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Bidding behavior in sequential cattle auctions

Christine Zulehner *

September 14, 2007

Abstract

The objectives of this study are to investigate the institutional specifics of sequential cattle auctions and their role as determinants of prices. Institutional specifics are the order of sale according to quality, a secret reserve price, bidders' multi-unit demand and different types of bidders. Prices decline and bidders with a higher demand pay on average lower prices. The estimation results show that declining prices are caused by the order of sale according to quality and the secret reserve price. The results further show that bidders take the strategic effect of sequential auctions and multi-unit demand into account.

JEL Classifications: D44, Q12

Keywords: Sequential auctions, private values, bidding behavior, applied econometrics

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Abstract

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1 Introduction

Standard models of auctions predict constant prices when identical units of a good are sequentially offered to bidders who demand at last one unit (Milgrom and Weber 2000, and Weber 1983). In reality, we often observe different price patterns, but also conditions that do not completely fulfill the assumptions of these models.[1,2] If bidders demand more than one unit, we expect bidders to shade their bids for earlier units and prices to increase over an auction day (Donald, Paarsch and Robert 2006). Reasons for declining prices might be the order of sale according to product quality (Beggs and Graddy 1997), supply uncertainty (Jeitschko 1999) or participation cost (von der Fehr 1994).

This is an empirical study of bidding behavior in sequential cattle auctions.[3] The objectives are to analyze the institutional specifics and their role as determinants of prices. Institutional specifics are the order of sale according to quality, a secret reserve price, bidders' multi-unit demand and different types of bidders. In Amstetten, a smaller city in Austria, dairy cows of different quality are offered in a sequence of ascending auctions. Every cow is offered by a different seller, who may reject the outcome of the auction immediately after the price has been hammered down by the auctioneer. Bidders are either representatives of wholesale firms, i.e. traders, or farmers from nearby regions. We observe two notable price patterns. Prices decline over auction days and traders pay on average lower prices than farmers.

[1]Examples for declining prices are wine auctions (Ashenfelter 1989), art auctions (Beggs and Graddy 1997), or rose auctions (van den Berg, van Ours and Pradhan 2001). Examples for increasing prices are rare book auctions (Deltas and Kosmopoulou, 2004). There are also other patterns like an inverse U-shaped pattern. An example are auctions of eggplants in France (Laffont, Loisel and Vuong 1997).

[2]More recent theoretical papers point out a number of conflicting effects that determine the outcome of expected prices. Deltas and Kosmopoulou (2004) provide a detailed list under which assumptions prices decrease, stay constant or increase. A discussion of sequential auctions in more detail is given in Krishna (2002).

[3]For surveys on empirical studies about auctions see Hendricks and Paarsch (1995) or Laffont (1997). For a survey on nonparametric identification and estimation of auction models see Athey and Haile (2005).

The addressed questions are, whether the decline in prices is caused by the order of sale according to quality or the secret reserve price that is equivalent to supply uncertainty. I further investigate, whether bidders consider the strategic effects generated by the sequential auctions and multi-unit demand, and whether the price difference between traders and farmers is caused by differences in bidders' preferences or differences in strategic behavior. Finally, I assess the relative contribution of the institutional specifics explaining the price decline over auction days and the price difference between traders and farmers.

To answer these questions, I discuss various bidding models that take the institutional specifics into account and describe the predictions of these models on prices. In particular, I describe the predictions of strategic variables like bidders' demand and the probability that the seller rejects the outcome of the auction. To test the predictions, I estimate hedonic price equations with ordinary least squares.[4] I further employ decomposition techniques known from labor economics to measure the relative importance of bidders' preferences, strategic behavior and the institutional specifics. I utilize a large data set on cattle auctions that covers 95 auction days from 1995 till 2003 with more than 25,000 dairy cows offered.

As all important characteristics of the auctioned cows are exposed in a catalogue, I discuss the bidding models within the independent private value paradigm.[5] The main characteristics of a dairy cow is the amount of milk it can produce and a categorization into quality classes provides this information. Sellers have to guarantee on this. The milk price is known, and thus the value of a cow. Bidders also tend to agree on the various characteristics of the animals in cattle auctions (Engelbrecht-Wiggans and Kahn, 1999). As a consequence bidders' preferences can be assumed to be purely of private nature.

[4]For a detailed analysis on the estimation of auctions with ordinary least squares see Rezende (2005).

[5]For an analysis whether a private or a common value model fits best the data see Paarsch (1992).

Every bidder ranks the different characteristics in another order depending on breeding program goals.

The second group of bidders are traders. They are agents of resale firms and buy cattle on behalf of their firms, but also on behalf of farmers who do not attend the auction, but who have placed orders for a particular animal(s) at specific prices before the opening of the market. These prices are then the valuations of the traders in the auction (Laffont, Ossard and Vuong, 1995). If traders only bid on behalf of their firms, their valuations are subject to demand in the resale market which may be correlated with demand in the current auction market. However, firms buy for geographically distinct markets which I assume to be independent.[6]

In ascending auctions with private values, it is a dominant strategy to "stay in" the auction as long as the price is lower than one's own valuation (Vickrey 1961). This also true for sequential ascending auctions (Milgrom and Weber 2000). The optimal stopping rule determines the price of each unit and we expect constant prices. If bidders demand more than one unit, bidders shade their bids for earlier units and we expect increasing prices (Donald, Paarsch and Roberts 2006). Prices further depend on bidders' own demand and other bidders' demand (Vickrey 1961, and Ausubel 2004). If the seller may reject the outcome of the auction, rational bidders adjust their strategic behavior to influence the probability that the seller rejects the outcome of the auction and guard themselves against a lower probability to win by bidding more aggressively for earlier units. By adjusting the model of Jeitschko (1999), I show that prices decline over an auction day and depend on the probability of rejection.[7]

[6]Haile (2001) provides an analysis of timber auctions, where firms bid for harvesting contracts in U.S. forests and have the opportunity to later resale their contracts.

[7]For an empirical analysis of a secret reserve price in static ascending auctions see Eklöf and Lunander (2003) and for an empirical analysis of a secret reserve price in static first-price auctions see Elyakime, Laffont, Ossard and Voung (1994).

The contribution of this paper is to consider multi-unit demand and a secret reserve price in sequential auctions and disentangle their opposite effects on prices. The contribution of it is, however, not a structural bidding model that already assumes optimal behavior, but to provide empirical evidence on bidders' behavior in sequential auctions. Similar papers are for example, Beggs and Graddy (1997), who analyze the order of sale in art auctions. They show that prices decline relative to a cost estimate when products are sorted according to their quality and the one with the highest quality is auctioned off first, and they provide empirical evidence for that effect. Deltas and Kosmopoulou (2004), in turn, use rare book auctions with floor and mail-in bidders as a natural experiment to distinguish between 'catalogue' and 'order-of-sale' effects. They find that bidding patterns are driven by non-strategic factors like the print order of lots in the catalogue.

An example for a structural model is Donald, Paarsch and Robert (2006). They analyze a sample of sequential, ascending, open-exit auctions of Siberian timber export-permits within the independent private value paradigm. They allow participants to desire more than one object and construct a theoretical model of participation and bidding. They estimate the participation process and underlying distribution of bidders' valuations with a structural simulation estimator based on the one proposed by Laffont, Ossard and Vuong (1995).

The remainder of the paper is organized as follows. Section 2 describes the auction market in Amstetten, the data and summary statistics. Section 3 discusses the sequential bidding process and its predictions on prices. Section 4 presents the empirical bidding model and construction of variables. Section 5 gives estimation and decomposition results. Section 6 concludes.

2 Cattle auctions in Amstetten, Austria

In this section I give a detailed description of Austrian cattle auctions. I also describe the data and provide summary statistics.

2.1 Description of the market

Cattle auctions in Amstetten[8] are conducted as ascending, closed-exit auctions. During a typical auction day 200-300 animals are sold. These are offered sequentially. The cattle auctioned are dairy cows and stock bulls. The second group is rather a complement to the former and will therefore not be investigated within this paper.

Auctions take place eleven times a year and each one lasts for two days. On the first day potential buyers have the opportunity to view the animals and a catalogue with a detailed description of every animal is available at a low price. In recent times this catalogue can also be downloaded from the organization's web page about two weeks before the market takes place. The description of the cattle includes various quality criteria such as milk production, milk components, the owner of the animal, its date of birth, names of parents and grandparents as well as some of the quality criteria of the parents and grandparents. The cattle are divided into three categories, namely "young female calves", "female calves" and "cows". The main difference between these three categories is the age of the cattle.[9] There are two different breeds, Fleckvieh, animals with a spotted coat, and Braunvieh, animals with a brown coat. The cattle are characterized by two quality criteria. The first criterion has six different classifications. For cows and female calves it gives the minimum requirements for the output and structure (fat, protein) of their milk. In the case of young female calves it gives the minimum requirements for their mother's milk. Cattle of the highest classification were not sold on one of the auction

[8] Amstetten is a small city in Lower Austria, a federal state in Austria.

[9] The label "calves" is slightly misleading as these animals have already given birth.

days. The second quality criterion has three classifications and it is a subclass of the other quality criterion.

Medical checks are carried out during the animals' stay in the auction stables and the results are published on the morning of the second day, when the auctions take place. Female calves and cows are also milked in the evening of the first day and in the morning of the second day. These results are published as well.[10] On the second day the animals are auctioned. The order of sale is sorted according to breed, category and the two quality criteria. Within each group the order is random.

