WORKING PAPERS

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March 2011

Working Paper No: 1106

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

Ownership structure and control in incomplete market economies with transferable utility. *

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March 1, 2011

Abstract

We consider an economy with incomplete markets and a single firm and assume that utility can be freely transferred in the form of the initially available good 0 (quasilinearity). In this particularly simple and transparent framework, the objective of a firm can be defined as the maximization of the total utility of its control group $\mathcal{C}$ measured in units of good 0. We analyze how the size and the composition of $\mathcal{C}$ influences the firm's market behavior and state conditions under which the firm sells its output at prices which are at, above, or below marginal costs, respectively. We discuss the assumption of competitive price perceptions and point out important differences between the concepts of a Drèze and of a Grossman-Hart equilibrium that occur in spite of the close similarity of the formulas which define them.

*This paper grew out of an extensive cooperation with Birgit Grodal, who passed away on May 4, 2004. She was Professor at the Økonomisk Institut, Københavns Universitet, Denmark. We would like to thank the participants of the micro jour fixe at the Institute for Advanced Studies in Vienna and the participants of the Workshop on General Equilibrium Theory in Krakow, June 2010. We are particularly grateful to Jacques Drèze and to Klaus Ritzberger for their valuable comments.

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1 Introduction

The theory of incomplete markets deals with intertemporal economies with uncertainty about future states of the world. Tradable assets can be used to transfer wealth across time and states. However, the trading possibilities are restricted because the asset span has less than full dimension. Once production is allowed this fact entails conceptual problems which we examine in a particularly simple setting.

We assume throughout the paper that utility can be transferred in the form of the initially available good 0. As a consequence, the objective of a firm can be defined as the maximization of the total utility sum (or welfare) of its control group measured in units of good 0. We focus on the case of transferable utility because of its transparency. Moreover, a general theory of incomplete markets with production should not fail to handle this particular case appropriately.

We deviate from the Arrow-Debreu tradition and allow a firm to take the influence of its production decision on its stock price into account. It may happen, though, that a firm is controlled by a group of consumers who aim to sell the firm's stock at marginal costs although the firm is a monopolist. In this case, the firm behaves in a similar way as a price taking firm in the Arrow-Debreu setting but not because it faces competition.

To avoid misinterpretations we do not refer to competitive behavior in this case but to price taking behavior. Price taking behavior is defined as profit maximization with respect to a given price system and entails that the output is sold at marginal costs. A price taking firm behaves as if it has adopted the marginal cost pricing rule for some reason.

There are two time periods, $t = 0$ and $t = 1$, and one good per state which bears the state's name. At $t = 0$, the economy is in state 0 and one of the states $s = 1, \dots S$ will obtain at $t = 1$. Consumer i is endowed with $\delta^i_j \geq 0$ original shares of firm j.[1] A stock market operates at $t = 0$ where original shares are exchanged against good 0. Consumer i's final shares of firm j are denoted by $\vartheta^i_j \geq 0$. The price of all shares of firm j is denoted q_j. The demand for shares depends on the production plans y_j and the stock prices q_j. In a *stock market equilibrium* the price system is such that the total demand $\sum_i \vartheta^i_j$ equals the supply, which is normalized to 1, for every j.

We adopt the framework of a corporation in the sense of Magill and Quinzii (1996), §32, and assume that the group $\mathcal{O}_j$ of owners of firm j's original shares receives firm j's net value. That is, consumer i receives $\delta^i_j q_j$ and pays the share δ^i_j of j's production costs. The final owners of shares obtain the state dependent dividends. Full insurance is impossible in incomplete markets and the success of an investment can be associated with substantial risk.[2]

There are several reasons why consumers do, in general, not agree on the objective of a firm. First, a net seller of shares gains from a high share price whereas a net buyer loses. Second, the original owners, who pay the production

[1] We will assume later that there is a single firm. Because the literature we refer to in the introduction typically deals with several firms we use the index j here.

[2] An instructive example is Daimler-Chrysler. Looking backwards, this firm has burned tens of billions of Euros/US dollars in its short history.

costs, tend to prefer lower production levels than the final owners, who receive the output without paying the cost. Third, shares are assets whose benefits depend on the production plans. Because production decisions influence the asset span and different consumers face different risks they do typically not agree on which production plans firms should carry out.

The importance of the three effects differs across consumers according to their original endowments of shares and goods as well as their preferences. The assumption of transferable utility allows us to capture all the aspects in a particularly simple and transparent way.

Assume that firm j is controlled by some group $\mathcal{C}_j$ of consumers. The members of $\mathcal{C}_j$ typically differ with respect to their individual characteristics and their original and final shareholdings. The ownership structure matters because different control groups tend to pursue different goals. Consider, for instance, the extreme case in which the group $\mathcal{O}_j$ of original owners of firm j never holds final shares so that $\mathcal{O}_j$ is disjoint from the group $\mathcal{F}_j$ of j's final shareholders for all production plans y_j. If the firm is controlled by $\mathcal{O}_j$ then the firm's goal is to maximize the net market value $q_j - c_j$ where c_j denotes firm j's cost. In this case, shares will typically be traded at prices that exceed marginal costs.

On the other hand, if the firm is controlled by its final rather than its original owners the production level will be high and it can very well be optimal for $\mathcal{C}_j = \mathcal{F}_j$ to let the share price q_j fall below marginal costs [cf. Section 4]. Deviations from marginal cost pricing are rarely considered in the literature on incomplete market economies with production and one would like to know when and why this is justified. Note that we do not assume that firms act as price takers.

To take another extreme case, suppose that all original and all final shareholders belong to $\mathcal{C}_j$ for any production plan. Then the stock market price q_j ceases to play any role in the firm's objective because a redistribution of good 0 among the members of $\mathcal{C}_j$ leaves $\mathcal{C}_j$'s aggregate utility unaffected. Since q_j becomes irrelevant in the case under consideration, the firm's task is to find a balance between today's cost c_j and the future benefits of its members. Such a balance is found in the case of a Drèze equilibrium.

The concept of a *Drèze equilibrium* can be based on infinitesimal transfers of good 0 and the following first order condition: The production plan of each firm j is such that the group $\mathcal{F}_j$ of j's final shareholders cannot change it infinitesimally and make infinitesimal transfers of good 0 among its members such that every $i \in \mathcal{F}_j$ makes a first order utility gain. Infinitesimal share adjustments need not be taken into account because of the envelope theorem. Observe, though, that the existence of infinitesimal utility gains is equivalent to the existence of infinitesimal utility losses at an interior stock market equilibrium.

Drèze equilibria can also be defined in other ways. Drèze (1974) aims at constrained efficiency of the whole economy. Consider a planner who cannot split assets to alleviate the market incompleteness but who can choose production plans, allocate shareholdings, and distribute the total endowments of good 0. An allocation is *constrained efficient* if this planner cannot make every consumer better off. Drèze equilibria can be characterized by the first order condition for constrained efficiency. If a stock market equilibrium is constrained efficient it must be a Drèze equilibrium.

