_t} + \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t}}$ represents the endogenous weight politicians attach to the adult-aged constituents. Unlike Eq. (21), Eq. (23) fully catches how the politicians internalize the feedback effects generated by the implemented fiscal bundle on the lifetime utility of the adult voters. Formally, the MRIS equals the relative political clout of the adult voters net of the expected benefit(cost) generated by a positive(negative) intertemporal correlation between the current and the subsequent level of intergenerational transfers and the expected induced variation in the marginal productivity of physical capital, $\frac{\Upsilon_{t+1}}{\phi}$. If the MRIS is greater than one, then the distribution of consumption is skewed in favor of the adults. Vice versa, a MRIS lower than one denotes an unbalanced distribution of consumption in favor of the old-aged. Finally, when MRIS is equal to one the two age-cohorts consume an equal share of the total output.

Proposition 4 (Necessary Condition) *In any intergenerational contract enforced as a Markov-perfect*

[15] See among others Grossman and Helpman (1996).

politico-economic equilibrium, the strategic effect generated by forward transfers is larger than the strategic effect generated by backward transfers.

Proof. *(See appendix).* ■

Proposition 4 reveals the intertemporal structural relations among policies, which make both forward and backward transfers to be simultaneously enforced through an intergenerational agreement. The marginal impact of current investment on next-period backward benefits and prices is required to be larger than the marginal impact of current backward transfers. By contradiction, if $\frac{d\mathcal{B}_{t+1}}{df_t} + \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t}$ were smaller or equal to $\frac{d\mathcal{B}_{t+1}}{db_t} + \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t}$, then in order to maximize the voters' utility, it would be sufficient to support intergenerational cooperation over backward transfers without investing in human capital. According to Eq. (23), when $\Upsilon_{t+1} \leq 0$ $\big($i.e. $\Delta^b(b_t) > 0\big)$, the current government would set the backward transfers to the maximum level, devoting no resources to productive investment.

From Eq. (23) we can also infer that: when the investment in human capital positively affects the return on physical capital, i.e. $\Theta_{k_{t+1}h_{t+1}} > 0$, politicians attach a larger endogenous weight to the adult constituents compared to a case with no technological complementarity. Additionally, this is the case in which the intergenerational conflict over public resources due to the difference in the ownership of productive factors as well as in the source of income turns out to be lower.

Remark 1 (Intergenerational Conflict) *Ceteris paribus, the greater the technological complementarity among factors of production and, in turn, the lower the degree of intergenerational conflicts, the smaller the amount of enforceable backward transfers.*

When the degree of imperfect substitutability is greater, i.e. ζ is lower, the politicians involved in a Markov game among successive generations deliver a smaller amount of backward transfers. This result stems from the politicians' strategic behavior: in setting the fiscal bundle, short-lived politicians internalize that the cost sustained by current adults for financing productive investment will be partially compensated by a larger amount of capitalized savings in their old-age. Following Eq. (23) it translates in a larger endogenous weight attached to the adult and, in turn, in a smaller amount of enforceable backward transfers.

Combining Eqs. (7) and (20) yields the political productive wedge:

$$\Delta^f(f_t) \equiv \Phi_{f_t} - \frac{\Theta_{k_{t+1}}}{\Theta_{h_{t+1}}}(1 - \varepsilon_{t+1}) = 0 \tag{24}$$

where $\varepsilon_{t+1} \equiv 1 - \frac{\nu(1-\mu_t)\gamma_{t+1}\Theta_{h_{t+1}}}{((1-\mu_t)\gamma_{t+1}+\mu_t)\frac{b_{t+1}}{h_{t+1}} + \nu\mu_t \frac{k_{t+1}}{h_{t+1}}\Theta_{k_{t+1}}}$ with $\mu_t \equiv 1 - \phi \frac{u_{c_t^o}}{u_{c_t^a}}$, which measures the human capital spillover internalized in the politico-economic equilibrium. Interestingly, even if short-lived politicians only care about the probability of being elected in the current political campaign, they can internalize the dynamic spillover generated by the public investment in forward transfers. Due to the overlapping demographic structure and the intertemporal linkage created by the dynastic human capital, politicians evaluate the utility of current living voters, internalizing the expectation of future generations over policies, which will be credibly proposed by future parties. Strikingly, backward and forward policies act as strategic complement: inspecting Eq. (24), the larger the expected backward transfers, i.e. the larger ε_{t+1}, the more discipline the current politicians show in investing in forward transfers. However, since the amount of enforceable backward transfers is called into question by technological complementarity, so does its discipline impact upon enforceable forward transfers.

Remark 2 (Strategic Complementarity) *Ceteris paribus, the greater the technological complementarity among factors of production, the weaker the strategic complementarity between backward and forward policies.*

To resume: the politico-economic equilibrium outcome is driven by two fundamental features of the model: (i) the prospect of follow-up intergenerational contracts, which serves as a discipline device to implement current policies as argued above, and (ii) the nature of short-term agreements among politicians and current living voters, i.e. the absence of commitment. Indeed, let us consider the possibility of the policy-makers being able to sign binding long-term agreements with current living voters. Since representatives are office-seeking and only aim to be elected during the current political campaign, then the intergenerational contract they commit on will be at most a two-period agreement. In the best scenario, they will promise current adults to fully expropriate the next-period generation and use the proceeds to subsidize their consumption when old. As a consequence, the pledgeable income devoted to productive investment would be such that the marginal rate of transformation equates the opportunity cost of savings disregarding the spillover effects associated with the forward transfers, i.e. $\varepsilon_{t+1} = 0$. On the contrary, in a politico-economic equilibrium without commitment, ε_{t+1} can differ from zero and attain higher economic growth. The next section provides an apt benchmark to evaluate how distant the Markov-perfect intergenerational contract lies from the Pareto frontier.

4 Social Planner

In this section, we characterize the efficient allocation chosen by a benevolent social planner, who chooses the sequence of allocations $\{c_t^a, c_t^o, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and policies $\{p_t\}_{t=0}^{\infty}$ so as to maximize the discounted utility of all generations. The planner attaches a Pareto weight $\delta \in (0,1)$ on the utility of each dynasty. Taking the initial level of physical and human capital $\{k_0, h_0\}$ as given, the sequential formulation of the planner's problem is:

$$\max_{\left\{c_t^a, c_{t+1}^o, f_t, h_{t+1}, k_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \left(u\left(c_t^a\right) + \beta u\left(c_{t+1}^o\right)\right) + \frac{\beta}{\delta} u\left(c_0^o\right)$$

subject to the aggregate resource constraint and the human capital technology:

$$c_t^a + \frac{c_t^o}{\nu} + \nu k_{t+1} + \nu f_t - \Theta\left(k_t, h_t\right) \leq 0, \ \forall t \ \left(\varkappa_t \delta^t\right)$$
$$h_{t+1} - \Phi\left(f_t, h_t\right) \leq 0, \ \forall t \ \left(\varrho_{t+1} \delta^{t+1}\right)$$

where $(\varkappa_t \delta^t)$ and $\left(\varrho_{t+1} \delta^{t+1}\right)$ are the associated Lagrangian multipliers. At each time, the utility of the elderly generation is weighted by $\frac{\beta}{\delta}$, reflecting the bias of the planner preference in favor of either adult $(\beta \leq \delta)$ or elderly $(\beta > \delta)$ agents. Varying δ yields all the allocation on the Pareto frontier. Removing the functional arguments, the first order conditions of the Lagrangian turn out to be equal to:

$$\begin{aligned}
c_t^a &: u_{c_t^a} = \varkappa_t \\
c_t^o &: \nu \beta u_{c_t^o} = \delta \varkappa_t \\
f_t &: \nu \varkappa_t = \delta \varrho_{t+1} \Phi_{f_t} \\
h_{t+1} &: \varrho_{t+1} = \varkappa_{t+1} \Theta_{h_{t+1}} + \varrho_{t+2} \delta \Phi_{h_{t+1}} \\
k_{t+1} &: \nu \varkappa_t = \delta \varkappa_{t+1} \Theta_{k_{t+1}}
\end{aligned}$$

together with the transversality conditions $\lim_{t\to\infty} \delta^t \varkappa_t k_{t+1} = 0$ and $\lim_{t\to\infty} \delta^{t+1} \varrho_{t+1} h_{t+1} = 0$. Rearranging the first order conditions, the following wedges for the optimal allocations must be satisfied:

$$\hat{\Delta}^k \left(k_{t+1}, b_t, f_t\right) \equiv u_{c_t^a} - \beta u_{c_{t+1}^o} \Theta_{k_{t+1}} = 0 \tag{25}$$

Eq. (25) describes the conventional inter-temporal consumption-savings optimal Euler condition and, in turn, the optimal accumulation of physical capital. The planner chooses k_{t+1} in order to equate the marginal cost, in terms of foregone consumption, to the discounted marginal benefits of savings.

$$\hat{\Delta}^b \left(b_t\right) \equiv \delta u_{c_t^a} - \nu \beta u_{c_t^o} = 0 \tag{26}$$

The second condition captures the intra-temporal redistribution wedge between current adult and elderly cohorts. Differently from Eq. (23), the social planner holds concern for future generations and does not deplete resources from unborn generations in order to redistribute to the current one. Thus, the redistributive wage is entirely determined by the Pareto weight.

$$\hat{\Delta}^f \left(f_t\right) \equiv \Phi_{f_t} - \frac{\Theta_{k_{t+1}}}{\Theta_{h_{t+1}}} \left(1 - \hat{\varepsilon}_{t+1}\right) = 0 \tag{27}$$

Finally, Eq. (27) describes the productive wedge where $\hat{\varepsilon}_{t+1} \equiv \frac{\nu}{\Theta_{k_{t+1}}} \Phi_{f_t} \frac{\Phi_{h_{t+1}}}{\Phi_{f_{t+1}}}$ denotes the education spillover expressed as a fraction of the education cost. It reflects the direct effect of the forward transfers on the utility of the adults in terms of current cost and discounted marginal benefits. $\hat{\varepsilon}_{t+1}$ fully quantifies the impact of productive spending on the future level of human capital through the channel of both parental investment and future policies.[16] Note that, despite the infinite persistent impact of the forward investment, only the current and the subsequent periods matter directly. Hence, Eq. (27) can be viewed as resulting from a variational (two-periods) problem. In other words, let us think of our variational argument as follows: given the state variables $\{k_t, h_t\}$ and $\{k_{t+2}, h_{t+2}\}$, let us vary $\{k_{t+1}, h_{t+1}\}$ through the controls f_t in order to obtain the highest possible utility.

Definition 5 (Social Planner Allocation) *For any initial conditions $\{k_0, h_0\}$ the optimal allocation $\left\{\hat{c}_t^a, \hat{c}_t^o, \hat{f}_t, \hat{k}_{t+1}, \hat{h}_{t+1}\right\}_{t=0}^{\infty}$ satisfies Eqs. (25), (26) and (27) for all $t \geq 0$ jointly with the transversality conditions.*

Eq. (27) jointly with the human capital transversality condition implies that in balanced growth $\nu\hat{\varphi} < \hat{\vartheta}_{\tilde{k}}$, where $\hat{\varphi}$ and $\hat{\vartheta}_{\tilde{k}}$ denote the optimal long-run growth rate and the associated rate of return on capital, respectively. Therefore, dynamic efficiency is always satisfied for any Pareto optimal allocation. On the contrary, no equilibrium conditions guarantee that the Cass sufficient requirement (1972) for dynamic efficiency is in general met in the Markov-perfect politico-economic equilibrium. Moreover, even if dynamic efficiency were attained, the lack of commitment technology and altruism could induce politicians to act shortsightedly and, as a consequence, to promote inefficient policies.

