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Excessive supplier pricing and high-quality foreclosure

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Abstract

This article shows that entry of a more input-efficient, but lower quality downstream producer, compared to a high-quality downstream incumbent, might be detrimental to social welfare. In particular, if the entrant is extremely efficient, a monopolist upstream supplier reacts by charging an excessive price, driving the high-quality incumbent out of the market and reducing social welfare. However, despite the entrant's low input requirement, the supplier's profit increases for all but the most efficient entrant technologies. Enabling the supplier to engage in third degree price discrimination may increase social welfare.

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1 Introduction

Imagine a monopolist mining company that extracts some rare precious metal. The firm chooses a price and sells its good to two downstream producers, which in turn use the metal as main input for manufacturing a high-tech electronic product. One of the downstream firms assembles a fast or highly reliable version that requires a large amount of the input per unit produced, whereas the other firm assembles a lower quality substitute by using less of the input. In the final goods market for the electronic product, consumers make their purchase decision based on their individual valuations for quality and the prices set by the downstream firms.[1]

This article shows that in a variety of related situations, the upstream supplier might have an incentive to raise its input price beyond the point where the high-quality, high-input-requirement downstream firm is forced to exit, and that this behavior is detrimental to social welfare. Put differently, I establish that entry of an input-efficient, low-quality downstream competitor into a formerly monopolistic downstream segment might decrease the economic surplus of each active consumer and both incumbent firms.

The reason why the upstream firm might charge an excessive input price is as follows. Clearly, a very input-efficient downstream producer will tend to obtain a large market share in the final goods market due to its lower cost. Hence, the supplier may want to charge a very high price that compensates for the efficient firm's low input demand, but drives out the high-quality producer. This may be optimal despite the fact that competition in the downstream market is beneficial for the upstream supplier, as it reduces the well-known double-marginalization externality it faces.[2]

At this point, it is crucial to note that the problem may only arise if the supplier is forced to charge a *linear* price. As is shown in this article, excessive supplier pricing will not emerge if the supplier can charge a fixed fee or may engage in third-degree price discrimination. Following Villas-Boas (1998) and the references therein, linear prices are plausible to assume if the supplier and the downstream firms are in an ongoing business relationship with frequently changing

[1]Two metals that might qualify for this motivating example are iridium and ruthenium. Both are among the rarest elements found in the Earth's crust, giving significant market power to the few worldwide upstream producers. According to the 2010 Minerals Yearbook on platinum-group metals published by the U.S. Geological Survey, ruthenium was mainly used in the hard disk industry, while "[i]ridium crucibles are used in the electronics industry to grow high-purity single crystals for use in various applications."
See `http://minerals.usgs.gov/minerals/pubs/commodity/platinum/` for details.

[2]The double-marginalization problem, as first formalized by Spengler (1950), describes the situation where in successive stages of manufacturing and distribution, independently operating firms with market power in their respective level of the supply chain add a mark-up over the competitive price level. This accumulation of margins leads to a loss of welfare, as final prices turn out to be higher than what a vertically integrated operation would charge. This is because when making their price choice, each firm does not take into account the negative externality imposed on all other firms' profits. For a thorough discussion of double marginalization and many contractual setups that may or may not reduce the problem, see Rey and Vergé (2008).

market parameters (e.g. costs and demand). In such scenarios, any contracts which are more complex than specifying a linear transaction price might be too costly to administer.

Keeping this restriction in mind, there are many examples other than processing of raw materials in which an upstream firm might rationally choose an excessive price and drive out a high-quality downstream firm. Namely, the logic extends to all situations where inputs can be utilized more efficiently by a low-quality operation, compared to a high-quality one. Most importantly, this includes many types of sharing and renting enterprises. Such firms do not need to purchase one unit of input to serve one final consumer (as their products may be shared among their consumers), but tend to offer a lower quality than their retail counterparts.

An increasingly successful example is given by carsharing, where consumers can borrow a vehicle out of some shared pool whenever they are in need. Typically, firms that offer such a service charge a hourly usage fee (sometimes combined with a monthly or yearly membership payment) that is more cost-effective to light users than owning a private car. Moreover, as several consumers can utilize one car, serving one final consumer requires less physical input for carsharing firms than for regular car dealers. In a study conducted with several North American carsharing companies, Martin et al. (2010) find that each car in a sharing firm's repertoire decreases the need for nine to thirteen privately owned cars. On the other hand, engaging in carsharing arguably provides consumers a lower *gross* utility than owning a car privately (e.g., because it is easier to access one's own car; there is no need to return it; etc.). Hence, if carsharing would attract a sufficiently large market share, the mechanism portrayed in this article might apply: dominant upstream manufactures would want to correct for the sharing firms' higher efficiency by charging an excessively high input price, driving out standard car retailers and reducing welfare.

Other potentially affected markets include the markets for information goods like movies, music, and books. For example, if entry of a legal (online) movie sharing service was so successful that standard DVD or Blu-ray sellers would lose a significant part of their market share, upstream copyright holders might react by considerably increasing their content prices, inducing the standard sellers to exit the market altogether. Then, as long as the shared movies' quality (gross utility) was perceived to be lower than their retail counterparts' quality (e.g., because the shared movies lack nice packaging or extra features), social welfare would be reduced.

Finally, the logic also applies to certain types of essential facilities (e.g., airport slots or harbors that, by cost-saving logistics, need to be employed less often by a low-quality operation), and even the availability of recycling opportunities (e.g., a firm might need less input for production than a competitor because part of its input can be generated from recycling returned final goods, resulting in a lower quality final product).[3]

Formalizing these examples, I introduce a stylized model of vertical interaction between a monopolist upstream supplier (U) of some essential input and a downstream, vertically differentiated goods duopoly engaging in price competition. In the downstream market, a traditional final good producer (P) and a more input efficient, lower quality alternative competitor (A) process U's intermediary good and strategically set prices. The key parameters of the model are thus the relative quality and relative input requirement of A, compared to P.

As a first result, I find that, depending on A's efficiency, U will choose from a number of different pricing strategies in equilibrium. These include charging the standard double marginalization monopoly price and only serving P (if A is highly inefficient), choosing an intermediate price to maintain competition in the downstream market (if P and A are comparably efficient), inducing limit pricing by either P or A (if either firm is moderately more efficient than its rival) or setting an excessive price where only A can profitably operate, foreclosing the high-quality producer P (if A is highly efficient).

The reason for U's rich strategic behavior is a combination of two countervailing effects. First, if A is active, it poses a threat to U's profit because it steals some market share from P. In turn, as A is able to serve its customers at a lower input requirement than P, U's total demand decreases. This is a negative direct effect on U's profit. However, as a second effect, the downstream competition that is caused by A is also beneficial to U, as lower prices in the downstream segment imply a higher demand by final consumers and thus more demand generated by both intermediaries. This is a positive indirect effect on U's profit. The relative magnitude of these effects ultimately affects which pricing strategy U employs. In particular, if A is very input-efficient, U's profit when choosing a moderate price that either keeps P in the market, or induces A to engage in limit pricing, is lower than when charging an excessive price. Such a price fully compensates for A's low input requirement, but also gives A monopoly power in the downstream market. This leads to a more severe form of the double-marginalization problem.

As emphasized above, the main contribution of this paper is not only to show that excessive supplier pricing might happen in equilibrium, but also, that such a conduct is harmful to social welfare, even compared to the inefficient standard double marginalization case. Intuitively, when the supplier charges an excessive price that optimally compensates for A's low input requirement, the consumers cannot benefit from low downstream prices, as A has to pass on its high production

[3] Many other potential applications are provided in Botsman and Rogers (2010).

cost. On the other hand, an excessive supplier price drives P out of the market, which essentially replaces the high-quality incumbent with a low-quality firm. It is not difficult to see that this must be detrimental to social welfare.

Another important result is that, despite A's lower input requirement, entry of A tends to increase U's equilibrium profit. This is because the increased demand caused by downstream competition typically more than offsets any losses in demand resulting from A's lower input requirement. In other words, A's indirect positive effect on U's demand usually dominates its direct negative effect. However, for very efficient A's, the direct effect may still dominate, leading to decreased upstream profits. Moreover, in situations where U finds it optimal to charge an excessive price, its profit is always *lower* than if A did not exist.

An interesting feature of the model is that allowing U to engage in third degree price discrimination can be welfare increasing. This is because, by charging an adequately higher input price for the more efficient firm, U can effectively maximize competition in the downstream market. This is better from a welfare point of view, compared to the situation where price discrimination is not feasible, if either U would otherwise decide to foreclose A or P by charging a high price (the standard double-marginalization case, or excessive supplier pricing), or if A is slightly more efficient than P. In all other cases, the aggregate deadweight loss that arises in the market is lower if price discrimination is not feasible, as sufficient incentives are provided for all (active) firms to charge relatively low prices.[4]

Finally, I show, partly using numerical methods, that the model's main qualitative insight is robust to two extensions. The finding that welfare-decreasing excessive supplier pricing might occur in equilibrium is not overturned by oligopolistic (Cournot) competition in the upstream market and a positive supplier production cost. In the case of upstream Cournot-competition, the range of A's technology parameters where excessive supplier-pricing is an equilibrium outcome merely becomes smaller. If the supplier has a positive production cost, not all parameter combinations where an excessive supplier price is chosen are harmful to society, as the aggregate production cost is reduced by A's efficient production technology. However, for any positive cost level, there still exists a parameter-region where excessive pricing is optimal, but total social-welfare is reduced.

There are two main strands of literature that are relevant to this article. Most closely related is

[4]Katz (1987), DeGraba (1990), and Yoshida (2000) all study third degree price discrimination in the context of vertical market structures. As the present model, these papers generally give ambiguous predictions regarding the welfare effects of price discrimination. Moreover, the usual necessary condition that output in the final goods market must increase in order for social welfare to rise (see Varian (1985)) may be violated.

the literature on vertical foreclosure, which, according to Rey and Tirole (2007)'s comprehensive survey, "refers to a dominant [upstream] firm's denial of proper access to an essential good it produces, with the intent of extending monopoly power from that segment of the market (the bottleneck segment) to an adjacent [downstream] segment (the potentially competitive segment)." Examples of vertical foreclosure that are often discussed in the literature include refusal to deal (see e.g., Bernheim and Whinston (1998)), vertical integration (see e.g. Salinger (1988), Hart et al. (1990)), and price squeezes or raising rivals' costs (see e.g., Salop and Scheffman (1983), Crocioni and Veljanovski (2003)).[5]

While the literature on vertical foreclosure is vast, I am only aware of a single paper that discusses the incentives of a *non-integrated* supplier to foreclosure a downstream firm by choosing a *uniform* price (rather than by exercising price discrimination, or engaging in a price squeeze). To this end, Spiegel and Yehezkel (2003) consider a setup, similar to the one presented in this paper, where an upstream monopolist U serves two quality differentiated downstream firms (say, P and A). While the authors' main focus lies on the effect of forced market segmentation by U, they also analyze the case where U may only charge a uniform price. However, there are two crucial differences to this article. First, Spiegel and Yehezkel do not account for the possibility that A might be more input-efficient than P. Thus, they ignore U's (potential) tradeoff between maintaining downstream competition and reducing double marginalization, or charging an excessive price and compensating for A's low input requirement. Second, they only consider the case where A is less cost efficient than P.[6] Due to these assumptions, Spiegel and Yehezkel find that U will always charge a price that forecloses the low quality firm.

Yehezkel (2008) considers the case of a downstream retailer that can either purchase a high-quality input from an upstream monopolist, or obtain a low-quality substitute, resulting in a lower quality final product, from an alternative source. He shows that under full information (as in the present model), the retailer offers both varieties if and only if it would also do so under vertical integration. Interesting frictions only arise if there is asymmetric information about the consumers' willingness to pay for quality. In contrast, an important contribution of this paper is to show that very problematic outcomes may also occur under full information, provided that the downstream firms' product quality is linked to their input requirement.

A somewhat different perspective is given by Villas-Boas (1998), who considers the choice of

[5] A price squeeze refers to the situation in which a vertically integrated supplier charges an input price that is above the selling price of its downstream operation, squeezing non-integrated downstream rivals out of the market.

[6] Clearly, in reality, the highest quality product is not always the most cost-efficient one.

product line (product qualities) and pricing of an upstream monopolist, given that a downstream retailer has to decide about the products it carries and which consumer segments (high or low valuation) to target. In some sense, the article endogenizes the quality levels that are being introduced to the market. However, different input-requirements in the transformation process are not considered and the focus lies on a downstream monopoly, rather than entry of an efficient low-quality firm.

The second most closely related literature deals with the economics of sharing, renting and copying of information goods. Typically, the main concern of papers in this area is whether the possibility of sharing and copying of information goods (e.g., photocopying of journals, copying of videos, sharing music over the Internet, etc.) harms the profits of respective copyright holders. There are two similarities to this article. First, consumers may also self-select into purchasing a high-quality "original" or low-quality "copy" (where the former is higher priced), depending on their valuation for quality. Second, if the copying technology is efficient, an (upstream) content producer may charge a higher price than without the existence of copying technologies, as it realizes that (downstream) "buying clubs" have a higher willingness to pay than individual consumers. Important contributions in this area have been made by Liebowitz (1985), Besen (1986), Bakos et al. (1999), Varian (2000), and Varian (2005).

Lastly, worth mentioning is also the literature on damaged goods, pioneered by Deneckere and McAfee (1996). The authors show that a monopolist manufacturer may want to artificially degrade (damage) parts of its products and sell them at a lower price in order to benefit from price-discriminating consumers. In the present model, A's good can be interpreted as damaged version of P's, but very different pricing incentives arise because of the vertical market structure.

The remaining article is structured as follows. The next section briefly describes the model setup. Section 3 examines consumers' demand and solves for the equilibrium of the downstream price game. In the core of this paper, Section 4, the supplier's optimal pricing decision is derived. In Section 5, I discuss third degree price discrimination by U. Section 6 contains the two above mentioned extensions, and Section 7 concludes. All proofs can be found in Appendix A, while tables for every relevant equilibrium expression can be found in Appendix B.

2 Model Setup

Consider a market where a monopolist upstream supplier (U) interacts with a standard downstream producer (P) of a high quality final good and an "alternative" downstream producer (A) that can provide a lower quality substitute at a lower input requirement. The relative quality $q \in (0,1)$ and the relative input requirement $r \in (0,1)$ at which A can produce, compared to P, are fixed exogenously and common knowledge. Without loss of generality, I normalize both the quality and input requirement of P to one, which means that the standard downstream producer can transform one unit of input obtained from the upstream supplier into one unit of the standard final good, whose quality is given by one. For simplicity, U can produce any amount at no cost.

The sequence of events is the following. In stage one, U sets the price c it charges for each unit of its input. In stage two, the downstream producers simultaneously compete in prices p (P) and a (A), given c, q, and r. Importantly, P and A face positive and different unit costs of c (P) and $d = r * c < c$ (A), respectively. Note that A faces a lower unit cost than P because it only needs to obtain $r \in (0,1)$ units of input from U to produce one unit of its substitute final good. The downstream firms do not incur any additional expenses other than the costs that arise from purchasing U's input.

Finally, in stage three, a unit mass of consumers, each having unit demand, decides at which firm (if any) to buy. Following Mussa and Rosen (1978), I specify that the utility of a consumer of type θ, who consumes a good of quality q at price p, is given by

$$U_\theta = \theta q - p, \tag{1}$$

where θ is uniformly distributed on the interval $[0,1]$. Note that by including zero to the range of consumers' valuation for quality, I ensure that the downstream market is uncovered.

In what follows, I solve for the (unique) subgame perfect Nash equilibrium for each parameter combination (q, r).

3 Consumer's Demand and Downstream Price Competition

Given the above setup, the type of consumer who is indifferent between purchasing at A and P is given by

$$\theta_h = \frac{p-a}{1-q}. \tag{2a}$$

Clearly, if $\theta_h < 0$ $(p < a)$, everybody prefers buying from P, whereas if $\theta_h > 1$ $(p > a + 1 - q)$, everybody prefers buying from A.

The type of consumer who is indifferent between buying at A and no consumption is given by

$$\theta_l = \frac{a}{q} > 0. \tag{2b}$$

If $\theta_l \geq 1$ $(a \geq q)$, no consumer would find it optimal to buy A's product even if P was not active.

Finally, the consumer θ_m who is indifferent between buying from P and not consuming at all is given by

$$\theta_m = p > 0. \tag{2c}$$

Thus, in the case of $p \geq 1$, P would not be able to attract consumers even if A was absent from the market.

Combining these observations and taking into account the uniform distribution of consumers' types θ, I arrive at the following lemma.[7]

Lemma 1. *The downstream firms' demand from final consumers, given prices $p \in (0,1)$ by P, $a \in (0,q)$ by A, and $q \in (0,1)$,[8] is given by*

$$D_P(p,a,q) = \begin{cases} 1-p & \textit{if} \quad p \leq \frac{a}{q} \\ 1 - \frac{p-a}{1-q} & \textit{if} \quad p \in (\frac{a}{q}, a+1-q) \\ 0 & \textit{if} \quad p \geq a+1-q \end{cases} \tag{3a}$$

$$D_A(a,p,q) = \begin{cases} 1-\frac{a}{q} & \textit{if} \quad a \leq p+q-1 \\ \frac{p-a}{1-q} - \frac{a}{q} & \textit{if} \quad a \in (p+q-1, pq) \\ 0 & \textit{if} \quad a \geq pq. \end{cases} \tag{3b}$$

Using the above demand functions for calculating the firms' best-response correspondences, one can show

Proposition 1. *The unique equilibrium of the downstream subgame, given c, d, and q, is characterized by five different cases (in increasing order of A's efficiency): monopoly pricing by P (I), limit pricing by P (II), duopoly competition between P and A (III), limit pricing by A (IV), and monopoly pricing by A (V).*

[7] Graphical intuition to a similar scenario can be found in Spiegel and Yehezkel (2003).

[8] For $p \geq 1$, P's demand is always zero and A's demand is equal to $\max\{1-\theta_l, 0\} = \max\{1-\frac{a}{q}, 0\}$. For $a \geq q$, A's demand is always zero and P's demand is equal to $\max\{1-\theta_h, 0\} = \max\{1-p, 0\}$. If both $p \geq 1$ and $a \geq q$, $D_P(p,a,q) = D_A(a,p,q) = 0$.

