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Quality-improving horizontal innovations

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Abstract: We modify the growth model with horizontal innovations from Romer (1990) and Jones (1995) by assuming that researchers can determine the quality of the products they invent. Because research effort increases with product quality, the researchers face a tradeoff between the quantity and the quality of their innovations. We show (i) that every product is slowly but steadily driven out of the market by superior goods, (ii) that the model does not exhibit a strong scale effect, (iii) that the long-run growth rate of the economy is not policy-invariant but can be controlled by quality-contingent R&D subsidies, and (iv) that the optimal solution can be decentralized using a mix of production subsidies, R&D subsidies, and lump-sum taxation.

Journal of Economic Literature **classification codes:** O41, O31, O23

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1 Introduction

Romer (1990) was the first to formally demonstrate within a dynamic general equilibrium framework how the excludability of ideas creates incentives for research and development (R&D), and how these incentives can lead to endogenous growth of per-capita output. He assumed that there exists a range of intermediate goods, each one produced according to a different design. The patent system ensures that every design can be used only by a single producer who is therefore able to make positive profits. The prospect of inventing new designs that generate positive profit flows induces households to devote part of their labor force (or human capital) to R&D. As a result, the range of intermediate goods expands over time (horizontal innovations, variety expansion) and per-capita output grows.

Romer (1990) as well as many other writers following his approach assumed that all intermediate goods have the same quality, that is, they are equally useful in the production of final output.[1] This homogeneity assumption rules out creative destruction: once a good has been invented, it is produced and used forever at the same rate as all other goods. Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992), on the other hand, introduced models in which R&D leads to the improvement of existing products rather than to the invention of new ones. In these models, the range of intermediate goods remains constant, but the quality of any given product increases over time (vertical innovations, quality improvements). As soon as an improved design for a product is invented, the innovator captures the whole market for that particular variety by setting a limit price. In other words, these so-called Schumpeterian growth models describe a process of creative destruction.

In the present paper we modify the model of horizontal innovations by assuming that intermediate goods can be heterogeneous with respect to their quality. Since the quality of an intermediate good determines its productivity in the final good sector, it follows that designs for high-quality products are more valuable than those for low quality products. Researchers can determine the quality of their designs at the time of invention. In doing so they face a trade-off between the quality (and, hence, the value) and the quantity of new inventions because designs of high quality require more research effort than those of low quality. The researchers select that point on the quality/quantity frontier which maximizes the return to their research effort.

Adding a quality dimension to the model of horizontal innovations gives it a distinctive Schumpeterian flavor. As the quality of intermediate goods increases, final good producers substitute the new and better goods for the old and inferior ones. Because different intermediate goods are *imperfect* substitutes for each other, this does not lead to an immediate replacement of the old inputs by the new ones, as in the above mentioned models of vertical innovation, but to a slow reduction of the utilization of low quality inputs. More specifically, the ratio between the production rate of any given intermediate good and the average production rate of all intermediate goods converges to zero at a finite rate. Instead of the creative *destruction* captured by models of vertical innovations, our model is therefore more appropriately described as one of creative *displacement*, whereby the process of displacement happens faster in quickly growing economies than in slowly growing ones.

[1] For a recent survey see Gancia and Zilibotti (2005).

Including the quality decision in the model of horizontal innovations has also another advantage, which has to do with the scale effect. What makes endogenous growth possible in all R&D driven growth models is an external effect of existing knowledge on the productivity of researchers. This externality arises because of the non-rival nature of knowledge. In Romer (1990), the effect takes the form $\dot{A}(t) = \text{const.} \times L_R(t)A(t)$, where $A(t)$ denotes the number of intermediate goods at time t and $L_R(t)$ measures research time (or human capital utilization in the R&D sector). The proportionality of research productivity, $\dot{A}(t)/L_R(t)$, to the number of existing designs, $A(t)$, creates the well-known strong scale effect according to which larger economies grow at higher rates than smaller ones. Jones (1995) emphasized that there is little empirical support for this scale effect and he suggested an alternative model that avoids it. According to that model, research output is described by an equation of the form $\dot{A}(t) = \text{const.} \times L_R(t)A(t)^\gamma$, where $\gamma < 1$. The productivity of researchers is therefore no longer proportional to existing knowledge but it is an increasing and strictly concave function of $A(t)$. Although this specification gets rid of the scale effect, it creates another unpleasant feature. The long-run growth rate of the economy depends only on the rate of population growth and on the externality parameter γ. Since neither of these two parameters can be easily influenced by policy, is is common to refer to this model as one of semi-endogenous growth (rather than truly endogenous growth).[2]

The model discussed in the present paper uses the same approach as Jones (1995) to avoid the strong scale effect but it retains the property that R&D subsidies have a long-run growth effect. We describe existing knowledge at time t not by the *number* of existing intermediate goods, $A(t)$, but by their *average quality* which we denote by $Q(t)$. Adapting the approach suggested by Jones (1995) to this scenario, the research equation takes the form $\dot{A}(t) = \text{const.} \times L_R(t)Q(t)^\gamma$. If the researchers choose quality optimally, it turns out that average quality $Q(t)$ is related to $A(t)$ according to $Q(t) = \text{const.} \times A(t)^\kappa$, where κ is a variable that can be influenced by R&D subsidies. This implies that growth is described by the equation $\dot{A}(t) = \text{const.} \times L_R(t)A(t)^{\gamma\kappa}$. Although formally equivalent to the main assumption made by Jones (1995), this equation does not imply policy-invariance because the exponent of $A(t)$ and, hence, the long-run growth rate of the economy is not a function of policy-independent parameters but can be controlled by the government.

At first glance it may appear that the mechanism by which the scale effect is eliminated in our model is the same as that in earlier papers combining aspects of horizontal and vertical innovations like Young (1998), Peretto (1998), or Dinopoulos and Thompson (1998).[3] This is, however, not the case because those authors deal essentially with models of vertical innovations even if they assume that the number of differentiated varieties is endogenously determined. Research effort is spread across the entire range of intermediate goods (or sectors), that is, all existing varieties can be improved by vertical innovations. If an increase in the scale of the economy increases also the rents that can be captured by successful innovators, then this leads to research on a broader range of goods without increasing the intensity of research in each sector: in the long-run, the growth rate of the economy remains the same. The model of the

[2] Other models that are based on similar ideas as those in Jones (1995) have been developed for example by Kortum (1997) and Segerstrom (1999).

[3] For some non-scale growth models, like Aghion and Howitt (1998, chap. 12), it is quite obvious that they are based on mechanisms different from the one in the present paper as they do not include an explicit choice of product quality.

present paper, on the other hand, is primarily one of horizontal innovations in which research is not sector-specific. Increased rents can therefore not be dissipated through increased entry and, as in Jones (1995), it is only the declining ratio of R&D productivity to existing knowledge that avoids the increase of the growth rate. To summarize this point, the contribution of the present paper to the discussion about the scale effect is not to show how the *strong scale effect* can be avoided in the endogenous growth model of Romer (1990) but to show how *policy-invariance* can be avoided in the semi-endogenous growth model of Jones (1995).

The question of why policy has long-run growth effects in the present paper but not in the semi-endogenous growth models of Jones (1995), Kortum (1997), or Segerstrom (1999) has a simple answer. The underlying reason is that R&D subsidies can be made quality-dependent.[4] Because of the positive external effect of average quality on the productivity of researchers, policy incentives for the invention of high quality products translate directly into high productivity in R&D which, in turn, leads to high growth of per-capita output. One policy implication of the present paper is therefore that, in order to promote high growth, governments should make their R&D subsidies dependent on how useful the resulting innovations are in final output production.

We compute both the market equilibrium under laissez-faire and the solution that would be chosen by a benevolent social planner. Both the growth rate of per-capita output and the research intensity are lower under laissez-faire than they are in the optimal solution. Using a simple growth accounting exercise we can show that quality improvements contribute more to overall economic growth in the optimal solution than in the market equilibrium, whereas the contribution of variety expansion is smaller in the social planner's solution than in equilibrium. Finally, we show that the social planner's solution can be implemented by the government via a combination of a production subsidy for intermediate goods producers (to compensate for monopolistic competition in that sector) and a quality-contingent R&D subsidy (to compensate for the external effects in R&D) provided that both subsidies are financed by a lump-sum tax on households.

The rest of this paper is organized as follows. In Section 2 we formulate the model. Apart from our description of R&D, this model is very similar to those in Romer (1990) and Jones (1995). In Section 3 we derive the market equilibrium of the economy, focussing on balanced growth paths. This allows us to discuss the process of creative displacement. Section 4 first presents the social planner's solution of the model and compares it to the market equilibrium. We then show how the government can affect the long-run growth rate by a quality-dependent R&D subsidy and how it can implement the optimal solution via taxes and subsidies. Finally, Section 5 concludes the paper.

2 Model formulation

We consider a continuous-time model of an economy that lasts from $t = 0$ to $t = +\infty$. The economy has two production sectors, one for final output and the other one for intermediate goods. In addition there exists a R&D sector. Final good producers operate under perfect

[4] This possibility has already been mentioned by Young (1998) although he did not include it in his model.

competition whereas the intermediate goods market is characterized by monopolistic competition. The labor supplied by the households is either used in production or used for research. Research leads to the invention of new intermediate goods. Contrary to the existing models of horizontal innovation, however, it is assumed that researchers can determine the quality of their designs, and intermediate goods are therefore heterogeneous with respect to quality.

