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Evolutionary stability and Nash equilibrium in finite populations, with an application to price competition [1]

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Abstract

Schaffer (1988) proposed a concept of evolutionary stability for finite-population models that has interesting implications in economic models of evolutionary learning, since it is related to perfectly competitive equilibrium. The present paper explores the relation of this concept to Nash equilibrium in particular classes of games, including constant-sum games, games with weak payoff externalities, and games where imitative decision rules are individually improving. An illustration of the latter is provided in the context of Bertrand oligopoly with homogeneous product which allows for a characterization of the set of evolutionarily stable prices.

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1 Introduction

In recent years, a renewed interest in decision rules based on imitation has emerged, partially motivated by the literature on evolutionary game theory. Björnerstedt and Weibull (1996) show that if a game is recurrently played by a continuum population of individuals who mimic the actions of better performing individuals observed at random, population play follows the solution trajectories of the replicator dynamics; imitation is thus one of the possible decision rules underlying the most prominent evolutionary dynamics. It is well known that Nash equilibria are rest points of the replicator dynamics. Moreover, evolutionarily stable strategies (henceforth ESS) as defined by Maynard Smith and Price (1973) for a continuum population are always Nash equilibrium strategies and asymptotically stable in the replicator dynamics. Further, if a state is the limit of a trajectory starting in the interior of the state space, then this limit is necessarily a symmetric Nash equilibrium. In summary, if individuals in a large population mimic successful behavior, when population play converges, it does so to a Nash equilibrium. Hence, evolutionary game theory provides a non rationalistic foundation to equilibrium play (see Weibull, 1995, chap. 2 and 3).

The relation between ESS, Nash equilibrium, and the long-run outcomes of imitative dynamics in *finite-population* models is not so well understood. This relation constitutes the main interest of the present paper. In particular, we identify classes of games where a finite-population ESS is always a Nash equilibrium strategy.

The concept of finite-population ESS as defined by Schaffer (1988) is not related to Nash equilibrium in general (cf. Section 3).[1] Stochastic models of evolutionary learning in games that postulate individual behavior driven by imitation also yield somewhat contradictory outcomes. A prominent example is provided by Vega-Redondo (1997), who shows that imitation of successful strategies leads to competitive equilibrium in a Cournot oligopoly. Alós-Ferrer et al. (2000), however, show that imitative behavior does lead to Nash equilibrium in a Bertrand oligopoly. The latter holds also in the case of convex costs where there is a large set of Nash

[1]Schaffer (1989) shows, for example, that Nash equilibrium and ESS differ in a Cournot duopoly and Hehenkamp et al. (2004) show this for rent-seeking games. Tanaka (1999; 2000) has similar results for oligopolies with asymmetric cost functions and differentiated product respectively.

equilibria beyond the competitive equilibrium (see Dastidar (1995)).[2] These results show that, in finite-population models, imitative behavior may or may not lead to Nash equilibrium, depending on the type of game. The present paper takes a further step in trying to understand how the properties of imitative rules are related to evolutionary stability of Nash equilibrium in finite-population models.

The results obtained so far point rather to a more general relation between evolutionary stability, the long-run outcomes of imitative behavior, and perfectly competitive (instead of Nash) equilibrium. In a recent paper, Alós-Ferrer and Ania (2005) study this relation for a class of games that includes the Cournot oligopoly. Their focus is on aggregative games, where payoffs to any player depend on own strategy and an aggregate of all players' strategies. For an aggregative game, aggregate-taking behavior can be defined as payoff maximization disregarding the own effect on the aggregate, which is the analogue to perfectly competitive behavior. It is shown that, if the game displays strategic substitutability between own strategy and the aggregate, aggregate-taking behavior has strong evolutionary stability properties. These properties, in turn, imply that aggregate-taking behavior is the long-run outcome of a stochastic learning process where individual decisions are based on imitation of successful strategies and random experimentation. Strategic substitutability between own strategy and the aggregate creates a tension between high relative performance, the dominating force behind evolutionary stability, and high absolute performance, which drives Nash equilibrium; as a consequence, imitation leads away from Nash equilibrium.

In contrast, our main focus here will be on games where imitation is improving; i. e. where mimicking successful strategies always results in a payoff improvement to the imitator. In such games the conflict between absolute and relative payoff maximization is weakened and we can show that a finite-population ESS always corresponds to a symmetric Nash equilibrium. Before that, we define imitative behavior formally and review the concept of finite-population ESS. We will argue that as a consequence of finite-population effects, the coincidence of ESS and Nash equilibrium constitutes the exception rather than the rule. That said, we proceed to show that ESS and Nash equilibrium do coincide in constant-sum games and in games with weak payoff externalities, where any deviation always affects the deviator's payoff more than the opponents' payoffs. We then turn to games where

[2] Note that the results for Cournot and Bertrand competition are compatible only in the particular case of homogeneous product and constant unit costs, where firms competing *à la* Bertrand always price competitively in equilibrium. For the case of increasing marginal costs, it is shown that imitative behavior leads to a subset of the set of Nash equilibria. That subset includes the competitive equilibrium only in certain cases.

imitation is improving and establish a link between the static equilibrium concepts and the properties of imitative dynamics. Finally, we give an example of a game in the latter class, namely a Bertrand oligopoly with homogeneous product and decreasing returns to scale. We show that mimicking the price of the best performing firm in the industry always increases the profits of the imitating firm. This implies that evolutionarily stable prices correspond to Nash equilibrium prices. In the context of this example, we also show that not all Nash equilibria are evolutionarily stable; evolutionary stability actually selects a subset of the equilibrium prices characterized by Dastidar (1995). Our results for the example also clarify some of the dynamic results obtained in Alós-Ferrer et al. (2000). The fact that ESS corresponds to Nash equilibrium only exceptionally, makes it even more remarkable that this is the case for price competition.

The results obtained here for Bertrand oligopoly are related to those in Qin and Stuart (1997) and Hehenkamp and Leininger (1999), who study evolutionary stability of Bertrand equilibrium in a market with constant unit costs and a continuum population. Whereas the former show that the Nash equilibrium where all firms price at marginal cost is not evolutionarily stable, the latter argue that, if the set of prices that firms are allowed to charge is discrete, then a new equilibrium appears where all firms set the smallest price above marginal cost, and this equilibrium is indeed evolutionarily stable. In the analysis, they use the notion of evolutionary stability for a continuum population — which corresponds in that case to a continuum of firms that are randomly matched in a continuum of n-firm independent Bertrand markets. This framework, traditionally used for evolutionary analysis, is difficult to reconcile with the interpretation of an oligopolistic market as a game with a small number of players. Partly, our contribution is to show that the finite-population definition of evolutionary stability is better suited for the analysis of evolutionary aspects in markets. For the case of constant unit costs, our results imply evolutionary stability of the Nash equilibrium.

