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# Endogenous Public Policy and Long-Run Growth: Some Simple Analytics

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# Endogenous Public Policy and Long-Run Growth: Some Simple Analytics

**Abstract**

We study the determinants of voting outcomes on the provision of public consumption through marginal income taxes in the context of the simple linear growth model. We provide analytical results on how the dynamic politicoeconomic equilibrium maps the economic fundamentals to policies and long-run growth. We find that in a deterministic growth environment voters internalize, although imperfectly, the deadweight losses of taxation and vote for lower taxes when the productivity of capital is higher. Therefore, the politicoeconomic channel reinforces the positive role of productivity for growth. In a stochastic linear-growth environment where business cycles are driven by productivity shocks, in line with existing evidence, we find that the level of endogenous public consumption is procyclical but its share of GDP is countercyclical.

*Keywords*: voting, second-best taxation, endogenous growth

*JEL classification*: C73, D72, E61, E62, O23

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## 1. Introduction

The link between public policy and growth has attracted a great deal of attention not only by economists and policy makers, but is also a matter of debate among the general public. On the one hand, the opportunity to achieve a high level of long-run growth rests on the ability of the economy to implement an optimal investment strategy, through the use of efficient capital markets. On the other hand, fiscal policies financed by means such as marginal income taxes, directly reduce the incentives for economic agents to follow growth-enhancing investment strategies. With marginal income taxes, markets are unable to yield the investment return that reflects the technological potential of the economy, since capital markets are distorted.

Marginal taxes differ vastly across countries.[1] If one views fiscal policies implemented by governments as exogenous, unrelated to the economic structure, there is a direct negative causality from marginal income taxes to growth. Several studies have examined the growth and welfare effects of such exogenous policy streams using calibrated or simulated models, predicting a very strong (negative) quantitative link.[2] On the other hand, empirical studies, using regression analyses, are unable to confirm the causality implied by the theory that treats policies as exogenous. Many conclusions from such empirical studies are conflicting.[3] This gap, between theory and evidence in the existing literature, does not allow an evaluation

[1] See, for example, Easterly and Rebelo (1993).

[2] See, for example, Lucas (1990), Stokey and Rebelo (1995), Mendoza et. al. (1997), Ortigueira (1998), Kneller et. al. (1999), and Rivas (2003). This approach to exogenous fiscal policies examines the impact of taxation on economic incentives and also how primitives may influence the sensitivity of economic activity to distortions.

[3] For example, Easterly and Rebelo (1993), and Mendoza et. al. (1997), have not given compelling evidence of a strong negative link between marginal income taxes and growth, at least across large samples of countries. Yet, Hall and Jones (1999) conclude that differences in institutions and government policies are behind the large cross-country differences in growth rates, productivity and per capita income. Moreover, Fölster and Henrekson (2001), whose study focuses on rich countries, exploit within-country variation over time, as opposed to cross-country variation, and report a negative link between government spending and growth.

of the contribution of, (i) economic primitives, such as productivity, and, (ii) levels of policy instruments, such as marginal taxes, to growth performance. Moreover, the question, "why are low-growth policies supported in many countries?" is left open.

The answer to why we observe vast differences in marginal taxes across countries must have both a political and economic background, and, especially, policies must be linked, somehow, to economic fundamentals. A line of more recent literature attempts to endogenize public policies, so that the impact of economic primitives and policies is studied through a concept of politicoeconomic equilibrium.[4] By endogenizing government policies, linking them to the economic fundamentals, one can shed light on the link between policies and growth. For example, the empirical conclusion of a weak link between marginal taxes and growth may be that taxes partly finance productive capital infrastructure and economic agents allow for higher taxes whenever they can observe such an opportunity to increase the return to private investment through more provision of infrastructure.[5] But, since in most countries the highest share of the government budget is spent on non-productive public consumption, the provision of public consumption is a core issue to explore, which is the focus of this paper.

A setup with forward-looking agents is essential to studying the relationship between public policies and capital markets. Yet, dynamic analysis with forward-looking economic agents and voters is very complex. So, we develop a parametric model that is able to explain the key mechanism of endogenous policy-making in dynamic environments using simple analytics. Moreover, we analyze the issue of the cyclical behavior of taxes and policies, an extension that is neglected in the literature.

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[4] See, for example, Krusell et al. (1997) and Krusell and Rios-Rull (1999), two papers that study voting in dynamic economies using numerical simulations.

[5] We examine the link between economic fundamentals and the provision of public infrastructure in Koulovatianos and Mirman (2004).

We provide a theory of public consumption goods provision in the simplest endogenous growth environment, the "linear growth" model without externalities in production. This class of linear growth models was proposed by Rebelo (1991) in order to study the link between exogenous policies, investment and growth in the long run. Without externalities in production, marginal taxation reduces the returns to accumulable capital. This reduction in the after-tax return on capital makes households act as if the available technological possibilities were shrunk in proportion to the level of the marginal tax rate. Thus, marginal taxes in the framework without externalities in production, always carry the competitive economy away from both production efficiency and Pareto efficiency.[6]

In our model, a public consumption good enters the utility function of households. Each household operates both as economic agent and voter. The public good can be financed only through marginal income taxes and under balanced fiscal budgets. The public-consumption externality generates a political demand for positive taxes that finance the provision of the public good. Hence, the occurence of a deadweight loss is inevitable, as marginal taxes generate disincentives for investment. But voters have the ability, depending on their economic fundamentals, to partly internalize the deadweight losses of taxation. Thus, we examine how the dynamic politicoeconomic equilibrium maps the economic fundamentals to policies and long-run growth.