2.1 Households

The economy is populated by a continuum of measure 1 of identical infinitely-lived households. At time t, the size of any given household is equal to $L(t)$, where $\dot{L}(t) = nL(t)$. The population growth rate n is assumed to be strictly positive. Households derive utility from consumption according to the utility functional[5]

$$\int_0^{+\infty} e^{-(\rho-n)t} \ln[c(t)] \, \mathrm{d}t, \tag{1}$$

where $c(t)$ denotes per-capita consumption in period t. The parameter ρ is the common time-preference rate of the households. It is assumed that $\rho > n$, which ensures that the utility functional takes finite values for all exponentially growing paths of per-capita consumption.

Each individual is endowed with one unit of homogeneous labor per time period. The labor endowment of the representative household at time t is therefore equal to $L(t)$. The labor supplied by the households is either used as an input in one of the two production sectors of the economy or for research. The competitive real wage in production is denoted by $w(t)$. The rate of return to one unit of labor used in research is denoted by $w_R(t)$. Total labor income by the representative household is therefore equal to $[L_Y(t) + L_X(t)]w(t) + L_R(t)w_R(t)$, where $L_Y(t)$, $L_X(t)$, and $L_R(t)$ denote the labor employed in final good production, intermediate goods production, and research, respectively.

Households can store wealth by holding shares in dividend paying firms.[6] Due to no-arbitrage conditions, all these assets have the same real rate of return (dividends plus capital gains), which we denote by $r(t)$. Let us denote by $a(t)$ the wealth owned by the representative household at time t. It evolves according to the flow budget constraint

$$\dot{a}(t) + L(t)c(t) = a(t)r(t) + [L_Y(t) + L_X(t)]w(t) + L_R(t)w_R(t). \tag{2}$$

Income consists of asset returns and labor income (right-hand side) and is used for asset accumulation and consumption (left-hand side).

The representative household maximizes its utility given in (1) subject to the flow budget constraint (2), the resource constraint

$$L_Y(t) + L_X(t) + L_R(t) = L(t), \tag{3}$$

[5] The analysis presented below can be generalized to the case where households have the instantaneous utility functional $[c(t)^{1-\theta} - 1]/(1-\theta)$ with $\theta > 0$. All key results of the paper carry over to this more general case. To simplify the algebra a bit, we restrict the presentation to the logarithmic case $\theta = 1$.

[6] These are the intermediate goods producing firms. Firms in the final good sector make zero profits due to perfect competition and constant returns to scale.

and the no-Ponzi game condition

$$\lim_{t\to+\infty} a(t)e^{-\int_0^t r(\tau)\,\mathrm{d}\tau} = 0. \tag{4}$$

A necessary and sufficient condition for an optimal consumption path is the Euler equation

$$\dot{c}(t)/c(t) = r(t) - \rho. \tag{5}$$

2.2 Final output

A single homogeneous final good is produced from labor and differentiated intermediate goods. The set of intermediate goods available at time t is the interval $[0, A(t)]$. The production function for final output is

$$Y(t) = L_Y(t)^{1-\alpha} \int_0^{A(t)} q_i^{1-\alpha} x_i(t)^\alpha \,\mathrm{d}i, \tag{6}$$

where $Y(t)$, $L_Y(t)$, and $x_i(t)$ denote the rate of final output in period t and the corresponding input rates of labor and intermediate good i, respectively. The number $\alpha \in (0,1)$ is an exogenously given technological parameter.[7] The parameter q_i describes the quality of good i which is an indicator of its productivity in the final good sector.[8] The quality of each intermediate good is determined at the time of its invention and does not change over time. Different intermediate goods, however, may be of different quality. We will explain in subsection 2.4 below how the measure of intermediate goods, $A(t)$, and the quality of an intermediate good, q_i, is determined by R&D.

Final output is chosen as the numeraire. Firms in the final good sector take the measure of available intermediate goods, $A(t)$, their respective quality levels, q_i, the real wage, $w(t)$, and the prices of intermediate goods, $p_i(t)$, as given and maximize their profit rates. The necessary and sufficient first-order conditions for this profit maximization problem are

$$w(t) = (1-\alpha)Y(t)/L_Y(t) \tag{7}$$

and

$$p_i(t) = \alpha[q_i L_Y(t)/x_i(t)]^{1-\alpha}. \tag{8}$$

2.3 Intermediate goods

All intermediate goods are produced by the same technology which uses labor as its only input. By a suitable choice of the units of measurement we may assume that the production of one

[7] The parameter α has multiple interpretations: it measures for example the labor share of income generated in final good production, the degree of market power, and the returns to specialization. It would be possible to disentangle (some of) the roles of α along the lines suggested by Benassy (1998) and Alvarez-Pelaez and Groth (2005), but for the sake of simplicity we refrain from doing so.

[8] The particular way how quality q_i and quantity $x_i(t)$ are combined in (6) follows Dinopoulos and Thompson (1998) and Aghion and Howitt (2005). Other formulations exist in the literature but using them would complicate the algebra without substantially altering the results of the present paper.

unit of any intermediate good requires one unit of labor (irrespective of the quality of the good). Monopoly rights for the production of intermediate good i are secured by a permanent patent. The firm holding that patent is therefore a monopolist and maximizes its profit rate subject to the technological constraint and the inverse demand function given in (8). Formally, in every period t, firm i chooses $x_i(t) \geq 0$ so as to maximize

$$\pi_i(t) = p_i(t)x_i(t) - w(t)x_i(t) = \alpha[q_i L_Y(t)]^{1-\alpha} x_i(t)^\alpha - w(t)x_i(t). \tag{9}$$

The necessary and sufficient first-order condition for profit maximization yields

$$x_i(t) = \left[\alpha^2/w(t)\right]^{1/(1-\alpha)} q_i L_Y(t) \tag{10}$$

and $p_i(t) = w(t)/\alpha$. All intermediate goods are sold for the same price (a constant markup on production costs) but their production rates are proportional to their quality levels. Intermediate goods of higher quality are therefore produced at higher rates. These results reflect the two assumptions that the production technology is the same for all intermediate goods and that the demand for an intermediate good is proportional to its quality.

Substituting (10) into (9) one finds that firm i's profit rate is given by

$$\pi_i(t) = (1-\alpha)\left[\alpha^{(1+\alpha)}/w(t)^\alpha\right]^{1/(1-\alpha)} q_i L_Y(t).$$

The present value as of time t of the profit flow for firm i over the interval $[t, +\infty)$ is therefore

$$V_i(t) = \int_t^{+\infty} e^{-\int_t^\tau r(\tau')\,\mathrm{d}\tau'} \pi_i(\tau)\,\mathrm{d}\tau = q_i v(t), \tag{11}$$

where

$$v(t) = (1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)} \int_t^{+\infty} e^{-\int_t^\tau r(\tau')\,\mathrm{d}\tau'} w(\tau)^{-\alpha/(1-\alpha)} L_Y(\tau)\,\mathrm{d}\tau. \tag{12}$$

$V_i(t)$ is the value of firm i or, equivalently, its share price at time t. Finally, we note that the total amount of labor used for the production of intermediate goods is given by

$$L_X(t) = \int_0^{A(t)} x_i(t)\,\mathrm{d}i = \left[\alpha^2/w(t)\right]^{1/(1-\alpha)} A(t)Q(t)L_Y(t), \tag{13}$$

where

$$Q(t) = A(t)^{-1} \int_0^{A(t)} q_i\,\mathrm{d}i \tag{14}$$

is the average quality of intermediate goods existing at time t (also called the quality index).

2.4 Research and development

Each intermediate good is produced according to a different design. These designs, in turn, are produced by researchers. The R&D sector is a perfectly competitive industry with no barriers to entry, that is, every person can create designs and sell them to potential intermediate good producers at competitive prices.

We assume that researchers can choose the quality levels of the product designs they create. This choice involves a tradeoff between the research effort they have to spend in order to create a new design on the one hand and the value (i.e., market price) of the new design on the other hand. The latter is determined by the present value of future profits that can be generated using the new product. As can be seen from (11), this value is proportional to the product quality.

As for the research effort necessary to develop a new design, we assume that it is an increasing function of the quality of the product and a decreasing function of the average quality of the already existing products. More specifically, the rate at which a single researcher makes innovations of quality q at time t is assumed to be equal to

$$F(q,t) = \begin{cases} [\beta - q/Q(t)]Q(t)^{\gamma} & \text{if } q \leq \beta Q(t), \\ 0 & \text{if } q > \beta Q(t), \end{cases} \tag{15}$$

where $\beta > 1$ and $\gamma \geq 0$ are fixed constants and where $Q(t)$ is the quality index defined in (14).

In this paper, $Q(t)$ is interpreted as a measure of total knowledge available in the economy at time t. The right-hand side of (15) is a decreasing function of the target quality q and an increasing function of the existing knowledge $Q(t)$. The negative dependence of $F(q,t)$ on q means that designs of high quality require more research effort than designs of lower quality. In particular, the specification in (15) implies that it is impossible to create designs of quality $q \geq \beta Q(t)$. The assumption $\beta > 1$ is therefore necessary in order to ensure that quality improvements are feasible.