Drèze (1974) shows by means of several examples that a lack of coordination among several firms can entail that some Drèze equilibria are constrained inefficient. Dierker et al. (2002) provide an example with a single firm, a single good per state (income), and a unique Drèze equilibrium that is constrained inefficient. Thus, there are economies in which the stock market cannot achieve a constraint efficient allocation.

E. and H. Dierker (2010a) focus on an example with von Neumann-Morgenstern utilities and the same properties. Due to the cardinal nature of the utility functions, this setting lends itself naturally to utilitarian welfare maximization, which requires cardinal unit comparability. Shareholders' welfare is defined as the sum of their utilities where every utility function is normalized such that the marginal utility of good 0 equals 1 at the Drèze equilibrium. It turns out that the unique Drèze equilibrium maximizes social welfare although it is constrained inefficient: The more the firm departs from the welfare maximizing Drèze equilibrium the more shareholders' utilities increase after transfers and share adjustments. The quasilinear case has the advantage that the goals of welfare maximization and of constrained efficiency are well aligned. E. and H. Dierker (2010b) examine the conflict between welfare maximization and minimal efficiency, which is a substantially weaker requirement than constrained efficiency.

In §31 of their book, Magill and Quinzii (1996) argue that Drèze equilibria should be considered within the framework of partnership economies, which differs from the present setting in the following way. A partnership economy has constant returns to scale, there are no original shares, and production costs are borne by the final shareholders in proportion to their shares.

Although we adopt the framework of a corporation and not that of a partnership economy, we can make the following assumption within our framework to reconcile the different settings. Assume that returns to scale are constant and that all firms are controlled by the grand coalition $\mathcal{G}$ of all consumers. Then every firm j acts as a price taker which takes the following firm specific price system as given:

$$\pi_j = \sum_{i \in \mathcal{F}_j} \vartheta_j^i DU^i(x^i) = \sum_{i \in \mathcal{G}} \vartheta_j^i DU^i(x^i), \tag{1}$$

where x^i is i's equilibrium consumption, and $DU^i(x^i)$ is i's utility gradient normalized such that the partial derivative with respect to today's consumption equals 1. Price taking behavior with respect to (1) provides an alternative characterization of a Drèze equilibrium [cf. Magill and Quinzii (1996), 31.5 Proposition].[3]

If a price taking firm has constant returns to scale then its profits are bound to vanish. Equation (1) illustrates the remaining conflict concerning its dividend. Because markets are incomplete there are no budget hyperplanes that make the individual utility gradients $DU^i(x^i)$ point into the same direction. If every final shareholder seeks to maximize profits with respect to some price system, then shareholder i would like the firm to maximize profits with respect to his state price vector $DU^i(x^i)$. The price system in (1) represents a compromise between different final shareholders.

[3] This characterization holds for general utility functions.

Drèze (1974) does not assume price taking behavior but derives it from economic principles. A firm that is controlled by a group containing at least all its customers has no reason to introduce a distortion that would harm them. This point becomes particularly obvious if the group equals the grand coalition $\mathcal{G}$ and the firm cannot hope to raise its welfare by charging a price above marginal costs. The argument tends to break down as soon as some customers are not included in the control group.

The second equality in equation (1) is a pure tautology because $\vartheta_j^i = 0$ for every $i \in \mathcal{G} \setminus \mathcal{F}_j$. However, there are reasons to focus on $\mathcal{G}$ rather than on $\mathcal{F}_j$. If firm j is controlled by $\mathcal{G}$ then nobody's interest is disregarded whatever the production plan y_j under consideration might be. The group $\mathcal{F}_j$, however, disregards potential shareholders, i.e., those consumers who would purchase shares at a different production plan. Furthermore, a common control group $\mathcal{G}$ can coordinate the production decisions of different firms, whereas Drèze equilibria can suffer from a coordination failure.

DeMarzo (1993) assumes that every firm acts as a price taker and resolves the conflict among the shareholders differently. The decision of which price the firm takes as given is made by majority voting where each final share has one vote. If an equilibrium outcome exists then firm j's equilibrium production plan is optimal for one, dominant shareholder i. That is to say, firm j maximizes profits with respect to the state price vector $\pi_j = DU^i(x^i)$ of shareholder i. DeMarzo's approach is positive and power-oriented whereas Drèze's original approach is normative and efficiency oriented. For the purposes of positive analysis, Drèze (1989) presents a new approach based on the following control principle: The decision of a firm must be approved by a majority of shareholders and by every member of the board of directors.

Demichelis and Ritzberger (2010) analyze a model with complete markets, imperfect competition and a single firm that is controlled through the possibility of shareholder voting. In contrast to the usual models, shares are traded before the production plan is determined. A major motive to trade shares is to gain control in the decision of the production plan. Active voters incur a small participation cost and non-pivotal voters abstain. The firm implements the preferred production plan of its dominant shareholder. As in the present paper, it depends on concentration of ownership whether shares are traded at or above marginal costs.

Bejan and Bidian (2010) take up the question of when price taking behavior arises in the limit if an economy with complete markets and imperfect competition and production is replicated many times and show that the answer depends on the distribution of shares.

We return to the case of incomplete markets. In an influential paper, Grossman and Hart (1979), or GH for short, consider an economy that lasts for $T \geq 2$ periods. The extension to $T > 2$ periods is important because the stock market opens more than once and future share prices enter the picture. GH assume that the group of original shareholders determines the firm's production plan.

Furthermore, GH write on p. 299: "We are making the assumption of "utility taking," as opposed to price taking, behavior. ... Ostroy has emphasized the equivalence between "no surplus" and competitive-like behavior. In Ostroy's terminology, our competitive price perception assumption implies that each consumer believes that he will get no additional surplus out of his consumption of the new security created by the change in the production plan."

GH have a specific setting in their minds and refer to Hart's paper on monopolistic competition in a large economy. However, they do not provide an explicit model of monopolistic competition with incomplete markets. Therefore, it is difficult to see which assumptions are needed to provide an appropriate basis for their approach.

The assumption of competitive price perceptions entails price taking behavior. In a GH equilibrium, every firm j maximizes profits with respect to the price system

$$\pi_j = \sum_{i \in \mathcal{O}_j} \delta^i_j DU^i(x^i). \tag{2}$$

The striking similarity between (1) and (2) suggests a close similarity between the cases in which the initial or the final shareholders control the firm. However, this viewpoint is deceptive.

GH explain the reason why final shares play the same role in equation (1) as the initial shares do in equation (2) by the fact that the manager of the firm is acting in the interest of the final shareholders in the first case and in the interest of the initial shareholders in the second. This leaves the following question open: What is the manager supposed to accomplish if $\mathcal{F}_j$ and $\mathcal{O}_j$ coincide.

The original shares δ^i_j serve to determine how firm j's net value $q_j - c_j$ is distributed. If returns to scale are constant and firms act as price takers then profits vanish at every stock market equilibrium. That is to say, j's net value is identically equal to zero. The only motive that $\mathcal{O}_j$ can have in this case is to insure its members against bad states at $t = 1$. This motive is captured by $\mathcal{O}_j$'s final and not by its original shares.

