WORKING PAPERS

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December 2008

Working Paper No: 0813

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

An explanation for the inverted-U relationship between competition and innovation

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December 5, 2008

Abstract

The Dixit-Stiglitz model is extended by the possibility for firms to undertake process innovation. The model can provide a new explanation to describe the relationship that research activity of firms is positively correlated with product market competition at low levels of competition, and negatively at high levels that has been found in the data. The initial positive relationship is caused by an increased business stealing opportunity with more competition, while the negative effect comes from the reduction of the markup due to higher competition (measured as elasticity of substitution). Also the ambiguous relationship of market entry barriers with respect to research activity is discussed using a less general form of the model. This framework may also be used to explain the inverted-U relationship found between competition and advertising expenditures.

JEL classification: L10, O3

*I want to thank my advisers Maarten Janssen and Klaus Gugler for their help and guidance. I also want to thank thank Georg Götz, Guy Michaels, Stephen Redding, John Van Reenen as well as participants at the graduate and staff seminar at the department of economics of the University of Vienna for helpful comments.

1 Introduction

Joseph Schumpeter [1943] is today associated with the idea, that firms should be allowed to exercise some market power to increase their incentive to innovate. Ever since the relationship between innovation and competition intensity of firms has created large interest in the literature. Despite vast recent progress in the empirical understanding of this question, the debate is far from being settled theoretically.[1] The main contribution of this paper is to suggest a new explanation for the observed empirical regularity of an inverted-U.

Already Dasgupta and Stiglitz [1980] summarized the ambiguous empirical findings that "up to a point industrial concentration is positively correlated with innovation activity, it is negatively correlated when an industry is too concentrated" (page 266). Given this observation, which largely rests on the work of Scherer [1970], a recent paper by Aghion, Bloom, Blundell, Griffith and Howitt [2005a] (furthermore ABBGH) suggested that empirically there may be an inverted-U relationship between the Lerner index (to measure the intensity of product market competition) and patents (as a measure of innovation). According to this hypothesis, the highest researching markets will be those that provide some market competition while still letting firms exercise a certain degree of market power. Other studies have presented evidence which confirms this finding for various datasets (for example Hashmi [2008]), and as well on industry level as on firm level (for a firm level investigation see for example Carlin et al. [2004], and a firm level case study for the automotive industry Van Biesebroeck and Hashmi [2007]).

In their paper ABBGH provide a theoretical explanation in the framework of two firms and three possible interrelations. A firm may either be innovation leader or follower, or firms may be on par. In their model increasing competition reduces the profits of firms on equal level of innovation, which has two effects: On the one hand it raises the incentive to "escape competition", to get ahead of ones competitors when firms are on equal level of innovation. On the other hand a follower has less incentive to catch up with the leader, since in the resulting state of equal innovation level the profits are less. The decreasing mechanism is called the "Schumpeterian effect of competition". In their model the positive and negative effect described balance each other in such a way that for some

[1] One of many good overviews of the theoretical debate on the relationship of industrial organization and innovative activity is given by Vives [2006], who also demonstrates that for Cournot and Bertrand markets a positive relationship should be expected.

parameters they indeed generate the inverted-U pattern.

Some empirical doubt has been cast on this model by Hashmi [2008] who found this inverted-U relationship to be robust in the data, but who rejected some other predictions of the ABBGH model. Particularly he found in the data that "it is possible that the relationship between competition and innovation is inverted-U regardless of the technology gap" (page 17), while the described "escape competition" effect is only sensible in markets in which firms are close in levels of technology.

In search for an additional explanation I introduce process innovation into a Dixit Stiglitz setting. This model of monopolistic competition is appealing because it creates a competitive environment for firms while still letting them exercise some market power. As Paul Krugman observed: "This framework, while admittedly special, is remarkably powerful in its ability to yield simple intuition-building treatments of seemingly intractable issues" (Krugman [1991]). This paper is written in the same spirit.

Some other attempts have already been made to analyze process innovation in the Dixit Stiglity framework. Some fist steps into this direction have been undertaken for example by Montagna [1995] and Melitz [2003], who have demonstrated the workings of the model using a certain form of heterogeneity in marginal costs. An example of a model using process innovation in this setting has been brought forward by Götz [1999] and by Ederington and McCalman [2008], who study the timing of innovation and technological diffusion in a related framework.

A further related work is the dissertation of Georg Götz [1996], who derives a related model involving a binary research decision of firms. Götz found a similar relationship between competition and innovation on aggregate market level. I extend his model by allowing firms to have continuous innovation efforts. In this generalized version of the model from Götz I can show that theoretically an inverted-U relationship may be found on firm level as well (as found by the authors mentioned above in the data). Further I will analyze the reaction of the innovative activity of firms with respect to the fixed costs.

Another attempt has been brought forward by Aghion and Griffith [2005b], who have analyzed the relationship between product innovation and competition in the Dixit Stiglitz model. They interpret product innovation to be equal to the number of firms in the market. They come to the conclusion that in the Dixit Stiglitz model "product market competition [...] reduces post-entry rents and therefore discourages entry (or innovation)" (page 12). I do not restrict innovation to describe solely the entry of new firms, but focus on process innovation. My results show that in fact, despite innovation forcing some firms to leave the market, cost reducing technologies may occur. The relationship of innovation and competition can have increasing and decreasing parts, and under fairly general assumptions the pattern will be of an inverted-U shape.

The model derived in the following pages unfolds as follows: At first firms with identical cost structures form the stable Dixit Stiglitz equilibrium. At one point firms simultaneously get the possibility to invest in a new cost reducing technology (innovate). Each firm maximizes its profits by choosing its innovation level and its prices before and after innovation. Additionally each firm has to form believes on the innovative activity of its competitors to determine its optimal innovation level. Doing so they can anticipate the unique Nash equilibrium of zero profits net of innovation costs.

The inverted-U found is created due to different reasons than in the paper by ABBGH. Their "escape competition" effect is not present in my model, and such incentives don't exist. Even without modeling such an effect, the inverted-U relationship on firm and market level can be found in my model, in which the increasing part is alternatively explained by a greater business stealing opportunity with more innovation. In a market with large substitutability one firm can attract customers from the other firms by reducing its price, which is less effective in a market with little or no competition.[2]

The decreasing part of the inverted-U is due to the same Schumpeterian effect of competition like in the model by ABBGH: Tough competition reduces the markup of firms, and therefore the post-innovation rents.