Remark 3 (Conflict and Efficiency) *Ceteris paribus, the higher the degree of intergenerational conflicts, the lower the inefficiency generated by the Markov-perfect politico-economic equilibrium.*

Intuitively, the positive relation between the degree of intergenerational conflict and the efficiency in allocations comes from the fact that the lower the technological substitutability among factors of

[16] Note that when $\varepsilon_{t+1} = 0$, Eqs. (26) and (27) are equivalent to the necessary conditions of a competitive equilibrium with no credit market constraints, where young can borrow money at the market interest rate.

production, the greater the strategic political power exerted by the adults compared to the elderly, and the weaker the strategic complementarity between forward and backward transfers (as already pointed out in Remark 1 and 2). Since in our framework this is the unique mechanism to discipline the politicians' behavior and enforce an efficient allocation, then weak strategic complementarity exacerbates even more the politicians' shortsighted behavior. Inefficiency vanishes only when the elasticity of substitution among productive factors tends to be infinite. Indeed, the resulting disappearance of pecuniary externalities induces both politicians and constituents to condition their strategies on the same information set and behave efficiently. In this circumstance, our result is in the spirit of Barro (1974), where the altruism motive is interpreted as the empowerment of the elderly and gifts/bequest as backward transfers. Similarly, in our OLG economy populated by selfish agents, current generations act effectively as though they were infinite-lived as long as they are linked to future generations by a chain of operative intergenerational transfers.

5 Illustrative Examples

We now explore the properties of the Markov-perfect equilibrium of our model in two limit cases by providing closed form solutions of the equilibrium policy rules and exploring their welfare properties. In the first case $\zeta = 1$, the imperfect substitutability among factors of production gives rise to general equilibrium effects via prices. In the second case, $\zeta = \infty$ implies perfect substitutability between the two factors of production and, in turn, the absence of pecuniary externalities. In both cases, the Markov-perfect politico-economic equilibrium is obtained as the limit of a finite-horizon equilibrium, whose characteristics do not significantly depend on the time horizon, as long as it is long enough. The resolution strategy consists in computing the first order conditions starting from a time $T < \infty$ and solving by backward for each time $T - j$ with $j = 0, 1, ...T$, subject to (i) the balanced budget constraint, Eq. (6), (ii) the economic Euler condition, Eq. (7), and (iii) the equilibrium policy rules of future periods. We obtain the equilibrium policy rules as the fixed point of the recursive problem in a multidimensional environment.

To provide the analytical characterization we parametrize (i) the preferences over private consumption as logarithmic, $u(c) = \log(c)$, (ii) the human capital technology as of the Cobb-Douglas type $h_{t+1} = Ah_t^{\theta} f_t^{1-\theta}$, with the parental human capital share $\theta \in (0,1)$ and the efficiency parameter $A \geq 1$,[17] and (iii) the production of the final good as a CES type $y_t = B\left[\alpha k_t^{\frac{1}{\epsilon}} + (1-\alpha) h_t^{\frac{1}{\epsilon}}\right]^{\epsilon}$, with the substitution parameter $\epsilon \geq 1$, the input share parameter $\alpha \in (0,1)$, and $B \geq 1$. Thus, the elasticity of substitution among factors of production amounts to $\zeta = \frac{\epsilon}{\epsilon - 1}$.

6 Example I: $\zeta = 1$

When $\zeta = 1$, the economy produces a single homogenous private good according to a Cobb-Douglas technology, i.e. $y_t = Bk_t^{\alpha} h_t^{1-\alpha}$. The imperfect substitutability among factors of production dampens the intergenerational conflicts arising from the divergent economic interests. By inspecting Eqs. (23) and (24), the following proposition yields:

Proposition 5 *A unique Markov-perfect politico-economic equilibrium exists and it is characterized by* $\Upsilon = \frac{\phi(1+\alpha\beta)}{\phi+\alpha\beta(\phi+\nu(1+\beta(1-\theta(1-\alpha))))}$ *and* $\varepsilon = -\frac{\nu(1+\beta(1-\theta(1-\alpha)))}{\phi+\alpha\beta(\phi+\nu(1+\beta(1-\theta(1-\alpha))))}$ *with* $\frac{d\Upsilon}{d\phi} > 0$ *and* $\frac{d\varepsilon}{d\phi} > 0$.

[17] A specification of this type is standard in the literature. For example, see Boldrin and Montes (2005) with f_t interpreted as public education.

Proof. *(See appendix)* ■

The cohorts' single-mindedness has a twofold impact on the shape of the equilibrium policy rules. On the one hand, the larger ϕ, the better the adults' opportunities to extract the political rent when old, and the larger the endogenous weight the politicians attach to the adults, $\frac{d\Upsilon}{d\phi} > 0$. On the other hand, the ideological bias positively affects the internalization of the dynamic spillover generated by the investment in human capital. From Proposition 5, the policy rules associated with the forward and backward intergenerational transfers are linear functions in the level of total income, as follows:

$$\mathcal{B}\left(h_t, k_t\right) = \frac{\nu\left(\phi - \alpha\left(\phi + \nu\left(1 + \beta\left(1 - \theta\left(1 - \alpha\right)\right)\right)\right)\right)}{\phi + \nu\left(1 + \beta\left(1 - \theta\left(1 - \alpha\right)\right)\right)} y_t \tag{28}$$

and

$$\mathcal{F}\left(h_t, k_t\right) = \frac{\beta\left(1 - \theta\right)\left(1 - \alpha\right)}{\phi + \nu\left(1 + \beta\left(1 - \theta\left(1 - \alpha\right)\right)\right)} y_t \tag{29}$$

As long as $\phi \leq \frac{\alpha\nu(1+\beta(1-\theta(1-\alpha)))}{1-\alpha}$, the equilibrium age-targeted fiscal bundle requires the elderly agents to subsidize both the adults' consumption and the productive investment devoted to the accumulation of the young skills. Adults have incentives to invest in human capital to increase the future return of their private savings. At the same time, they do not have the right incentives to transfer public resource backward: agents perfectly anticipate that when old, due to their weak political power, they will be not sufficiently rewarded for their sacrifices. As a consequence, in equilibrium, productive transfers are financed exclusively through taxes paid by the elderly. When $\phi > \frac{\alpha\nu(1+\beta(1-\theta(1-\alpha)))}{1-\alpha}$, the equilibrium political platform reverses. Both backward and forward transfers are financed through lump-sum taxes paid by the middle-aged cohort: due to the elderly political activism, the adults must satisfy their claim for positive transfers, but at the same time, since they also expect that future generations will do the same, they invest a fraction of the collected taxes in forward transfers in order to grab a bigger stake when old.

To summarize: for any initial level of physical and human capital, a Markov-perfect politico-economic equilibrium is characterized by the policy functions described in Eqs. (28) and (29) and, in balanced growth, by the following growth rate and associated return on capital: $\varphi = A\left(\frac{\chi^{1-\alpha}}{\xi^{\alpha}}\right)^{\frac{1-\theta}{1-\alpha\theta}}$ and $\vartheta_{\tilde{k}} = \alpha B\left(\xi\chi^{1-\theta}\right)^{\frac{1-\alpha}{1-\alpha\theta}}$ where $\xi \equiv \frac{A}{B}\frac{\phi+\alpha\beta(\phi+\nu(1+\beta(1-\theta(1-\alpha))))}{\alpha\beta(1+\alpha\beta)}$ and $\chi \equiv \frac{B\beta(1-\theta)(1-\alpha)}{\phi+\nu(1+\beta(1-\theta(1-\alpha)))}$. Replicating the exercise, the Pareto optimal allocation attains $\hat{\varphi} = A\left(\frac{\hat{\chi}^{1-\alpha}}{\hat{\xi}^{\alpha}}\right)^{\frac{1-\theta}{1-\alpha\theta}}$ and $\hat{\vartheta}_{\tilde{k}} = \alpha B\left(\hat{\xi}\hat{\chi}^{1-\theta}\right)^{\frac{1-\alpha}{1-\alpha\theta}}$ as the economy's growth rate and rate of return on capital respectively, with $\hat{\xi} \equiv \frac{A}{B}\frac{\nu}{\delta\alpha}$ and $\hat{\chi} \equiv \frac{B\delta(1-\theta)(1-\alpha)}{(1-\delta\theta)\nu}$.

Proposition 6 *There exist no parameters values $\delta \in (0, 1)$ and $\phi > 0$ such that the Markov-perfect politico-economic equilibrium replicates the social planner allocation.*

Proof. *(See appendix).* ■

Figure 9 simultaneously depicts the optimal growth rate and rental price of capital as a function of δ and the corresponding Markov-perfect equilibrium rates as a function of ϕ. Under the 45-degree line - dotted line - the marginal product of capital exceeds the economy's growth rate and the allocation exhibits dynamic efficiency. Clearly, the social planner allocation is dynamically efficient for any level of δ, whereas the politico-economic equilibrium is dynamically efficient only in a subset of ϕ, i.e. for ϕ large enough. Moreover, both $\tilde{k}$ and $\tilde{f}$ are, on the one hand, increasing functions in the dynasty discount factor and converge to zero when δ goes to zero, and, on the other hand, decreasing functions in the elderly political clout and converge to zero when ϕ goes to infinity. As illustrated in Figure 9, these structural properties imply that the Markov-perfect equilibrium can never replicate the social planner allocation for any parametric configuration (ϕ, δ): in steady state, there always exists a fiscal policy that would make everyone better off by increasing growth and transferring to the old (at the expense of current output).

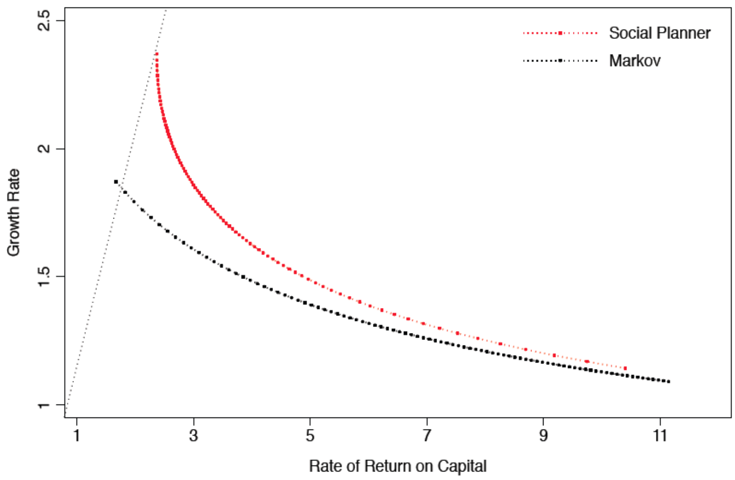

Fig. 9: Long-run Equilibrium vs Efficient Allocation

The source of inefficiency can be traced back to the absence of a long term commitment device and the low degree of substitutability among factors of production. When adult voters anticipate that, by investing in forward transfers, they will gain larger private benefits when old in terms of return on capital. In the specific parametric examples, it translates in a large adults' endogenous weight, Υ, and, in turn, in a low level of enforceable backward transfers. Therefore, the fiscal burden sustained by current taxpayers for any additional unit of investment in productive spending will be never credibly compensated through the implementation of larger expected backward benefits. The distance between the Pareto frontier and the Markov allocation along any ray from the origin reflects the degree of inefficiencies caused by the resulting distortions in the accumulation of both physical and human capital.