Competitive price perceptions entail that the objective of a firm is determined by an exogenously given distribution of shares that can be arbitrarily chosen. The concept of a Drèze equilibrium has the advantage that it does not rely on any perceptions and that the objective of a firm depends on the optimal choice of every consumer.

In Section 3, we present a simple, numerical example. We consider the case of a single firm and drop the index j. Thus, we do not try to model a situation which GH had in their minds. Our conclusions are, however, to some extent independent of the supply side. If the manager of the firm acts in $\mathcal{O}$'s interest by maximizing $\mathcal{O}$'s total utility then the firm's goal is independent of how the original shares δ^i are allocated across $\mathcal{O}$ as long as the support of the distribution does not change. We find it puzzling that the δ^is are the important variables in (2) if their redistribution leaves $\mathcal{O}$'s welfare invariant.

The paper is organized as follows. In Section 2, we describe our framework and focus on the Drèze rule. Grossman-Hart equilibria are discussed in Section 3. In Section 4 we analyze the conflict between original and final shareholders and its impact on market power. Section 5 deals with competitive price perceptions and Section 6 concludes.

2 Social welfare maximization and the Drèze rule

To keep matters as simple as possible we use the following framework. As usual, the subscript 0 refers to the state $s = 0$ and the subscript $\mathbf{1}$ refers to all states $s = 1, \cdots, S$ at $t = 1$. A consumption vector is written as $x = (x_0, x_{\mathbf{1}}) \in \mathbb{R}_+^{S+1}$. Every utility function takes the form $U^i(x_0, x_{\mathbf{1}}) = x_0 + V^i(x_{\mathbf{1}})$. The initial endowment of consumer i is $e^i = (e_0^i, e_{\mathbf{1}}^i) \geq 0$, where e_0^i is a (sufficiently large) positive number. For convenience, we will set $e_{\mathbf{1}}^i = 0$ in specific examples. As a consequence, there is no need for short sales.

There is a single firm so that we can drop the index j. The firm has the technology $Y \subset \mathbb{R}_- \times \mathbb{R}_+^S$ which allows it to convert good 0 into a state dependent output at $t = 1$. The only asset in the economy consists of shares in this firm. The firm chooses an output vector $y_{\mathbf{1}}$ and incurs a cost of $c(y_{\mathbf{1}})$ units of good 0. The production vector $y = (y_0, y_{\mathbf{1}})$ with $y_0 = -c(y_{\mathbf{1}})$ lies on the efficient boundary of the production set Y.

The cost $c(y_{\mathbf{1}})$ is borne by the original shareholders in proportion to their initial shares δ^i. Consumer i's demand for shares is denoted $\vartheta_i(y_{\mathbf{1}})$. The firm's value $q(y_{\mathbf{1}})$ is determined by a market clearing condition. At a *stock market equilibrium*, $q(y_{\mathbf{1}})$ is such that the total demand for shares $\sum_{i\in\mathcal{G}} \vartheta_i(y_{\mathbf{1}})$ equals 1. A priori, it is *not* ruled out that the firm possesses market power.

If the firm produces $y_{\mathbf{1}}$ then i consumes $x^i(y_{\mathbf{1}}) = (e_0^i + (\delta^i - \vartheta^i(y_{\mathbf{1}}))\, q(y_{\mathbf{1}}) - \delta^i c(y_{\mathbf{1}}), e_{\mathbf{1}}^i + \vartheta^i(y_{\mathbf{1}}) y_{\mathbf{1}})$. The welfare of any control group $\mathcal{C}$ of consumers equals

$$W^{\mathcal{C}}(y_{\mathbf{1}}) = e_0^{\mathcal{C}} + (\delta^{\mathcal{C}} - \vartheta^{\mathcal{C}}(y_{\mathbf{1}}))q(y_{\mathbf{1}}) - \delta^{\mathcal{C}} c(y_{\mathbf{1}}) + \sum_{i\in\mathcal{C}} V^i(e_{\mathbf{1}}^i + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}), \quad (3)$$

where $e_0^{\mathcal{C}}$ is $\mathcal{C}$'s initial endowment of good 0 and $\delta^{\mathcal{C}}$ its endowment of original shares. The firm aims to maximize the welfare of its control group $\mathcal{C}$. We want to relate welfare maximization to price taking behavior and the Drèze rule.

For simplicity's sake, we assume that (3) consists of C^1 functions.[4] If the firm produces $y_{\mathbf{1}}$ and the associated technology gradient is $\pi(y_{\mathbf{1}}) = (1, \pi_{\mathbf{1}}(y_{\mathbf{1}}))$ then the firm's output is evaluated at marginal cost prices. That is to say, $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = Dc(y_{\mathbf{1}})y_{\mathbf{1}}$. Furthermore, because the final shareholder i maximizes utility given the stock market price $q(y_{\mathbf{1}})$, i's utility gradient at the optimum is orthogonal to the ray through $(-q(y_{\mathbf{1}}), y_{\mathbf{1}})$ along which agents trade. Thus, $(1, DV^i(x_{\mathbf{1}}^i(y_{\mathbf{1}})))(-q(y_{\mathbf{1}}), y_{\mathbf{1}}) = 0$ and we obtain $DV^i(x_{\mathbf{1}}^i)y_{\mathbf{1}} = q(y_{\mathbf{1}})$. All final shareholders attribute the value $DV^i(x_{\mathbf{1}}^i)y_{\mathbf{1}} = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = q(y_{\mathbf{1}})$ to $y_{\mathbf{1}}$.

Proposition 1. *Assume that the control group $\mathcal{C}$ of the monopolistic firm contains $\mathcal{F} \cup \mathcal{O}$. If $y_{\mathbf{1}}$ maximizes $\mathcal{C}$'s welfare then $y_{\mathbf{1}}$ maximizes profits given the*

[4]It is well-known that multiple equilibria can arise and that the equilibrium correspondence need not possess a continuous selection. However, our goal is to shed light on economically relevant issues without unnecessary complications.

price system

$$\pi(y_{\mathbf{1}}) = \sum_{i \in \mathcal{F}} \vartheta^i(y_{\mathbf{1}}) DU^i(x^i) = \sum_{i \in \mathcal{G}} \vartheta^i(y_{\mathbf{1}}) DU^i(x^i), \tag{4}$$

where x^i is i's optimal consumption bundle. The optimal bundle $y_{\mathbf{1}}$ is sold at marginal costs, that is to say, $q(y_{\mathbf{1}}) = Dc(y_{\mathbf{1}})y_{\mathbf{1}}$.