To demonstrate the low innovation incentive at the extremes, consider the extremes: In markets with tough competition the firms operate close to the margin and can hardly generate profits. In the Dixit Stiglitz environment this will also be true for post-innovation profits, which reduces the incentive to undertake expensive research efforts. If on the other hand competition is low, the firms do not feel the pressure from substitution, and serve one market segment with large room for pricing flexibility. Innovation and the resulting price cuts in such markets will not attract much additional demand, which makes cost

[2]The idea of a business stealing effect has been expressed in other contexts for example by Aghion and Howitt [1992], Grossman and Helpman [1991], Segerstrom [1998]. In a more similar context the idea has been expressed by Raith [2003]. In these models an innovator can fully crowd out a competitor by innovating. The business stealing effect described here differs, since undercutting other firms will not necessarily drive them out of the market, but just decrease their demand.

reductions less valuable to firms.

While elements of this understanding of the problem have been suggested in the literature[3], the model combines these to create a compact framework that generates the inverted-U pattern.

This paper is organized as follows: After stating the assumptions of the model, a general version of it is derived. After the discussion of some properties (most noteworthy the conditions to find an inverted-U), the model is solved using an example for a particular innovation function. Further it is shown that the model could be also interpreted as a model of advertising. A conclusion concludes.

2 The model

The framework in which I study innovation is an extension of the Dixit Stiglitz model of monopolistic competition, which is additionally extended by the option for firms to undertake process innovation. Like in their model, the aggregated demand from consumers follows a CES utility function. Let x_i denote the total quantity of good i that is consumed, and θ be an exogenous parameter. Then the utility function is:

$$U(x_1, x_2, ..., x_n) = \left[\sum_{j=1}^{n} x_j^{\theta}\right]^{\frac{1}{\theta}}. \tag{1}$$

It is assumed that $0 < \theta < 1$, which ensures that the goods are substitutes. If θ approaches one, the demand function becomes linear in x_i, and the goods become perfect substitutes. A reduction of θ reduces the willingness of consumers to substitute one good for the other. It follows that θ is a parameter that can be used to smoothly adjust competition in this model. Another possibility to adjust competition would be to change the number of firms; given however that in this model the number of firms is endogenous, I will use the parameter θ as the parameter to adjust competition, and interpret changes of θ as changes of competition. Also the Lerner index, which was used in the ABBGH paper and others to measure competition, is in this model a function of θ only. The case of

[3]For example Vives (2006) has found that the relationship between the elasticity of substitution and process innovation should be positive in Cournot and Bertrand settings. Raith (2003) showed that with more elastic firm - level demand functions there is a greater business stealing opportunity (page 1246).

changing barriers to entry (a change of the fixed costs f) will also be discussed later.

Further I assume that the total consumer income is I in every period, and that in each period the full amount of income is spent on the consumption goods x. It may be argued that the assumption of constant income is in this context overly restrictive, since it is widely believed that innovation increases income. But innovation does still increase the relative income in this model despite assumption two, since it lowers prices and with a fixed income therefore increases purchasing power. Also the constancy of income is not crucial to the result; the model could be modified in a way to increase I directly in the post-innovation equilibrium. If this increase is not assumed to be too large, the shape of the competition - innovation relationship and all the qualitative predictions would stay the same (only the numerical results would shift as a consequence, if the model is solved using an explicit cost function).[4]

In the initial equilibrium, the cost function of firms is fully characterized by a fixed cost f and a variable cost c, which are both constant for all firms in the initial innovation. Innovation can then serve to lower the marginal cost, in the following way: A one time research investment of ω_i today lowers the marginal costs of firm i from c to $c(\omega_i)$ in the future. Innovation thus only affects the per-unit production costs and not the entry to market. It can be thought of as the purchase of new machines, which work faster than the ones they replace but also require a one time investment. It is this assumption by which the model discussed here differs from the Dixit-Stiglitz model.

It is reasonable to assume that a cost reducing investment (a process innovation) of ω should have the following relationship with marginal costs c: $c(\omega_i) > 0$, $c^{'}(\omega_i) < 0$ and $c^{''}(\omega_i) > 0$. This description requires that marginal costs will always be positive, decrease as a consequence of research effort, but this decrease happens at a declining rate; the impact of the marginal dollar spent on research is smaller the larger the research effort. An example of the form $c(\omega_i) = 1/\omega_i$ (which is one of the most simple functions to fulfill these three properties) is derived later.

Hence there are three cost elements that a firm faces: marginal costs $c(\omega_i)$, which are initially identical for all firms, but can be lowered, fixed costs f, which are constant in all every period. Finally a firm has the option to pay research costs ω_i, which have to be paid once and are sunk costs thereafter.

[4]Since the assumption will be made that the number of firms is large, such an increase of income due to innovation would be of little strategic relevance to one single firm.

In this model, innovation may also be thought of as producing a higher quality variety after a one time investment while leaving marginal costs unchanged. Another possible view of the model would be, to interpret it as a model of advertising. The interpretation of the model as a model of advertising will be developed below.

Next I assume that the number of firms is large. This assumption may be restated such that firms ignore the impact of their price on the income of consumers (whose purchasing power changes as a result of a price change). This assumption is necessary to justify a certain Dixit Stiglitz simplification when computing the price elasticity. See also the derivation of equation 3 in the appendix. Finally, there is free entry to the market at any point in time, which validates the use of a zero profits condition.

3 Equilibrium

First I will derive the initial Dixit Stiglitz equilibrium, then analyze the innovation incentive, and then analyze the new stable equilibrium and the conditions under which it will be observed. For most of the following equations a more detailed derivation is given in the appendix. From the utility function and the income of the consumers follows, that the optimal demand for good x_i is fully determined by the prices of all firms and the income of the consumers I (see appendix).

$$x_i = \frac{I p_i^{\frac{1}{\theta-1}}}{\sum_{j=1}^{n} p_j^{\frac{\theta}{\theta-1}}} \tag{2}$$

From this result, and the assumption on the cost structure follows the optimal pricing strategy of a monopolist, using further the assumption that the number of firms is large. Initially all firms have the same constant marginal costs equal to c. Then (see appendix):

$$p_i = \frac{c}{\theta}. \tag{3}$$

Combining equation 2, equation 3, and the zero profit condition which must hold given that free entry is assumed, the number of firms in the initial equilibrium is determined by (see appendix):

$$n = \frac{(1-\theta)I}{f} \tag{4}$$

So far these results follow Dixit Stiglitz [1977]. Now the model is extended by the introduction of the possibility to innovate, which implies also an extension to two periods. Let n_1 denote the number of firms that will be selling a positive quantity in the new equilibrium (in the second period). Let further r denote the constant interest rate at which firms can obtain credit. Then the profits of an innovating firm Π^I will be (see appendix):

$$\Pi^I = \left(\frac{1}{1+r}\right)\left(\frac{(1-\theta)Ic(\omega_i)^{\frac{\theta}{\theta-1}}}{\sum_{j=1}^{n_1} c(\omega_j)^{\frac{\theta}{\theta-1}}} - f\right) - \omega_i \tag{5}$$

The expression $\left(\sum_{j=1}^{n_1} c(\omega_j)^{\frac{\theta}{\theta-1}}\right)$ will be called the innovation index. The innovative activity of firm j influences the profit of firm i only through this innovation index, and it does so negatively. This externality of research is capturing the negative side of the business stealing effect; the loss of consumers due to rivals research activities. This effect also mathematically operates through demand (compare with equation 2).

