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New Evidence from Nonparametric Tests

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Abstract

We apply range unit-root tests to OECD unemployment rates and compare the results to conventional tests. By simulations, we find that unemployment is represented adequately by a new nonlinear transformation of a serially-correlated I(1) process.

Keywords:
Unemployment Rates, Unit-Root Tests, Hysteresis, Nonlinear Transformations, Serial Correlation

JEL classification: C12, C22

1 Introduction

We test for the null hypothesis of a unit root in unemployment of 13 countries. Unemployment rates are unlikely to fit the maintained models of traditional unit-root tests, such as the (augmented) Dickey-Fuller or (A)DF tests. Therefore, such tests may yield unreliable conclusions concerning the hypotheses of hysteresis and of the natural rate, which are often identified with the statistical null and alternative, respectively. Definitionally bounded, unemployment rates may rather correspond to nonlinear transformations of I(0) or I(1) processes, a class introduced by Granger and Hallman (1991). Unemployment additionally faces a lower bound due to limited employability of workers in prosperity and an upper bound representing the level of minimum employment maintained in recession. We contrast parametric unit-root tests with range unit-root (RUR) tests introduced by Aparicio et al. (2006), which are robust to nonlinear transformations, and we find that their results conflict with DF-type tests: RUR tests reject more often, although they have less power in linear models and Granger and Hallman (1991) point out size distortions and the danger of DF overrejection in the presence of nonlinearity. Gustavsson and Österholm (2006) conduct a comparable study that provides evidence supporting the natural rate hypothesis. Their maintained model class, however, is a special class of smooth-transition models where they allow for nonlinearity under the alternative only. We propose a nonlinear transformation of the unemployment rate that can explain the observed features, and we support this claim by some Monte Carlo evidence. A nonlinearly transformed I(1) process can reproduce the observed patterns best, hence our results tend to support the hysteresis hypothesis.

In Section 2, we describe the data and compare the results of the RUR tests to the results of DF and ADF test. To evaluate the results, we run some size and power simulations in Section 4. A new nonlinear transformation is introduced in Section 3. That is, in our simulations we test for a unit root against a stationary alternative where the series are transformed adequately to fit the real data according to the minimum sum of squared errors. Finally, we may draw some conclusions.

2 Data and Unit-Root Tests

We use survey-based, seasonally-adjusted, monthly data on unemployment rates of 13 countries provided by the OECD: Australia, Brazil, Canada, Finland, France, Ireland, Japan, Korea, Mexico, Netherlands, Sweden, UK, and US.

Table 1 reports the results of four tests with the unit root assumed under the null hypothesis. We apply the range unit-root (RUR) and the forward-backward range unit-root (FB-RUR) test by Aparicio et al. (2006) where the test statistics are equal to

$$J_0^{(n)} = \frac{1}{\sqrt{n}} \sum_{t=2}^{n} \mathbb{1} \left(\Delta R_t^{(x)} > 0 \right) \tag{1}$$

Country	Period	Obs.	RUR	FB-RUR	DF	ADF
Australia	02/78–01/08	360	1.897	2.907	-0.748	-0.690 (14)
Brazil	01/81–01/08	325	1.775	2.550	-0.924	-1.091 (4)
Canada	01/55–02/08	638	1.386^{*}	2.715	-0.265	-0.525 (5)
Finland	01/59–01/08	589	2.349	4.458	0.067	-0.880 (14)
France	01/78–07/07	355	2.813	4.353	1.031	-0.186 (10)
Ireland	01/83–01/08	301	4.381	6.440	-2.493^{**}	-1.386 (5)
Japan	01/55–12/07	636	1.467^{*}	2.075^{*}	0.071	0.018 (12)
Korea	01/90–01/08	217	1.154^{**}	2.304	-0.302	-0.778 (7)
Mexico	01/87–12/07	252	1.323^{*}	2.094	-0.774	-0.591 (4)
Netherlands	01/01–09/07	81	2.556	3.693	0.280	0.037 (2)
Sweden	01/70–12/07	456	1.218^{**}	2.252	-0.242	-0.535 (7)
UK	02/71–11/07	442	2.949	4.473	0.243	-0.762 (6)
US	01/55–02/08	638	1.109^{***}	2.044^{*}	-0.414	-0.506 (12)

*, **, *** denotes significance at a 10%, 5%, 1% level, respectively.