The positive dependence of $F(q,t)$ on $Q(t)$ reflects an intertemporal knowledge spillover effect. In the present model, this effect has two components corresponding to the two appearances of $Q(t)$ on the right-hand side of (15). To explain them, it is useful to depict equation (15) in the form of a quality/quantity frontier; see figure 1. For a fixed value of $Q(t)$, the set of all pairs $(q, F(q,t))$ satisfying equation (15) forms the downward sloping line AB in the quality/quantity space.[9] An increase of existing knowledge $Q(t)$ makes the frontier flatter (AC) and it shifts it outwards (DE). The flattening of the frontier corresponds to the appearance of $Q(t)$ in the brackets on the right-hand side of (15) and implies that less quantity has to be sacrificed in order to achieve a given quality increment. In other words, researchers can invent better products at any given rate of innovations when they start from a higher average quality of existing products. The outward shift from AC to DE comes from the term $Q(t)^{\gamma}$ in (15) and reflects a general improvement of the quality/quantity tradeoff (without any effect on the rate of transformation between quality and quantity): innovations occur faster and they lead to better products.

It is worth pointing out that the invention of a new product increases total knowledge only to the extent that it improves the average quality $Q(t)$ of all intermediate goods. If a new design of low quality is created, total knowledge as measured by $Q(t)$ decreases. While this possibility may appear counterintuitive at first glance, it is not if one assumes that researchers can only

[9] The linearity of the quality/quantity frontier is imposed for simplicity. None of the main insights derived in this paper depend on this linearity assumption.

sample the existing designs but are unable to study *all* of them. In this case, adding a design of low quality reduces the chances to find and, hence, to profit from good ideas.[10]

Researchers take the productivity function F (or, equivalently, the quality/quantity frontier) as given and choose the quality level q optimally. A researcher who designs a good of quality q at time t will be able to sell it for $qv(t)$; see equation (11). The rate of return to one unit of time spent for designing a good of quality q at time t is therefore equal to

$$W_R(q,t) = qv(t)F(q,t) = q[\beta - q/Q(t)]v(t)Q(t)^{\gamma}. \tag{16}$$

This rate of return has a unique maximum at $q = (\beta/2)Q(t)$. For the sake of a more compact notation, let us define $\kappa = \beta/2 - 1$. Using that definition and noting that all researchers try to make the most profitable innovations, it follows that

$$q_{A(t)} = (1+\kappa)Q(t). \tag{17}$$

Because of (16)-(17), the resulting rate of return to research time is given by

$$w_R(t) = W_R(q_{A(t)}, t) = (1+\kappa)^2 v(t)Q(t)^{1+\gamma}. \tag{18}$$

2.5 Market clearing

Having described the behavior of all agents in the economy, let us now turn to market clearing. Market clearing on intermediate goods markets has already been taken into account by substituting the inverse demand functions in the profit maximization problems of intermediate goods producers. This leaves us with the markets for labor, assets, and final output.[11]

The labor market clearing condition coincides obviously with the representative household's resource constraint (3). However, equilibrium on the labor market imposes also another constraint. Since labor is homogeneous it must receive the same remuneration in all its uses. Labor used as input in final good or intermediate goods production earns the real wage $w(t)$, whereas labor used to do research yields the return $w_R(t)$. If labor is used both in production and research, then it must hold that $w_R(t) = w(t)$. On the other hand, if $w_R(t) < w(t)$, then households will not do any research, that is, $L_R(t) = 0$. The final case $w_R(t) > w(t)$ would lead to $L_Y(t) = L_X(t) = 0$, which cannot occur in equilibrium as labor is required for any form of production. We can therefore summarize these observation in the following condition:

$$w_R(t) \leq w(t) \text{ with equality if } L_R(t) > 0. \tag{19}$$

Asset market clearing requires that the wealth of the representative household at time t is equal to the total value of all intermediate goods producing firms, i.e.,

$$a(t) = \int_0^{A(t)} V_i(t)\,\mathrm{d}i \tag{20}$$

[10] At this stage it is also worth mentioning that all of the results of this paper remain qualitatively correct if one replaces the term $Q(t)^{\gamma}$ in (15) by the more general term $A(t)^{\gamma_A}Q(t)^{\gamma_Q}$ with $\gamma_A \geq 0$ and $\gamma_Q \geq 0$. In this more general setting, the knowledge spillover effect depends on both product variety and product quality. For the sake of simplicity we restrict ourselves to the special case $\gamma_A = 0$.

[11] According to Walras' law, one of the market clearing conditions is redundant.

with $V_i(t)$ given in (11).

The market for final output is in equilibrium if

$$c(t) = Y(t)/L(t). \tag{21}$$

This is the case because the only demand for final output is the consumption demand of the households.

2.6 The knowledge spillover effect

Before we analyze the market equilibrium of the economy, we need to make a remark regarding the size of the parameter γ that describes the knowledge spillover effect in (15). To this end, let us introduce some notation that will be used throughout the rest of this paper. For any variable $y(t)$, we denote by $g_y(t)$ its growth rate defined by $g_y(t) = \dot{y}(t)/y(t)$. The relative quality of the latest intermediate good at time t is denoted by $z(t)$, that is, $z(t) = q_{A(t)}/Q(t)$.

Now suppose that all researchers at time t choose the relative quality $z(t)$.[12] Because every researcher produces new designs at the rate $F(z(t)Q(t), t)$, and the total measure of researchers is given by $L_R(t)$, it follows that the measure of intermediate goods evolves according to

$$\dot{A}(t) = L_R(t)F(z(t)Q(t), t) = [\beta - z(t)]L_R(t)Q(t)^\gamma. \tag{22}$$

Differentiating the definition of the quality index (14) with respect to time t and using $q_A(t) = z(t)Q(t)$ yields

$$g_Q(t) = [z(t) - 1]g_A(t). \tag{23}$$

Defining $S(t) = Q(t)^\gamma/A(t)$, it follows from (22)-(23) that

$$\dot{S}(t) = [\beta - z(t)][\gamma z(t) - (1 + \gamma)]L_R(t)S(t)^2.$$

In order for the model to be well defined we must ensure that all variables take finite values at all times t. This implies in particular that the above differential equation for $S(t)$ must have a globally defined solution for all feasible paths of relative quality $z(t)$ and research time $L_R(t)$. It follows that $\gamma z - (1 + \gamma) \leq 0$ must hold for all $z \in [0, \beta]$.[13] Obviously this is the case if and only if $\gamma \leq 1/(\beta - 1)$. This assumption will therefore be maintained throughout the paper. It imposes an upper bound on the parameter γ, which measures the strength of the intertemporal knowledge spillover effect.[14]

[12] We know from (17) that $z(t) = \beta/2 = 1 + \kappa$ must hold in equilibrium but, for the moment, let us consider all possible values $z(t) \in [0, \beta]$.

[13] This is the case because the solution of the differential equation $\dot{S}(t) = \zeta S(t)^2$ and the initial value $S(0) = \zeta_0$ diverges to $+\infty$ in finite time whenever $\zeta > 0$ and $\zeta_0 > 0$.

[14] The assumption $\gamma \leq 1/(\beta - 1)$ corresponds to the assumption that $\varphi < 1$ holds in equation (6) of Jones (1995); see also footnote 15 below.

3 Laissez-faire equilibrium

In the present section we discuss the decentralized market equilibrium of the economy without government intervention. We start by studying how the quality of intermediate goods evolves over time. Then we derive the growth rates of the most important endogenous variables along a balanced growth path. Finally, we discuss creative displacement, that is, the process by which intermediate goods become obsolete because they are slowly but continually replaced by goods of higher quality.

3.1 Equilibrium quality

We start with the following simple but important lemma relating the quality index $Q(t)$ to the measure of intermediate goods $A(t)$. Recall the definition $\kappa = \beta/2 - 1$.

Lemma 1 *In every equilibrium it holds for all $t \geq 0$ that*

$$Q(t) = BA(t)^{\kappa}, \tag{24}$$

where $B = A(0)^{-\kappa}Q(0)$. Furthermore, $q_i = (1+\kappa)Bi^{\kappa}$ holds for all $i \geq A(0)$.

PROOF: Combining (17) and (23) yields $g_Q(t) = \kappa g_A(t)$. Equation (24) follows immediately from this result. Combining (17) and (24) yields $q_{A(t)} = (1+\kappa)BA(t)^{\kappa}$. Since this has to hold for all $t \geq 0$ we obtain $q_i = (1+\kappa)Bi^{\kappa}$ for all $i \geq A(0)$. □

Let us add two remarks regarding the equilibrium quality. First, it is easy to see from (17) that a newly developed product has a quality level exceeding the average quality of existing products if and only if $\kappa > 0$ (which is equivalent to $\beta > 2$). In other words, if large quality improvements are possible (that is, if β is larger than 2), then it follows that newly developed products are of superior quality. If innovations cannot lead to large quality jumps (i.e., if β is smaller than 2), then product quality in equilibrium must be decreasing over time. Correspondingly, we see from lemma 1 that, in the case $\beta > 2$, average quality $Q(t)$ and product quality q_i are increasing functions of $A(t)$ and i, respectively.

The second remark is of a more technical nature. Lemma 1 shows that $A(t)^{-\kappa}Q(t)$ is invariant with respect to time t along every equilibrium. For different equilibria, however, the value of $A(t)^{-\kappa}Q(t)$, which we have denoted by B, is generally a different one. If an equilibrium converges towards a balanced growth path equilibrium (BGP equilibrium), then it must be the case that $A(t)^{-\kappa}Q(t) = B$ holds also along the BGP equilibrium. It follows that equilibria starting in different initial states do not in general converge to the same BGP equilibrium. We can therefore not expect uniqueness of a BGP equilibrium. Instead, it can be conjectured that, for every value $B > 0$, there exists a unique BGP equilibrium along which $A(t)^{-\kappa}Q(t) = B$ holds. This conjecture will be shown to be true in the next subsection.