The maximization problem of this profit function with respect to ω_i may not yield a positive solution for ω_i. In this case no innovation would take place. Given the assumptions that $c(\omega_i) > 0$ and $c^{'}(\omega_i) < 0$, as can be observed from the first order condition of this problem, the solution for ω_i will be unique, provided that it is positive for all values of θ.

This feature is visible in figure 1, which shows this profit function for a firm versus its research effort, taking the behavior of the other firms as exogenously given. The innovation function used is the one from the special case presented later as an example. For all values of θ larger than one half this function will look similar, for the values of θ less then half it has just the single peak without the initial decline.

The innovation index of the initial equilibrium is equal to $nc^{\frac{\theta}{\theta-1}}$. Given the free entry assumption the innovation index can never be smaller than that in the new equilibrium. If it were, the entry of non-innovators would increase the index. Hence the equilibrium innovation index can only be greater or equal to the original one. Thus the number of firms in the new equilibrium n_1 must be less or equal to the number of firms in the initial equilibrium n. Due to this negative externality from innovation in this model (the business stealing effect), some firms may be forced to leave the market. Also due to the zero profit condition in the initial equilibrium all non innovators will be driven out of the market if the innovation index is greater than the one of the initial equilibrium, since at this initial value firms just break even.

In equation 5 the positive and the two negative effects of competition (measured by θ) on profits are visible. On the one hand an increase of θ decreases the markup $(1-\theta)$. This is what has been called earlier the Schumpeterian competition effect. On the other hand the exponent of $c(\omega_i)$ will be more negative as a consequence of an increase of θ. Hence the impact of a reduction of marginal costs on profits is larger, and a decrease of costs more valuable to firms. This second effect comes from the fact that price differences result in larger business stealing possibilities if competition is tougher. If a good has many close substitutes, a price reduction of that good results in a larger increase in demand as compared to the situation when there are no substitutes.

When deciding how much to innovate, each firm can only influence its own research output, but takes the research effort from the other firms as given. Let $V = \sum_{j \neq i} c(\omega_j)^{\frac{\theta}{\theta-1}}$ be the innovation index of all firms but innovating firm i. Then firm i will take V as constant, and solve the following profit maximization problem:

$$\omega_i^* = \arg\max_{\omega_i} \left[\left(\frac{1}{1+r} \right) \left(\frac{I(1-\theta)c(\omega_i)^{\frac{\theta}{\theta-1}}}{c(\omega_i)^{\frac{\theta}{\theta-1}} + V} - f \right) - \omega_i \right] \tag{6}$$

Setting the first derivative of this problem with respect to ω equal to zero implies:

$$\frac{\theta I}{1+r} \left(\frac{c(\omega_i)^{\frac{\theta+1}{\theta-1}} c^{'}(\omega_i)}{(c(\omega_i)^{\frac{\theta}{\theta-1}} + V)^2} - \frac{c(\omega_i)^{\frac{1}{\theta-1}} c(\omega_i)^{'}}{c(\omega_i)^{\frac{\theta}{\theta-1}} + V} \right) - 1 = 0 \tag{7}$$

As demonstrated in the appendix, a sufficient condition for the second order derivative to be smaller than zero is that $c(\omega_i) > 0$, $c^{''}(\omega_i) > 0$, $c^{'}(\omega_i) < 0$ and $n_1 > \max\{1, 2\theta\}$. The three conditions on the cost function are reasonable qualities, and when describing the assumptions of the model the cost function was defined to have these properties. The requirement on the number of firms to be large is not restrictive by assumptions. Hence the solution to equation 6 will be a maximum in all feasible situations.

To come to a proper solution firm i must form beliefs on the innovation of the other firms (the value of V). Given the initial symmetry of firms and the unique solution to the problem of maximizing firm profits with respect to ω_i (provided that this solution is positive), firm i can anticipate that other firms who face the identical maximization problem will come to the same solution like itself. Analogue to the usual derivation of the Cournot equilibrium, a symmetry condition on the first order condition derived can be imposed. Then a firm can anticipate that $V = (n_1 - 1)c(\omega_i)^{\frac{\theta}{\theta-1}}$, where again n_1 is the number of firms that will be present in the new equilibrium. Let ω^* be the optimal research effort in equilibrium. Imposing this symmetry condition on the first order condition 7 implies that:

$$-\frac{c(\omega^*)}{c(\omega^*)'} = \frac{\theta I}{1+r}\left(\frac{n_1 - 1}{n_1^2}\right). \tag{8}$$

A second condition that can be anticipated is that in the emerging equilibrium the free entry will again lead to the abolishment of positive profits. As shown before, each innovator reduces the profits of the other firms. If expected profits after innovation are greater than zero there would be money left on the table and more firms would innovate. Each innovator reduces the expected profits after innovation, in the evolving Nash equilibrium profits are equal to zero.

If too many firms innovate, profits including research costs may be negative. In this case firms may still decide to stay in the market, since research costs are sunk, and in every following period the fixed costs may be covered by sales. Ex post the innovation decision is not optimal in this case. Hence profits including research costs in the new equilibrium will be less or equal to zero. Despite having a possible solution with negative long run profits, the only relevant ex ante assumption is that innovating firms assume long run profits to be exactly equal to zero, since no firm would innovate given expected negative profits.

From the profit function 5, noting again that $V = (n_1 - 1)c(\omega_i)^{\frac{\theta}{\theta-1}}$ this zero profit condition requires that:

$$\omega^* = \frac{1}{1+r}\left[\frac{(1-\theta)I}{n_1} - f\right] \tag{9}$$

The situation described by condition 8 and condition 9 defines the unique Nash equilibrium in two equations with two unknowns (the research effort ω^* and the endogenous number of firms n_1), which determine the equilibrium of the model. These equations can be solved once a particular form of the cost function $c(\omega_i)$ is assumed. One example is given later in this paper.