Proof. If $\mathcal{C}$ instructs the firm to produce $y_{\mathbf{1}}$ then $\mathcal{C}$'s consumption of good 0 becomes $e_0^{\mathcal{C}} - c(y_{\mathbf{1}})$. The market value $q(y_{\mathbf{1}})$ is irrelevant for $\mathcal{C}$'s consumption at $t = 0$ since the members of $\mathcal{C}$ pay and receive $q(y_{\mathbf{1}})$ and the utility functions are quasilinear. The original shares play no role since $\mathcal{C}$'s aggregate utility does not depend on how $q(y_{\mathbf{1}})$ and $c(y_{\mathbf{1}})$ are allocated. The firm maximizes

$$W^{\mathcal{C}}(y_{\mathbf{1}}) = e_0^{\mathcal{C}} - c(y_{\mathbf{1}}) + \sum_{i \in \mathcal{C}} V^i(e_{\mathbf{1}}^i + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}).$$

Let $v^i(y_{\mathbf{1}}) = V^i(e_{\mathbf{1}}^i + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}})$. The first order condition for welfare maximization can be stated as $Dc(y_{\mathbf{1}}) = \sum_{i \in \mathcal{C}} Dv^i(y_{\mathbf{1}}) = \sum_{i \in \mathcal{C}} \vartheta^i(y_{\mathbf{1}}) DV^i(x_{\mathbf{1}}^i)$ where $x^i = (x_0^i, x_{\mathbf{1}}^i) = (e_0^i + (\delta^i - \vartheta^i(y_{\mathbf{1}}))\, q(y_{\mathbf{1}}) - \delta^i c(y_{\mathbf{1}}), e_{\mathbf{1}}^i + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}})$ denotes i's optimal consumption. If the firm maximizes $W^{\mathcal{C}}(y_{\mathbf{1}})$ then it maximizes profits with respect to $\pi(y_{\mathbf{1}}) = \sum_{i \in \mathcal{C}} \vartheta^i(y_{\mathbf{1}}) DU^i(x^i)$. As shown in the paragraph preceding Proposition 1, $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = Dc(y_{\mathbf{1}})y_{\mathbf{1}}$. □

The Drèze rule says that the marginal costs paid today equal the marginal benefits consumed tomorrow. Consumer i's marginal benefits are proportional to i's final shares $\vartheta^i(y_{\mathbf{1}})$. If $\mathcal{F} \cup \mathcal{O}$ is contained in $\mathcal{C}$ the distribution of original shares serves only to determine who belongs to $\mathcal{O}$. Revenues play no role for $\mathcal{C}$ and $\mathcal{C}$ bears the production costs. The insurance motive remains and is captured by the Drèze rule.

Observe that the inclusion of $\mathcal{F}$ in $\mathcal{C}$ is a fixed point condition because $\mathcal{F}$ depends on $y_{\mathbf{1}}$. Independence obtains if $\mathcal{C}$ equals $\mathcal{G}$. The first order condition for the maximization of $\mathcal{G}$'s welfare coincides with the first order condition for constrained efficiency. Therefore, our reasoning is in line with Drèze (1974).

The fact that the group $\mathcal{F}$ of final shareholders also appears in (4) should not be misinterpreted. If $\mathcal{F} \subsetneqq \mathcal{O}$ and $\mathcal{C} = \mathcal{F}$ then $\mathcal{F}$ will typically not let the firm act according to (4) [see Proposition 3 in Section 4]. The reason is that we have not adopted the framework of a partnership economy and the original shares are an obligation to contribute to the production cost. Thus, $\mathcal{C} = \mathcal{F}$ does not take the production costs fully into account if $\mathcal{F} \subsetneqq \mathcal{O}$.

The Drèze rule presents a benchmark for the case in which not all original or not all final shareholders belong to the control group. Both cases are investigated in Section 4. The following corollary states that the *original* shareholders may deviate from the GH rule, which is discussed in the next section.

Corollary 1. *Assume the firm is controlled by its* original *shareholders and $\mathcal{F} \subseteq \mathcal{O} = \mathcal{C}$. Then the firm follows the Drèze rule.*

3 Grossman-Hart equilibria in a quasilinear example

In the example, there are two types $\tau = 1, 2$ of consumers, N^τ persons of each type and a single firm. We do not aim to provide a framework with monopolistic competition in the spirit of Grossman and Hart (1979) but want to explain some difficulties arising in the present simple setting.

For convenience, we use the following terminology. Consumers of the same type have the same preferences and initial endowments of goods. However, they may differ with respect to their original shares which can be varied parametrically. S equals 2 and the utility functions for the two types are given by

$$\begin{aligned} U^1(x_0, x_1, x_2) &= x_0 + 2\log(x_1) + \log(x_2), \\ U^2(x_0, x_1, x_2) &= x_0 + \log(x_1) + 2\log(x_2), \end{aligned} \tag{5}$$

respectively. Every consumer has the initial endowment $(e_0, 0, 0)$.

We assume that the costs to produce (y_1, y_2) are $c(y_1, y_2) = y_1^r + y_2^r$, where the scale elasticity $r \geq 1$. This allows us to consider constant and strictly decreasing returns to scale.

We determine the asset demand for both types and the market clearing asset price. Assume that the firm's output equals (y_1, y_2). Shares of (y_1, y_2) can be bought on the stock market. A consumer i of type 1 who decides to buy the share ϑ^i consumes the bundle $(e_0^i + \delta^i(q - c) - \vartheta^i q,\ \vartheta^i y_1,\ \vartheta^i y_2)$ and obtains the utility $e_0^i + \delta^i(q - c) - \vartheta^i q + 2\log(\vartheta^i y_1) + \log(\vartheta^i y_2)$. The utility maximizing amount of ϑ^i is obtained if $-q + 2/\vartheta^i + 1/\vartheta^i = 0$. Therefore, the demand for shares of a consumer of type 1 is given by $\vartheta^i = 3/q$. Due to the symmetry of the types, the demand for shares of a consumer of type 2 also equals $\vartheta^i = 3/q$. Since there are $N = N^1 + N^2$ consumers, market clearing requires $\sum^i \vartheta^i = 3N/q = 1$. The firm's market value is $q = 3N$.

The fact that q is constant in (y_1, y_2) is remarkable for the following reason. According to the assumption of competitive price perceptions in Grossman and Hart (1979), the original shareholders, who do not know the function q, feel that small output changes induce linear changes of q. Every shareholder uses his normalized utility gradient at his optimal consumption plan to evaluate the change. In the example, the assumption of competitive price perceptions is violated everywhere for every shareholder.

Competitive price perceptions have undesirable consequences. *In the quasilinear case, a redistribution of original shares within $\mathcal{O}$ is irrelevant for $\mathcal{O}$'s welfare.* However, the original shares δ^i play a decisive role in a GH equilibrium where the firm maximizes profits with respect to the price system $\sum_{i \in \mathcal{O}} \delta^i DU^i(x^i)$. In contrast to the Drèze rule, the GH rule is not oriented towards welfare and constrained efficiency.

An important difference between final and original shares is the following. *Final shareholdings are chosen by economic agents, whereas original shareholdings can be assigned arbitrarily.* In our example, this fact has the following implication. Consider two economies, $\mathcal{E}$ and $\tilde{\mathcal{E}}$. In economy $\mathcal{E}$, the Drèze rule (1) is used whereas the GH rule (2) is applied in economy $\tilde{\mathcal{E}}$. There are N^τ consumers of type τ in $\mathcal{E}$ and $\tilde{N}^\tau$ in $\tilde{\mathcal{E}}$. The economies are of the same size, that is to say, $N^1 + N^2 = \tilde{N}^1 + \tilde{N}^2 = N$. We have $\mathcal{G} = \mathcal{F} = \mathcal{O}$ in both economies so that price taking behavior is well founded. Proposition 2 shows that the GH rule aims to maximize welfare in an artificial way because it is based on the original shares. In the Grossman-Hart equilibrium of $\tilde{\mathcal{E}}$, the firm aims to maximize the social welfare in the wrong economy $\mathcal{E}$.