3.2 Balanced growth

Substituting (17) and (24) into (22) we obtain

$$\dot{A}(t) = (1+\kappa)B^{\gamma}L_R(t)A(t)^{\gamma\kappa}. \tag{25}$$

This equation resembles the central assumption in Jones (1995) according to which the rate of change of the measure of intermediate goods is proportional to the measure of researchers and to a power of the measure of intermediate goods itself.[15] Note that our assumption $\gamma \leq 1/(\beta-1)$ implies $\gamma\kappa < 1$. Based on the above equation, Jones (1995) shows that his model allows for a BGP equilibrium along which $L_R(t)$ grows at the rate n and $A(t)$ grows at the rate

$$g = n/(1-\gamma\kappa). \tag{26}$$

It is straightforward to derive an analogous result for the present model. This result is stated in the following theorem. The proof of the theorem as well as some other properties of BGP equilibria can be found in appendix A.

Theorem 1 *For every fixed value $B > 0$ and every initial population $L(0) > 0$, there exists a unique BGP equilibrium satisfying $A(t)^{-\kappa}Q(t) = B$ for all t. Moreover, along every BGP equilibrium it holds that*

$$g_A(t) = g > 0\,,\ g_Q(t) = \kappa g\,,\ g_{Y/L}(t) = (1-\alpha)(1+\kappa)g > 0, \tag{27}$$

and

$$L_R(t)/L(t) = K/(1-\alpha+\alpha^2+K), \tag{28}$$

where g is given by (26) and $K = \alpha(1-\alpha)(1+\kappa)/[1+\kappa+(\rho-n)/g] > 0$.

As in Jones (1995), positive population growth is necessary for economic growth. More precisely, from (26) and (27) it follows that all the growth rates mentioned in theorem 1 are proportional to the population growth rate n. Whereas the measure of intermediate goods, $A(t)$, and per-capita output, $Y(t)/L(t)$, always grow at positive rates, the same need not be true for the quality index $Q(t)$. Whether $Q(t)$ grows or not depends on whether $\kappa > 0$ or $\kappa \leq 0$, that is, on whether $\beta > 2$ or $\beta \leq 2$; see also the discussion in subsection 3.1.

Since the growth rates in (27) are independent of the initial value of the population, $L(0)$, there is no strong scale effect.[16] The reason for the absence of a strong scale effect is exactly the same as in the model from Jones (1995), namely that existing knowledge exhibits decreasing returns in the aggregate production function for ideas. In the present model, this property is ensured by $\gamma\kappa < 1$ which, in turn, follows from the assumption $\gamma < 1/(\beta-1)$.

[15] The assumption under consideration is equation (6) in Jones (1995). It is more general than our equation (25) in the sense that it is of the form $\dot{A}(t) = \text{const.} \times L_R(t)^{\mu}A(t)^{\varphi}$, where μ and φ are real numbers satisfying $\varphi < 1$. Our case corresponds to $\mu = 1$ and $\varphi = \gamma\kappa$. For further discussion see also Jones (2005).

[16] This means that there is no effect of the scale of the economy, $L(0)$, on the long-run growth rate g. There is, however, a weak scale effect, that is, the level of $A(t)$ (and other variables) along the BGP equilibrium depends on $L(0)$. See appendix A for details and Jones (2005) for a general discussion of weak and strong scale effects.

For the purpose of illustration of theorem 1, we present a few numerical results in table 1 below. The results have been produced using the following parameter specifications: the population growth rate n is set to 1%, the time-preference rate ρ to 3%, and the technology parameter α is equal to 1/3. Similar parameter specifications are often used in calibration exercises for yearly data. To pin down reasonable values for the parameters β and γ describing the external effects in R&D, we report the results for a grid of parameter values where $\beta \in \{2, 4, 6\}$ and $\gamma \in \{0.05, 0.1, 0.15\}$. Note that these parameter values satisfy all our restrictions including $\gamma < 1/(\beta - 1)$. The first entry in each cell is the growth rate of per-capita output $g_{Y/L}(t)$, the second one is the research intensity $L_R(t)/L(t)$, the third one is a growth accounting measure D which will be explained shortly, and the last entry measures the speed of creative displacement to be defined in the next subsection.

β \ γ	0.05	0.10	0.15
2	0.67% 8.70% 0.00% $+\infty$	0.67% 8.70% 0.00% $+\infty$	0.67% 8.70% 0.00% $+\infty$
4	1.40% 12.78% 50.00% 65.8	1.48% 13.07% 50.00% 62.4	1.57% 13.38% 50.00% 58.9
6	2.22% 15.15% 66.67% 31.2	2.50% 15.71% 66.67% 27.7	2.86% 16.30% 66.67% 24.3

Table 1: The market equilibrium for the parameter values $n = 1\%$, $\rho = 3\%$, $\alpha = 1/3$, $\beta \in \{2, 4, 6\}$, and $\gamma \in \{0.05, 0.1, 0.15\}$. First line in each cell is $g_{Y/L}(t)$, second line is $L_R(t)/L(t)$, third line is D, and last line is H.

The growth rates of per-capita output for the chosen parameter values range from two thirds of a percentage point to almost three percentage points. These numbers are roughly in line with empirical data for many countries. US per-capita GDP, for example, has grown on average at 1.8% per year during the last 125 years, which is very close to the average growth rate reported in table 1. As for the research intensity $L_R(t)/L(t)$ we obtain values between 8% and a bit over

16%, which are too high to be realistic.[17] The simplifications of our model, in particular the homogeneity of labor, are likely to be responsible for this misfit. It can be conjectured that the introduction of different skill levels would improve the quantitative predictions of the model along this dimension.

We know from theorem 1 that $g_{Y/L}(t) = (1-\alpha)[g_A(t) + g_Q(t)]$ and $g_Q(t) = \kappa g_A(t)$; see also equation (45) in appendix A. The contribution of quality improvements to overall economic growth is therefore equal to $(1-\alpha)g_Q(t)/g_{Y/L}(t) = \kappa/(1+\kappa) = (\beta-2)/\beta$. We denote this contribution by D and report its value as the third entry in each cell of table 1. Note that this contribution is independent of the intertemporal knowledge spillover parameter γ. The first row of the table corresponds to $\beta = 2$, which implies $\kappa = 0$. In this case, quality remains constant and, hence, quality improvements do not contribute to economic growth, that is, $D = 0$. In the second row of the table, which corresponds to $\beta = 4$, it holds that $\kappa = 1$ and $D = 1/2$. Variety expansion and quality improvements contribute equally to the growth rate of per-capita output. Finally, in the third row it holds that $\beta = 6$, $\kappa = 2$, and $D = 2/3$. In this case, quality improvement is the dominant source of economic growth.

Whenever $D > 0$, the growth rates and the research intensity depend positively on the externality parameter γ. This shows that, as long as quality improvement contributes to economic growth, the strength of the knowledge spillover has a positive growth effect. If quality improvement does not happen ($D = 0$), on the other hand, then there is no effect of knowledge spillovers on the long-run growth rate of the economy or on the research intensity.[18] This follows of course from our assumption that knowledge is represented entirely by average quality $Q(t)$.

3.3 Creative displacement

We have already noted before that the average quality of intermediate products increases over time if and only if $\beta > 2$ or, equivalently, $\kappa > 0$. For the present subsection we assume this condition to hold. Newly invented products are therefore of a higher quality than old ones. It can be expected that, in this situation, new products will drive the old ones out of the market. To see this formally, we use (7), (10), (26), and (27) to get

$$g_{x_i}(t) = -(1+\gamma)\kappa g. \tag{29}$$

If $\kappa > 0$, as we presently assume, then it follows that the production rate of any given product i converges exponentially to 0. This alone, however, does not prove that intermediate good i is driven out of the market but partly reflects a process of expenditure diversion. The increase of the measure of intermediate goods dilutes the expenditure of final good producers over a larger mass of inputs, thereby reducing the demand for any given product.

[17] According to Jones (2002), the research intensity in the US is smaller than 1%. Because this empirical estimate is based on a narrow definition of "research", it is probably biased downwards; see footnote 9 in Jones (2002).

[18] If $\beta < 2$, then $\kappa < 0$ and quality deteriorates over time, i.e., $g_Q(t) < 0$. In this case we would have $D < 0$ and higher values of γ would correspond to smaller growth rates. The knowledge spillover effect would be negative. Since we do not regard this case as an interesting one, we do not report any results for $\beta < 2$.