For the equilibrium to be valid, it has to be verified that the new innovation index is larger than the old one. If that was not the case, non-innovating firms would enter the market, thereby increasing the innovation index, until it reaches the initial level. For this condition to hold, certain parameter values have to be excluded. At these parameters, no innovation will take place. This condition can not be stated for the general case, but will be derived for an explicit solution of this model later.

4 Equilibrium Properties

The inverted-U on firm level: There are several ways to measure competition (Boone et al. 2007). As noted before, in this paper the elasticity of substitution θ is interpreted to measure competition. The same interpretation of θ is found often in the literature; among others by ABBGH, who write in the footnote on page 710: "Increased product market competition is modeled [...] as an increase in the substitutability between differentiated products in Dixit and Stiglitz [1977]."

As can easily be verified, the Lerner index to measure market power is given by $(1-\theta)$, and hence depends directly and only on θ. The Lerner index was also the number used to measure competition in the empirical investigation of ABBGH in which the inverted-U was described and in many other studies.

For this reason the relationship between ω^* and θ is analyzed further. Differentiating equation 8 and equation 9 with respect to θ gives the following two conditions:

$$
\begin{aligned}
\frac{\partial \omega}{\partial \theta} &= \frac{I}{1+r}\left(-\frac{1}{n_1} - \frac{(1-\theta)}{n_1^2}\frac{\partial n_1}{\partial \theta}\right) \\
\frac{\partial n_1}{\partial \theta} &= \frac{n_1(n_1-1) - \frac{(1+r)n_1^3}{I}\left(\frac{c(\omega)c(\omega)''}{(c(\omega)')^2} - 1\right)\frac{\partial \omega}{\partial \theta}}{(n_1-2)\theta}.
\end{aligned}
$$

Combining these two equations yields

$$
\frac{\partial \omega}{\partial \theta} = -\left(\frac{I}{(1+r)n_1}\right)\frac{n_1 - 1 - \theta}{(n_1-2)\theta - (1-\theta)n_1\left(\frac{c(\omega)c(\omega)''}{(c(\omega)')^2} - 1\right)}. \tag{10}
$$

This equation gives the impact of a change in competition on innovation, thus it describes the relationship of interest. If it is positive for a small value of

θ, and negative for a large value of it, its form can be described as an inverted-U.

The first observation is that when θ approaches 1, this equation will be negative for all parameter values. If θ approaches to 1 the second fraction converges to $(n_1 - 2)/(n_1 - 2) = 1$ if $c^{'}$ is not equal to zero, which was assumed. The fraction in brackets is always positive, hence the equation is indeed negative as θ approaches 1.

By assumption 4 it should hold for all valid parameters that $n_1 > 2$. Then as θ approaches zero, it can be easily verified that the equation will be positive iff:

$$c(\omega)c(\omega)^{''} > \left(c(\omega)^{'}\right)^2. \tag{11}$$

Equation 11 is fulfilled for all cost functions of the form $c(\omega) = \omega^x$ with x being a negative number (the model will be solved for the case of $x = -1$ next), for $-log(x)$ and also for a large number of other functions.

Hence given equation 11 holds for all values of ω, the relationship between innovation and competition can be expected to be increasing in the area of $\theta = 0$, and decreasing in the area of $\theta = 1$, with a single change of slope in between. If parameter restrictions don't cut off the maximum, this would result in the observation of what could be called an inverted-U.

If condition 11 is not fulfilled at all feasible values of θ then the relationship of competition and innovation will be decreasing only. The interpretation of this finding is, that the slope is so steep (the cost reduction as a consequence of an increase of ω is so high), that this reduction outweights the negative effect of little competition.

It may be with some functions $c(\omega)$ that equation 11 holds in the area of $\theta = 0$ (note that ω depends on θ), but not for all possible values of θ. Then the relationship may be characterized by more than one change of direction. It would still be increasing at the minimal and decreasing at the maximal value of θ, but could be of a shape that changes the sign of the slope more than once in between.

The inverted-U relationship on firm level is shown with varying I in figure 2 and with varying f in figure 3 for the situation of the example analyzed in the next section. The inverted-U pattern is clearly visible. An increase of I will ceteris paribus unambiguously increase research effort, while the role of an increase of f is less clear. This role is discussed shortly later in this section.

The inverted-U on market level: Also the joint output of firms will follow this inverted-U pattern. This is shown in figure 4. Market research effort in this graph is measured as the number of firms (n) times the research output of one firm (ω^*). This feature is similar to the one described by Götz [1996] in his dissertation, who found the same result in a related model with a binary choice for innovation of firms.

The role of f: The parameter f describes the fixed costs of firms, and hence also the market entry barriers. In the model developed in this paper, the effect of a variation of entry barriers on innovation is ambiguous. This can be seen directly in equation 9, which holds for general research functions.

The first observation which follows from this equation is that the partial derivative $\partial\omega^*/\partial f = -(1+r)^{-1}$, which is strictly smaller than zero. This observation simply acknowledges that fixed costs and research costs can be substituted one for the other to make the zero profits condition hold, and that hence a fixed cost increase would tend to reduce research activity.

However, there are more effects at work. The second observation from this equation is that also the derivative $\partial\omega^*/\partial n_1 = -(1+r)^{-1}(1-\theta)I/{n_1}^2$ is smaller than zero. Hence a partial increase of the number of firms will decrease the research activity of each firm. If there are less firms they can spread the research costs over a larger base, and hence have an incentive to innovate more.

In the general setting n_1 can not explicitly be stated. Given however that f is the barrier to market entry, it can be assumed that an increase of the market entry fee f leads to an equilibrium consisting of less firms. Therefore $\partial n_1/\partial f$ should also be smaller than zero. In the example given in the next section this is clearly the case, and it should be expected in general for sensible parameters.

Since $\partial\omega^*/\partial n_1 < 0$ and $\partial n_1/\partial f < 0$, there is also a positive effect of f on the research activity of firms via its influence on the number of firms. The effect from lowering or raising the entry barriers for firms to access a certain market on the research activity of firms operating in that market can thus be ambiguous.

The intuition is that on the one hand an increase of entry barriers will create less and therefore larger firms that can more easily afford large research spendings, but on the other hand increase the costs of these firms and hence make the zero profits condition harder to comply.

A graphical representation of this relationship is given for the example analyzed in figure 3. The lines intersect, which means that ceteris paribus it is not clear if an increase of f will increase or reduce the research output of a given firm. Examples for both effects can easily be constructed using figure 3.