6.1 Example II: $\zeta = \infty$

When $\zeta = \infty$, the perfect substitutability among factors of production exacerbates the intergenerational conflicts due to the divergent economic interests between the adults (i.e. workers) and the old (i.e. capitalists). The production function at time t is as $y_t = Rk_t + h_t$, where R denotes the gross interest rate.[18]

According to Eq. (20) the political productive wedge collapses to $\frac{d\mathcal{B}_{t+1}}{df_t} = \nu R$. Further, from Eq. (19), the endogenous weight attached to the adults reduces to $\Upsilon_{t+1} = 1 - \frac{\nu}{R}\frac{d\mathcal{B}_{t+1}}{db_t}$. Modified Proposition 4 predicts that $\frac{d\mathcal{B}_{t+1}}{db_t} < \frac{d\mathcal{B}_{t+1}}{df_t}$: the marginal impact of current investment on next-period backward benefits is required to be larger than the marginal impact of current backward transfers on future ones.

Proposition 7 *Let $m : \mathbb{R}_+ \to \mathbb{R}_+$ be a differentiable single-valued function $m(\psi_{(j)}) \equiv \left(\frac{\nu A\theta}{R}\psi_{(j)} + \frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$ with $\psi_{(1)} = \left(\frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$ as the initial condition.*[19] *For $R > A\nu^{\theta}$, the solutions of the first-order nonlinear*

[18] The linearity of the production function can be derived as an equilibrium outcome in a context of perfect international capital mobility and factor price equalization in the presence of goods trade, where we normalize $B = \frac{1}{1-\alpha}$ and $R = \frac{\alpha}{1-\alpha}$.

[19] The subscript in the parenthesis denotes the number of iterations.

equation $\psi_{(j+1)} = m(\psi_{(j)})$ converge to the fixed points $\psi_x = m(\psi_x)$ with $x \in \{1, 2\}$, where $\psi_1 \leq \psi_2$ and $\psi_1 \equiv \psi$ is the unique locally stable fixed point.

Proof. *(See appendix).* ■

The series $\psi_{(j+1)} = m(\psi_{(j)})$ quantifies the dynamic spillover generated by human capital production. According to Proposition 7, there exists a unique feasible allocation supported as a Markov-perfect politico-economic equilibrium, as follows:[20]

$$\mathcal{B}(k_t, h_t) = \begin{cases} -\frac{\nu^2(1+\beta)R}{\phi+\nu(1+\beta)}k_t + \frac{\phi\nu\left(1+\frac{\theta\nu}{1-\theta}\psi\right)}{\phi+\nu(1+\beta)}h_t & \text{if} \quad \phi \leq \hat{\phi} \\ \\ \frac{\phi\nu}{\phi+\nu(1+\beta(1-\theta))}h_t & \text{if} \quad \phi > \hat{\phi} \end{cases} \tag{30}$$

and

$$\mathcal{F}(h_t) = \begin{cases} \psi h_t & \text{if} \quad \phi \leq \hat{\phi} \\ \\ \frac{\beta(1-\theta)}{\phi+\nu(1+\beta(1-\theta))}h_t & \text{if} \quad \phi > \hat{\phi} \end{cases} \tag{31}$$

where $\hat{\phi} \equiv \beta\left(\frac{R}{A\psi^{1-\theta}} - \nu\right)$ identifies the threshold level of the elderly single-mindedness at which human and physical capital grow at the same rate. Therefore, three possible equilibrium regimes emerge depending on the relative political clout attached to the old: (i) if $\phi > \hat{\phi}$, then $\frac{h_{t+1}}{h_t} > \frac{k_{t+1}}{k_t}$, (ii) if $\phi = \hat{\phi}$, then $\frac{h_{t+1}}{h_t} = \frac{k_{t+1}}{k_t}$, and (iii) if $\phi < \hat{\phi}$, then $\frac{h_{t+1}}{h_t} < \frac{k_{t+1}}{k_t}$. In each regime, the policy rules associated with both forward and backward intergenerational transfers are linear functions in the asset variables, but differently from the case discussed in section 5.1, they possibly differ in the identification of the payoff-relevant state variables.

The equilibrium predictions for forward transfers are easily illustrated. Assumption 4 suggests that the sole way to maximize the utility of the adult constituents requires a positive correlation between human capital and forward transfers, i.e. $\mathcal{F}_{h_t} > 0$. Moreover, since prices are exogenous and the productive investment does not affect the preferences of current old-aged agents, physical capital is never a payoff-relevant state variable for the implementation of forward-oriented policies, i.e. $\mathcal{F}_{k_t} = 0$.

Conversely, the payoff-relevant state variables for backward transfers vary according to the equilibrium regimes. When the elderly political power is low, $\phi \leq \hat{\phi}$, and consequently, the expected rent extraction possibilities for current adults, agents substitute self-insurance through their private savings to social insurance through public spending. Therefore, in equilibrium, backward transfers turn out to be a non-increasing function in the stock of physical capital. Indeed, suppose $\mathcal{B}(\cdot)$ were increasing in k_t, then adults would have incentives to save in order to receive higher pork-barrel transfers when old. However, this expectation cannot be self-fulfilling, since at the same time agents become richer and due to their relatively low political power, they will be required to partially use their proceeds to subsidize consumption of the next generation. Nevertheless, in equilibrium, agents still have incentives to implement forward transfers: by making productive investment, adults would increase their offsprings' productivity and, in turn, they would reduce their next-period fiscal burden, $\mathcal{B}_{h_t} > 0$. To recap, in this scenario, each government, after some initial ones, requires the old to partially transfer resources to their offspring in order to subsidize both the consumption of the adults and the productive investment for the young. In the long run, since income grows, so do the subsidies to the young, the adults' savings, and the capital stock.

[20] Our equilibrium predictions are directly comparable with those explored by Grossman and Helpman (1998), Azariadis and Galasso (2002), and Razin (2002) in a linear environment similar to ours.

The second scenario is characterized by $\phi > \hat{\phi}$. In this circumstance, there exists a unique and stationary equilibrium in which parties tax away a share of the income from the adults and use the tax revenue to invest in forward transfers and to subsidize backward. The initial old generation consumes $R_0 k_0 + b_0$, which is more than it would have consumed in the absence of redistributive policies. But the capital stock converges to zero after one period: the adults anticipate that they will exert a large political power when old, which induces them to substitute public savings to private ones. Therefore, backward intergenerational transfers in each period do not depend on the current value of physical capital and politics fosters economic growth through only human capital.

To summarize, for any initial level of physical and human capital, a Markov-perfect politico-economic equilibrium is characterized by the set of policy functions (30) and (31) and by the following steady-state equilibrium conditions: (i) when $\phi < \hat{\phi}$, the economy diverges to an unbounded and stable long-run at a rate $\frac{k_{t+1}}{k_t} = \frac{\beta R}{\phi + \nu\beta}$, (ii) if $\phi = \hat{\phi}$, the long-run allocation depends on the initial condition of physical capital, i.e. indeterminacy in the initial condition, and the balanced growth rate is $\frac{k_{t+1}}{k_t} = \frac{h_{t+1}}{h_t} = A\psi^{1-\theta}$, and (iii) if $\phi > \hat{\phi}$, the economy grows at the human capital growth rate, $\frac{h_{t+1}}{h_t} = A\left(\frac{\beta(1-\theta)}{\phi+\nu(1+\beta(1-\theta))}\right)^{1-\theta}$. Guessing and verifying that the social planner rules are structurally equivalent to Eqs. (30) and (31), we determine three different long-run paths, which depends on the level of the social welfare weight with respect to $\hat{\delta} = \frac{\nu A\psi^{1-\theta}}{R}$, i.e. the threshold Pareto weight at which the asset variables grow at the same rate. Specifically, (i) when $\delta > \hat{\delta}$, the economy diverges to an unbounded and stable long-run at a rate $\frac{k_{t+1}}{k_t} = \frac{R\delta}{\nu}$, (ii) if $\delta = \hat{\delta}$, the long-run allocation resembles the political balanced growth path with $\frac{k_{t+1}}{k_t} = \frac{h_{t+1}}{h_t} = A\psi^{1-\theta}$, and (iii) if $\delta < \hat{\delta}$, the economy grows at the human capital growth rate, $\frac{h_{t+1}}{h_t} = A\left(\frac{\delta(1-\theta)}{\nu(1-\delta\theta)}\right)^{1-\theta}$.

Proposition 8 *For any $\delta \in (0,1)$ and $\phi = \frac{(1-\delta)\beta\nu}{\delta}$ if $\phi \le \hat{\phi}$, or $\phi = \frac{\nu(\beta-\nu(1+\beta))}{\delta}$ if $\phi > \hat{\phi}$, the Markov-perfect politico-economic equilibrium corresponds to the social planner allocation.*

Proof. *(See appendix).* ■

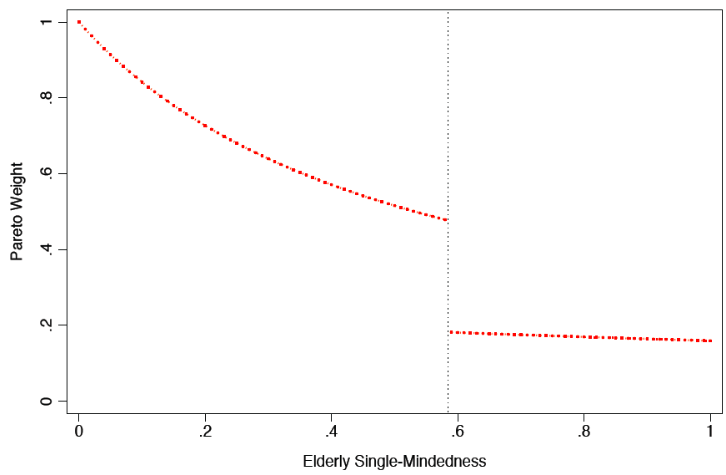

Fig. 10: Politico-Economic vs Efficient Growth Rate

Figure 10 graphically reports the main finding of Proposition 8: for any Pareto weight, a Markov-perfect equilibrium parameterized in ϕ which attains the efficient allocation always exists. Due to the perfect substitutability among factors of production, current investment in productive spending does not generate pecuniary externalities by affecting the rate of return on capital. Therefore, underaccumulation of human capital might potentially be the unique source of distortion. Nevertheless, parties succeed through democratic competition in proposing the efficient long-term productive program, since they fully reward current generation for their sacrifices suffered when adult. Such a reward is quantified by the redistributive wedge, which works as a self-enforcing leverage to provide the adequate incentives for all generations in supporting the efficient productive platform.[21]

7 Conclusions

In this paper, we propose a novel mechanism that enforces the emergence of growth-enhancing intergenerational contracts, given the lack of commitment technology, reputational mechanisms, and altruism. Embedding a repeated probabilistic voting setup in a standard OLG model with human capital accumulation, we point out that the empowerment of elderly constituents is key in order to enforce forward productive transfers. When old-aged voters actively participate to the public debate, they exert their influence by extracting a political rent in terms of backward transfers. Therefore, in their adult-age, they support forward-oriented policies in order to raise their children's labor productivity and, in turn, grab a larger share of the next-period production. As an equilibrium outcome, the intergenerational government expenditures are strategic complement.