The type of improving property exploited in the present paper is related to the optimality properties of imitative rules defined by Schlag (1998) for the case of multi-arm bandit problems; i. e. problems of individual decision making under uncertainty. There, it is shown that individuals in a large population can learn the best strategy by following certain forms of sophisticated imitation. Conlisk (1980) and, more recently, Rhode and Stegeman (2001) and Schipper (2002) provide dynamic models with two types of decision-makers, optimizers and imitators, in a stable environment. In different contexts, they show that imitators survive and perform at least as well as optimizers in the long run. The idea that absolute payoff maximizers do not necessarily obtain higher payoffs in equilibrium than individuals with other objectives was also proposed by Fershtman and Judd (1987) and, more recently, Koçkesen et al. (2000) show that strategic decision makers that maximize relative instead of absolute payoffs may have an absolute-payoff advantage in equilibrium in a large class of interesting economic games that satisfy structural conditions related to super- or submodularity, payoff monotonicity, and payoff externalities.

Imitative behavior has often been justified not on the grounds of optimality in decision making, but because it saves decision-making costs. Pingle and Day (1996) report on a number of experimental settings where decision costs have been explicitly incorporated. They find that subjects use imitation along with other modes of economizing behavior in order to avoid those costs. Finally, Huck et al. (1999) and Apesteguia et al. (2003) provide theoretical and experimental support showing that the use and dynamic properties of imitative behavioral rules depend crucially on the informational setting in which they take place. Our work is complementary to theirs, showing that even under the same informational assumptions, imitative rules have different properties depending on the game where they are used. Moreover, our emphasis is on the fact that the dynamic properties of imitation are directly related to the properties of the static concept of finite-population evolutionary stability.

The rest of the paper is organized as follows. In Section 2 we make a formal description of imitative and improving rules. In Section 3 we review the concept of finite-population ESS and establish the relation to Nash equilibrium in particular classes of games. In Section 4 we turn to Bertrand oligopoly and show that price imitation is always improving, which allows to calculate the set of evolutionarily stable prices easily; at the end of the section we also explore the effects of imitative behavior on industry profits. In Section 5 we make some concluding remarks.

2 Simple behavioral rules for games

In the present section we give a definition of imitative behavior. A basic premise will be that behavior is adaptively driven by observation of past actions and the performance associated to those actions.[3] As will become apparent later on, behavior based on imitation requires symmetry to a certain extent; for example, imitation is only possible if the same set of actions is available to all decision makers. Therefore our focus will be on symmetric games and on symmetric behavioral rules.

[3]Our formal description of behavioral and, in particular, imitative rules will be akin to those in Apesteguia et al. (2003), Josephson and Matros (2004), and Selten and Ostmann (2001).

2.1 Better reply correspondence

Consider a normal-form game Γ with set of players $I = \{1, \ldots, n\}$, set of strategies S, common to all players, and payoffs to player $i \in I$ given by the function $\pi_i : S^n \to \mathbb{R}$. The game is symmetric if there exists a function $\pi : S \times S^{n-1} \to \mathbb{R}$ such that, for any strategy profile $\mathbf{s} = (s_1, \ldots, s_n)$, $\pi_i(\mathbf{s}) = \pi(s_i|s_{-i}) = \pi(s_i|s'_{-i})$ where s_i is player i's strategy in the profile $\mathbf{s}$, s_{-i} is the vector of all players' strategies except i in the profile $\mathbf{s}$, and s'_{-i} is any permutation of s_{-i}. I. e. the game is symmetric if payoffs to any strategy are independent of the players' names and invariant to permutations of the opponents' strategies.

Denote (s'_i, s_{-i}) the strategy profile where all players but i choose strategies according to a given profile $\mathbf{s}$ and player i chooses $s'_i \in S$. Given $\mathbf{s}$, the *better reply* set of player i is given by

$$B_i(\mathbf{s}) = \{s'_i \in S \mid \pi_i(s'_i, s_{-i}) \geq \pi_i(\mathbf{s})\}. \tag{1}$$

The set $B_i(\mathbf{s})$ contains the strategies that would weakly improve i's payoff at $\mathbf{s}$. Obviously, $s_i \in B_i(\mathbf{s})$ for all $\mathbf{s}$. This concept was introduced by Ritzberger and Weibull (1995).

2.2 Imitative and improving rules

Let us now consider any decision problem where the individuals in $I = \{1, \ldots, n\}$ have to choose strategies from the set S, knowing the sets I and S, and knowing that their payoffs depend on the strategies chosen by others in I. Suppose, however, that they do not have precise information about the payoff function and are thus not able to calculate best responses. Instead, they decide on the basis of observed past performance. Since they do not behave strategically, we do not refer to them as players, but rather as individuals or decision makers. We now define what we mean by an imitative behavioral rule in this context.

A *behavioral rule* for decision maker i, denoted $F_i : X_i \twoheadrightarrow S$, is a correspondence mapping i's set of possible observations X_i into the set of strategies S. Given that individual i observes $x_i \in X_i$, $F_i(x_i) \subseteq S$ is the set of strategies that i may take next period. A *system of behavioral rules* $\mathbf{F} = (F_1, \ldots, F_n)$ is *symmetric* if $X = X_1 = \ldots = X_n$ and if $F_1(x) = \ldots = F_n(x)$ for all $x \in X$; i. e. if all decision makers have the same set of possible observations, and if the individual behavioral rules prescribe the same to all of them, provided that they observed the same. For the purpose of this paper it will be enough to focus on symmetric systems where all decision makers use the same behavioral rule F.