The auctioneer starts the auction at a fixed price and raises the price by fixed amounts. The bidders have so-called "Winkers". These are paddles with a number on their front. Everyone who wants to bid has to pay a small fee for such a "Winker". With these, the bidders can bid for a particular animal. They raise and drop the paddle to indicate that they accept the announced price.[11] The auction lasts until no one is willing to accept the next highest bid. When the bidding stops, the animal for sale is hammered down, but not necessarily sold as the seller has the possibility to reject the price. During the auction the respective seller represents the animal in front of the bidders. If nobody is willing to accept the starting price the auctioneer lowers the price as in a descending (Dutch) auction until someone accepts it. From there on, the auctioneer again starts to raise the price by the same fixed amount as before and the procedure continues as described above.

In this market, sellers are farmers and buyers are representatives of wholesale firms, i.e. traders, or farmers from nearby regions. Traders regularly attend and bid in auctions.

[10]If the milking output does not coincide with the information provided by the seller, the auction house depreciates the quality of the cattle to the lowest quality and announces that before the auctions take place. Thus, it is in the very interest of the sellers to correctly report the cattle's quality.

[11]When bulls are auctioned, bidders sometimes also outcry their bids. I did not observe this phenomenon when dairy cows were auctioned. Avery (1998) showed that in the case of jump bidding the equilibrium bids change. This is another reason why I do not consider bulls in the empirical analysis.

Farmers may also regularly attend auctions, but due their lower demand do not bid that often. This may create a difference in experience. All bidders arrive at the auction place with trucks to transport the cattle afterwards. As the cattle has to be brought away from the market place within the next day, the size of these trucks determines bidders' capacity on an auction day. Some of the traders even use truck tractors, whereas farmers usually use small tumbrils. The trucks are parked in front of the auction hall and are thus visible for all. Bidders' overall capacities depend, of course, more likely on the size of their firms or their farms. The animals, however, have to be removed from the auction stable within one day. Besides, some cattle produce milk and need daily care. Thus, bidders' capacity on an auction day is determined by their trucks. Furthermore, traders tend to bid for pregnant cows only, as lactating cows cannot be transported long distances.

2.2 Data and summary statistics

The data, which were kindly provided by "NOE Genetik", cover 95 auction days from 1995 till 2003. For each animal the winning bid, breed, category, two quality criteria, weight, date of the auction, (anonymous) identity of the seller, and (anonymous) identity of the winning bidder (the number of the "Winker") are known. The identity of the sellers can be traced across auction days, those of the bidders only within auction days. However, some of the regular bidders always obtain a "Winker" with a particular number. The data also include the outcome of each auction. It may one of the following four cases, "sold in the auction", "sold after the auction", "the seller does not accept the price obtained in the auction" and "no bidder is willing to accept the initial price". Only few objects were sold after the auction and not during the auction. Usually, in these cases the seller had an outside offer which he or she accepted. These deals also go through the auction house as every seller commits himself or herself to report any sale of a registered animal. Finally, for each auction day the order of sale is known.

Table 1 presents summary statistics. From 1995 till 2003, 27183 cows were registered for sale; 25125 (92%) cows were sold, for 1497 (6%) cows the seller rejected the outcome of the auction and for 561 (2%) cows the initial offer was not accepted. The average winning bid of all registered objects was Euro 1275.[12] For the cases when the seller did not accept the offer or there was no initial offer, the winning bid is equal to the last submitted offer. The average winning bid of those cows that were sold in the auction was equal to Euro 1296, that of those cows that were sold after the auction was equal to Euro 1381. There was variation in the mean winning bids across various subgroups like breed, category or quality. The two most often offered product groups are Fleckvieh, female calves of quality 2B and Fleckvieh, female calves of quality 3A. They amount for 66% and 16% of the overall sample.

Table 1 about here

Within an auction day, bidders can be identified by the number of their "Winker". With the help of this variable I define two groups of bidders. As representatives of wholesale firms usually get a "Winker" number ending with a zero like 10, 20, and so on, these bidders are defined as traders. The other bidders are defined as farmers. In contrast to the group of sellers it is not possible to follow different bidders across auctions days. The average winning bids of the two bidder groups differ. The mean price of traders equals Euro 1197, that of farmers Euro 1352, a price difference of roughly 12%. One reason for this difference could be that these two groups bid for cattle of different quality, another that the bidding behavior is different across bidders.

To investigate how prices evolve over an auction day, I divide each auction day into ten intervals and we observe that the mean price decreases from Euro 1263 in the morning to Euro 1054 in the afternoon, a price difference of roughly 17%. Actually, we

[12] Prices are constant as of 1996. Prices before the introduction of the Euro are divided by 13.7603 which denotes the reference value for the Austrian Schilling in the Euro currency board.

first observe increasing and then decreasing prices, which can be explained by the order of sale according to breed, category and the two quality criteria. I therefore also calculated the price difference over an auction day within particular product groups (same breed, same category and same qualities). Here, I observe monotone price declines of about 20% depending on the product group. Then I calculated the price decline of traders' and farmers' prices. The first is equal to 5%, whereas the later is of about 22%.

Table 2 presents bidders' demand schedules. Most of the cattle were sold to bidders who bought one or two animals. Within this group most buyers were farmers. Generally, we notice a large difference between traders and farmers, not only with respect to their demand, but also with respect to the prices they pay. The majority of traders buys between 11 and 40 cattle per auction day. They pay lower prices and prices are (relatively) constant. The majority of farmers buys one cattle and those, who buy a second one, pay a lower price for that.

Table 2 about here

The descriptive statistics show a falling price sequence. It may be caused by the order of sale according to observed product characteristics or by bidders' strategic behavior. The descriptive statistics further show that bidders demand not only one unit and that sellers' rate to reject the outcome of the auction is not negligible. The next section studies bidders' strategic behavior taking into account the results of the descriptive analysis.

3 Theoretical considerations

This section outlines the sequential bidding process on which the empirical bidding model is build. I first describe bidders' demand and their valuations. I then describe bidders' strategic behavior when they demand one unit, when they demand more than one unit, and when there is a secret reserve price available to sellers who may reject the outcome of the auction.

3.1 Bidders' demand and valuations

There is a sequence of ascending auctions.[13] Assume there are $l = 1, \dots, L$ units for sale and $i = 1, \dots, I$ risk neutral bidders who demand at least one unit. Let d_i be bidder i's capacity on an auction day. It is determined by the size of bidder i's truck and thus determines bidder i's demand on an auction day. Total demand is denoted by D and is equal to the sum of the demand of all bidders, i.e. $D = \sum_{i=1}^{I} d_i$. The values d_i are independent draws from a continuous probability distribution F with the associated density function f. Let v_{il} be bidder i's valuation for the l-th unit. Bidder i's valuation v_{il} is a linear function of observed product characteristics z_l and bidder i's demand schedule $\gamma_i(u_i, q_{il})$, such that

$$v_{il} = \begin{cases} \beta z_l + \gamma_i(u_i, q_{il}) & \text{if } q_{il} \leq d_i \\ 0 & \text{otherwise} \end{cases}, \qquad (1)$$

where β is a coefficient to be estimated. Bidder i's demand schedule γ_i is a function of bidder i's private valuation u_i and the number of units q_{il} bidder i has bought until l. Bidder i's valuation depends on the number of bought units q_{il} as long as it is smaller than bidder i's demand d_i. Otherwise bidder i's valuation is equal to zero. The values u_i are independent draws from a continuous probability distribution G with the associated density function g.

[13] There are some forms of ascending auctions. Milgrom and Weber (1982, 2000) describe button (clock, Japanese) auctions when they analyze this format. There, the price raises continuously and bidders press a button as long as they wish to stay in the auction. Once they have left the auction, they have to stay out. These auctions are also called open-exit auctions, as the exit of one bidder can be observed by other bidders. The auctions in Amstetten are ascending, closed-exit auctions with a fixed step size announced by the auctioneer. The bidders use their "Winkers" to indicate their willingness to accept an announced offer. If they wish to do so, they raise their "Winker". Afterwards they put it down. Thus, one cannot perfectly observe whether other bidders have already left the auction or are still participating. However, I assume that the button auction is a sufficient model of the auctions in Amstetten.

Without specifying the optimal behavior of bidders and sellers, the winning bid b_{wl} can be expressed as a function of product characteristics and a function a_{wl} that measures all game theoretic relevant components, such that

$$b_{wl} = \beta z_l + a_{wl}, \tag{2}$$

where the subscript w denotes the winning bidder. In the following, I describe the functional form of a_{wl} for some special cases.

3.2 One-unit demand

With independent private values, the outcome of an one-unit ascending auction is equivalent to the outcome of an one-unit second-price sealed bid auction. In the latter, bidders submit sealed bids to a seller. As Vickrey (1961) shows the dominant strategy for each bidder in the one-unit second-price auction is to bid one's own valuation regardless of the other bidders' valuations, i.e. $b_{opt} = \beta z_l + u_i$. In the ascending auction, the dominant strategy is to bid up to one's own valuation. This result does not depend on the symmetry of bidders' valuations or risk aversion. The highest bid wins, but the price is only equal to the second highest bid. If there is a sequence of independent one-unit auctions , the function a_{wl} is equal to

$$a_{wl} = u_{[2:I]} \ \forall l, \tag{3}$$

where $u_{[2:I]}$ denotes the second-order statistic of the distribution G.