The power of this mechanism lies on the ability of short-lived institutions to generate long-run growth. In this institutional environment, the sacrifices suffered by the current constituents will be credibly and at least partially rewarded by future ones. Two fundamental features of the model drive our results: (i) the nature of short-term agreements among politicians and voters, and (ii) the prospect of follow-up intergenerational contracts, which serve as discipline devices to implement current policies.

Throughout the paper, we have assumed that the government runs a balanced budget and does not have access to financial credit markets. In a theoretical environment in which the operating budget is used to finance both durable good, like forward transfers, and non durable ones, like backward transfers, balanced budget restrictions necessarily generate distortions in the provision of public spending. Indeed, voters have incentives to pay solely for the services they get from the government, but do not have the same incentives to optimally finance durable goods, whose benefits have a long-lasting impact. It implies that it would be fair to ask voters to share the fiscal burden by using debt financing, which, in turn, would alleviate the inefficiency due to the underinvestment in productive assets. Assessing the relative impact of government indebtedness on the equilibrium intergenerational contracts is left to future research.

[21] Note that, when $\phi \geq \hat{\phi}$, the environment with $\zeta = \infty$ resembles the economy with only human capital. Thus, the efficiency results of section 6.1 also apply to section 3.4.

8 Appendix A

Proposition 1. To prove uniqueness of the Markov-perfect equilibrium we consider a finite-horizon economy. We start from a final period $T < \infty$ and proceed by backward induction. In the last period, Eqs. (21) and (22) yields:

$$b_T : \Delta^b (b_T, h_T) = 0 \tag{32}$$

$$f_T : \Delta^f (f_T, h_T) < 0 \tag{33}$$

where $\Delta^b (b_T, h_T) \equiv \phi u_{c^o_T} - u_{c^a_T}$ and $\Delta^f (f_T, h_T) \equiv -\nu u_{c^a_T}$. Eqs. (32) and (33) imply $f_T = 0$ and $b_T = \mathcal{B}(h_T) > 0$ as a function of human capital. By implicit function, it immediately follows that $\mathcal{B}_{h_T} > 0$. Furthermore, since the second order condition is smaller than zero for any $b_T > 0$, i.e. $\frac{d\Delta^b(b_T,h_T)}{db_T} < 0$, then there exists a unique $\mathcal{B}(h_T)$, satisfying condition (32). At time $T-1$, the following system of Euler condition holds:

$$b_{T-1} : \Delta^b (b_{T-1}, h_{T-1}) = 0$$

$$f_{T-1} : \Delta^f (f_{T-1}, h_{T-1}) = 0$$

where $\Delta^b (b_{T-1}, h_{T-1}) \equiv \phi u_{c^o_{T-1}} - u_{c^a_{T-1}}$ and $\Delta^f (f_{T-1}, h_{T-1}) \equiv -\nu u_{c^a_{T-1}} + \beta u_{c^o_T} \mathcal{B}_{h_T} \Phi_{f_{T-1}}$. It is straightforward to show that $f_{T-1} = \mathcal{F}(h_{T-1}) > 0$ and $b_{T-1} = \mathcal{B}(h_{T-1}) > 0$. The second order conditions are satisfied for each $b_{T-1},\ f_{T-1} > 0$, i.e. $\frac{d\Delta^b}{db_{T-1}} < 0$ and $\frac{d\Delta^f}{df_{T-1}} < 0$, which imply the uniqueness of the Markov-perfect equilibrium policies at time $T-1$. Replicating the above procedure for each time $t < T-1$ yields the final result. ■

Proposition 2. Let us restate the political Euler conditions, Eqs. (21) and (22), as $\Xi(f_t, \phi) \equiv \Phi_{f_t} - \frac{\nu}{\beta} \frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}} \frac{1}{\mathcal{B}_{h_{t+1}}}$. The implicit function yields $\frac{df_t}{d\phi} = -\frac{\Xi_\phi}{\Xi_{f_t}}$, where $\Xi_{f_t} \equiv \Phi_{f_t f_t} + \frac{\nu}{\beta} \Phi_{f_t} \frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}} (\frac{\mathcal{B}_{h_{t+1}h_{t+1}}}{\mathcal{B}^2_{h_{t+1}}} + \frac{u_{c^o_{t+1}c^o_{t+1}}}{u_{c^o_{t+1}}})$ and $\Xi_\phi \equiv -\frac{\nu}{\beta} \left(\frac{d}{d\phi} \left(\frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}} \right) \frac{1}{\mathcal{B}_{h_{t+1}}} - \frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}} \frac{\mathcal{B}_{h_{t+1},\phi}}{\mathcal{B}^2_{h_{t+1}}} \right)$. Assumptions 2 and 4 guarantee the negativity of Ξ_{f_t}. Therefore, $\frac{df_t}{d\phi} \geq (<)\, 0$ if and only if $\Xi_\phi \geq (<)\, 0$. Let us denote by $\rho_{\mathcal{B}_{h_{t+1},\phi}} \equiv \frac{\phi \mathcal{B}_{h_{t+1},\phi}}{\mathcal{B}_{h_{t+1}}}$ and $\rho_{\frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}},\phi} \equiv \frac{d}{d\phi} \left(\frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}} \right) \frac{u_{c^o_{t+1}}}{u_{c^o_t}}$. Hence, $\Xi_\phi \geq (<)\, 0$ if and only if $\rho_{\frac{\phi u_{c^o_t}}{u_{c^o_{t+1}}},\phi} \leq (>)\, \rho_{\mathcal{B}_{h_{t+1},\phi}}$. Thus, the final result yields. ■

Proposition 3. The proof relies on two sections. We first show that there exists a continuum of undetermined differentiable policy functions if $\phi = 0$ (section 1). Second, we point out how a positive value of the ideological bias breaks the indeterminacy of the optimal policy, yielding uniqueness (section 2). For simplicity let us denote by $\mu_t \equiv 1 - \phi \frac{u_{c^o_t}}{u_{c^a_t}}$ and $\Gamma_{t+1} \equiv \gamma \Theta_{k_{t+1}} I_t + \frac{1-\gamma_{t+1}}{\Theta_{k_{t+1}}} (\frac{\nu k_{t+1} \Theta_{k_{t+1}} + b_{t+1}}{\Theta_{k_{t+1}}})$.

Section 1 (Indeterminacy): Applying implicit function theorem to Eq. (17) yields:

$$\mathcal{K}_{b_t} \equiv -\frac{\Delta^k_{b_t}}{\Delta^k_{k_{t+1}}} = -\frac{\gamma_{t+1}}{\nu \left(\nu - \Gamma_{t+1} \Theta_{k_{t+1}k_{t+1}} + \frac{1-\gamma_{t+1}}{\Theta_{k_{t+1}}} \left(\nu k_{t+1} \Theta_{k_{t+1}k_{t+1}} + \mathcal{B}_{k_{t+1}} \right) \right)} \tag{34}$$

and

$$\mathcal{K}_{f_t} \equiv -\frac{\Delta^k_{f_t}}{\Delta^k_{k_{t+1}}} = -\frac{\gamma_{t+1} - \frac{\Phi_{f_t}}{\nu} \left(\Gamma_{t+1} \Theta_{k_{t+1}h_{t+1}} - \frac{1-\gamma_{t+1}}{\Theta_{k_{t+1}}} \left(\nu k_{t+1} \Theta_{k_{t+1}h_{t+1}} + \mathcal{B}_{h_{t+1}} \right) \right)}{1 - \frac{1}{\nu} \left(\Gamma_{t+1} \Theta_{k_{t+1}k_{t+1}} - \frac{1-\gamma_{t+1}}{\Theta_{k_{t+1}}} \left(\nu k_{t+1} \Theta_{k_{t+1}k_{t+1}} + \mathcal{B}_{k_{t+1}} \right) \right)} \tag{35}$$

Plugging Eq. (34) into Eq. (19) we obtain:

$$\mathcal{B}_{k_{t+1}} = -\frac{\nu\mu_t}{\mu_t\left(1-\gamma_{t+1}\right)+\gamma_{t+1}}\Theta_{k_{t+1}} - \left(\nu k_{t+1} - \frac{\mu_t}{\mu_t\left(1-\gamma_{t+1}\right)+\gamma_{t+1}}\Gamma_{t+1}\Theta_{k_{t+1}}\right)\Theta_{k_{t+1}k_{t+1}} \tag{36}$$

Combining Eqs. (19) and (20) yields:

$$\left(\mathcal{B}_{k_{t+1}} + \nu k_{t+1}\Theta_{k_{t+1}k_{t+1}}\right)\left(\frac{1}{\nu}\mathcal{K}_{f_t} - \frac{\nu}{\mu_t}\mathcal{K}_{b_t}\right) + \left(\mathcal{B}_{h_{t+1}} + \nu k_{t+1}\Theta_{k_{t+1}h_{t+1}}\right)\frac{1}{\nu}\Phi_{f_t} = 0 \tag{37}$$

Inserting Eqs. (34) and (35) into (37) we get:

$$\mathcal{B}_{h_{t+1}} = \frac{\nu\left(1-\mu_t\right)\gamma_{t+1} + \mu_t\Gamma_{t+1}\Theta_{k_{t+1}h_{t+1}}\Phi_{f_t}}{\mu_t\left(1-\gamma_{t+1}\right)+\gamma_{t+1}}\frac{\Theta_{k_{t+1}}}{\Phi_{f_t}} - \nu k_{t+1}\Theta_{k_{t+1}h_{t+1}} \tag{38}$$

Therefore, Eqs. (34) and (35) reduce to $\mathcal{K}_{b_t} = -\frac{\mu_t(1-\gamma_{t+1})+\gamma_{t+1}}{\nu\left(\nu-\Gamma_{t+1}\Theta_{k_{t+1}k_{t+1}}\right)}$ and $\mathcal{K}_{f_t} = -\frac{\nu-\Phi_{f_t}\Gamma_{t+1}\Theta_{k_{t+1}h_{t+1}}}{\nu-\Gamma_{t+1}\Theta_{k_{t+1}k_{t+1}}}$, respectively. Both equations belong to the interval $(-1, 0)$. Rearranging Eq. (38) and exploiting the homogeneity property of the politico-economic setting, i.e. $b_{t+1} = k_{t+1}\mathcal{B}_{k_{t+1}} + h_{t+1}\mathcal{B}_{h_{t+1}}$, yields:

$$\Phi_{f_t} = \frac{\Theta_{k_{t+1}}}{\Theta_{h_{t+1}}}\frac{\nu\left(1-\mu_t\right)\gamma_{t+1}\Theta_{h_{t+1}}}{\left(\mu_t\left(1-\gamma_{t+1}\right)+\gamma_{t+1}\right)\frac{b_{t+1}}{h_{t+1}} + \nu\mu_t\frac{k_{t+1}}{h_{t+1}}\Theta_{k_{t+1}}} \tag{39}$$

Eqs. (36) and (39) jointly describe the necessary conditions, which are required to be satisfied in equilibrium. To show indeterminacy of the solution, let us inspect Eq. (38). Since $\frac{u_{c_t^o}}{u_{c_t^a}} > 0$ for any level of ϕ, if $\phi \to 0$ then $\mu_t \to 1$. Therefore, it is straightforward to verify that, since Φ_{f_t} wipes out, the necessary conditions for Markov-perfect equilibrium are satisfied for any level of forward transfers. Finally, combining Eqs. (36) and. (37) yields:

$$\begin{cases} \mathcal{B}_{k_{t+1}} = -\nu\Theta_{k_{t+1}} + \left(\Gamma_{t+1}\Theta_{k_{t+1}} - \nu k_{t+1}\right)\Theta_{k_{t+1}k_{t+1}} \\ \mathcal{B}_{h_{t+1}} = \left(\Gamma_{t+1}\Theta_{k_{t+1}} - \nu k_{t+1}\right)\Theta_{k_{t+1}h_{t+1}} \end{cases}$$

Any solution of the above system of differential equations is a candidate equilibrium policy. The general solution turns out to be characterized by multiple stationary equilibria indexed by self-fulfilling expectation-driven parameters.