Let $C(\mathbf{s})$ be the set of strategies currently chosen at any profile $\mathbf{s}$. Formally,

$$C(\mathbf{s}) = \{s \in S \mid s = s_i, \text{ for some } i \in I\}. \tag{2}$$

A behavioral rule $F : X \twoheadrightarrow S$ is *imitative* if $X = S^n \times \mathbb{R}^n$ and $F(x) \subseteq C(\mathbf{s})$ for all $x = (\mathbf{s}, \mathbf{u}) \in X$ where $\mathbf{u} = (u_1, \ldots, u_n)$ is an arbitrary vector of observed payoffs.[4] Note the two parts of the definition. First, current strategies chosen and payoffs to all individuals constitute the set of possible observations. Second, the rule prescribes to choose strategies that are observed in the current profile only. We explicitly introduce the first requirement because imitation is possible only if other strategies are observed. Observability of payoffs is not necessary in general for the definition of an imitative rule. Rules like 'imitate the most popular strategy' do not require that payoffs are observed. It is more likely, however, that imitation is based on some measure of success associated to each strategy and that this success is related to payoffs obtained and not only to popularity.[5]

Given $x = (\mathbf{s}, \mathbf{u})$, let the *reference set* at x be given by

$$R(x) = \{s \in S \mid s = s_i \text{ for some } i \in I \text{ and } u_i \geq u_j \text{ for all } j \in I\}. \tag{3}$$

The reference set $R(x) \subseteq C(\mathbf{s})$ contains the strategies chosen at $\mathbf{s}$ that gave highest observed payoffs. We say that a behavioral rule F corresponds to *imitate the best* if $F(x) = R(x)$ for all $x \in X$; i. e. if the strategies that may be chosen by any decision maker next period are those that gave highest payoffs in the current profile.

Finally, suppose that the underlying decision problem can be modelled through the game Γ, although individuals do not know the payoff functions and follow some simple behavioral rule F. Given $\mathbf{s} \in S^n$, denote $\pi(\mathbf{s}) = (\pi_1(\mathbf{s}), \ldots, \pi_n(\mathbf{s}))$ the associated vector of payoffs. We say that a behavioral rule F is *improving* in Γ if for all $x(\mathbf{s}) = (\mathbf{s}, \pi(\mathbf{s}))$, $F(x(\mathbf{s})) \subseteq B_i(\mathbf{s})$ for all $\mathbf{s} \in S^n$ and all $i \in I$. I. e. starting at any $\mathbf{s}$ with associated observed payoffs given by $\mathbf{u} = \pi(\mathbf{s})$ according to Γ, following F will result in a weak payoff improvement to any decision maker.

[4] We use the notation π for payoffs resulting from the game Γ and $\mathbf{u}$ for arbitrary vectors of observed payoffs. Recall that decision makers cannot infer payoffs when they observe strategies.

[5] A more general definition of an imitative rule could allow the elements of X to be arbitrary sets instead of vectors, provided that each element of X intersects $S \times \mathbb{R}$; i. e. imitation is only possible if at least some strategy and payoff are observed. It is not necessary that individuals observe *all* strategies *currently* used and their associated payoffs. Alternatively, they could observe the current strategies and payoffs of some randomly chosen sample of individuals, or they could also have information about past behavior. This would require a richer structure for the set X that we avoid here.

3 Evolutionary stability in finite populations

In the present section we establish a relation between improving imitative rules and the concept of evolutionary stability. It is customary in evolutionary game theory to consider an infinite population of individuals who are randomly matched to play some given game recurrently. In that context, a strategy is evolutionarily stable if, once adopted by all individuals in the population, it cannot be outperformed by any other mutant strategy coming in the population in a sufficiently small fraction. The notion of evolutionary stability is based on relative performance; that is, on payoff comparisons between the status quo and the mutant strategy in the post-entry population profile. It is well known that in the context of an infinite population an evolutionarily stable strategy always constitutes a symmetric Nash equilibrium (see Weibull, 1995, chap. 2). In the same spirit, Schaffer (1988) proposed a definition of evolutionary stability for n-player games played within a finite population, which seems more suitable for application to economic problems. In this context, however, due to finite-population effects, a strategy may be successful in relative terms even if it is not a Nash equilibrium strategy (see Vega-Redondo, 1996, sec. 2.7).

Consider the symmetric game Γ. The set of players I is the finite population of individuals choosing strategies from S. Payoffs to individual $i \in I$ at the profile $\mathbf{s} = (s_i, s_{-i})$ are given by $\pi_i(\mathbf{s}) = \pi(s_i|s_{-i})$.

We say that $s \in S$ is an *evolutionarily stable strategy* (ESS) if for all $s' \in S$,

$$\pi(s|s', s, \overset{n-2}{\dots}, s) \geq \pi(s'|s, s, \overset{n-1}{\dots}, s). \tag{4}$$

An ESS is *strict* if the last inequality holds strictly for all $s' \neq s$. That is, once adopted by all individuals in the population, an ESS cannot be outperformed by any alternative strategy after any single deviation. Note that, in a population where all but one deviant choose s, the single deviant choosing s' faces the opponents' profile $(s, s, \overset{n-1}{\dots}, s)$ while those still choosing s face the profile $(s', s, \overset{n-2}{\dots}, s)$; the deviant never confronts another deviant as in the standard definition of ESS for an infinite population where a small positive mass of mutants enters the population.

The focus of evolutionary stability is not on the usual comparison of payoffs to s and s' before and after deviation as in a Nash equilibrium, but on the comparison of simultaneous payoffs to s and s' in the resulting profile after deviation. An ESS has a relative, not necessarily an absolute, advantage. If an ESS, s, has been adopted by all individuals in the population, it may be profitable for an individual to deviate to some s', but that deviation would result in an even larger increase in the payoffs of s, which has *ex-post* a relative advantage.

An ESS can be viewed as a Nash equilibrium strategy of a transformed game, where the players' objective is to maximize relative instead of absolute payoffs. Rewriting expression (4), we say that s is an ESS if it solves the problem

$$\max_{s' \in S} \pi(s'|s, s, \overset{n-1}{\ldots}, s) - \pi(s|s', s, \overset{n-2}{\ldots}, s). \tag{5}$$

Instead, a strategy played in a symmetric Nash equilibrium of the original game (with payoff function π) would only maximize the first part of the objective function in problem (5). Therefore, it is not surprising that the equivalence of ESS and Nash equilibrium strategies is more the exception than the rule.

On the other hand, the concept of ESS is related to another well-known equilibrium concept. In particular, Schaffer (1989) shows for a symmetric Cournot duopoly with constant unit costs that the output level corresponding to the competitive equilibrium is evolutionarily stable. When all firms behave competitively and price equals marginal cost, any firm deviating from the competitive output may strategically improve its profits, but in that case the profits of the competitive firms will increase even more. This result was generalized by Alós-Ferrer and Ania (2005) to a large class of economic games, where finite-population ESS is related to perfectly competitive behavior. Here perfectly competitive behavior refers to aggregate-taking behavior; that is, payoff maximizing behavior disregarding the individual effect on some payoff-relevant aggregate.

