WORKING PAPERS

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November 2009

Working Paper No: 0904

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

The Open Method of Coordination (OMC) as an Evolutionary Learning Process*

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November 23, 2009

Abstract

We interpret the Open Method of Coordination (OMC), recently adopted by the EU as a mode of governance in the area of social policy and other fields, as an imitative learning dynamics of the type considered in evolutionary game theory. The best-practise feature and the iterative design of the OMC correspond to the behavioral rule "imitate the best." In a redistribution game with utilitarian governments and mobile welfare beneficiaries, we compare the outcomes of imitative behavior (long-run evolutionary equilibrium), decentralized best-response behavior (Nash equilibrium), and coordinated policies. The main result is that the OMC allows policy coordination on a strict subset of the set of Nash equilibria, favoring in particular coordination on *intermediate* values of the policy instrument.

Keywords: Open Method of Coordination, Finite-population Evolutionarily Stable Strategy, Imitation, Mobility, Redistribution.

JEL Classification: H77, H75, C73, I38.

*We are grateful to Carlos Alós-Ferrer, Simon Weidenholzer, Guttorm Schjelderup, and seminar participants in Bayreuth, Hannover, Munich, Innsbruck, Trento, Zürich, and Maastricht for helpful comments and suggestions.

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1 Introduction

Over the last fifteen years, a new mode of governance has emerged within the European Union (EU). Since the European Council summit in Lisbon (March 2000), it has been coined as the Open Method of Coordination (OMC). Initially designed for, and applied in, social policy (social inclusion, health care, pensions and long-term care), various European Councils approved of the extension of the OMC to a broad spectrum of policy areas, encompassing for example migration, technology, environment, and R&D.

The Presidency Conclusions of the Lisbon European Council define the OMC as "a means of spreading best practice and achieving greater convergence towards the main EU goals" (European Council, 2000). The OMC is best described as an iterative process of benchmarking and mutual learning among the EU member states.[1]

Roughly, the OMC proceeds as follows (see Figure 1 for a sketch): Having agreed on EU-wide common objectives and indicators, EU member states individually design and implement their national policies. After a certain period, these national strategies are jointly evaluated and compared within the EU. Best practices are identified and member states are encouraged (but not forced) to adopt them. The process is then iterated.

The OMC induces member states to systematically compare themselves to one another in terms of their policy performance. It promotes the imitation of successful policies, thus aiming at policy convergence (Trubek and Mosher, 2001). Diversity, though, is not disallowed (Zeitlin, 2005; Daly, 2007). The OMC is a soft-law method that leaves to member states control of their policies (Pochet, 2005) in an attempt to keep agency costs and losses in national sovereignty minimal (Borrás and Jacobsson, 2004). The rationale behind the OMC is the hope that the quality of policy decisions improves and that policy-learning through benchmarking is enhanced.

In this paper, we provide a theoretical model of the OMC and an assessment of its performance in a social policy (redistribution) setting. Following the design depicted in Figure 1, we model the OMC as an iterative process with an emphasis on mimicking best practices

[1] While the OMC still lacks a unique and precise definition, there seems to be a consensus among its commentators that "learning" and "imitation of best practise" are core ingredients of the method. Detailed information on the OMC in the area of European social policy is provided by EU Commission (2005, 2008).

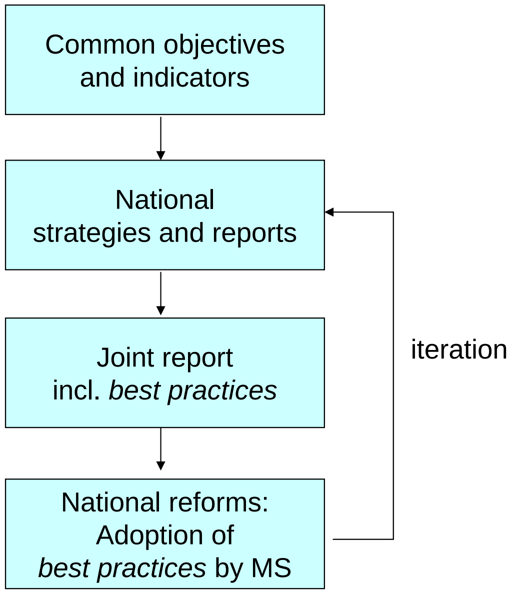

Figure 1: The iterative OMC process.

that, at the same time, allows for country-specific deviations. The resulting political process exhibits evolutionary learning with imitation and experimentation. This allows us to use concepts and results from evolutionary game theory and stochastic learning in games.

More specifically, we embed the OMC into a standard game of redistribution from rich to poor in a multi-country setting with labour mobility. As a stylized example of social policy, this application seems reasonably close to the contexts for which the OMC was originally designed. In our model, the poor beneficiaries of transfers are internationally mobile and settle where the welfare state is most generous. Governments are inequality-averse utilitarians and, thus, have a preference for redistribution. With decentralization, transfer policies are best responses to the policies of other jurisdictions. The decentralized redistribution game has a large set of Nash equilibria. With the OMC, however, governments follow an imitative behavioral rule, choosing policies that perform best *relative* to what other governments do. As we show, this leads them to evolutionarily stable strategies (ESS). An ESS is a strategy which, once chosen by all players, cannot be invaded by any competing alternative strategy. ESS are also the outcome of imitative learning. For an iterated process like the OMC with its emphasis on copying best practices evolutionary stability, thus, appears to be the appropriate solution concept.

We want to stress the importance of the strategic effects arising in a setting with a finite number of players. In our analysis we compare the ESS resulting from the OMC to Nash equilibria and co-operative solutions of the policy competition game. These two solution concepts represent the traditional modes of governance in the EU: they are the outcomes when, respectively, policies remain fully decentralized (fiscal competition) or are fully coordinated (integration or community method). In general, all these concepts differ in the presence of payoff externalities (in our model: fiscal externalities). It therefore makes sense to put the OMC into an explicitly game-theoretic context, even though the literature on the OMC hardly ever mentions strategic interdependencies as a relevant impact factor on the method's performance.[2]

In Section 3 we find that the ESS coming out of the OMC are a strict subset of the set of Nash equilibrium strategies of the decentralized redistribution game. Hence, in our model the OMC cannot achieve anything that could not be obtained also via a decentralized approach. However, the OMC avoids some extreme outcomes that are possible under decentralization.

As our main contribution, we provide in Section 4 a dynamic approach that reflects the iterative and imitative gist of the OMC. For commonly agreed-upon objectives (represented in the model by a social welfare function), governments choose policies mimicking what was observed to be successful in previous periods; this captures the idea of learning from other's experience. The "open" nature of the OMC process is modeled as experimentation: there is no binding commitment to adopt best practices and countries are free to implement policies as they wish. We show that this process of imitation and experimentation leads to the emergence of ESS, although not all ESS will be equally robust to experimentation. In particular, we show that long-run equilibria emerge from the "medium" range of ESS, resulting in a convergence towards moderate redistribution policies where transfers to the poor are neither extremely low nor overly generous. As we illustrate in Section 5 long-run equilibria are not necessarily related to efficient outcomes and the OMC may result in underprovision as well as in overprovision of redistribution.

[2] A few exceptions should be mentioned: Pestieau (2005) and Coelli et al. (2008) relate the OMC to yardstick competition and its idea that information spill-overs would enable citizens to compare the performance of their governments with that of goverments elsewhere and then to punish and reward politicians. However, no formal analysis is provided. Büchs (2008) informally analyzes the OMC as a "two-level" (more precisely: two-stage) game where governments first agree on objectives and then implement policies to meet these objectives.

Empirically, a trend of convergence of social policies and the absence of a race-to-the-bottom have been observed for OMC participants by Coelli et al. (2008). This fits well to our theoretical observations. Formally, our analysis of the OMC is closely related to that of price competition in an oligopolistic market for a homogeneous good with convex costs. Akin to consumers in such markets, poor individuals in our model settle down in the country with most generous transfer policies. Alós-Ferrer et al. (2000) show that imitative learning perturbed by occasional price experimentation results in coordination on *central* prices which correspond to our predicted intermediate subsidy levels. Technically, our analysis follows the work on stochastic learning models proposed by Foster and Young (1990), Young (1993), Kandori et al. (1993), and Samuelson (1994). In the proofs we use the radius-coradius technique developed by Ellison (2000).

Implicit in our analysis is the assumption that OMC participants have a symmetric payoff function. Taken literally, this assumption implies that those countries are identical, which is clearly unrealistic in view of the large diversity among EU member states. Yet it makes sense in an analysis of the OMC. Recall that the OMC postulates that countries agree on common objectives. Moreover, it operates under the premise that successful policy measures adopted in one country also work in a favorable way when adopted in another country. Without the assumptions of common objectives and identical effects of policy measures the idea of imitating best practises is called into question. Our assumptions of identical countries and a symmetric game, thus, capture an essential feature of the OMC.

To summarize, our paper makes the following points. Models of learning and evolution in games provide a suitable framework for an analysis of the OMC. Evolutionary stability captures both the static (relative performance) and the dynamic features (learning process) of the OMC. The imitative process of the OMC converges and settles at intermediate transfer levels. Finally, the OMC does not necessarily lead to efficient outcomes, although the degree of inefficiency may be greatly reduced thanks to the strong refinement result that enables countries to coordinate in a small subset of policies relative to the great number of coordination possibilities that are compatible with complete descentralization.

The rest of the paper is organized as follows. Section 2 sets up a game of decentralized redistribution. Section 3 derives Nash equilibria and ESS for that game (static analysis). Section 4 presents a formal analysis of the imitation dynamics induced by the OMC. Section 5 discusses efficiency issues and some basic extensions of the model. Section 6 concludes. All proofs are relegated to the Appendix.

2 The Model

2.1 Mobility and redistribution

There are $n \geq 2$ identical countries that form an integrated economic area with free mobility. Countries, indexed by $i = 1, \ldots, n$, decide on whether and, if so, to what degree to engage in redistribution among their residents. In each country there is one very rich and immobile resident earning w_R; this normalization to one rich per country is innocuous in our framework. A large population of poor individuals, each earning $w_P < w_R$, can benefit from redistribution. Poor individuals are perfectly mobile and decide where to establish their residence based on the generosity of social policy. Let $\nu \geq n$ be the total size of the population of poor individuals in the economic area. Each individual inelastically supplies one unit of labor. Thus, labor supply and basic earnings do not depend on social policy; this guarantees that the total size of the population of individuals affected by redistribution is constant.

We denote by ℓ_i (with $0 \leq \ell_i \leq \nu$) the amount of mobile poor living in country i. Redistribution from rich to poor is organized as follows. Each country i implements a non-negative lump-sum transfer, s_i, payable to each poor within its jurisdiction and financed by a lump-sum tax t_i on its rich resident. Government budgets are required to balance; i.e., the sum of transfers equals the amount of revenues raised:

$$s_i \cdot \ell_i = t_i.$$

With such a redistribution scheme, consumption levels of the poor and the rich residing in country i, respectively, amount to

$$c_i^P = w_P + s_i \quad \text{and} \quad c_i^R = w_R - t_i = w_R - \ell_i \cdot s_i.$$

The set of possible subsidies is restricted to the interval $S = [0, w_R]$. We henceforth write $\mathbf{s} = (s_1, \ldots, s_n) \in S^n$ for vectors of redistributive policies. It is convenient to use notation $\mathbf{s} = (s_i | \, \mathbf{s}_{-i})$, where s_i is the subsidy chosen by country i and (with some abuse of notation) $\mathbf{s}_{-i}$ is the vector of subsidies chosen by countries other than country i or any permutation thereof. Finally, denote $\overline{s}_i = \max_{j \neq i} s_j$; for any given i, $\overline{s}_i$ is the maximum subsidy chosen by any country other than i.