Let us therefore denote the average production rate of all intermediate goods by $X(t)$, that is,

$$X(t) = \frac{1}{A(t)} \int_0^{A(t)} x_i(t)\,\mathrm{d}i.$$

From (13) it follows that $X(t) = L_X(t)/A(t)$. Using $g_{L_X}(t) = n$ (see equation (47) in appendix A), (26), and (27) we obtain $g_X(t) = -\gamma\kappa g$. Combining this with (29) we get $g_{x_i}(t) - g_X(t) = -\kappa g$. This shows that the ratio between the production rate of any given intermediate good, $x_i(t)$, and the average production rate of all intermediate goods, $X(t)$, converges at the rate κg towards 0 (remember that $\kappa > 0$ is assumed throughout this subsection). Old goods are therefore slowly but steadily displaced by new and better ones. We call this process creative displacement because it captures Schumpeter's idea of creative destruction but differs from the instantaneous replacement of inferior goods typically found in models of vertical innovation like those from Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992). The difference arises of course because the horizontal innovations in our model create *imperfect* substitutes to existing products of lower quality, whereas vertical innovations create *perfect* substitutes of existing products of lower quality.

The half-life of the variable $x_i(t)/X(t)$ is given by $H = -\ln(2)/[g_{x_i}(t) - g_X(t)] = \ln(2)/(\kappa g)$. We report this number as the last entry in each cell of table 1. If quality remains constant ($\beta = 2$), then there is no creative displacement and the half-life is equal to $+\infty$. In the cases $\beta = 4$ and $\beta = 6$, the half-life periods are finite but rather large (recall that the unit of time is assumed to be one year). In general, the half-life is negatively related to the growth rate of per-capita output. This follows from the observation that g and κ are positively related to each other (see equation (26)) and that H is a decreasing function of κg. The process of creative displacement happens therefore faster in quickly growing economies than in slowly growing ones.

4 Optimal growth

In the present section we first derive the allocation that would be chosen by a benevolent social planner. We then discuss how this optimal solution can be implemented by subsidies and taxation. We shall demonstrate in particular how the government can affect the long-run growth rate of the economy using quality-dependent R&D subsidies.

4.1 The social planner's solution

In order to solve the social planner's optimization problem we proceed in three steps. The first two steps deal with the optimal allocation of resources within any given period t. Step 3 finally treats the intertemporal tradeoff that arises from the optimal choice of quality and research intensity.

Consider the economy at time t. In step 1 we study how, given $A(t)$, $\{q_i \,|\, i \in [0, A(t)]\}$, $L_Y(t)$, and $L_X(t)$, the social planner allocates the $L_X(t)$ units of labor to the $A(t)$ intermediate goods producers that exist at time t. Since all final output is consumed and since the production of one unit of any type of intermediate good requires one unit of labor, the social planner must solve the problem of maximizing the final output rate as given in (6) subject to the constraint $\int_0^{A(t)} x_i(t)\,\mathrm{d}i = L_X(t)$. The solution of this problem is given by $x_i(t) = q_i L_X(t)/\left[A(t)Q(t)\right]$ and the resulting optimal rate of final output at time t is

$$Y(t) = [A(t)Q(t)L_Y(t)]^{1-\alpha} L_X(t)^{\alpha}. \tag{30}$$

In step 2 we take $A(t)$, $Q(t)$, and $L_R(t)$ as given and determine the optimal allocation of labor between its two uses in the production of intermediate and final goods. This allocation can be found by maximizing $Y(t)$ as given in (30) with respect to $L_Y(t)$ and $L_X(t)$ and subject to the resource constraint (3). The solution to this problem is given by $L_X(t) = \alpha[L(t) - L_R(t)]$ and $L_Y(t) = (1-\alpha)[L(t) - L_R(t)]$ and the corresponding optimal rate of final output is

$$Y(t) = \alpha^{\alpha}[(1-\alpha)A(t)Q(t)]^{1-\alpha}[L(t) - L_R(t)]. \tag{31}$$

We are now ready for step 3 of the solution of the social planner's optimization problem. The social planner wants to maximize the representative household's objective functional in (1). Since the only use of final output is consumption, it must hold that $c(t) = Y(t)/L(t)$; see also (21). Substituting this relation together with (31) into (1), using $\dot{L}(t)/L(t) = n$ and cancelling constant terms, we can write the social planner's objective functional as

$$\int_0^{+\infty} e^{-(\rho-n)t}\left\{(1-\alpha)\ln[A(t)Q(t)] + \ln[L(t) - L_R(t)]\right\}\,\mathrm{d}t. \tag{32}$$

The social planner maximizes this objective functional subject to the differential equations (22)-(23), which are repeated here for convenience:

$$\begin{aligned}\dot{A}(t) &= [\beta - z(t)]L_R(t)Q(t)^{\gamma},\\ \dot{Q}(t) &= [\beta - z(t)][z(t) - 1][L_R(t)/A(t)]Q(t)^{1+\gamma}.\end{aligned}$$

The control constraints are $0 \leq L_R(t) \leq L(t)$ and $0 \leq z(t) \leq \beta$.

This is a standard optimal control problem with state variables $A(t)$ and $Q(t)$ and control variables $L_R(t)$ and $z(t)$. The fact that the social planner internalizes all externalities is reflected by non-convexities of that problem which imply that the first-order conditions of the maximum principle are not automatically sufficient for optimality. The following theorem assumes therefore that an optimal solution with balanced growth (i.e., an optimal BGP) exists and it uses the necessary optimality conditions to characterize the properties of that solution.

Theorem 2 *There exists a unique value $\bar{z} \in (\beta/2, \beta)$ satisfying $G(\bar{z}) = 0$, where $G(z)$ is the second-order polynomial*

$$G(z) = 2\gamma(\rho - n)z^2 - [(2 + 2\gamma + \beta\gamma)\rho - (1 + \gamma + \beta\gamma)n]\,z + \beta(1+\gamma)\rho.$$

Along an optimal BGP it holds that

$$g_A(t) = \bar{g}\,,\ g_Q(t) = \bar{\kappa}\bar{g}\,,\ g_{Y/L}(t) = (1-\alpha)(1+\bar{\kappa})\bar{g}, \tag{33}$$

and

$$L_R(t)/L(t) = \bar{K}/(1+\bar{K}), \tag{34}$$

where $\bar{\kappa} = \bar{z} - 1$, $\bar{g} = n/(1 - \gamma\bar{\kappa})$, and $\bar{K} = (1-\alpha)(\beta - \bar{z})/[\beta - \bar{z} + (1 + \beta - 2\bar{z})(\rho - n)/\bar{g}]$.

The proof of the theorem can be found in appendix B. To compare the equilibrium BGP described in theorem 1 with the optimal BGP from theorem 2 we first note that $\bar{z} > \beta/2$ implies $\bar{\kappa} > \kappa$ and, hence, $\bar{g} > g$. It follows that all the optimal growth rates in (33) are higher than the corresponding equilibrium growth rates in (27). To illustrate the extent to which equilibrium growth rates fall short of the optimum, we report in table 2 the characteristics of the optimal solution for the same parameter values that have been used to generate table 1. Table 2 is organized in the same way as table 1, that is, the first entry in each cell is the growth rate of per-capita output $g_{Y/L}(t)$, the second entry is the research intensity $L_R(t)/L(t)$, the third entry is the contribution of quality improvement to growth D, and the last entry is the half-life period H.

β \ γ	0.05	0.10	0.15
2	0.81% 21.36% 17.33% 327	0.83% 21.70% 18.00% 309	0.85% 22.05% 18.66% 292
4	1.75% 36.89% 59.03% 45	1.95% 39.38% 59.81% 40	2.21% 42.40% 60.70% 34
6	2.85% 48.72% 72.96% 22	3.56% 54.29% 73.90% 18	4.94% 62.23% 75.23% 12

Table 2: The social planner's solution for the parameter values $n = 1\%$, $\rho = 3\%$, $\alpha = 1/3$, $\beta \in \{2, 4, 6\}$, and $\gamma \in \{0.05, 0.1, 0.15\}$. First line in each cell is $g_{Y/L}(t)$, second line is $L_R(t)/L(t)$, third line is D, and last line is H.

The numerical results confirm that the social planner solution features higher growth rates than the market equilibrium. More specifically, optimal growth rates are between 20% and 70% higher than their equilibrium counterparts. The most striking difference between tables 1 and 2, however, is that the social planner employs roughly 2-4 times more labor in the R&D sector than would be the case in the market equilibrium. Even if the absolute numbers of R&D intensity in our model are much higher than they are in reality, the *ratio* between the optimal R&D intensity and its equilibrium value closely matches the estimates that Jones and Williams (1998) have drawn from a large body of empirical work in the productivity literature.[19] From (31) and theorem 2 we see that the contribution of quality improvement to overall economic growth, D, is again given by $(1-\alpha)g_Q(t)/g_{Y/L}(t) = \bar{\kappa}/(1+\bar{\kappa})$. Contrary to the case of the market equilibrium, in the optimal solution this number depends also on γ. A comparison of tables 1 and 2 shows that D is on average (across the chosen values of γ) somewhat higher than it was in the market equilibrium especially for small values of β. For the case $\beta = 2$, for example, growth in the market equilibrium is entirely driven by variety expansion ($D = 0$), whereas quality improvements contribute roughly 18% to per-capita output growth in the optimal solution. For higher values of β, the difference between D in the optimal solution and the market equilibrium is not as striking. Finally, the half-life period H in the command economy is considerably smaller than in the decentralized one. Thus, creative displacement happens much faster under the control of a social planner than in laisse-faire equilibrium.

4.2 Optimal policy

There are two reasons why the market equilibrium is not optimal: firstly, there is monopolistic competition in the intermediate goods market and, secondly, the average quality has external effects on the research productivity. In the present subsection we show how the government can compensate these distortions and decentralize the optimal solution derived in subsection 4.1 by means of appropriate subsidies and taxes.