More precisely,

$$
\begin{array}{llr}
(I) & p^{**}_{m} = \dfrac{1+c}{2} \quad \textit{and} \quad a^{**}_{00} = \{a \mid a \geq d\} & \textit{if} \quad c \leq \overline{c} \quad \textit{and} \quad d > \hat{d} \\
(II) & p^{**}_{lim} = \dfrac{d}{q} \quad \textit{and} \quad a^{**}_{0} = d & \textit{if} \quad c \leq \overline{c} \quad \textit{and} \quad d \in (\overline{d}, \hat{d}] \\
(III) & p^{**}_{acc} = \dfrac{2-2q+d+2c}{4-q} \quad \textit{and} \quad a^{**}_{acc} = \dfrac{q-q^2+cq+2d}{4-q} & \textit{if} \quad c \leq \overline{c} \quad \textit{and} \quad d \leq \overline{d} \\
(IV) & p^{**}_{0} = c \quad \textit{and} \quad a^{**}_{lim} = c+q-1 & \textit{if} \quad c \in (\overline{c}, \hat{c}] \quad \textit{and} \quad d \leq \overline{d} \\
(V) & p^{**}_{00} = \{p \mid p \geq c\} \quad \textit{and} \quad a^{**}_{m} = \dfrac{q+d}{2} & \textit{if} \quad c > \hat{c} \quad \textit{and} \quad d \leq \overline{d},
\end{array}
$$

where

$$
\begin{aligned}
\overline{c} &= \frac{2-q-(q-d)}{2-q} \in (0,1), \quad \hat{c} = \frac{2-q+d}{2} < 1, \\
\overline{d} &= \frac{q(1-q+c)}{2-q} \in (0,q), \quad \hat{d} = \frac{q(1+c)}{2} < q.
\end{aligned}
$$

For the following intuition to Proposition 1 and the analysis of the supplier's optimal pricing decision in Section 4, it is convenient to draw the unit cost parameter space with the different downstream equilibrium regions in a plane. Doing so, I exploit the fact that for given q, the region boundaries $\overline{c}$ and $\hat{c}$ ($\overline{d}$ and $\hat{d}$) are linear functions of d (c).[9] The resulting graph is depicted in Figure 1.

Proposition 1 can be explained as follows. If A is very inefficient compared to P, it has an overall high demand for the input and hence high cost. This implies that even if P charges the monopoly price, A cannot profitably operate in the market. This is case (I). As A becomes more efficient, P cannot charge the monopoly price anymore and keep A from entering. For a sufficiently efficient A, P reduces its price below the monopoly price in such a way that A cannot profitably operate (i.e., at best make losses of ϵ) when entering. In this equilibrium, labeled (II) above, P charges some $p < p^m$ and A prices at marginal cost, making zero sales. But if A is even more efficient compared to P, P finds it too costly to keep A out of the market. In such equilibria (III), the downstream firms engage in duopoly competition. For A's that are even more efficient, A either optimally engages in limit pricing (IV), or charges its monopoly price (V), and P exits.

Inserting the Nash-equilibrium expressions for p and a (as derived in Proposition 1) into the demand functions given in Lemma 1, I find the following corollary.

[9] Note that for both $c \geq 1$ and $d \geq q$, none of the firms will be active in equilibrium and the market is not served.

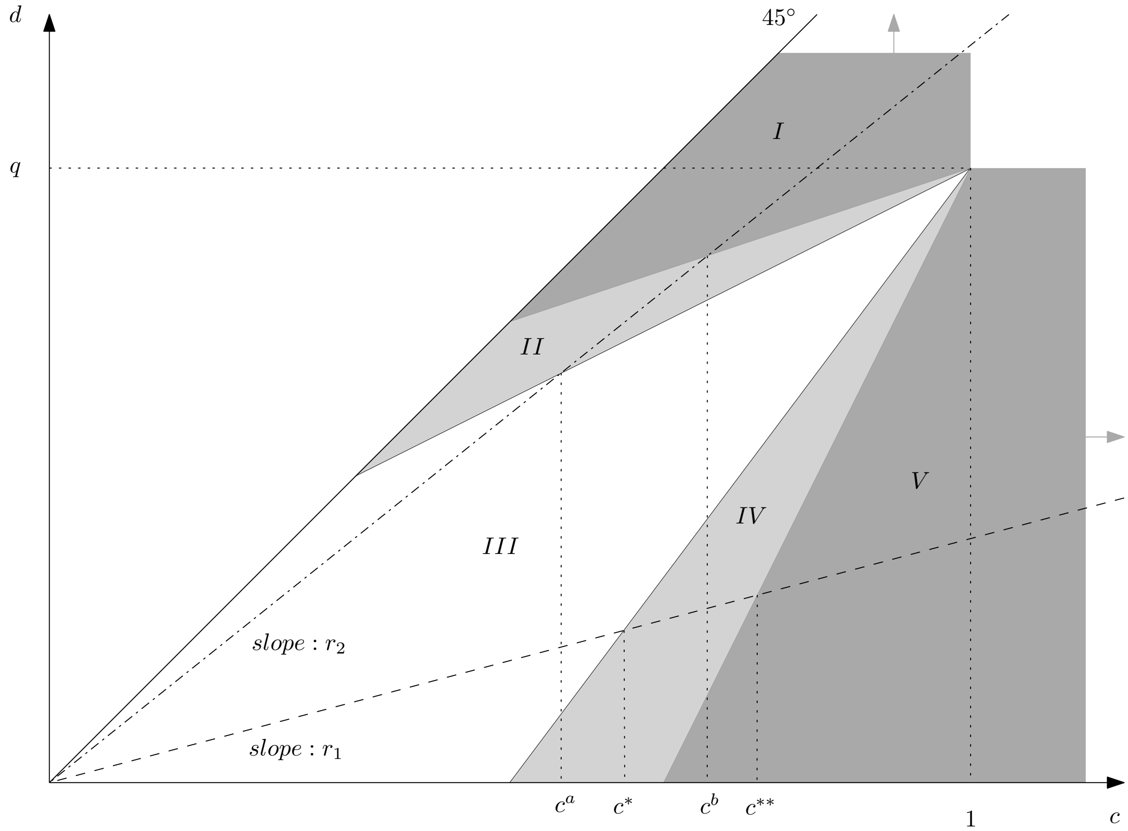

Figure 1: Equilibrium regions of the downstream subgame drawn in (c, d) space for a given q. The boundaries between regions (I,II), (II,III), (III,IV) and (IV,V) are given by $\hat{d}(c)$, $\overline{d}(c)$, $\overline{c}(d)$ and $\hat{c}(d)$, respectively. For the analysis in Section 4, the dashed line indicates the case of a relatively input efficient A ($r_1 < q$), whereas the dashed-dotted line indicates the case of a relatively input inefficient one ($r_2 > q$). Points above the 45° line are not attainable, since $r < 1$ by assumption.

Corollary 1. *The equilibrium demand functions of the downstream subgame are given by*

$$D_P^*(c,d,q) = \begin{cases} \frac{1-c}{2} & \text{if} \quad c \leq \bar{c} \quad \text{and} \quad d > \hat{d} & (I) \\ \frac{q-d}{q} & \text{if} \quad c \leq \bar{c} \quad \text{and} \quad d \in (\bar{d}, \hat{d}] & (II) \\ \frac{2+d-2c+cq-2q}{(4-q)(1-q)} & \text{if} \quad c \leq \bar{c} \quad \text{and} \quad d \leq \bar{d} & (III) \\ 0 & \text{if} \quad c > \bar{c} \quad \text{and} \quad d \leq \bar{d} & (IV)+(V) \end{cases} \tag{4a}$$

and

$$D_A^*(c,d,q) = \begin{cases} 0 & \text{if} \quad d > \bar{d} \quad \text{and} \quad c \leq \bar{c} & (I)+(II) \\ \frac{d(q-2)+(1+c-q)q}{(4-q)(1-q)q} & \text{if} \quad d \leq \bar{d} \quad \text{and} \quad c \leq \bar{c} & (III) \\ \frac{1-c}{q} & \text{if} \quad d \leq \bar{d} \quad \text{and} \quad c \in (\bar{c}, \hat{c}] & (IV) \\ \frac{q-d}{2q} & \text{if} \quad d \leq \bar{d} \quad \text{and} \quad c > \hat{c}. & (V) \end{cases} \tag{4b}$$

In the subsequent section, I will use these equilibrium demand schedules to solve for U's optimal pricing decision. The equilibrium profits, consumer surplus and total social welfare of the downstream subgame are found in Appendix B.

4 Optimal Supplier Pricing

Before starting the analysis, the following definition will turn out to be useful.

Definition 1 (Relative Input Efficiency)**.** *A is called relatively input efficient (relatively input inefficient) if it has a lower (higher) input requirement per unit of quality than P, i.e., if $r < q$ ($r > q$).*

In this section, I assume that the supplier cannot price-discriminate between P and A. Hence, given c, A's effective input price (per unit of final consumers' demand served) is $d = r * c$. That is, for given r, the supplier may only choose a point on the line through the origin with slope r in the cost parameter space depicted in Figure 1 (cf. Section 3). For the purpose of analyzing U's optimal price choice, one has to distinguish two cases:

(i) (*dashed-dotted line* in Figure 1) If A is relatively input inefficient, P is always active if A is, since the unit cost pair (c, rc) must lie in regions I, II or III.[10] Moreover, as A is relatively

[10] There is also a direct argument for this: Suppose A is active ($d = rc < q$) and relatively input inefficient ($r < q$). For P not to be active in equilibrium, it must hold that $p > a + 1 - q$. Now, for the lowest possible price $a = d$ firm A can choose in equilibrium, this inequality becomes $p > 1 - q + rc$. As $1 - q + rc > c$ if A is relatively input inefficient, it must hold that $p > 1 - q + rc > c$ for P not to be active in equilibrium. But this cannot be

input inefficient, it can be driven out of the market if U chooses a sufficiently high input price. Denoting the solutions to $rc = \overline{d}(c)$ and $rc = \hat{d}(c)$ by c^a and c^b, respectively, one finds that the upstream firm has three options: (a) price at $c \in [0, c^a)$ and face demand by both downstream firms (b) price at $c \in [c^a, c^b)$, keep A out of the market and induce limit pricing by P or (c) price at $c \in [c^b, 1)$ and induce monopoly pricing by P.

Using Corollary 1, the supplier's demand can thus be written as

$$D_U^{(1)}(c,q,r) = \begin{cases} x_U(c,q,r) & \text{if} \quad c < c^a, \quad \text{with } c^a := \frac{q(1-q)}{2r-rq-q} \in (0,1) & (III) \\ \frac{q-cr}{q} & \text{if} \quad c \in [c^a, c^b), \quad \text{with } c^b := \frac{q}{2r-q} \in (c^a,1) & (II) \\ \frac{1-c}{2} & \text{if} \quad c \in [c^b, 1) & (I) \\ 0 & \text{if} \quad c \geq 1, \end{cases} \tag{5a}$$

with

$$x_U(c,q,r) := \frac{r+2}{4-q} - \frac{c(2r^2 - qr^2 - 2rq + 2q - q^2)}{(4-q)(1-q)q}.[11]$$

The first line of equation (5a) is given by $D_P^*(c,d,q) + rD_A^*(c,d,q)$ for $c \leq \overline{c}$ and $d = rc \leq \overline{d}$, the second line is given by $D_P^*(c,d,q)$ for $c \leq \overline{c}$ and $d = rc \in (\overline{d}, \hat{d}]$, and the third line is given by $D_P^*(c,d,q)$ for $c \leq \overline{c}$ and $d = rc > \hat{d}$. Note that the lower input requirement of A is taken into account in the first line, as firm A only needs to purchase $r < 1$ units of input from U for any final consumer it serves.

(ii) (*dashed line* in Figure 1) If A is relatively input efficient, it must always be active in the market if P is, since the unit cost pair (c, rc) lies in regions III, IV or V.[12] Denoting the solutions to $c = \overline{c}(d)$ and $c = \hat{c}(d)$ by c^* and c^{**}, respectively, one finds that the upstream firm has again three options: (a) price at $c \in [0, c^*)$ and face demand by both P and A (b) price at $c \in [c^*, c^{**})$, keep P out of the market and induce limit pricing by A or (c) price at $c \in [c^{**}, \frac{q}{r})$, which induces monopoly pricing by A.

part of an equilibrium strategy combination, because P could price at some intermediate $\tilde{p} \in (c, 1-q+rc)$, get positive demand and make a positive profit.

[11] Note that the nominator of the second term is positive. This is because $2r^2 - qr^2 - 2rq + 2q - q^2 = q(1-r)^2 + (1-q)(q+2r^2)$, which is certainly positive for $q \in (0,1)$.

[12] Again, a direct argument for this runs as follows: Suppose that P is active (thus, $c < 1$) and A is relatively input efficient ($r > q$). In order for P to price A out of the market, it must hold that $p < \frac{a}{q}$. But for a given cost level c, the standard producer cannot price below it in equilibrium. Thus, suppose P chooses its lowest possible price: $p = c$. Then, A would not be active if $p = c < \frac{a}{q}$. Now, because $d = rc$, this inequality becomes $a > \frac{dq}{r}$. For a relatively input efficient alternative producer, $r > q$ and hence $a > \frac{dq}{r} > d$ must hold for A not to be active in equilibrium. But this cannot be part of an equilibrium strategy combination, because A could price at some intermediate $\tilde{a} \in (d, \frac{dq}{r})$, get positive demand and make a positive profit.

Using once again Corollary 1, it follows that the supplier's demand can be written as

$$D_U^{(2)}(c,q,r) = \begin{cases} x_U(c,q,r) & \text{if} \quad c < c^*, \quad \text{with } c^* := \frac{2(1-q)}{2-q-r} \in (0,1) & (III) \\ \frac{r(1-c)}{q} & \text{if} \quad c \in [c^*, c^{**}), \quad \text{with } c^{**} := \frac{2-q}{2-r} \in (c^*,1) & (IV) \\ \frac{r(q-cr)}{2q} & \text{if} \quad c \in [c^{**}, \frac{q}{r}) & (V) \\ 0 & \text{if} \quad c \geq \frac{q}{r}, & \end{cases} \tag{5b}$$

where the first line is given by $D_P^*(c,d,q) + rD_A^*(c,d,q)$ for $d = rc \leq \overline{d}$ and $c \leq \overline{c}$, the second line is given by $rD_A^*(c,d,q)$ for $d = rc \leq \overline{d}$ and $c \in (\overline{c}, \hat{c}]$, and the third line is given by $rD_A^*(c,d,q)$ for $d = rc \leq \overline{d}$ and $c > \hat{c}$. For all of these cases, the lower input requirement of A is taken into account by multiplying A's demand from final consumers by r, which gives its effective input requirement from U. Clearly, the supplier cannot even sell to the more efficient producer A if it chooses an input price $c \geq \frac{q}{r}$, as this implies $d \geq q$.

As U has to incur zero cost by assumption, the firm's profit is given by

$$\Pi_U(c,q,r) = \begin{cases} c\ D_U^{(1)}(c,q,r) & \text{if} \quad r > q & (I)+(II)+(III), \\ c\ D_U^{(2)}(c,q,r) & \text{if} \quad r \leq q & (III)+(IV)+(V). \end{cases} \tag{6}$$

Comparing U's maximum profits in the different downstream equilibrium regions, I obtain

Proposition 2. *The unique subgame perfect Nash equilibrium of the full game is characterized by five different supplier pricing strategies. In increasing order of A's efficiency, U sets c such to induce (I) monopoly pricing by P, (II) limit pricing by P, (III) duopoly competition, (IV) limit pricing by A, and (V) monopoly pricing by A. In region (V), U's optimal input price is "excessive", i.e., higher than the standard double marginalization monopoly price.*

The intuition for Proposition 2 is as follows. Clearly, by choosing a sufficiently low c, the supplier can always induce both downstream firms to be active in equilibrium. The competition that ensues reduces prices in the final goods market and hence eliminates part of the double-marginalization externality imposed on U. However, for a very inefficient alternative producer A, U does not benefit from inducing competition in the downstream market, as it would have to reduce its price to a very low level in order for A to be active. Hence, it is best for U to choose the standard double marginalization upstream price and ignore the inefficient A, which cannot even compete if P charges the downstream monopoly price.

If A is slightly more efficient, U can do better by choosing an input price that is somewhat lower than the standard monopoly price, as it can create an artificial competitive threat for P. At such an input price, if P would continue to charge the corresponding monopoly price in the downstream market, A could profitably operate and would steal a fraction of P's consumers. Hence, P prefers to engage in limit pricing. For some technology parameters (q, r), the resulting increased demand for U's input more than offsets U for the reduction of its margin.

If P and A are comparably efficient, U finds it optimal to charge a moderate input price that induces duopoly competition in the downstream market. At such an input price, both P and A are active downstream.

Next, if A is sufficiently *more* input efficient than P, the limit-pricing situation reverts. By moderately increasing its input price from the level that maximizes U's profits for downstream duopoly competition, U can induce A to engage in limit pricing and drive P out of the market. U also benefits from this, as the demand from final consumers stays relatively high, causing a high input demand from A.

Finally, U can find it optimal to charge an input price above the standard double marginalization monopoly price in which case A monopolizes the downstream market. The supplier prefers this situation if A is much more efficient than P, as in that case, compensating for A's low input requirement by sufficiently increasing the price for its input gives U higher profits than maintaining competition in the downstream market or inducing A to engage in limit pricing. Put differently, for very input-efficient A's, U prefers a large margin for each unit it sells to A, even though A's demand is low.

In the proof of Proposition 2, the exact technology parameters in which either of these five different pricing conducts are optimal are derived. Figure 2 depicts these findings graphically. In particular, it can be observed that U finds it optimal to charge an excessive price if A is highly efficient, that is, if it has low r and sufficiently high q.

Some further graphical insight can be obtained when plotting the firms' equilibrium prices for changing technology parameters. Figure 3 depicts the firms' equilibrium price paths for three different levels of A's relative input requirement r, as a function of A's relative quality q. It is easy to see that for sufficiently input-efficient A's (middle and bottom panel), the supplier charges an excessive price in equilibrium, given that A's relative quality exceeds a certain threshold. Note moreover that only in the middle panel, all five different pricing strategies can be observed (compare with Figure 2).