Yet a natural question is whether the concept of finite-population ESS is sometimes related to Nash equilibrium, and whether we can say anything general about the games where that is the case. We turn to this question in what follows.

3.1 Equivalence to Nash equilibrium in constant-sum games

We first focus on constant-sum games; i. e. games such that $\sum_i \pi_i(\mathbf{s})$ is constant for all $\mathbf{s}$. The next proposition shows that ESS and Nash equilibrium strategies coincide in that case.

Proposition 1. *Let Γ be a symmetric, constant-sum game. A strategy s is ESS if and only if $\mathbf{s} = (s, \ldots, s)$ is a symmetric Nash equilibrium in Γ.*

Proof. Consider the symmetric profile $(s, \ldots, s)$ and a unilateral deviation to any strategy $s' \neq s$. The sum of payoffs before and after deviation has to be the same, since the game is of constant sum. It follows that

$$\left(\frac{n-1}{n}\right) \pi(s|s', s, \overset{n-2}{\ldots}, s) + \left(\frac{1}{n}\right) \pi(s'|s, \overset{n-1}{\ldots}, s) = \pi(s|s, \overset{n-1}{\ldots}, s) \tag{6}$$

By equation (6) the payoff to s before deviation must lie between the payoffs to s and s' after deviation. If s is an ESS, then $\pi(s|s', s, \overset{n-2}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s)$ for all s';

i. e. after any deviation, the deviator to s' must have a lower payoff that those still choosing s. Equation (6) then implies that $\pi(s|s, \overset{n-1}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s)$ for all s'. Thus $(s, \ldots, s)$ is a symmetric Nash equilibrium. Analogously, if $(s, \ldots, s)$ is a Nash equilibrium, then $\pi(s|s, \overset{n-1}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s)$ for all s'; i. e. the deviator to any s' must have a lower payoff than before deviation. Equation (6) then implies that, if the payoff to the deviator decreases, that of the non-deviators must increase to keep the sum of payoffs constant; thus $\pi(s|s', s, \overset{n-2}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s)$ for all s', so that s is an ESS. □

Intuitively, in a constant-sum game if the payoffs to a player decrease after a deviation, the opponents' payoffs must increase, leaving the deviator in a worse relative position. This shows that any strategy played at a symmetric Nash equilibrium must be evolutionarily stable. Conversely, the payoffs to any player in a symmetric profile must equal the average payoff across players in a non-symmetric profile. Thus if a single deviator from a symmetric profile is worse off in relative terms after deviation, this must result from a worsening in absolute terms. Therefore, any evolutionarily stable strategy is played at a symmetric Nash equilibrium.

3.2 Games with weak-payoff externalities

The focus in this section is on games where the effect of any unilateral deviation on the deviator's payoff is always greater than the effect on the opponents' payoffs. Formally, we say that Γ has *weak-payoff externalities*, if for all $\mathbf{s}, \mathbf{s}' \in S^n$ with $\mathbf{s} = (s_i, s_{-i})$, $\mathbf{s}' = (s_i', s_{-i})$, and $s_i \neq s_i'$ and for all $i, j \in I,\ i \neq j$ we have

$$|\pi_i(\mathbf{s}') - \pi_i(\mathbf{s})| > |\pi_j(\mathbf{s}') - \pi_j(\mathbf{s})|.$$

Proposition 2. *Let Γ be a symmetric game with weak-payoff externalities. A strategy s is ESS if and only if $\mathbf{s} = (s, \ldots, s)$ is a Nash equilibrium.*

Proof. Suppose s is ESS but $\mathbf{s} = (s, \ldots, s)$ is not a Nash equilibrium. It follows that there exists $s' \neq s$ such that

$$\pi(s|s', s, \overset{n-2}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s) > \pi(s|s, \overset{n-1}{\ldots}, s).$$

That is, it is profitable to deviate to some s' for some player, but payoffs to the non-deviators increase at least as much, so that the deviator ends up in a worse relative position. This obviously contradicts the fact that the game has weak-payoff externalities.

Analogously, suppose that $\mathbf{s} = (s, \ldots, s)$ is a Nash equilibrium, but s is not ESS. It follows that there exists $s' \neq s$ such that

$$\pi(s|s, \overset{n-1}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s) > \pi(s|s', s, \overset{n-2}{\ldots}, s).$$

That is, a deviation to s' results in a decrease in absolute payoffs for the deviator, but payoffs to the non-deviators decrease at least as much, so that the deviator ends up in a better relative position. This again contradicts the property of weak-payoff externalities. □

3.3 Games where imitation is improving

So far we have identified classes of games where the relation between ESS and Nash equilibrium can be established directly. We now turn to a dynamic approach. In this section we relate ESS and Nash equilibrium through the properties of imitative behavioral rules of the type defined in Section 2.2. In particular, we show that in games where imitation is improving an ESS is always played in a symmetric Nash equilibrium. We will also see later by means of an example, that the reciprocal implication does not hold and thus there may be Nash equilibria that are not evolutionarily robust. In this class of games, evolutionary stability indeed serves as a selection criterion.

Proposition 3. *Consider the decision problem modelled by the game* Γ*. Suppose decision makers do not behave strategically but follow a simple behavioral rule* F *corresponding to* imitate the best*. Assume that* F *is* improving *in* Γ*. If* s *is an ESS of* Γ*, then* $\mathbf{s} = (s, \ldots, s)$ *is a Nash equilibrium of the game.*

Proof. Suppose $s \in S$ is an ESS and let $s' \in S$ be any other strategy. Then $\pi(s|s', s, \overset{n-2}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s)$ holds by definition. Let $\mathbf{s}' = (s', s, \overset{n-1}{\ldots}, s)$ be the resulting strategy profile after a single deviation to s' with associated vector of payoffs $\pi(\mathbf{s}')$. Let F correspond to imitate the best, then at $\mathbf{s}'$ all individuals observe $x(\mathbf{s}') = (\mathbf{s}', \pi(\mathbf{s}'))$ and $s \in R(x(\mathbf{s}'))$. If F is improving, then $R(x(\mathbf{s}')) \subseteq B_i(\mathbf{s}')$ for all $i \in I$. This implies that $\pi(s|s, \overset{n-1}{\ldots}, s) \geq \pi(s'|s, \overset{n-1}{\ldots}, s)$; i. e. the deviant choosing s' obtains a weak improvement by choosing s, implying that s must correspond to a symmetric Nash equilibrium of Γ. □

Example 1. *Minimum-effort games*

Consider the class of games where each player $i \in I$ chooses an effort level $s_i \in S \equiv \mathbb{R}_+$. Player i's payoff is given by

$$\pi_i(s_1, \ldots, s_n) = a \cdot \min\{s_1, \ldots, s_n\} - b \cdot s_i + c,$$

where a, b, and c are constants with $a > b \geq 0$. These are referred to as minimum-effort coordination games or Stag Hunt games (see Crawford (1991)).