Individuals care only about their consumption. Thus, mobile individuals establish their residence in the country with the most generous redistribution policy. Given $\mathbf{s} = (s_i | \, \mathbf{s}_{-i})$, denote by $M_i(\mathbf{s}_{-i}) = \{j \neq i | s_j = \bar{s}_i\}$ the set of countries offering the highest subsidy when we exclude i and let $m_i(\mathbf{s}_{-i}) = |M_i(\mathbf{s}_{-i})|$ be its cardinality. Given a vector $\mathbf{s}$ of subsidies, we denote the distribution of mobile poor by $(\ell_1(\mathbf{s}), \dots, \ell_n(\mathbf{s}))$, where $\ell_i(\mathbf{s})$ denotes the amount of poor residing in country i. Assume that whenever two countries, i and j, choose the same transfer level, they attract the same amount of poor; i.e.

$$s_i = s_j \quad \Longrightarrow \quad \ell_i(\mathbf{s}) = \ell_j(\mathbf{s}).$$

As the mobile poor settle only in the most generous countries, their distribution across countries follows the pattern in (1):

$$\ell_i(\mathbf{s}) = \ell(s_i|\ \mathbf{s}_{-i}) := \begin{cases} 0 & \text{if } s_i < \bar{s}_i \\ \frac{\nu}{1+m_i(\mathbf{s}_{-i})} & \text{if } s_i = \bar{s}_i \\ \nu & \text{if } s_i > \bar{s}_i. \end{cases} \tag{1}$$

Clearly, $0 \leq \ell_i(\mathbf{s}) \leq \nu$ and $\sum_{i=1}^{n} \ell_i(\mathbf{s}) = \nu$. Two observations about (1) will become important later on. First, the fraction of the poor residing in country i is invariant to permutations of other countries' subsidies. Second, so expressed, $\ell(s_i|\ \mathbf{s}_{-i})$ is also the amount of poor that would reside in *any* country (not only i) choosing $s = s_i$ when all other countries choose subsidies according to $\mathbf{s}_{-i}$.

We postulate our model in terms of governments choosing subsidies that attract mobile poor. This is in line with models by Wildasin (1991, 1994), Cremer and Pestieau (2003), and many others. Alternatively, we could have chosen to make poor individuals immobile, let rich individuals be mobile, and governments choose taxes instead of subsidies. This would not change the essence of our analysis.[3] A potentially more critical modeling choice is the assumption that *all* poor are mobile. In Section 5 we discuss the robustness of our results to the introduction of some immobile poor. Due to costless mobility, migration responses in our model are extremely sensitive and discontinuous: a slight change in transfers might cause a complete reshuffling of the population in the economic area. This assumption, which is similarly made in other papers (see, e.g., Cremer and Pestieau, 2003; Kolmar, 2007), gives our approach a Bertrand-type flavor. In a companion paper, Ania and Wagener (2009) consider a model with smooth migration flows.

[3] It is well-known, however, that results for decentralized redistribution change if *both* tax payers and welfare recipients are mobile. See e.g. Leite-Monteiro (1997).

2.2 Policy objectives

At least since Mansoorian and Myers (1997), it is well-known that government objective functions play an important role in decentralized redistribution games where population sizes are endogenous (also see Cremer and Pestieau, 2004). We consider here utilitarian governments that evaluate individuals' utility derived from consumption by some utility function, $u(c)$, which is twice continuously differentiable and such that $u'(c) > 0 > u''(c)$ for all $c \geq 0$. We assume that $u(w_P) \geq 0$, and there exists $K \geq 0$ such that $u(0) < -K$.

The government of any country $i = 1, \ldots, n$ assesses different policies by comparing the sum of the utilities of those currently living in i under such policies; i.e.

$$\pi_i(\mathbf{s}) = \ell_i(\mathbf{s}) \cdot u(c_i^P) + u(c_i^R) = \ell_i(\mathbf{s}) \cdot u(w_P + s_i) + u(w_R - \ell_i(\mathbf{s}) \cdot s_i). \tag{2}$$

The fact that $\ell_i(\mathbf{s})$ is invariant to permutations of other countries' subsidies allows to write payoffs also as:

$$\pi_i(\mathbf{s}) = \pi(s_i|\, \mathbf{s}_{-i}) = \ell(s_i|\, \mathbf{s}_{-i}) \cdot u(w_P + s_i) + u(w_R - \ell(s_i|\, \mathbf{s}_{-i}) \cdot s_i), \tag{3}$$

where now $\pi(s_i|\, \mathbf{s}_{-i})$ is the payoff to *any* country choosing $s = s_i$ when *all other* countries choose subsidies according to the vector $\mathbf{s}_{-i}$.

If all governments set identical transfers, the poor will be equally distributed across countries (i.e., $\ell_i(\mathbf{s}) = \nu/n$). Then, the optimal subsidy is given by[4]

$$s^0 \;:=\; \arg\max_{s \in S} \left\{ \frac{\nu}{n} \cdot u(w_P + s) + u(w_R - \nu/n \cdot s) \right\} = \frac{w_R - w_P}{1 + \nu/n}.$$

We refer to s^0 as the *efficient symmetric* solution; transfers s^0 lead to an egalitarian income distribution.

The objective function (2) is called *generalized utilitarianism.* In settings with variable population sizes, it is one out of many utilitarian-type social welfare functions (Blackorby et al., 2009). A serious flaw of generalized utilitarianism is that it gives rise to the so called *repugnant conclusion* (Parfit, 1982; Blackorby et al., 2009) – for every population

[4]To see this, first observe that s^0 is strictly positive since the objective function is strictly increasing at $s = 0$ by strict concavity of $u(c)$ and the fact that $w_P < w_R$. Moreover, s^0 must satisfy the first order condition $u'(w_P + s^0) = u'(w_R - \nu/n \cdot s^0)$ which gives the expression in the second line of (4); clearly $s^0 < n/\nu \cdot w_R$, so that the rich is not completely expropriated in the symmetric efficient allocation.

of arbitrary well-offs, there exists another, suitably larger population of paupers such that utilitarians will strictly prefer the latter to the former. This substitutability of population size for quality of life is ethically questionable. Obviously, the set of Nash equilibria, and our results, depend crucially on the choice of objectives. Our focus here is on the workings of the OMC *given a particular type of objective function,* which is meant to reflect the common objectives and target indicators that member states agreed upon.

3 Static Analysis: Nash equilibria vs. ESS

The model presented in Section 2 defines a game where the players (countries $i = 1, \ldots, n$) simultaneously choose subsidies out of a common strategy set given by the feasible set of subsidies $S = [0, w_R]$. Migration decisions as summarized by expression (1) determine payoffs $\pi_i : S^n \to \mathbb{R}$ given by expression (2). The game is symmetric, since payoffs can be written as $\pi_i(\mathbf{s}) = \pi(s_i | \ \mathbf{s}_{-i})$, shown in (3). Before proceeding with the analysis, let us recall here the definitions of a *symmetric* Nash equilibrium and a *finite-population* evolutionarily stable strategy and shortly comment on the difference between the two concepts. By focusing directly on symmetric equilibria we can write both definitions using the same notation.

Definition 1 *A strategy s^N is played in a* symmetric Nash equilibrium *if*

$$\pi(s^N | \ s^N, s^N, \ldots, s^N) \geq \pi(s | \ s^N, s^N, \ldots, s^N) \qquad \text{for all } s \in S.$$

A strategy $s^E \in S$ is said to be an evolutionarily stable strategy *(ESS) if*

$$\pi(s^E | \ s, s^E, \ldots, s^E) \geq \pi(s | \ s^E, s^E, \ldots, s^E) \qquad \text{for all } s \in S.$$

We say that a Nash equilibrium or an ESS is strict *if the corresponding inequality holds strictly for all $s' \neq s$.*

Schaffer (1988) gives a definition of evolutionary stability for a finite population of N individuals who are randomly matched to play an n-person game. We take here Schaffer's definition for the case of $n = N$.[5]

[5] See also Vega-Redondo (1996, pp. 31-33) for a discussion of this concept. See Crawford (1991) and Tanaka (2000) for closely related concepts. See Nowak et al. (2004) for a recent dynamic concept of evolutionary stability for finite populations.

In a Nash equilibrium no player would strictly benefit from a deviation, given what other players are doing. In an evolutionarily stable profile no player would be able to gain a strict *relative* advantage by deviating. Note that for a Nash equilibrium we compare the deviator's payoffs before and after deviation. In an evolutionarily stable profile, instead, we compare payoffs to the deviator, choosing s, with payoffs to the non-deviators, choosing s^E, *after* a unilateral deviation. For this reason, when the population is finite and each player has a non-negligible impact on the payoffs of all other players, it may pay in relative terms to deviate from a Nash equilibrium, if the loss imposed on non-deviators is bigger than the loss suffered by the deviators themselves. This is referred to as *spiteful* behavior (Hamilton, 1970).[6]

Before we characterize the Nash equilibria and the ESS of the game, let us introduce the following family of auxiliary functions.

$$f(k,s) = \frac{\nu}{k} \cdot u(w_P + s) + u\left(w_R - \frac{\nu}{k} \cdot s\right)$$

where $k \in \{1, \ldots, n\}$. The value of $f(k,s)$ can be interpreted as the payoff to any of k countries equally sharing all the poor at subsidy level s, of course provided they attract the poor with that subsidy (i.e. s is currently the maximum subsidy). Note that $f(k,0) \geq u(w_R)$. By the strict concavity of $u(c)$ and since $w_P < w_R$, we get that $f(k,s)$ is strictly increasing at $s = 0$. Moreover, for every k, $f(k,s)$ is strictly concave in s. Let

$$s^*(k) = \arg\max_{s \geq 0} f(k,s).$$

The properties of f guarantee that $s^*(k)$ is strictly positive for all k and it must satisfy the first order condition

$$u'(w_P + s^*(k)) = u'(w_R - \nu/k \cdot s^*(k)),$$

which yields

$$s^*(k) = \frac{w_R - w_P}{1 + \frac{\nu}{k}}.$$

Given that k countries share the burden of redistribution, they maximize social welfare by choosing $s = s^*(k)$. Clearly, $0 < s^*(k) < k/\nu \cdot w_R$ is increasing in k; i.e., social policy should optimally be more generous as more countries engage in redistribution.

[6] Such considerations would not play a role in a continuum population, since each player has a negligible impact on the payoffs of others in that case. It is well known that ESS are always Nash equilibrium strategies in a continuum population. See, e.g., Fudenberg and Levine (1998, p. 59).

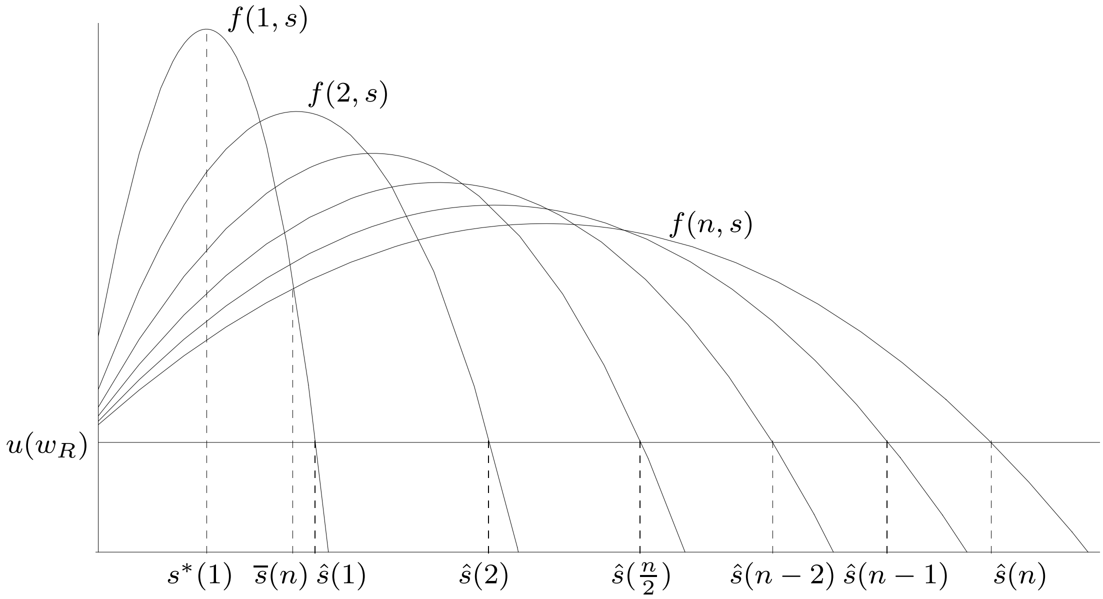

Figure 2: Properties of auxiliary welfare functions $f(k, s)$.