If there is more than one unit and bidders demand only one unit, Milgrom and Weber (2000) show that the optimal strategy in a sequence of ascending auctions is also to bid up to one's own valuation for each object, i.e. $b_{opt} = \beta z_l + u_i$. An optimal stopping rule determines the price of each unit. The L bidders with the L-th highest valuations are going to obtain one unit. In the first auction, they stay in the auction till the bidder with the $L+1$-th highest valuation has left. The remaining L bidders leave simultaneously and the unit is assigned to the bidder with the highest valuation at a price equal to the value of the $L+1$ highest bidder. In the next auction, there is one bidder less and one object less for sale. The $L-1$ bidders with the $L-1$-th highest valuations are going to obtain one unit. They stay in the auction till the bidder with the $L+1$-th highest valuation has left. The remaining $L-1$ bidders leave simultaneously and the unit is assigned to the bidder with the second highest valuation at a price equal to the value of the $L+1$ highest bidder. This procedure is applied until the last unit. The function a_{wl} is then equal to

$$a_{wl} = u_{[L+1:I]} \ \forall l, \tag{4}$$

where $u_{[L+1:I]}$ denotes the $L+1$-th order statistic of the distribution function G. Prices are expected to be constant over an auction day and to depend negatively on L. The more units L are offered on an auction day, the lower the price.

3.3 Multi-unit demand

If bidders demand more than one unit, Donald, Paarsch and Roberts (2006) show that expected prices increase. In general, there are multiple equilibria and there might be even no closed form solution for a_{wl}. To still illustrate such a case, I describe the generalized Vickrey auction with L units, where bidders are assumed to bid demand schedules. In a generalized Vickrey auction, the auctioneer sorts all the submitted demand schedules and allocates the L units to the L-th highest bids at prices equal to the bids of the highest losing bidders. A bidder who wins q_{wL} units on an auction day pays the q_{wL} highest losing bids of the other bidders.[14] The optimal strategy in the generalized Vickrey auction is to bid one's own demand schedule, $b_{opt} = \beta z_l + \gamma_i(u_i, q_{il})$. For this auction, the function a_{wl} is equal to

$$a_{wl} = \gamma_{-w}(u_{[L+q_{wL}-q_{wl}:D]}, q_{-wl}), \tag{5}$$

[14]See for example, Krishna (2002) or Paarsch and Hong (2006).

where γ_{-w} is the residual supply that each winning bidder w is facing, i.e. the aggregated demand schedule of all other bidders. Prices depend on the arguments of γ_{-w}. These are the valuations $u_{[L+q_{wL}-q_{wl}:D]}$ of the highest losing bids (out of D positive valuations) and the demand q_{-wl} of other bidders, where q_{wL} is the total number of units bought by the winning bidder on an auction day, q_{wl} is the cumulative number of units bought by the winning bidder on an auction day.

Prices are expected to depend positively on q_{wL} and q_{wl} and negatively on L and q_{-wl}. The more units the winning bidder q_{wL} demands, the higher the price. Winning bidders pay the lowest price for their first unit and the highest price for their last unit or equivalently, they shade their bids for earlier units. Prices are therefore expected to increase with the cumulative number of units bought by the winning bidder q_{wl}. According to standard arguments, we expect that the more units L are offered on an auction day, the lower the price. The function γ_{-w} is decreasing in q_{-wl} and we therefore also expect prices to decrease with the cumulative number of units bought by other bidders q_{-wl}.

3.4 Sellers' valuations and a secret reserve price

In Amstetten, a seller may reject the outcome of an auction. Although some sellers offer more than one unit, I assume for simplicity that there are L sellers each of them offering one unit. Let r_l be seller l's valuation of the l-th unit and let it be an independent draw from a continuous probability distribution H with the associated density function h. In contrast to bidders, each seller faces a static decision problem.

Seller l optimally rejects the outcome of the l-th auction, i.e. the adjusted winning bid a_{wl}, if a_{wl} is lower than his valuation r_l, and accepts the outcome of the auction, if a_{wl} is equal or higher than his valuation r_l. The possibility to reject the outcome of the auction effects the number of offered units and adds uncertainty to total supply. Rational bidders are aware of this. If they leave the auction too early – compared to a model without a secret reserve price, sellers with a high valuation will reject the outcome of the auction and fewer units will be available. Bidders therefore adjust their strategic behavior to influence the probability of rejection.

The sequence of winning bids changes due to the change in bidders' optimal behavior. If we assume that there are two units for sale and bidders demand only one unit, bidder i's optimal strategy in the second auction is to bid up to her valuation u_i. Her expected profit π_2 in the second auction is equal to

$$\pi_2(u_i) = u_i - \rho_2\{\rho_1 E[u_{[3]}|u_i = u_{[2]}] + (1-\rho_1)E[u_{[2]}|u_i = u_{[1]}]\}, \tag{6}$$

where ρ_l is the probability that the outcome of the l-th auction is not rejected, i.e. $\rho_l = \text{Prob}(r_l < a_{wl})$. Bidder i's profit in the second auction is equal to bidder i's valuation u_i minus the price paid in the second auction given that second unit is sold. This price is a weighted average of the third-order statistic and the second-order statistic with the weight to be the probability that the first unit is sold, ρ_1.

Bidder i's optimal strategy in the first auction is to bid her valuation u_i minus the expected payoff in the second auction (6):

$$b_1(u_i) = u_i - \pi_2(u_i) = (1-\rho_2)u_i + \rho_2\{\rho_1 E[u_{[3]}|u_i = u_{[2]}] + (1-\rho_1)E[u_{[2]}|u_i = u_{[1]}]\}. \tag{7}$$

To guard themselves against a lower probability to win, bidders react by bidding more aggressively. They bid a weighted average of their valuation u_i and the price in the second auction, i.e. $\rho_1 E[u_{[3]}|u_i = u_{[2]}] + (1-\rho_1)E[u_{[2]}|u_i = u_{[1]}]$. The higher the probabilities of rejection, the more aggressive bidders bid, as otherwise some of the bidders do not obtain a unit although their valuation is higher than the valuation of the seller. If the probabilities of rejection are very large, bidders stay in the first auction even until the bidder with the second highest valuation has left.[15]

[15] Jeitschko (1999) formally describes the equilibrium price paths of a model that also comes close to the situation in Amstetten. In his model, there are two units for sale and with some probability ρ there

If the outcome of the first auction and the second auction are not rejected by the respective sellers and both units are sold, we can show that the price of the second unit is expected to be lower than the price of the first unit, i.e. $E[P_2|p_1] < p_1$. The proof is analogous to the proof in Jeitschko (1999).[16] If both units are sold, the function a_{wl} is equal to

$$a_{wl} = \begin{cases} (1-\rho_2\rho_1)u_{[2:I]} + \rho_2\rho_1 u_{[3:I]} & \text{if l=1} \\ u_{[3:I]} & \text{if l=2} \end{cases}, \tag{8}$$

where the price in the first auction is equal to a weighted average of the second and the third order statistic. Prices are expected to decline and to depend positively on the probability of rejection $(1-\rho_2\rho_1)$.

4 Empirical bidding model

This section presents the empirical bidding model. I describe the econometric specification, the estimation method and the construction of variables.

4.1 Price equations

To assess the effect of bidders' preferences and strategic behavior on prices, I assume that a_{wl} can be approximated by a linear function such that the bidding model (2) becomes

$$p_l = \beta z_l + \gamma_1 L + \gamma_2 q_{wL} + \gamma_3 q_{wl} + \gamma_4 q_{-wl} + \gamma_5 \rho_l + \gamma_6 s_l + \epsilon_l, \tag{9}$$

where $l = 1, \ldots, L$ and $L = 1, \ldots, 96$ the number of auction days. The dependent variable p_l is the logarithm of the observed winning bid b_{wl}. The independent variables are product

is no second auction. The probability $\rho \in [0, 1]$ is common knowledge and can be viewed as an estimate of sellers' valuations that is known to all bidders in advance. In the second auction, bidders bid up to their valuation u_i and in the first auction, bidders bid a weighted average of their valuation u_i and the price in the second auction, the third-order statistic $u_{[3:I]}$. Beggs and Graddy (1997) obtain a similar outcome, when the order of sale is according to quality and bidders' valuations are assumed to decline with a common factor $\rho \in [0, 1]$.

[16] A detailed proof is given in Appendix B.

characteristics z, where z also includes product-specific, auction day-specific and seller-specific fixed effects. Other independent variables are the arguments of the function a_{wl} given by (4), (5) and (8). The number of offered units L is used to test the strategic effect of sequential auctions (see equation 4). The total number of units bought by the winning bidder on an auction day q_{wL}, the cumulative number of units bought by the winning bidder q_{wl} and the cumulative number of units bought by other bidders q_{-wl} are used to test for the strategic effect of multi-unit demand (see equation 5). The probability of rejection ρ_l and the order of units s_l are used to test the effect of the secret reserve price (see equation 8). The number of bidders I is not known from the data. In line with the theoretical considerations, it is assumed to be constant over an auction day. Its effect is therefore not identified from the auction day-specific fixed effects. ϵ_l is the error term.

I estimate equation (9) with ordinary least squares. To account for differences in bidders' preferences, I also estimate it separately for traders and farmers. To explore the determinants of the price patterns, I decompose the price difference between traders and farmers with techniques developed by Blinder (1973), Oaxaca (1973) and Neumark (1988). These techniques are used in labor economics to analyze wage differentials and distinguish between differences in human capital endowment and differences in the valuation of human capital across groups of individuals. Here, these techniques help to understand whether the difference in prices across bidders can be explained by differences in preferences or differences in strategic behavior.[17]

4.2 Probability of rejection

To model the probability of rejection ρ_l, I use data on units for which the seller has rejected the outcome of the auction. There are 1497 such observtions (see Table 1). I estimate a probit model with an indicator variable that is equal to one for unsold cows

[17] A description of the Blinder-Oaxaca decomposition is given in Appendix C.

and equal to zero otherwise as the dependent variable. Explanatory variables are product characteristics z, including product-specific, auction day-specific and seller-specific dummy variables, the number of offered units L, total number of units bought by the winning bidder on an auction day q_{wL}, cumulative number of units bought by the winning bidder q_{wl}, cumulative number of units bought by other bidders q_{-wl} and order of units s_l. I then construct the inverse Mill's ratio to be eventually used as an estimate for the probability of rejection ρ_l in the price equations.[18] As there is no obvious choice for an exclusion restriction, the probit and price equations are only identified through the nonlinearity of the inverse Mill's ratio.