Proposition 2. *The original shares in $\tilde{\mathcal{E}}$ can be assigned in such a way that the unique Grossman-Hart equilibrium of $\tilde{\mathcal{E}}$ coincides with the unique Drèze equilibrium of $\mathcal{E}$.*

Proof. To prove the claim, consider $\mathcal{E}$ and assume that the bundle (y_1, y_2) is produced. The utility gradient of a consumer of type 1 is $DU^1(x_0, x_1, x_2) = (1, 2/x_1, 1/x_2)$ and that of a consumer of type 2 is $DU^2(x_0, x_1, x_2) = (1, 1/x_1, 2/x_2)$. Since the Drèze rule (1) is used in $\mathcal{E}$ and the consumption at $t = 1$ is the same for all consumers, that is to say $(x_1, x_2) = (y_1/N, y_2/N)$, the firm maximizes profits with respect to

$$\pi = \frac{N^1}{N}\left(1, \frac{2N}{y_1}, \frac{N}{y_2}\right) + \frac{N^2}{N}\left(1, \frac{N}{y_1}, \frac{2N}{y_2}\right) = \left(1, \frac{2N^1 + N^2}{y_1}, \frac{N^1 + 2N^2}{y_2}\right). \tag{6}$$

In equilibrium, π equals the technology gradient $(1, ry_1^{r-1}, ry_2^{r-1})$. Hence, the equilibrium output is $y_1 = ((2N^1 + N^2)/r)^{1/r}$ and $y_2 = ((N^1 + 2N^2)/r)^{1/r}$. The output is independent of how the original shares are distributed because the utility functions are quasilinear.

Consider now economy $\tilde{\mathcal{E}}$, which implements a Grossman-Hart equilibrium, and assign to each of the $\tilde{N}^\tau$ consumers of type τ the original shares $\delta^\tau = N^\tau/(\tilde{N}^\tau N)$. Then the weight of the utility gradient of type τ in the GH rule (2) is $\tilde{N}^\tau\delta^\tau = N^\tau/N$. Therefore, the firm maximizes profits with respect to the price system π as given by (6) and we obtain the same equilibrium production. □

What matters for $\mathcal{O}$'s welfare is $\mathcal{O}$'s size and composition. However, our example illustrates the following problem. The size of $\mathcal{O}$ can be changed without any effect on the stock market by redistributing original shares within types. For each type $\tau = 1, 2$, let $\bar{\delta}^\tau$ be the total number of original shares held by all consumers of type τ and denote the normalized utility gradient of any such consumer by π^τ. Then the GH rule (2) becomes

$$\pi = \bar{\delta}^1\pi^1 + \bar{\delta}^2\pi^2. \tag{7}$$

The degree of concentration of the original shares cannot be deduced from (7). Suppose that all original shares are in the hands of one consumer of each type and that $\mathcal{C} = \mathcal{O}$. Then the original owners can exploit the others by raising prices above marginal costs.

In the next section we analyze how the distribution of original and final shares influences the stock market price. In the first part the firm is controlled by the final, in the second part by the original shareholders.

4 The conflict between final and original shareholders.

We have shown that the Drèze rule (1) results if the distributional conflicts at $t = 0$ are internalized by a sufficiently large control group, for instance by $\mathcal{G}$.[5] However, an original shareholder $i \notin \mathcal{F}$ receives $\delta^i(q(y_\mathbf{1}) - c(y_\mathbf{1}))$ units of good 0 whereas a final shareholder $i \notin \mathcal{O}$ pays $\vartheta^i(y_\mathbf{1})q(y_\mathbf{1})$ and neglects the costs when he can influence the choice of $y_\mathbf{1}$. We argue that the market price $q(y_\mathbf{1})$ tends to *fall below* its perfectly competitive level if the corporation is controlled by $\mathcal{F}$.

For simplicity's sake, we assume that there are consumers who always buy shares while the others never do so. That is to say, $\mathcal{C} = \mathcal{F}$ is supposed to be independent of the choice of the production plan $y_\mathbf{1}$. Deviations from the Drèze rule occur if there are owners who do not belong to $\mathcal{C}$.

We analyze these deviations. If the corporation produces $y_\mathbf{1}$ then i consumes $(e_0^i + (\delta^i - \vartheta^i(y_\mathbf{1}))\, q(y_\mathbf{1}) - \delta^i c(y_\mathbf{1}), e_\mathbf{1}^i + \vartheta^i(y_\mathbf{1})y_\mathbf{1})$. Let $\delta^\mathcal{F}$ denote the total amount of original shares owned by $\mathcal{F}$ and $e_0^\mathcal{F}$ its initial endowment of good 0. Define $v^i(y_\mathbf{1}) = V^i(e_\mathbf{1}^i + \vartheta^i(y_\mathbf{1})y_\mathbf{1})$. Then $\mathcal{F}$'s welfare equals

$$W^\mathcal{F}(y_\mathbf{1}) = e_0^\mathcal{F} + (\delta^\mathcal{F} - 1)q(y_\mathbf{1}) - \delta^\mathcal{F} c(y_\mathbf{1}) + \sum_{i\in\mathcal{F}} v^i(y_\mathbf{1}). \tag{8}$$

We obtain the first order condition

$$(1 - \delta^\mathcal{F})Dq(y_\mathbf{1}) + \delta^\mathcal{F} Dc(y_\mathbf{1}) = \sum_{i\in\mathcal{F}} \vartheta^i(y_\mathbf{1})DV^i(x_\mathbf{1}^i), \tag{9}$$

where $x_\mathbf{1}^i$ is i's optimal consumption at $t = 1$.

As shown immediately before Proposition 1, $DV^i(x_\mathbf{1}^i)y_\mathbf{1} = q(y_\mathbf{1})$ for every $i \in \mathcal{F}$. That is to say, if i uses his utility gradient to evaluate $y_\mathbf{1}$ then the resulting value $DV^i(x_\mathbf{1}^i)y_\mathbf{1}$ coincides with $y_\mathbf{1}$'s market value $q(y_\mathbf{1})$.

Therefore, if we use (9) to evaluate $y_\mathbf{1}$ we obtain the following relationship between the market value $q(y_\mathbf{1})$ and $y_\mathbf{1}$'s value $\pi_\mathbf{1}(y_\mathbf{1})y_\mathbf{1}$ at marginal cost prices:

$$(1 - \delta^\mathcal{F})Dq(y_\mathbf{1})y_\mathbf{1} + \delta^\mathcal{F} Dc(y_\mathbf{1})y_\mathbf{1} = q(y_\mathbf{1}). \tag{10}$$

In the numerical example in Section 3, $q(y_\mathbf{1})$ is constant so that equation (10) reduces to $\delta^\mathcal{F}\pi_\mathbf{1}(y_\mathbf{1})y_\mathbf{1} = q(y_\mathbf{1})$. If $\delta^\mathcal{F} = 1$ we obtain a Drèze equilibrium and $y_\mathbf{1}$ is sold at marginal costs. However, if $\delta^\mathcal{F}$ falls below 1 then $\pi_\mathbf{1}(y_\mathbf{1})y_\mathbf{1} = q(y_\mathbf{1})/\delta^\mathcal{F}$ exceeds $q(y_\mathbf{1})$.