Given any $k \in \{1, \ldots, n\}$, define $\widehat{s}(k)$ as the strictly positive value of s that solves

$$f(k, \widehat{s}(k)) = u(w_R),$$

and $\bar{s}(k)$ as the strictly positive value that solves

$$f(k, \bar{s}(k)) = f(1, \bar{s}(k)).$$

At $\widehat{s}(k)$, a country is indifferent between paying transfers $\widehat{s}(k)$ to ν/k poor and not attracting any poor at all. At $\bar{s}(k)$, a country is indifferent between paying transfers $\bar{s}(k)$ to ν/k poor and, at the same transfer, hosting all poor. In Appendix A we show that the family of functions $\{f(k, s)\}_{k=1,\ldots,n}$ and the corresponding values of $\widehat{s}(k)$ and $\bar{s}(k)$ have the properties depicted in Figure 2. In particular, we show that

$$s^*(1) < \bar{s}(2) < \bar{s}(3) < \ldots < \bar{s}(n) < \widehat{s}(1) < \widehat{s}(2) < \ldots < \widehat{s}(n). \tag{4}$$

This observation allows an easy characterization of Nash equilibria and ESS.[7] We first show that our game has a large set of symmetric, pure-strategy Nash equilibria:

[7] Our game has a structure akin to that of an oligopolistic market of the type analyzed by Dastidar (1995), where firms have decreasing returns to scale and compete in prices.

Proposition 1 *Under generalized utilitarianism the set of pure-strategy Nash equilibria is given by*

$$\Sigma^N = \{\mathbf{s} = (s, \ldots, s) \mid \bar{s}(n) \leq s \leq \widehat{s}(n)\}.$$

Observe that transfer levels at Nash equilibria are quite generous; even overprovision of transfers (i.e., values of s^N larger than $s^0 = s^*(n)$) is possible in a Nash equilibrium. This is in contrast with the widespread fear of an erosion of the welfare state due to migration pressures (Cremer and Pestieau, 2004). It is a consequence of the government objective which entails a strong preference for large population sizes (recall the *repugnant conclusion*). Hence, the widely feared demise of the welfare state does not occur in our model (see Secton 5 for modifications).

Our next proposition characterizes the set of ESS. It shows that ESS are a strict subset of the set of Nash equilibrium strategies found in Proposition 1:

Proposition 2 *Under generalized utilitarianism the set of ESS is the interval*

$$S^E = [\widehat{s}(1), \widehat{s}(n-1)].$$

Proposition 2 conveys that whatever can be achieved at an ESS could also be achieved through decentralization at a Nash equilibrium. The set of ESS, however, is a strict subset of the set of Nash equilibria, precluding some extremely low and extremely high subsidies that can be rationalized as a Nash equilibrium. In particular, for $s \in [\bar{s}(n), \widehat{s}(1))$, subsidies are still too low and a single country could still achieve a relative advantage through more redistribution even if it attracted all poor. The situation with $s \in (\widehat{s}(n-1), \widehat{s}(n)]$ is also too unstable; if a single country were to lower its subsidy, all others would be left with too high subsidies given the number of countries sharing the burden of redistribution.

It is worth pointing out the special feature of the upper bound in the interval S^E. Starting at the symmetric profile where all countries set $s = \widehat{s}(n-1)$, a deviation downwards to some $s' < s = \widehat{s}(n-1)$ would result in a relocation of all poor among the remaining $n-1$ non-deviating countries; their payoff, however, would be exactly $u(w_R)$ by definition of $\widehat{s}(n-1)$ and the deviator would have no strict disadvantage. This cannot happen with any other ESS in the interval S^E — from any other ESS a deviator would suffer a strict disadvantage. All ESS in the interval are strict except the upper bound $\widehat{s}(n-1)$. This will make a subtle difference in the dynamics analysis below.

4 The OMC as a Dynamic Imitative Process

In the present section we come to what we consider is the spirit of the OMC. We now explicitly take a dynamic approach, allowing countries to observe each others' subsidies and welfare levels and to make sequential decisions based on this information. We assume that countries tend to adopt subsidy levels associated with the highest welfare levels currently observed. Occasionally, countries can experiment with random subsidies. However, such experiments are followed by other countries only when they prove to be successful compared to other currently observed subsidy levels and will not persist otherwise. The model intends to capture the main features of the OMC as stated by the European Commission (see Figure 1 again). In our model, the (symmetric) welfare function stands for the commonly agreed-upon objectives; subsidies are the policy instrument and each period welfare levels constitute the target indicator that is reported by each country. Our imitation dynamics intends to capture the iterative loop by which countries learn from each other's experience. Finally, experimentation captures the open nature of the process; namely, the most successful policies observed and the recommendations of the Commission are not binding – countries are allowed to adopt other policies based on their own motivations to do so, which may range from mistakes to national political interests. We now proceed to introduce and analyze the dynamic model.

The analysis is applied to a discretized version of the model presented in Section 3. Specifically, we assume that countries choose subsidies from a finite set $\Gamma \subset S$. For simplicity of exposition, we assume that Γ contains the values $\widehat{s}(k)$ for all $k = 1, \dots, n$.[8] The state space of the process is Γ^n, the state at $t = 1, 2, \dots$ is given by the vector of subsidies chosen by all countries at t denoted

$$\mathbf{s}(t) = (s_1(t), \dots, s_n(t)) .$$

Subsidies at t determine welfare levels given by the vector

$$\pi(\mathbf{s}(t)) = (\pi_1(\mathbf{s}(t)), \dots, \pi_n(\mathbf{s}(t))) ,$$

where $\pi_i(\mathbf{s}(t))$ is the welfare attained by country i in state $\mathbf{s}(t)$ and is defined as in expression (2) in Section 3. In any period $t = 1, 2, \dots$ all countries observe $\mathbf{s}(t)$ and the vector $\pi(\mathbf{s}(t))$. Given any $\mathbf{s}(t)$, define the set

$$\widehat{B}(\mathbf{s}(t)) = \{ s \in \Gamma \mid s = s_i(t) \text{ for some } i \text{ and } \pi_i(\mathbf{s}(t)) \geq \pi_j(\mathbf{s}(t)) \text{ for all } j \} .$$

[8]Note that this assumption may preclude Γ from being a regular grid.

$\widehat{B}(\mathbf{s}(t))$ contains all subsidy levels that earned highest welfare in period t. Following the OMC guideline to adopt best practices, only subsidies in $\widehat{B}(\mathbf{s}(t))$ should be followed with positive probability. This gives a clear policy guideline whenever $\widehat{B}(\mathbf{s}(t))$ is a singleton. For the case that there is more than one best-performing policy in t, it is, however, unclear what further motives should determine policy choice. To the end of reaching a unique policy recommendation, let $\overline{s}(t) = \max_j s_j(t)$ and define

$$B(\mathbf{s}(t)) = \widehat{B}(\mathbf{s}(t)) \cap \{\overline{s}(t)\}.$$

The set $B(\mathbf{s}(t))$ singles out the highest subsidy in t whenever this was also a best-performing policy. However, $B(\mathbf{s}(t))$ is empty when countries that offered the highest subsidies in t were not among the most successful then.[9] Let us now describe the imitative behavioral rule that we propose as a model of the OMC.

Iterative OMC process

Strategy revision takes place according to the following *imitative behavioral rule.* With probability $0 < \lambda < 1$ a country may be called to revise its subsidy for the next period. A revising country chooses the policy recommendation contained in $B(\mathbf{s}(t))$, whenever this set is not empty. I.e., a revising country will choose the most generous subsidy level of the previous period, provided that this level was among the best-performing policies then. If $B(s(t)) = \emptyset$, i.e., if the most generous subsidy level was not among the most successful ones in period t, we assume that best-performing countries (i.e., those with subsidies in $\widehat{B}(\mathbf{s}(t))$) keep their strategies unchanged while all other countries choose any of the subsidies in $\widehat{B}(\mathbf{s}(t))$ with positive probability. With probability $1 - \lambda$ a country does not get a revision opportunity.

To formalize the open, non-binding character of the OMC, we allow countries to engage in *experimentation* as follows. At the end of each period t, each country has probability $1-\varepsilon$ to follow the imitative rule just described and no experimentation takes place. With probability ε, however, the country gets an experimentation draw and it may choose any subsidy $s \in \Gamma$ at random.

Both, imitation and experimentation opportunities, are drawn independently across countries and across periods. A country keeps its subsidy unchanged if no imitation or experimentation opportunity arrives at t.

[9] For example, that would be the case if only a few EU member states actively engaged in sufficiently generous redistribution policies. This would trigger too much immigration into those countries, rendering their social policies unprofitable. In the model this occurs when less than k countries set subsidy $s \geq \widehat{s}(k)$ and $s = \max_j s_j$.

A few comments on the behavioral rule that we postulate are in order. First, our imitative rule incorporates a certain predilection for transfer generosity. If high subsidies do not cause harm to social welfare, then countries will grant them. Out of several policies that give identical levels of overall welfare, countries are recommended to choose the one with highest transfers to the poor. Such behavior appears plausible when the OMC is applied in the context of social policy. Our imitative rule also incorporates some conservative upholding. If countries with less generous transfer policies are performing best, in particular better than countries with the highest subsidies, then they will keep their policies.

The inertia incorporated by λ in our behavioral rule captures asynchronous evaluation and policy adoption in the different countries which may be due for example to restrictions in the administrative or political decision-making process that are left out of the model. Finally, experimentation captures the soft-low features of the OMC. It enables countries to ignore the policy guidelines without an explicit punishment and opens the door to the implementation of tailor-made policies that respond exclusively to individual national interests. It also captures errors and true trial and error experimentation. We are now ready to proceed with the analysis.

The iterative OMC process described by the imitative behavioral rule with experimentation defines a stationary Markov process. Given λ and ε, let $P^{\lambda,\varepsilon}_{\mathbf{s},\mathbf{s}'}$ be the probability of a direct transition of the process from state $\mathbf{s}$ to state $\mathbf{s}'$. The transition matrix of the process is given by

$$P^{\lambda,\varepsilon} = \left(P^{\lambda,\varepsilon}_{\mathbf{s},\mathbf{s}'}\right)_{\mathbf{s},\mathbf{s}'\in\Gamma^n}.$$

We refer to the process with $\varepsilon = 0$ as the *unperturbed imitation dynamics*. A state $\mathbf{s}$ is called *absorbing* if $P^{\lambda,0}_{\mathbf{ss}} = 1$; i.e. absorbing states are the steady states of the unperturbed dynamics. An obvious property of imitation is that it leads to *monomorphic* states in our setup, states where all countries choose the same subsidy level, defined by

$$M = \{\mathbf{s} \in \Gamma^n \mid \mathbf{s} = (s, \ldots, s),\ s \in \Gamma\}.$$

From any state $\mathbf{s}(t) \notin M$, imitation alone cannot increase the number of different strategies chosen. Eventually, a unique subsidy will be singled out as the best performing policy and will be adopted by all countries. Once a monomorphic state is reached, imitation alone cannot lead the system away. This gives us the following lemma.