We first study a deterministic growth environment. We find that in economies where the productivity of capital is higher, voters choose lower taxes, because they (optimally) select a lower ratio of current public goods to future private consumption. This happens because, in equilibrium, marginal income taxes create a strong substitution effect, driven by the fact

[6] On the contrary, in a linear growth model with externalities in production, as this of Romer (1986), the production possibility frontier is not pre-determined, but depends on private investment choices. A fiscal policy transfering resources from consumption to investment may lead to both higher production efficiency and to a Pareto improvement.

that it is not the stock of capital that is taxed over time, but only the capital income flow. Because of this voting behavior, the politicoeconomic channel reinforces the positive role of productivity for growth. Higher productivity gives voters the opportunity to choose lower taxes and to achieve higher growth. We provide the full intuition behind this result using a graphical approach.

Our findings are in accordance with the conclusions of Hall and Jones (1999) for the link between government policies, growth and long-run differences in cross country per-capita incomes. Yet, our results do not suggest that reducing marginal income taxes exogenously leads to necessarily large improvements in growth. It is, after all, institutions that create the ability for private markets to compete with the provisions of public goods and to sustain optimal growth policies. Democracy and the pure ability of private markets to operate and compete with the public-goods provision mechanism are the ingredients of our model. Since in our model endogenous policies amplify growth differences that stem from economic fundamentals, such as preference and technology parameters, we provide an explanation for the large cross-country development differences.

We go further, by examining public policies in a stochastic environment. We find that if business cycles are driven by productivity shocks in the endogenous growth framework, equilibrium policies imply that taxes should fall in high growth periods and rise in low growth periods. Exploiting growth opportunities further by decreasing taxes in periods of booms goes against the usual idea of stabilizing countercyclical policies. Moreover, the public consumption share of GDP is countercyclical in our model, also in accordance with evidence for OECD countries (only for the public consumption component of the fiscal budget as a share of GDP) reported by Hercowitz and Strawczynski (2002).

At the same time, we find that the level of public consumption is procyclical. Several empirical studies find evidence for procyclical public expenditures in general, in Latin American countries (see, for example, Gavin et. al. (1996), Gavin and Perotti (1997), Stein et. al. (1999)), and also in other economies, including many OECD countries (see Talvi and Vegh (2000) and Lane (2003)).

We stress, however, that our study is not a public-finance exercise. The policy problem posed is not the choice of public-finance instruments over time for an exogenously given stream of public goods.[7] The problem we study is the voters' choice of the public goods level over time, given that the policy instrument is a marginal income tax satisfying a balanced fiscal budget. An interesting finding of Lane (2003) is that countries with more dispersed political power (a proxy for strong democratic institutions), are more likely to exhibit procyclical fiscal policies. This is in accordance with our model, which expresses a strong democratic represantation of voters' political preferences in policy making.

Related studies focusing, as we do, on time-consistent policies and introducing an externality in the form of a public consumption good, also appearing in additively-separable utility functions, are Xie (1997) and Karp and Ho Lee (2003). Xie (1997) discusses the technical issue of when optimal policies with commitment at time 0 (open-loop policies) may be time-consistent. Karp and Ho Lee (2003) examine the public finance question of setting optimal marginal taxes for a given exogenous stream of public goods, showing the way of constructing models exhibiting time-consistent open-loop policies.

In contrast to these studies, in the context of our model, all optimal and equilibrium policies are time consistent. Using our framework, we focus on the economic background of the linkages between fiscal policies and growth. In our model, governments are elected, or

[7] For example, Klein and Rios-Rull (2003) study the cyclical properties of flat marginal taxes on capital and labor in a calibrated version of a neoclassical growth model, in the case where public goods are exogenous (and they can also be random) and in the presence of productivity shocks. Unlike the topic of our study, Klein and Rios-Rull (2003) focus on the public-finance question: how to finance exogenous public spending using balanced fiscal budgets and flat marginal tax rates.

supported, by voters who are aware of the potential negative impact of taxes. These voters internalize the deadweight losses of taxation and electoral process in the most fundamental way, that is, they make perfect-foresight voting decisions.

The concept of perfect foresight voting was first proposed by Denzau and Mackay (1981) and was further articulated by Epple and Kadane (1990) in a spatial model. Simulation-based studies with perfect-foresight voting models are Krusell et al. (1997) and Krusell and Rios-Rull (1999). Moreover, Klein et al. (2003) suggest a generalized method for simulating models of time-consistent public-goods provision with perfect foresight.

As all the above literatures are quite specialized and rather technical, our study aims at pointing out intuitive results and raising null hypotheses. The observations we make in this paper capture key mechanisms through which fiscal policies depend on the fundamentals of economic environments that exhibit endogenous sustainable growth, especially the link between productivity and flat income taxes.

The plan of the paper is as follows. In section 2 we present the deterministic economy, its politico-economic equilibrium and the link between economic fundamentals and growth paths of economies with endogenous policies. In section 3 we extend our model to having stochastic productivity and we comment on the cyclical behavior of taxes.

## 2. The Deterministic Economy

The benchmark of our analysis is a deterministic economic environment in which a reproducible and partially storable capital good is accumulated. This capital good may be a composite of various types of physical and/or human capital. For the purpose of tractability, we focus on the dynamics of this single capital variable.

Time is discrete and the time horizon is infinite, $t = 0, 1, ....$ Households comprise a set

$\mathcal{I}$ and may differ only with respect to their initial endowment of capital claims (assets). We denote assets for household $i \in \mathcal{I}$ at time 0 as $a_{i,0}$.[8] At any point in time, the aggregate capital level is given by,

$$\mathbf{k}_t = \int_{\mathcal{I}} a_{i,t} \mu_t (i) \, di \ , \quad t = 0, 1, ... \tag{1}$$

where $\mu_t (i)$ is the measure of individuals of type $i$ at time $t$. We denote all aggregate variables by bold characters in order to distinguish them from variables pertaining to individual agents. We also assume that $\mathbf{k}_0 > 0$. All households are infinitely-lived. There is a single private consumable good produced through a linear aggregate production function,

$$\mathbf{y}_t = A\mathbf{k}_t \ , \tag{2}$$

with $A > 0$. The marginal return to capital is,

$$R_t = A \quad \text{and} \quad r_t = A - \delta \ , \tag{3}$$

where $\delta$ is the depreciation rate of capital, and $r_t$ is the interest rate, which is constant over time.