To see that imitation is improving note that at the profile $\mathbf{s}$, individuals observe $x(\mathbf{s}) = (\mathbf{s}, \pi(\mathbf{s}))$ and the reference set is $R(x(\mathbf{s})) = \{s_i \in \mathbb{R}_+ | s_i = \min\{s_1, \ldots, s_n\}\}$. Take any individual j with $s_j \neq \min\{s_1, \ldots, s_n\}$, j's payoff always improves after imitation since

$$(a - b) \cdot \min\{s_1, \ldots, s_n\} + c \geq a \cdot \min\{s_1, \ldots, s_n\} - b \cdot s_j + c$$

It is easy to see that although any level of effort constitutes a symmetric Nash equilibrium, only $s = 0$ is an ESS.

3.4 Discussion on supermodular and potential games

We conclude this section with a discussion on supermodular and potential games. At first glance, these well behaved classes of games seem good candidates that would satisfy the kind of properties we have been looking at. It turns out that games where imitation is improving (and thus ESS always corresponds to Nash equilibrium) are neither a special case of, nor do they include, supermodular or potential games.

Intuitively, strategic complementarities in the case of supermodular games seem to provide a framework where imitation should have good strategic properties. The reason being that imitation has the effect of pooling decision makers in the same direction. Given that best response correspondences are increasing in this case (see e. g. Vives, 1999, sec. 2.2.3), 'moving together' seems as the right thing to do from a strategic perspective. However, imitation not only determines the direction in which a mimicking decision maker moves, but makes decision makers move to exactly the same strategy. Although it may be correct that the better reply correspondence lies in the direction of better performers, copying the opponent strategy may take the decision maker 'too far' with respect to the better reply set. An example of a supermodular game where the ESS and the Nash equilibrium differ is given by Tanaka (2000) for the case of price competition with differentiated product. Proposition 3 above then implies that imitation can not be improving. Thus, supermodular games are not a subclass of the games where imitation is improving. Reciprocally, it is easy to construct examples of games which are not supermodular (not even ordinally supermodular) but where imitation is improving. Thus, supermodularity is unrelated to the property of improving imitation.

Monderer and Shapley (1996) show that certain examples of Cournot oligopoly are (cardinal) potential games. However, as mentioned above, ESS there corresponds to competitive and not to Nash equilibrium. Proposition 3 then implies that imitation is not improving in Cournot oligopolies. On the other hand, the property of improving imitation seems to be related intuitively to the finite improvement property that characterizes generalized ordinal potential games (see Monderer and Shapley (1996)). However, we present here an example of a game where imitation is improving and the finite improvement property is violated. This example shows that the class of games where imitation is improving is not a subclass of generalized ordinal potential games.

Example 2. Consider the two-player game with payoff matrix

	X	Y	Z
X	0,0	0,0	-1,1
Y	0,0	0,0	1,-1
Z	1,-1	-1,1	0,0

It is easy to see that imitation is improving in this game.[6] However, the game has a cycle in the following improvement path

$$(Y,X) \to (Z,X) \to (Z,Z) \to (Y,Z) \to (Y,X);$$

hence, it is not a generalized ordinal potential game. This example shows that games where imitation is improving are not a subclass of the most general class of potential games. Additionally, given that not even the most restrictive class of potential games (namely cardinal potential games) is a subclass of the games where imitation is improving, we conclude that these two classes of games are unrelated.

4 Price imitation

In this section we provide an economic example of a class of n-person games where imitate the best is always improving. Therefore, by Proposition 3 all ESS are Nash equilibrium strategies. As we will show below, this fact is useful to find all ESS.

4.1 The industry

Let the game Γ model an industry where identical firms $I = \{1, \ldots, n\}$ set prices from $S = [0, \overline{p}]$. All firms face the same cost function $C(q)$, where q is the individual output level. Assume that C is strictly increasing and convex and, for simplicity, that $C(0) = 0$. Suppose customers buy from the firm with lowest price only and that firms

[6] It is also easy to check that this game is not ordinally supermodular under any reordering of the strategies.

adjust production to demand. Let $D(p)$ be a strictly decreasing demand function, which we assume strictly positive for all $p \in [0, \overline{p}]$. In case of ties, demand splits equally. Given the strategy profile $\mathbf{p} = (p_1, \ldots, p_n)$, let $P(\mathbf{p}) = \min\{p_1, \ldots, p_n\}$ and $M(\mathbf{p}) = \{i \in I \mid p_i = P(\mathbf{p})\}$. Thus $|M(\mathbf{p})|$ is the number of firms that set lowest price at $\mathbf{p}$. The profits to firm i are given by

$$\pi_i(\mathbf{p}) = \begin{cases} P(\mathbf{p})\frac{D(P(\mathbf{p}))}{|M(\mathbf{p})|} - C\left(\frac{D(P(\mathbf{p}))}{|M(\mathbf{p})|}\right) & \text{if} \quad i \in M(\mathbf{p}) \\ 0 & \text{if} \quad i \notin M(\mathbf{p}) \end{cases}$$

and the game is obviously symmetric.

Contrary to the case of constant unit costs, where market price always equals marginal cost in equilibrium, Dastidar (1995) shows that a symmetric Bertrand oligopoly with homogeneous product and convex costs has a large set of pure-strategy Nash equilibria. In order to state the set of equilibria, define for any $k = 1, \ldots, n$, and any $p \in \mathbb{R}_+$

$$\pi(p, k) = p\frac{D(p)}{k} - C\left(\frac{D(p)}{k}\right).$$

Let $P_k \in \mathbb{R}_+$ be such that $\pi(P_k, k) = 0$ and $D(P_k) > 0$; i. e. P_k is such that k active firms make zero profits. Let $P'_k \in \mathbb{R}_+$ be such that $\pi(P'_k, k) = \pi(P'_k, 1)$; i. e. P'_k is such that each of the k active firms are indifferent between sharing the market and being a monopolist at price P'_k. It can be shown that P_k is decreasing with k, $P'_n > P_1$, and that, for any price $p \in [P_n, P'_n]$, the profile where all firms set price equal to p is a Nash equilibrium (see Dastidar, 1995, Lem. 6 and 7, and Prop. 1). Note that at the Nash equilibrium where all firms set price P_n firms make zero profits.