Lemma 1 *The set M of monomorphic states is the set of absorbing states of the unperturbed imitation dynamics $P^{\lambda,0}$.*

With experimentation, $\varepsilon > 0$, there is positive probability to exit every state. We refer to the Markov process with experimentation as the *perturbed imitation dynamics*. This process is ergodic since there is positive probability that any subset of countries experiment simultaneously with any subsidy so that $P^{\lambda,\varepsilon}_{\mathbf{ss}'} > 0$ for all $\mathbf{s}, \mathbf{s}' \in \Gamma^n$. Moreover, the process is aperiodic, since at any time there is positive probability that no country revises its subsidy due to inertia and the process stays at the same state for one period which gives $P^{\lambda,\varepsilon}_{\mathbf{ss}} > 0$ for all $\mathbf{s}$. For every λ and ε, this guarantees convergence to a unique invariant distribution denoted

$$\mu^{\lambda,\varepsilon} = \left(\mu^{\lambda,\varepsilon}(\mathbf{s})\right)_{\mathbf{s}\in\Gamma^n},$$

which satisfies $\mu^{\lambda,\varepsilon} = \mu^{\lambda,\varepsilon} P^{\lambda,\varepsilon}$, where $\mu^{\lambda,\varepsilon}(\mathbf{s})$ gives both, the average frequency with which the process visits any state $\mathbf{s} \in \Gamma^n$ along any sample path (Ergodic Theorem) as well as the probability that the system is at any state $\mathbf{s} \in \Gamma^n$ in the long run (Fundamental Theorem of Markov Chains).

Following the literature on stochastic evolutionary learning models, the analysis in this paper focusses on the *limit* invariant distribution, $\mu^\lambda = \lim_{\varepsilon\to 0} \mu^{\lambda,\varepsilon}$. States with positive probability in μ^λ are called *stochastically stable* (alternatively, *long-run equilibria*). These are the states that are observe almost always as the probability of experimentation goes to zero. The limit invariant distribution exists and it only gives positive probability to absorbing states of the unperturbed dynamics, the monomorphic states in M from Lemma 1[10]. We now provide an intuitive explanation of our main result which appears in Proposition 4 at the end of this section. Formal proofs are relegated to Appendix C.

Define the set E as the subset of monomorphic states where all countries choose an ESS,

$$E = \{\mathbf{s} \in M \mid \widehat{s}(1) \leq s \leq \widehat{s}(n-1)\}.$$

E corresponds essentially to the interval S^E from Proposition 2. By definition, from states in E a single deviating country would perform worse than non-deviators and would not be followed by imitation. The disadvantage of a single deviator is strict everywhere except at the upper bound of the interval S^E (cf. Section 3). If all countries currently choose $\widehat{s}(n-1)$ and a single country lowers its subsidy, both the deviator and non-deviators

[10]See e.g. Young (1993, Theorem 2), or Fudenberg and Levine (1998, Chapter 5)

attain payoff $u(w_R)$, but non-deviators offer highest subsidy $\widehat{s}(n-1)$, which is still the single element of $B(\mathbf{s}(t))$; thus, the deviating country will not be followed.

All other states in M can be abandoned after a single deviation in favor of some state in E. To see this, suppose the process is currently at a monomorphic state with $s < \widehat{s}(1)$ and, in period t, one of the countries deviates to $\widehat{s}(1)$. The deviating country is the only one actively performing redistribution and attains welfare level exactly equal to $u(w_R)$. Thus, $B(\mathbf{s}(t)) = \{\widehat{s}(1)\}$ and the deviator will be followed at the next revision opportunity. The case of high subsidies is analogous. Suppose the process is currently at a state where all countries set subsidy $s > \widehat{s}(n-1)$. If at t a single country lowers its subsidy (e.g. to any subsidy in the interval S^E), the welfare of the remaining $n-1$ non-deviating countries falls strictly below $u(w_R)$. In this case $B(\mathbf{s}(t))$ is empty and $\widehat{B}(\mathbf{s}(t)) = \{s\}$; the deviator would not change its subsidy, if it was called to review its strategy again, while all other countries will follow the deviator at the next revision opportunity. These arguments show that E can be reached from outside if a single country experiments with a subsidy that is ESS, but E cannot be abandoned with a single deviation. This makes it more likely for the process to move in than out of the set E.Recall from Section 3 that the interval S^E is itself a subset of the set of strategies played in any Nash equilibrium. Our prediction, summarized in the following proposition, is that the process eventually moves into the set E and, thus, some Nash equilibrium is always observed in the long run.

Proposition 3 *If* $\mathbf{s}^*$ *is stochastically stable, then* $\mathbf{s}^* \in E \subset \Sigma^N$*.*

By definition of ESS, single experiments are not enough to move the process from one state to another within the set E. Yet it is possible to abandon states in E after the simultaneous deviation of several experimenting countries. We will argue that the number of simultaneous experiments needed to disturb states in E increases as we move towards intermediate levels of the subsidy. In our construction, it will be enough to argue with simultaneous deviations to the *same* alternative subsidy level. It is easy to see that if a transition between two monomorphic states is possible after the simultaneous deviation of at least two countries experimenting with different subsidies, then the same transition is possible with the same number of deviations to the same subsidy.[11]

[11] Intuitively, this is due to $\widehat{s}(k)$ being increasing with k, so that subsidies that can be successfully sustained by k deviators would have always be successful if more deviators had originally chosen them.

We now proceed to refine our prediction within the set E. To that purpose, define the collection of sets

$$E^k = \{\mathbf{s} \in M \mid \widehat{s}(k-1) \le s \le \widehat{s}(k)\}, \qquad k = 2, \dots, n-1.$$

States in E^k are *monomorphic* states with subsidies in the interval $[\widehat{s}(k-1), \widehat{s}(k)]$ and we have that $E = \bigcup_{k=2}^{n-1} E^k$. It is also convenient to introduce some notation for monomorphic states with subsidy equal to $\widehat{s}(k)$; often this *boundary* states of the sets E^k have to be treated separately in our analysis. We denote

$$\mathbf{s}^k = (\widehat{s}(k), \dots, \widehat{s}(k)).$$

Note that $E^k \cap E^{k+1} = \{\mathbf{s}^k\}$ for $k = 2, \dots, n-2$. Subsidies in the states of E^k can be profitably sustained with at least k countries, where *profitably* hear means with welfare higher than or equal to $u(w_R)$. Instead, if strictly less than k countries actively engaging in redistribution set a subsidy in $(\widehat{s}(k-1), \widehat{s}(k)]$, welfare will fall strictly below $u(w_R)$. We want to understand what it takes for the process to abandon states in E. Moving to higher subsidies requires that enough countries coordinate to sustain this profitably. Moving to lower subsidies requires that too few countries are left to sustain high subsidies profitably. We now argue this in more detail.

- *Moving from lower to higher subsidies.*

  The process moves away from any state in $E^2 \setminus \{\mathbf{s}^2\}$ if *two* countries simultaneously deviate to the same higher subsidy in the interval $(\widehat{s}(1), \widehat{s}(2)]$, so that two simultaneous deviations suffice to move the process from any state in $E^2 \setminus \{\mathbf{s}^2\}$ to any other state in E^2 with a strictly higher subsidy and, in particular, to the *boundary* state $\mathbf{s}^2$. Analogously, the process moves from any state in $E^3 \setminus \{\mathbf{s}^3\}$ to any other state in E^3 with strictly higher subsidy if *three* countries simultaneously deviate to the same higher subsidy in $(\widehat{s}(2), \widehat{s}(3)]$, and so on. In general, given any pair $\mathbf{s}, \mathbf{s}' \in E^k$ with $s < s'$, state $\mathbf{s}'$ can be reached from $\mathbf{s}$ if k countries simultaneously deviate to s'. Moreover, for all k, state $\mathbf{s}^k$ can be reached from *a*ny monomorphic state with lower subsidy either directly with k simultaneous deviations to $\widehat{s}(k)$ or through a chain of single transitions going through all $E^{k'}$ with $k' < k$. It is also important to note that less than k deviators are not enough to move the process to some $\mathbf{s} \in E^k \setminus \{\mathbf{s}^{k-1}\}$ from *lower* states. Coordinating on higher and higher subsidies, thus, requires more and more countries and, hence, *transitions upwards* become less likely for higher k. Moreover, the most likely upward transitions are stepwise transitions from any subsidy level to the next one in Γ.

- *Moving from higher to lower subsidies.*

  Since migration takes place to countries with the highest subsidies, experimenting with lower subsidies means losing all poor and giving up active redistribution policy. At the same time, if countries currently choosing different subsidies attain (the same) highest welfare level, then our decision rule prescribes to mimic the most generous subsidy. It follows that, deviations to lower subsidies can only be successful if non-deviators are left with welfare strictly lower than $u(w_R)$. Note that all subsidies associated to states in E^{n-1} can be sustained by $n-1$ countries with welfare higher than or equal to $u(w_R)$. Therefore, we need at least *two* countries simultaneously lowering their subsidies in order to move downwards from states in $E^{n-1} \setminus \{\mathbf{s}^{n-2}\}$. Analogously, we need *three* countries simultaneously lowering their subsidies to move downwards from states in $E^{n-2} \setminus \{\mathbf{s}^{n-3}\}$, and so on. In general, the process will move *downwards* from any state in $E^k \setminus \{\mathbf{s}^{k-1}\}$ if $n-k+1$ countries lower their subsidies. Note that, since the number of deviations needed for a downward transition is independent of the state we intent to reach, there is no particular advantage of step-by-step transitions in this case. In general, lowering subsidies further requires more simultaneous deviations the lower subsidies already are and, hence, *transitions downwards* become less likely for lower k. Analogously, reaching any state $\mathbf{s} \in E^k$ from any *higher* state $\mathbf{s}' \in E^{k'}$ with $k' \geq k$ requires more simultaneous deviations the lower k' is.

These findings can be roughly summarized as follows. From states in E^k with low k the process is more likely to move upwards; for high k the process is more likely to move downwards. In the intermediate states these probabilities are balanced and the process is likely to stay. Detailed accounting of transition probabilities has to be done carefully depending also on whether n is odd or even and, in particular, for values of k around $n/2$. This is done in the proof in Appendix C.

In Figure 3 we illustrate the case of n odd, the relevant case for the current EU-27. Our prediction for n odd is the set $E^{\lceil \frac{n}{2} \rceil}$ (e.g. E^5 with $n = 9$). The lower and the upper bounds of the interval associated with this set can be reached from outside the set with at most $\lceil \frac{n}{2} \rceil - 1$ deviations. The most likely way to exit the set is downwards, which requires at least $\lceil \frac{n}{2} \rceil$ deviations. Within $E^{\lceil \frac{n}{2} \rceil}$ it is then possible to move up and down with exactly $\lceil \frac{n}{2} \rceil$ deviations. A typical path of the process is likely to move into $E^{\lceil \frac{n}{2} \rceil}$ eventually and then stay bouncing up and down in this set.

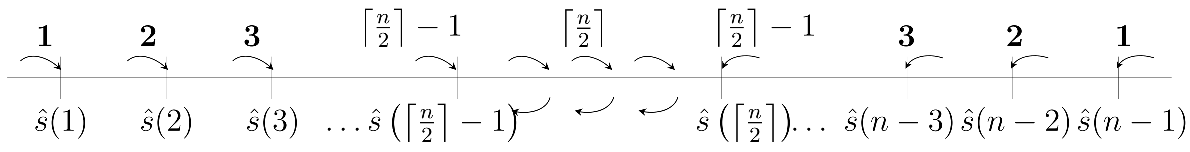

Figure 3: Most likely transition paths. The case of n odd.