A government, elected by the households each period, collects taxes in order to provide a single composite public good (parks, hospitals, theaters, art schools, defense, etc.). Nothing is returned to the individuals in the form of a direct monetary transfer. The constitution allows only one type of taxation, the use of a common flat marginal tax on personal income, denoted by $\tau_t$, for period t. Moreover, the fiscal budget should be balanced every period, so the government revenues (and expenditures) are given by,

$$\mathbf{g}_t = \tau_t (A - \delta) \mathbf{k}_t \ , \tag{4}$$

[8] The set of households can be countable, finite, or a continuum. It can also be that all households have the same initial endowment (representative-agent economy), but in all cases there is a "large" number of households, making each of them having a negligible impact on the aggregate economy.

i.e., capital depreciation is tax-exempt. In order to obtain closed-form solutions, we assume log preferences. In particular, the momentary utility function for all households is,

$$u(c_t, \mathbf{g}_t) = \chi \ln(c_t) + (1-\chi)\ln(\mathbf{g}_t) \ , \tag{5}$$

with $\chi \in (0,1]$.[9] The assumption of additive separability and homotheticity of preferences over private and public consumption is standard. Amano and Wirjanto (1998) estimate a utility function of the average American household from aggregate data with constant intertemporal and intratemporal elasticity of substitution. Their regressions are based on Euler equations that stem from a permanent-income model with lump-sum taxes.[10] They find that private and public consumption are unrelated in the Edgeworth-Pareto sense (they are neither substitutes nor complements). They also find that the elasticities of intertemporal and intratemporal substitution are both about 1.56. The latter means that the assumption of natural-log preferences (unitary elasticities of intertemporal and intratemporal substitution) is close to their findings.

## 2.1 Competitive equilibrium under any stream of political outcomes

We assume, for the moment, that a stream of taxes and aggregate capital, $\{(\tau_t, \mathbf{k}_t)\}_{t=0}^{\infty}$, is pre-determined exogenously. Chatterjee (1994) proves that, for a general class of neoclassical growth setups, households need to know *only* the future stream of aggregate capital levels and not the future distributions of physical capital claims in order to calculate their optimal path of savings accurately.[11] The argument of Chatterjee (1994) is valid for the linear-growth

[9] Superscripts $i$ are dropped throughout the text unless necessary.

[10] Even though the permanent-income Euler equations of Amano and Wirjanto (1998) differ from the politico-economic equilibrium conditions of our model, their findings should imply similar estimates for the intra- and inter-temporal elasticities of substitution with these implied by a model distorted by marginal taxes.

[11] Using Chatterjee's (1994) argument, Krusell and Rios-Rull (1999) prove the same result in an economy with proportional taxation and lump-sum transfers. Interestingly, Krusell and Smith (1997) find that in

model that we examine. Although it is not necessary that there be a *representative agent* in our framework (i.e., all households possess the same initial wealth), due to the fact that preferences are of the "Gorman form," implying linear demand aggregation, our setup leads to the presence of a *representative consumer*.[12]

Consider the problem of an individual household $i$.

**Household Problem**

$$\max_{\{(c_t, a_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ \chi \ln(c_t) + (1-\chi) \ln(\mathbf{g}_t) \right] ,$$

subject to:

$$a_{t+1} = \left[1 + r_t (1 - \tau_t)\right] a_t - c_t \ , \tag{6}$$

$$\text{(3) and (4),}$$

$$\text{given } a_0, \ \ \{(\tau_t, \mathbf{k}_t)\}_{t=0}^{\infty} \ ,$$

here $\beta \in (0,1)$. In equilibrium, $\{\mathbf{k}_t\}_{t=0}^{\infty}$ consistent with the household's solution to the maximization problem as well as to market clearing conditions. Since individuals have negligible economic weight, they assume that their decisions do not have any impact on aggregate variables and prices. For this reason, the optimality conditions of a household $i$ are given by,

$$\frac{c_{t+1}}{c_t} = \beta \left[1 + (A - \delta)(1 - \tau_{t+1})\right] \ , \tag{7}$$

equation (6), and the transversality condition $\lim_{t \to \infty} \frac{a_{t+1}}{\prod_{s=0}^{t} [1 + (A-\delta)(1-\tau_s)]} = 0$. We obtain a closed-form solution for this problem, which is given in Proposition 1.

neoclassical models modified in ways such that Chatterjee's (1994) theoretical argument fails, at a numerical level, households calculating distribution moments that capture future individual asset distributions, can very approximate their optimal path very well by relying only on the future stream of first moments, namely on the future sequence of aggregate capital. Similar results to Chatterjee (1994) are pointed out by Caselli and Ventura (2000) in more general continuous-time frameworks.

[12]A representative consumer is a consumer whose demand functions coincide with the corresponding aggregate demand functions. Thus, Chatterjee (1994) proves the existence of the representative consumer for dynamic decision rules and aggregate laws of motion in a large class of optimal growth economies.

**Proposition 1** *The competitive equilibrium decision rules for the* ***Household Problem*** *of any individual household* $i$*, are given by,*

$$a_{t+1} = \beta \left[1 + (A - \delta)(1 - \tau_t)\right] a_t \ , \tag{8}$$

*and,*

$$c_t = (1 - \beta) \left[1 + (A - \delta)(1 - \tau_t)\right] a_t \ . \tag{9}$$

*The aggregate-economy law of motion for capital is,*

$$\mathbf{k}_{t+1} = \beta \left[1 + (A - \delta)(1 - \tau_t)\right] \mathbf{k}_t \ . \tag{10}$$

**Proof.** See the Appendix.

Note that the assets of all households grow at the same rate in equilibrium. Thus, as is the case for the class of models with homothetic preferences in Chatterjee (1994), the *relative household wealth distribution* is invariant over time.