This observation can be generalized as follows. Consider a given output vector $y_\mathbf{1} \gg 0$ and vary the scale λ of production. We assume that an infinitesimal increase of λ decreases the profit, that is to say, $\partial_\lambda(q(\lambda y_\mathbf{1}) - c(\lambda y_\mathbf{1}))|_{\lambda=1} < 0$. Hence,

$$\partial_\lambda q(\lambda y_\mathbf{1})|_{\lambda=1} = Dq(y_\mathbf{1})y_\mathbf{1} < \partial_\lambda c(\lambda y_\mathbf{1})|_{\lambda=1} = Dc(y_\mathbf{1}))y_\mathbf{1}\,. \tag{11}$$

[5] There is also no conflict at $t = 0$ in a partnership economy because there are no original owners.

Then we conclude from (10) that

$$q(y_{\mathbf{1}}) < (1-\delta^{\mathcal{F}})Dc(y_{\mathbf{1}})y_{\mathbf{1}} + \delta^{\mathcal{F}}Dc(y_{\mathbf{1}})y_{\mathbf{1}} = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}. \tag{12}$$

Competitive pricing in the sense of Magill and Quinzii (1996), p.382, means that q is a linear function. Then $\partial_\lambda q(\lambda y_{\mathbf{1}})|_{\lambda=1} = 1$ and the assumption $\partial_\lambda(q(\lambda y_{\mathbf{1}}) - c(\lambda y_{\mathbf{1}}))|_{\lambda=1} < 0$ is satisfied if and only if we have decreasing returns to scale so that $\partial_\lambda(c(\lambda y_{\mathbf{1}}) > 1$.

Proposition 3. *If* $y_{\mathbf{1}}$ *maximizes* $W^{\mathcal{F}}$*,* $\partial_\lambda(q(\lambda y_{\mathbf{1}}) - c(\lambda y_{\mathbf{1}}))|_{\lambda=1} < 0$*, and* $\mathcal{C} = \mathcal{F} \supsetneqq \mathcal{O}$ *then the market value* $q(y_{\mathbf{1}})$ *is* below *its value* $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$ *at marginal cost prices.*

The question of who controls the firm and the question of who has the power to exploit whom are intrinsically related. If the firm maximizes $\mathcal{F}$'s welfare and there are original shareholders outside $\mathcal{F}$ then these shareholders are exploited.

Now we assume that the market power rests with $\mathcal{O}$ and argue that the market price $q(y_{\mathbf{1}})$ tends to *rise above* its perfectly competitive level $\pi_{\mathbf{1}}(y_{\mathbf{1}})$. Let $\vartheta^{\mathcal{O}}$ denote the total amount of final shares owned by $\mathcal{O}$ and $e_0^{\mathcal{O}}$ its initial endowment of good 0. Then $\mathcal{O}$'s welfare equals

$$W^{\mathcal{O}}(y_{\mathbf{1}}) = e_0^{\mathcal{O}} + (1-\vartheta^{\mathcal{O}}(y_{\mathbf{1}}))q(y_{\mathbf{1}}) - c(y_{\mathbf{1}}) + \sum_{i\in\mathcal{O}} V^i(e_{\mathbf{1}}^i + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}) \tag{13}$$

and we obtain the first order condition

$$(\vartheta^{\mathcal{O}}(y_{\mathbf{1}}) - 1)Dq(y_{\mathbf{1}}) + Dc(y_{\mathbf{1}}) = \sum_{i\in\mathcal{O}} \vartheta^i(y_{\mathbf{1}})DV^i(x_{\mathbf{1}}^i). \tag{14}$$

If we take the inner product with $y_{\mathbf{1}}$ we obtain

$$(\vartheta^{\mathcal{O}}(y_{\mathbf{1}}) - 1)Dq(y_{\mathbf{1}})y_{\mathbf{1}} + \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = \sum_{i\in\mathcal{O}} \vartheta^i(y_{\mathbf{1}})q(y_{\mathbf{1}}) = \vartheta^{\mathcal{O}}(y_{\mathbf{1}})q(y_{\mathbf{1}}). \tag{15}$$

Consider the extreme case in which no member of $\mathcal{O}$ wants to hold final shares so that $\vartheta^{\mathcal{O}}(y_{\mathbf{1}}) = 0$. Then (13) becomes $W^{\mathcal{O}}(y_{\mathbf{1}}) = e_0^{\mathcal{O}} + q(y_{\mathbf{1}}) - c(y_{\mathbf{1}}) + \sum_{i\in\mathcal{O}} V^i(e_{\mathbf{1}}^i)$ and the firm aims to maximize its net market value.

In our example, Dq vanishes and (15) becomes $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = \vartheta^{\mathcal{O}} q(y_{\mathbf{1}}) \leq q(y_{\mathbf{1}})$, where the inequality is strict provided $\vartheta^{\mathcal{O}} < 1$. In this case, $y_{\mathbf{1}}$ is sold with a positive mark-up.

We assume now that $\partial_\lambda q(\lambda y_{\mathbf{1}})/q(\lambda y_{\mathbf{1}})|_{\lambda=1} < 1$. That is to say, q grows by less than 1% if $y_{\mathbf{1}}$ is increased by 1% and the boundary case of competitive pricing is ruled out. Equation (15) is equivalent to

$$q(y_{\mathbf{1}}) - \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}} = (\vartheta^{\mathcal{O}}(y_{\mathbf{1}}) - 1)(Dq(y_{\mathbf{1}})y_{\mathbf{1}} - q(y_{\mathbf{1}})). \tag{16}$$

Because $\vartheta^{\mathcal{O}}(y_{\mathbf{1}}) < 1$ and $Dq(y_{\mathbf{1}})y_{\mathbf{1}} = \partial_\lambda(q(\lambda y_{\mathbf{1}}) < q(y_{\mathbf{1}}))$ by assumption we obtain that (16) is positive, that is to say $q(y_{\mathbf{1}}) > \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$.

Proposition 4. *If* $y_{\mathbf{1}}$ *maximizes* $W^{\mathcal{O}}$*,* $\partial_\lambda(q(\lambda y_{\mathbf{1}})/q(\lambda y_{\mathbf{1}})|_{\lambda=1} < 1$*, and* $\mathcal{C} = \mathcal{O} \supsetneqq \mathcal{F}$ *then* $q(y_{\mathbf{1}}) > \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$*.*

To summarize, if the control group is so large that it contains $\mathcal{O}$ and $\mathcal{F}$, for instance if $\mathcal{C} = \mathcal{G}$, then $\mathcal{C}$'s welfare takes its maximum at a Drèze equilibrium and $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. However, $q(y_{\mathbf{1}})$ can be below or above $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. The first case arises if the firm is controlled by $\mathcal{F}$, the second if it is controlled by $\mathcal{O}$.