Detailed counting gives a slight difference between the odd and even cases. If n is even, the central state, $\mathbf{s}^{\frac{n}{2}}$, can be sustained profitably by exactly half of the countries. This state can be reached from both, states with higher and with lower subsidies, with at most $\frac{n}{2}$ simultaneous deviations to $\widehat{s}(\frac{n}{2})$. Instead, it would take at least $\widehat{s}(\frac{n}{2})+1$ simultaneous deviations to exit the central state in favor of any lower or higher subsidy.[12] Therefore, for n even our prediction concentrates on $\mathbf{s}^{\frac{n}{2}}$. Our results are summarized in next proposition.

Proposition 4 *The set of stochastically stable states is $E^{\lceil\frac{n}{2}\rceil}$ if n is odd and $\{s^{\frac{n}{2}}\}$ if n is even.*

Proposition 4 shows that the most likely outcomes of the OMC applied to decentralized redistribution lie in the intermediate range of the ESS. Intuitively, low subsidy levels can be easily destabilized by a small number of countries coordinating on the same higher subsidy; high subsidy levels become *unprofitable* if only a small number of countries cut their subsidies. Only the intermediate subsidies, which correspond to intermediate values of the ESS are robust to such small-group experimentation. The learning process of the OMC, thus, eschews ESS with very low or overly generous support for the poor. *A fortiori*, since ESS are a subset of the set of Nash equilibria in our model, the OMC also avoids potentially extreme states that could emerge in the traditional setting of decentralized choice of redistribution policies. In that sense, the learning process in the OMC can be viewed as a moderating device.

[12] Higher subsidies cannot be profitably sustained with less than $\widehat{s}(\frac{n}{2})+1$ countries, while if only half of the countries lower their subsidies, $\widehat{s}(\frac{n}{2})$ would still be the highest, most profitable policy.

5 Discussion of the results

5.1 Speed of convergence

Our main result in Proposition 4 implies that member states in the EU adopting the OMC as a mode of governance in the area of social policy when policy beneficiaries are mobile will succeed in coordinating their policies at "intermediate" transfer values, where intermediate means that these transfer values require the coordiantion of half of the participating states in order to be sustainable. The discussion and proof of our main result reveals that moving from extreme to intermediate transfer values requires simultaneous experimentation of additional member states, up to $n/2$ for transfers in the central interval. Our prediction should be interpreted as follows. Along any sample path of our stochastic learning process, the fraction of periods in which subsidies in the central interval are observed converges to one as $\varepsilon \to 0$; i.e., if we observe the process long enough, we will essentially only observe intermediate subsidies. A natural question to ask is how long we would have to wait to actually observe these subsidies. We can use Ellison (2000, Theorem 2) to give an estimate for the expected waiting time until the process hits our prediction for the first time (cf. Lemma 2 in the Appendix). Note that the set E, containing symmetric profiles where all countries choose some evolutionarily stable subsidy, can be reached from everywhere else with a single deviation; in Ellison's terminology the coradius of E equals 1. This implies that the expected waiting time to reach E is of order ε^{-1}. Analogously, the modified coradius[13] of the intermediate interval is $[n/2]$ and, thus, the expected waiting time to reach intermediate subsidies is of order $\varepsilon^{-[n/2]}$, which increases as the number of member states participating in the OMC becomes larger. In general, the process will lead quickly to ESS values of the transfers and stay there most of the time. Coordination in states closer to the central interval, respectively in central values of the transfers may require a long time, depending on the number of countries that decide to follow the OMC.

[13]Note that intermediate transfer values in our model can be reached through step-by-step evolution moving gradually toward the central interval. The cost of these intermediate transitions is subtracted when computing the value of the modified coradius.

5.2 Efficiency

Given that, in general, an important motivation for policy coordination is the avoidance of externality-induced efficiency failures of decentralization, it seems natural to ask whether the learning process of the OMC indeed leads to efficient policy outcomes. Formally, we have to check for the relationship between the symmetric efficient solution, s^0, and the set of stochastically stable states characterized in Proposition 4. Intuitively, $s^0 = s^*(n)$ maximizes the joint welfare function $f(n,s)$, whereas the values of $\widehat{s}\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ and $\widehat{s}\left(\left\lceil \frac{n}{2} \right\rceil\right)$, relevant to define $E^{\left\lceil \frac{n}{2} \right\rceil}$, refer to properties of the functions $f(\frac{n-1}{2}, s)$ and $f(\frac{n}{2}, s)$. Technically, there is no good reason why s^0 should be related to our prediction in the set $E^{\left\lceil \frac{n}{2} \right\rceil}$. Indeed, it is possible to construct examples where the efficient outcome is in our prediction. In general, however, the predictions of our model entail subsidies which could be higher or lower than the efficient outcome. This is illustrated in Example 1 bellow. It should still be stressed that the convergence to "intermediate" subsidy levels precludes the more extreme outcomes that could emerge under decentralization in a Nash equilibrium.

Example 1 *Consider a utility function of the form $u(x) = \sqrt{x} - 1$. Take $w_P = 1$ and assume that $w_R \leq 4$. We have that*

$$\widehat{s}(k) = \frac{4\left(\sqrt{w_R} - 1\right)\left(\frac{\nu}{k} + \sqrt{w_R}\right)}{\left(\frac{\nu}{k} + 1\right)^2}$$

Take $n = 9$, in which case our prediction is always the interval $[\widehat{s}(4), \widehat{s}(5)]$. Table 1 summarizes our results for different values of w_R and ν. We see that the efficient outcome, s^0, may be lower than $\widehat{s}(4)$, in which case we would predict inefficiently high redistribution; s^0 may be higher that $\widehat{s}(5)$, in which case we would predict inefficiently low redistribution; finally, s^0 may be contained in our prediction.

w_R	ν	s^0	$\widehat{s}(4)$	$\widehat{s}(5)$
2	12	0.43	0.46	0.55
2	21	0.30	0.28	0.34
4	27	0.75	0.58	0.72

Table 1: Efficiency results

5.3 Mobility of the poor

Let us now briefly discuss the implications of our assumption on perfect mobility of the poor. We will argue that our qualitative results are robust to the introduction of a small fraction of immobile poor in each country. Consider a small variation of our model, where all countries have some fixed amount β of immobile (native) poor. The parameter β measures the relative mass of immobile poor to rich in any given country. Symmetry is preserved if β is the same for all countries. Assume $\nu > n\beta$; that is the total amount of immobile poor is smaller than the amount of mobile poor. All other features of our model are as before. The total amount of poor living in country i is now given by $\widetilde{\ell}_i(\mathbf{s}) := \beta + \ell_i(\mathbf{s})$, where $\ell_i(\mathbf{s})$ is determined as in (1). Governments are still assumed to adhere to generalized utilitarianism. Thus, payoffs can be written as in expression (2) by replacing $\ell_i(\mathbf{s})$ with $\widetilde{\ell}_i(\mathbf{s})$. Countries now always have an incentive to actively pursue some transfer policy, at least for their native poor. If no mobile poor moves to country i; i.e., if $\tilde{\ell}_i = \beta$, its government would set the transfer to maximize the following welfare function

$$g(\beta, s) = \beta \cdot u(w_P + s) + u(w_R - \beta s).$$

Note that $g(\beta, s)$ is a strictly concave function of s with $g(\beta, 0) > u(w_R)$ and $g'(\beta, 0) > 0$. The optimal transfer in the absence of mobile poor is then given by

$$\widehat{s}^0 = \frac{w_R - w_P}{1 + \beta}.$$

Our assumptions guarantee that $g(\beta, s)$ has the same properties as our auxiliary functions $f(k, s)$, derived in Appendix A. Thus, for $\beta < \nu/n$, we have that $\widehat{s}^0 > s^*(n) = s^0$. Intuitively, when the fraction of immobile poor is not too large, optimal transfer policy can be more generous when a country only has to provide social policy for the native immobile. Moreover, define $\widetilde{s}$ as the value of the subsidy that satisfies $g(\beta, \widetilde{s}) = u(w_R)$. This is the analogous to our previous $\widehat{s}(k)$, which we proved to be strictly increasing in k. Again, since $\beta < \nu/n$, we have that $\widetilde{s} > \widehat{s}(n)$. An intuitive picture can be obtained from Figure 2 by replacing the horizontal line at the value $u(w_R)$ by a humped g curve with the same features as the f curves and cutting through $u(w_R)$ further to the right. The points at which $g(\beta, s)$ cuts trough $f(1, k)$ and $f(n-1, s)$ will now be the boundaries of the ESS interval. Note finally that $g(\beta, \widehat{s}(n-1)) > u(w_R) = f(n-1, \widehat{s}(n-1))$; analogously, $g(\beta, \widehat{s}(1)) > u(w_R) = f(1, \widehat{s}(1))$. This indicates that the bounds of the ESS interval will be lower than for the case without immobile poor. All other elements of the analysis are as before.

5.4 Average utilitarianism

Finally, we turn our attention to the welfare function. Obviously, payoffs and, thus, the equilibria of our game depend crucially on the choice of welfare function (Mansoorian and Myers, 1997). We discussed at the end of Section 2 that *generalized utilitarianism*, given in expression (2), as a government objective has the drawback of allowing for the repugnant conclusion. With decentralized redistribution, this strong predilection for large population sizes leads to quite generous subsidies to the mobile poor – an observation that is at odds with the widespread fear of a decline of the welfare state in the presence of labor mobility. There exist social welfare functions that avoid the repugnant conclusion (for a survey, see Blackorby et al., 2009). One alternative that has captured some attention is *average utilitarianism*. As the name suggests, government payoffs given by

$$\pi_i^{AU}(\mathbf{s}) = \frac{1}{\ell_i(\mathbf{s})+1} \cdot \left[\ell_i(\mathbf{s}) \cdot u(w_P + s_i) + u(w_R - \ell_i(\mathbf{s}) \cdot s_i)\right]. \tag{5}$$

It can be easily checked that the *symmetric efficient* solution under average utilitarianism coincides with the one obtained for generalized utilitarianism, s^0, given by expression (4). However, average utilitarianism gives a large welfare weight to well-off people, providing strong incentives to cut back transfers to the poor. This actually results in a remarkable efficiency failure both in a Nash equilibrium and in an ESS.

To see this, note first that for $\ell_i = 0$, we have $\pi_i^{AU} = u(w_R)$. Denote $\lambda_i = \frac{\ell_i}{1+\ell_i}$. By strict monotonicity and strict concavity of u we have that, for any $\ell_i > 0$,

$$\pi_i^{AU} = \lambda_i u(w_P + s_i) + (1-\lambda_i)u(w_R - \ell_i s_i) < \\ u(\lambda_i(w_P + s_i) + (1-\lambda_i)(w_R - \ell_i s_i)) = u\left(\frac{w_R + \ell_i w_P}{1+\ell_i}\right) < u(w_R). \tag{6}$$

This implies that at any profile with a subset of countries sharing the burden of redistribution at some strictly positive subsidy level, there is an incentive to cut down transfers (leading to $\ell_i = 0$). The only Nash equilibrium will have all countries setting $s_i = 0$. Expression (6) also shows that $s = 0$ is the only ESS of the game, since at any symmetric profile with $s > 0$ a relative advantage can be obtained by cutting down s; alternatively, starting at the symmetric profile with $s_i = 0$ for all i, any increase in s results in a relative disadvantage. Clearly, the imitative process of the OMC will also not deliver any improvement over the inefficient Nash equilibrium.

6 Conclusions

We propose to analyse the Open Method of Coordination (OMC), which the EU has adopted since its Lisbon Summit in 2000 for social policy and elsewhere, as a dynamic stochastic learning process of the type studied in evolutionary game theory. The OMC is based on the idea that, for certain commonly agreed policy objectives, national policies emerge from a process where governments compare themselves to one another in terms of policy performance, learn from each other, and imitate what they perceive as best practices. If convergence occurs under the OMC, then not due to express legislation but by the force of example.