## 2.2 Politicoeconomic equilibrium

Since the constitution restricts governments to having a balanced budget, the only voting issue each period is the current level of the income tax rate. Majority voting over taxes takes place at the beginning of every period, i.e. *before* households and producers make their economic decisions.

Households vote simultaneously, before they act, again simultaneously, in the markets in order to make their economic decisions given the current period's electoral outcome. So, while each household decides, independently, about its best voting strategy, it takes account of the full impact of its voting action on the current electoral outcome and how this electoral outcome affects its own current and future economic decisions, its own future voting decisions, and the current and future economic and voting decisions of other households in politicoeconomic equilibrium over the infinite horizon.

The timing of actions mimics the fact that, in most presidential and parliamentary democracies, in the beginning of each period, a government pre-announces a certain fiscal budget and commits to it. This fits the specification of Cohen and Michel (1988), about the time-consistent making of fiscal policies.[13] Our politicoeconomic equilibrium concept can be linked observationally to the real world in two ways. Either, (i) elections are held every four years (so, the duration of one period in our model is four years) and elected governments commit to their pre-announced policies for four years, or, (ii) independently from whether there are elections or not, voter preferences are reflected into the pre-announced fiscal budget in the beginning of each fiscal year.

The solutions given by equations (8), (9) and (10) enable us to calculate the analytical form of the value function for household $i \in \mathcal{I}$. The value function at time $t \in \{0, 1, ...\}$ is given by,

$$V\left(a_{i,t}, \mathbf{k}_t, \{\tau_s\}_{s=t}^{\infty}\right) = \kappa_D + \chi \frac{\ln\left(a_{i,t}\right)}{1-\beta} + (1-\chi)\frac{\ln\left(\mathbf{k}_t\right)}{1-\beta} + $$
$$+ \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\ln\left[1 + (A-\delta)\left(1-\tau_{t+s}\right)\right]}{1-\beta} + (1-\chi)\ln\left(\tau_{t+s}\right) \right\}, \qquad (11)$$

where $\kappa_D$ is a constant.[14] The optimal tax rate of a household at time $t$ is derived from setting the partial derivative of the value function with respect to $\tau_t$ equal to zero, yielding,

$$\tau_t = \tau^D = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left(1 + \frac{1}{A-\delta}\right), \quad t = 0, 1, .... \qquad (12)$$

Note that all households agree upon $\tau^D$ at all times.[15]

### 2.2.1 Endogenous Unanimity

Unanimity across households comes from the fact that the term $(1-\chi)\frac{\ln(a_{i,t})}{1-\beta}$ in the value function of household $i$ and the current and future policies $\{\tau_s\}_{s=t}^{\infty}$ are additively separable.

[13] In particular, Cohen and Michel (1988), in the context of their continuous-time framework, call this equilibrium concept "feedback Stackelberg equilibrium with instantaneous precommitment."

[14] In particular,

$$\kappa_D = \chi \frac{\ln(1-\beta)}{1-\beta} + \frac{\beta \ln(\beta)}{(1-\beta)^2} + (1-\chi)\frac{\ln(A-\delta)}{1-\beta}.$$

[15] The symbol "$\tau^D$" represents the winning tax rate in the deterministic economy.

Two important remarks should be made about this endogenous unanimity.

First, unlike models with redistributive policies, for public-goods provision it is not necessary that less wealthy voters 'free ride' on the wealth of richer voters. In particular, in our model marginal taxes distort capital markets and all indiduals are affected in the same way. Moreover, in our model, the revenues from higher taxation do not benefit less wealthy voters in the form of lump-sum tranfers, as it is, for example, in Meltzer and Richard (1981), Krusell et al. (1997) and Krusell and Rios-Rull (1999).

Second, in the case of public-goods provision the link between the ordering of wealth and the ordering of preferences over the public good can be positive, depending on the utility function. This is easy to see in a static version of our model, for which log preferences also imply unanimity, whereas if the elasticity of substitution between private and public goods is less than one (log implies that this elasticity is one), wealthier agents prefer higher taxes.[16]

The way preferences affect the politicoeconomic equilibrium in a dynamic framework, especially the impact of the distribution of wealth on the political outcome, is beyond the

[16]For example, consider a static version of our model where the preferences of individual $i \in \mathcal{I}$ are

$$u(c_i, \mathbf{g}) = \chi \frac{c_i^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} + (1-\chi) \frac{\mathbf{g}^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} \ .$$

It is straightforward that the voting preferences of $i \in \mathcal{I}$ will be,

$$v^i(\tau) = \chi \frac{\left[(1-\tau) A a_i\right]^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} + (1-\chi) \frac{\left[\tau A \mathbf{k}\right]^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} \ ,$$

with a best-preferred tax rate for $i \in \mathcal{I}$ given by,

$$\tau_i = \frac{1}{1 + \left(\frac{\chi}{1-\chi}\right)^{\eta} \left(\frac{a_i}{\mathbf{k}}\right)^{\eta - 1}} \ ,$$

so,

$$\frac{\partial \tau_i}{\partial a_i} \lesseqqgtr 0 \Leftrightarrow \eta \gtreqqless 1 \ .$$

In our case, $\eta = 1$, which leads to unanimity.

scope of our analysis. Our analysis sheds light on the importance of productivity differences for growth through a simple analytical perspective of endogenous policy making.

Unanimity also leads to the conclusion that the tax rate, $\tau^D$, given by (12), coincides with the solution of a benevolent social planner who chooses period-by-period second-best policies, given that the constitution is restricted to the use of marginal taxes and a balanced fiscal budget in each period. The social planner's objective at time $t \in \{0, 1, ...\}$ is given by $\int_{\mathcal{I}} V\left(a_{i,t}, \mathbf{k}_t, \{\tau_s\}_{s=t}^{\infty}\right) \omega(i) \mu_t(i)\, di$, for any set of weights $\omega(i) \geq 0$ on household $i$'s utility. Note that equation (11) implies that $\tau^D$ is the second-best policy for the social planner as well.