Another representation is given in figure 5, where f is drawn on the horizontal axis, and the research output on the vertical one, allowing to see the variation of research output with respect to f in greater detail. Also this relationship seems to be best described as an inverted-U for the assumptions and parameters in the example used. Hence in the case of this example, there is a single entry barrier that maximizes firms research output.

Welfare: Innovation in this model has two impacts on utility. On the one hand it lowers costs, and therefor prices, which benefits consumers. On the other hand it reduces the number of firms and hence reduces the number of varieties. It is therefor interesting to compare the utility of the two equilibria. This comparison yields, that the utility is higher in the situation allowing process innovation than in the initial equilibrium if

$$n_1 c(\omega^*)^{\frac{\theta}{\theta-1}} > nc^{\frac{\theta}{\theta-1}}. \tag{12}$$

This is exactly comparing the old and the new innovation index, utility will be larger if the innovation index is. But as established before, the new innovation index can not be smaller than the initial one, since otherwise non-innovators have an incentive to enter the market. Their entry would increase the innovation index until it reaches the initial level. Hence in all situations in which innovation takes place, it will do so to the benefit of the consumers.

The market solution need not be socially optimal, and can differ from the choice of a social planner. For example for certain non-convex cost functions $c(\omega)$ the social planner decision need not yield a symmetrical solution. If the solution is symmetric, the social planner would maximize utility minus costs:

$$\max_{\omega,n,x} \left[\left(\frac{1}{1+r} \right) \left(\left(x^\theta n \right)^{\frac{1}{\theta}} - n\omega - nf \right) - nc(\omega) \right].$$

The partial derivative with respect to ω yields, that a social planner would determine the research effort per firm according to $-c(\omega)' = 1/(1+r)$. This can be larger or smaller or identical to the market solution. The number of firms and the quantity of good produced can vary accordingly. Hence only for a specific set of parameters the market solution is socially optimal, and generally it is not.

5 An example

In this section I will assume that $c(\omega_i) = \min\{c, c/\omega_i\}$. This means that for example a research investment by a firm of 2 allows that firm to halve its marginal costs. Obviously under this assumption the optimal innovation effort can only be a value larger than one if it is positive, or zero otherwise.

Combining the equilibrium conditions 8 and 9 gives a quadratic equation for the number of firms n_1, that always has a positive and a negative solution. Since the number of firms can not be negative, only the positive solution is relevant. Then there is a unique solution for the number of firms in equilibrium. It is given by:

$$n_1 = \frac{I}{2f}\left(1 - 2\theta + \sqrt{(1-2\theta)^2 + \frac{4\theta f}{I}}\right). \tag{13}$$

Just like in the initial equilibrium, the number of firms depends on the ratio of consumer income over fixed cost. The corresponding research effort of one firm is also unique, and given by the following equation (which is also shown graphically in figures 2 and 3:

$$\omega^* = \frac{f}{1+r}\left[\frac{1 - \sqrt{(1-2\theta)^2 + \frac{4\theta f}{I}}}{1 - 2\theta + \sqrt{(1-2\theta)^2 + \frac{4\theta f}{I}}}\right]. \tag{14}$$

Partial derivatives show that a higher interest rate has a negative effect on innovation effort, which is expected given the assumptions. An increase of the income of the consumer income will increase the research effort, which should also be expected. Finally the already discussed ambiguous effect of the fixed costs f on research effort is clearly visible in this equation. In the equation as presented, f shows up three times. An increase of the first f has a positive effect on ω^*, while an increase of the other two fs has a negative effect. The partial derivative is given by:

$$\frac{\partial \omega^*}{\partial f} = \frac{\omega^*}{f} - \frac{f^2}{(1+r)^2\omega^*}.$$

For any ω^* this relationship will be positive if f is small. Hence in markets with low entry barriers an increase of them will increase the research output of the remaining firms. For larger values of f the relationship may become negative.

Innovation as described will not take place for all possible parameters. The new innovation index has to be larger than the old one for this solution to hold. This is fulfilled if

$$r \leq f \frac{\left(1 - \sqrt{(1-2\theta)^2 + \frac{4\theta f}{I}}\right)\left(1 - 2\theta + \sqrt{(1-2\theta)^2 + \frac{4\theta f}{I}}\right)^{\frac{1-2\theta}{\theta}}}{(2-2\theta)^{\frac{1-\theta}{\theta}}} - 1. \quad (15)$$

If equation 15 is fulfilled, additionally it must hold that ω^* has to be larger than one. This comes from the definition of the research function. Equilibria which predict a research effort of less then one have to be ruled out, which requires additionally

$$f > (1+r)(1-2\theta) \quad (16)$$

$$I > \left(\frac{\theta}{1-\theta}\right)(1+r)\frac{\left(\left(\frac{1}{1+r}\right)f+1\right)^2}{\left(\theta f\left(\frac{1}{1+r}\right)+2\theta-1\right)}. \quad (17)$$

The condition on f is not overly restrictive given that r is small. It is also fulfilled for all values of $\theta \geq 1/2$. The threshold of I is potentially large, particularly if f is large, but this is in line with assumption four that the number of firms in the initial equilibrium $n = (1-\theta)I/f$ is large.

If condition 15 is an equality, non-innovating firms may survive. Then all firms assume that the new innovation index will equal the initial one. If in this case too few firms innovate, profits would still be zero for the innovators. A new equilibrium consisting of both, innovating and non-innovating firms could establish. For all other parameters for which innovation takes place, non-innovators will be forced to leave the market, given the increase of the innovation index.

For this example the inverted-U, and how it changes with I or f is graphically presented in figures 2 and 3, market research effort is given in figure 4.

6 A note on advertising

In the introduction it was already remarked that advertising could be modeled in a similar way like process innovation is described in this model, and that hence this model may be alternatively interpreted as a model of advertising. Process innovation as interpreted here and throughout the literature allows firms to lower production costs permanently after a one time payment. An interpretation for advertising could be, that it allows firms to increase demand in every period through the payment of a certain sum (denoted by $C(\psi)$) in every period. Thus $C(\psi)$ denotes the per period advertising expenditure of a firm, and ψ is a parameter, to adjust how consumers react to it.