4.3 Construction of variables

Product characteristics like breed, category and quality are transformed into dummy variables, which are equal to one, when a particular classification is met and zero otherwise. For example, there is a dummy variable that is equal to one for Fleckvieh and zero for Braunvieh or one that is equal to one for Quality 1B and zero else. Additionally, I construct product-specific dummy variables. For each product, defined by the characteristics breed, category, quality and subquality, a dummy variable is constructed that is equal to one for the product and zero else. There are four products that account for the largest proportion of the sample. These are Fleckvieh, female calves of quality 2B, subquality 1 (2 or 3) and Fleckvieh, female calves of quality 3A, subquality 1.

Auction days are also transformed into auction day specific dummy variables. The data include an identifier for sellers that allows me to follow them across auction days. I construct personal specific dummy variables for six sellers who supply more than 100 animals in the years 1995 to 2003. There are further 80 sellers who sold between 50 and 100 objects. These are summarized to one group specific dummy variable.

[18] For a discussion on the inverse Mill's ratio see for example, Wooldridge (2001, chapter 17).

L is the number of offered cows on an auction day. This variable does not vary within auction days, but only over auction days. Its effect can therefore only be tested, if there is data from a sufficient number of auction days. For each winning bidder, the variable q_{wL} is the total number of units bought by the winning bidder on an auction day. For each bidder, this variables does not vary within auction days, but only over auction days. Its effect can therefore only be tested, if there is enough variation across bidders. The variable q_{wl} is equal to the cumulative number of units bought by the winning bidder within an auction day. It is zero for the first bought unit on each auction day, equal to one for the second bought unit and so on. The variable q_{-wl} is equal to the cumulative number of units bought by the other bidders on an auction day. The latter two variables vary over bidders and auction days.

The order of units s_l on a particular auction day is constructed as a variable that measures the relative time elapsed on an auction day. The number of units offered at various auction days differs. I normalize these numbers to one and construct a variable that is equal to zero before the auction starts, that linearly increases in the course of the auction day and is equal to one at the end of an auction day.

5 Estimation results

In this section I present the estimation results of the probit model and the hedonic price equations. I report the results of basic and some further specifications. Finally, I describe the decomposition results.

5.1 Basic specifications

Table 3 presents the estimation results. Column 1 reports the results of the probit model. It reports the change in the probability for an infinitesimal change in independent, continuous variables and the discrete change in the probability for dummy variables. We find that the explanatory power is of 12% and that the probability of rejection significantly depends on some but not all product characteristics and bidder's own demand. Important product characteristics are a cow's quality and weight. A higher quality and more weight makes it more unlikely that the cow stays unsold. The contrary is true for bidders' own demand. The more cows a bidder has already bought, the more likely that a cow stays unsold. The product-specific, auction day-specific and seller-specific effects also have a significant effect on the probability of rejection.

Columns 2-8 present the estimation results for various basic specifications of the price equation (9). Column (2) uses data on all bidders, whereas all other specifications separate between traders and farmers. The first specification (columns 2, 3 and 4) includes product characteristics and the order of units. It constitutes that specification to which I am going to compare the other specifications. It is also the most common specification in the literature.[19] The second specification (columns 5 and 6) additionally includes the number of offered units, the total number of units bought by the winning bidder on an auction day, cumulative number of units bought by the winning bidder and cumulative number of units bought by other bidders. This specification accounts for the strategic effect of sequential auctions and bidders' multi-unit demand. The third specification (columns 7 and 8) additionally includes the inverse Mill's ratio to account for the strategic effect of the secret reserve price. Product-specific, auction day-specific and seller-specific dummy variables are used and prove to be highly significant in all specifications. To avoid a potential misclassification of traders and farmers, I drop observations when traders bought less than ten cows or when farmers bought more than two cows (see also Table 2). We find that the explanatory power of the conducted regressions is quite good. Between 62 and 70% of the variance in the dependent variable can be explained.

When we compare column 2 with columns 3 and 4, we observe that some of estimated

[19] See for example, Beggs and Graddy (1997) or Deltas and Kosmopoulou (2004).

coefficients are different across specifications. Examples are quality 2B, weight and the order of units. To test their statistical significance, I conduct a Chow test. The F-statistic has (116, 22148) degrees of freedom and a value of 24.29. The null hypothesis of equal coefficients can be rejected. This indicates the importance to distinguish between traders and farmers in the regressions. It also indicates that bidders' preferences are different. Although there are differences in the estimated coefficients, there are no differences in the general perception which characteristics are worth most.

Table 3 about here

The estimated values for the constant are 5.7630 and 5.7778 for traders and between 5.9796 and 6.0501 for farmers. As the coefficients indicate, to control for product quality does not necessarily provide an explanation for the price difference between traders and farmers. Traders pay between 20 and 30% less for a cattle with basic product characteristics (Braunvieh, cow, quality 3B, subquality 3) than farmers do. To recall, the raw price differential is of 12%. This might be an indication that the valuations of traders are on average lower than those of farmers. The constant is, however, not identified from the constant of the linear function approximating the strategic behavior.

The estimated coefficients of the product characteristics are jointly significant in all equations. Their signs and order of magnitude depict the same general pattern for traders and farmers. We observe, as earlier mentioned, differences in the coefficients for category, quality 2B and weight. The difference in the coefficient for young female calves might account for differences in risk aversion as in contrast to female calves and cows, young female calves do not produce milk yet and thus their quality, which is based on their mothers' quality, is afflicted with some uncertainty. The higher coefficient for quality 2B in the equation for farmers, reflects their preference for this quality. The higher coefficient of weight in the equation of traders mirrors their preference for cows with more weight. As traders prefer pregnant cows over lactating cows as the latter can not be transported long distances, it is most likely that the higher coefficient of weight accounts for the additional value of the unborn calf.

The price decline over auction days observed in the raw data is mainly explained by the behavior of farmers. Still, both groups of bidders pay declining prices over an auction day. The extent is different. Everything equal, the average price a trader (farmer) pays for the first cow on an auction day is about 3% (13%) higher than that he or she pays for the last cow (see columns 3 and 4). This result provides evidence that part of the raw price decline of 17% (5% for traders and 22% for farmers) can be explained by the fact that the order of sale is according to quality.[20]

When we add the number of offered units, the total number of units bought by the winning bidder on an auction day, cumulative number of units bought by the winning bidder and cumulative number of units bought by other bidders to the regression equations, the estimated coefficients of the order of units change. It becomes a positive value of about 1% for traders (column 5). It is, however, insignificant. It decreases to significant 5% for farmers (column 6). When we add the probability of rejection to the regression equations, the estimated coefficients of the order of units change only slightly (columns 7 and 8). These results indicate that the estimated price decline is actually less pronounced compared to a standard specification used in the literature, once we take the institutional aspects of the auctions into account. For some bidders we even do not observe a price

[20]By normalizing the variable the order of units to be zero at the beginning and to be one at the end of each action day, it is defined to measure the relative time elapsed on an auction day. If instead, I use a variable that measures the absolute time elapsed, i.e. the number of remaining objects, I obtain qualitatively the same results (in absolute values only, as the two variables are defined in the opposite way). However, to be able to compare the estimated coefficients, I calculated the elasticities evaluated at the sample means for both variables. I obtained 1.8% and 1.8% for traders and 6.0% and 6.2% for farmers. The first numbers are for the relative time elapsed and the second numbers are for the absolute time elapsed on an auction day. All these numbers should be interpreted as such prices decline over an auction day.

decline anymore.

In turn, we observe that prices also depend on the variables that explain bidders' strategic behavior with respect to the sequential aspect of the auctions, bidders' multi-unit demand and the secret reserve price. The estimated coefficient of the number of offered units tests the prediction of the sequential aspect of the auctions (see equation 4). It is insignificant for traders and significantly negative for farmers. Traders do no adjust their bids to different market situations. This result also indicates that their aggregate demand function is highly elastic. Farmers, whereas, adjust their bids in a way as is expected. The more units are offered the lower the price.

The estimated coefficients of the total number of units bought by the winning bidder on an auction day, cumulative number of units bought by the winning bidder and cumulative number of units bought by other bidders test the prediction of multi-unit demand (see equation 5). The estimated coefficient of the number of units bought by the winning bidder on an auction day is significantly positive for traders and significantly negative for farmers. The values are 0.0004 and -0.0264. The more units traders buy on an auction day, the higher the price. A higher demand let them bid more aggressively. We observe the opposite for farmers. The reason for this result is that there are only two groups of farmers buying either one or two units, and therefore, there is not enough variation in the data to identify the effect of this variable. The more units bidders have already bought within an auction day, the higher the price. Bidders shade their bids and take the strategic effect of multi-unit demand into account. The estimated coefficients correspond to a 1.0% elasticity for traders and a 0.2% elasticity for farmers (each evaluated at the sample mean). The effect of bid shading – assuming it to be linear – is given but small. This is in particular true when we compare these values to the estimated elasticities for the cumulative number of units bought by other bidders. The more objects other bidders have already bought the lower the price. The estimated coefficients correspond to a

3.2% elasticity for traders and a 4.0% elasticity for farmers (each evaluated at the sample mean).