## 2.3 Economic fundamentals, public policy and long-run growth

Since the equilibrium tax rate in the deterministic political economy is constant, the equilibrium growth rate is also constant.[17] In particular, from equation (8), after substituting the tax rate (12), the growth rate of all economic variables in this economy is,

$$\gamma^D = \frac{\beta(1 + A - \delta)}{1 + (1 - \beta)(1 - \chi)} - 1 . \tag{13}$$

The endogenous tax rate given by equation (12) has many intuitive features. If the relative weight of utility derived from the consumption of public goods is higher (lower $\chi$), then $\tau^D$ is higher, and therefore growth is lower. More patient households (higher $\beta$) choose a lower ratio of current public goods to future private consumption (by choosing a lower $\tau^D$), because they can compensate for relatively less public goods today, by achieving a higher growth rate. In the case of higher $\beta$, this happens because households are more willing to save. A higher capital depreciation rate, $\delta$, leads to a lower interest rate and less incentives

[17] So, from period 0 the political economy is in a steady state, hence there are no transitional dynamics leading to it.

for saving, so voters choose a higher $\tau^D$ in order to derive more utility from a higher public consumption today, a choice leading to lower growth.

One of the most important economic fundamentals for explaining cross-country differences in living standards and growth rates is cross-country productivity differences. Hall and Jones (1999) stress that physical and human capital accumulation alone are inadequate to explain the biggest part of cross-country differences in per-capita income. They find that large productivity differences across countries explain most of cross-country differences in development.

In the context of our endogenous growth model, if we treat capital as a composite good, consisting of physical and human capital, differences in the productivity parameter, $A$, are crucial for growth because they also influence the level of marginal taxes. This can be seen by equation (12). When $A$ is higher, voters internalize the growth opportunities generated by a higher capital productivity, and vote for a lower $\tau^D$.

### 2.3.1 Higher productivity implies lower taxes

In our politicoeconomic equilibrium, marginal income taxes create a strong substitution effect. Namely, voters substitute current public consumption with future private consumption. We present the intuition behind this political choice in Figure 1.

Recall that the political choice is made subject to the economic decision rules that are conditional upon the potential current and future tax rates. Therefore, equations (4), (8) and (9) hold for all policy streams $\{\tau_s\}_{s=t}^{\infty}$ from period $t \in \{0, 1, ...\}$. Moreover, at time $t$ voters choose only the current tax rate, $\tau_t$, while future taxes are determined by future elections given the future level of state variables in the economy.

Given the nature of the decision rules (8) and (9), and also due to the additive separability of the value function with respect to the tax rates over time, c.f. (11), it suffices to present the savings choice between any two periods. For this reason, Figure 1 presents four graphs: (i) the space $(c_t, c_{t+1})$, denoted by "**I**," (ii) the space $(c_t, \mathbf{g}_t)$ indicated by "**II**," (iii) the space $(\tau_t, \mathbf{g}_t)$ denoted by "**III**," and, (iv) $(\tau_t, c_{t+1})$, captured by "**IV**."

We consider an economy with productivity $A$ with the optimal tax rate, $\tau_t^D$, given by (12). We include another economy with productivity level $\hat{A} > A$, and we show that if the high-productivity country follows the tax rate $\tau_t^D$ of the low-productivity country, this is a suboptimal voting choice: the high-productivity country must reduce taxes to the level $\hat{\tau}_t^D < \tau_t^D$ as this is given by (12) with $A$ replaced by $\hat{A}$.

The politicoeconomic equilibrium of the low-productivity country (with productivity $A$ and tax rates $\tau_t^D$ and $\tau_{t+1}^D$), is given by the points $\left(\varepsilon_1^I, \varepsilon_1^{II}, \varepsilon_1^{III}, \varepsilon_1^{IV}\right)$. The point $\varepsilon_1^I$ lies on a straight line that passes through the origin, with slope $\beta\left[1 + (A - \delta)\left(1 - \tau_{t+1}^D\right)\right]$, implied by equation (7). $\varepsilon_1^{II}$ also lies on a straight line that passes through the origin, with slope,

$$\frac{\mathbf{g}_t}{c_t} = \frac{\tau_t^D (A - \delta)\,\mathbf{k}_t}{(1 - \beta)\left[1 + (A - \delta)\left(1 - \tau_t^D\right)\right] a_t}\ , \tag{14}$$

which is the result of combining equations (4) and (9). Notice that the slope, given by (14), is increasing in both $A$ and $\tau_t^D$. Yet, substituting (12) into (14), yields,

$$\frac{\mathbf{g}_t}{c_t} = \frac{\tau_t^D (A - \delta)\,\mathbf{k}_t}{(1 - \beta)\left[1 + (A - \delta)\left(1 - \tau_t^D\right)\right] a_t} = (1 - \chi)\,\frac{\mathbf{k}_t}{a_t}\ . \tag{15}$$

Next, notice that in both spaces, **I** and **II**, we can draw indifference curves for the voter. The indifference curves passing through points $\varepsilon_1^I$ and $\varepsilon_1^{II}$, provide an accurate account of the utility level of the voter, despite the fact that the voter also derives utility from future periods. This is because the value function given by (11) implies that the continuation of the well being of the voter from period $t+1$ is increasing in $a_{t+1}$. It is easy to see the impact of changes in productivity and taxes on $a_{t+1}$, from equation (8). And conclusions on how $a_{t+1}$ influences the utility derived by the indifference curve in space I, are also sufficient for deriving conclusions about the voter's lifetime utility.