Using Proposition 2 combined with equations (5a), (5b), (6), Proposition 1, Corollary 1 and the first part of Appendix B, one can now solve for the equilibrium prices, demand levels, profits,

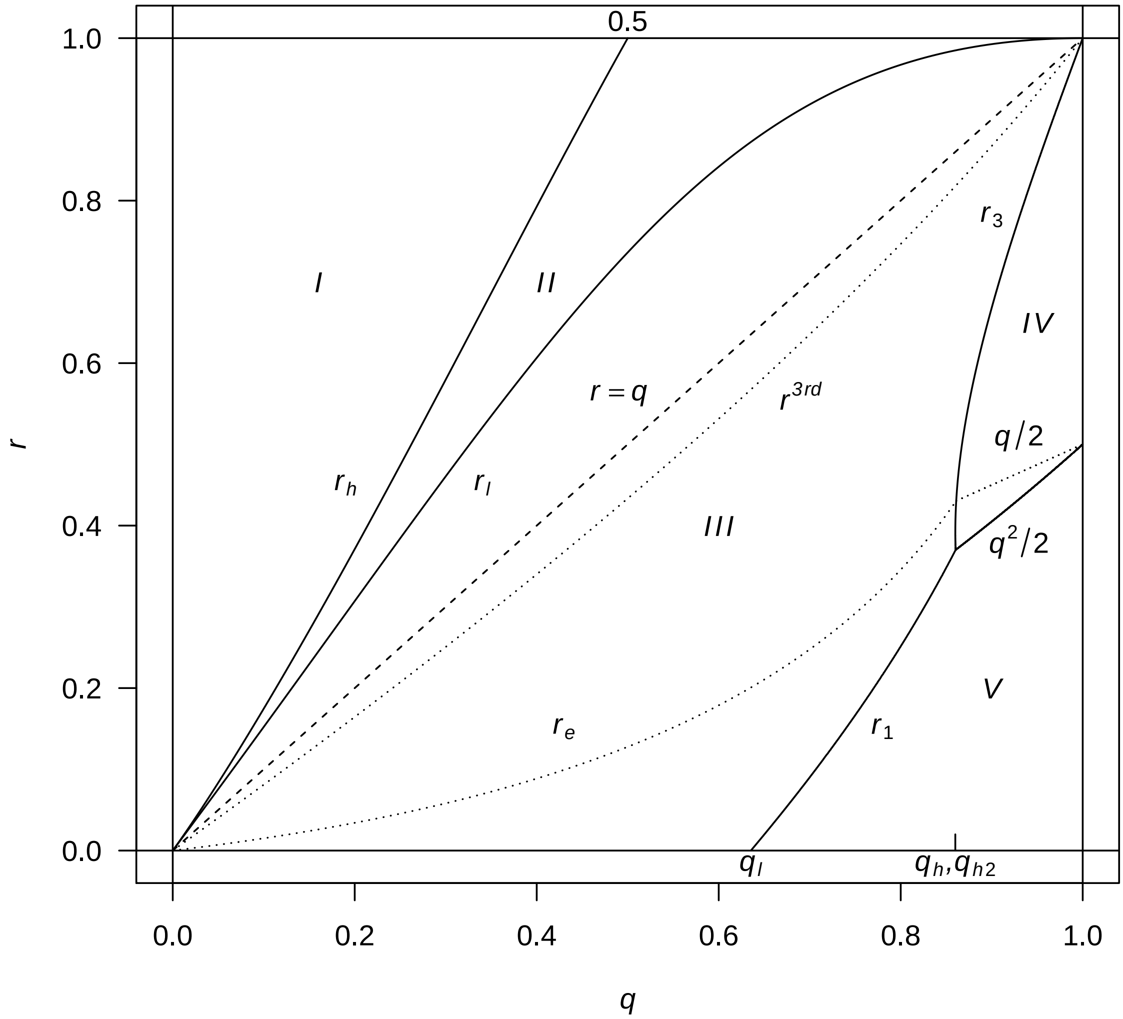

Figure 2: Characterization of the supplier's optimal pricing decision across the technology parameter space. The horizontal axis measures A's relative quality, while the vertical axis measures its relative input requirement. In region I, the supplier charges the standard double marginalization monopoly price. In region II, it induces limit pricing by P. In region III, downstream duopoly competition is enabled. In region IV, U induces limit pricing by A. Finally, in region V, U charges an excessive price, permitting A to charge its monopoly price downstream.

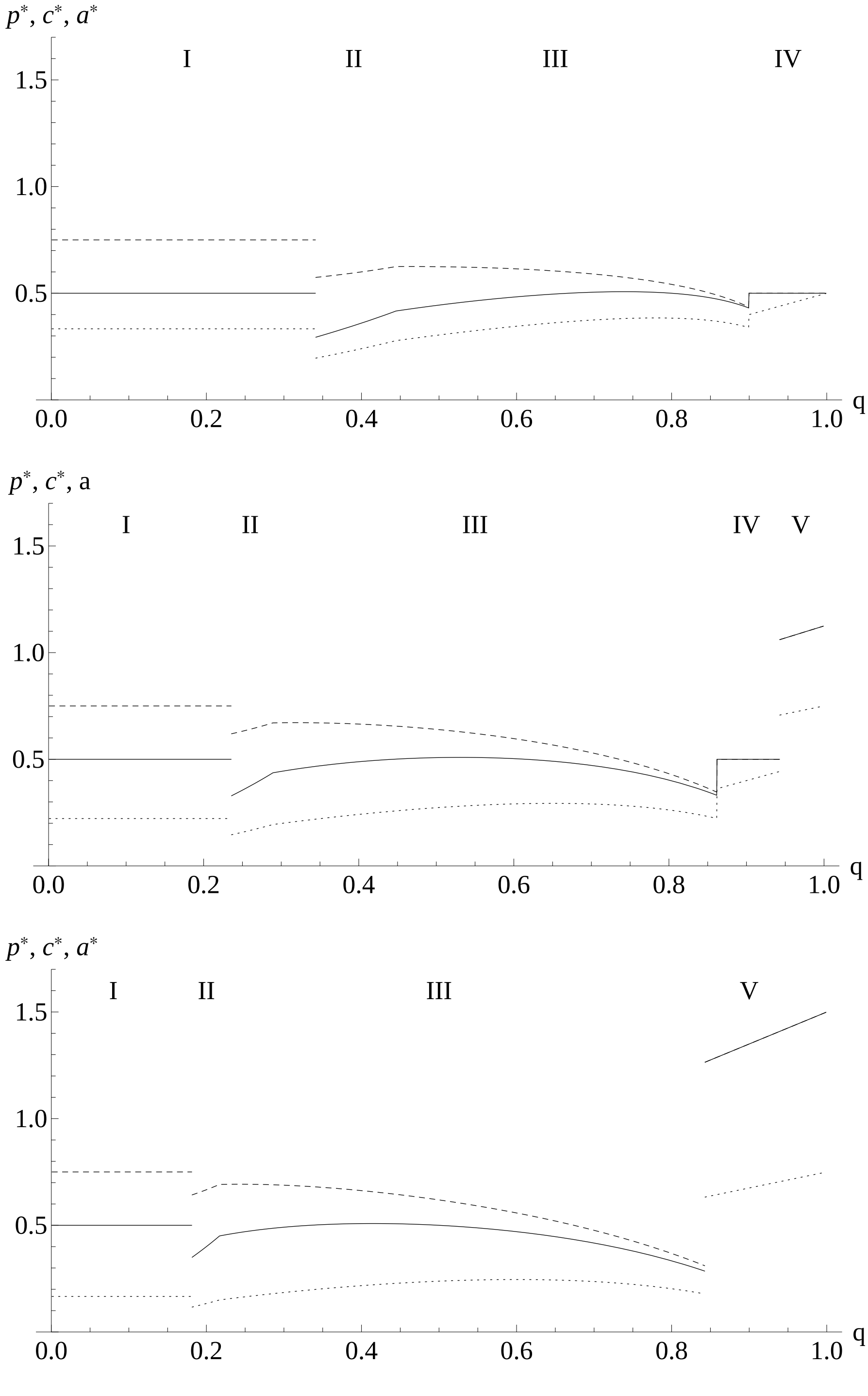

Figure 3: Equilibrium prices for $r = 2/3$ (top), $r = 4/9$ (center) and $r = 1/3$ (bottom) as a function of q. The solid (dashed) [dotted] line depicts the supplier's (standard producer's) [alternative producer's] equilibrium price, respectively.

consumer surplus, producer surplus and total social welfare in each of the five different optimal pricing regions. For the two main qualitative findings of this paper, the expressions for total social welfare W and supplier profit Π_U in each of the regions are needed. These are provided in the following table, while all other equilibrium expressions can be found in Appendix B.

Region	c^*	W^*	Π_U^*
I	$\frac{1}{2}$	$\frac{7}{32}$	$\frac{1}{8}$
II	$\frac{q(1-q)}{2r-rq-q}$	$\frac{(r-q)(3r-2rq-q)}{2(2r-rq-q)^2}$	$\frac{q(1-q)(r-q)}{(2r-rq-q)^2}$
III	$\frac{(r+2)(1-q)q}{2(2r^2-qr^2-2rq+2q-q^2)}$	W_{III}	$\frac{(r+2)^2(1-q)q}{4(4-q)(2r^2-qr^2-2rq+2q-q^2)}$
IV	$\frac{1}{2}$	$\frac{1}{2}-\frac{1}{8q}$	$\frac{r}{4q}$
V	$\frac{q}{2r}$	$\frac{7}{32}q$	$\frac{q}{8}$

Where $W_{III} = S + T$, with

$$S = \frac{192r^4 + (304r^2 - 432r^3 - 212r^4)q + (112 - 368r - 24r^2 + 340r^3 + 47r^4)q^2}{8(4-q)^2(2r^2 - qr^2 - 2rq + 2q - q^2)^2},$$

$$T = \frac{(-36 + 236r - 117r^2 - 46r^3 + 13r^4)q^3 + (-84 + 65r^2 - 6r^3 - 4r^4)q^4 + (52 - 12r - 12r^2)q^5 - 8q^6}{8(4-q)^2(2r^2 - qr^2 - 2rq + 2q - q^2)^2}.$$

Using the above table and keeping in mind that in equilibrium, all five pricing strategies may be chosen by the supplier (see Proposition 2), it is possible to state the two main propositions of this article.

Proposition 3. *For $q > q_l \approx 0.635$, there exists $\overline{r}(q) \in (0,1)$ such that for all $r < \overline{r}(q)$, the total social welfare in the market is reduced, compared to the standard double-marginalization case.*[13] *This is true if the supplier charges an excessive price in equilibrium (region V), and the associated welfare loss is given by $\frac{7}{32}(1-q)$. For all other technology pairs (q,r), the total social welfare is (weakly) higher than in the standard double-marginalization case.*

An excessive supplier price implies that only the more input efficient firm A can profitably operate, and that it may charge its monopoly price in the downstream market without losing sales to the standard producer. What essentially happens in region V, compared to the standard double marginalization case of region I, is that the high quality downstream monopolist is replaced by a low quality one, although the higher input efficiency of A does not benefit final consumers (i.e., does not reduce deadweight loss). This is because the supplier corrects for the lower input requirement of A by increasing its price in inverse proportion. That is, as A only

[13]Formally, $\overline{r}(q) = \frac{q^3-6q^2+16q-8}{4-q^2+\sqrt{(4-q)^2(4-6q+3q^2-q^3)}}$ for $q \in (q_l, q_{h2}] \approx (0.635, 0.8603]$ and $\overline{r}(q) = \frac{q^2}{2}$ for $q > q_{h2}$.

needs r units of input to serve one final consumer, the supplier simply increases its price by the factor $\frac{1}{r}$. Considering the effective prices that are paid in the market, everything becomes as if there was a standard producer in the downstream market, with full input requirement, that can only provide less than full quality of $q < 1$. It is easy to check that this implies a lower surplus for every agent in the market.

On the other hand, as long as the supplier does *not* find it optimal to charge an excessive price, existence of A is (weakly) welfare-increasing. In particular, whenever A is sufficiently efficient such that P has to change its pricing behavior, compared to the standard double marginalization case, the decreased downstream prices due to competition (or potential competition under limit pricing) in the downstream market lead to a reallocation of goods that is socially desirable. This is always true, although for some parameter combinations, there are two countervailing effects at work. Namely, competition in the downstream market allows some new consumers to participate in the market (which generates additional surplus), but if A offers a very appealing price, some high valuation consumers are led to switch to the low-quality firm, which generates less net surplus in these transactions. The proof of Proposition 3 shows that the former effect always dominates.

The next main question is whether the supplier may benefit from the existence of an alternative, lower input downstream firm. This is answered in

Proposition 4. *Existence of A decreases the supplier's profit, compared to the standard double marginalization case, if and only if A is highly efficient, that is, if and only if* $q < q_{h3} = \frac{4}{3}\left(-2+\sqrt{7}\right) \approx 0.861$ *and* $r < r_e := \frac{q(2+q)}{8-5q+\sqrt{3(4-q)^2(1-q)}} < \frac{q}{2}$, *or* $q \geq q_{h3}$ *and* $r < \frac{q}{2}$. *Whenever U charges an excessive price, its profit is lower than in the standard double marginalization case.*

The intuition behind Proposition 4 is that the effect of increased demand because of (a) decreased downstream prices and (b) a new segment of low valuation consumers that can be served by the low quality producer (if A is active), typically more than offset the losses in demand due to the decreased input requirement that characterizes the alternative production technology. However, if this technology is very efficient, the second effect may still dominate, leading to lower supplier profits than in the case where no alternative technology exists. In particular, a necessary condition for this is that A needs at most half of P's input per unit of quality provided. In region V (where an extremely input efficient technology is available), U's profit is always lower than if A did not exist, but the firm can minimize its losses by charging an excessive input price.

Another interesting finding is given by the following.

Remark. *Total social welfare is maximized if U induces competition in the downstream market, but is actually indifferent between doing so, inducing limit pricing by A, or charging an excessive input price. The unique parameter point where this is true is given by* $q = q_{h2} \approx 0.8603$ *and* $r = \frac{(q_{h2})^2}{2} \approx 0.37$. *There, the total social welfare is equal to* $W_{max} \approx 0.4284$.

Corollary 2. *There exists a non-degenerate parameter region where total social welfare exceeds the social surplus that would be obtained in the case where the monopolist supplier serves a competitive (Bertrand) downstream market with identical full quality producers.*

Thus, in the model, alternative downstream production technologies are best for society if they are so efficient that the supplier induces competition in the downstream market, but is (almost) indifferent between doing so and inducing limit pricing or even monopoly pricing by A. In other words, the alternative technology is most beneficial to welfare if the supplier has no strong preference over A's standing in the downstream market (as long as A is active), but opts for preserving competition between A and P.[14] Surprisingly, there is a range of technology parameters where it is better for society to have asymmetric downstream firms with some market power, as in the present model, than perfect competition in the downstream market. Intuitively, this is the case because, if A is significantly more efficient than P, the supplier maximizes its profit by charging an input price that is lower than if there was Bertrand competition downstream, as it wants to maintain competition in the downstream market. This reduction in the supplier's price can more than offset the efficiency loss that stems from double marginalization externalities that are present if the downstream firms have market power.

5 (Third Degree) Price Discrimination

In contrast to Section 4, suppose now that the institutional framework is such that the supplier is able to price discriminate between P and A. In principle, several different forms of price discrimination are possible: individual tariffs (e.g., depending on the firms' product qualities and input requirements), two-part tariffs, other forms of non-linear pricing like quantity discounts and rebates, etc. Moreover, the supplier could be vertically integrated with the incumbent P, implying that even if price discrimination is not legally feasible, the firm might charge a prohibitively high input price and subsidize its high quality downstream branch. However, it is easy to see that

[14] Whether this feature is robust to changes in the distribution of consumers' valuation for quality could be an interesting topic for future research.

U's problem is uninteresting if it can charge two-part tariffs (second degree price discrimination) or is vertically integrated with P. In both cases, the supplier can extract all of the downstream surplus and earn the full monopoly profit.

To see this, note first that profits are certainly highest for both downstream producers if they are monopolists in their market. Then, using Proposition 1 combined with Corollary 1 (or using Appendix B directly), one can observe that $\Pi^*_P(c,d) = \frac{(1-c)^2}{4}$ and $\Pi^*_A(c,d) = \frac{(q-d)^2}{4q}$.[15] Now, in the two-part-tariff case, suppose that U charges a zero per-unit cost, i.e., $c = d = 0$. Maximal downstream profits are then given by $\frac{1}{4}$ and $\frac{q}{4} < \frac{1}{4}$ for P and A, respectively. Thus, by charging $F = \frac{1}{4}$, the supplier can certainly drive out A of the market, allowing P to make a profit of exactly $\Pi_P = \frac{1}{4}$, which is fully extracted by U.

On the other hand, suppose that U is vertically integrated with P. Then, it can drive out the non-integrated firm A by charging an appropriately high unit cost (engaging in a *price squeeze*) and subsidizing P. As the latter has to pay a *de facto* unit cost of zero, the monopoly profit can again be achieved.

Hence, in the following, I will concentrate on the more interesting case where the supplier is not able to charge two-part tariffs and is not vertically integrated with P. For simplicity, I will analyze a situation of maximum freedom in price discrimination, namely the polar case to Section 4: unrestricted third degree price discrimination by U.

As it turns out, the consequences of third degree price discrimination on the equilibrium outcome are more straightforward to analyze than the baseline model. The following proposition summarizes the main findings.

Proposition 5. *If U can engage in third degree price discrimination, it will choose prices such as to equalize the relative efficiency (cost per unit of quality provided) of P and A. In particular, it will set* $c_1 = \frac{1}{2}$ *for P and* $c_2 = \frac{q}{2r}$ *for A.*

Corollary 3. *Under third degree price discrimination by U, both downstream firms are always active, irrespective of* q *and* r*. Equilibrium prices, demands, profits, consumer surplus, producer surplus and total social welfare are summarized in the following two tables.*

Firm	Price	Demand	Profit
U	$c_1 = \frac{1}{2}$, $c_2 = \frac{q}{2r}$	$\frac{2+r}{2(4-q)}$	$\frac{2+q}{4(4-q)}$
P	$\frac{6-3q}{2(4-q)}$	$\frac{1}{4-q}$	$\frac{1-q}{(4-q)^2}$
A	$\frac{q(5-2q)}{2(4-q)}$	$\frac{1}{2(4-q)}$	$\frac{q(1-q)}{4(4-q)^2}$

PS	CS	W
$\frac{12-q-2q^2}{4(4-q)^2}$	$\frac{4+5q}{8(4-q)^2}$	$\frac{28+3q-4q^2}{8(4-q)^2}$

[15] Recall that d denotes A's unit cost.

Intuitively, the supplier always prefers to induce competition in the downstream market because it reduces the double marginalization externality caused by P's and A's market power in the downstream segment. In particular, it is optimal for U to *maximize* competition in the downstream market, which is achieved by equalizing the downstream firms' relative cost efficiencies.[16]

Having determined the supplier's optimal price tuple, it follows

Proposition 6. *Enabling the upstream supplier to engage in third degree price discrimination increases total social welfare if and only if (a) q and r are such that an undiscriminating supplier would induce monopoly pricing by P or A (regions I and V) or (b) A is slightly more input efficient than P.*

The above proposition is true because of two reasons. First, compared to the case of a downstream monopoly by either P or A, the total output in the market greatly expands once the supplier maximizes competition in the downstream segment. Hence, less deadweight loss is created and social welfare increases.