To model the effect of ψ in this model, one possibility to model advertising would be to rewrite the utility function as: $U = \left[\sum_{i=1}^{n} (\psi_i x_i)^\theta\right]^{\frac{1}{\theta}}$. Analogue to the case of process innovation, which was analyzed so far, a firm can increase the demand for its product linearly by paying $C(\psi)$ with $\psi > 1$. Hence a payment of ψ would raise utility and further also demand. Solving the consumer maximization problem, demand is then given by:

$$x_i = I \frac{p_i^{\frac{1}{\theta-1}} \psi_i^{\frac{\theta}{1-\theta}}}{\sum_{j=1}^{n} \left(\frac{p_j}{\psi_j}\right)^{\frac{\theta}{\theta-1}}} \tag{18}$$

The price elasticity stays the same, and also the firm pricing condition, which will be independent of ψ. Then the cost of an advertising firm can be expressed as $C(\psi)$, whereby it is assumed that advertising costs have to be paid every period to achieve a permanent effect (contrary to research expenditures, which have to be paid once).

$$\Pi_i = \frac{1}{1+r} \left(\frac{I(1-\theta)\psi_i^{\frac{\theta}{1-\theta}}}{\sum_{j=1}^{n_1} \psi_j^{\frac{\theta}{1-\theta}}} - f - C(\psi) \right) \tag{19}$$

The function $C(\psi)$ denotes the cost from advertising an amount ψ in every period. If the costs of advertising are thought to be similar to the way in which costs to innovation were modeled, this equation is mathematically equivalent to equation 6. Hence this model could also be used to predict an inverted-U relationship between competition and advertising. In the same way the model may also be interpreted as a model of increased product quality, which could also be thought of as leading to an increased demand as a consequence of some investment.

Indeed some evidence for an inverted U-relationship between advertising and competition has been found in the data. First Sutton [1974] found empirically "support for the inverted-U hypothesis" (page 62) between competition and advertising. More recently Lee [2002] remarked on the literature on the relationship between advertising and competition that it has become conventional wisdom to assume "the so called inverted U hypothesis, which implies that moderately concentrated industries engage more intensively in advertising than both atomistically competitive and highly concentrated industries" (page 89). Further also he confirmed this inverted-U relationship between competition and advertising in consumer good industries.

Hence a second testable prediction of the model described (that we should expect an inverted-U relationship between innovation and advertising) is confirmed by empirical investigations of the subject.

7 Conclusion

In this paper a model is presented that provides an alternative explanation for the observed inverted-U relationship between product market competition and innovation. It is derived on the basis of the standard Dixit-Stiglitz model. The model predicts that ceteris paribus markets with medium competition should exhibit a tendency to create more innovation than those markets on the extremes. For a large general set of innovation functions this result holds theoretically for firm level innovation as well as for aggregate innovation activity.

The increasing part of this inverted-U is explained by a larger business stealing effect (the possibility to attract consumers from opponents by lowering costs), which is higher in markets with more competition. The decreasing part is explained by the common "Schumpeterian effect of competition", which states that more competition decreases post-innovation monopoly rents.

The ambiguous relationship of innovation with respect to market entry barriers is discussed using an explicit research function. While in markets with low entry barriers an increase of these barriers should raise the innovation effort of firms, the effect may be opposite in markets that already have large barriers to entry.

Hence the model suggests a new hypothesis to understand the empirical regularity of an inverted-U relationship between competition and innovation.

Additionally the model can also be used to explain the inverted-U hypothesis found between product market competition and advertising expenditures.

Further it provides a new compact framework to study the effect of various policies affecting market competition on innovative activity and advertising in those markets.

Possible further extensions may analyze the case with less restrictive assumptions on the symmetry of firms, for example by introducing heterogeneity in marginal costs. The results may look similar, with an indifferent firm that has zero profits after innovation, but with other firms that operate with lower marginal costs and can generate positive profits. Also the model may be extended to overcome the static nature and have a more dynamic structure with innovation in different periods.

8 Appendix 1: Derivations and Proofs

Derivation of equation 2: In this model, the budget constraint is given by $\sum_j x_j p_j \leq I$. The utility maximization problem yields then the following relationship between any two goods consumed: $x_i/x_j = (p_i/p_j)^{\frac{1}{\theta-1}}$. Substituting this relationship back into the budget constraint gives the demand stated.

Derivation of equation 3: From the derivative of the solution for x_i with respect to its own price p_i follows the exact price elasticity ϵ_i:

$$\epsilon_i = \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = -\frac{1}{1-\theta} + \left(\frac{\theta}{1-\theta}\right)\frac{x_i p_i}{I}$$

Given the assumption that the number of firms is large, the second term in this equation can be dropped. This simplification has attracted quite some controversy in the literature,[5] and I do acknowledge that the results only holds if the number of firms is large (assumption four). Then from the Lerner pricing condition given the price elasticity $\epsilon_i = -\frac{1}{1-\theta}$ follows the price setting equation stated.

Derivation of equation 4: Since initially all firms have identical fixed and marginal costs c and f, they all charge the same prices $p = c/\theta$ and produce the same quantity x, which equals:

[5] See for example the critical comments by Yang and Heijdra [1993], or d'Aspremont et al. [1996], who directly address this point.

$$x = \frac{I p^{\frac{1}{\theta-1}}}{n p^{\frac{\theta}{\theta-1}}} = \frac{I}{np}.$$

Profits are then given by $\Pi = xp - xc - f = xp(1-\theta) - f = (1-\theta)I/n - f$, and since they have to be zero the number of firms n can be determined from this last equation.

Derivation of equation 5: Using equation 2 and 3 the profit in a single period can be calculated. I assume that the fixed and marginal costs have to be paid in the second period also, but the research costs only in the first period. To account for this time asymmetry I discount the transfers that occur in the second period by interest rate r, such that an amount of a in 1 period is of current value $a/(1+r)$. Then profits of a firm in the period after innovation from perspective of the date when the investment decision has to be made can be developed as follows:

$$\begin{aligned}
\Pi^I &= \left(\frac{1}{1+r}\right)(x_i p_i - x_i c(\omega_i) - f) - \omega_i \\
&= \left(\frac{1}{1+r}\right)(x_i p_i (1-\theta) - f) - \omega_i \\
&= \left(\frac{1}{1+r}\right)\frac{(1-\theta) I \left(\frac{c(\omega_i)}{\theta}\right)^{\frac{\theta}{\theta-1}}}{\sum_{j=1}^{n_1}\left(\frac{c(\omega_j)}{\theta}\right)^{\frac{\theta}{\theta-1}}} - f - \omega_i \\
&= \left(\frac{1}{1+r}\right)\left(\frac{(1-\theta) I c(\omega_i)^{\frac{\theta}{\theta-1}}}{\sum_{j=1}^{n_1} c(\omega_j)^{\frac{\theta}{\theta-1}}} - f\right) - \omega_i.
\end{aligned}$$