The distortion by monopoly power can be easily offset by a production subsidy for intermediate goods producers. Since the markup on production costs is equal to $1/\alpha - 1$, the government can use a constant production subsidy $\sigma_X = 1/\alpha - 1$ in order to induce firms to produce at the efficient output level.

Let us now turn to the more complicated question of how to compensate the external effects in the R&D sector. These externalities distort both the quality decision of the researchers and the total amount of labor employed in that sector. Correspondingly, the government needs a R&D subsidy depending on two policy parameters that can be set independently from each other to offset the two distortions. The basic mechanism of such an optimal R&D subsidy is easy to demonstrate. Suppose that the government grants researchers a subsidy at the rate of

$$\sigma_R(q,t) = \frac{\tilde{\delta}\tilde{\beta} - \beta + (1-\tilde{\delta})q/Q(t)}{\beta - q/Q(t)} \tag{35}$$

when they successfully develop a good of relative quality $q/Q(t)$ at time t. Here, $\tilde{\beta}$ and $\tilde{\delta}$ are the policy parameters which can be arbitrary positive constants. The proof of the following lemma is trivial and therefore omitted.

Lemma 2 *If $\tilde{\beta} > \beta$, then it follows that $\sigma_R(q,t)$ is strictly increasing with respect to q and satisfies $\lim_{q\to\beta Q(t)} = +\infty$. The sign of $\sigma_R(0,t)$ coincides with the sign of $\tilde{\delta}\tilde{\beta} - \beta$.*

The lemma shows that $\sigma_R(q,t)$ is indeed a non-negative subsidy for any possible relative quality level $q/Q(t) \in [0,\beta]$ provided that the conditions $\tilde{\beta} \geq \beta$ and $\tilde{\delta} \geq \beta/\tilde{\beta}$ hold. If $\tilde{\beta} > \beta$ and

[19] Jones and Williams (1998) define R&D intensity as R&D spending as a share of GDP. In the present paper, in contrast, we use employment in R&D as a share of total employment.

$\tilde{\delta} < \beta/\tilde{\beta}$, on the other hand, then $\sigma_R(q,t)$ is negative for small values of q, in which case low-quality innovations are taxed rather than subsidized.[20]

In the presence of the R&D subsidy (35), the return on labor used for designing an intermediate good of quality q at time t is no longer given by (16) but by

$$\tilde{W}_R(q,t) = [1+\sigma_R(q,t)][\beta - q/Q(t)]qv(t)Q(t)^\gamma = \tilde{\delta}[\tilde{\beta} - q/Q(t)]qv(t)Q(t)^\gamma. \tag{36}$$

Maximizing $\tilde{W}_R(q,t)$ with respect to $q \in [0, \beta Q(t)]$ yields $q = \min\{\tilde{\beta}/2, \beta\}Q(t)$. Since $\tilde{\beta}$ can be freely chosen by the government, $\min\{\tilde{\beta}/2, \beta\}$ can take any feasible value in the interval $[0, \beta]$. It follows therefore that, by a suitable choice of the policy parameter $\tilde{\beta}$, the government can perfectly control the quality decisions of researchers. In what follows, we assume that $\tilde{\beta} \leq 2\beta$ such that $\min\{\tilde{\beta}/2, \beta\}$ can be replaced by $\tilde{\beta}/2$. Moreover, in analogy to subsection 2.4 we define $\tilde{\kappa} = \tilde{\beta}/2 - 1$. By the same reasoning that has been applied in subsection 2.4, one can see that the above assumptions imply

$$q_{A(t)} = (1+\tilde{\kappa})Q(t) \tag{37}$$

for all t.

Before we discuss the role of the other policy parameter $\tilde{\delta}$, let us show that the policy parameter $\tilde{\beta}$ (or, equivalently, $\tilde{\kappa}$) has a long run growth effect. Indeed, since relative quality $z(t) = q_{A(t)}/Q(t) = 1+\tilde{\kappa}$, it follows from (22)-(23) that

$$\dot{A}(t) = (1+2\kappa-\tilde{\kappa})L_R(t)Q(t)^\gamma \tag{38}$$

and $g_Q(t) = \tilde{\kappa}g_A(t)$. Lemma 1 holds also in the present case with κ replaced by $\tilde{\kappa}$. It follows that $Q(t) = BA(t)^{\tilde{\kappa}}$. Combining this with (38), we obtain the equation

$$\dot{A}(t) = (1+2\kappa-\tilde{\kappa})B^\gamma L_R(t)A(t)^{\gamma\tilde{\kappa}}, \tag{39}$$

which is the correct modification of equation (25) in the presence of a research subsidy of the form (35). Applying the same arguments as in the proof of theorem 1 (which are those originally put forward by Jones (1995)) it follows that the growth rate of $A(t)$ along any balanced growth path must be equal to

$$\tilde{g} = n/(1-\gamma\tilde{\kappa}). \tag{40}$$

Since $\tilde{\kappa}$ can be controlled by the policy parameter $\tilde{\beta}$, we conclude that the government can affect the growth rate of the economy. The policy-invariance proposition of semi-endogenous growth models is therefore not applicable to the present model.

The above arguments did not involve the policy parameter $\tilde{\delta}$ at all. What matters for the quality decisions of the researchers is the incentive controlled by $\tilde{\beta}$. But in order to implement the optimal solution, the government must also be able to affect the aggregate research intensity $L_R(t)/L(t)$. This can be done by an appropriate choice of $\tilde{\delta}$. Under a R&D subsidy of the form (35), the rate of return to research time is given by

$$w_R(t) = \tilde{W}_R(q_{A(t)},t) = \tilde{\delta}(1+\tilde{\kappa})^2 v(t)Q(t)^{1+\gamma}, \tag{41}$$

[20] In the following discussion we shall, for simplicity, always refer to $\sigma_R(q,t)$ as a research subsidy although we do not exclude the case where $\sigma_R(q,t)$ is negative for some values of q.

which makes it obvious that the government can use $\tilde{\delta}$ to control $w_R(t)$.

We assume that both the production subsidy for intermediate good producers and the R&D subsidy are financed by a lump-sum tax on households. Denoting the tax per household at time t by $T(t)$, the government's budget constraint can be expressed as

$$T(t) = \sigma_X \int_0^{A(t)} p_i(t)x_i(t)\,\mathrm{d}i + \sigma_R((1+\tilde{\kappa})Q(t),t)\dot{A}(t)V_{A(t)}(t).$$

The first term on the right-hand side is the total production subsidy paid to intermediate good producers and the second term is the research subsidy.

Following the same arguments as in appendix A, one can show that the market equilibrium under the tax/subsidy regime (σ_X, σ_R, T) described above has the following properties.

Theorem 3 *Consider the market equilibrium under the tax/subsidy policy described above. For every fixed value $B > 0$ and every initial population $L(0) > 0$, there exists a unique BGP equilibrium satisfying $A(t)^{-\tilde{\kappa}}Q(t) = B$ for all t. Moreover, along every BGP equilibrium it holds that*

$$g_A(t) = \tilde{g} > 0\,,\ g_Q(t) = \tilde{\kappa}\tilde{g}\,,\ g_{Y/L}(t) = (1-\alpha)(1+\tilde{\kappa})\tilde{g} > 0,$$

and

$$L_R(t)/L(t) = \tilde{\delta}\tilde{K}/(1+\tilde{\delta}\tilde{K}),$$

where

$$\tilde{K} = \frac{(1-\alpha)(1+\tilde{\kappa})^2}{(1+2\kappa-\tilde{\kappa})[1+\tilde{\kappa}+(\rho-n)/\tilde{g}]}.$$

Comparing theorems 1 and 3 it is obvious that the optimal BGP can be decentralized by choosing the policy parameters $\tilde{\beta}$ and $\tilde{\delta}$ in such a way that the two equations $\tilde{g} = \bar{g}$ and $\tilde{\delta}\tilde{K}/(1+\tilde{\delta}\tilde{K}) = \bar{K}/(1+\bar{K})$ hold. It is easy to see that this is the case if and only if $\tilde{\beta} = 2\bar{z}$ and $\tilde{\delta} = \bar{K}/\tilde{K}$. In table 3 we report the required settings of the policy parameters under the same parameter constellations that have already been used in tables 1 and 2. The first entry in each cell is $\tilde{\beta}$ and the second one is $\tilde{\delta}$.

We know from the above discussion and from theorem 2 that $\tilde{\beta} = 2\bar{z} > \beta$. This can also be seen from table 3. According to lemma 2 this demonstrates that the optimal R&D subsidy $\sigma_R(q,t)$ is increasing with respect to quality. The condition $\tilde{\delta} > \beta/\tilde{\beta}$, on the other hand, is satisfied for $\beta = 4$ and $\beta = 6$ but fails to hold for $\beta = 2$. Because of lemma 2 this shows that the R&D subsidy is positive whenever $\beta = 4$ or $\beta = 6$ but that it becomes negative for small relative quality levels when $\beta = 2$. The interpretation of this result is that, in the case $\beta = 2$, where researchers choose to make no quality improvements under laissez-faire, the government has to take the drastic measure of penalizing (i.e., taxing) low-quality innovations in order to implement the optimal solution.