Second, if the technology is such that without discrimination, the supplier would optimally induce limit pricing (by either of the downstream firms) or downstream competition, total social welfare (typically) declines if price discrimination is enabled. In some sense, such technology parameters anyway provide sufficient incentives for low prices, as both U and the downstream firm(s) charge prices that are well below the monopoly price in order to maximize profits. But interestingly, in the case that A is just slightly more input efficient than P, it is still welfare enhancing to allow the supplier to price discriminate. This is because in such situations, P's drop in profit, compared to the case of equal cost-efficiency of P and A, is larger than A's increase in profit and the consumers' gain. At the same time, U is (almost) indifferent, as its loss, compared to the optimum of equal cost-efficiency, is only of second order. Hence, price discrimination by the supplier can correct for P's abundant loss in profits.[17]

[16] An interesting consequence is that a supplier might want to support a hopelessly inefficient low or high quality producer by adjusting the downstream firms' input costs adequately. Hence, if third degree price discrimination is possible, firms might be active in the downstream market that could never profitably operate in markets where price discrimination is not feasible. For example, downstream firms that provide a much lower quality at a marginally lower input requirement than some standard producer, or firms that can produce a marginally higher quality at a much higher input requirement, might both be active in equilibrium.

[17] In fact, in the second case, the usual necessary condition that output must increase in order for third-degree price discrimination to improve social welfare (see e.g. Varian (1985)) is not fulfilled, as the supplier's total output remains unchanged. Clearly, this stems from the specific modeling framework employed in this article.

6 Extensions

The baseline model as presented in Sections 2 to 4 can be extended in numerous dimensions. In this section, I will analyze the robustness of the model's main qualitative insight, namely that suppliers might charge an excessive price in equilibrium, foreclosing high-quality downstream firms and reducing social welfare, to changes in the model setup. To this end, I will consider an alternative market structure (oligopolistic competition in the upstream market) and an alternative supplier technology (positive unit cost of production). It will be seen that in both cases, welfare reducing excessive pricing might still occur in equilibrium.[18]

Oligopolistic Competition in the Upstream Market

So far, I have analyzed the case of a monopolist upstream firm supplying the downstream market. One may now wonder whether (welfare reducing) excessive pricing and high quality foreclosure can also happen if there is competition in the upstream market. However, note that if there is perfect (Bertrand) competition upstream, this cannot be the case, as every upstream firm must price at marginal cost ($c_i = 0$ for all upstream firms), which implies that existence of an alternative downstream producer will always increase social welfare by introducing competition in the downstream market and reducing deadweight loss.

In order to make things interesting, I will thus consider the simple case where two upstream firms $1, 2$ compete in Cournot, setting quantities S_1, S_2. Given the resulting aggregate output S, a Walrasian mechanism selects the price $\tilde{c}$ that leads to market clearing behavior in the downstream market, given the demand functions specified in equations (5a) and (5b).

Inverting this demand system for the case of a relatively input efficient firm A ($r < q$), it is found that the market clearing price, for some aggregate output S, is given by the following schedule:

$$\tilde{c} = \begin{cases} \frac{rq-2qS}{r^2} & \text{if} \quad S < \frac{r(q-r)}{q(2-r)} \\ 1 - \frac{qS}{r} & \text{if} \quad \frac{r(q-r)}{q(2-r)} \leq S < \frac{r(q-r)}{q(2-q-r)} \\ \frac{(1-q)q(2+r-(4-q)S)}{2r^2-qr^2-2rq+2q-q^2} & \text{if} \quad \frac{r(q-r)}{q(2-q-r)} \leq S < \frac{r+2}{4-q} \\ 0 & \text{if} \quad S \geq \frac{r+2}{4-q}. \end{cases} \tag{7}$$

[18]Moreover, it is easy to see that the model is robust to the introduction of a small fixed cost of production, or an initial entry decision coupled with a small entry cost, for either downstream firm. In both cases, the supplier's equilibrium decision is modified in such a way that regions II and IV of the downstream price game are integrated to regions I and V, respectively. This is because if either A or P make zero operational profits in the equilibrium of the downstream game, they will not enter (or shut down) if they face entry costs (fixed costs of production).

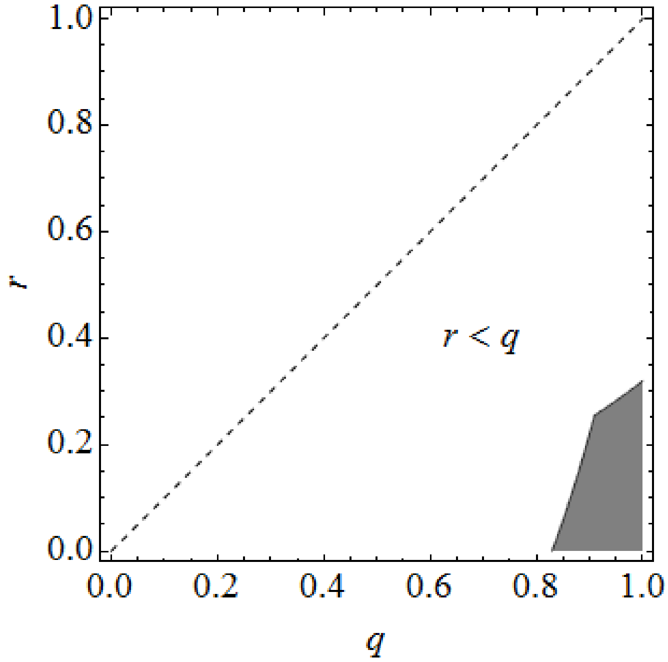

Figure 4: Region of technology parameters (gray) where the upstream duopolists may restrict their output in equilibrium.

Using this schedule, it is straightforward to observe that *if* there exists a symmetric equilibrium in which both firms restrict their output in such a way that the resulting input price for the downstream market will be "excessive" ($S^* < \frac{r(q-r)}{q(2-r)}$), it must be characterized by $S_1^* = S_2^* = \frac{r}{6}$, with associated firm profits of $\Pi_i^* = \frac{q}{18}$, $i = 1, 2$. However, at such low supply levels, the firms might have an incentive to expand their outputs in order to accrue larger profits. Only if there exists a parameter range where this is not the case, an excessive upstream price might be observed in equilibrium.

As comparing the firms' restricted-supply profit to all potentially profit-increasing deviations is rather cumbersome, I will resort to a numerical simulation in order to determine the range of technology parameters where excessive pricing might occur in equilibrium. I do so by fixing $S_j = \frac{r}{6}$ and checking whether $S_i = \frac{r}{6}$ is optimal, given (q, r), for all possible combinations of these parameters that may support excessive pricing (i.e., where $r < q$). The resulting graph is depicted in Figure 4.

It can be seen that there exists a non-degenerate parameter region in which an excessive supplier price might be found in equilibrium. However, contrasting Figure 4 with Figure 2, it is immediate that this region is smaller than in the case of a monopolist upstream firm. Thus, the downstream technologies have to be more asymmetric, in the sense of higher relative quality and lower relative input requirement of A, in order for excessive pricing and high-quality foreclosure to take place in equilibrium. Note moreover that excessive-pricing equilibria will never be unique, as if both firms choose a large output that leads to competition in the downstream market, unilateral deviation to a lower output level never pays.

The question which remains open is whether excessive-pricing equilibria under upstream duopoly competition are inferior, from a welfare point of view, to upstream duopoly equilibria where just P is active downstream. A straightforward comparison shows that this is indeed the case. These observations are summarized in

Proposition 7. *If there is duopoly Cournot-competition in the upstream market, a range of technology parameters exists such that in equilibrium, an excessive input price for the downstream firms might result. Whenever this is the case, welfare is reduced, compared to the case where only P is active downstream.*

Positive Supplier Production Cost

For simplicity and in order to obtain closed-form solutions for all optimal-pricing-region boundaries, the main model only dealt with the case of a zero unit-cost supplier. But this assumption implies that A can only have an *indirect* welfare increasing role by introducing competition in the downstream market. The efficient outcome would completely ignore A, as production of its input is costless anyway, but the firm's final good has lower quality than P's.

However, this observation no longer holds if there is a positive cost of producing the downstream firms' input. In that case, if A is active in equilibrium, it will also have a *direct* welfare-increasing role by reducing aggregate production costs. If A is highly input efficient, less of the costly input needs to be manufactured.

The purpose of this subsection is not to give a full solution to the supplier's problem for positive production costs, but rather to show whether and to what extent excessive supplier pricing might still harm social welfare, *despite* the lower production costs. For this, note that *if* the supplier chooses an excessive price in equilibrium, A will behave like a monopolist in the downstream market, creating an upstream demand of $D_U(c) = \frac{r(q-cr)}{2q}$ (as $c \in [c^{**}, \frac{q}{r}]$). Denoting the supplier's unit cost by $e > 0$, its profit can be written as

$$\tilde{\Pi}_U(c;e) = (c-e)\frac{r(q-cr)}{2q}, \quad c \in [c^{**}, \frac{q}{r}].$$

An interior maximizer of this expression, if it exists, is given by $\tilde{c}^*(e) := \frac{q}{2r} + \frac{e}{2}$,[19] leading to an upstream profit of $\tilde{\Pi}_U^* = \frac{(q-re)^2}{8q}$. Given $\tilde{c}^*(e)$ as input price, A will charge a final price of $\tilde{a}^* = \frac{q+r\tilde{c}}{2} = \frac{3q+re}{4}$, resulting in a downstream profit of $\tilde{\Pi}_A^* = (\tilde{a}^* - r\tilde{c}^*)(1 - \frac{\tilde{a}^*}{q}) = \frac{(q-re)^2}{16q}$. Adding these profits to the market's consumer surplus of $\tilde{CS} = \int_{\frac{\tilde{a}^*}{q}}^{1} (\theta q - \tilde{a}^*) d\theta = \frac{(q-re)^2}{32q}$, the resulting total social welfare is $\tilde{W} = \frac{7(q-re)^2}{32q}$.

In the standard double marginalization case, U's profit can be written as

$$\bar{\Pi}_U(c; e) = (c - e)\frac{1-c}{2},$$

with unique maximizer $\bar{c}^* = \frac{1+e}{2}$ and an associated profit of $\bar{\Pi}_U^* = \frac{(1-e)^2}{8}$. Given $\bar{c}^*(e)$ as input price, P will charge a final price of $\bar{p}^* = \frac{1+\bar{c}^*}{2} = \frac{3+e}{4}$, implying a downstream profit of $\bar{\Pi}_P^* = (\bar{p}^* - \bar{c}^*)(1 - \bar{c}^*) = \frac{(1-e)^2}{16}$. Adding the upstream and downstream profit to the final consumers' surplus of $\bar{CS} = \int_{\bar{p}^*}^{1} (\theta - \bar{p}^*) d\theta = \frac{(1-e)^2}{32}$, a total social welfare of $\bar{W} = \frac{7(1-e)^2}{32}$ can be computed. Solving $\bar{W} > \tilde{W}$, it easily follows

Proposition 8. *If the supplier has a positive unit cost of $e > 0$, existence of A reduces social welfare, compared to the standard double marginalization case, if (i) q and r are such that the supplier charges an excessive price in equilibrium, and (ii) A is not sufficiently efficient, i.e., $r > \frac{q-\sqrt{q}(1-e)}{e}$.*

Note that $r > \frac{q-\sqrt{q}(1-e)}{e}$ is definitely the case whenever $q \leq (1-e)^2$, as r is strictly positive. Moreover, it can be shown that for values of r that are close to zero, any supplier marginal cost of $e < 1$ does in fact give rise to some non-empty interval of q's where the supplier charges an excessive price in equilibrium *and* total social welfare is reduced.[20] Hence, even if the supplier faces a high marginal cost, as long as r is sufficiently low, there always exists a region in (q, r)-space such that the firm optimally forecloses the high-input producer P by charging an excessive price, but total social welfare is reduced. Put differently, the direct efficiency gain of an extremely efficient alternative producer (that reduces the aggregate production cost to almost zero) cannot always offset the welfare loss that is associated with an (optimal) excessive supplier price.

A numerical simulation of the supplier's optimal pricing regions for $e = 0.1$ can be found in Figure 5.

[19] For every $e \geq 0$, the boundary value c^{**} cannot be a maximizer of U's overall profit function (see equation (6) for $e \geq 0$), as $\lim_{c\uparrow c^{**}} \frac{\partial \Pi_U(c;e)}{\partial c} > 0$ and $\lim_{c\downarrow c^{**}} \frac{\partial \Pi_U(c;e)}{\partial c} < 0$ cannot hold at the same time. This is because $\lim_{c\uparrow c^{**}} \frac{\partial \Pi_U(c;e)}{\partial c} > 0$ implies $r - \frac{2r(2-q)}{2-r} + re > 0$, whereas $\lim_{c\downarrow c^{**}} \frac{\partial \Pi_U(c;e)}{\partial c} < 0$ implies $q - \frac{2r(2-q)}{2-r} + re < 0$, which cannot both be the case for $r < q$.

[20] A somewhat tedious proof can be obtained from the author upon request.

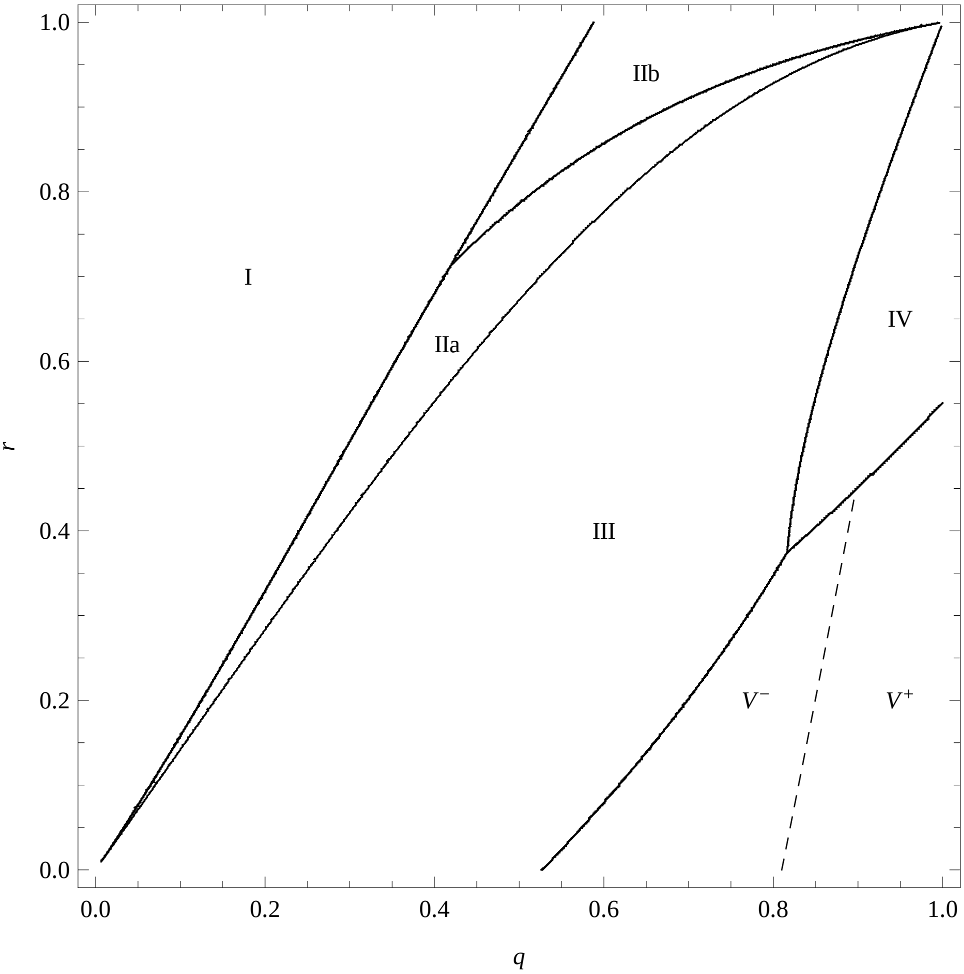

Figure 5: The supplier's optimal pricing decision for $e = 0.1$. In region I, the supplier charges the standard double marginalization monopoly price. In regions IIa and IIb, it induces limit pricing by P. In region III, downstream duopoly competition is enabled. In region IV, U induces limit pricing by A. Finally, in region V, U charges an excessive price, permitting A to charge its monopoly price downstream. For values of q that are to the right (left) of the dashed line, total social welfare under an excessive supplier price is higher (lower) than in the standard double marginalization case.

Two things are noticeable. First, the dashed lined defined by $r = \frac{q-\sqrt{q}(1-e)}{e}$ separates region V, where U chooses an excessive price in equilibrium, into two subregions. Depending on whether q is to the right or left of the dashed line (regions V^+ and V^-, respectively), total social welfare is higher (lower) than in the standard double marginalization case. Second, the supplier's pricing behavior becomes even richer with positive production cost, as the limit pricing region II is split up into two subregions. This is because both boundary limit pricing (by choosing $c = c^a$) and interior limit pricing (by choosing some $c \in (c^a, c^b)$) can be optimal for the supplier.[21]

7 Conclusion

In many industries, an essential intermediary good can be processed to final goods of different qualities. In particular, a large quantity of some intermediary good might be transformed to a high-quality final product, whereas a smaller quantity of that good might be transformed to a lower quality substitute. Examples include the transformation of raw materials to durables (e.g., precious metals that are processed to electronics), various types of renting and sharing (e.g., cars that are sold directly to individual final consumers, versus cars that are shared among a group of consumers, organized by a carsharing enterprise), essential facilities like harbors or airport slots (where a low quality company might use less of that resource for providing its service – e.g., because fewer, but more crowded cruises are offered), and final goods markets where imperfect recycling is possible (allowing recycling firms to offer a lower-quality, lower-input-requirement final good).

As the upstream segment of such markets is often characterized by significant economies of scale, or large sunk costs that make entry of new firms difficult, it is important to understand how upstream firms with market power would behave in such a context. This article establishes that a monopolist upstream supplier might price-foreclose a *high-quality*, *high-input-requirement* final good producer if a single lower-quality, lower-input-requirement competitor is active in the downstream segment. Intuitively, the supplier does so if correcting for the efficient firm's low input requirement by charging a very high price dominates reducing double-marginalization externalities by maintaining downstream competition. This reduces total social welfare even compared to the highly inefficient double-marginalization case.

Besides this main finding, the considered model has several other interesting properties. First, varying the relative input-requirement and relative quality of the low-quality producer, it

[21] If e is sufficiently large, the same is true for the other limit pricing region IV.

is found that a monopolist upstream supplier might choose from a number of different pricing strategies. These include charging the standard double marginalization price and only serving the high-quality producer, inducing limit pricing by either downstream firm, enabling duopoly competition in the downstream market, or charging an excessive price where only the low-quality firm can profitably operate.

Another result is that the existence of a low-input-requirement, low-quality downstream producer typically increases the upstream firm's profit, despite the low-input threat that arises. In the model, the monopolist's profit may only decrease if the alternative producer is at least twice as input efficient as the standard producer. Intuitively, this is the case because the intermediaries' increased input demand caused by competition (or potential competition) and lower prices in the downstream market typically more than offsets any losses in the upstream firm's demand caused by the low-quality producer's low input requirement. Only if the latter is very efficient, this may be reversed.