4.2 Price imitation is improving

We now proceed to show that copying the price of the most successful firm in the industry can only improve the profit of the imitating firm, therefore imitate the best is improving in this game. This result provides a strategic rationale for the imitation of competitors' prices.

Given $\mathbf{p}$, all firms observe $x(\mathbf{p}) = (\mathbf{p}, \pi(\mathbf{p}))$, where $\pi(\mathbf{p})$ is the vector of profits. Then the reference set is given by

$$R(x(\mathbf{p})) = \{p \in [0, \overline{p}] \mid p = p_i \text{ for some } i \in I \text{ and } \pi_i(\mathbf{p}) \geq \pi_j(\mathbf{p})) \quad \forall j \in I\}.$$

The rule imitate the best prescribes to copy any of the prices charged by the firms with highest profits; i. e. $F(x(\mathbf{p})) = R(x(\mathbf{p}))$ for all $\mathbf{p}$.

Proposition 4. *In a symmetric Bertrand oligopoly with decreasing demand $D(p)$, increasing and convex costs $C(q)$, and equal splitting in case of ties, $R(x(\mathbf{p})) \subseteq B_i(\mathbf{p})$ for all $\mathbf{p} \in \mathbb{R}^n$ and all $i \in I$ with $x(\mathbf{p}) = (\mathbf{p}, \pi(\mathbf{p}))$.*

Proof. For all strategy profiles of the type $\mathbf{p}^n = (p, p, \overset{n}{\dots}, p)$, when all firms set the same price, firms share the market and obtain the same profits. By following imitate the best, none of them will change the price and profits cannot change after any price revision, implying that $R(x(\mathbf{p}^{(n)})) = \{p\} \subset B_i(\mathbf{p}^{(n)})$ for all $i \in I$.

All other states are of the form $\mathbf{p}^{(m)} = (p_1, p_2, \dots, p_n)$ where $1 \leq m < n$ firms set price p and all other firms set a higher price. Without loss of generality, assume that $p_1 = p_2 = \dots = p_m = p$ and that $p_i > p$ for $i = m+1, \dots, n$. The profits of the firms $i = m+1, \dots, n$, with higher than minimum price, are $\pi_i(\mathbf{p}^{(m)}) = 0$ while the profits of the firms $i = 1, \dots, m$, with minimum price p are given by

$$\pi_i(\mathbf{p}^{(m)}) = \pi(p, m) = p\frac{D(p)}{m} - C\left(\frac{D(p)}{m}\right) = \frac{D(p)}{m}\left[p - AC\left(\frac{D(p)}{m}\right)\right]$$

where $AC(q) = C(q)/q$ denotes average cost and the last equality is understood to hold only if $D(p) > 0$. To show that starting at $\mathbf{p}^{(m)}$ imitate the best is improving, we have to consider the following cases separately.

First, if $\pi(p, m) > 0$, then $R(x(\mathbf{p}^{(m)})) = \{p\}$. Convexity of $C(q)$ implies that that $AC(q)$ is increasing in q.[7] After strategy revision, profits of idle firms do not change and for any $i = 1, \dots, m+1$

$$\begin{aligned} \pi_i(\mathbf{p}^{(m+1)}) = \pi(p, m+1) = \frac{D(p)}{m+1}\left[p - AC\left(\frac{D(p)}{m+1}\right)\right] \geq \\ \frac{D(p)}{m+1}\left[p - AC\left(\frac{D(p)}{m}\right)\right] > 0 \end{aligned} \tag{7}$$

This implies that $p \in B_i(\mathbf{p}^{(m)})$ for all $i \in I$.

Second, if $\pi(p, m) < 0$, then $R(x(\mathbf{p}^{(m)})) = \{p_{m+1}, \dots, p_n\}$, since highest profits equal zero and are attained by idle firms. Imitation of any price in $R(x(\mathbf{p}^{(m)}))$ would not change profits for idle firms $i = m+1, \dots, n$. What happens to the

[7]Consider $q' \geq q > 0$, by convexity of $C(q)$,

$$C(q) \leq \frac{q}{q'}C(q') + \left(1 - \frac{q}{q'}\right)C(0) = \frac{q}{q'}C(q')$$

Thus, $AC(q') \geq AC(q)$. If $C(0) > 0$ and costs are convex, then $AC(q)$ is U-shaped, but average variable costs are increasing. Actually, this is all we need here, although for simplicity we assumed zero fixed costs.

profits of active firms, $i = 1, \ldots, m$, when any of them copies a price in $R(x(\mathbf{p}^{(m)}))$? Obviously, if $m > 1$, any unilateral deviation to a price in $R(x(\mathbf{p}^{(m)}))$ will yield the deviating firm zero profits, which is better than losses. Consider now the case $m = 1$ with $\pi(p, 1) = D(p)[p - AC(D(p))] < 0$. In this case $R(x(\mathbf{p}^1)) = \{p_2, \ldots, p_n\}$. Call $p' = \min R(x(\mathbf{p}^1))$. If firm 1 now imitates any $p_i > p'$, it will face no demand and avoid losses. If it imitates p', its profits after imitation will be

$$\pi(p', m') = \frac{D(p')}{m'} \left[p' - AC\left(\frac{D(p')}{m'}\right)\right] \tag{8}$$

where $m' \geq 2$ is the number of firms setting price p' after imitation. If the expression in square brackets in equation (8) is positive, then after imitation profits instead of losses are achieved. If it is still negative, since $p' > p$, the demand faced after imitation is smaller $(D(p')/m' \leq D(p') \leq D(p))$, thus also $AC(D(p')/m') \leq AC(D(p))$; that is, less is sold to a lower loss per unit which results in lower total losses. In any case, profits of firm 1 after imitation will increase.

Last, if $\pi(p, m) = 0$, then $R(x(\mathbf{p}^{(m)})) = \{p_1, \ldots, p_n\} = C(\mathbf{p})$, since all firms active or idle make zero profits. That is, at these strategy profiles, imitate the best prescribes to imitate any of the observed prices. Again we distinguish the cases $m > 1$ and $m = 1$. Suppose $m > 1$, then for any $i = 1, \ldots, m$ imitate the best will not change profits, and for any $i = m + 1, \ldots, n$ that imitates p the new profits will be as in equation (7) positive. Suppose $m = 1$, then for all $i = m + 1, \ldots, n$ everything is analogous to the case $m > 1$, and for firm 1 everything is analogous to the case $\pi(p, 1) < 0$ considered above. □

Remark 1. It is straightforward to check that the proof of Proposition 4 extends to the case of constant unit costs; i. e. if $AC(q) = c \geq 0$ for all q.