The second order condition of problem 6

Writing $W = V + c(\omega_i)^{\frac{\theta}{\theta-1}}$ as the total innovation index, the second order derivative is smaller than zero if

$$c''(\omega_i)\left(\frac{c(\omega_i)^{\frac{\theta+1}{\theta-1}}}{W^2} - \frac{c(\omega_i)^{\frac{1}{\theta-1}}}{W}\right) + \frac{c(\omega_i)'(\omega_i)}{(1-\theta)W^2}\left(\frac{2\theta c(\omega_i)^{\frac{2+\theta}{\theta-1}}}{W} + c(\omega_i)^{\frac{2-\theta}{\theta-1}} W - (2\theta+1) c(\omega_i)^{\frac{2}{\theta-1}}\right) < 0$$

After deriving the second order condition, the symmetry condition can be imposed, such that W can be written as $n_1 c(\omega_i)^{\frac{\theta}{\theta-1}}$. Rearranging yields

$$-\frac{c^{''}(\omega_i)}{c(\omega_i)}\left(\frac{n_1-1}{n_1^2}\right)+\frac{(n_1-1)(n_1-2\theta)}{(1-\theta)n_1^3c(\omega)^2}<0.$$

This is fulfilled provided the conditions stated on the cost function and the number of firms hold.

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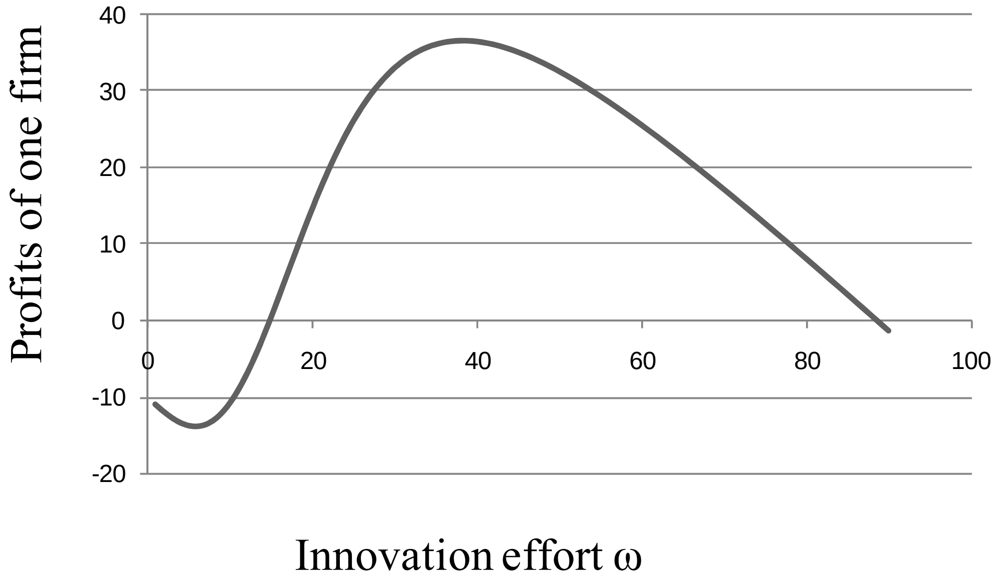

Figure 1: Profits of one firm taking the behavior of other firms as constant. The following parameters are assumed: $\theta = 3/4$, $I(1 - \theta) = 100$, a value for the innovation index of the other firms of 10000, r = 0:5 and f = 10.

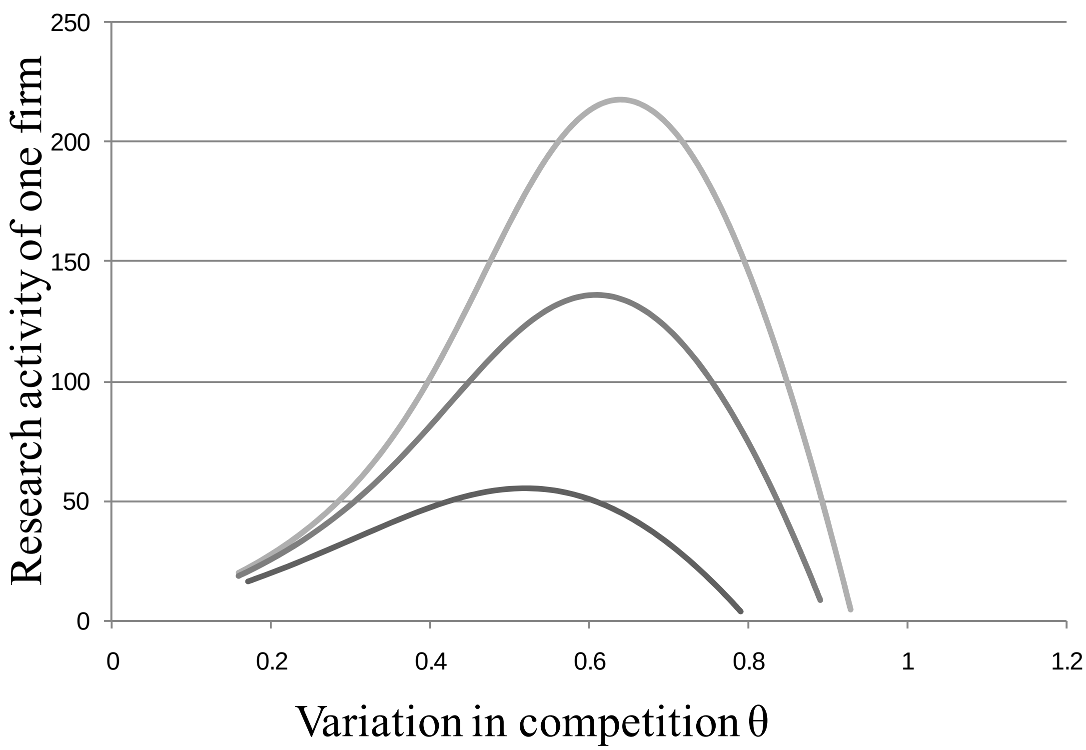

Figure 2: The research effort of an innovative firm as function of market competition. The following parameters are assumed: I is given by 1500, 1000 and 500 (from top to bottom), f is given by 100. Interest rate r is given by 0.05. Values for which either of conditions 11, 12 or 13 are not fulfilled are excluded from the representation.