The estimated coefficient of the probability of rejection tests the prediction of the secret reserve price (see equation 8). We expect it to be positively related with prices. If it is significant, we also expect prices to decline over an auction day. If, however, only the coefficient of the order of units is significant, declining prices are not explained by the secret reserve price but might be due to other reasons like participation cost. As columns 7 and 8 show, the estimated coefficient of the probability of rejection is insignificantly different from zero for traders and significantly different from zero for farmers. Traders do not adjust their bids to the secret reserve price, whereas farmers do. The higher the probability that the seller rejects the outcome of the auction, the higher the prices of farmers. In line with the predictions, the estimated price decline is at the same time insignificant for traders and significant for farmers. The joint occurrence of these results let us believe that there is no explanation for the price decline other than the secret reserve price. To evaluate the strength of this hypothesis, I conduct robustness checks and estimate further specifications to account for bidders' participation.

5.2 Bidders' participation

Other reasons for declining prices may be a declining participation due to the order of sale according to quality or due to participation cost. Up to now, I have assumed that the number of bidders I, although unknown and only jointly identified with auction day-specific fixed effects, is constant over an auction day. Cows of different quality are imperfect substitutes. If bidders' substitution patterns are different, different product qualities may attract a different number of bidders. The order of sale according to quality then automatically induces a decreasing participation, as bidders who only buy higher quality do not bid for lower quality. If there are participation cost, any order of sale induces a decreasing participation.[21] Both effects would be measured by the order of units.

I estimate further specifications of the price equation. The first specification adds the order of units multiplied with particular product groups to the specification presented in columns 7 and 8 of Table 3. The second specification additionally includes the order of units multiplied with the deciles in which each auction day can be divided (see also Table 1). The first set of variables tests for different participation behavior within particular product groups, whereas the second set of variables tests whether participation is driven by particular times of an auction day. All specifications are again estimated with product-specific, auction day-specific and seller-specific dummy variables. To support the hypothesis that declining prices are primarily caused by the secret reserve price, we expect the previously obtained results to sustain. The added variables are either insignificant or, if they are significant, do not not change the signs and significance of the variables that describe bidders' strategic behavior with respect to the secret reserve price, i.e. the probability of rejection and order of units.

Table 4 presents the estimation results. Columns 1 and 2 report the results of the extended probit models. Columns 3-8 present the results of the extended price equations for all bidders, traders and farmers. We find that the explanatory power of the regressions does not improve anymore. Columns 3 and 4 present the results for all bidders. They are depicted for comparison reasons only. I also conduct Chow tests and can reject the null hypotheses of equal coefficients for both specifications.

Table 4 about here

Columns 5 and 6 present the results when we add the order of units multiplied with

[21]For explicit models with participation cost see for example, von der Fehr (1994) or Menezes and Monteiro (1997). For structural models that account for bidders' participation see Bajari and Hortacsu (2003), Donald, Paarsch and Robert (2006) or Song (2004).

particular product groups for traders and farmers. We observe a larger than previously estimated price decline over the auction day and significant price movements for each product group. The overall price decline is of about 6.0%, although still insignificant, for traders and 26.2% for farmers, whereas the price decline for the product group "Braunvieh" is stronger and the price decline for the product group Fleckvieh, female calves, quality 2B, sub-quality 2 is weaker than the overall decline. The estimated coefficient of the inverse Mill's ratio is insignificant for traders and it is significant for farmers.

Columns 7 and 8 present the last specification for traders and farmers. The estimated coefficients of the order of units multiplied with ten intervals in which each auction day can be divided are jointly significant for traders, but none of the coefficient is individually significant. The estimated coefficients are jointly and individually insignificant for farmers.

The estimation results of the specifications that take the bidders' participation behavior into account support the hypothesis that declining prices are mainly caused by the secret reserve price. For both further specifications, we observe an insignificant probability of rejection combined with an insignificant price decline for traders and a significant probability of rejection combined with a significant price decline for farmers. The added variables do not change the previously obtained results. The results, however, further show that different product groups attract different numbers of bidders.

5.3 Decomposition results

To evaluate the price difference between traders and farmers, I employ decomposition techniques. There are three sources that may explain this difference: bidders' preferences for particular products, strategic behavior and participation. Bidders' preferences are measured by product characteristics, product-specific and seller-specific fixed effects. Strategic behavior is measured by the number of offered units, total number of units bought by the winning bidder, cumulative number of units bought by the winning bidder, cumulative number of units bought by other bidders and order of units. Participation behavior is measured by the order of units multiplied with particular product groups and order of units multiplied with the deciles in which each auction day can be divided.

Table 5 presents the decomposition results. The basic results are obtained using the specifications presented in columns 7 and 8 in Table 3, but without the variable the number of units bought by the winning bidder on an auction day. I leave out this variable as it does not measures the same for traders and farmers (see section 5.1). I calculate the decomposition results for three additional specifications. The first of these specifications drops the variables that describe bidders' preferences; the second drops the variables that describe strategic behavior; and the third adds the variables that describes participation behavior. In all specifications, the distribution of reference is the distribution of all bids.

The (logarithmic) price difference between traders and farmers is equal to 0.129. The decomposition results of the basic specification show that the unexplained part of the price difference is equal to 0.034 (27.3%) and the explained part of the price difference is equal to 0.091 (72.7%). About three quarters third of the raw price difference can be attributed to bidders' endowment, i.e. which product qualities bidders go for. The other part is due to differences in coefficients that may reflect differences in the willingness to pay, strategic behavior or participation.

Table 5 about here

The results for the other specifications show that bidders' preferences are the most important source of the unexplained price difference between traders and farmers. Bidders' strategic behavior is the second most important source, whereas a contribution of variables that describe bidders' participation is not given.

Particular variables that contribute most to the unexplained price difference between traders and farmers are quality 2B, weight, the number of offered units, cumulative number of units bought by the winning bidder, order of units and the inverse Mill's ratio. These results indicate that differences in strategic behavior are mainly driven by differences in the reaction to supply and uncertainty of supply.

6 Summary and concluding remarks

I presented an empirical analysis of bidding behavior in sequential cattle auctions. The objectives were to investigate the role of institutional specifics as determinants of prices. Institutional specifics are the order of sale according to quality, a secret reserve price, bidders' multi-unit demand and the existence of two bidder groups, i.e. traders and farmers. Prices decline over auction days and bidders with a higher demand pay on average lower prices.

I discussed various bidding models and their predictions on prices. I showed that a secret reserve price in sequential auctions is equivalent to uncertainty about the number of offered units and that prices are expected to decline. If bidders demand more than one unit, prices are expected to increase (Donald, Paarsch and Roberts 2006). I then estimated reduced form price equations and employed decomposition techniques to find out, whether the decline in prices is caused by the order of sale according to quality or the secret reserve price. Further questions were, whether bidders consider the strategic effects generated by the sequential auctions and multi-unit demand, and whether the price difference between traders and farmers is caused by differences in bidders' preferences or differences in strategic behavior. For the analysis, I utilized a large data set on cattle auctions that covers 95 auction days from 1995 till 2003 with more than 25,000 dairy cows offered.

The main estimation results are summarized as follows. Product characteristics are the most important source of variation in prices. This is not astonishing given that a cow's quality determines its (known) common value on the market. Nevertheless, institutional aspects and their effect on bidders' strategic behavior are non-negligible determinants of prices. Declining prices are caused by the order of sale according to quality and the secret reserve price. This is evidence in line with Beggs and Graddy (1997) and Deltas and Kosmopoulou (2004), who can also explain observed price patterns by specific institutions. Further, bidders take the strategic effects of sequential auctions and multi-unit demand into account and shade their bids for earlier units.

The price difference between traders and farmers is mainly explained by differences in preferences and only to a lesser extent by differences in strategic behavior. Traders are in general more insensitive to institutional aspects. They strategically react to sequential auctions and multi-unit demand, but do not adjust their bids to the secret reserve price. Their prices are on average constant over auction days and their aggregate demand is highly elastic – a result in line with treasury auctions, where the aggregate demand of internationally acting banks is also highly elastic (Nyborg, Rydqvist and Sundaresan 2002, and Elsinger and Zulehner 2007). Farmers react to all institutional aspects, in particular to supply and to the uncertainty of supply generated by the secret reserve price. This reflects the difference of these two groups. Traders regularly attend auctions not only in Amstetten but also in other places. Farmers most likely have to rely on the auction market in Amstetten and rationally react to supply in this market.

Institutional aspects may have opposite effects on prices in sequential auctions. If the empirical analysis does not or due to lack of data cannot take all aspects into account, the evidence of some strategic behavior might be buried under a general price decline. Once we take all aspects into account, the empirical evidence might be in accordance to the predictions of the theoretical models.

References

Ashenfelter, O., 1989. How auctions work for wine and art. Journal of Economic Perspectives 3, 39–64.

Athey, S., Haile, P., 2005. Nonparametric approaches to auctions. in J.J. Heckman: The Handbook of Econometrics, Volume 6, Elsevier Science.

Ausubel, L. M., 2004. An efficient ascending-bid auction for multiple objects. American Economic Review 94 (5), 1452–1475.

Avery, C., 1998. Strategic jump bidding in English auctions. Review of Economic Studies 65 (2), 185–210.

Bajari, P., Hortacsu, A., 2003. The winner's curse, reserve prices and endogenous entry: Empirical insights from eBay auctions. Rand Journal of Economics 34 (2), 329–355.