Table 1: Unit-root tests.

and

$$J_*^{(n)} = \frac{1}{\sqrt{2n}} \sum_{t=2}^{n} \left[\mathbb{1}\left(\Delta R_t^{(x)} > 0\right) + \mathbb{1}\left(\Delta R_t^{(x')} > 0\right) \right], \tag{2}$$

respectively. $\Delta R_t^{(x)}$ denotes the first differences of the running ranges of the realization x_t, $t = 1, \ldots, n$ and $\Delta R_t^{(x')}$ those of the reversed series x_t, $t = n, \ldots, 1$. $\mathbb{1}$ is the indicator function. Running ranges are formed by the differences of the ith extremes, $\max_{t=1,\ldots,i} x_t$ and $\min_{t=1,\ldots,i} x_t$, for each $i = 1, \ldots, n$. $J_0^{(n)}$ counts the number of increases of the sequence of ranges, and the forward-backward version $J_*^{(n)}$ also considers the range increases in the reversed series. Critical values for each sample size are calculated from 10000 replications of the null model of a random walk with normal increments. Columns 6 and 7 contain the test statistics corresponding to the conventional DF test (Dickey and Fuller, 1979) and the ADF test (Said and Dickey, 1984), respectively. The optimum number of lags given in parentheses is selected by AIC with the maximum number determined according to Schwert (1989).

The RUR test rejects the null hypothesis for six countries at least at a 10% level of significance whereas the FB-RUR test, which is supposed to be more powerful (Aparicio et al., 2006), rejects the null only twice. In contrast, the DF test rejects the null of a unit root only for Ireland which may be due to serial correlation as the ADF test does not even reject for that country.

3 Nonlinear Transformations

To assess the results of the unit-root tests, we implement some size and power experiments where we generate artificial unemployment data. For this purpose, we fit an autoregressive model to the Canadian data where we have the largest number of observations available. Canadian unemployment rates exhibit several characteristics of unemployment rates in general. Particularly, autocorrelation and partial autocorrelation functions as well as the near-unit-root behavior are quite representative. The original series is linearized by a new transformation function. Autocorrelation and partial autocorrelation functions of most unemployment series indicate AR(1) models. We first demean the data and estimate AR(1) by least squares without intercept. The demeaned unemployment rate u is not bounded between 0 and 1 any more, rather between $-(n-1)/n$ and $(n-1)/n$[1]. We introduce the following two-parts nonlinear transformation function

$$f(u) = \left[\ln\left(\frac{n-1}{n}\right) - \ln\left(\frac{n-1}{n} - u\right)\right]^{1/\alpha} \cdot \beta \qquad \text{if } u \geq 0 \tag{3}$$

and

$$f(u) = -\left[\ln\left(\frac{n-1}{n}\right) - \ln\left(\frac{n-1}{n} + u\right)\right]^{1/\alpha} \cdot \beta \qquad \text{if } u < 0 \tag{4}$$

where $\alpha \in (0, 1]$ and $\beta > 0$. The function defined by (3) and (4) forms a sigmoid curve over the first and the third quadrant, bounded between $-(n-1)/n$ and $(n-1)/n$ for $n \geq 2$. A lower value α corresponds to higher nonlinearity whereas a higher α comes close to linearity in the relevant range. The demeaned Canadian data ranges from –0.045 to 0.057. The parameter β allows for scaling.

4 Size and Power Simulations

By running RUR, FB-RUR, and DF test on nonlinear series generated according to the suggested transformation, we elaborate differences in size and power performance. We simulate 10000 series of a stationary AR(1) process to obtain power estimates. That is, we apply transformations (3) and (4) to the Canadian data, estimate an AR(1) model and save the autoregressive coefficient. Starting with the first observation of the Canadian series, we generate 638 observations. The errors variance is calibrated to match the sample variance. The resulting series are transformed by the inverse functions of transformations (3) and (4). As the nonlinear transformation also scales the variance, the scaling parameter β becomes redundant after applying the inverse functions and is set equal to

[1] This corresponds to the situations where either one rate is equal to one and the other rates are equal to zero or one rate is equal to zero and the other rates are equal to one, respectively.

one. Correspondingly, size estimates are calculated from 10000 replications of a random walk where the variance of disturbances is calibrated as before.