β \ γ	0.05	0.10	0.15
2	2.42 0.70	2.44 0.69	2.46 0.68
4	4.88 0.99	4.98 1.00	5.09 1.01
6	7.40 1.30	7.66 1.39	8.07 1.53

Table 3: Policy parameters implementing the social planner's solution for the parameter values $n = 1\%$, $\rho = 3\%$, $\alpha = 1/3$, $\beta \in \{2, 4, 6\}$, and $\gamma \in \{0.05, 0.1, 0.15\}$. First line in each cell is $\tilde{\beta}$, second line is $\tilde{\delta}$.

5 Conclusion

We have studied a growth model with horizontal innovations à la Romer (1990), in which the quality of new products is endogenously determined. The optimal quality decisions of the researchers balance the required research effort against the value of their inventions. Under suitable parameter constellations, the model predicts that existing products are slowly but steadily driven out of the market by new products of superior quality.

The paper adds also a new aspect to the discussion about the scale effect of endogenous growth models. Indeed, our model assumes a scale-free research equation similar in nature to that introduced by Jones (1995). Contrary to Jones (1995), however, our model describes endogenous growth (as opposed to semi-endogenous growth) because the government can affect the growth rate of per-capita output by making research subsidies quality-dependent. In particular, we were able to compute the optimal solution of the model and to demonstrate how it can be decentralized using a combination of a production subsidy for intermediate goods producers, a research subsidy, and a lump-sum tax on households.

A very rough calibration of the model shows that the most striking difference between the market equilibrium and the social planner's solution is that, in the latter, a much larger fraction of the labor force is employed in the R&D sector. The extent to which optimal R&D intensity exceeds its equilibrium value in our theoretical model matches closely the predictions derived by Jones and Williams (1998) from empirical studies of productivity. The policy recommendation of our paper is therefore that governments should provide incentives for R&D but that these incentives have to be made contingent on the quality of the resulting innovations.

Appendix A

This appendix presents the proof of theorem 1 as well as some additional results about market equilibria (in particular about BGP equilibria). Let us start with some preliminary observations.

From (11), (14), and (20) we get

$$a(t) = A(t)Q(t)v(t). \tag{42}$$

From (6), (10), and (14) it follows that

$$Y(t) = \left[\alpha^2/w(t)\right]^{\alpha/(1-\alpha)} A(t)Q(t)L_Y(t). \tag{43}$$

Substituting (43) into (7) one gets

$$w(t) = \alpha^{2\alpha} \left[(1-\alpha)A(t)Q(t)\right]^{1-\alpha}. \tag{44}$$

Substituting this result back into (43) we obtain

$$Y(t) = \alpha^{2\alpha}(1-\alpha)^{-\alpha} \left[A(t)Q(t)\right]^{1-\alpha} L_Y(t). \tag{45}$$

Combining (13), (14), and (44) yields

$$L_X(t) = [\alpha^2/(1-\alpha)]L_Y(t). \tag{46}$$

Since labor is necessary for production, the inequalities $L_Y(t) > 0$ and $L_X(t) > 0$ must hold for all t in any equilibrium. If this equilibrium is a BGP, then it follows obviously that

$$g_{L_Y}(t) = g_{L_X}(t) = g_L(t) = n. \tag{47}$$

Along a BGP equilibrium, the growth rate of per-capita consumption is constant. In this case, it follows therefore from the Euler equation (5) that the real interest rate is also constant, that is, $g_r(t) = 0$.

Let us now turn to the research effort $L_R(t)$. Along a BGP equilibrium, this variable must either be equal to 0 or growing at the population growth rate n. The following lemma rules out the former case.

Lemma 3 *Along a BGP equilibrium it must hold that $L_R(t) > 0$ and $g_{L_R}(t) = n$.*

Proof: Suppose that $L_R(t) = 0$ holds for all t. Then it follows from (24), (25), and (44) that $g_A(t) = g_Q(t) = g_w(t) = 0$. Using $g_w(t) = g_r(t) = 0$, (47), and (12) it follows that $g_v(t) = n$. Together with $g_Q(t) = 0$ and (18) this implies $g_{w_R}(t) = n$. Because of $n > 0$, the results $g_w(t) = 0$ and $g_{w_R}(t) = n$ are inconsistent with (19). This contradiction proves the lemma. $\square$

Proof of Theorem 1: From (25) we get

$$g_A(t) = (1+\kappa)B^\gamma L_R(t)A(t)^{\gamma\kappa-1}.$$

Because of lemma 3 it follows that $g_A(t)$ must be strictly positive. Differentiating the above equation with respect to t and using $g_{L_R}(t) = n$ we obtain $\dot{g}_A(t) = g_A(t)[n-(1-\gamma\kappa)g_A(t)]$. Since, along a BGP equilibrium, $g_A(t)$ must be a strictly positive constant we obtain $g_A(t) = g > 0$, where g is defined in (26). The remaining growth rates in (27) are easily derived from this result and equations (24), (45), and (47).

As for the interest rate it follows from (5), (21), and (27) that

$$r(t) = (1-\alpha)(1+\kappa)g + \rho. \tag{48}$$

Let us now turn to the proof of (28). From (27) we know that $A(t) = A_0 e^{gt}$ and $Q(t) = Q_0 e^{\kappa g t}$ for some constants A_0 and Q_0. Dividing (25) by $A(t)$, evaluating the resulting equation at $t = 0$, and using the definition $B = A_0^{-\kappa} Q_0$ yields therefore

$$A_0/Q_0^\gamma = (1+\kappa)L_R(0)/g. \tag{49}$$

Equation (44), on the other hand, implies that

$$w(t) = \alpha^{2\alpha}\left[(1-\alpha)A_0 Q_0\right]^{1-\alpha} e^{(1-\alpha)(1+\kappa)gt}. \tag{50}$$

Substituting this result together with (48) and $L_Y(\tau) = L_Y(t)e^{n(\tau-t)}$ into (12) it follows that

$$v(t) = \frac{\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha}(A_0 Q_0)^{-\alpha}}{(1+\kappa)g + \rho - n} L_Y(t) e^{-\alpha(1+\kappa)gt}. \tag{51}$$

Substituting this and $L_Y(t) = L_Y(0)e^{nt}$ into (18) and using (26) yields

$$w_R(t) = \frac{\alpha^{1+2\alpha}(1-\alpha)^{1-\alpha}(1+\kappa)^2 A_0^{-\alpha} {Q_0}^{1-\alpha+\gamma} L_Y(0)}{(1+\kappa)g + \rho - n} e^{(1-\alpha)(1+\kappa)gt}.$$

Substituting this result and (50) into condition (19) we get

$$A_0/Q_0^\gamma = \alpha(1+\kappa)^2 L_Y(0)/[(1+\kappa)g + \rho - n].$$

Together with (49) this implies $L_Y(0) = (1-\alpha)L_R(0)/K$, where K is defined in theorem 1. Substituting this and (46) into (3) yields after simplifications equation (28).

Equations (47) and (51) imply that $g_v(t) = n - \alpha(1+\kappa)g$. Using this result and (27) in (42), we obtain $g_a(t) = g_A(t) + g_Q(t) + g_v(t) = n + (1-\alpha)(1+\kappa)g$. Together with (48) and the assumption $\rho > n$ this shows that the no-Ponzi game condition (4) is satisfied.

Finally, it remains to show that the BGP equilibrium is indeed uniquely determined for any fixed pair $(B, L(0))$, where $B = A(0)^{-\kappa}Q(0)$. Substituting (28) into (49) we obtain

$$A_0/Q_0^\gamma = (1+\kappa)KL(0)/[(1-\alpha+\alpha^2+K)g].$$

Because of $\gamma\kappa < 1$, there exists exactly one pair (A_0, Q_0) satisfying this equation as well as $A_0^{-\kappa}Q_0 = B$. Thus, for any fixed pair $(B, L(0)) \in (0, +\infty)^2$, there exists a unique BGP equilibrium. In addition, it is easy to see that higher values of $L(0)$ translate into higher values of both A_0 and Q_0. Thus, the BGP equilibrium exhibits a weak scale effect.

Appendix B

The Hamiltonian function for the social planner's optimal control problem can be written as

$$J(A, Q, L_R, z, \lambda_A, \lambda_Q, t) = (1-\alpha)\ln(AQ) + \ln[L(t) - L_R] + J_1(A, Q, z, \lambda_A, \lambda_Q)L_R,$$

where $J_1(A, Q, z, \lambda_A, \lambda_Q) = (\beta - z)Q^\gamma \lambda_A[1 + (z-1)\lambda_Q Q/(\lambda_A A)]$, and where λ_A and λ_Q are the costate variables corresponding to A and Q, respectively. The adjoint equations are

$$g_{\lambda_A}(t) = \rho - n - (1-\alpha)/[\lambda_A(t)A(t)] + [z(t) - 1]g_A(t)/M(t) \tag{52}$$

and

$$g_{\lambda_Q}(t) = \rho - n - (1-\alpha)/[\lambda_Q(t)Q(t)] - g_A(t)\left\{\gamma M(t) + (1+\gamma)[z(t) - 1]\right\}, \tag{53}$$

where

$$M(t) = [A(t)\lambda_A(t)]/[Q(t)\lambda_Q(t)]. \tag{54}$$

We start by proving a few auxiliary lemmas.