We formalize and explore the workings of OMC for the particular case of income redistribution in an integrated economic area with perfectly mobile social welfare beneficiaries. Our main observation is that the OMC strongly favors coordination on a subset of Nash equilibria. In a dynamic interpretation, the OMC results in a powerful equilibrium refinement. In particular, intermediate values of subsidies that can be sustained by coordination of approximately half of the countries are the most likely ones to be observed in the long run.

To our knowledge, this is the first paper that provides a formal, game theoretic analysis of the OMC. Both opponents and advocates of the OMC will, with good reason, argue that our stylized analysis ignores many of the OMC's advantages (e.g., the higher degree of legitimacy), defines away a number of problems (e.g., the definition and measurement of performance indicators, communication procedures etc.) and discusses the OMC in an artificial setting (decentralized redistribution) to which it may not at all be suited. Notwithstanding these concerns, our analysis entails important messages for policy-makers and mechanism designers in the EU where welfare and redistribution policies are still in the domain of national governments. The hope that the OMC will "recalibrate" European welfare states (Ferrera et al., 2000) seems justified. On a first pass, the OMC does indeed provide a successful way to attain policy coordination and to avoid extreme, undesirable outcomes. This appears to be in line with preliminary empirical evidence compiled in Coelli et al. (2008).

A Properties of the payoff function

We show here the main properties of the family of functions $\{f(k,s)\}_{k=1,\dots,n}$ that are used in the proofs of our results. Recall

$$f(k,s) = \frac{\nu}{k} \cdot u(w_P + s) + u(w_R - \nu/k \cdot s) \qquad k = 1, \dots, n.$$

Notice $f(k,0) \geq u(w_R)$ and $f_s(k,0) = \nu/k \cdot (u'(w_P) - u'(w_R)) > 0$, since $u'' < 0$ and $w_P < w_R$ implies $u'(w_P) > u'(w_R)$. Moreover, $f_{ss}(k,s) < 0$.[14]

Recall $s^*(k) = \arg\max_{s \geq 0} f(k,s) = \frac{w_R - w_P}{1+\frac{\nu}{k}}$ satisfies

$$f_s(k, s^*(k)) = \frac{\nu}{k}\left(u'(w_P + s^*(k)) - u'\left(w_R - \nu/k \cdot s^*(k)\right)\right) = 0.$$

Clearly, $s^*(k) < \frac{k}{\nu} w_R$ and $s^*(k)$ is strictly increasing with k.

Given $k \in \{1, \dots, n\}$, let $\widehat{s}(k)$ be a strictly positive value of s such that $f(k, \widehat{s}(k)) = u(w_R)$. The properties of f imply that $f(k,s) > u(w_R)$ for all $s \in (0, s^*(k)]$. By definition of $s^*(k)$ and $f_{ss} < 0$, f is strictly decreasing for all $s > s^*(k)$. Moreover, for $s = \frac{k}{\nu} w_R$ we have that

$$f(k, \frac{k}{\nu} w_R) = \frac{\nu}{k} u(w_P + \frac{k}{\nu} w_R) + u(0) < u(w_R)$$

for $u(0)$ sufficiently low. In particular, $\frac{\nu}{k} u(w_P + \frac{k}{\nu} w_R)$ is strictly decreasing with k.[15] Let $u(0) < -K := u(w_R) - \nu \cdot u(w_P + 1/\nu \cdot w_R)$ to obtain the desired inequality. This implies existence and uniqueness of $\widehat{s}(k)$ for all k and it shows that $s^*(k) < \widehat{s}(k) < \frac{k}{\nu} w_R$. Finally, we have that $\widehat{s}(k') > \widehat{s}(k)$ for all $k, k' \in \{1, \dots, n\}$ and $k' > k$. To see this, note that, by definition, $\widehat{s}(k)$ satisfies

$$\Gamma(k,s) := \frac{k}{\nu} \cdot \left[u(w_R) - u\left(w_R - \frac{\nu}{k} s\right)\right] = u(w_P + s). \tag{7}$$

The right hand side of expressions (7) is strictly increasing and strictly concave with s and it equals $u(w_P)$ for $s = 0$. For any k, $\Gamma(k,0) = 0 \leq u(w_P)$ and $\Gamma(k,s)$ is strictly increasing and strictly convex in s with

$$\Gamma_s(k,s) \;\; = \;\; u'\left(w_R - \frac{\nu}{k} s\right) > 0.$$

[14] We adopt the conventional notation $f_i(k,s) = \frac{\partial f(k,s)}{\partial i}$ and $f_{ij}(k,s) = \frac{\partial f_i(k,s)}{\partial j \partial i}$ with $i, j = k, s$.

[15] Define $F(x) = \nu/x \cdot u(w_P + x/\nu \cdot w_R)$ with $x \in \mathbb{R}_{++}$. We have that

$$F'(x) = -\frac{\nu}{x^2} u(w_P + \frac{x}{\nu} w_R) + \frac{w_R}{x} u'(w_P + \frac{x}{\nu} w_R) < 0,$$

since, by strict concavity of $u(c)$, $u(w_P + y) - y u'(w_P + y) > u(w_P) \geq 0$ for all $y > 0$.

Furthermore, $u'' < 0$ implies that $\Gamma_s(k', s) < \Gamma_s(k, s)$ for all $s > 0$ and $k' > k$; i.e. the Γ functions become flatter with s as we increase k. Thus, $\Gamma(k, s) > \Gamma(k', s)$ for $s > 0$ and $k' > k$. It follows that $\widehat{s}(k') > \widehat{s}(k)$ for all $k, k' \in \{1, \ldots, n\}$ and $k' > k$.

Given $k \in \{1, \ldots, n\}$, let $\bar{s}(k)$ be a strictly positive s such that $f(k, \bar{s}(k)) = f(1, \bar{s}(k))$. We now proceed to show existence and uniqueness of $\bar{s}(k)$. To this purpose define

$$\begin{aligned} \Delta(k, s) &:= f(1, s) - f(k, s) \\ &= \nu \left(1 - \frac{1}{k}\right) \cdot u(w_P + s) + u(w_R - \nu s) - u\left(w_R - \frac{\nu}{k} s\right). \end{aligned} \tag{8}$$

Clearly, $\Delta(k, 0) \geq 0$. Moreover, strict concavity of u implies that[16]

$$\Delta(k, s) > \nu \left(1 - \frac{1}{k}\right) \{u(w_P + s) - s \cdot u'(w_R - \nu \cdot s)\}. \tag{9}$$

For $s \leq s^*(1)$ we have $w_P + s \leq w_R - \nu \cdot s$ and, by $u'' < 0$, then $u'(w_P + s) \geq u'(w_R - \nu \cdot s)$. For $s \leq s^*(1)$, we can then replace the right hand side of expression (9) by the following smaller expression

$$\Delta(k, s) > \nu \left(1 - \frac{1}{k}\right) \{u(w_P + s) - s \cdot u'(w_P + s)\} > \nu \left(1 - \frac{1}{k}\right) u(w_P) \geq 0. \tag{10}$$

The second inequality in (10) follows again from strict concavity of u (cf. Footnote 15). This shows that $\Delta(k, s) > 0$ for $s \in (0, s^*(1)]$ and thus, if $\bar{s}(k)$ exists, we must have $\bar{s}(k) > s^*(1)$. Furthermore,

$$\begin{aligned} \Delta_s(k, s) &= \nu \left(1 - \frac{1}{k}\right) \cdot u'(w_P + s) - \nu \cdot u'(w_R - \nu s) + \frac{\nu}{k} \cdot u'\left(w_R - \frac{\nu}{k} s\right) \\ &= \nu \left(1 - \frac{1}{k}\right) \cdot [u'(w_P + s) - u'(w_R - \nu s)] + \frac{\nu}{k} \cdot \left[u'\left(w_R - \frac{\nu}{k} s\right) - u'(w_R - \nu s)\right] \\ &< \nu \left(1 - \frac{1}{k}\right) \cdot [u'(w_P + s) - u'(w_R - \nu s)]. \end{aligned}$$

The inequality follows from $u'\left(w_R - \frac{\nu}{k} s\right) - u'(w_R - \nu s) < 0$ by $u'' < 0$. Recall that $u'(w_P + s) - u'(w_R - \nu s) \leq 0$ for $s \geq s^*(1)$. Thus, $\Delta_s(k, s) < 0$ for all $s \geq s^*(1)$. Finally, take $s = \widehat{s}(1)$. We know that $s^*(1) < \widehat{s}(1) < \widehat{s}(k)$ for $k > 1$ and we have that

$$\Delta(k, \widehat{s}(1)) = f(1, \widehat{s}(1)) - f(k, \widehat{s}(1)) = u(w_R) - f(k, \widehat{s}(1)) < 0. \tag{11}$$

Existence and uniqueness of $\bar{s}(k)$ for all $k > 1$ follows. Furthermore, (10) and (11) also imply that $s^*(1) < \bar{s}(k) < \widehat{s}(1)$ for all k. Finally, re-writing (8), we have that $\bar{s}(k)$ must

[16]By strict concavity $u(w_R - \nu/k \cdot s) < u(w_R - \nu \cdot s) + \left(1 - \frac{1}{k}\right) \nu \cdot s \cdot u'(w_R - \nu \cdot s)$.

satisfy

$$\Omega(k,s) := \frac{k}{\nu(k-1)} \cdot \left[u\left(w_R - \frac{\nu}{k}s\right) - u(w_R - \nu s)\right] = u(w_P + s).$$

It is easy to check that strict concavity of u implies $\Omega(k,s)$ is strictly decreasing with k and, thus, $\bar{s}(k)$ increases with k.

B Proofs of Section 3

Proof of Proposition 1. We proceed in two steps.

Step 1: *All Nash equilibria are symmetric.*

Consider any non-symmetric profile $(s_1, \ldots, s_n)$ with $s_i \neq s_j$ for some $i \neq j$. Without loss of generality suppose $s_i < s_j$. Country i attracts currently no poor and gets payoff $u(w_R)$. The payoff to all countries currently choosing maximum subsidy can be expressed as $f(m, \bar{s})$, where $\bar{s} = \max_{k=1,\ldots,n} s_k$ and m is the number of countries currently choosing maximum subsidy. If $\bar{s} > \widehat{s}(m)$, then $f(m, \bar{s}) < u(w_R)$. This cannot be an equilibrium, since any of the countries currently choosing maximum subsidy could strictly increase its payoffs by lowering the subsidy. If, alternatively, $\bar{s} \leq \widehat{s}(m)$, then $f(m, \bar{s}) \geq u(w_R)$. Since $\widehat{s}(m) < \widehat{s}(m+1)$, we have that $f(m+1, \bar{s}) > u(w_R)$. Thus, if country i would deviate from s_i and choose $\bar{s}$, it would strictly improve its payoffs.

Step 2: *Characterization of Nash equilibria*

Consider any symmetric profile $\mathbf{s} = (s, \ldots, s)$. Payoffs to all countries at $\mathbf{s}$ can be expressed as $f(n, s)$.

(i) Suppose $s < \bar{s}(n)$. By definition of $\bar{s}(n)$, $f(1, s) > f(n, s)$; i.e. a single country offering subsidy s would attain strictly greater payoffs. By continuity, there exists s' with $s < s' < \bar{s}(n)$ and $f(1, s') > f(n, s)$. Thus, there are incentives to deviate.

(ii) Suppose $s > \widehat{s}(n)$. By definition of $\widehat{s}(n)$, $f(n, s) < u(w_R)$; i.e. the subsidy is unprofitable and countries could strictly improve by any $s' < s$.