Section 2 (Break of Indeterminacy and Uniqueness): Similarly to the Proof of Proposition 1, when $\phi > 0$ the uniqueness of the Markov-perfect equilibrium is easily proved by backward induction. In the last period T, using Eq. (7), the following system of first order conditions for b_T and f_T characterizes the Markov-perfect equilibrium:

$$b_T : \Delta^b\left(b_T, k_T, h_T\right) = 0 \tag{40}$$

$$f_T : \Delta^f\left(f_T, k_T, h_T\right) < 0 \tag{41}$$

where $\Delta^b\left(f_T, k_T, h_T\right) \equiv \phi u_{c_T^o} - u_{c_T^a}$ and $\Delta^f\left(f_T, k_T, h_T\right) \equiv -\nu u_{c_T^a}$. Eqs. (40) and (41) imply $f_T = 0$ and $b_T = \mathcal{B}\left(k_T, h_T\right) \neq 0$ as a function of physical and human capital. It is straightforward to show that at time T the political objective function, Eq. (18), is concave in b_T, i.e. $\frac{d\Delta^b}{db_T} \equiv \phi u_{c_T^o c_T^o} + \frac{1}{\nu}u_{c_T^a c_T^a} < 0$. Hence, there exists a unique $\mathcal{B}\left(k_T, h_T\right)$, satisfying condition (40). At time $T-1$, the following system of Euler condition holds:

$$b_{T-1} : \Delta^b (b_{T-1}, k_{T-1}, h_{T-1}) = 0$$
$$f_{T-1} : \Delta^f (b_{T-1}, k_{T-1}, h_{T-1}) = 0$$

where $\Delta^b (b_{T-1}, k_{T-1}, h_{T-1}) \equiv \phi u_{c^o_{T-1}} - \beta u_{c^o_T} \left(\Theta_{k_T} - \nu \left(\frac{d\mathcal{B}_T}{db_{T-1}} + \nu k_T \frac{d\Theta_{k_T}}{db_{T-1}} \right) \right)$ and $\Delta^f (f_{T-1}, h_{T-1}) \equiv -\nu u_{c^a_{T-1}} + \beta u_{c^o_T} \left(\frac{d\mathcal{B}_T}{df_{T-1}} + \nu k_T \frac{d\Theta_{k_T}}{df_{T-1}} \right)$. From the above equations $f_{T-1} = \mathcal{F}(k_{T-1}, h_{T-1}) > 0$ and $b_{T-1} = \mathcal{B}(k_{T-1}, h_{T-1}) \neq 0$. Next, we control for the second order conditions that imply uniqueness of the Markov-perfect equilibrium policies at time $T-1$. After some tedious algebra, $\frac{d\Delta^b}{db_{T-1}} < 0$ and $\frac{d\Delta^f}{df_{T-1}} < 0$ if $\frac{d^2 \mathcal{C}^o_T}{dk_T dk_T} - \frac{\Theta_{k_T k_T}}{\Theta_{k_T}} \frac{d\mathcal{C}^o_T}{dk_T} \leq 0$ for each b_{T-1} and f_{T-1}. Replicating the above procedure for each time $t < T-1$ yields the final result. ■

Proposition 4. Using the Euler condition for saving and combining Eqs. (19) and (20) yields $\nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t} + \frac{d\mathcal{B}_{t+1}}{df_t} = \frac{\nu^2}{\mu_t} \left(\nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t} + \frac{d\mathcal{B}_{t+1}}{db_t} \right)$ with $\nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t} + \frac{d\mathcal{B}_{t+1}}{df_t} = \Theta_{k_{t+1}} > 0$. We distinguish two cases: (i) if $\mu_t < 0$ then $\nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t} + \frac{d\mathcal{B}_{t+1}}{db_t} < 0 < \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t} + \frac{d\mathcal{B}_{t+1}}{df_t}$; (ii) if $\mu_t \in [0, 1]$ then $\nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t} + \frac{d\mathcal{B}_{t+1}}{db_t} \geq 0$. Since $\frac{\nu^2}{\mu_t} > 1$, we obtain the following rank: $0 \leq \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{db_t} + \frac{d\mathcal{B}_{t+1}}{db_t} < \nu k_{t+1} \frac{d\Theta_{k_{t+1}}}{df_t} + \frac{d\mathcal{B}_{t+1}}{df_t}$. ■

Proposition 5. The resolution strategy consists in three steps. We first compute the first order conditions starting from a time $T < \infty$ large enough and solving backward for each time $T - j$ with $j = 0, 1, ...T$, subject to (i) the economic Euler condition, Eq. (7), (ii) the balanced budget constraint, Eq. (6), and (iii) the equilibrium policy rules of future periods. Second, we recursively determine the conditions for the existence of the fixed points. As final step, we characterize the Politico-Economic policy rules. Recall that $I^a_t \equiv (1-\alpha) y_t - \nu f_t - \frac{b_t}{\nu}$ denotes the present value of after-tax lifetime for the adult. Let consider a finite horizon economy for any $T < \infty$. The politicians' maximization problem is equal to $\max_{f_T, b_T} \nu u(\mathcal{C}^a_T) + \phi u(\mathcal{C}^o_T)$, where $\mathcal{C}^a_T \equiv I^a_T$ and $\mathcal{C}^o_T \equiv \nu\alpha y_T + b_T$. In the last period $f_T = 0$ and the Euler condition for the backward transfers collapses to $u_{c^a_T} - \phi u_{c^o_T} = 0$. Under logarithmic preferences it implies:

$$b_T = \frac{\nu (\phi - \alpha (\phi + \nu))}{\phi + \nu} y_T \tag{42}$$

Thus, the resulting consumption levels are $\mathcal{C}^a_T = \frac{\nu}{\phi+\nu} y_T$ and $\mathcal{C}^o_T = \frac{\nu\phi}{\phi+\nu} y_T$. Combining the Euler condition on savings and Eq. (42) yields:

$$k_T = \frac{\alpha\beta (\phi + \nu)}{\nu (\phi + \alpha\beta (\phi + \nu))} I^a_{T-1} \tag{43}$$

It follows that at time $T-1$ the individual consumptions are equal to $\mathcal{C}^a_{T-1} \equiv \frac{\phi}{\phi+\alpha\beta(\phi+\nu)} I^a_{T-1}$ and $\mathcal{C}^o_{T-1} \equiv \nu\alpha y_{T-1} + b_{T-1}$. The politicians' maximization problem is $\max_{f_{T-1}, b_{T-1}} \nu u(\mathcal{C}^a_{T-1}) + \beta\nu u(\mathcal{C}^o_T) + \phi u(\mathcal{C}^o_{T-1})$. Applying the envelope condition and using Eq. (43), the Euler for interior policies are as follows:

$$f_{T-1} : \Phi_{f_{T-1}} = \frac{R_T}{w_T} (1 - \varepsilon_T)$$
$$b_{T-1} : \phi u_{c^o_{T-1}} = \Upsilon_{T-1} u_{c^a_{T-1}}$$

where $\varepsilon_T \equiv -\frac{\nu}{\phi+\alpha\beta(\phi+\nu)}$ and $\Upsilon_{T-1} \equiv \frac{\phi(1+\alpha\beta)}{\phi+\alpha\beta(\phi+\nu)}$. Under the technology of the Cobb-Douglas type, solving the above system yields:

$$f_{T-1}=\frac{\beta\left(1-\theta\right)\left(1-\alpha\right)}{\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)}y_{T-1} \tag{44}$$

$$b_{T-1}=\frac{\nu\left(\phi-\alpha\left(\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)\right)\right)}{\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)}y_{T-1} \tag{45}$$

Further, plugging Eqs. (44) and (45) into the individual consumption functions and the Euler condition on savings we get $\mathcal{C}^a_{T-1}=\frac{\phi\nu(1+\alpha\beta)}{(\phi+\alpha\beta(\phi+\nu))(\phi+\nu(1+\beta(1-(1-\alpha)\theta)))}y_{T-1}$, $\mathcal{C}^o_{T-1}=\frac{\nu\phi}{\phi+\nu(1+\beta(1-(1-\alpha)\theta))}y_{T-1}$, and

$$k_{T-1}=\frac{\alpha\beta\left(\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)\right)}{\nu\left(\phi+\alpha\beta\left(\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)\right)\right)}I^a_{T-2} \tag{46}$$

Using Eq. (46), at time $T-2$ the individual consumption are equal to $\mathcal{C}^a_{T-2}\equiv\frac{\phi}{\phi+\alpha\beta(\phi+\nu(1+\beta(1-(1-\alpha)\theta)))}I^a_{T-2}$ and $\mathcal{C}^o_{T-2}\equiv\nu\alpha y_{T-2}+b_{T-2}$. The politicians' maximization problem is $\max\limits_{f_{T-2},b_{T-2}}\ \nu u(\mathcal{C}^a_{T-2})+\beta\nu u(\mathcal{C}^o_{T-1})+\phi u(\mathcal{C}^o_{T-2})$. Replicating the same argument of time $T-1$, the Euler conditions for interior policies are:

$$f_{T-2}:\Phi_{f_{T-2}}\left(f_{T-2}\right)=\frac{R_{T-1}}{w_{T-1}}\left(1-\varepsilon_{T-1}\right)$$

$$b_{T-2}:\phi u_{c^o_{T-2}}=\Upsilon_{T-2}u_{c^a_{T-2}}$$

where $\varepsilon_{T-1}\equiv-\frac{\nu(1+\beta(1-(1-\alpha)\theta))}{\phi+\alpha\beta(\phi+\nu(1+\beta(1-(1-\alpha)\theta)))}$ and $\Upsilon_{T-2}\equiv\frac{(1+\alpha\beta)\phi}{\phi+\alpha\beta(\phi+\nu(1+\beta(1-(1-\alpha)\theta)))}$ with $\frac{d\varepsilon_{T-1}}{d\phi}>0$ and $\frac{d\Upsilon_{T-2}}{d\phi}>0$. The policy rules that solve the above system are identical to those of period $T-1$. Due to the specific parametric form, after only two recursions the policies converge, i.e. $b_{T-2}=b_{T-1}$ and $f_{T-2}=f_{T-1}$. Hence, the individual consumption at time $T-2$ amounts to $\mathcal{C}^o_{T-2}=\frac{\nu\phi}{\phi+\nu(1+\beta(1-(1-\alpha)\theta))}y_{T-2}$ and $\mathcal{C}^a_{T-2}=\frac{\phi\nu(1+\beta\alpha)}{(\phi+\alpha\beta(\phi+\nu(1+\beta(1-(1-\alpha)\theta))))(\phi+\nu(1+\beta(1-(1-\alpha)\theta)))}y_{T-2}$. We conclude that for a generic time t the equilibrium policies are given by Eqs. (44) and (45). Furthermore, the laws of motion of the state variables are as follows:

$$k_{t+1}=\frac{B\alpha\beta\left(1+\alpha\beta\right)}{\phi+\alpha\beta\left(\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)\right)}k_t^{\alpha}h_t^{1-\alpha}$$

$$h_{t+1}=A\left(\frac{B\beta\left(1-\theta\right)\left(1-\alpha\right)}{\phi+\nu\left(1+\beta\left(1-\left(1-\alpha\right)\theta\right)\right)}\right)^{1-\theta}k_t^{(1-\theta)\alpha}h_t^{1-(1-\theta)\alpha}$$