4.3 Evolutionarily stable prices

It follows from Propositions 3 and 4 that all ESS prices must correspond to Nash equilibrium. Thus the set of ESS must be a subset of $[P_n, P'_n]$. The next proposition identifies the set of prices that are evolutionarily stable.

Proposition 5. *In a symmetric Bertrand oligopoly with decreasing demand $D(p)$, increasing and convex costs $C(q)$, and equal splitting in case of ties, the set of prices that are evolutionarily stable is the interval $[P_{n-1}, P_1]$.*

Proof. Recall that, by definition, at any P_k we have that $D(P_k) > 0$ and the following holds:

$$\pi(P_k, k) = \frac{D(P_k)}{k}\left[P_k - AC\left(\frac{D(P_k)}{k}\right)\right] = 0$$

Note first that, starting at P_k if all k firms increase their price to $p > P_k$ simultaneously, demand per firm will decrease and, by decreasing returns to scale, unit costs will decrease with $AC\left(\frac{D(p)}{k}\right) < AC\left(\frac{D(P_k)}{k}\right)$. At the new price, profits per unit sold are now strictly positive. Since demand is positive by assumption, profits will increase above zero. Analogously, if all firms simultaneously decrease their price, profits will fall below zero. I. e. for $0 < p < P_k$ we have $\pi(p, k) < 0$ and for $P_k < p < \overline{p}$ we have $\pi(p, k) > 0$.

Now, for all strategy profiles $\mathbf{p} = (p, \ldots, p)$ where all firms set $p \in [P_n, P_{n-1})$, if a firm deviates upwards to any $p' > p$ the profits of the deviator are zero because consumers buy only at the minimum price p. Given that $p < P_{n-1}$ the remaining $n-1$ firms still charging p will make losses; i. e. $\pi(p, n-1) < 0$. This means that a single deviation to a higher price can destabilize the profile where all firms set price p, implying that p is not an ESS.

Note also that for all strategy profiles $\mathbf{p} = (p, \ldots, p)$ where all firms set $p \in (P_1, P'_n]$, if a firm deviates downwards to any $P_1 < p' < p$, the deviating firm, now the one with lowest price in the market, still makes profits because $p' > P_1$, while all competitors make zero profits; i. e. $\pi(p', 1) > 0$. This means that a single deviation to a lower price can destabilize p, and thus p is not an ESS.

It remains to check that all prices $p \in [P_{n-1}, P_1]$ are ESS. Consider any symmetric profile $\mathbf{p} = (p, \ldots, p)$ with $p \in [P_{n-1}, P_1]$. Note that any deviant setting price $p' > p$ earns zero profits while non-deviants get $\pi(p, n-1) \geq 0$ since $p \geq P_{n-1}$. Alternatively, any deviant with price $p' < p$ obtains $\pi(p', 1) < 0$ since $p' < P_1$, while non-deviants earn zero profits. □

Proposition 5 shows that only a strict subset of Nash equilibrium prices are evolutionarily stable. In particular neither very low prices like P_n not very high prices in that interval, like P'_n, are robust to single deviations. If the market price is lower than P_{n-1}, the profit margin is so small that any firm that would experiment with a higher price would leave competitors with losses. If the market price is higher than P_1, a monopoly would be profitable at that and slightly lower prices, thus any firm that would experiment with a slightly lower price would make profits and leave competitors with zero profits. If relative performance is relevant to firms we can rule out some of the Nash equilibria; in particular those equilibria with zero or low profits. In a dynamic setting with firms that imitate the price of the best-performing firms in the industry and sporadically experiment with unobserved prices, Alós-Ferrer et al. (2000) show that precisely the prices in the interval $[P_{n-1}, P_1]$ are the long-run prices observed in a Bertrand oligopoly with homogeneous product and decreasing returns to scale. As we see here, the fact that all prices in the interval $[P_{n-1}, P_1]$ are ESS underlies their dynamics results, since none of these prices can be destabilized with a single mutation.

the fact that these prices are evolutionarily stable is crucial for their result, since single deviations to other prices are not enough to destabilize profiles where all firms set the same price in that interval.

Example 3. Consider a symmetric duopoly with demand function $D(p) = 10 - p$ and cost function $C(q) = \frac{1}{2}q^2$. Any profile where both firms set the same price in $[P_2, P_2']$ with $P_2 = 2$ and $P_2' = 4.285$ is a Nash equilibrium. The only price which is evolutionarily stable is $P_1 = 3.33$, which entails strictly positive profits for both firms and turns out to be the competitive equilibrium also.

Remark 2. Note that if unit costs are constant, i. e. if $AC(q) = c \geq 0$ for all q, then $P_k = c$ for all k. Thus, the only ESS is $p = c$.

4.4 Industry profits

The results obtained so far focus on profits to individual firms. We turn our focus now to industry profits. We show that when all firms follow imitate the best simultaneously, average payoff in the industry does not necessarily increase, although individual payoff always increases. Interestingly, we will argue at the end of the section that some imitative rules could even have both properties, always improving individual and average industry payoff.

Following the notation introduced previously in this section, at any strategy profile $\mathbf{p}^{(m)}$, average industry profits are given by

$$\overline{\pi}(\mathbf{p}^{(m)}) = \frac{m \cdot \pi(p, m)}{n} = \frac{D(p)}{n}\left[p - AC\left(\frac{D(p)}{m}\right)\right] \tag{9}$$

Note first that, if $\pi(p, m) > 0$, then $R(x(\mathbf{p}^{(m)})) = \{p\}$. Thus, if all firms follow imitate the best simultaneously, all will set price p. Since $AC(\cdot)$ is increasing we have

$$\overline{\pi}(\mathbf{p}^{(n)}) - \overline{\pi}(\mathbf{p}^{(m)}) = \frac{D(p)}{n}\left[AC\left(\frac{D(p)}{m}\right) - AC\left(\frac{D(p)}{n}\right)\right] \geq 0 \tag{10}$$

That is, from any strategy profile where all active firms in the industry obtain strictly positive profits, imitate the best will increase average payoff in the industry.