(iii) Suppose now that $\bar{s}(n) \leq s \leq \widehat{s}(n)$. By definition of $\bar{s}(n)$ we have $f(n, s) \geq f(1, s)$ and by definition of $\widehat{s}(n)$ we have $f(n, s) \geq u(w_R)$. Any country reducing the subsidy would get payoff $u(w_R)$ with no strict improvement. Any country increasing the subsidy to $s' > s$ would get payoff $f(1, s')$. Recall $\bar{s}(n) > s^*(1)$ and, thus, $f(1, s') < f(1, s) \leq f(n, s)$. Therefore, there are no incentives to deviate.

It follows from (i)-(iii) that $[\bar{s}(n), \widehat{s}(n)]$ is the interval of equilibrium subsidies. ■

Proof of Proposition 2. Consider any symmetric profile $\mathbf{s} = (s, \ldots, s)$. Suppose a single country deviates to some subsidy $s' \neq s$. The relevant payoffs to characterize ESS are now the payoffs obtained *after* deviation.

(i) Suppose first $s < \widehat{s}(1)$. A deviation upwards with $s < s' < \widehat{s}(1)$ gives payoff $f(1, s') > u(w_R)$ for the deviator while all others get $u(w_R)$ after deviation. Thus, it is possible to obtain a strict relative advantage and $s < \widehat{s}(1)$ is not ESS.

(ii) Consider now $s > \widehat{s}(n-1)$. A deviation downwards to any $s' < s$ gives the deviating country payoff $u(w_R)$. After deviation, non-deviators get payoff $f(n-1, s) < u(w_R)$ by definition of $\widehat{s}(n-1)$. Thus, the deviator has a strict advantage and $s > \widehat{s}(n-1)$ is not ESS.

(iii) Let now $s \in [\widehat{s}(1), \widehat{s}(n-1)]$. Deviations to $s' < s \leq \widehat{s}(n-1)$ will earn payoff $u(w_R)$ while those countries sticking to s obtain $f(n-1, s) \geq u(w_R)$. Deviations to $s' > s \geq \widehat{s}(1)$ will earn the deviator a payoff $f(1, s') < u(w_R)$ while all others get $u(w_R)$. Thus, it is not possible to attain a strict relative advantage through deviation.

It follows from (i)-(iii) that only $s \in [\widehat{s}(1), \widehat{s}(n-1)]$ are ESS. ■

C Proofs of Section 4

Proof of Lemma 1. At any $\mathbf{s}(t) \notin M$, $B(\mathbf{s}(t))$ is either a singleton, or it is empty. If it is a singleton, there is positive probability that all countries simultaneously revise their subsidies, reaching a monomorphic state. If it is empty, then there is positive probability that all countries with subsidies that are not in $\widehat{B}(\mathbf{s}(t))$ get a revision opportunity choosing some subsidy in $\widehat{B}(\mathbf{s}(t))$ and, thus, reducing the number of different subsidies chosen. We can repeat this argument until $B(\mathbf{s}(t))$ is a singleton. At any $\mathbf{s}(t) = (s, \ldots, s) \in M$, $B(\mathbf{s}(t)) = \{s\}$ and imitation alone is not enough to move the process away from $\mathbf{s}(t)$. ■

Let us now briefly review the main technical results we use to characterize the support of the limit invariant distribution (see Ellison (2000) for details). The support of the limit invariant distribution is contained in the set M of absorbing states of $P^{\lambda,0}$. Henceforth, we restrict analysis to this set. Given any two absorbing states $\mathbf{s}, \mathbf{s}' \in M$, define the *transition cost* from $\mathbf{s}$ to $\mathbf{s}'$, $c(\mathbf{s}, \mathbf{s}')$, as the minimum number of simultaneous deviations needed for a transition from $\mathbf{s}$ to $\mathbf{s}'$. Let $\Omega \subset M$ be a collection of absorbing states. The *basin of attraction* of Ω, $D(\Omega)$, is the set of states in Γ^n from which the process $P^{\lambda,0}$ converges to Ω with probability one. Define a *path* from a set $X \subset \Gamma^n$ to a set $Y \subset \Gamma^n$ as a finite sequence of distinct states $(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T)$ with $\mathbf{s}_1 \in X$, $\mathbf{s}_t \neq Y$ for all $2 \leq t \leq T-1$, and $\mathbf{s}_T \in Y$. Let $S(X, Y)$ be the set of all paths from X to Y. The cost of a path is given by $c(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T) = \sum_{t=1}^{T-1} c(s_t, s_{t+1})$. Define

$$C(X, Y) = \min_{(\mathbf{s}_1, \ldots, \mathbf{s}_T) \in S(X,Y)} c(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T)$$

Then we define the **radius** of Ω as $R(\Omega) = C(\Omega, \Gamma^n \setminus D(\Omega))$; i.e. the minimum cost from Ω out of $D(\Omega)$ or, alternatively, the minimum number of deviations needed for the process to leave Ω. The **coradius** of Ω is defined as $CR(\Omega) = \max_{\mathbf{s} \notin D(\Omega)} C(\mathbf{s}, D(\Omega))$; roughly, the coradius puts a bound on the cost of reaching Ω from every other state. A tighter bound can be obtained using the following alternative definition of costs over paths. Given any path $(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T)$ from some state $\mathbf{s} \in \Gamma^n$ to a subset of absorbing states Ω, let $\{\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_k\} \subset M$ be a sequence of absorbing states through which the path passes consecutively. Note that $\mathbf{s}$ could be $\mathbf{r}_1$, $\mathbf{r}_i \notin \Omega$ for $i < k$, and $\mathbf{r}_k \in \Omega$. Define the *modified cost function* as $c^*(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T) = c(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T) - \sum_{i=2}^{k-1} R(\mathbf{r}_i)$. For a transition that goes through a chain of absorbing states, the modified cost counts the number of deviations needed to set the transition in motion, discounting the costs of leaving the intermediate absorbing states. Accordingly, let

$$C^*(\mathbf{s}, \Omega) = \min_{(\mathbf{s}_1, \ldots, \mathbf{s}_T) \in S(\mathbf{s},\Omega)} c^*(\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_T).$$

The **modified coradius** of Ω is defined as $CR^*(\Omega) = \max_{\mathbf{s} \notin D(\Omega)} C^*(\mathbf{s}, D(\Omega))$. Note that $CR(\Omega) \geq CR^*(\Omega)$.

Lemma 2 *If for some set Ω which is a collection of absorbing states $R(\Omega) > CR^*(\Omega)$, then the set of stochastically stable states is contained in Ω and the expected waiting time to reach Ω from any other state is of order $\varepsilon^{-CR^*(\Omega)}$. If for some absorbing state $\mathbf{s}$ and some $\mathbf{s}' \neq \mathbf{s}$ we have that $R(\mathbf{s}) > C^*(\mathbf{s}', \mathbf{s})$ then $\mu^\lambda(\mathbf{s}') = 0$. If $R(\mathbf{s}) = C^*(\mathbf{s}', \mathbf{s})$ then $\mu^\lambda(\mathbf{s}') > 0$ implies $\mu^\lambda(\mathbf{s}) > 0$. If $R(\mathbf{s}) \geq CR^*(\mathbf{s})$, then $\mathbf{s}$ is stochastically stable.*

Proof. The lemma follows from Theorems 2 and 3 in Ellison (2000). To see the last part note that by definition of modified coradius $C^*(\mathbf{s}', \mathbf{s}) \leq CR^*(\mathbf{s})$ for all $\mathbf{s}' \neq \mathbf{s}$. Thus, for any absorbing state $\mathbf{s}$ if $R(\mathbf{s}) > CR^*(\mathbf{s})$, then $\mu^\lambda(\mathbf{s}') = 0$ for all $\mathbf{s}'$ and $\mathbf{s}$ must be the only stochastically stable state. Finally, let X be a set of stochastically stable states, which always exist. If for any $\mathbf{s}' \in X$ we have that $R(\mathbf{s}) = CR^*(\mathbf{s}) = C^*(\mathbf{s}', \mathbf{s})$, then $\mathbf{s}$ is also stochastically stable. ■

Proof of Proposition 3. Consider the set $E = \{\mathbf{s} \in M \mid \widehat{s}(1) \leq s \leq \widehat{s}(n-1)\}$. Denote $\mathbf{s}^k = (\widehat{s}(k), \ldots, \widehat{s}(k))$ with $k = 1, \ldots, n$. Let $\mathbf{s} = (s, \ldots, s) \in M$ be a monomorphic state at which the process starts in any period t and call $\mathbf{s}' \in \Gamma^n$ the resulting state when one of the countries deviates to subsidy $s' \in \Gamma$ in period $t+1$ while the remaining $n-1$ countries still choose s. Consider the following cases separately:

(i) Suppose $s < \widehat{s}(1)$ and $s' = \widehat{s}(1)$. The deviating country attracts all poor and gets payoff $u(w_R)$ by definition of $\widehat{s}(1)$. Non-deviating countries with lower subsidies attract no poor and also get $u(w_R)$. Only the country with highest best-performing policy is imitated. Thus, $B(\mathbf{s}'(t+1)) = \{s'\}$.

(ii) Suppose $s > \widehat{s}(n-1)$ and $s' \in [\widehat{s}(1), \widehat{s}(n-1)] \cap \Gamma$. The deviating country with lower subsidy attracts no poor and gets payoff $u(w_R)$. The remaining $n-1$ non-deviating countries must now share all the poor at $s > \widehat{s}(n-1)$ which results in payoff $f(n-1, s) < u(w_R)$ by definition of $\widehat{s}(n-1)$. It follows that $B(\mathbf{s}'(t+1)) = \{s'\}$.

Both in (i) and (ii) there is positive probability that all countries revise their subsidies and choose s' at the end of $t+1$. This shows that from any state $\mathbf{s} \notin E$ a state in E can be reached after a single deviation, so that $CR(E) = CR^*(E) = 1$.

(iii) Suppose $\widehat{s}(1) \leq s \leq \widehat{s}(n-1)$. If $s' < s$ (deviation downwards), the deviating country gets $u(w_R)$, while the remaining $n-1$ non-deviating countries with $s \leq \widehat{s}(n-1)$ get payoffs $f(n-1, s) \geq u(w_R)$ by definition of $\widehat{s}(n-1)$ and offer maximum subsidy s. Alternatively, if $s' > s$ (deviation upwards), the deviating country with $s' > s \geq \widehat{s}(1)$ attracts all poor and gets payoffs $f(1, s') < u(w_R)$, while the non-deviating countries get $u(w_R)$. In both cases $B(\mathbf{s}'(t+1)) = \{s\}$ and the deviation is not followed.

By (iii) states in E cannot be abandoned with a single deviation and $R(E) > CR^*(E) = 1$. By Lemma 2, the set of stochastically stable states must be contained in E. ■

Proof of Proposition 4. By definition, states in E cannot be abandoned with a single deviation. We will show, however, that it is possible to abandon states in E with an ever increasing number of simultaneous deviations as we move to intermediate states. In a first step we show that it is sufficient to consider simultaneous deviations to the *same* alternative subsidy. To see this, take any two states $\mathbf{s}, \mathbf{s}' \in E$ and suppose that the process moves from $\mathbf{s}$ to $\mathbf{s}'$ after multiple deviations with at least two different subsidies s' and s''. This transition requires that the process visits some intermediate state of the form $\tilde{\mathbf{s}} = (s, \ldots, s, s', \ldots, s', s'', \ldots, s'')$. If the process is to reach the monomorphic state $\mathbf{s}'$ eventually from state $\tilde{\mathbf{s}}$, we must have that $s' \in \widehat{B}(\tilde{\mathbf{s}})$ so that s' survives revision. Two cases are possible. Either $s' = \max\{s, s', s''\}$ and then $B(\tilde{\mathbf{s}}) = \{s'\}$, or else $B(\tilde{\mathbf{s}})$ is empty. In both cases the number of countries choosing s' after revision is at least as high as in state $\tilde{\mathbf{s}}$. If $B(\tilde{\mathbf{s}}) = \{s'\}$, this is because enough deviators had chosen s' so as to sustain this subsidy profitably. Since $\widehat{s}(k)$ is increasing with k, as more countries choose s' welfare cannot fall bellow $u(w_R)$ and s' will still be the unique best-performing policy. If, instead, $B(\tilde{\mathbf{s}})$ is empty, then countries currently choosing s' will continue to do so after revision and all other revising countries have positive probability to choose s', resulting in a state analogous to $\tilde{\mathbf{s}}$ where we can repeat the same arguments as long as there are some countries still choosing s''. In any case, the state resulting after revision from $\tilde{\mathbf{s}}$ could have been reached directly with all deviating and revising countries choosing s' in the first place.