Moreover, I find that if the upstream supplier is allowed to engage in third-degree price-discrimination, the total social welfare in the market may increase. This is because a price-discriminating monopolist will always try to equalize the cost-efficiency of the downstream firms, maximizing competition in the downstream market. This is beneficial to social welfare if the downstream firms are either very asymmetric in their input-efficiencies (such that the downstream segment would be monopolized in the equilibrium without price discrimination), or very similar (with the low-quality firm being slightly more efficient than the high-quality one).

Finally, I show that the model is robust to two extensions. Both (Cournot-) competition in the upstream market, and a positive production cost of the supplier, do not turn over the result that welfare-decreasing excessive-supplier-pricing might occur in equilibrium. However, in both cases, the region of problematic technology-parameters becomes smaller.

As this article's main intent is to show that excessive supplier-pricing might happen in equilibrium, there are numerous things left open for future research. One interesting point would be to allow for multiple quality and input-differentiated downstream firms, analyzing how the upstream supplier's optimal pricing strategies would be affected. Moreover, for technical simplicity, the present model only considered a uniform distribution of the consumers' valuation for quality. However, in many markets, these valuations may have totally different shapes. Hence, it could be an interesting undertaking to match the model to various consumer distributions and examine whether qualitative changes would arise.

However, the most important task is an empirical one. This paper gives a key testable prediction: as alternative downstream production technologies emerge and become more input-efficient or provide higher quality, the optimal pricing conduct of an upstream monopolist should vary considerably. In particular, once an alternative, lower input-requirement producer becomes so efficient that maintaining downstream competition is not profitable anymore to an upstream monopolist, the latter should choose an excessive price. This price must be higher than the input price that was charged before the alternative technology became available, and should drive out all traditional, high-quality final goods producers. If it can be established that such a conduct does in fact take place in real markets, counter-measures need to be considered by antitrust authorities.

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Appendix A: Technical Proofs

Proof of Lemma 1. Consider firm P first. If $\theta_h \geq 1$, which implies $p \geq a+1-q$, every consumer prefers buying at A over buying at P and P's demand is zero. This explains the last part of P's demand. If $\theta_h < 1$, P's demand is given by $1-\max\{\theta_h, \theta_m\}$, where the maximum operator reflects the fact that P's marginal consumer can either be indifferent between going to A or not buying at all. Hence, if $\theta_m \geq \theta_h$, which implies $p \leq \frac{a}{q}$, P's demand can be written as $1-\theta_m = 1-p$. This explains the first part of P's demand. On the other hand, if $\theta_h > \theta_m$ ($p > \frac{a}{q}$), P's demand is $1-\theta_h = 1-\frac{p-a}{1-q}$. This explains the second part of P's demand.

Next, consider firm A. If $\theta_h \geq 1$, which implies $a \leq p+q-1$, every consumer prefers buying at A over buying at P and A's demand is $1-\theta_l = 1-\frac{a}{q}$. This explains the first part of A's demand. If $\theta_h < 1$, there are two cases. First, if $\theta_h > \theta_l$, which implies $a < pq$, A's demand is given by $\theta_h - \theta_l = \frac{p-a}{1-q} - \frac{a}{q}$. This explains the second part of A's demand. Second, if $\theta_h \leq \theta_l$ ($a \geq pq$), A's demand is zero, as every consumer that gets positive utility from buying at A prefers purchasing at P. □

Proof of Proposition 1. I start by deriving the firms' best response functions $p^*(a;q)$ and $a^*(p;q)$. In the following, this will be sketched exemplary for $p^*(a;q)$; the derivation for $a^*(p;q)$ is analogous.

Given P's demand function specified in equation (3a), the firm has two possibilities for maximizing its profit. First, by pricing in the range $[0, \frac{a}{q}]$, it can prevent A from getting positive demand, implying a demand of $1-p$. Thus, for such low prices, the firm maximizes its profit either at

$$p^m := \arg\max_p (p-c)(1-p) = \frac{1+c}{2},$$

or, if $p^m > \frac{a}{q}$ ($a < \overline{a} := \frac{q(1+c)}{2}$), at the boundary $p^{lim} := \frac{a}{q}$.

If, on the other hand, P prices in the range $(\frac{a}{q}, \infty)$, it enables A to be active. In that case, P should either price at

$$p^{acc} := \arg\max_p (p-c)(1-\frac{p-a}{1-q}) = \frac{a+1-q+c}{2},$$

or, if this value is lower or equal to the firm's unit cost c ($a \leq \underline{a} := c+q-1$), it should price at any $p \geq 1-q+a$ in order to get a demand of zero. However, since pricing below marginal cost is a weakly dominated strategy in the subgame defined by the price competition stage, and is not robust to minor perturbations in consumers' demand function, I restrict the firms' strategy space to prices which are at or above marginal cost. Thus, in the case of $a \leq \underline{a}$, P can choose any price $p \in [c, \infty)$ and make zero profit.

Now, it is apparent that if $a \geq \overline{a}$ ($p^m \leq \frac{a}{q}$; P can choose its monopoly price without inducing A to be active) it must be optimal for P to pick its monopoly price. If the inequality does not hold, one has to compare P's profit when choosing the limit price p^{lim} with its profit when allowing for positive demand of A, i.e., when choosing the accommodation price p^{acc}. Again using the demand schedule summarized in equation (3a) for the corresponding profit expressions, one can see that

$$\Pi_P(p^{lim}) > \Pi_P(p^{acc}) \Longleftrightarrow \frac{(q-a)(a-c)}{q^2} > \frac{(1-q+a-c)^2}{4(1-q)}.$$

After some calculation, this simplifies to

$$\Pi_P(p^{lim}) > \Pi_P(p^{acc}) \Longleftrightarrow a > \tilde{a} := \frac{q(1+c-q)}{2-q}.$$

Given the assumption of $c < 1$, it is easy to show that $\underline{a} < \tilde{a} < \overline{a}$ holds. All in all, the argument thus implies the following best reply correspondence for P.[22] For $a \leq \underline{a}$, price at any $p \geq c$. For $a \in (\underline{a}, \tilde{a})$, price at p^{acc}. For $a \in [\tilde{a}, \overline{a})$, price at p^{lim}. And finally, for $a \geq \overline{a}$, price at p^m.

The above best response function and its equivalent for A is summarized in the following equations. Namely, for $c < 1$, $d < q$, $p \geq c$ and $a \geq d$, P's and A's best reply correspondences are given by

$$p^*(a;q) = \begin{cases} \{p \mid p \geq c\} & a \leq \underline{a} = c + q - 1 \\ p^{acc} := \frac{1-q+a+c}{2} & a \in (\underline{a}, \tilde{a}), \text{ where } \tilde{a} := \frac{q(1+c-q)}{2-q} > \underline{a} \\ p^{lim} := \frac{a}{q} & a \in [\tilde{a}, \overline{a}), \text{ where } \overline{a} := \frac{q(1+c)}{2} > \tilde{a} \\ p^m := \frac{1+c}{2} & a \geq \overline{a} \end{cases}$$

and

$$a^*(p;q) = \begin{cases} \{a \mid a \geq d\} & p \leq \underline{p} := \frac{d}{q} \\ a^{acc} := \frac{pq+d}{2} & p \in (\underline{p}, \tilde{p}), \text{ where } \tilde{p} := \frac{2-q-(q-d)}{2-q} > \underline{p} \\ a^{lim} := p + q - 1 & p \in [\tilde{p}, \overline{p}), \text{ where } \overline{p} := \frac{2-q+d}{2} > \tilde{p} \\ a^m := \frac{q+d}{2} & p \geq \overline{p}, \end{cases}$$

respectively.

[22] Since $p^{acc}(\tilde{a}) = p^{lim}(\tilde{a})$ and $p^{lim}(\overline{a}) = p^m$, it does not matter how to specify these borderline cases.

Case by case, I will now show that each of the five parameter regions outlined in the proposition consists of a unique type of equilibrium. Moreover, each of these five equilibria will be characterized.

Before doing so, note that if $c \geq 1$ and $d < q$, P can never get a positive demand for any $p \geq c$ it chooses. Hence, the unique type of equilibrium is such that P prices at or above its cost and A chooses its monopoly price $a^m = \frac{q+d}{2}$ (region V). Conversely, if $c < 1$ and $d \geq q$, A can never get a positive demand for any $a \geq d$ it chooses. Hence, the unique type of equilibrium is such that A prices at or above its cost and P chooses its monopoly price $p^m = \frac{1+c}{2}$ (region I). If both $c \geq 1$ and $d \geq q$, it is obvious that both P and A will be inactive in equilibrium. In the following, the remaining case were $c < 1$ *and* $d < q$ is considered.

Region I: $d > \hat{d} := \frac{q(1+c)}{2}$.
Since A has to price at or above marginal cost by assumption, it must hold that $a \geq d > \frac{q(1+c)}{2}$ in equilibrium. Hence, $a > \overline{a}$ is always satisfied, which, according to P's best-reply schedule, implies that P finds it optimal to price at p^m for every $a \geq d$. On the other hand, $p^m < \underline{p} = \frac{d}{q}$, as easily follows from $d > \hat{d}$. That is, by the above best-reply schedule, A's best reply to p^m is in fact given by any price that satisfies $a \geq d$. Thus, in region I, every pair of prices (p, a) such that $p = p^m$ and $a \geq d$ forms an equilibrium.

Region II: $d \in (\overline{d}, \hat{d}]$, where $\overline{d} := \frac{q(1-q+c)}{2-q}$.
Suppose first that some $a' > d$ could be chosen by A in equilibrium. Hence, $a' > \frac{q(1+c-q)}{2-q} = \tilde{a}$ must hold. According to P's best reply schedule, it must thus follow that either (a) $p^{lim}(a') = \frac{a'}{q}$ (if $a' < \overline{a}$), or (b) $p^m = \frac{1+c}{2}$ (if $a' \geq \overline{a}$) should be chosen by P.

For case (a), $p^{lim}(a') = \frac{a'}{q} > \frac{d}{q} = \underline{p}$, where the inequality follows from $a' > d$, as assumed. On the other hand, it must be true that $p^{lim}(a') < \overline{p}$. To see this, note that as $a' < \overline{a}$ for case (a), it is sufficient to show that $\frac{\overline{a}}{q} < \overline{p}$. This easily implies $d > c + q - 1$. Since $d > \overline{d}$, it is straightforward to show that this is in fact true for $c < 1$. Overall, since $p^{lim}(a') \in (\underline{p}, \overline{p})$, A's best reply to this price must either be given by (a1) $a^{acc}(p^{lim}(a'))$ or (a2) $a^{lim}(p^{lim}(a'))$. For subcase (a1), A's best reply is given by $a^{acc}(p^{lim}(a')) = \frac{p^{lim}(a')q+d}{2} = \frac{a'+d}{2}$. However, as $d < a'$ by assumption, $\frac{a'+d}{2} < a'$. This shows that $a' > d$ cannot be part of an equilibrium in subcase (a1). For subcase (a2), A's best reply is given by $a^{lim}(p^{lim}(a')) = p^{lim}(a') + q - 1 = \frac{a'}{q} + q - 1$. This expression can only be equal to a' if $a' = q$. However, this contradicts $a' < \overline{a}$, as was assumed for case (a). This establishes that $a' > d$ cannot be part of an equilibrium in subcase (a2).

For case (b), note that $p^m > \underline{p}$ must as well be the case, since this relation implies $d < \frac{q(1+c)}{2} = \hat{d}$, which is true by assumption. Moreover, $p^m < \overline{p}$ reduces to $d > c + q - 1$, which has already been proven in case (a). Overall, since $p^m \in (\underline{p}, \overline{p})$, A's best reply to this price must either be given by (b1) $a^{acc}(p^m)$ or (b2) $a^{lim}(p^m)$. For subcase (b1), it holds that $a^{acc}(p^m) = \frac{qp^m+d}{2}$. But as $a' \geq \overline{a} = qp^m$ in case (b), which implies $p^m \leq \frac{a'}{q}$, it follows that $a^{acc}(p^m) < \frac{a'+d}{2}$. And since $d < a'$ by assumption, one finds that $a^{acc}(p^m) < a'$. Again, this establishes that $a' > d$ cannot be part of an equilibrium price pair. For subcase (b2), A's best response is $a^{lim}(p^m) = p^m + q - 1$. It is then easy to show that $a^{lim}(p^m) < \overline{a}$, contrary to the assumption of case (b). It is thus proven that A may never price above its marginal cost d in equilibria of region II.

One can subsequently focus on the case $a = d \in (\frac{q(1-q+c)}{2-q}, \frac{q(1+c)}{2}] = (\tilde{a}, \overline{a}]$. According to the above best-reply schedule, P's best response to this is given by $p^{lim}(d) = \frac{d}{q}$. Clearly, $p^{lim} \geq \frac{d}{q} = \underline{p}$. On the other hand, $p^{lim} = \frac{d}{q} \leq \tilde{p}$, as easily follows from $d \leq \hat{d}$. Thus, according to A's best reply schedule, the firm should choose $a^{acc}(p^{lim}(a))$. Now $a^{acc}(p^{lim}(a)) = \frac{p^{lim}(a)q+d}{2} = \frac{a+d}{2} = a$, as $a = d$ by assumption. This implies that the price pair $(p^{lim}(d), d) = (\frac{d}{q}, d)$ must constitute the unique equilibrium of the downstream subgame for $d \in (\overline{d}, \hat{d}]$.

Region III: $d \leq \overline{d}$ and $c \leq \overline{c}$, where $\overline{c} = \frac{2-q-(q-d)}{2-q}$.
First, it is convenient to establish that neither $a \geq \overline{a}$ nor $p \geq \overline{p}$ can be part of an equilibrium for $d \leq \overline{d}$ and $c \leq \overline{c}$. This ensures that each firm's best reply function is essentially a weakly increasing and weakly convex function with respect to the other firm's price.

To see this, suppose first that some $a' \geq \overline{a}$ might be chosen by A in equilibrium. P's best reply is then given by $p^m = \frac{1+c}{2}$. However, the condition $d \leq \overline{d}$ directly implies $c \geq \frac{d(2-q)}{q} - 1 + q$. Hence, $p^m \geq \frac{2d-dq+q^2}{2q}$ must hold. It is now straightforward to see that $\frac{2d-dq+q^2}{2q} > \underline{p}$, as follows from $d > q$. In turn, A's best response to p^m should either be given by (a) $a^{acc}(p^m)$ (if $p^m \in (\underline{p}, \tilde{p})$), (b) $a^{lim}(p^m)$ (if $p^m \in [\tilde{p}, \overline{p})$), or (c) a^m (if $p^m \geq \overline{p}$), depending on how large p^m is. For case (a), it is easy to establish that $a^{acc}(p^m) < \overline{a}$, as follows from $d < \hat{d}$. For case (b), $a^{lim}(p^m) < \overline{a}$ is also fulfilled, as this inequality implies $c < 1$. For case (c), note that a^m can only be a best response to p^m if $p^m \geq \overline{p}$, that is, $\frac{1+c}{2} \geq \frac{2-q+d}{2}$. Rearranging this conditions yields $d \leq c - 1 + q$. Hence, $a^m = \frac{q+d}{2} \leq \frac{q+(c-1+q)}{2}$. But $\frac{q+(c-1+q)}{2}$ is strictly smaller than $\overline{a} = \frac{q(1+c)}{2}$, as easily follows from $c < 1$. In all three cases, it thus holds that $a^*(p^*(a')) = a^*(p^m) < a'$. This confirms that $a' \geq \overline{a}$ cannot be an equilibrium price in the first place.

Next, suppose that some $p' \geq \overline{p}$ might be chosen by P in equilibrium. A's best reply is then given by $a^m = \frac{q+d}{2}$. However, the condition $c \leq \overline{c}$ directly implies $d \geq c(2-q) - 2 + 2q$. Hence, $a^m \geq \frac{2c-cq-2+3q}{2}$ must hold. It is now straightforward to see that $\frac{2c-cq-2+3q}{2} > \underline{a}$, as follows from $c < 1$. In turn, P's best response to a^m should either be given by (a) $p^{acc}(a^m)$ (if $a^m \in (\underline{a}, \tilde{a})$), (b) $p^{lim}(a^m)$ (if $a^m \in [\tilde{a}, \overline{a})$), or (c) p^m (if $a^m \geq \overline{a}$), depending on how large a^m is. For case (a), it is easy to establish that $p^{acc}(a^m) < \overline{p}$, as follows from $c < \hat{c}$. For case (b), $p^{lim}(a^m) < \overline{p}$ is also fulfilled, as this inequality implies $d < q$. For case (c), note that p^m can only be a best response to a^m if $a^m \geq \overline{a}$, that is, $\frac{q+d}{2} \geq \frac{q(1+c)}{2}$. Rearranging this conditions yields $c \leq \frac{d}{q}$. Hence, $p^m = \frac{1+c}{2} \leq \frac{1+\frac{d}{q}}{2}$. But $\frac{1+\frac{d}{q}}{2}$ is strictly smaller than $\overline{p} = \frac{2-q+d}{2}$, as easily follows from $d < q$. In all three cases, it thus holds that $p^*(a^*(p')) = p^*(a^m) < p'$. This confirms that $p' \geq \overline{p}$ cannot be an equilibrium price in the first place.

Having established that $a \geq \overline{a}$ or $p \geq \overline{p}$ can never be chosen in equilibrium in region III, the next step is to reexamine the firms' best response functions. To simplify things, suppose for the moment that for $a \leq \underline{a}$ ($p \leq \underline{p}$), P's (A's) best response is simply given by pricing at their respective marginal cost, rather than at any arbitrary price not lower than that. Moreover, suppose that even for $a \geq \overline{a}$ ($p \geq \overline{p}$), P (A) continues pricing at its respective limit price. It is then easy to see that each firm's (modified) best response is a weakly increasing and weakly convex function in the other firm's price. Drawn in a single diagram with a on the horizontal and p on the vertical axis, this implies that $p^*(a)$ must be weakly increasing and weakly convex in a, whereas the inverse of $a^*(p)$, $(a^*)^{-1}(a)$, must be strictly increasing and weakly concave in a. Now, if a weakly convex and weakly concave function intersect in two single points (rather than sharing the same value for some non-empty interval), these points must constitute the only intersections of the two functions.

Solving $p^{lim}(a) = (a^{lim})^{-1}(a)$, or $\frac{a}{q} = a + 1 - q$, yields $a^* = q$ and $p^* = 1$ as a possible point of intersection. As $a^* > \tilde{a}$ (which follows from $c < 1$), and $p^* > \tilde{p}$ (which follows from $d < q$), the firms' modified best replies are in fact given by p^{lim} and a^{lim} in this point. Hence, one proper intersection of the firms' modified best replies is found in the point $(p^*, a^*) = (1, q)$. However, note that $p^* > \overline{p}$ and $a^* > \overline{a}$ holds, as is again implied from $c < 1$ and $d < q$. Because such equilibria have been ruled out above, this proper intersection of the firms' best reply functions is meaningless.