5 Competitive price perceptions

The assumption of competitive price perceptions says that every individual $i \in \mathcal{O}$ uses his own utility gradient $DV^i(x^i_{\mathbf{1}})$ at his optimal consumption given $y_{\mathbf{1}}$ to evaluate alternative production plans. Thus, i thinks that $q(y'_{\mathbf{1}}) = DV^i(x^i_{\mathbf{1}})y'_{\mathbf{1}}$.

If transfers of good 0 are used to enable the winners of a potential change $\Delta y_{\mathbf{1}}$ of $y_{\mathbf{1}}$ to compensate the losers one faces the following difficulty. Since markets are incomplete $DV^i(x^i_{\mathbf{1}})\Delta y_{\mathbf{1}}$ will typically be positive for some consumers and negative for others. The first group feels that the share price will go up while the other group feels that it will go down if production is changed by $\Delta y_{\mathbf{1}}$. If $\mathcal{O}$ knows how to transfer good 0 from the members of the first group to the members of the second group then $\mathcal{O}$ must be informed about which member has which characteristics. In particular, *it is known within $\mathcal{O}$ that the individual price perceptions $q(y'_{\mathbf{1}}) = DV^i(x^i_{\mathbf{1}})y'_{\mathbf{1}}$ of $\mathcal{O}$'s members are incompatible with each other*. The members of $\mathcal{O}$ agree to disagree.

The assumption of competitive price perceptions serves the following purpose. Suppose the firm changes its output slightly from $y_{\mathbf{1}}$ to $\hat{y}_{\mathbf{1}} = y_{\mathbf{1}} + \Delta y_{\mathbf{1}}$. In the quasilinear case, i's utility at $y_{\mathbf{1}}$ is $e^i_0 + \delta^i[q(y_{\mathbf{1}}) - c(y_{\mathbf{1}})] - \vartheta^i q(y_{\mathbf{1}}) + V^i(e^i_{\mathbf{1}} + \vartheta^i y_{\mathbf{1}})$. The output change $\Delta y_{\mathbf{1}}$ induces the first order utility change

$$\Delta U^i = \delta^i[Dq(y_{\mathbf{1}}) - Dc(y_{\mathbf{1}})]\Delta y_{\mathbf{1}} - \vartheta^i(y_{\mathbf{1}})[Dq(y_{\mathbf{1}}) - DV^i(x^i_{\mathbf{1}})]\Delta y_{\mathbf{1}}, \tag{17}$$

where $x^i_{\mathbf{1}} = e^i_{\mathbf{1}} + \vartheta^i(y_{\mathbf{1}})y_{\mathbf{1}}$. According to Grossman and Hart (1979), consumer i, who does not know the function q, feels that, for any $\Delta y_{\mathbf{1}}$, the utility change $DV^i(x^i_{\mathbf{1}})\Delta y_{\mathbf{1}}$ at $t = 1$ is exactly offset by the associated price change $Dq(y_{\mathbf{1}})\Delta y_{\mathbf{1}}$. That is to say, i feels that the second bracket $Dq(y_{\mathbf{1}}) - DV^i(x^i_{\mathbf{1}})$ in equation (17) vanishes.

Competitive price perceptions have three important consequences for the conflict among the original shareholders. First, they assume away their conflict as final shareholders because they annihilate the second bracket in (17). Second, they create a new conflict among the members of $\mathcal{O}$ in their role as original owners since the objective market value $q(y_{\mathbf{1}} + \Delta y_{\mathbf{1}})$ is replaced by a family of subjective perceptions $DV_i(x^i_{\mathbf{1}})\Delta y_{\mathbf{1}}$. Third, whenever a firm has market power it is deprived of this power by the perceptions of its shareholders. That is to say, in equilibrium we have $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. In the present setting this equality follows immediately from the first order condition (16) and the definition of competitive price perceptions. If $Dq(y_{\mathbf{1}}) = DV^i(x^i_{\mathbf{1}})$ then $Dq(y_{\mathbf{1}})y_{\mathbf{1}} = DV^i(x^i_{\mathbf{1}})y_{\mathbf{1}} = q(y_{\mathbf{1}})$. Therefore, the right hand side $(\vartheta^{\mathcal{O}}(y_{\mathbf{1}}) - 1)(Dq(y_{\mathbf{1}})y_{\mathbf{1}} - q(y_{\mathbf{1}}))$ of (16) vanishes and we obtain $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$.

In order to apply the assumption of competitive price perceptions we write the first order condition (14) for $\mathcal{O}$'s welfare maximum explicitly as a sum of derivatives of individual utility functions and obtain

$$\sum_{i \in \mathcal{O}} (\delta^i - \vartheta^i(y_{\mathbf{1}}))Dq(y_{\mathbf{1}}) - Dc(y_{\mathbf{1}}) + \sum_{i \in \mathcal{O}} \vartheta^i(y_{\mathbf{1}})DV^i(x^i_{\mathbf{1}}) = 0. \tag{18}$$

When we replace $(\delta^i - \vartheta^i(y_{\mathbf{1}}))Dq(y_{\mathbf{1}})$ by $(\delta^i - \vartheta^i(y_{\mathbf{1}}))DV^i(x^i_{\mathbf{1}})$ equation (18) reduces to the GH rule

$$\sum_{i\in\mathcal{O}} \delta^i DV^i(x^i_{\mathbf{1}}) - Dc(y_{\mathbf{1}}) = 0. \tag{19}$$

Drèze equilibria have the merit that they do not rely on perceptions. The control group is so large that it wants the firm to act as a price taker. Thus, the firm needs to know the value $q(y_{\mathbf{1}})$ but not the function q. In a GH equilibrium, $\mathcal{C} = \mathcal{O}$ can be a small group which would gain by selling $y_{\mathbf{1}}$ at a price above $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. We ignore this problem and consider only stock market equilibria where firms act as price takers. This fact as well as the firm's technology are assumed to be known to the members of $\mathcal{C} = \mathcal{O}$. We ask whether this knowledge can be helpful.

The optimal choice of a stock market equilibrium in which firms act as price takers by assumption presents a constrained optimization problem. Its analysis requires at least some local knowledge about the set of feasible equilibria. If such knowledge is not available because only the value of q at $\bar{y}_{\mathbf{1}}$ is known one can either resort to some kind of perceived knowledge or one can ask whether there is a desirable goal that can be reached given information that is actually available to $\mathcal{O}$.

To illustrate the latter approach, consider an economy with quasilinear and quasiconcave utilities and a technology with constant scale elasticity $r \geq 1$ as in our example. Suppose $\mathcal{O}$ has to evaluate the stock market equilibrium associated with the production plan $(\bar{y}_0, \bar{y}_{\mathbf{1}})$ and the technology gradient $(1, \bar{\pi}_{\mathbf{1}})$. Consider the fixed cost level $\bar{c} = |\bar{y}_0|$. Then all $y_{\mathbf{1}}$ with $c(y_{\mathbf{1}}) = \bar{c}$ have the same value $\bar{q} = r\bar{c}$ and yield the same profit $(r-1)\,\bar{c}$ at marginal costs prices (normalized technology gradients). Among these bundles, the original shareholders prefer the bundle $y_{\mathbf{1}}$ that constitutes their optimal output mix.