Therefore, in balanced growth we obtain $\tilde{k}=\left(\frac{1}{\xi}\right)^{\frac{1}{1-\alpha\theta}}\left(\frac{1}{\chi}\right)^{\frac{1-\theta}{1-\alpha\theta}}$ and $\tilde{f}=\left(\frac{1}{\xi}\right)^{\frac{\alpha}{1-\alpha\theta}}\left(\chi\right)^{\frac{1-\alpha}{1-\alpha\theta}}$, where $\xi\equiv\frac{A}{B}\frac{\phi+\alpha\beta(\phi+\nu(1+\beta(1-(1-\alpha)\theta)))}{\alpha\beta(1+\alpha\beta)}$ and $\chi\equiv\frac{B\beta(1-\theta)(1-\alpha)}{\phi+\nu(1+\beta(1-(1-\alpha)\theta))}$. Hence, the growth rate and associated rental price of capital are $\vartheta_{\tilde{k}}=\alpha B\left(\xi\chi^{1-\theta}\right)^{\frac{1-\alpha}{1-\alpha\theta}}$ and $\varphi=A\left(\frac{\chi^{1-\alpha}}{\xi^{\alpha}}\right)^{\frac{1-\theta}{1-\alpha\theta}}$, respectively. ■

Proposition 6. We separate the proof in two sections. Section 1 characterizes the efficient growth rate of the economy and the rental price of capital in balanced growth path. Section 2 states the efficient properties of the Markov-perfect equilibrium.

Section 1 (Efficient allocation): Along the balanced growth path $\frac{c^o_{t+1}}{c^o_t}=\varphi\left(\tilde{f}\right)$. Combining Eqs. (25) and (26) yields:

$$\varphi\left(\tilde{f}\right)=\frac{\delta\vartheta_{\tilde{k}}}{\nu} \tag{47}$$

Further, Eq. (27) collapses to:

$$\varphi_{\tilde{f}}-\frac{\vartheta_{\tilde{k}}}{\vartheta-\tilde{k}\vartheta_{\tilde{k}}}\left(1-\hat{\varepsilon}\right) \tag{48}$$

where $\hat{\varepsilon} = \frac{\nu}{\vartheta_{\tilde{k}}}\left(\varphi - \tilde{f}\varphi_{\tilde{f}}\right)$. Under human capital production as of Cobb-Douglas type and Eq. (47), we obtain $\hat{\varepsilon} = \delta\theta$. Using Eq. (47) and rewriting Eq. (48) as $\tilde{f} = \frac{1-\theta}{1-\delta\theta}\frac{1-\alpha}{\alpha}\tilde{k}\varphi\left(\tilde{f}\right)$ yields:

$$\frac{\tilde{f}}{\tilde{y}} = \frac{(1-\theta)(1-\alpha)\delta}{(1-\delta\theta)\nu} \tag{49}$$

The aggregate consumption and saving are equal to $\tilde{C} \equiv \tilde{c}^a + \frac{\tilde{c}^o}{\nu} = \tilde{y} - \nu\frac{h_{t+1}}{h_t}\tilde{k} - \nu\tilde{f}$ and $\tilde{s} = \tilde{y} - \tilde{C}$, respectively. Using the Cobb-Douglas properties, i.e. $\vartheta_{\tilde{k}}\tilde{k} = \alpha\tilde{y}$, and Eq. (47), we get:

$$\tilde{k} = \frac{\delta\alpha\tilde{y}}{\nu\varphi\left(\tilde{f}\right)} \tag{50}$$

Rearranging the terms, we obtain $\tilde{C} = \left(\frac{(1-\delta\theta)(1-\delta\alpha)-(1-\theta)(1-\alpha)\delta}{(1-\delta\theta)}\right)\tilde{y}$. To identify how aggregate consumption is shared between the first and the second period consider the relation $\vartheta_{\tilde{k}}\left(\tilde{k}\right) = \frac{u_{c_t^a}}{\beta u_{c_{t+1}^o}} = \frac{\varphi\left(\tilde{f}\right)}{\beta}\frac{\tilde{c}^o}{\tilde{c}^a}$, which yields $\tilde{c}^a = \frac{\delta}{\beta+\delta}\tilde{C}$ and $\tilde{c}^o = \frac{\beta\nu}{\beta+\delta}\tilde{C}$. From the human capital and final good productions, Eq. (49) yields the efficient growth rate as:

$$\varphi\left(\tilde{f}\right) = A\left(\frac{B(1-\theta)(1-\alpha)\delta}{(1-\delta\theta)\nu}\right)^{1-\theta}\left(\tilde{k}\right)^{\alpha(1-\theta)} \tag{51}$$

Likewise, from Eq. (50) we obtain:

$$\tilde{k} = \left(\frac{B}{A}\frac{\delta\alpha}{\nu}\right)^{\frac{1}{1-\alpha\theta}}\left(\frac{(1-\delta\theta)\nu}{B(1-\theta)(1-\alpha)\delta}\right)^{\frac{1-\theta}{1-\alpha\theta}} \tag{52}$$

Eqs. (51) and (52) simultaneously imply $\hat{\varphi} = A\left(\frac{\hat{\chi}^{1-\theta}}{\hat{\xi}^{\alpha}}\right)^{\frac{1-\alpha}{1-\alpha\theta}}$ and $\hat{\vartheta}_{\tilde{k}} = \alpha B\left(\hat{\xi}\hat{\chi}^{1-\theta}\right)^{\frac{1-\alpha}{1-\alpha\theta}}$ where $\hat{\xi} \equiv \frac{A}{B}\frac{\nu}{\delta\alpha}$ and $\hat{\chi} \equiv \frac{B\delta(1-\theta)(1-\alpha)}{(1-\delta\theta)\nu}$.

Section 2 (Efficiency vs Markov): The Markov-perfect equilibrium corresponds to the social planner allocation if and only if there exists a non empty parametric space of Pareto weights and ideological bias (δ, ϕ) such that $\hat{\varphi} = \varphi$ and $\hat{\vartheta}_{\tilde{k}} = \vartheta_{\tilde{k}}$. It translates in solving the system of equations $\chi_p = \chi_o$ and $\xi_o = \xi_p$, which has no real solutions. ■

Proposition 7. Let us follow the same procedure proposed in the Proof of Proposition 5.

First Step (First Order Conditions): In the last period $T < \infty$, the adults have one period temporal-horizon. Thus, the political objective function is as $\nu u\left(\mathcal{C}_T^a\right) + \phi u\left(\mathcal{C}_T^o\right)$, where $\mathcal{C}_T^a \equiv h_T - \nu f_T - \frac{b_T}{\nu}$ and $\mathcal{C}_T^o \equiv \nu Rk_T + b_T$. In the absence of future the agents have no incentives to privately save and to invest in productive transfers, i.e. $f_T = 0$. The Euler condition of backward transfers turns out to be $u_{c_T^a} - \phi u_{c_T^o} = 0$. Under logarithmic utility it implies:

$$b_T = -\frac{\nu^2}{\phi+\nu}Rk_T + \frac{\nu\phi}{\phi+\nu}h_T \tag{53}$$

At time $T-1$ the agents born at time $T-2$ live up three periods. Thus, the political objective function is as follows:

$$\nu\left(u\left(\mathcal{C}_{T-1}^a\right) + \beta u\left(\mathcal{C}_T^o\right)\right) + \phi u\left(\mathcal{C}_{T-1}^o\right) \tag{54}$$

where $\mathcal{C}_{T-1}^a \equiv h_{T-1} - \nu f_{T-1} - \frac{b_{T-1}}{\nu} - \nu k_T$ and $\mathcal{C}_{T-1}^o \equiv \nu Rk_{T-1} + b_{T-1}$. After plugging the equilibrium policy rules of period T into Eq. (54), we maximize with respect to the political platform $p_{T-1} = \{b_{T-1}, f_{T-1}\}$. By applying the envelope theorem, we obtain the following system of Euler conditions for the intergenerational transfers:

$$\begin{aligned} b_{T-1}: \; & \frac{u_{c^o_{T-1}}}{u_{c^a_{T-1}}} = \frac{1+\beta}{\phi + \beta(\nu + \phi)} \\ f_{T-1}: \; & \frac{dh_T}{df_{T-1}} = R \end{aligned}$$

Let us denote by $\psi_{(1)} \equiv \left(\frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$, where into the brackets we report the number of iterations. Under human capital function of the Cobb-Douglas type, the equilibrium policy rules are:

$$\begin{aligned} b_{T-1} &= -\frac{(1+\beta)\nu^2}{\phi + \nu(1+\beta)} Rk_{T-1} + \frac{\nu\phi}{\phi + \nu(1+\beta)}\left(1 + \frac{\theta\nu}{1-\theta}\psi_{(1)}\right)h_{T-1} \\ f_{T-1} &= \psi_{(1)} h_{T-1} \end{aligned}$$

Starting from period $T-2$ onwards, the political objective function turns out to be structurally equivalent to Eq. (54). Taking the maximization with respect to $p_{T-2} = \{b_{T-2}, f_{T-2}\}$ and replicating the same argument of $T-1$, the following system of Euler conditions holds:

$$\begin{aligned} b_{T-2}: \; & \frac{u_{c^o_{T-2}}}{u_{c^a_{T-2}}} = \frac{1}{\phi + \nu\beta} \\ f_{T-2}: \; & \frac{dh_{T-1}}{df_{T-2}} = R\left(1 - \frac{\psi_{(1)}}{\frac{1-\theta}{\theta\nu} + \psi_{(1)}}\right) \end{aligned}$$

Let us denote by $\psi_{(2)} = m(\psi_{(1)}) \equiv \left(\frac{A\theta\nu}{R}\psi_{(1)} + \frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$. The corresponding policy rules (solutions of the system above) are:

$$\begin{aligned} b_{T-2} &= -\frac{(1+\beta)\nu^2 R}{\phi + \nu(1+\beta)} k_{T-2} + \frac{\nu\phi}{\phi + \nu(1+\beta)}\left(1 + \frac{\theta\nu}{1-\theta}\psi_{(2)}\right)h_{T-2} \\ f_{T-2} &= \psi_{(2)} h_{T-2} \end{aligned}$$

It is straightforward to show that from $T-2$ on the policy rules that solve the Euler conditions for intergenerational transfers are structurally equivalent to those of period $T-2$.