Lemma 4 *Along every optimal BGP it holds for all t that $L_R(t) > 0$ and $g_{L_R}(t) = n$.*

Proof: Suppose to the contrary that $L_R(t) = 0$ holds for all t along an optimal BGP. From the state equations we see that $A(t)$ and $Q(t)$ must be constant (hence, $g_A(t) = 0$) and that the choice of $z(t)$ has no influence on the value of the objective functional. We can therefore conclude that the constant control path $z(t) = 1$ is optimal. Using these results in (52)-(53) and noting that $g_{\lambda_A}(t)$ and $g_{\lambda_Q}(t)$ must be constant along a BGP, it follows that $\lambda_A(t)$ and $\lambda_Q(t)$ themselves must be constant. Substituting $g_A(t) = g_{\lambda_A}(t) = g_{\lambda_Q}(t) = 0$ into (52)-(53) and using the assumption $\rho > n$ we conclude that $\lambda_A(t)$ and $\lambda_Q(t)$ are strictly positive constants. A necessary condition for $L_R(t) = 0$ to be optimal is that the derivative of the Hamiltonian with respect to L_R evaluated at $L_R = 0$ is non-positive. This means that

$$J_1(A(t), Q(t), z(t), \lambda_A(t), \lambda_Q(t)) \leq 1/L(t).$$

The results mentioned above imply that the left-hand side of this inequality is a positive constant. Since $n > 0$, the right-hand side converges to 0 and the inequality is therefore violated for all sufficiently large t. This contradiction proves the lemma. $\square$

Lemma 5 *Along every optimal BGP $z(t)$ is constant and it holds for all t that $0 < z(t) < \beta$.*

Proof: Along a BGP both $g_A(t)$ and $g_Q(t)$ are constant. Because of (23) this implies that $z(t)$ is constant as well. Now suppose that $z(t) = 0$ for all t. This means that newly invented intermediate goods have quality $q_i = 0$ and, hence, do not contribute anything to final good production. R&D is therefore useless and it is optimal to choose $L_R(t) = 0$. Since this is a contradiction to lemma 4, it follows that $z(t) > 0$ holds.

Now suppose that $z(t) = \beta$ for all t. In this case it follows from the state equations that $A(t)$ and $Q(t)$ are constant and independent of the choice of $L_R(t)$. From (32) we see therefore that $L_R(t) = 0$ must be optimal. This contradicts lemma 4 again, and it follows that $z(t) < \beta$. $\square$

Proof of Theorem 2: From lemma 5 we know that $z(t)$ must be a constant. Let us denote this constant by $\bar{z}$. From (23) we obtain immediately that

$$g_Q(t) = (\bar{z} - 1)g_A(t). \tag{55}$$

Dividing equation (22) by $A(t)$ and noting that $g_A(t)$ and $z(t)$ must be constant along a balanced growth path, one sees that $L_R(t)Q(t)^\gamma/A(t)$ must remain constant. Because of lemma 4 this implies that $n + \gamma g_Q(t) - g_A(t) = 0$. Together with (55) this yields the first two statements in (33). The third one follows now easily from $g_L(t) = g_{L_R}(t) = n$ and (31).

Let us now turn to the determination of $\bar{z}$. From lemma 5 we know that $0 < \bar{z} < \beta$, that is, $\bar{z}$ is an interior maximum of the Hamiltonian function J. The corresponding first-order condition is $L_R(t)Q(t)^\gamma\lambda_A(t)[1 + \beta - 2\bar{z} - M(t)]/M(t) = 0$, which implies that

$$M(t) = 1 + \beta - 2\bar{z}. \tag{56}$$

This shows that $M(t)$ must be constant.

Now consider the adjoint equation (52). Since $z(t)$, $M(t)$, $g_{\lambda_A}(t)$, and $g_A(t)$ are constants along a BGP, it follows that $\lambda_A(t)A(t)$ must also be constant. Hence, we obtain $g_{\lambda_A}(t) = -g_A(t) = -\bar{g}$. Equation (53) implies in a completely analogous way that $g_{\lambda_Q}(t) = -g_Q(t) = -(\bar{z} - 1)\bar{g}$. Substituting these results back into (52)-(53), we get

$$-\bar{g} = \rho - n - (1 - \alpha)/[\lambda_A(t)A(t)] + (\bar{z} - 1)\bar{g}/M(t) \tag{57}$$

and

$$-(\bar{z} - 1)\bar{g} = \rho - n - (1 - \alpha)/[\lambda_Q(t)Q(t)] - \bar{g}\left[\gamma M(t) + (1 + \gamma)(\bar{z} - 1)\right].$$

Multiplying the former equation by $M(t)$ and subtracting it from the latter, we obtain after some rearrangement

$$M(t) = 1 - (1 + \gamma)\bar{z}\bar{g}/[(1 + \gamma)\bar{g} + \rho - n].$$

Substituting for $M(t)$ and $\bar{g}$ from (56) and $\bar{g} = n/[1 - \gamma(\bar{z} - 1)]$, we finally obtain the equation $G(\bar{z}) = 0$, with G as defined in the theorem. Now note that $G(0) = \beta(1 + \gamma)\rho > 0$ and $G(\beta) = -\beta(\rho - n)[1 - \gamma(\beta - 1)] < 0$, where we have used the assumptions $\rho > n$ and $\gamma < 1/(\beta - 1)$. Since $G(\bar{z})$ is a quadratic polynomial in $\bar{z}$, these properties prove that there exists exactly one value $\bar{z} \in (0, \beta)$ satisfying $G(\bar{z}) = 0$. Finally, because of $G(\beta/2) = \beta(1 + \gamma)n/2 > 0$, this solution must also satisfy $\bar{z} > \beta/2$.

It remains to determine the optimal research intensity $L_R(t)/L(t)$. From lemma 4 we know that $L_R(t) > 0$. Since $L_R(t) = L(t)$ leads to an objective value of $-\infty$, we can also conclude that $L_R(t) < L(t)$. The optimal value of $L_R(t)$ must therefore be an interior maximum of the Hamiltonian function. The corresponding first-order condition is $J_1(t) = 1/[L(t) - L_R(t)]$, which can also be written as $L_R(t)/L(t) = J_1(t)L_R(t)/[1 + J_1(t)L_R(t)]$.[21] The proof of the theorem is therefore complete if we can show that $J_1(t)L_R(t) = \bar{K}$ with $\bar{K}$ as defined in theorem 2.

From the definitions of $J_1(t)$ and $M(t)$, from $z(t) = \bar{z}$, and from $g_A(t) = \bar{g}$ we know that

$$J_1(t)L_R(t) = \bar{g}\lambda_A(t)A(t)[1 + (\bar{z} - 1)/M(t)].$$

[21] Here and in what follows we write $J_1(t)$ instead of $J_1(A(t), Q(t), z(t), \lambda_A(t), \lambda_Q(t))$.

Solving (57) for $\lambda_A(t)A(t)$ and substituting the result into the above equation yields after rearrangements

$$J_1(t)L_R(t) = \frac{(1-\alpha)\bar{g}[M(t)+\bar{z}-1]}{[M(t)+\bar{z}-1]\bar{g}+M(t)(\rho-n)}.$$

Replacing $M(t)$ by its value given in (56) one obtains $J_1(t)L_R(t) = \bar{K}$. This completes the proof of the theorem.

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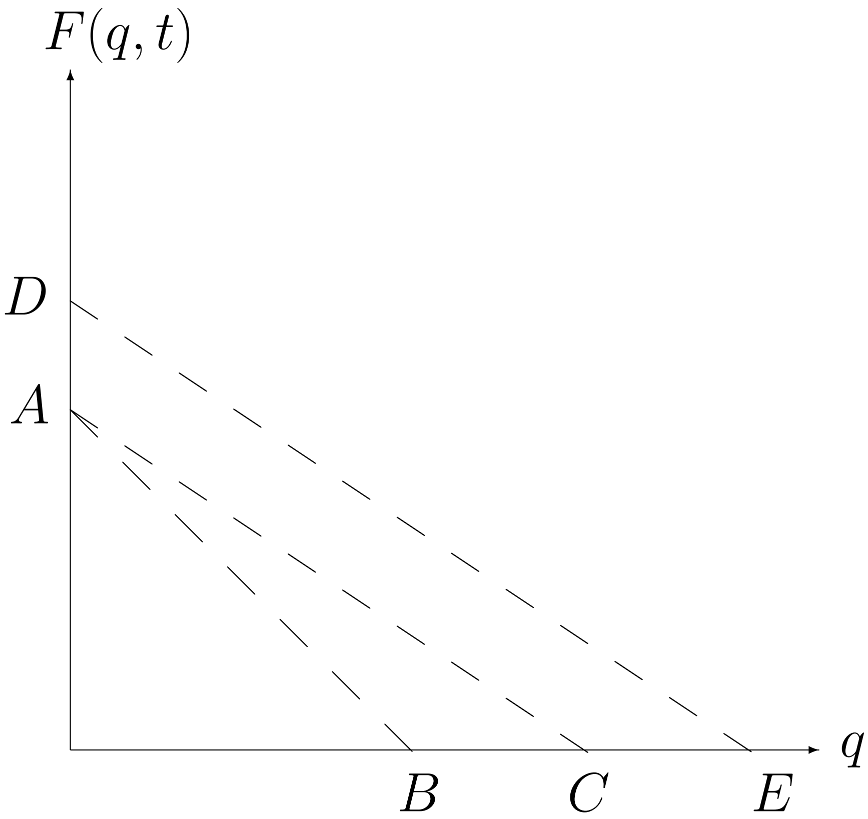

Figure 1: The quality/quantity frontier.