Tables 2 and 3 present size and power estimates, respectively, corresponding to different degrees of nonlinearity where we assume a nominal size of 5%. We use the minimum mean sum of squared errors over all 10000 series as a measure of fit. The best fit of the random-walk model occurs at $\alpha = 0.6$. There, RUR and FB-RUR test yield a higher probability of correctly accepting the null hypothesis than the DF test. The size of the RUR and the FB-RUR test seems to be definitely robust to nonlinear transformations whereas the size of the DF test is not. That is, the tests by Aparicio et al. (2006) are close to the nominal size whereas the DF test overrejects as already reported in Granger and Hallman (1991) for various nonlinear transformations. Including five lagged differences shifts the size close to the 5% level. However, in case of sufficient nonlinearity, augmenting cannot preserve the size. For $\alpha = 0.2$, ADF size is equal to 20.61% whereas RUR and FB-RUR size are equal to 5.72% and 5.48%, respectively. In contrast, the DF test applied to untransformed simulated data achieves a size equal to 4.82%. To obtain size estimates located nearby the empirical rejection frequency of 23% corresponding to the 5% level of significance in Table 1, we may allow for serial correlation which is supported by the autocorrelation function of the residuals. Negative MA coefficients result in different size increases of DF test and RUR test.

α	RUR	FB-RUR	DF
0.1	no results due to collinearity		
0.2	5.59	5.22	50.62
0.3	5.50	5.29	32.02
0.4	5.72	5.48	18.79
0.5	5.66	5.41	11.39
0.6	5.50	5.29	7.47
0.7	5.35	5.26	5.60
0.8	5.61	5.59	4.63
0.9	5.59	5.22	4.79
1	5.71	5.34	4.75

Table 2: Size estimates in %.

The best fit of the stationary model is obtained at $\alpha = 0.9$ which is associated with near-linear behavior and thus comes close to the fit of the untransformed process. Both tests experience strong power losses with increasing α. However, RUR and FB-RUR test suffer from even higher declines than the conventional DF test at all levels of nonlinearity. Applying the ADF test with 5 lagged differences seems natural, but does not result in higher power. By using a simulated ARMA(1,1) process at $\alpha = 0.9$, we obtain similar power estimates for RUR and FB-RUR test and a small power decrease for DF test.

α	RUR	FB-RUR	DF
0.1	no results due to collinearity		
0.2	90.03	96.72	100
0.3	65.24	73.48	99.92
0.4	41.58	46.26	95.17
0.5	28.93	31.27	74.57
0.6	22.14	22.10	50.82
0.7	16.95	16.84	33.89
0.8	14.20	14.37	27.41
0.9	12.53	12.41	24.84
1	11.96	11.90	24.55

Table 3: Power estimates in %.

5 Conclusions

The RUR test seems to reject the null hypothesis of a unit root in unemployment rather easily. However, it achieves only half of the power of the conventional DF test. In contrast, the DF and the ADF test tend to accept the null hypothesis of a random walk. As the DF test exhibits serious size problems in presence of nonlinearities, its prospects of correctly identifying a transformed random walk are poor. In presence of serial correlation, all tests face strong power declines. Augmentations in the DF test improve the size performance but do not countervail increases caused by nonlinearity. The Aparicio et al. (2006) tests still lack a sufficient solution concerning serial correlation. In summary, our simulations support the conjecture that a nonlinearly-transformed serially-correlated I(1) process has generated the Canadian data.

We note that Aparicio et al. (2006) report higher power and smaller size estimates for the RUR test than for the DF test in case of near-unit-root stationary processes. Moreover, they claim the FB-RUR test to have more power than the simple RUR test. These results conflict with the present work.

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