Beggs, A., Graddy, K., 1997. Declining values and the afternoon effect: Evidence from art auctions. RAND Journal of Economics 28 (3), 544–565.

Blinder, A. S., 1973. Wage discrimination: Reduced form and structural estimates. Journal of Human Resources 18:4, 436–455.

Deltas, G., Kosmopoulou, G., 2004. 'Catalogue' vs. 'order-of-sale' effects in sequential auctions: Theory and evidence from a rare book sale. Economic Journal 114, 28–54.

Donald, S. D., Paarsch, H. J., Robert, J., 2006. An empirical model of the multi-unit, sequential, clock auction. Journal of Applied Econometrics 21, 1221–1247.

Eklöf, M., Lunander, A., 2003. Open outcry auctions with secret reserve prices: An empirical application to executive auctions if tenant owner's apartments in sweden. Journal of Econometrics 114, 243–260.

Elsinger, H., Zulehner, C., 2007. Bidding behavior in Austrian treasury bond auctions. Monetary Policy and the Economy Q2/07, 109–125.

Elyakime, B., Laffont, J.-J., Ossard, H., Vuong, Q., 1994. First-price sealed-bid auctions with secret reservation prices. Annales d'Econmie et de Statistique 34, 115–141.

Engelbrecht-Wiggans, R., Kahn, C. M., 1999. Calibration of a model of declining prices in cattle auctions. Quarterly Review of Economics and Finance 39 (1), 113–28.

Halie, P., 2001. Timber auctions with resale markets. American Economic Review 91 (3), 399–427.

Hendricks, K., Paarsch, H. J., 1995. A survey of recent empirical work concerning auctions. Canadian Journal of Economics 28 (2), 403–426.

Jeitschko, T. D., 1999. Equilibrium price paths in sequential auctions with stochastic supply. Economic Letters 64, 67–72.

Krishna, V., 2002. Auction Theory. Academic Press, San Diego.

Laffont, J.-J., 1997. Game theory and empirical economics: The case of auction data. European Economic Review 41, 1–35.

Laffont, J.-J., Loisel, P., Robert, J., 1997. Intra-day dynamics in sequential auctions: Theory and estimation, mimeo, IDEI Toulouse.

Laffont, J.-J., Ossard, H., Vuong, Q., 1995. Econometrics of first-price auctions. Econometrica 63, 953–980.

Menezes, F. M., Monteiro, P. K., 1997. Sequential asymmetric auctions with endogenous participation. Theory and Decision 43, 187–202.

Milgrom, P. R., Weber, R. J., 1982. A theory of auctions and competitive bidding. Econometrica 50, 1089–1122.

Milgrom, P. R., Weber, R. J., 2000. A theory of auctions and competitive bidding II. in Paul Klemperer, ed.: The Economic Theory of Auctions, Edgar Elgar, 179–194.

Neumark, D., 1988. Employers' discriminatory behavior and the estimation of wage discrimination. Journal of Human Resources 22, 279–295.

Nyborg, K. G., Rydqvist, K., Sundaresan, S. M., 2002. Bidder behavior in multiunit auctions: Evidence from Swedish treasury auctions. Journal of Political Economy 110 (2), 394–424.

Oaxaca, R. L., 1973. Male-female wage differentials in urban labor markets. International Economic Review 14, 693–709.

Paarsch, H. J., 1992. Deciding between the common and private value paradigms in empirical models of auctions. Journal of Econometrics 51, 192–215.

Paarsch, H. J., Hong, H., 2006. An Introduction to the Structural Econometrics of Auction Data. MIT Press, London, with contributions by M. Ryan Haley.

Rezende, L., 2005. Auction econometrics by least squares, mimeo, University of Illionois at Urbana Champaign.

Song, U., 2004. Nonparametric estimation of an eBay auction model with an unknown number of bidders, mimeo, University of British Columbia.

van den Berg, G. J., van Ours, J. C., Pradhan, M. P., 2001. The decling price anomaly in Dutch Dutch rose auctions. American Economic Review 91 (4), 1055–1062.

Vickrey, W., 1961. Counterspeculation, auctions and competitive sealed tenders. Journal of Finance 16, 8–37.

von der Fehr, N.-H., 1994. Predatory pricing in sequential auctions. Oxford Economic Papers 46, 345–356.

Weber, R. J., 1983. Multiple-obejct auctions. in Richard Engelbrecht-Wiggans, Martin Shubik, and Robert M. Stark, eds.: Auctions, Bidding, and Contracting: Uses and Theory, New York University Press, New York, 165–191.

Wooldridge, J. M., 2001. Econometric Analysis of Cross Section and Panel Data. The MIT Press, Cambridge.

A Appendix: Tables

Table 1: Summary statistics for prices (winning bids)

Variable	Number of observations	Mean	Standard error	Minimum	Maximum
Winning bid in Euro	27,183	1275.20	271.1	85.2	4798.2
Outcomes					
Objects sold in the auction	25,036	1296.00	263.4	427.9	4798.2
Objects sold after the auction	89	1380.46	323.4	753.0	2695.8
Seller did not accept the price	1,497	1064.79	222.8	239.9	3125.0
Initial offer was not accepted	561	891.97	174.3	85.2	3088.1
Bidders					
Representatives of wholesale firms	8,982	1196.72	209.7	427.9	3215.3
Farmers from nearby regions	16,143	1351.70	274.2	498.3	4798.2
Breed					
Braunvieh	3,034	1223.13	248.8	237.4	2121.1
Fleckvieh	24,149	1281.75	273.1	85.2	4798.2
Category					
Female Calves	26,563	1283.98	265.1	85.2	4798.2
Young Female Calves	338	762.55	123.3	427.9	1053.0
Cows	282	1062.88	281.9	589.6	2379.7
Quality					
Quality 1B	505	1844.19	303.6	1203.7	4798.2
Quality 2A	782	1632.76	258.7	710.4	2379.7
Quality 2B	20,341	1304.30	235.1	85.2	3215.3
Quality 3A	5,141	1080.75	207.4	242.0	2203.9
Quality 3B	414	890.77	206.3	427.9	1530.7
Subquality 1	14,523	1326.16	299.2	242.0	4798.2
Subquality 2	9,758	1236.88	221.0	85.2	3125.0
Subquality 3	2,902	1149.05	206.2	498.3	2207.6
Most often offered product groups					
Fleckvieh, female calves, quality 2B	17899	1315.26	234.2	85.2	3215.3
Fleckvieh, female calves, quality 3A	4402	1100.44	201.6	242.0	2203.9
Order of sale					
Decile 1 (Morning)	2,665	1263.48	271.3	434.7	4798.2
Decile 2	2,712	1347.22	389.0	237.4	4022.0
Decile 3	2,723	1426.74	247.6	443.1	3125.0
Decile 4	2,713	1437.05	218.8	242.0	2695.8
Decile 5	2,758	1372.54	230.3	242.0	2402.7
Decile 6	2,680	1299.99	203.3	596.7	3125.0
Decile 7	2,737	1255.59	200.6	241.3	2207.6
Decile 8	2,699	1192.27	202.2	85.2	3125.0
Decile 9	2,736	1106.64	188.2	242.0	1948.5
Decile 10 (Afternoon)	2,760	1053.47	201.5	242.0	2203.9

Table 1 presents summary statistics of the winning bids. Prices are in constant Euros as of 1996. Prices before the introduction of the Euro are divided by 13.7603, which was the reference value for the Austrian Schilling in the Euro currency board.

Table 2: Demand schedules

Number of units bought on one auction day	Number of bidders All	Number of bidders Traders	Number of bidders Farmers	Mean price in Euro All	Mean price in Euro Traders	Mean price in Euro Farmers
1-10	15,811	465	15,346	1352.1	1118.9	1359.2
1	9,910		9,888	1378.0		1378.7
2	3,882		3,860	1338.0		1339.8
11-20	1,612	1,346	266	1213.0	1214.1	1207.7
21-30	3,462	2,964	498	1198.0	1195.8	1211.2
31-40	2,645	2,612	33	1191.9	1192.4	1154.3
41-50	679	679		1199.9	1199.9	
51-60	327	327		1240.5	1240.5	
61-100	589	589		1214.5	1214.5	
Total	25,125	8,982	16,143	1296.3	1196.7	1351.7

Table 2 presents the demand schedules of all bidders, traders and farmers. For each bidder group, the number of units bought and the mean prices is given in frequency intervals. Prices are in constant Euros as of 1996. Prices before the introduction of the Euro are divided by 13.7603, which was the reference value for the Austrian Schilling in the Euro currency board.