Point $\varepsilon_1^{III}$ lies on a line that reflects the fiscal budget given by (4). For the slope of the line on which point $\varepsilon_1^{IV}$ lies, denoted as $\omega\left(A, \tau_{t+1}^D\right)$, we have combined (8) and (9), to get,

$$c_{t+1} = \beta\left(1-\beta\right)\left[1+\left(A-\delta\right)\left(1-\tau_{t+1}^D\right)\right]\left[1+\left(A-\delta\right)\left(1-\tau_t\right)\right]a_t \ ,$$

so,

$$\omega\left(A, \tau_{t+1}^D\right) \equiv \frac{\partial c_{t+1}}{\partial \tau_t} = \beta\left(1-\beta\right)\left[1+\left(A-\delta\right)\left(1-\tau_{t+1}^D\right)\right]\left(A-\delta\right)a_t \ ,$$

where the current tax rate is a "free" parameter and the future tax rate is fixed at the level $\tau_{t+1}^D$, given by (12).

The equilibrium points $\left(\varepsilon_2^I, \varepsilon_2^{II}, \varepsilon_2^{III}, \varepsilon_2^{IV}\right)$ reveal the levels of well-being if the high-productivity country, with productivity $\hat{A} > A$, follows the policies $\left(\tau_t^D, \tau_{t+1}^D\right)$ of the low-productivity country, instead of its optimal policies $\left(\hat{\tau}_t^D, \hat{\tau}_{t+1}^D\right)$, as (12) would dictate if $A$ was replaced by $\hat{A}$. While it is still the case that the high-productivity country would enjoy a higher level of welfare, as indicated by the indifference curves that pass through the points $\varepsilon_2^I$ and $\varepsilon_2^{II}$, switching to another political choice, namely $\left(\hat{\tau}_t^D, \tau_{t+1}^D\right)$, with a lower current public consumption but higher current and future private consumption, as the equilibrium points $\left(\varepsilon_3^I, \varepsilon_3^{II}, \varepsilon_3^{III}, \varepsilon_3^{IV}\right)$ indicate, the high-productivity country can do better, and this is perceived by voters. Finally, the high-productivity country can do even better if it votes for a lower future tax rate, namely if it chooses $\left(\hat{\tau}_t^D, \hat{\tau}_{t+1}^D\right)$, the case that corresponds to $\left(\varepsilon_4^I, \varepsilon_4^{II}, \varepsilon_4^{III}, \varepsilon_4^{IV}\right)$. Lower taxes today, also lead to higher savings, i.e. to a higher $a_{t+1}$. And the value function given by (11) reveals that the continuation of the well being of the voter from period $t+1$ and on is increasing in $a_{t+1}$.

For this result, the linear technology is important. The curve of the space **IV** is a straight line that captures the permanent effect of productivity changes on capital accumulation and consumption choices. With decreasing marginal returns to capital, this curve would be concave, leaving the possibility of attaining higher future private consumption with lower taxes ambiguous. This is because, in the case of decreasing marginal product of capital in production, the curve of orthant **I** would be concave in equilibrium.

Moreover, this substitution effect is driven by the fact that it is not the stock of capital that is taxed over time, but only the capital income flow. The balanced fiscal budget ties, directly, the quantity of public consumption to the *taxed* capital income flow. But the prices of future private consumption depend (negatively) also on the whole stock of capital, not only on its capital income flow. A higher productivity, $A$, gives better opportunities for capital accumulation and, in turn, for a sharper decline in the prices of private consumption over time.[18]

In other words, the tax-driven mechanism of public goods provision is dominated by the competitive price mechanism of private goods provision. Due to the presence of endogenous growth in the model, and the critical dependence of growth on productivity, the higher the productivity, $A$, the higher the dominance of the competitive price mechanism over the public-goods provision mechanism. This is why the deadweight loss of taxation is higher when production possibilities and growth opportunities are higher, and voters perceive this. Higher productivity gives the opportunity to voters to choose optimally lower taxes and achieve higher growth.

[18]To see this point, consider the case where the fiscal budget was given, instead, by,

$$\mathbf{g}_t = \tau_t \left(1 + A_t - \delta\right) \mathbf{k}_t \ ,$$

i.e. both the capital stock and income were taxed in every period. It is very easy to verify that, in that case, taxes would be given by,

$$\tau_t^D = \tau^D = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)} \ ,$$

for all $t \in \{0, 1, ...\}$, i.e. taxes would be unrelated to productivity, $A$. In our Figure 1, this taxation system would tend to move the line of orthant **II** counterclockwise for every lower tax, a case that would require much higher $c_{t+1}$ in compensation. But any reduction in taxes would be unable to deliver the required level of $c_{t+1}$, because with taxation of the capital stock a higher $A$ would not deliver enough future capital accumulation to support a sufficient higher level of future private goods.

### 2.3.2 Productivity and growth disparities

Consider a world of economies in which policies are exogenous and suppose that the marginal tax rate $\tau^{exog}$, in this world, is constant over time. A rise in the productivity parameter, $A$, causes $\gamma^{exog}$, to increase by,

$$\frac{\partial \gamma^{exog}}{\partial A} = \beta \left(1 - \tau^{exog}\right) \ , \tag{16}$$

as can be verified from equation (8). The gradient given by (16) determines the link between cross-country productivity differences and cross-country growth differences. Now, consider a world of political economies, where policies are endogenous in each country. A rise in the productivity parameter, $A$, causes growth to increase by,

$$\frac{\partial \gamma^{D}}{\partial A} = \beta \left[\left(1 - \tau^{D}\right) - (A - \delta) \frac{\partial \tau^{D}}{\partial A}\right] \ ,$$

so, using (12),

$$\frac{\partial \gamma^{D}}{\partial A} = \beta \left[\left(1 - \tau^{D}\right) + \frac{(1-\beta)(1-\chi)}{[1 + (1-\beta)(1-\chi)](A - \delta)}\right] \ . \tag{17}$$

If we pick two countries, one from each of the two different worlds, both characterized by the same productivity level, $A$, *and* such that taxes are at the same level, namely $\tau^{exog} = \tau^{D}$, equations (16) and (17) imply that,

$$\frac{\partial \gamma^{D}}{\partial A} > \frac{\partial \gamma^{exog}}{\partial A} \ .$$

Thus, the politicoeconomic channel generates higher cross-country development differences that arise from structural cross-country differences in productivity.