Second Step (Fixed Point): The political Markov-perfect equilibrium exists if and only if the limits for $j \to \infty$ of the series $\psi_{(j+1)} = m(\psi_{(j)}) \equiv \left(\frac{A\theta\nu}{R}\psi_{(j)} + \frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$ exists and is finite. The differentiable single-valued function $m(\psi_{(j)})$ is characterized by $m(0) > 0$, $m_\psi > 0$, and $m_{\psi\psi} > 0$. Let us denote by $\bar{\psi} \equiv \frac{1}{\theta}\left(\left(\frac{R}{A\nu}\right)^{\frac{1}{1-\theta}} - \frac{1-\theta}{\nu}\right)$ the value of $\psi_{(j)}$ such that $m_\psi(\bar{\psi}) = 1$. The corresponding value of the series evaluated at $\bar{\psi}$ is $m(\bar{\psi}) \equiv \left(\frac{R}{A\nu}\right)^{\frac{1}{1-\theta}}$. For $R > A\nu^\theta$, we obtain $m(\bar{\psi}) < \bar{\psi}$ and, in turn, the first-order nonlinear equation $\psi_{(j+1)} = m(\psi_{(j)})$ converges to the fixed points $\psi^x = m(\psi^x)$, with $x \in \{1, 2\}$, where $\psi^1 \leq \psi^2$ and $\psi^1 \equiv \psi$ is the unique locally stable fixed point. By using implicit function theorem, $\psi_\theta < 0$, $\psi_R < 0$, $\psi_A > 0$, and $\psi_\nu < 0$ characterize the fixed point.

Third Step (Equilibrium Policy Rules): Let us denote by $\hat{\phi}$ the solution of the first order condition for backward transfers in balanced growth path, i.e. $\frac{1}{\hat{\phi}}\left(1 - \frac{\nu}{R}\frac{d\mathcal{B}_{t+1}(\hat{\phi})}{db_t}\right)\beta R = \frac{c^o_{t+1}}{c^o_t}$ with $\frac{c^o_{t+1}}{c^o_t} = \frac{d\mathcal{B}_{t+1}(\hat{\phi})}{db_t} = \varphi$. It implies $\hat{\phi} = \frac{\beta(R-\nu\varphi)}{\varphi}$. When $\phi \leq \hat{\phi}$, the Markov-perfect politico-economic equilibrium of the intergenerational voting game is equal to:

$$\mathcal{B}(h_t, k_t) = -\frac{\nu^2(1+\beta)R}{\phi+\nu(1+\beta)}k_t + \frac{\nu\phi}{\phi+\nu(1+\beta)}\left(1+\frac{\theta\nu}{1-\theta}\psi\right)h_t$$

and

$$\mathcal{F}(h_t) = \psi h_t$$

Therefore, the laws of motion of human and physical capital are as follows:

$$k_{t+1} = \frac{\beta R}{\phi+\nu\beta}k_t + \frac{1}{1+\beta}\left(\frac{(1-\nu\psi)\beta(\phi+\nu(1+\beta))}{\phi\nu} - \left(\frac{\beta}{\nu}+\frac{A\psi^{1-\theta}}{R}\right)\left(1+\frac{\theta\nu}{1-\theta}\psi\right)\right)h_t$$

and

$$h_{t+1} = A\psi^{1-\theta}h_t$$

with $\frac{h_{t+1}}{h_t} = A\psi^{1-\theta} \leq \frac{k_{t+1}}{k_t} = \frac{\beta R}{\phi+\nu\beta}$. On the contrary, if $\phi > \hat{\phi}$, then $\frac{k_{t+1}}{k_t} < \frac{h_{t+1}}{h_t}$. In this case the unique Markov-perfect equilibrium is characterized by human capital as the sole payoff-relevant state variable. Replicating the backward procedure as in Step (1), we achieve the result:

$$\mathcal{B}(h_t) = \frac{\phi\nu}{\phi+\nu(1+\beta(1-\theta))}h_t$$

and

$$\mathcal{F}(h_t) = \frac{\beta(1-\theta)}{\phi+\nu(1+\beta(1-\theta))}h_t$$

The resulting dynamic for human capital is as $h_{t+1} = A\left(\frac{\beta(1-\theta)}{\phi+\nu(1+\beta(1-\theta))}\right)^{1-\theta}h_t$. ■

Proposition 8. We sketch the proof following the steps of Proposition 6.

Section 1 (Efficient allocation): To analytically characterize the social planner solution we guess and verify that the equilibrium policy rules are structural equivalent to Eqs. (30) and (31). If the guesses are the equilibrium, then they must simultaneously satisfy the first order conditions, i.e. Eqs. (26) and (27). Under human capital production of Cobb-Douglas type, we obtain $\frac{\Phi_{h_{t+1}}}{\Phi_{f_{t+1}}} = \frac{\theta}{1-\theta}\frac{f_{t+1}}{h_{t+1}}$. Hence, the optimal first order condition for forward transfers reduces to $\Phi_{f_t} = R\frac{\Phi_{f_{t+1}}}{\Phi_{f_{t+1}}+\nu\Phi_{h_{t+1}}}$ or, equivalently, to $\frac{f_t}{h_t} = \left(\frac{\nu\theta A}{R}\frac{f_{t+1}}{h_{t+1}} + \frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$. Using the guess of the optimal forward policy, i.e. $\hat{f}_t = \hat{\psi}h_t$, yields $\hat{\psi} = \left(\frac{\nu\theta A}{R}\hat{\psi} + \frac{A(1-\theta)}{R}\right)^{\frac{1}{\theta}}$, whose zero is equivalent to the politico-economic solution, i.e. $\hat{\psi} = \psi$. As a result, the law of motion of human capital is as:

$$h_{t+1} = A\psi^{1-\theta}h_t \tag{55}$$

Let us denote by $\hat{\delta}$ the solution of the first order condition for the optimal backward transfers, $\hat{b}_t$, in balanced growth path, i.e. $\frac{c^o_{t+1}}{c^o_t} = \frac{\delta R}{\nu}$ with $\frac{c^o_{t+1}}{c^o_t} = \frac{d\mathcal{B}_{t+1}(\hat{\delta})}{db_t} = \hat{\varphi}$. It implies $\hat{\delta} = \frac{\nu A\psi^{1-\theta}}{R}$. When $\delta \geq \hat{\delta}$, let us guess the backward policy as $\hat{b}_t = \pi_1 k_t + \pi_0 h_t$. Under logarithmic utility, the optimal saving decision yields $k_{t+1} = \frac{\beta R}{\nu(1+\beta)R+\pi_1}\left(h_t - \nu\hat{f}_t - \frac{\hat{b}_t}{\nu}\right) - \frac{\pi_0}{\nu(1+\beta)R+\pi_1}h_{t+1}$. Plugging the guess and the law of motion of physical and human capital into Eq. (26), we obtain:

$$\hat{b}_t = -\frac{\delta(\nu(1+\beta)R+\pi_1)\nu R}{\delta(\nu(1+\beta)R+\pi_1)+\beta(\nu R+\pi_1)}k_t + \frac{\nu\beta(\nu R+\pi_1)(1-\nu\psi)+\nu\beta\nu\pi_0 A\psi^{1-\theta}}{\delta(\nu(1+\beta)R+\pi_1)+\beta(\nu R+\pi_1)}h_t$$

where

$$\begin{cases} \pi_1 = -\frac{\delta(\nu(1+\beta)R+\pi_1)\nu R}{\delta(\nu(1+\beta)R+\pi_1)+\beta(\nu R+\pi_1)} \\ \pi_0 = \frac{\nu\beta(\nu R+\pi_1)(1-\nu\psi)+\nu\beta\nu\pi_0 A\psi^{1-\theta}}{\delta(\nu(1+\beta)R+\pi_1)+\beta(\nu R+\pi_1)} \end{cases}$$

Solving for undetermined coefficients, two solutions yield: (i) $\pi_1 = -R\nu$ and $\pi_0 = 0$, and (ii) $\pi_1 = -R\delta\nu\frac{1+\beta}{\beta+\delta}$ and $\pi_0 = R\beta\nu\frac{1-\delta}{\beta+\delta}\frac{\psi^\theta(1-\nu\psi)}{R\psi^\theta-A\nu\psi}$. Therefore, the optimal dynamics for physical capital is as follows:

$$k_{t+1} = \frac{\delta R}{\nu}k_t + \frac{\left(R\delta\psi^\theta - A\nu\psi\right)(1-\nu\psi)}{\nu\left(R\psi^\theta - A\nu\psi\right)}h_t$$

with $\frac{h_{t+1}}{h_t} = A\psi^{1-\theta} \leq \frac{k_{t+1}}{k_t} = \frac{\delta R}{\nu}$. On the contrary, if $\delta < \hat{\delta}$, then $\frac{k_{t+1}}{k_t} < \frac{h_{t+1}}{h_t}$. In this case the efficient allocation is characterized by human capital as the sole payoff-relevant state variable. Replicating the guess and verify procedure with $\hat{b}_t = \pi h_t$, we achieve $\hat{f}_t = \frac{\delta(1-\theta)}{\nu}\left(1+\nu\frac{\theta}{1-\theta}\psi\right)h_t$ and $\hat{b}_t = \frac{\delta\left(1-\delta(1-\theta)\left(1+\nu\frac{\theta}{1-\theta}\psi\right)\right)}{\nu\beta+\frac{\delta}{\nu}}h_t$. The resulting optimal dynamic for human capital is as $h_{t+1} = A\left(\frac{\delta(1-\theta)}{\nu(1-\delta\theta)}\right)^{1-\theta}h_t$.

Section 2 (Efficiency vs Markov): Similarly to Proposition 6, the Markov-perfect equilibrium corresponds to the social planner allocation if and only if there exists a non-empty parametric space of Pareto weights and ideological bias (δ, ϕ) such that $\hat{\varphi} = \varphi$. Therefore, when $\phi \leq \hat{\phi}$ and $\delta \geq \hat{\delta}$ then $\hat{\varphi} = \varphi$ if $\delta = \frac{\beta\nu}{\phi+\nu\beta}$. Whereas, when $\phi > \hat{\phi}$ and $\delta < \hat{\delta}$ then $\hat{\varphi} = \varphi$ if $\delta = \frac{\beta\nu}{(\phi+\nu(1+\beta(1-\theta)))+\beta\nu\theta}$. ■

9 Appendix B

In this section, we provide supplementary material of the probabilistic model for the paper "A Dynamic Politico-Economic Model of Intergenerational Contracts" by Francesco Lancia and Alessia Russo. Section B1 presents the politico-economic microfoundation of the model. Finally, section B2 shows the details of the numerical analysis of an economy with only human capital such as the one discussed in the text.