Finally, solving $p^{acc}(a) = (a^{acc})^{-1}(a)$, or $\frac{1-q+a+c}{2} = \frac{2a-d}{q}$, yields $a^{**} = \frac{q-q^2+cq+2d}{4-q}$ and $p^{**} = \frac{2-2q+d+2c}{4-q}$. It remains to show that $p^{**} \in [\underline{p}, \tilde{p}]$ and $a^{**} \in [\underline{a}, \tilde{a}]$, such that it is ensured that the firms' modified best replies are appropriately chosen. This is true because $p^{**} \in [\underline{p}, \tilde{p}]$ implies $d \leq \overline{d}$ and $c \leq \underline{c}$, whereas $a^{**} \in [\underline{a}, \tilde{a}]$ implies $c \leq \overline{c}$ and $d \leq \overline{d}$, as is the case in region III.

As the other intersection of the best reply schedules is not valid, this ensures that (p^{**}, a^{**}) forms the unique intersection of the two modified best-reply functions. Moreover, since there are two proper intersections of the firms' modified best replies, it is geometrically clear that the first intersection between the weakly concave function and the weakly convex function must occur from below (that is, $(a^*)^{-1}(a)$ cuts $p^*(a)$ from below at a^{**}). This shows that the set-valued best-reply portions can never overlap, rendering the price pair (p^{**}, a^{**}) the unique equilibrium of region III.

The uniqueness proof for regions IV and V is completely analogous to the proof for regions II and I, respectively, and will not be reported here. It can be obtained from the author upon request.

□

Proof of Proposition 2. The following can be shown:

(A) If A is relatively input inefficient $(r > q)$, U's optimal price is given by

$$c^{opt} = \begin{cases} (I) \quad c^{m,P} = \frac{1}{2} & \text{if} \quad r \geq r_h \\ (II) \quad c^{a} = \frac{q(1-q)}{2r-qr-q} & \text{if} \quad r \in [r_l, r_h] \\ (III) \quad c^{acc} = \frac{(r+2)(1-q)q}{2(2r^2-qr^2-2rq+2q-q^2)} & \text{if} \quad r \leq r_l, \end{cases} \tag{8}$$

where $r_h = \frac{q(9-8q)}{6-5q-\sqrt{8q(1-q)^2}}$ and $r_l = \frac{4q(3-q)}{4+q+\sqrt{16-40q+41q^2-8q^3}}$.

(B) If A is relatively input efficient $(r \leq q)$, U's optimal price is given by

$$c^{opt} = \begin{cases} (III) \quad c^{acc} & \text{if} \quad q \leq q_l \quad \text{or} \quad q \in (q_l, q_h] \quad \text{and} \quad r \geq r_1 \quad \text{or} \\ & \quad q \in [q_h, q_{h2}] \quad \text{and} \quad r \in [r_1, r_2] \cup [r_3, 1] \quad \text{or} \quad q \geq q_{h2} \quad \text{and} \quad r \geq r_3 \\ (IV) \quad c^{lim,A} = \frac{1}{2} & \text{if} \quad q \in [q_h, q_{h2}] \quad \text{and} \quad r \in [r_2, r_3] \quad \text{or} \quad q \geq q_{h2} \quad \text{and} \quad r \in \left[\frac{q^2}{2}, r_3\right] \\ (V) \quad c^{m,A} = \frac{q}{2r} & \text{if} \quad q \in (q_l, q_{h2}] \quad \text{and} \quad r \leq r_1 \quad \text{or} \quad q \geq q_{h2} \quad \text{and} \quad r \leq \frac{q^2}{2}, \end{cases} \tag{9}$$

where $r_1 = \frac{q^3-6q^2+16q-8}{4-q^2+\sqrt{(4-q)^2(4-6q+3q^2-q^3)}}$, $r_2 = \frac{8(1-q)}{2+q+\sqrt{\frac{q^3-12q^2+84q-64}{q}}}$, $r_3 = \frac{8(1-q)}{2+q-\sqrt{\frac{q^3-12q^2+84q-64}{q}}}$, $q_l \approx 0.635$ is given by the real root to $q^3 - 6q^2 + 16q - 8 = 0$, $q_h \approx 0.86$ is given by the real root to $q^3 - 12q^2 + 84q - 64 = 0$ and $q_{h2} \approx 0.8603$ is given by the (positive) root to $\frac{q^2}{2} = r_2$.[23]

In particular, as $q > r$ for a relatively input efficient A, $c^{m,A}$ must be larger than $c^{m,P} = \frac{1}{2}$,

[23] It can be shown that $\frac{q^2}{2}$, r_1 and r_2 all intersect in q_{h2}. Thus, q_{h2} alternatively solves $\frac{q^2}{2} = r_1$ and $r_1 = r_2$.

which is the standard double marginalization monopoly price (the equilibrium price that U would charge if only P was active downstream). This implies that in region V, an excessive supplier price is chosen in equilibrium.

In what follows, cases (A) and (B) will be proven successively.

Case (A): $r > q$.

Because for $r > q$, $\Pi_U(c)$ is a piecewise quadratic function, with $\Pi_U(0) = 0$ and $\Pi_U(c) = 0$ for $c \geq 1$, it has at most five potential maximizers: the three local maxima c^{acc}, $c^{lim,P}$ and $c^{m,P}$, as well as the two regime boundaries c^a and c^b. As a first step, it will be shown that $c^{lim,P}$ and c^b cannot be global maximizers of $\Pi_U(c)$.

For the former, note that $c^{lim,P}$ can only be a (local) maximum of $\Pi_U(c)$ if $c^{lim,P} \geq c^a$ (otherwise, the first part of U's demand schedule has to be used, and $c^{lim,P}$ is no longer a local maximum in that demand portion). Inserting $c^{lim,P}$ and c^a, this implies that $\frac{q}{2r} \geq \frac{q(1-q)}{r(2-q)-q}$ must hold for $c^{lim,P}$ to constitute a local maximum of U's profit function. But this inequality implies $r \geq 1$, which is outside the relevant parameter range. Hence, $c^{lim,P}$ can never be a maximizer of $\Pi_U(c)$.

Next, consider c_b. Clearly, this value can only be a maximizer of $\Pi_U(c)$ if (a) $\lim_{c \uparrow c^b} \frac{\partial \Pi_U(c)}{\partial c} > 0$ and (b) $\lim_{c \downarrow c^b} \frac{\partial \Pi_U(c)}{\partial c} < 0$ hold at the same time. But using U's demand function given in equation (5a), it is easy to see that (a) requires $-\frac{q}{2r-q} > 0$, which can never be the case for $r > q$. This also rules out c^b as maximizer to $\Pi_U(c)$.

Thus, only three possible maximizers to $\Pi_U(c)$ for the case that $r > q$ remain:

$$\begin{aligned}
c^{acc} &= \arg\max_c c\, x_U(c,q,r) = \frac{(r+2)(1-q)q}{2(2r^2 - qr^2 - 2rq + 2q - q^2)}, \\
c^a &= \frac{q(1-q)}{2r - qr - q}, \quad \text{or} \\
c^{m,P} &= \arg\max_c c\left(\frac{1-c}{2}\right) = \frac{1}{2}.
\end{aligned}$$

One can now show that the following implications hold:[24]

$$\begin{aligned}
c^{acc} x_U(c^{acc}, q, r) &\geq c^a \left(\frac{q - rc^a}{q}\right) \Longleftrightarrow \text{always true, since } c^{acc} = \arg\max_c c\ x_U(c, q, r). \\
c^{acc} x_U(c^{acc}, q, r) &\geq c^{m,P} \left(\frac{1 - c^{m,P}}{2}\right) \Longleftrightarrow \Pi_U^{acc} := \frac{(r+2)^2(1-q)q}{4(4-q)(2r^2 - qr^2 - 2rq + 2q - q^2)} \geq \frac{1}{8} \Longleftrightarrow r \in [r_d, r_e] \\
c^a \left(\frac{q - rc^a}{q}\right) &\geq c^{m,P} \left(\frac{1 - c^{m,P}}{2}\right) \Longleftrightarrow \frac{q(1-q)(r-q)}{(2r - rq - q)^2} \geq \frac{1}{8} \Longleftrightarrow r \in [r_g, r_h]
\end{aligned}$$

where $r_d = \frac{q(2+q)}{8-5q+\sqrt{3(4-q)^2(1-q)}}$, $r_e = \frac{q(2+q)}{8-5q-\sqrt{3(4-q)^2(1-q)}}$, $r_g = \frac{q(9-8q)}{6-5q+\sqrt{8q(1-q)^2}}$, $r_h = \frac{q(9-8q)}{6-5q-\sqrt{8q(1-q)^2}}$.

Note that the lower boundary r_d for c^{acc} to yield a higher profit than $c^{m,P}$ is irrelevant, since it is easy to show that $r_d < q \quad \forall q \in (0,1)$, but $r > q$ by assumption in the considered case (A). As c^{acc} always generates a higher profit than c^a for $c^{acc} \leq c^a$, it thus follows that c^{acc} might be optimal if $r \leq r_e$. However, this only follows if it holds that $c^{acc} \leq c^a$. The latter implies $r \in \left[\frac{q}{2-q}, r_l\right]$ or $r \geq r_k$, where $r_l = \frac{4q(3-q)}{4+q+\sqrt{16-40q+41q^2-8q^3}}$ and $r_k = \frac{4q(3-q)}{4+q-\sqrt{16-40q+41q^2-8q^3}}$. As it can be shown that $r_k > 2 \quad \forall q \in (0,1)$, $r \geq r_k$ cannot be satisfied; hence one can focus on the first range. Now $r \geq \frac{q}{2-q}$ is certainly true, since $\frac{q}{2-q} < q$ and $r > q$ by assumption. But because $r_l < r_e$, a more stringent bound for c^{acc} to be optimal is found. Namely, this is the case if $r \leq r_l$.

If $r > r_l$, either c^a or $c^{m,P}$ must be optimal. As derived above, c^a is better if $r \in [r_g, r_h]$. But since it can be shown that r_g is smaller than r_l, the lower boundary is meaningless and c^a is optimal if and only if $r \in [r_l, r_h]$. In the remaining parameter space where $r \geq r_h$, $c^{m,P}$ is optimal. However, note that since $r_h > 1$ for $q \in (\frac{1}{2}, 1)$, this can only be the case for $q \leq \frac{1}{2}$. Combining these results, three optimal pricing regions for U are found:

(I) For $r \geq r_h$ and $q \leq \frac{1}{2}$, $c^{m,P}$ is optimal.

(II) For $r \in [r_l, r_h]$, c^a is optimal.

(III) Finally, for $r \leq r_l$, c^{acc} is optimal.

However, the above is only true if (i) for the region where $c^{m,P}$ is optimal, it holds that $c^{m,P} \geq c^b$, (ii) for the region where c^a is optimal, it holds that $c^a \in [c^a, c^b)$ (which is obviously true), (iii) for the region where c^{acc} is optimal, it holds that $c^{acc} \leq c^a$ (the condition for this has already been derived and used above).

Thus, the first statement remains to be proven. $c^{m,P} \geq c^b$ implies $q \leq \frac{2r}{3}$, which has to be fulfilled whenever $c^{m,P}$ is optimal. Now in the region where this is the case, it holds that $r \geq r_h$ and $q \leq \frac{1}{2}$. Thus, for the lowest possible r (where the condition is hardest to fulfill), $q \leq \frac{2r_h}{3} = \frac{2q(9-8q)}{3(6-5q-\sqrt{8q(1-q)^2})}$ has to be fulfilled. This is equivalent to $72q^2 - 145q + 72 \geq 0$, which

[24] Here, I ignore the possibility that $c^{acc} > c^a$ or $c^{m,P} < c^b$ could be true. This is ruled out later in the proof.

is true for every $q \leq \frac{8}{9}$, in particular $q \leq \frac{1}{2}$. Hence, the proof of case (A) is complete.

Case (B): $r \leq q$.

Since also for $r \leq q$, $\Pi_U(c)$ is a piecewise quadratic function, with $\Pi_U(0) = 0$ and $\Pi_U(c) = 0$ for $c \geq \frac{q}{r}$, it has at most five potential maximizers: the three local maxima c^{acc}, $c^{lim,A}$ and $c^{m,A}$, as well as the two regime boundaries c^* and c^{**}. The first step is to prove that c^* and c^{**} cannot be global maximizers of $\Pi_U(c)$.

Begin with c^*. A necessary condition for this value to be a maximizer of $\Pi_U(c)$ is that (a) $\lim_{c \uparrow c^*} \frac{\partial \Pi_U(c)}{\partial c} > 0$ and (b) $\lim_{c \downarrow c^*} \frac{\partial \Pi_U(c)}{\partial c} < 0$. Using U's demand function given in equation (5b), it is straightforward to show that this implies

$$\begin{aligned} &(a) \qquad f(q,r) := q^2(2-r) + q(3r^2 + 8r - 4) - 8r^2 > 0, \text{ and} \\ &(b) \qquad 3q - r - 2 < 0. \end{aligned}$$

It is easy to see that $f(q,r)$ is strictly convex in q over the relevant parameter range, and that $f(0,r) < 0$. Hence there exists some unique $\tilde{q}$ such that $f(q,r) < 0$ for $q < \tilde{q}$ and $f(q,r) > 0$ for $q > \tilde{q}$. After some calculation, one finds that $f(q,r)$ is equal to $\frac{8}{9}(1-r)^2(r-2) < 0$ for $q = \frac{r+2}{3}$. However, due to inequality (b), it cannot hold that $q \geq \frac{r+2}{3}$. It follows that both inequalities cannot be fulfilled at the same time, which shows that c^* can never be a maximizer of $\Pi_U(c)$.

Next, consider c^{**}. Similar to before, a necessary condition for this value to be a maximizer of $\Pi_U(c)$ is that (a) $\lim_{c \uparrow c^{**}} \frac{\partial \Pi_U(c)}{\partial c} > 0$ and (b) $\lim_{c \downarrow c^{**}} \frac{\partial \Pi_U(c)}{\partial c} < 0$. Using once again U's demand function given in equation (5b), (a) can only be true if $-2(1-q) - r > 0$. Clearly, this is impossible, which proves that c^{**} cannot be a maximizer of $\Pi_U(c)$.

Hence, also for case (B) where $r \leq q$, only three possible maximizers to $\Pi_U(c)$ remain:

$$\begin{aligned} c^{acc} &= \arg\max_c c\, x_U(c,q,r) = \frac{(r+2)(1-q)q}{2(2r^2 - qr^2 - 2rq + 2q - q^2)}, \\ c^{lim,A} &= \arg\max_c c\left(\frac{r(1-c)}{q}\right) = \frac{1}{2}, \quad \text{or} \\ c^{m,A} &= \arg\max_c c\left(\frac{r(q-cr)}{2q}\right) = \frac{q}{2r}. \end{aligned}$$

Again, it is a fairly straightforward task to show that the following implications hold:[25]

$$\begin{aligned}
c^{lim,A}\left(\frac{r(1-c^{lim,A})}{q}\right) &\geq c^{acc}x_U(c^{acc},q,r) \Longleftrightarrow \frac{r}{4q} \geq \Pi_U^{acc} \Longleftrightarrow r \geq \frac{q}{2-q} \quad \text{or} \quad q > q_h \quad \text{and} \quad r \in [r_2, r_3] \\
c^{lim,A}\left(\frac{r(1-c^{lim,A})}{q}\right) &\geq c^{m,A}\left(\frac{r(q-c^{m,A}r)}{2q}\right) \Longleftrightarrow \frac{r}{4q} \geq \frac{q}{8} \Longleftrightarrow r \geq \frac{q^2}{2} \\
c^{m,A}\left(\frac{r(q-c^{m,A}r)}{2q}\right) &\geq c^{acc}x_U(c^{acc},q,r) \Longleftrightarrow \frac{q}{8} \geq \Pi_U^{acc} \Longleftrightarrow q > q_l \quad \text{and} \quad r \leq r_1,
\end{aligned}$$

where $r_1 = \frac{q^3-6q^2+16q-8}{4-q^2+\sqrt{(4-q)^2(4-6q+3q^2-q^3)}}$, $r_2 = \frac{8(1-q)}{2+q+\sqrt{\frac{q^3-12q^2+84q-64}{q}}}$, $r_3 = \frac{8(1-q)}{2+q-\sqrt{\frac{q^3-12q^2+84q-64}{q}}}$, $q_l \approx 0.635$ is given by the (single) real root to $q^3-6q^2+16q-8=0$ and $q_h \approx 0.86$ is given by the (single) real root to $q^3-12q^2+84q-64=0$.

Note that the condition $r \geq \frac{q}{2-q}$ for $c^{lim,A}$ to be optimal can never be fulfilled, as this would imply values of r that are greater than q, which are not considered in case (B). Hence, if $c^{lim,A}$ is optimal, it must be the case that $q > q_h$ and $r \in [r_2, r_3]$.

Using this observation and combining it with the above results, again three optimal pricing regimes are found: (I) For $q > q_h$ and $r \in \left[\max\left\{r_2, \frac{q^2}{2}\right\}, r_3\right]$, $c^{lim,A}$ is optimal.
(II) For $q > q_l$ and $r \leq \min\left\{r_1, \frac{q^2}{2}\right\}$, $c^{m,A}$ is optimal.
(III) For all other combinations of (q,r) where $r \leq q$, c^{acc} is optimal.

Examining the conditions $r_2(q) > \frac{q^2}{2}$ and $r_1(q) < \frac{q^2}{2}$, the former is equivalent to $16(1-q)(2+q)(8-12q+4q^2-q^3) > 0$, whereas the latter is equivalent to $(2+q)\left(2-2q+q^2\right)\left(-8+16q-6q^2+q^3\right)\left(8-12q+4q^2-q^3\right) > 0$. Since $q > q_l$ (which must be true in the considered case) implies $-8+16q-6q^2+q^3 > 0$, both inequalities boil down to $8-12q+4q^2-q^3 > 0$, which has the solution $q < q_{h2} \approx 0.8603$. Hence the above optimal pricing regimes can be rewritten as
(Ia) For $q \in [q_h, q_{h2}]$ and $r \in [r_2, r_3]$, $c^{lim,A}$ is optimal.
(Ib) For $q \geq q_{h2}$ and $r \in \left[\frac{q^2}{2}, r_3\right]$, $c^{lim,A}$ is optimal.
(IIa) For $q \in (q_l, q_{h2}]$ and $r \leq r_1$, $c^{m,A}$ is optimal.
(IIb) For $q \geq q_{h2}$ and $r \leq \frac{q^2}{2}$, $c^{m,A}$ is optimal.
(III) For all other combinations of (q,r) where $r \leq q$, c^{acc} is optimal.
This is identical to the optimal pricing schedule that was claimed above.