Table 3: Determinants of sale and prices

Data set	Offered units	All bidders	Traders	Farmers	Traders	Farmers	Traders	Farmers
Number of observations	23740	22265	8517	13748	8517	13748	8517	13726
Pseudo / Adjusted R-squared	0.12	0.64	0.69	0.62	0.70	0.62	0.70	0.62
Dependent variable:	Prob(Rejection)	Ln(Winning bid)						
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	1.0951	5.9869	5.7658	6.0501	5.7695	6.0126	5.7683	6.0275
	(1.73)	(249.70)**	(175.04)**	(175.28)**	(116.53)**	(121.26)**	(96.98)**	(51.77)**
Fleckvieh	-0.0200	0.0644	0.0289	0.0806	0.0307	0.0820	0.0307	0.0837
	(-0.11)	(7.13)**	(1.93)	(7.54)**	(2.06)*	(7.69)**	(2.05)*	(7.83)**
Female calves	-0.5013	0.2452	0.2522	0.2573	0.2529	0.2521	0.2528	0.2403
	(-2.46)*	(21.02)**	(12.01)**	(18.93)**	(12.07)**	(18.61)**	(12.00)**	(17.31)**
Young female calves	-2.2570	-0.1740	-0.0836	-0.1949	-0.0832	-0.1964	-0.0836	-0.2449
	(-7.12)**	(-12.80)**	(-4.20)**	(-8.62)**	(-4.19)**	(-8.71)**	(-3.70)**	(-9.91)**
Quality 1B	-0.7403	0.5285	0.4751	0.5452	0.4755	0.5475	0.4753	0.5045
	(-3.04)**	(41.32)**	(22.40)**	(28.33)**	(22.47)**	(28.53)**	(21.97)**	(24.04)**
Quality 2A	-0.5831	0.4412	0.3447	0.4690	0.3450	0.4720	0.3449	0.4319
	(-2.80)**	(37.77)**	(21.42)**	(25.25)**	(21.50)**	(25.48)**	(21.14)**	(21.42)**
Quality 2B	-0.7036	0.2552	0.1436	0.3106	0.1444	0.3127	0.1443	0.2732
	(-3.11)**	(21.66)**	(11.05)**	(15.83)**	(11.13)**	(15.98)**	(10.94)**	(12.82)**
Quality 3A	-0.7604	0.0858	0.0459	0.1203	0.0469	0.1208	0.0468	0.0810
	(-3.36)**	(7.47)**	(3.88)**	(5.78)**	(3.98)**	(5.82)**	(3.87)**	(3.60)**
Sub-quality 1	-0.1644	0.1205	0.0752	0.1221	0.0784	0.1252	0.0783	0.1224
	(-1.06)	(11.81)**	(6.49)**	(8.26)**	(6.78)**	(8.50)**	(6.73)**	(8.31)**
Sub-quality 2	-0.1578	0.0367	0.0272	0.0465	0.0291	0.0486	0.0291	0.0466
	(-1.01)	(3.53)**	(2.34)*	(3.06)**	(2.51)*	(3.22)**	(2.49)*	(3.09)**
Weight	-2.5259	0.6376	1.0847	0.5127	1.0882	0.5124	1.0876	0.4391
	(-9.06)**	(42.16)**	(51.49)**	(24.84)**	(51.77)**	(24.93)**	(41.93)**	(17.80)**

Table 3: Determinants of sale and prices continued

Data set	Offered units	All bidders	Traders	Farmers	Traders	Farmers	Traders	Farmers
Number of observations	23740	22265	8517	13748	8517	13748	8517	13726
Pseudo / Adjusted R-squared	0.12	0.64	0.69	0.62	0.70	0.62	0.70	0.62
Dependent variable:	Prob(Rejection)				Ln(Winning bid)			
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Order of units	-0.5561	-0.0724	-0.0322	-0.1336	0.0103	-0.0595	0.0102	-0.0669
	(-1.99)*	(-7.11)**	(-2.39)*	(-9.73)**	(0.52)	(-2.83)**	(0.51)	(-3.14)**
Number of offered units	-0.0009				-0.0001	0.0002	-0.0001	-0.0017
	(-0.58)				(-0.48)	(1.69)	(-0.48)	(-19.28)**
Total number of units bought	0.0366				0.0004	-0.0264	0.0004	-0.0252
by winning bidder	(26.14)**				(2.91)**	(-8.43)**	(1.33)	(-8.03)**
Cumulative number of units bought	-0.0210				0.0003	0.0343	0.0003	0.0337
by winning bidder	(-9.79)**				(1.56)	(8.39)**	(1.18)	(8.24)**
Cumulative number of units bought	0.0035				-0.0002	-0.0003	-0.0002	-0.0002
by other bidders	(5.21)**				(-3.73)**	(-5.34)**	(-3.42)**	(-3.23)**
Inverse Mill's ratio							0.0023	0.8872
							(0.04)	(5.35)**
Product-specific fixed effects (4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	(9.65)**	(11.53)**	(4.92)**	(6.16)**	(5.08)**	(6.42)**	(4.78)**	(3.22)**
Auction day-specific fixed effects (94)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	(330.58)**	(141.21)**	(80.25)**	(79.51)**	(73.16)**	(68.34)**	(58.39)**	(56.22)**
Seller-specific fixed effects (7)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	(39.81)**	(63.54)**	(13.52)**	(45.27)**	(13.62)**	(46.04)**	(13.49)**	(48.20)**

Table 3 presents the estimation results from the probit model and the price equations for all bidders and separately for traders and farmers. In the probit model (column 1), the dependent variable is an indicator variable, which is equal to one when the object was sold and zero otherwise. Explanatory variables are product characteristics, the order of units, number of offered units, total number of units bought by winning bidder, cumulative number of units bought by winning bidder and cumulative number of units bought by other bidders. In the price equations (columns 2-8), the dependent variable is the logarithm of the wining bid. The first specification (columns 2-4) includes product characteristics and the order of units; the second specification (columns 5 and 6) additionally includes the number of offered units, total number of units bought by winning bidder, cumulative number of units bought by winning bidder and cumulative number of units bought by other bidders; and the third specification (columns 7 and 8) additionally includes the inverse Mill's ratio. The reference group with respect to product characteristics is Braunvieh, cows, quality 3B and subquality 3. All specifications are estimated with product-specific, auction day-specific and seller-specific dummy variables. Absolute values of t-statistics are shown in parentheses below the parameter estimates. In the probit equation, the value of the χ^2-statistic is shown for the fixed effects and in the hedonic price equations, it is the value of the F-statistic.
** (*) denotes a 95% (90%) level of significance.

Table 4: Robustness checks

Data set	Offered units	Offered units	All bidders	All bidders	Traders	Farmers	Traders	Farmers
Number of observations	23740	23740	22243	22243	8517	13726	8517	13726
Pseudo / Adjusted R-squared	0.12	0.12	0.65	0.65	0.70	0.63	0.70	0.63
Dependent variable:	Prob(Rejection)		Ln(Winning bid)					
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.2328	1.7992	6.6950	6.6908	5.7676	6.3817	5.7490	6.4066
	(0.33)	(2.08)*	(155.54)**	(138.93)**	(96.82)**	(56.39)**	(87.02)**	(53.93)**
Fleckvieh	-0.5541	-0.7741	0.1025	0.0958	0.0607	0.1040	0.0632	0.0891
	(-2.39)*	(-2.72)**	(8.86)**	(6.70)**	(2.63)**	(7.11)**	(2.51)*	(4.79)**
Female calves	-0.8064	-0.7322	0.2995	0.3004	0.2820	0.2793	0.2856	0.2799
	(-3.59)**	(-3.25)**	(23.32)**	(23.38)**	(11.07)**	(18.24)**	(11.18)**	(18.21)**
Young female calves	-1.6444	-1.6447	-0.1682	-0.1653	-0.0989	-0.2686	-0.0908	-0.2692
	(-4.85)**	(-4.84)**	(-10.99)**	(-10.78)**	(-4.55)**	(-10.59)**	(-4.13)**	(-10.57)**
Quality 1B	0.6347	0.5817	0.3858	0.3854	0.4249	0.3805	0.4213	0.3784
	(1.35)	(1.23)	(18.58)**	(18.56)**	(13.25)**	(10.06)**	(13.12)**	(9.94)**
Quality 2A	0.6688	0.6367	0.3096	0.3088	0.3030	0.3131	0.2995	0.3109
	(1.57)	(1.48)	(16.33)**	(16.29)**	(11.75)**	(8.69)**	(11.60)**	(8.54)**
Quality 2B	0.3272	0.4028	0.1684	0.1690	0.1245	0.1804	0.1214	0.1813
	(0.84)	(1.02)	(10.36)**	(10.35)**	(7.55)**	(5.18)**	(7.24)**	(5.18)**
Quality 3A	0.1856	0.1738	0.0476	0.0474	0.0360	0.0292	0.0347	0.0265
	(0.50)	(0.46)	(3.25)**	(3.24)**	(2.66)**	(0.85)	(2.55)*	(0.76)
Sub-quality 1	-0.1208	-0.1199	0.0931	0.0941	0.0731	0.0780	0.0750	0.0785
	(-0.69)	(-0.67)	(8.36)**	(8.44)**	(5.69)**	(4.72)**	(5.83)**	(4.74)**
Sub-quality 2	-0.1115	-0.0620	0.0213	0.0218	0.0228	0.0271	0.0228	0.0287
	(-0.68)	(-0.37)	(2.02)*	(2.06)*	(1.88)	(1.75)	(1.88)	(1.85)
Weight	-2.5622	-2.5524	0.7105	0.7155	1.0863	0.4769	1.1032	0.4753
	(-9.16)**	(-9.11)**	(42.31)**	(42.29)**	(43.57)**	(20.04)**	(42.77)**	(19.50)**