B1. PROBABILISTIC VOTING MODEL

The political equilibrium discussed in the paper has an explicit microfoundation in terms of the voting model based on Lindbeck and Weibull (1987), and it is applied to a dynamic voting setting in an OLG environment with intergenerational transfers.[22] The electoral competition takes place between two office-seeking candidates, denoted by $\iota_t \in \{\mathcal{L}_t, \mathcal{R}_t\}$. Parties and voters move sequentially: first, parties announce their own multidimensional political platform constrained to the per-period balanced budget. Since the election takes place each time, the candidates cannot make a credible commitment over future policies. Second, voters belonging to each cohort, $i \in \{a, o\}$, choose the preferred candidate based on the fiscal announcements and their ideology. Agents vote for party $\mathcal{R}_t$ as long as the idiosyncratic ideological bias, σ_j^i, is larger than the difference in the indirect utility achieved from voting for any alternative platforms, net of the aggregate shock η. It implies:

$$\sigma_j^i \geq \sigma^i (k_t, h_t) \equiv \left(\mathcal{W}_{\mathcal{L}_t}^i - \mathcal{W}_{\mathcal{R}_t}^i\right) - \eta \tag{B1.1}$$

where $\sigma^i (k_t, h_t)$ represents the voter in cohort i, who is indifferent between the two parties. Without loss of generality, we assume σ_j^i is drawn from a symmetric and cohort-specific uniform distribution in the support, $\left[-\frac{1}{2\boldsymbol{\sigma}^i}, \frac{1}{2\boldsymbol{\sigma}^i}\right]$. Similarly, η is uniformly distributed over the interval $\left[-\frac{1}{2\boldsymbol{\eta}}, \frac{1}{2\boldsymbol{\eta}}\right]$. By Eq. (B1.1) the share of voters belonging to cohort i and supporting party $\mathcal{R}_t$ is equal to $\lambda_t^i \equiv \frac{1}{2} - \boldsymbol{\sigma}^i \left(\mathcal{W}_{\mathcal{L}_t}^i - \mathcal{W}_{\mathcal{R}_t}^i - \eta\right)$. Under majoritarian rule, party $\mathcal{R}_t$ wins the election if and only if it obtains the largest share of votes, namely if $N_t^{t-1}\lambda_t^a + N_t^{t-2}\lambda_t^o > \frac{1}{2}\left(N_t^{t-1} + N_t^{t-2}\right)$, which implies that η must be larger than the threshold level:

$$\eta (k_t, h_t) \equiv \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{W}_{\mathcal{L}_t}^o - \mathcal{W}_{\mathcal{R}_t}^o\right) + (1+n)\frac{\boldsymbol{\sigma}^a}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{W}_{\mathcal{L}_t}^a - \mathcal{W}_{\mathcal{R}_t}^a\right)$$

As a result, the objective function of party $\mathcal{R}_t$, i.e. $\max_{p_{\mathcal{R}_t}} \Pr\left(\eta_t \geq \eta (k_t, h_t)\right)$, simplifies to:

$$\max_{p_{\mathcal{R}_t}} \frac{1}{2} - \boldsymbol{\eta}\eta (k_t, h_t) \tag{B1.2}$$

Likewise, for party $\mathcal{L}_t$ the objective function, i.e. $\max_{p_{\mathcal{L}_t}} \Pr\left(\eta_t \leq \eta (k_t, h_t)\right)$, collapses to:

$$\max_{p_{\mathcal{L}_t}} \frac{1}{2} + \boldsymbol{\eta}\eta (k_t, h_t)$$

[22] Due to the multidimensionality of the political platform, a Condorcet winner generally fails to exist. As a consequence, the median voter theorem does not hold (Plot, 1967). The literature has proposed three main alternative influential approaches. The first is the implementation of structure-induced equilibria. By following Shepsle (1979), agents vote simultaneously, yet separately (i.e. issue by issue), on the issues at stake. Votes are then aggregated over each issue by the median voter. The second is the legislative bargaining approach, which stems from the seminal work of Baron and Ferejohn (1989). This approach applies when legislators' first loyalty is to their constituents and when legislative coalitions are fluid across time and issues. The latter concerns the adoption of the probabilistic voting rule. While this model of voting dates back to the 1970s, its resurgence in popularity stemed from Lindbeck and Weibull (1987). It applies to political environments where party discipline is strong and the winning political party simply implements its platform.

To show that the two parties' platforms converge in equilibrium to the same fiscal policy maximizing the weighted average utility of adults and old, we adopt a backward procedure. Let us consider a two-periods economy, $t = 1, 2$. In period 2, the political maximization program for party $\mathcal{R}_2$ described in Eq. (B1.2) simplifies to:

$$\max_{p_{\mathcal{R}_2}} \frac{1}{2} - \boldsymbol{\eta} \left(\frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o} \left(\mathcal{W}^o_{\mathcal{L}_2} - \mathcal{W}^o_{\mathcal{R}_2} \right) + \nu \frac{\boldsymbol{\sigma}^a}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o} \left(\mathcal{W}^a_{\mathcal{L}_2} - \mathcal{W}^a_{\mathcal{R}_2} \right) \right) \tag{B1.3}$$

In the last period, adults have no future and $\mathcal{W}^a_{\iota_2}$ is equal to $u\left(\mathcal{C}^a_{\iota_2}\right)$. As a consequence, Eq. (B1.3) reduces to:

$$\max_{p_{\mathcal{R}_2}} \nu u\left(\mathcal{C}^a_{\mathcal{R}_2}\right) + \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a} u\left(\mathcal{C}^o_{\mathcal{R}_2}\right)$$

In the same spirit, the political objective function for party $\mathcal{L}_2$ turns out to be:

$$\max_{p_{\mathcal{L}_2}} \nu u\left(\mathcal{C}^a_{\mathcal{L}_2}\right) + \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a} u\left(\mathcal{C}^o_{\mathcal{L}_2}\right)$$

It immediately follows that in equilibrium, the office-seeking parties propose the same platform, i.e. $p_{\mathcal{L}_2} = p_{\mathcal{R}_2}$. Replicating the same argument, at time $t = 1$ the maximization program for party $\mathcal{R}_1$ is:

$$\max_{p_{\mathcal{R}_1}} \frac{1}{2} - \boldsymbol{\eta} \left(\frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o} \left(\mathcal{W}^o_{\mathcal{L}_1} - \mathcal{W}^o_{\mathcal{R}_1} \right) + \nu \frac{\boldsymbol{\sigma}^a}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o} \left(\mathcal{W}^a_{\mathcal{L}_1} - \mathcal{W}^a_{\mathcal{R}_1} \right) \right) \tag{B1.4}$$

where $\mathcal{W}^a_{\iota_1}$ is equal to $u\left(\mathcal{C}^a_{\iota_1}\right) + \Pi_{\iota_1}\left(\mathcal{W}^o_{\mathcal{R}_2} + \sigma^o_{j,2} + \eta\right) + \left(1 - \Pi_{\iota_1}\right)\mathcal{W}^o_{\mathcal{L}_2}$, with $\Pi_{\iota_1} \equiv \frac{1}{2} - \boldsymbol{\eta}\eta\left(k_2, h_2; \iota_1\right)$ defined as the probability of $\mathcal{R}_2$ to be elected conditioned to the incumbent ι_1. Given that $p_{\mathcal{L}_2} = p_{\mathcal{R}_2}$ and $\mathcal{W}^i_{\mathcal{L}_2} = \mathcal{W}^i_{\mathcal{R}_2}$, it follows that $\Pi_{\mathcal{L}_1} = \Pi_{\mathcal{R}_1} = \frac{1}{2}$. As a result, the maximization program given by Eq. (B1.4) reduces to:

$$\max_{p_{\mathcal{R}_1}} \nu \left(u\left(\mathcal{C}^a_{\mathcal{R}_1}\right) + \beta u\left(\mathcal{C}^o_{\mathcal{R}_2}\right)\right) + \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a} u\left(\mathcal{C}^o_{\mathcal{R}_1}\right)$$

Replicating the same argument for party $\mathcal{L}_1$ the objective turns out to be:

$$\max_{p_{\mathcal{L}_1}} \nu \left(u\left(\mathcal{C}^a_{\mathcal{L}_1}\right) + \beta u\left(\mathcal{C}^o_{\mathcal{L}_2}\right)\right) + \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a} u\left(\mathcal{C}^o_{\mathcal{L}_1}\right)$$

As a result, in period $t = 1$, both candidates propose the same platform $p_{\mathcal{L}_1} = p_{\mathcal{R}_1}$. To conclude, in equilibrium, the platforms of the two candidates converge to the same fiscal policy that maximizes a weighted utility of current adults and old:

$$\max_{p_t} \nu \left(u\left(\mathcal{C}^a\left(f_t, b_t, h_t, k_{t+1}\right)\right) + \beta u\left(\mathcal{C}^o\left(\mathcal{B}_{t+1}, k_{t+1}\right)\right)\right) + \frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a} u\left(\mathcal{C}^o\left(b_t, k_t\right)\right)$$

We conclude by noting that under the assumption of Markov-perfect equilibria, the probabilistic voting outlined in this appendix applies equally to both static and dynamic models.

B2. ECONOMY WITH SOLE HUMAN CAPITAL

In this section, we study the robustness of the results of subsection 3.4 computing the equilibrium policy functions under general CRRA utility, i.e. $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ where $\gamma > 0$, and human capital technology as of the Cobb-Douglas type $h_{t+1} = A h_t^{\theta} f_t^{1-\theta}$, where $\theta \in (0, 1)$ and $A \geq 1$. Assuming no saving choice yields $c^a_t = h_t - \nu f_t - \frac{b_t}{\nu}$ and $c^o_{t+1} = b_{t+1}$. To characterize the politico-economic equilibrium as described in Definition 4, we guess and verify the equilibrium policy rules as $f_t = \pi h_t$ and $b_t = \rho h_t$. Substituting the guess into the political first order conditions for interior b_t and f_t, Eqs. (21) and (22), we obtain $f_t = \frac{1}{\nu} h_t - \frac{\nu + \phi^{\frac{1}{\gamma}}}{\nu^2 \phi^{\frac{1}{\gamma}}} b_t$ and $b_t = \left(\rho^{\gamma-1} A^{\gamma-1} \pi^{\theta+(1-\theta)\gamma} \left(\frac{\nu\phi}{\beta(1-\theta)} \right) \right)^{\frac{1}{\gamma}} h_t$. Solving for the undetermined coefficient yields:

$$\begin{cases} \pi = \frac{1}{\nu} - \frac{\nu + \phi^{\frac{1}{\gamma}}}{\nu^2 \phi^{\frac{1}{\gamma}}} \rho \\ \rho = A^{\gamma-1} \pi^{\theta+(1-\theta)\gamma} \left(\frac{\nu\phi}{\beta(1-\theta)} \right)^{\frac{1}{\gamma}} \end{cases}$$

Hence, the backward and forward transfers per income, i.e. $\bar{\mathcal{B}} \equiv \frac{b_t N_t^{t-2}}{h_t N_t^{t-1}}$ and $\bar{\mathcal{F}} \equiv \frac{f_t N_t^t}{h_t N_t^{t-1}}$, are equal to $\frac{\pi_b}{\nu}$ and $\nu \pi_f$, respectively. For $\gamma \neq 1$ a full analytical solution is not available; therefore, we must resort to numerical analysis. Let us take one period in the model to correspond to 30 years (one generation's life span). We set $\nu = 1$, implying an equal mass of population across periods, $\beta = 0.99^{30}$, $A = 2.5$, $\theta = 0.66$, and $\gamma = 0.3$, corresponding to a sufficiently low degree of risk aversion as reported in Proposition 2. Figures 7 and 8 show graphically the outcome of the experiment, i.e. the equilibrium policy rules parameterized on the relative political clout of the elderly agents. The experiment confirms the qualitative prediction of Proposition 2. The forward transfers display a U-shape with respect to the degree of the elderly single-mindedness.

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