More precisely, let $Y_{\bar{c}}$ denote the image of the technology section $\{(y_0, y_{\mathbf{1}}) \in Y | y_0 = -\bar{c}\}$ under the projection $\text{proj}_S : \mathbb{R}^1 \times \mathbb{R}^S \to \mathbb{R}^S$. Similarly, consider the set of $\mathcal{O}$'s aggregate preferred net trades $P^{\mathcal{O}} = \{x^i - e^i | x^i \succ x^i(\bar{y}_{\mathbf{1}}))\}$ and denote the image of $P^{\mathcal{O}} \cap (\{-\bar{c}\} \times \mathbb{R}^S)$ under proj_S by $P^{\mathcal{O}}_{\bar{c}}$. We multiply $Y_{\bar{c}}$ with $\mathcal{O}$'s final shares $\bar{\vartheta}^{\mathcal{O}} = \sum_{i\in\mathcal{O}} \vartheta^i(y_{\mathbf{1}})$ at $y_{\mathbf{1}}$. Observe that $\bar{y}_{\mathbf{1}}$ lies on the boundaries of the convex and disjoint sets $Y_{\bar{c}}$ and $P^{\mathcal{O}}_{\bar{c}}$. The optimality of $\mathcal{O}$'s output mix at $\bar{y}_{\mathbf{1}}$ requires that the boundary of $\bar{\vartheta}^{\mathcal{O}} Y_{\bar{c}}$ does not cut into $P^{\mathcal{O}}_{\bar{c}}$ (separating hyperplane).[6] Therefore, $\sum_{i\in\mathcal{O}}(\vartheta^i(\bar{y}_{\mathbf{1}}) DV^i(\bar{x}^i_{\mathbf{1}})$ must be proportional to $\bar{\pi}_{\mathbf{1}}$. In analogy to the Drèze rule, the technology gradient $(1, \bar{\pi}_{\mathbf{1}})$ is set equal to the convex combination $\sum_{i\in\mathcal{O}}(\vartheta^i(y_{\mathbf{1}})/\bar{\vartheta}^{\mathcal{O}}) DU^i(\bar{x}^i)$. On the basis of this rule, $\mathcal{O}$ can instruct the firm to implement its optimal output mix at a marginal cost pricing equilibrium without knowledge of the function q.

This goal is particularly natural if the scale elasticity r is equal to 1 since profits are identically equal to 0 in the case of price taking behavior so that the profit motive vanishes completely. In our numerical example, the above procedure entails that the original shareholders maximize their welfare subject to the constraint of price taking behavior because all stock market equilibria are associated with the same cost level.

[6]The assumption of quasiconcavity allows us to formulate the argument within $\mathbb{R}^S$.

If one wants to base the objective of a firm on the original shares then these shareholdings should be derived from economic considerations. Suppose that every consumer i had a chance to acquire original shares at period $t = -1$ and that i knows that his interests will be included in the firm's objective at $t = 0$ provided $\vartheta^i > 0$. Then every i has an incentive to possess at least one zillionth of these shares. In this case, the firm is controlled by $\mathcal{O} = \mathcal{G}$ and $\vartheta^{\mathcal{O}} = 1$. Then q drops out of the objective function (13) and the problem disappears.

6 Conclusions

This paper aims to examine the role of the ownership structure and the control rights in a particularly simple and transparent setting. Because good 0 can be used to transfer utility the firm's objective can be defined as the maximization of a function, the welfare or total utility of its control group $\mathcal{C}$. The fact that the welfare of the whole group $\mathcal{C}$ rather than each individual utility gain or loss matters has important implications.

Assume that the firm is controlled by $\mathcal{O}$. If $\mathcal{C} = \mathcal{O}$ is so large that it contains all final shareholders then the market value q does not enter $\mathcal{C}$'s welfare even if all its individual members are affected by q. As a consequence, there is no need for price perceptions in this case and $\mathcal{O}$'s welfare maximum is attained at a Drèze equilibrium and not at a GH equilibrium.

Furthermore, a redistribution of the original shares δ^i within $\mathcal{C} = \mathcal{O}$ does not have any effect on $\mathcal{O}$'s welfare as long as nobody loses all his original shares so that $\mathcal{O}$ shrinks. Therefore, the weights δ^i of the utility gradients in the definition of a GH equilibrium do not enter the firm's objective in the transferable utility case beyond the fact that they determine the members of $\mathcal{O}$. The fact that the welfare neutral original shares serve as weights of the utility gradients in a GH equilibrium is a pure consequence of the shareholders' perceptions of their problem.

The assumption of competitive price perceptions combines two aspects. The first, competitive pricing, says that the stock market value q is supposed to be a linear function. Second, each consumer i feels that $q(y_{\mathbf{1}})$ coincides with i's marginal utility evaluation $DV^i(x^i)y_{\mathbf{1}}$ of $y_{\mathbf{1}}$. This implies that $y_{\mathbf{1}}$ is sold at marginal costs independently of whether this lies in $\mathcal{C}$'s genuine interest.

We have presented a numerical example in which no original shareholder satisfies the assumption of competitive price perceptions for any production decision of the firm. In the example, the function q is constant rather than linear.

The question of whether the firm should be priced at, above, or below marginal costs depends on the ownership structure and control in the following way. If the firm is controlled by its *original shareholders* and $\mathcal{F}$ is not fully contained in $\mathcal{O}$ then $\mathcal{C}$ has an incentive to sell its stock at a price $q(y_{\mathbf{1}})$ *above* marginal costs $\pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$.

However, if the firm is controlled by its *final shareholders* and $\mathcal{O}$ is not contained in $\mathcal{F}$ then $\mathcal{C}$ has an incentive to charge a price *below* marginal costs since all costs are borne by the original shareholders in our setting so that a positive fraction of the costs is not accounted for by $\mathcal{C}$.

The Drèze rule results if $\mathcal{C} \supseteq (\mathcal{F} \cup \mathcal{O})$ and $q(y_{\mathbf{1}}) = \pi_{\mathbf{1}}(y_{\mathbf{1}})y_{\mathbf{1}}$. The case of $\mathcal{C} = \mathcal{G}$ is particularly important from a welfare perspective because every consumer's interest is taken into account. If $\mathcal{C} = \mathcal{F}$ is smaller than $\mathcal{G}$ then those consumers who would hold final shares at an alternative production plan are ignored.[7]

The last three paragraphs shed light on the need for price perceptions in the quasilinear case. Assume it is optimal for $\mathcal{C} = \mathcal{O}$ to sell its output at marginal costs although the firm is not forced to do so by fierce competition. Then $\mathcal{O}$ must contain $\mathcal{F}$. Therefore, $\mathcal{C}$ does not need to know q and $\mathcal{O}$'s welfare optimum is a Drèze equilibrium.

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[7] The Drèze rule is typically used in the framework of a partnership economy without initial shares. In this case, the Drèze rule relies on the condition $\mathcal{C} \supseteq \mathcal{F}$.