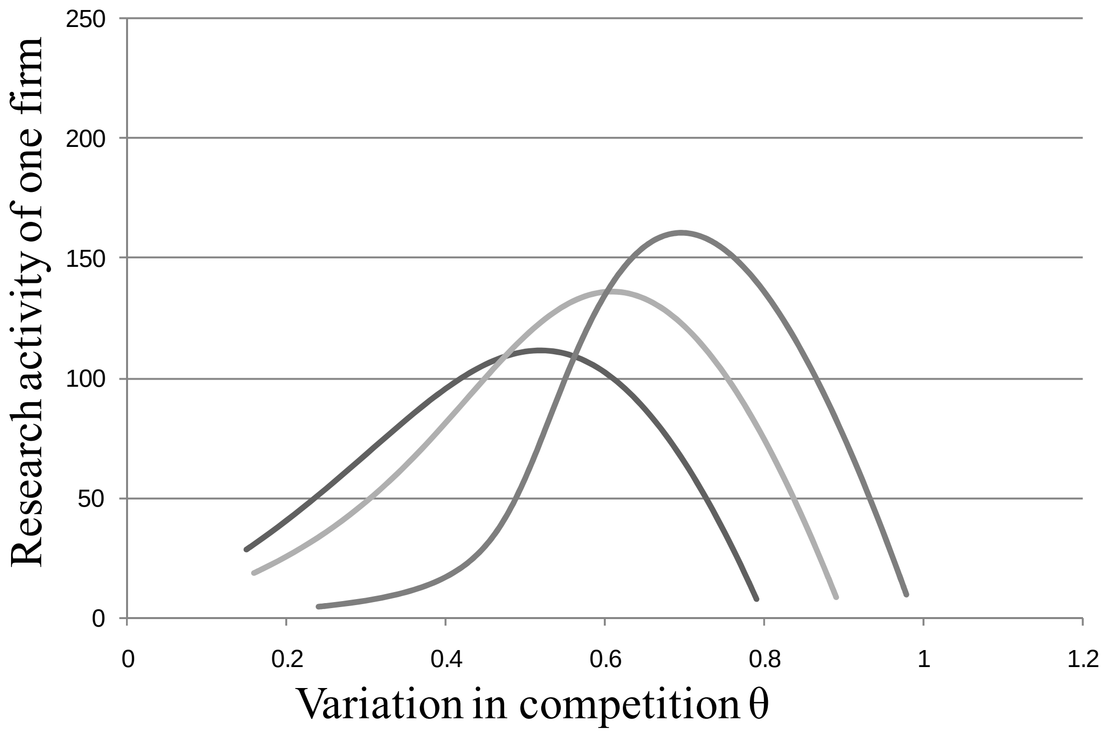

Figure 3: The research effort of an innovative firm as function of market competition. The following parameters are assumed: I is given by 1000, f is (from highest to lowest maximum): 10, 100 and 200. Interest rate r is given by 0.05. Values for which either of conditions 11, 12 or 13 are not fulfilled are excluded from the representation.

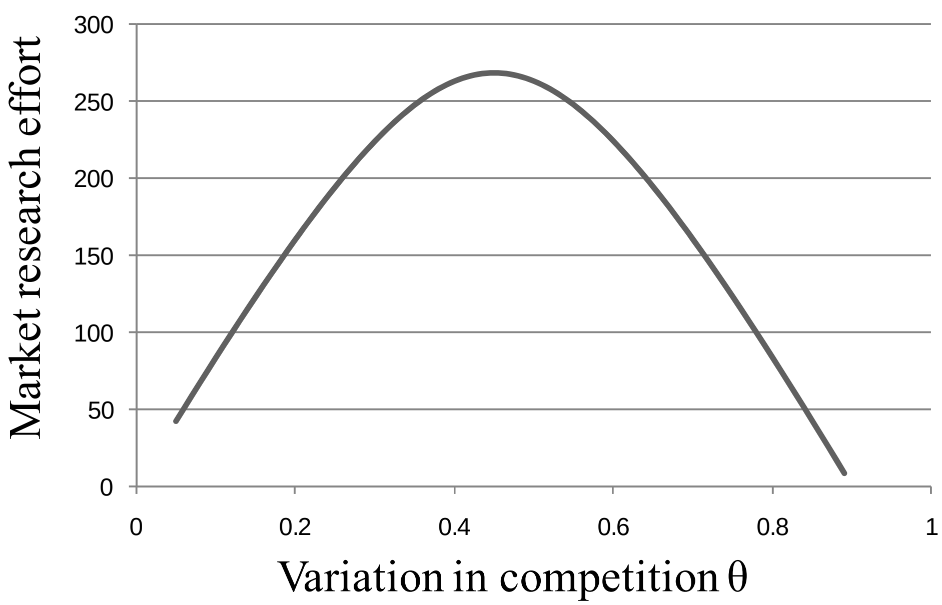

Figure 4: The research effort of all firms as a function of θ. The following parameters are assumed: m=1000, f=100 and r=0.05. Parameters for which the parametric conditions are not fulfilled are excluded from the representation.

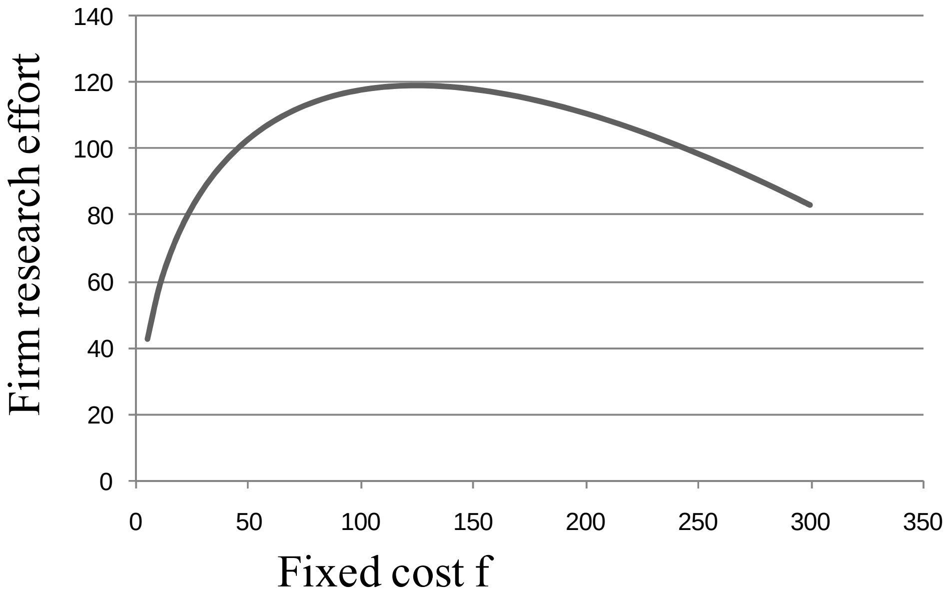

Figure 5: The research effort of one firm varying with fixed costs f. I is assumed to be 1000, r is 5 percent, theta is 0.5.