## 3. The Stochastic Economy

The cyclical relationship between the gross national product and the fiscal policy of a government e.g., taxation policy, is an important subject from both a political and an economic point of view. Indeed, the movement of fiscal policies in relationship to the cyclical behavior of the gross national production of an econmy is an open research topic, both theoretically and empirically. For example, the traditional Keynesian view suggests that policies should be stabilizing, namely marginal taxes should be procyclical, while government spending should be even countercyclical. On the other hand, Barro (1979) suggests tax smoothing over the business cycle, so if the fiscal budget is not restricted to be balanced, the public surplus should be procyclical.

Several empirical studies find evidence for procyclical fiscal policies. Gavin et. al. (1996), Gavin and Perotti (1997), Stein et. al. (1999) find that both taxes and government expenditures in Latin American countries are procyclical. Talvi and Vegh (2000) and Lane (2003) report procyclical policies for public consumption, the focus of this paper, for rich countries. Hercowitz and Strawczynski (2002) study the cyclical behavior of GDP shares of several components of government spending in OECD countries and test for possible cyclical asymmetries of the fiscal budget in periods of expansion, versus periods of contraction. Although public consumption is procyclical in the OECD (see Lane (2003)), Hercowitz and Strawczynski (2002) find that its GDP share is generally countercyclical over the cycle.

On the theoretical side, Lane (2003) claims that the fiscal cyclicality implications of endogenized public consumption in neoclasssical frameworks rest upon the specification of the utility function with respect to private versus public consumption (see p. 2664). In particular, Lane (2003) notes that if the utility function is additively separable between public and private goods and, also, these goods are neither substitutes nor complements, then endogenous public consumption in a neoclassical framework should be smooth over the business cycle.[19]

[19]Thus, in order to address the empirically observed procyclicality of government components, political economy models of the "voracity effect" have been suggested by Lane and Tornell (1996) and (1998) and Tornell and Lane (1998) and (1999).

These are exactly the conditions on the utility function under which we study this issue. We show that, in a stochastic endogenous growth environment (as opposed to an exogenous long-run growth neoclassical environment), endogenizing public consumption through perfect-foresight voting reconciles the theory with the empirical observation of public-consumption procyclicality and the countercyclicality of the public consumption GDP share. Moreover, we examine the effect of higher capital productivity volatility on average taxes.

We extend the deterministic model of the previous section to the case in which the capital productivity is random. The model is the same as in the deterministic case, with the sole difference that aggregate production is now given by,

$$\mathbf{y}_t = A_t \mathbf{k}_t \ , \tag{18}$$

where the capital productivity shock is serially uncorrelated over time, with distribution,

$$\ln(A_t) \sim N\left(\mu - \frac{\sigma^2}{2} \ , \ \sigma^2\right) \ ,$$

where $\mu > 0$ and $\sigma \geq 0$. The special case $\sigma = 0$, coincides with the model of the previous section. Parametrizing the distribution of the capital-productivity shock in this way, allows us to compare the impact of volatility on decision rules and election outcomes. In particular,

$$E(A_t) = e^{\mu} \ ,$$

while

$$Var(A_t) = e^{2\mu}\left(e^{\sigma^2} - 1\right) \ ,$$

i.e. if two countries differ only with respect to parameter $\sigma$, they have the same average capital productivity but different capital-productivity variances.

## 3.1 Competitive equilibrium in the stochastic economy

The interest rate in this stochastic model is,

$$r_t = A_t - \delta \ , \tag{19}$$

and the fiscal budget is,

$$\mathbf{g}_t = \tau_t \left(A_t - \delta\right) \mathbf{k}_t \ . \tag{20}$$

The problem of an individual household $i$, is described by the following constrained problem,

**Household Problem in the Stochastic Economy**

$$\max_{\{(c_t, a_{t+1})\}_{t=0}^{\infty}} \ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \chi \ln\left(c_t\right) + \left(1-\chi\right) \ln\left(\mathbf{g}_t\right) \right] \right\} \ ,$$

subject to:

$$a_{t+1} = \left[1 + r_t \left(1 - \tau_t\right)\right] a_t - c_t \ , \tag{21}$$

$$\ln\left(A_t\right) \sim N\left(\mu - \frac{\sigma^2}{2} \ , \ \sigma^2\right) \ ,$$

equations (19) and (20),

given $a_0$, $\{(\tau_t, \mathbf{k}_t)\}_{t=0}^{\infty}$ .

In competitive equilibrium, $\{\mathbf{k}_t\}_{t=0}^{\infty}$ should conform to every household's solution to the same problem and to market-clearing conditions. The necessary conditions of household $i$ are given by,

$$\frac{1}{c_t} = \beta E_t \left[ \frac{1 + \left(A_{t+1} - \delta\right)\left(1 - \tau_{t+1}\right)}{c_{t+1}} \right] \ , \tag{22}$$

equation (21), and the transversality condition $\lim_{t\to\infty} E_t \left\{ \frac{a_{t+1}}{\prod_{s=0}^{t} [1+(A_s-\delta)(1-\tau_s)]} \right\} = 0$. It turns out that we get the same solution as in the deterministic economy,

$$a_{t+1} = \beta \left[1 + \left(A_t - \delta\right)\left(1 - \tau_t\right)\right] a_t \ , \tag{23}$$

and,

$$c_t = (1 - \beta)\left[1 + (A_t - \delta)(1 - \tau_t)\right] a_t \ , \tag{24}$$

which satisfy both necessary (and sufficient) conditions (22) and (21), and the trasversality condition.