Table 4: Robustness checks continued

Data set	Offered units	Offered units	All bidders	All bidders	Traders	Farmers	Traders	Farmers
Number of observations	23740	23740	22243	22243	8517	13726	8517	13726
Pseudo / Adjusted R-squared	0.12	0.12	0.64	0.64	0.70	0.62	0.70	0.62
Dependent variable:	Prob(Rejection)		Ln(Winning bid)					
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Order of units	1.1978	-0.2514	-0.2249	-0.2023	-0.0574	-0.2321	-0.0100	-0.2587
	(2.08)*	(-0.32)	(-7.72)**	(-5.04)**	(-1.49)	(-4.54)**	(-0.19)	(-4.18)**
Order of objects ×								
Braunvieh	-0.5227	-2.5693	-0.3214	-0.3610	-0.0891	-0.7110	-0.0808	-0.7914
	(-0.31)	(-1.25)	(-3.26)**	(-3.20)**	(-0.74)	(-4.72)**	(-0.57)	(-4.79)**
Sub-quality 1	-2.3953	-1.4090	0.4061	0.3841	0.1470	0.3931	0.2227	0.3486
	(-3.65)**	(-1.46)	(12.46)**	(8.20)**	(3.23)**	(7.21)**	(3.26)**	(5.14)**
Sub-quality 2	-1.8326	-2.5858	0.0862	0.0958	0.0184	0.0764	-0.0362	0.0878
	(-3.05)**	(-3.27)**	(2.78)**	(2.30)*	(0.44)	(1.44)	(-0.62)	(1.31)
Sub-quality 3	-1.6585	-3.2349	-0.1078	-0.0341	0.0671	-0.1793	0.1548	-0.1467
	(-2.05)*	(-3.26)**	(-2.37)*	(-0.61)	(1.06)	(-2.63)**	(1.98)*	(-1.75)
Number of offered units	-0.0007	-0.0007	-0.0017	-0.0017	-0.0001	-0.0017	-0.0001	-0.0017
	(-0.46)	(-0.45)	(-23.56)**	(-23.48)**	(-0.65)	(-19.38)**	(-0.53)	(-19.35)**
Total number of units bought	0.0366	0.0366	-0.0027	-0.0028	0.0004	-0.0253	0.0002	-0.0253
by winning bidder	(25.91)**	(25.89)**	(-18.98)**	(-18.95)**	(1.73)	(-8.12)**	(0.62)	(-8.13)**
Cumulative number of units bought	-0.0208	-0.0209	0.0034	0.0034	0.0002	0.0334	0.0004	0.0335
by winning bidder	(-9.61)**	(-9.64)**	(20.64)**	(20.77)**	(1.05)	(8.23)**	(1.67)	(8.26)**
Cumulative number of units bought	0.0035	0.0035	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
by other bidders	(5.22)**	(5.26)**	(-5.78)**	(-5.93)**	(-3.35)**	(-3.97)**	(-3.77)**	(-3.87)**
Inverse Mill's ratio			-0.0936	-0.1182	0.0187	0.5744	-0.0462	0.5951
			(-2.16)*	(-2.62)**	(0.34)	(3.90)**	(-0.75)	(3.71)**
Order of objects × Deciles (9)	No	Yes	No	Yes	No	No	Yes	Yes
		(37.17)**		(1.19)			(2.55)**	(1.32)
Product-specific fixed effects (4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	(34.06)**	(18.58)**	(47.43)**	(15.31)**	(6.27)**	(29.32)**	(3.91)**	(7.65)**
Auction day-specific fixed effects (94)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	(324.44)**	(321.54)**	(91.08)**	(89.44)**	(51.93)**	(41.92)**	(50.93)**	(40.63)**
Seller-specific fixed effects (7)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	(40.01)**	(39.15)**	(62.20)**	(62.17)**	(13.43)**	(44.51)**	(13.39)**	(44.85)**

Table 4 presents the estimation results from the probit model and the price equations for all bidders and separately for traders and farmers. There are two specifications. In the probit models (columns 1 and 2), the dependent variable is an indicator variable, which is equal to one when the object was sold and zero otherwise. Explanatory variables in column (1) are product characteristics, the order of units, number of offered units, total number of units bought by winning bidder, cumulative number of units bought by winning bidder, cumulative number of units bought by other bidders and the order of units multiplied with particular product groups. Explanatory variables in column (2) additionally include the order of units multiplied with the deciles in which each auction day can be divided. In the price equations (columns 3-8), the dependent variable is the logarithm of the wining bid. The first specification (columns 3, 5 and 7) includes product characteristics, the order of units, number of offered units, number of units bought by winning bidder, number of units bought by winning bidder until l, number of units bought by other bidders and the order of units multiplied with particular product groups; and the second specification (columns 4, 6 and 8) additionally include the order of units multiplied with the deciles in which each auction day can be divided. The reference group with respect to product characteristics is Braunvieh, cows, quality 3B and subquality 3. All specifications are estimated with product-specific, auction day-specific and seller-specific dummy variables. Absolute values of t-statistics are shown in parentheses below the parameter estimates. In the probit equation, the value of the χ^2-statistic is shown for the fixed effects and in the hedonic price equations, it is the value of the F-statistic. *** (**, *) denotes a 99% (95%, 90%) level of significance.

Table 5: Blinder-Oaxaca decomposition results

Price difference Distribution of reference: all bids	Total	Unexplained	Explained
Basic specification	0.129	0.034	0.091
		27.3%	72.7%
Additional specifications			
Without variables that describe product characteristics	0.129	0.077	0.048
		61.5%	38.5%
Without variables that describe strategic behavior	0.129	0.037	0.087
		29.8%	70.2%
With variables that describe participation behavior	0.129	0.034	0.090
		27.6%	72.4%
Contribution of selected variables			
Quality 2B		0.118	0.043
Weight		-0.423	-0.016
Order of units		-0.027	0.005
Number of offered units		-0.451	0.032
Number of bought units		-0.101	0.010
Inverse Mill's ratio		0.677	-0.005

Table 5 presents decomposition results based on separate price regressions for traders and farmers. The distribution of reference is the distribution of all bids. The dependent variable is the logarithm of the winning bid. The explanatory variables of the basic specification are a constant, product characteristics, the order of units, total number of units bought by winning bidder, cumulative number of units bought by winning bidder and cumulative number of units bought by other bidders, product-specific, auction day-specific and seller-specific fixed effects. The variables that describe bidders' preferences are product characteristics, product-specific and seller-specific fixed effects. The variables that describe the strategic behavior are the order of units, the number of bought units, the number of units bought by other bidders and the number of offered units. Variables that describe the participation behavior are the order of units multiplied with particular product groups and the order of units multiplied with the deciles in which each auction day can be divided. The reference group with respect to product characteristics is Braunvieh, cows, quality 3B and subquality 3.

B Appendix: Proposition and proof

Proposition 1. *Suppose the outcomes of the first and the second auction are not rejected by the respective sellers and both units are sold, then the price of the second unit is expected to be lower than the price of the first unit, i.e.* $E[P_2|p_1] < p_1$.

Proof. The expected price P_2 in the second auction given that both units are sold is equal to

$$E[P_2|p_1] = E[b_2(u_{[3]})|b_1(u_{[2]}) = p_1] = E[u_{[3]}|b_1^{-1}(p_1) = u_{[2]}]. \tag{10}$$

By rearranging the first period bidding function (7) and replacing u_i by $b_1^{-1}(p_1)$, equation (7) becomes

$$E[P_2|p_1] = \frac{p_1 - (1-\rho_2)b_1^{-1}(p_1) - \rho_2(1-\rho_1)E[u_{[2]}|b_1^{-1}(p_1) = u_{[1]}]}{\rho_1\rho_2}, \tag{11}$$

and we notice that

$$p_1 - \rho_1\rho_2\{p_1 - E[u_{[2]}|b_1^{-1}(p_1) = u_{[1]}]\} < b_1^{-1}(p_1) - \rho_2\{b_1^{-1}(p_1) - E[u_{[2]}|b_1^{-1}(p_1) = u_{[1]}]\} \tag{12}$$

is true as $p_1 > b_1(p_1)$ and $E[u_{[2]}|b_1^{-1}(p_1) = u_{[1]}] > p_1$. Therefore, $E[P_2|p_1] < p_1$ as asserted.

□

C Appendix: Blinder-Oaxaca decomposition

Blinder-Oaxaca decompositions are usually used in labor economics to analyze wage differentials across sex and/or race. With the help of these techniques one can divide the raw wage differential in an explained and an unexplained part. The first is due to differences in the endowment of human capital and other explanatory variables. The second part is due differences in coefficients and in labor economics often denoted discrimination. When applying this method to price differences in auctions, the difference in the endowment reflects a difference in preferences. For example, which qualities bidders go for. The unexplained part mirrors a difference in coefficients. Controlling for preferences, i.e. product characteristics, differences in coefficients may be perceived as a difference in strategic behavior or relative demand.

There are two groups of individuals, $i = 1, 2$. For each group a regression equation is estimated by ordinary least squares:

$$Y_i = X_i \; \theta_i + u_i, i = 1, 2,$$

with Y_i to be logarithmic price of object i and X_i to be the respective explanatory variables. The Blinder-Oaxaca decomposition uses the property of the ordinary least square estimator $\hat{\theta}_i$, that the means of observations $\overline{Y}_i$ and $\overline{X}_i$ lie on the regression plane:

$$\overline{Y}_i = \overline{X}_i \; \hat{\theta}_i, \; i = 1, 2.$$

The mean difference in the dependent variable can then be decomposed into two weighted differences:

$$(\overline{Y}_1 - \overline{Y}_2) = (\overline{X}_1 - \overline{X}_2) \; \hat{\theta}_1 + \; \overline{X}_2(\hat{\theta}_1 - \hat{\theta}_2).$$

It is a weighted sum of differences in explanatory variables and of differences in coefficients. The first difference is called the explained part. The second one is called the unexplained part. The reference distribution is the distribution of the first group. This decomposition is not unique, as there is an index problem. Another form is to weight the above described differences with the regression coefficient of the second group and the mean vector of explanatory variables of the first group, respectively:

$$(\overline{Y}_1 - \overline{Y}_2) = (\overline{X}_1 - \overline{X}_2)\ \hat{\theta}_2 +\ \overline{X}_1(\hat{\theta}_1 - \hat{\theta}_2).$$

The reference distribution is the distribution of the second group. There are also other forms of decomposition using other distributions as the reference distribution like the overall population mean.