However, this is only correct if (i) for the region where $c^{lim,A}$ is optimal, it holds that $c^{lim,A} \in [c^*, c^{**}]$, (ii) for the region where $c^{m,A}$ is optimal, it holds that $c^{m,A} \geq c^{**}$, (iii) for the region

[25] Similar to above, I first ignore the possibility that $c^{acc} > c^*$, $c^{lim,A} \notin [c^*, c^{**}]$, or $c^{m,A} < c^{**}$ could hold. This is ruled out later in the proof.

where c^{acc} is optimal, it holds that $c^{acc} \leq c^{*}$. This is verified in the following.

(i) It is trivial to see that $c^{lim,A} \leq c^{**}$ is always fulfilled, whereas $c^{lim,A} \geq c^{*}$ implies $r \leq 3q-2$, which can only be fulfilled for $q > \frac{2}{3}$. Clearly, where $c^{lim,A}$ is (potentially) optimal, q does in fact exceed $\frac{2}{3}$, since q must be larger than $q_h \approx 0.86$ by the above findings. Moreover, $r \leq 3q-2$ is fulfilled, as optimality of $c^{lim,A}$ implies $r \leq r_3$, and r_3 can easily be shown to be smaller than $3q-2$ for every candidate $q > q_h$.

(ii) $c^{m,A} \geq c^{**}$ implies $r \leq \frac{2q}{4-q}$. For $c^{m,A}$ to be optimal, it must hold that $q > q_l$ and $r \leq \max\left\{r_1, \frac{q^2}{2}\right\}$. Now, for $q > q_l$, this maximum is given by r_1 for $q < q_h$ and by $\frac{q^2}{2}$ for $r \geq q_h$. Thus, for $q < q_h$, $\frac{q^2}{2}$ is a more stringent bound than the other, as it is lower: if one can show that $r \leq \frac{q^2}{2}$ implies $r \leq \frac{2q}{4-q}$ for any $q > q_l$, then any candidate $c^{m,A}$ in region (II) is in fact optimal. This is straightforward to establish.

(iii) $c^{acc} \leq c^{*}$ implies $q \leq q_{hh}$ or $q > q_{hh}$ and $r \notin (r_4, r_5)$, where $r_4 = \frac{8-4q}{8-q+\sqrt{\frac{q^3-40q^2+176q-128}{q}}}$, $r_5 = \frac{8-4q}{8-q-\sqrt{\frac{q^3-40q^2+176q-128}{q}}}$, and $q_{hh} = 2\left(9-\sqrt{73}\right) \approx 0.912$ is the (lowest) root to $q^3 - 40q^2 + 176q - 128 = 0$. Thus, one needs to show that no optimum candidate c^{acc} exists where $q > q_{hh}$ and r lies in the inappropriate range specified above.

As $r \geq r_3$ must hold for c^{acc} to be optimal whenever $q > q_{h2}$ (in particular, when $q > q_{hh}$), and $r_4 < r_5$, it is sufficient to show that $r_3 \geq r_5$ for all $q > q_{hh}$. After a lengthy calculation, one can find that this is indeed the case. This completes the proof of case (B). □

Proof of Proposition 3. The first part of the statement can easily be seen, since the total social welfare in region V is given by $\frac{7}{32}q$, which is strictly less than the total social welfare in the standard double marginalization case of $\frac{7}{32}$, as found in region I (the boundary of region V, $\overline{r}(q)$, has already been determined in the proof of Proposition 2). For the second part, one has to show that the total social welfare is at least as high as $\frac{7}{32}$ for the remaining parameter regions I, II, III and IV. Clearly, this is true for region I, as nothing changes compared to the standard double marginalization case. It remains to be shown that the total social welfare in regions II, III and IV exceeds the total social welfare of $W = \frac{7}{32}$ that emerges in the standard double marginalization case (region I).

Consider an arbitrary triple of prices (c, p, a). If a type θ consumer purchases at P, her surplus is given by $\theta - p$, P's profit for that unit is given by $p - c$, and U's profit for that unit is given by c. In sum, a total surplus of θ is created. In contrast, if a type θ consumer purchases at A, her surplus is given by $\theta q - a$ (as A can only offer a quality of $q < 1$), A's profit for that unit is given by $a - rc$ (as A only needs r units of input for every final consumer that is served), and

U's profit for that unit is given by rc. Hence, a total surplus of θq is created. Overall, it follows that depending on whether only P, both downstream firms, or only A are active in equilibrium, the total social welfare in the market can be written as $\int_{p^*}^1 \theta d\theta$, $\int_{\frac{p^*-a^*}{1-q}}^1 \theta d\theta + \int_{\frac{a^*}{q}}^{\frac{p^*-a^*}{1-q}} \theta q d\theta$, and $\int_{\frac{a^*}{q}}^1 \theta q d\theta$, respectively.

In both regions I and II, only P is active downstream. Hence, in either case the total social welfare in the market is equal to $\int_{p^*}^1 \theta d\theta$, where p^* denotes the respective equilibrium price of P. It is then apparent that the total social welfare in region II strictly exceeds the total social welfare in region I if and only if $p^*_{II} < p^*_I$ for all (q,r) in region II. This condition is equivalent to $\frac{r(1-q)}{2r-qr-q} < \frac{3}{4}$. Noting that $2r - qr - q > 0$ as $r > q$ must hold in region II, the condition can be simplified to $r(2+q) - 3q > 0$, which is strictly increasing in r. It follows that the condition is fulfilled for all (q,r) in region II if and only if it does hold for the lowest possible r in region II, $r = r_l$, for every $q \in (0,1)$. Now $r_l(2+q) - 3q > 0$ is equivalent to $-3 + \frac{4(3-q)(2+q)}{4+q+\sqrt{16-40q+41q^2-8q^3}} > 0$. After some calculation, this simplifies to $f(q) := q^3 + 4q^2 - 29q + 24 > 0$. It is easy to see that $f(q)$ is strictly decreasing in q, reaching its minimum for $q = 1$. As $f(1) = 0$, the condition does in fact hold for every (q,r) in region II. Thus, the social welfare in region II strictly exceeds the total social welfare in the standard double marginalization case (region I).

For region III, it is clearly sufficient to show that $W_{III}(q,r) \geq \frac{7}{32}$ must hold for *all* $q \in (0,1)$, $r \in (0,1)$ (that is, even for those parameter pairs that do not lie in region III). In order to do so, note first that since each consumer has the choice between purchasing at P, purchasing at A, or not purchasing at all, the aggregate consumer surplus in the market is bounded below by $\int_{p^*_{III}}^1 (\theta - p^*_{III}) d\theta = \frac{(1-p^*_{III})^2}{2}$. Hence, it is sufficient to show that the aggregate producer surplus in the market does not fall short of $\frac{p^*_{III}}{4}$, since it is easy to see that $\frac{(1-p)^2}{2} + \frac{p}{4} \geq \frac{7}{32}$ for all $p \in \mathbb{R}$. Moreover, because $\Pi^*_{A,III}$ is clearly non-negative (compare with the expression in Appendix B), it is sufficient to establish that $\Pi^*_{U,III} + \Pi^*_{P,III} \geq \frac{p^*_{III}}{4}$ for all $q \in (0,1)$, $r \in (0,1)$. Algebraically, this is equivalent to

$$\frac{\Omega(q,r)(1-q)}{8(4-q)^2\left(2r^2 - qr^2 - 2rq + 2q - q^2\right)^2} \geq 0$$

for all $q \in (0,1)$, $r \in (0,1)$, where

$$\Omega = 24q^3 - 20q^4 + 4q^5 + \left(-32q^3 + 12q^4\right) r + \left(32q + 64q^2 - 50q^3 + 11q^4\right) r^2 + \left(-96q + 10q^3\right) r^3 + \left(64 - 8q - 20q^2 + 5q^3\right) r^4.$$

Hence, it remains to show that $\Omega(q,r) \geq 0$ for all $q \in (0,1)$, $r \in (0,1)$. Now, a simple bijection between the positive reals $(0,+\infty)$ and the open interval $I := (0,1)$ is given by $f(x) := \frac{1}{1+x}$, where f maps from the positive reals to $(0,1)$. Thus, let $q := \frac{1}{1+y}$ and $z := \frac{1}{1+z}$, which leads to the equivalent condition

$$\Delta(y,z) := \frac{\Lambda(y,z)+\Psi(y,z)}{(1+y)^5(1+z)^4} \geq 0$$

for all $y > 0$, $z > 0$, where

$$\Lambda(y,z) := \left(45z^2+12z^3+8z^4\right)+y\left(81+54z+243z^2+60z^3+28z^4\right)+y^2\left(297+102z+382z^2+64z^3+24z^4\right),$$

$$\Psi(y,z) := 12y^3\left(33+16z^2\right)+8y^4\left(31-4z+4z^2\right)+64y^5.$$

Observe that the only term that could potentially become negative in $\Delta(y,z)$ is given by $8y^4\left(31-4z+4z^2\right)$. But it is trivial to see that $31-4z+4z^2 > 0$ for all $z > 0$, hence $\Delta(y,z) \geq 0$ for all $y > 0$, $z > 0$. This in turn implies that $\Omega(q,r) \geq 0$ for all $q \in (0,1)$ and $r \in (0,1)$, and ultimately $PS_{III} \geq \frac{p^*_{III}}{4}$, leading to $W_{III} \geq \frac{7}{32}$ for all $q \in (0,1)$, $r \in (0,1)$.

Finally, for region IV, one has to show that $W_{IV} := \frac{1}{2} - \frac{1}{8q}$ strictly exceeds $\frac{7}{32}$. This is equivalent to $q > \frac{4}{9}$. But as the lowest possible value of q in region IV is given by $q_h \approx 0.86$, this is clearly fulfilled. □

Proof of Proposition 4. In region V, U's profit is given by $\frac{q}{8} < \frac{1}{8}$, which proves the second part of the proposition. For the rest, it still needs to be shown that U's profit is lower (higher) than in the standard double marginalization case whenever $q < q_{h3} := \frac{4}{3}\left(-2+\sqrt{7}\right) \approx 0.861$ and $r < r_e < \frac{q}{2}$ $(r > r_e)$, or $q \geq q_{h3}$ and $r < \frac{q}{2}$ $(r > \frac{q}{2})$.

To see this, note first that solving $\frac{1}{8} < \Pi_{U,III}$ leads to $r \in (r_e, r_f)$, where $r_e = \frac{q(2+q)}{8-5q+\sqrt{3(4-q)^2(1-q)}}$ and $r_f = \frac{q(2+q)}{8-5q-\sqrt{3(4-q)^2(1-q)}}$. It is now straightforward to show that $r_f > r_l$ for every $q \in (0,1)$, implying that the upper boundary r_f is not relevant, as the supplier does not allow for downstream competition for $r > r_l$. But taking the lower boundary r_e into account, one can observe that region III is split into two subregions: for $r \in (r_e, r_l]$, the supplier's profit is higher under downstream competition, compared to the standard double marginalization case; for $r < r_e$, the supplier's profit is lower.

It is now necessary to consider the region boundaries between regions III, IV and V (see Figure 6 below for a graphical illustration). First, it is straightforward to show that $r_1 < r_e$ must hold for every $q \in (0,1)$. Thus, the boundary between region III and region V is never relevant when determining whether the supplier's profit increases in region III, compared to the standard double marginalization case. Second, note that r_2 only constitutes a boundary between regions III and IV if $q \in (q_h, q_{h2})$ (where both r_2 and r_3 form the boundary between regions III and IV). But as it can be shown that r_e is a strictly increasing function of q for every $q \in (0,1)$, and r_2 is a strictly decreasing function of q for every $q \in (q_h, 1)$, r_2 is an irrelevant boundary if $r_e(q_h) > r_2(q_h)$. This can easily be confirmed. The boundary r_3 between regions III and IV remains to be checked. Namely, for $q > q_h$, only values of r that are not smaller than $r_3(q)$ are part of region III. Now, $r_e(q) \geq r_3(q)$ easily implies $q \leq q_{h3} = \frac{4}{3}\left(-2 + \sqrt{7}\right) \approx 0.861$. Moreover, it is not difficult to check that $r_e(q) < \frac{q}{2}$ for every $q \in (0, q_{h3})$, as claimed in the proposition.

Next, solving $\frac{1}{8} < \Pi_{U,IV}$, it trivially follows that the supplier's profit is higher (lower) in region IV, compared to the standard double marginalization case, whenever $r > \frac{q}{2}$ ($r < \frac{q}{2}$). Again, one has to consider the region boundaries of region IV (see Figure 6). First, as $\frac{q}{2} > \frac{q^2}{2}$, the boundary between region IV and region V is never relevant when determining whether the supplier's profit increases in region IV, compared to the standard double marginalization case. Second, r_2 only constitutes a boundary between regions IV and III if $q \in (q_h, q_{h2})$. As $\frac{q}{2}$ is strictly increasing in q, while r_2 is strictly decreasing in q for every $q \in (q_h, 1)$, r_2 is an irrelevant boundary if $\frac{q_h}{2} > r_2(q_h)$. This is in fact fulfilled. As above, the boundary r_3 remains to be examined. For $q > q_h$, only values of r that are not larger than $r_3(q)$ are part of region IV. But solving $\frac{q}{2} \leq r_3(q)$ leads to $q \geq q_{h3}$, the same value as was found when solving $r_e(q) \geq r_3(q)$.

Overall, it is thus established that for $q < q_{h3}$, $r_e(q)$ marks U's indifference curve between the standard double marginalization case and existence of A, whereas for $q > q_{h3}$, $\frac{q}{2}$ marks this boundary. At $q = q_{h3}$, both indifference curves touch each other at the region boundary between regions III and IV.

The final step is to prove that U's profit is (weakly) higher in region II, compared to the standard double marginalization case. It has already been shown in the proof of Proposition 2 (case A) that this condition is equivalent to $r \in [r_g, r_h]$. As $r_g < r_l$, it follows that region II (where $r \in [r_l, r_h]$) is a subset of the region where the profit inequality holds. This completes the proof.

□

Proof of Remark (page 19). An analytical proof of the optimality of $(q_{h2}, \frac{(q_{h2})^2}{2})$ in terms of social welfare is difficult because of the complexity of the total social welfare function in region III. However, this result can easily be confirmed using numerical methods. W_{max} is then calculated by inserting $q = q_{h2}$ and $r = \frac{(q_{h2})^2}{2}$ into the social welfare function relevant for region III. □

Proof of Corollary 2. If the downstream market is characterized by Bertrand competition between identical producers (leading to downstream equilibrium prices that are equal to marginal cost), it is easy to prove that total social welfare is given by $W = \frac{3}{8} = 0.375$. But at the point

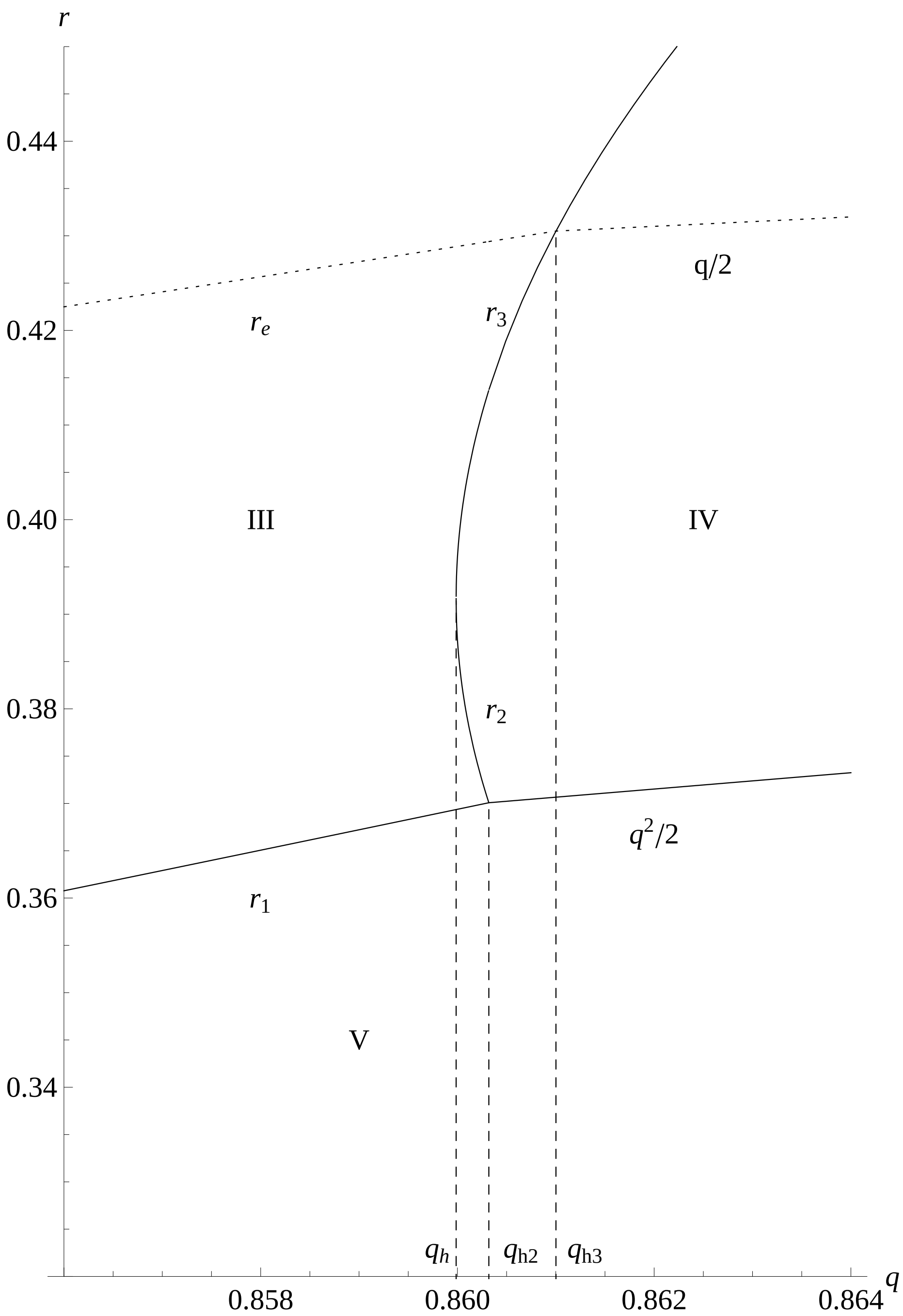

Figure 6: Zoom of the supplier's optimal pricing regions. The dotted line represents the firm's profit indifference curve, where U has a higher (lower) profit than in the standard double marginalization case whenever r is above (below) the dotted line.

in parameter space where social welfare is maximized in the case of a duopolistic downstream market as analyzed above, social welfare is given by $W_{max} \approx 0.4284 > 0.375$. By continuity of W_{III}, there must exist a non-degenerate parameter region around the socially optimal parameter combination where social welfare strictly exceeds 0.375. $\square$

Proof of Proposition 5. First, notice that if U is able to price discriminate, it is free to choose any point in the (c, d) space of the downstream price competition game (compare with Figure 1). This is because the supplier can ignore the comparatively lower revenues it gets from the more input efficient firm by setting its input price appropriately. Hence, it faces the problem $\max_{c_1,c_2} \Pi_U(c_1, c_2; q, r) = c_1 D_P^*(c_1, rc_2) + c_2 r D_A^*(c_1, rc_2)$, which is equivalent to $\max_{c,d} cD_P^*(c, d) + dD_A^*(c, d)$.