Henceforth, we focus on simultaneous deviations of at least two countries choosing the same alternative subsidy. To compute transition costs between states in E, we first define the following collection of sets:

$$E^k = \{\mathbf{s} \in M \mid \widehat{s}(k-1) \leq s \leq \widehat{s}(k)\}, \qquad k = 2, \ldots, n-1.$$

We let the process start at some $\mathbf{s} = (s, \ldots, s) \in E^k$ at any period t and denote $\mathbf{s}' \in \Gamma^n$ the resulting state after deviation. We now consider the following cases separately:

(i) Suppose $\mathbf{s}'$ has k' countries choosing $s' > s$ with $\widehat{s}(k'-1) < s' \leq \widehat{s}(k')$ and $k' \geq k$. Deviating countries get payoff $f(k', s') \geq u(w_R)$ by definition of $\widehat{s}(k')$ and offer the highest subsidies. The remaining non-deviating countries are inactive and get $u(w_R)$. It follows that $B(\mathbf{s}'(t+1)) = \{s'\}$.

(ii) Suppose $\mathbf{s}'$ has $n-k+1$ deviating countries choosing $s' < s \neq \widehat{s}(k-1)$. Deviating countries get payoff $u(w_R)$. The remaining $k-1$ non-deviating countries get payoff $f(k-1, s) < u(w_R)$. It follows that $B(\mathbf{s}'(t+1)) = \{s'\}$. If $s = \widehat{s}(k-1)$ the same conclusion holds with $n-k+2$ deviations to a lower subsidy.

Both in (i) and (ii) there is positive probability that all countries revise their subsidies and choose s' at the end of $t+1$ reaching state $\mathbf{s}'$.

(iii) If $\widetilde{k} < k'$ countries coordinate on $s' > s$ with $s' \in (\widehat{s}(k'-1), \widehat{s}(k')]$, their payoff is $f(\widetilde{k}, s') < u(w_R)$, implying $\widehat{B}(\mathbf{s}') = \{s\}$ and $B(\mathbf{s}')$ is empty. Non-deviators keep their strategies if they are called to revise. Deviators go back to s at the first revision opportunity. The process will not exit state $\mathbf{s}$. Analogously, let $\widetilde{k} < n-k+1$ countries choose $s' < s$. Deviating countries get $u(w_R)$; non-deviating countries get payoff $f(n-\widetilde{k}, s) \geq u(w_r)$, since $n-\widetilde{k} \geq k$. Since non-deviators offer highest subsidy, we have $B(\mathbf{s}') = \{s\}$ and the process cannot leave $\mathbf{s}$. Finally, note that $n-k+1$ deviators choosing a lower subsidy are not enough to leave $\mathbf{s}^{k-1}$.

Since $\widehat{s}(k')$ increases with k', it follows from (i) and (iii) that the minimum number of deviations needed to exit $\mathbf{s} \in E^k$ upwards is $k' = k$ for $s < s' \leq \widehat{s}(k)$ and the process needs at least $k+1$ deviations to exit $\mathbf{s}^k$ upwards. From (ii) and (iii) at least $n-k+1$ deviations are needed to exit states in $E^k \setminus \mathbf{s}^{k-1}$ downwards and at least $n-k+2$ to exit $\mathbf{s}^{k-1}$ downwards. Therefore, $R(\mathbf{s}^k) = \min\{k+1, n-k+1\}$ for all k. Furthermore, for any $\mathbf{s} = (s, \ldots, s)$ with $s \in (\widehat{s}(k-1), \widehat{s}(k))$, $R(\mathbf{s}) = \min\{k, n-k+1\}$. This gives the following values for the *radius* of states in E:

a) For any $\mathbf{s}^k = (\widehat{s}(k), \ldots, \widehat{s}(k))$, $k = 1, \ldots, n-1$, we have

$$R(\mathbf{s}^k) = \begin{cases} k+1 & \text{if } k \leq n/2 \\ n-k+1 & \text{if } k > n/2 \end{cases} \tag{12}$$

b) For any $\mathbf{s} = (s, \ldots, s)$ with $\widehat{s}(k-1) < s < \widehat{s}(k)$, $k = 2, \ldots, n-1$, we have

$$R(\mathbf{s}) = \begin{cases} k & \text{if } k \leq \frac{n+1}{2} \\ n-k+1 & \text{if } k > \frac{n+1}{2} \end{cases} \tag{13}$$

The radius gives the minimum costs necessary to leave states in E. Observe in (12) and (13) that this value is highest around $k = \lceil \frac{n-1}{2} \rceil$. More precisely, we have that:

- If n is even, the maximum radius is $\frac{n}{2}+1$ attained at $\mathbf{s}^{\frac{n}{2}}$.
- If n is odd, the maximum radius is $\frac{n+1}{2}$ attained at states in $E^{\frac{n+1}{2}}$.

The case of even n: We compute the costs of reaching $\mathbf{s}^{\frac{n}{2}}$. From (i) and (iii) we have that $\mathbf{s}^{\frac{n}{2}}$ can be reached *directly* from states $\mathbf{s} = (s, \ldots, s)$ with $s < \widehat{s}(n/2)$ with no less than

$n/2$ deviations. Alternatively, from states $\mathbf{s} \in E^k \setminus \{\mathbf{s}^k\}$ with $k < n/2$, we can construct a path from $\mathbf{s}$ to $\mathbf{s}^{\frac{n}{2}}$ going through absorbing states $\mathbf{s}^k, \mathbf{s}^{k+1}, \ldots, \mathbf{s}^{\frac{n}{2}-1}$ consecutively. By (i) and (13), $c^*(\mathbf{s}, \mathbf{s}^k, \mathbf{s}^{k+1}, \ldots, \mathbf{s}^{\frac{n}{2}-1}, \mathbf{s}^{\frac{n}{2}}) = k < n/2$. Also transitions to $\mathbf{s}^{\frac{n}{2}}$ from $\mathbf{s} \in E^{\frac{n}{2}}$ with $\widehat{s}(\frac{n}{2} - 1) \leq s < \widehat{s}(\frac{n}{2})$ can either be done directly or through a chain of transitions through monomorphic states $\mathbf{s}' \in E^{\frac{n}{2}}$ with $s < s' < \widehat{s}(\frac{n}{2})$. In any case the modified costs of these transitions is $n/2$. Therefore, $C^*(\mathbf{s}, \mathbf{s}^{\frac{n}{2}}) = k$ for $\mathbf{s} \in E^k \setminus \{s^k\}$ and $k \leq n/2$ which reaches a maximum at $k = n/2$. On the other hand, from (ii) and (iii) we have that $\mathbf{s}^{\frac{n}{2}}$ can be reached directly from $\mathbf{s} \in E^k \setminus \{s^{k-1}\}$ with $k > n/2$ with $n - k + 1$ deviations. Alternatively, this can be done with a path through absorbing states $\mathbf{s}^{k-1}, \mathbf{s}^{k-2}, \ldots, \mathbf{s}^{\frac{n}{2}+1}$ at modified cost also equal to $n - k + 1$ (step-by-step transitions are not more likely in this case). Therefore, $C^*(\mathbf{s}, \mathbf{s}^{\frac{n}{2}}) = n - k + 1$ for $\mathbf{s} \in E^k \setminus \{s^{k-1}\}$ and $k > n/2$ which reaches a maximum at $k = \frac{n}{2} + 1$. In summary, the minimum cost of reaching $\mathbf{s}^{\frac{n}{2}}$ is highest from states in $E^{\frac{n}{2}}$ and from states in $E^{\frac{n}{2}+1}$. In both cases, the costs of these transitions are $n/2$. It follows that $CR^*(\mathbf{s}^{\frac{n}{2}}) = n/2$. We thus have

$$R(\mathbf{s}^{\frac{n}{2}}) = \frac{n}{2} + 1 > \frac{n}{2} = CR^*(\mathbf{s}^{\frac{n}{2}})$$

and by Lemma 2 $\mathbf{s}^{\frac{n}{2}}$ is the only stochastically stable state in this case.

The case of odd n: In this case we speak of a *set of central states* given by $E^{\frac{n+1}{2}} = E^{\lceil \frac{n}{2} \rceil}$. We want to show that this is the set of stochastically stable states. Take state $\mathbf{s}^{\frac{n-1}{2}}$, the 'lower' bound of the set. By (12) we know that $R(\mathbf{s}^{\frac{n-1}{2}}) = (n + 1)/2$. We can follow the same arguments that we used for state $\mathbf{s}^{\frac{n}{2}}$ in the even case to show that for all $\mathbf{s} \in E^k \setminus \{s^k\}$ with $k \leq \frac{n-1}{2}$ we have $C^*(\mathbf{s}, \mathbf{s}^{\frac{n-1}{2}}) = k$ and for all $\mathbf{s} \in E^k \setminus \{s^{k-1}\}$ with $k > \frac{n+1}{2}$ we have $C^*(\mathbf{s}, \mathbf{s}^{\frac{n-1}{2}}) = n - k + 1$. We thus have that

$$R(\mathbf{s}^{\frac{n-1}{2}}) = \frac{n+1}{2} > \frac{n-1}{2} \geq C^*(\mathbf{s}, \mathbf{s}^{\frac{n-1}{2}}) \qquad \text{for all } \mathbf{s} \notin E^{\lceil \frac{n}{2} \rceil},$$

and by Lemma 2 this implies that states $\mathbf{s} \notin E^{\lceil \frac{n}{2} \rceil}$ cannot be stochastically stable. Stochastically stable states must therefore be contained in the set of central states. We know from (12) and (13) that $R(\mathbf{s}) = (n + 1)/2$ for all $\mathbf{s} \in E^{\lceil \frac{n}{2} \rceil}$. Note also that for any pair $\mathbf{s}, \mathbf{s}' \in E^{\lceil \frac{n}{2} \rceil}$, we have $c(\mathbf{s}, \mathbf{s}') = (n + 1)/2$. Take for example $s' > s$; if the process starts at $\mathbf{s}$, we need at least $(n + 1)/2$ countries coordinating to sustain s'; if the process starts at $\mathbf{s}'$, we need at least $(n + 1)/2$ countries lowering their subsidies, so that subsidy s' cannot be sustained profitably anymore. Furthermore, it is not possible to reduce the costs of a transition from $\mathbf{s}$ to $\mathbf{s}'$ by a chain of transitions going through states out of the set $E^{\lceil \frac{n}{2} \rceil}$. Note that $c(\mathbf{s}, \mathbf{s}'') \geq \frac{n+3}{2}$ for all $s'' > \widehat{s}(\frac{n+1}{2})$ and $c(\mathbf{s}, \mathbf{s}'') \geq \frac{n+1}{2}$ for all $s'' < \widehat{s}(\frac{n-1}{2})$. It follows that $C^*(\mathbf{s}, \mathbf{s}') = \frac{n+1}{2}$. Therefore,

$$R(\mathbf{s}) = (n+1)/2 = CR^*(\mathbf{s}) \qquad \text{for all } \mathbf{s} \in E^{\lceil \frac{n}{2} \rceil}.$$

By Lemma 2 all states in $E^{\lceil \frac{n}{2} \rceil}$ are stochastically stable. ■

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