To see that average industry payoff may decrease when all firms imitate the prices of the best performing firms consider the following example. Take $I = \{1, \dots, 4\}$ and consider the strategy profile $\mathbf{p}^{(2)} = (p, p, p', p'')$ with $p < p' = p + \epsilon < p''$ for small $\epsilon > 0$. Assume the demand and cost functions are such that[8]

$$\pi(p, 2) = \frac{D(p)}{2} \left[p - AC \left(\frac{D(p)}{2} \right) \right] < 0; \qquad i \text{ such that } p_i = p$$

Then $\overline{\pi}(\mathbf{p}^{(2)}) = \frac{1}{2}\pi(p, 2) < 0$

Now it could happen that firms with maximum profits (choosing p' and p'' in our example) do not change their price and firms with lower than maximum profits (currently choosing p) copy the highest price observed, p''. The resulting strategy profile after imitation would be of the type $\mathbf{p}'^{(1)} = (p', p'', p'', p'')$. After imitation, the profit of the firm with price p' will be given by

$$\pi(p', 1) = D(p + \epsilon) \left[p + \epsilon - AC \left(D(p + \epsilon) \right) \right]$$

For $\epsilon > 0$ small enough,

$$\overline{\pi}(\mathbf{p}'^{(1)}) - \overline{\pi}(\mathbf{p}^{(2)}) = \frac{1}{4}\pi(p', 1) - \frac{1}{2}\pi(p, 2) < 0.$$

That is, in an industry where the current active firms make losses, copying the price set by firms which are not active, which can be interpreted as exiting the industry, leaves the industry with fewer active firms. Decreasing returns to scale imply that profits can be even lower, and thus average industry profits may decrease.

Consider instead a further specification of the imitative rule used so far, assuming that firms copy only the lowest price among those that gave maximum payoffs. This rule is plausible if we believe that firms understand at least the broad structure of the game they are playing, even if they do not know the demand and cost functions exactly. We refer to this rule as *imitate the minimum best* and it is formally defined as follows. Given $x = (\mathbf{p}, \mathbf{u})$, $F(x) = \min R(x)$. As the following proposition shows, this rule has both properties; it increases individual as well as average industry profits.

Proposition 6. *In a symmetric Bertrand game with decreasing demand $D(p)$, increasing and convex costs $C(q)$, and equal splitting in case of ties,* imitate the minimum best *is improving; i. e.* $\min R(x(\mathbf{p})) \subseteq B_i(\mathbf{p})$ *for all* $\mathbf{p} \in \mathbb{R}^n$ *and all* $i \in I$ *with* $x(\mathbf{p}) = (\mathbf{p}, \pi(\mathbf{p}))$. *Moreover, when all firms use imitate the minimum best simultaneously, average industry profits always increase.*

[8] Take for example $D(p) = 12 - 2p$, $C(q) = q^2$, and $\mathbf{p}^{(2)} = (2, 2, 2.5, 3)$.

Proof. The first part of the proposition is a straightforward corollary from Proposition 4. To see that, when all firms follow the rule, it also increases average industry payoffs define let $\mathbf{p}^{(m)}$ be a state of the type defined above, where $1 \leq m < n$ firms set price p and all other firms set higher prices. Note first that, at states where all active firms have strictly positive profits $\min R(x(\mathbf{p})) = R(x(\mathbf{p})) = \{p\}$, and at states where active firms obtain exactly zero profits $R(x(\mathbf{p})) = \{p_1, \ldots, p_n\}$ and $\min R(x(\mathbf{p})) = \{p\}$ also. By (10) in all those states average payoff increases when all firms copy p. At states where active firms make losses, the reference set is given by $R(x(\mathbf{p}^{(m)})) = \{p_{m+1}, \ldots, p_n\}$. Let $p' = \min\{p_{m+1}, \ldots, p_n\}$. If all firms copy p', the industry moves to a state of the form $\mathbf{p}^{(n)} = (p', \overset{n}{\ldots}, p')$. By (9),

$$\overline{\pi}(\mathbf{p'}^{(n)}) - \overline{\pi}(\mathbf{p}^{(m)}) = \frac{D(p')}{n}\left[p' - AC\left(\frac{D(p')}{n}\right)\right] - \frac{D(p)}{n}\left[p - AC\left(\frac{D(p)}{m}\right)\right] >$$
$$> \frac{D(p')}{n}\left[(p' - p) + AC\left(\frac{D(p)}{m}\right) - AC\left(\frac{D(p')}{n}\right)\right] > 0$$

The first inequality holds because $D(p') < D(p)$ and $\overline{\pi}(\mathbf{p}^{(m)}) < 0$. The second inequality holds because $p' > p$ and AC is increasing. □

5 Conclusions

Evolutionary game theory emphasizes the role of relative performance in determining the outcomes that we should expect to observe in games. In the framework of a continuum population, developed and extensively applied in Biology, the notion of evolutionary stability provides a stability check for Nash equilibrium. The application of evolutionary principles to finite-population models has shown some surprising effects. In particular, high relative performance is often in accordance with perfectly competitive behavior, and not necessarily with the strategic behavior inherent to Nash equilibrium.

The present paper explores the relation between finite-population evolutionary stability and Nash equilibrium. First, we show that ESS and symmetric Nash equilibrium strategies coincide in zero-sum games and in what we call games with weak payoff externalities. Then we point out that in some games (e. g. minimum-effort games) imitate the best is improving, meaning that individuals who copy the best-performing among the observed actions can only improve their payoffs. If that is the case, imitate the best is strategically justified and gives rise to a dynamics related to the better-reply dynamics. We then show that in these kind of games finite-population evolutionarily stable strategies always correspond to Nash equilibrium strategies, but not vice-versa.

We illustrate these properties in the context of a Bertrand oligopoly with homogeneous product and decreasing returns to scale. There, we show that price imitation is always individually improving; a firm that mimics the price of the best performing competitor can only improve its profits. This, in turn, implies that all evolutionarily stable prices are Nash equilibrium prices. This result allows easy identification of the set of evolutionarily stable prices. Moreover, we find that not all Nash equilibria are evolutionarily stable. Sufficiently low prices, that can be part of a Nash equilibrium, can easily lead to losses, if a boundedly rational firm experiments with a higher price. Analogously, sufficiently high Nash equilibrium prices leave room for an individual firm to dominate the market at a slightly lower price. Even if these deviations imply a reduction of the deviator's profits by definition, they have an even stronger negative effect on competitors and result in a relative advantage.

Note finally that the evolutionarily stable prices characterized here would also be the prediction of a model where firms care for both, absolute and relative performance, as in Koçkesen et al. (2000). Given that evolutionarily stable strategies are Nash equilibria, any deviation would be associated with a worse absolute and relative position. Therefore, this set of prices is extremely robust.

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