Next, consider the demand functions given by Corollary 1 for the case of competition in the downstream market and, for the moment, ignore the constraints under which downstream competition can take place (take the demand functions for region III irrespective of c and d). Then, it is straightforward to show that the unique global maximizing pair (c, d) of

$$\Pi_{3rd}^U := c\left(\frac{2 + d - 2c + cq - 2q}{(4 - q)(1 - q)}\right) + d\left(\frac{d(q - 2) + (1 + c - q)q}{(4 - q)(1 - q)q}\right)$$

is given by $c_{1,3rd}^* = \frac{1}{2}$ and $d_{2,3rd}^* = rc_{2,3rd}^* = \frac{q}{2}$, with an associated profit of $\Pi_{3rd}^{U*} = \frac{q+2}{4(4-q)}$. Since $d_{2,3rd}^* = qc_{1,3rd}^*$, this optimal pair of prices always lies in region III of the downstream price game (compare with Figure 1).

What remains to be shown is that it cannot be optimal for U to choose its prices in such a way that only one firm is active downstream (regions I, II, IV and V). To see this, consider regions I and V first. In these regions, U's maximal profit is given by $\frac{1}{8}$ and $\frac{q}{8} < \frac{1}{8}$, respectively (compare with the table in Section 4). But as it is easy to show that $\frac{1}{8} < \Pi_{3rd}^{U*}$ for $q \in (0, 1)$, a higher profit could be achieved.

For the limit pricing case, notice that the *active* downstream firm's demand in regions II and IV is independent of its own cost and *decreasing* with the other firm's cost (compare with Corollary 1). Because U can freely choose the downstream cost levels, it wants to set the inactive firm's cost as low as possible, given that limit pricing still occurs in equilibrium. This maximizes the limit pricing demand that arises from the active firm. Thus, one may insert $d = \overline{d}$ ($c = \overline{c}$) for the inactive firm's cost in regions II (IV), which leads to a final consumers' demand of $\frac{1-c}{2-q}$ (region II) and $\frac{q-d}{(2-q)q}$ (region IV), respectively. Hence, for a given active firm's cost of c (d), U's maximal limit pricing profit can be written as $c\left(\frac{1-c}{2-q}\right)$ ($d\left(\frac{q-d}{(2-q)q}\right)$). Maximizing these expressions with respect to c and d yields a maximal limit pricing profit of U that is equal to $\frac{1}{4(2-q)}$ and $\frac{q}{4(2-q)}$, respectively. It is then straightforward to show that $\frac{1}{4(2-q)} < \Pi_{3rd}^{U*}$ for $q \in (0,1)$, implying that U will never want to induce limit pricing downstream. □

Proof of Corollary 3. Insert $c = \frac{1}{2}$ and $d = \frac{q}{2}$ into the downstream equilibrium expressions given in the first part of Appendix B (accounting for the supplier's profit when calculating total social welfare). □

Proof of Proposition 6. The following properties need to be shown: (a) Under third degree price discrimination by U, the total social welfare in the market is (i) always higher than in regions I and V, but (ii) always lower than in regions II and IV. (b) If A is slightly more input-efficient than P, third degree price discrimination by U yields a higher total social welfare than in the case were such discrimination is not feasible.

Start with (a). It is easy to show that $W_{3rd} = \frac{28+3q-4q^2}{8(4-q)^2}$ is strictly increasing in q for $q \in (0,1)$. But for $q=0$, $W_{3rd} = \frac{7}{32}$, which is already as high as total social welfare in region I and maximum total social welfare in region V. This proves part (i) of (a). For part (ii), it needs to be shown that $W_{3rd} < W_{IV}$ and $W_{3rd} < W_{II}$ for all parameter-pairs (q,r) that lie in the respective regions.

To see that $W_{3rd} < W_{IV}$, note that this condition is equivalent to $2q^3 - 9q^2 + 11q - 4 > 0$, or $q \in \left(\frac{7-\sqrt{17}}{4}, 1\right)$, which must always be fulfilled in region IV, as $\frac{7-\sqrt{17}}{4} \approx 0.72 < q_h \approx 0.86$.

In order to prove that $W_{3rd} < W_{II}$, first observe that the total social welfare in region II, $W_{II} = \frac{(r-q)(3r-2rq-q)}{2(2r-rq-q)^2}$, is a strictly *increasing* function in r.[26] Thus, if it can be shown that even at the boundary of region II to region III, where $r = r_l$ is lowest in that region (compare with Figure 2), it holds that $W_{3rd} < W_{II}(r_l)$ for every $q \in (0,1)$, the claim is verified. Inserting r_l into W_{II}, it follows that $W_{3rd} < W_{II}(r_l)$ is equivalent to $\frac{28+3q-4q^2}{8(4-q)^2} < \frac{\left(-8+5q+\sqrt{16-40q+41q^2-8q^3}\right)\left(-32+37q-8q^2+\sqrt{16-40q+41q^2-8q^3}\right)}{2\left(-20+21q-4q^2+\sqrt{16-40q+41q^2-8q^3}\right)^2}$. The LHS and RHS of this inequality coincide for $q=0$ or, in the limit, as q tends to 1. Finally, for $q \in (0,1)$, it can be shown numerically that the inequality does in fact hold.[27] This finishes the proof of part (a).

For part (b), one can check that W_{3rd} coincides with W_{III} for $r = q$, that is, if the downstream firms are equally input efficient. Now, W_{3rd} is independent of r. On the other hand,

[26] This is true because $\frac{\partial W_{II}}{\partial r} = \frac{(1-q)^2 qr}{(2r-qr-q)^3} > 0$, where the inequality follows from $r > q$, as must be the case in region II.

[27] A lengthy analytic proof can be obtained from the author upon request.

differentiating W_{III} with respect to r and evaluating this expression at $q = r$ yields

$$\left.\frac{\partial W_{III}}{\partial r}\right|_{q=r} = \frac{1}{4(8+2r-r^2)} > 0.$$

Hence, starting from identical relative cost efficiency (where downstream duopoly competition must always be the equilibrium outcome), marginally decreasing A's input requirement r reduces total social welfare, whereas it stays constant if the supplier can price discriminate. It follows that there must exist a (small) parameter region slightly below $r = q$ where total social welfare is higher under third degree price discrimination.[28] This confirms claim (b), completing the proof. $\square$

Proof of Proposition 7. The first part of the proposition has been proven numerically above. Consider the second part. If only P is active downstream, the equilibrium of the downstream game is characterized by $p^*(c) = \frac{1+c}{2}$, implying an upstream demand of $D_U^* = \frac{1-c}{2}$. Inverting this demand function, it follows that the market clearing price, given an aggregate output S in the upstream market, can be written as $\tilde{c} = 1-2S$. It is then trivial to see that the resulting Cournot-equilibrium in the upstream market is characterized by $S_i^* = \frac{1}{6}$, with individual upstream profits of $\Pi_i^* = \frac{1}{18}$, a market clearing upstream price of $\tilde{c}^* = \frac{1}{3}$, an equilibrium downstream price of $p^* = \frac{2}{3}$, and a downstream profit of $\Pi_P^* = (p^* - \tilde{c}^*)(1-p^*) = \frac{1}{9}$. Summing up the firms' profits and the aggregate consumer surplus $CS^* := \int_{p^*}^{1}(\theta - p^*)d\theta = \frac{1}{18}$, one arrives at a total social welfare of $W^* = \frac{5}{18}$.

On the other hand, take equilibria where an excessive upstream price is found. In such equilibria, A will behave like a downstream monopolist, implying $a^* = \frac{q+d}{2} = \frac{q+rc}{2}$, and an upstream demand of $D_U^* = \frac{r(q-rc)}{2q}$. Inverting this demand function, it follows that the market clearing price, given an aggregate output S in the upstream market, is equal to $\tilde{c} = \frac{rq-2qS}{r^2}$. Again, it is easy to compute that the resulting Cournot-equilibrium in the upstream market is characterized by $S_i^* = \frac{r}{6}$, with individual upstream profits of $\Pi_i^* = \frac{q}{18}$, a market clearing upstream price of $\tilde{c}^* = \frac{q}{3r}$, an equilibrium downstream price of $a^* = \frac{2q}{3}$, and a downstream profit of $\Pi_A^* = (a^* - r\tilde{c}^*)(1 - \frac{a^*}{q}) = \frac{q}{9}$. Summing up the firms' profits and the aggregate consumer surplus $CS^* := \int_{\frac{a^*}{q}}^{1}(\theta q - a^*)d\theta = \frac{q}{18}$, the resulting total social welfare is given by $W^* = \frac{5q}{18} < \frac{5}{18}$, which proves the statement. $\square$

[28] Solving $W_{III} = W_{3rd}$, a complicated explicit solution for the parameter range where $W_{3rd} < W_{III}$ can be given. This is depicted in Figure 2: in region III, W_{3rd} exceeds W_{III} if and only if $r \in (r_{3rd}, q)$, where r_{3rd} is strictly smaller than q. The formula for r_{3rd} can be obtained from the author upon request.

Appendix B: Tables

(A) Downstream Subgame:

Let $q \in (0,1)$, $c < 1$, and $d < q$. Then, the equilibrium values of P's and A's profit, consumer surplus, and total social welfare, are given by the following.

Region	Condition	$\Pi_P^*(c,d,q)$	$\Pi_A^*(c,d,q)$	$CS(c,d,q)$	$W(c,d,q)$
I	$c \leq \bar{c} \wedge d > \hat{d}$	$\frac{(1-c)^2}{4}$	0	$\frac{(1-c)^2}{8}$	$\frac{3}{8}(1-c)^2$
II	$c \leq \bar{c} \wedge d \in (\bar{d}, \hat{d}]$	$\frac{(q-d)(d-cq)}{q^2}$	0	$\frac{(q-d)^2}{2q^2}$	$\frac{(q-d)(q+d-2cq)}{2q^2}$
III	$c \leq \bar{c} \wedge d \leq \bar{d}$	$\frac{(2+d-2c+cq-2q)^2}{(4-q)^2(1-q)}$	$\frac{(d(q-2)+(1+c-q)q)^2}{(4-q)^2(1-q)q}$	CS^{acc}	$\Pi_{P,III}^* + \Pi_{A,III}^* + CS^{acc}$
IV	$c \in (\bar{c}, \hat{c}] \wedge d \leq \bar{d}$	0	$\frac{(c+q-1-d)(1-c)}{q}$	$\frac{(1-c)^2}{2q}$	$\frac{(1-c)(1+c+2q-2-2d)}{2q}$
V	$c > \hat{c} \wedge d \leq \bar{d}$	0	$\frac{(q-d)^2}{4q}$	$\frac{(q-d)^2}{8q}$	$\frac{3}{8}(q-d)^2$

where

$$\bar{c} = \frac{2-q-(q-d)}{2-q}, \quad \hat{c} = \frac{2-q+d}{2}, \quad \bar{d} = \frac{q(1-q+c)}{2-q}, \quad \hat{d} = \frac{q(1+c)}{2},$$

$$CS^{acc} := \frac{d^2(4-3q) - 2dq[4+(c-4)q] + q[4+c^2(4-3q)+q-5q^2+2c(q^2+3q-4)]}{2(4-q)^2(1-q)q}.$$

To calculate the above reduced form profit expressions, one may simply multiply the equilibrium prices as found in Proposition 1, net of the firms' respective unit costs, with the reduced form demand expressions as found in Corollary 1. Moreover, using the firms' demand functions specified in equations (3a) and (3b), it holds that for arbitrary prices p, a (such that $p \geq 1$ and $a \geq q$ do not hold simultaneously), consumer welfare is given by

$$CS = \begin{cases} \int_p^1 (\theta - p) f(\theta) d\theta = \frac{(1-p)^2}{2} & \text{if} \quad p \leq \frac{a}{q} \\ \int_{\frac{a}{q}}^{\frac{p-a}{1-q}} (\theta q - a) f(\theta) d\theta + \int_{\frac{p-a}{1-q}}^1 (\theta - p) f(\theta) d\theta = \frac{a^2 - 2apq + [1+p^2-2p(1-q)-q]q}{2(1-q)q} & \text{if} \quad p \in (\frac{a}{q}, a+1-q) \\ \int_{\frac{a}{q}}^1 (\theta q - a) f(\theta) d\theta = \frac{(q-a)^2}{2q} & \text{if} \quad p \geq a+1-q. \end{cases}$$

By inserting the corresponding equilibrium prices into the above equation, one can directly calculate the equilibrium consumer surplus of the downstream subgame, as found in the table. Finally, total social welfare is defined as the sum of P's and A's profit and consumer surplus, which is straightforward to calculate.

(B) Full Game:

Region	Condition
I	$q \leq \frac{1}{2}$ and $r \geq r_h$
II	$r \in [r_l, r_h]$
III	$q \leq q_l \approx 0.635$ or $q \in (q_l, q_h] \approx (0.635, 0.86)$ and $r \geq r_1$ or $q \in [q_h, q_{h2}] \approx [0.86, 0.8603]$ and $r \in [r_1, r_2] \cup [r_3, 1]$ or $q \geq q_{h2}$ and $r \geq r_3$
IV	$q \in [q_h, q_{h2}] \approx [0.86, 0.8603]$ and $r \in [r_2, r_3]$ or $q \geq q_{h2}$ and $r \in \left[\frac{q^2}{2}, r_3\right]$
V	$q \in (q_l, q_{h2}] \approx (0.635, 0.8603]$ and $r \leq r_1$ or $q \geq q_{h2}$ and $r \leq \frac{q^2}{2}$

where $r_h = \frac{q(9-8q)}{6-5q-\sqrt{8q(1-q)^2}}$, $r_l = \frac{4q(3-q)}{4+q+\sqrt{16-40q+41q^2-8q^3}}$,
$r_1 = \frac{q^3-6q^2+16q-8}{4-q^2+\sqrt{(4-q)^2(4-6q+3q^2-q^3)}}$, $r_2 = \frac{8(1-q)}{2+q+\sqrt{\frac{q^3-12q^2+84q-64}{q}}}$, $r_3 = \frac{8(1-q)}{2+q-\sqrt{\frac{q^3-12q^2+84q-64}{q}}}$.

Region	c^*	p^*	a^*
I	$\frac{1}{2}$	$\frac{3}{4}$	$\{a \mid a \geq \frac{r}{2}\}$
II	$\frac{q(1-q)}{2r-qr-q}$	$\frac{r(1-q)}{2r-qr-q}$	$\frac{qr(1-q)}{2r-qr-q}$
III	$\frac{(1+2r)(1-q)q}{2(2r^2-qr^2-2rq+2q-q^2)}$	$\frac{(1-q)[8r^2+(12-4r-3r^2)q-4q^2]}{2(4-q)(2r^2-qr^2-2rq+2q-q^2)}$	$\frac{(1-q)q[2r^2(3-q)+r(4-3q)+2(3-q)q]}{2(4-q)(2r^2-qr^2-2rq+2q-q^2)}$
IV	$\frac{1}{2}$	$\frac{1}{2}$	$q-\frac{1}{2}$
V	$\frac{q}{2r}$	$\{p \mid p \geq \frac{q}{2r}\}$	$\frac{3}{4}q$

Region	D_U^*	D_P^*	D_A^*
I	$\frac{1}{4}$	$\frac{1}{4}$	0
II	$\frac{r-q}{2r-qr-q}$	$\frac{r-q}{2r-qr-q}$	0
III	$\frac{r+2}{2(4-q)}$	$\frac{8r^2+(4-8r-3r^2)q-(2-r)q^2}{2(4-q)(2r^2-qr^2-2rq+2q-q^2)}$	$\frac{2r^2-qr^2-r(4+q)+2(3-q)q}{2(4-q)(2r^2-qr^2-2rq+2q-q^2)}$
IV	$\frac{r}{2q}$	0	$\frac{1}{2q}$
V	$\frac{r}{4}$	0	$\frac{1}{4}$

Region	Π_U^*	Π_P^*	Π_A^*
I	$\frac{1}{8}$	$\frac{1}{16}$	0
II	$\frac{q(1-q)(r-q)}{(2r-qr-q)^2}$	$\frac{(1-q)(r-q)^2}{(2r-qr-q)^2}$	0
III	$\frac{(r+2)^2(1-q)q}{4(4-q)(2r^2-qr^2-2rq+2q-q^2)}$	$\frac{(1-q)[8r^2+(4-8r-3r^2)q-(2-r)q^2]^2}{4(4-q)^2(2r^2-qr^2-2rq+2q-q^2)^2}$	$\frac{q(1-q)[2r^2-qr^2-r(4+q)+2(3-q)q]^2}{4(4-q)^2(2r^2-qr^2-2rq+2q-q^2)^2}$
IV	$\frac{r}{4q}$	0	$\frac{2q-r-1}{4q}$
V	$\frac{q}{8}$	0	$\frac{q}{16}$

Region	PS	CS	W
I	$\frac{3}{16}$	$\frac{1}{32}$	$\frac{7}{32}$
II	$\frac{r(1-q)(r-q)}{(2r-qr-q)^2}$	$\frac{(r-q)^2}{2(2r-qr-q)^2}$	$\frac{(r-q)(3r-q-2qr)}{2(2r-qr-q)^2}$
III	$\Pi_{U,III}^* + \Pi_{P,III}^* + \Pi_{A,III}^*$	CS_{III}	$\Pi_{U,III}^* + \Pi_{P,III}^* + \Pi_{A,III}^* + CS_{III}$
IV	$\frac{1}{2} - \frac{1}{4q}$	$\frac{1}{8q}$	$\frac{1}{2} - \frac{1}{8q}$
V	$\frac{3}{16}q$	$\frac{q}{32}$	$\frac{7}{32}q$

where

$$CS_{III} := \frac{U + Vq + Wq^2 + Xq^3 + Yq^4 + Zq^5}{8(4-q)^2(2r^2 - qr^2 - 2rq + 2q - q^2)^2},$$

with

$U = 64r^4, \quad V = 80r^2 - 208r^3 - 12r^4, \quad W = 16 - 144r + 216r^2 + 44r^3 - 23r^4,$

$X = 68 - 44r - 99r^2 + 22r^3 + 7r^4, \quad Y = -60 + 48r + 19r^2 - 2r^3, \quad Z = 12 - 4r.$