## 3.2 Politicoeconomic equilibrium in the stochastic economy

As in the deterministic case, the only voting issue is the current level of the income tax rate. In the beginning of each period, the shock $A_t$ is revealed, and then majority voting over taxes follows. Afterwards households and producers make their economic decisions, given the electoral outcome. Each household decides, independently, about its best voting strategy, taking account of the full impact of its voting action on the current electoral and economic outcome. That is, given the probability distribution of future shocks, the household considers how the electoral outcome affects its own current and future economic decisions, its own future voting decisions, as well as the current and future economic and voting decisions of others in politicoeconomic equilibrium over the infinite horizon.

Given the analytical solutions in equations (23) and (24), the value function for a household $i \in \mathcal{I}$ at time $t \in \{0, 1, ...\}$ is,

$$\begin{aligned} V\left(a_{i,t}, \mathbf{k}_t, \{\tau_s\}_{s=t}^{\infty}\right) = \kappa_S + \chi \frac{\ln(a_{i,t})}{1-\beta} + (1-\chi)\frac{\ln(\mathbf{k}_t)}{1-\beta} + \\ + \frac{\ln\left[1 + (A_t - \delta)(1-\tau_t)\right]}{1-\beta} + (1-\chi)\ln(\tau_t) + \\ + E_t\left\{\sum_{s=1}^{\infty} \beta^s \left\{\frac{\ln\left[1 + (A_{t+s} - \delta)(1 - \tau_{t+s})\right]}{1-\beta} + (1-\chi)\ln(\tau_{t+s})\right\}\right\} \ , \end{aligned} \tag{25}$$

where $\kappa_S$ is a constant.[20] Setting the partial derivative of the value function, with respect to the voting choice, $\tau_t$, equal to zero yields the preferred tax rate for the single household, namely,

$$\tau_t^S = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left(1+\frac{1}{A_t-\delta}\right), \quad t = 0, 1, .... \tag{26}$$

As in the deterministic model, there is unanimity about $\tau_t^S$, the winning tax rate in the stochastic economy, at all times. The rationale behind unanimity across voters is the same as the one presented in the deterministic case. Moreover, following the argument made for the deterministic model, the tax policy given by (26) is also the second-best policy of a benevolent utilitarian social planner.

### 3.2.1 Countercyclical Marginal Tax Rates

From equation (26) taxes are *countercyclical* since high (low) productivity, $A_t$, households vote in favor of a decrease (increase) of the current public to future private consumption ratio, by reducing (increasing) the tax rate.

As in the deterministic economy, in the stochastic endogenous growth model, marginal income taxes create a strong substitution effect between current public and future private consumption. While current and future consumption is provided by private markets, the public goods provision mechanism is public, paid for through taxes. Accumulating more capital is the key to having more of both private and public goods in the future. From the gross effective interest rate, $1+(A_t-\delta)(1-\tau_t)$, and the tax policy of the government, it is obvious that only capital income and not the capital stock is taxed over time. Thus, it is the substitution effect between current public and future private consumption that leads to

[20]In particular,

$$\kappa_S = \chi\frac{\ln(1-\beta)}{1-\beta} + \frac{\beta\ln(\beta)}{(1-\beta)^2} + (1-\chi)\frac{E_t[\ln(A_{t+1}-\delta)]}{1-\beta}.$$

lower taxes when productivity is higher. In other words, the politicoeconomic equilibrium in the stochastic endogenous growth model implies that growth opportunities are exploited by decreasing taxes in periods of booms (and not by increasing taxes, as suggested by a countercyclical policy).

Substituting equation (26) into (20), yields the politicoeconomic equilibrium value for public consumption, namely,

$$\mathbf{g}_t = \frac{(1-\beta)(1-\chi)(1+A_t-\delta)}{1+(1-\beta)(1-\chi)}\mathbf{k}_t \ , \tag{27}$$

which implies that *government consumtion is procyclical* in equilibrium, i.e., $\frac{d\mathbf{g}_t}{dA} > 0$. Moreover, combining (27) with (18) yields,

$$\frac{\mathbf{g}_t}{\mathbf{y}_t} = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left(1+\frac{1-\delta}{A_t}\right) \ , \tag{28}$$

which means that *the GDP share of government consumption is countercyclical* in equilibrium, i.e., $d\left(\frac{\mathbf{g}_t}{\mathbf{y}_t}\right)/dA < 0$.

The intuition behind these results is provided by Figure 2. In Figure 2, $\bar{A}$ is the average productivity, whereas $A_t$ is a realization of the stochastic process, with $A_t > \bar{A}$. $\bar{\tau}^S$ is the average optimal tax rate, from (26), chosen by the voters, with $A_t$ replaced by $\bar{A}$, whereas $\tau_t^S$ is the optimal tax rate of equation (26), corresponding to $A_t$. The graph shows that since $A_t > \bar{A}$, $\tau_t^S < \bar{\tau}^S$, and that the optimal level of public consumption is higher than the average level of public consumption, i.e. $\mathbf{g}_t^S > \bar{\mathbf{g}}^S$.

Notice that in orthant **I** there is only one line (c.f., the deterministic case). This is because the savings choice of the individual, and hence $E_t(c_{t+1})$, adjusts one-to-one to the choice of $c_t$, as the decision rules given by (23) and (24) reveal. Notice that, in orthant **IV**, the two lines cross where $\tau = 1$, which is an implication of equation (24).

The equilibrium points $\left(\varepsilon_1^I, \varepsilon_1^{II}, \varepsilon_1^{III}, \varepsilon_1^{IV}\right)$ correspond to the politicoeconomic equilibrium when the current productivity shock is $\bar{A}$ and taxes are set optimally at the level $\bar{\tau}^S$. The points $\left(\varepsilon_2^I, \varepsilon_2^{II}, \varepsilon_2^{III}, \varepsilon_2^{IV}\right)$ denote the case where the current productivity shock is $A_t > \bar{A}$ and taxes are kept at the (nonoptimal) level $\bar{\tau}^S$. From the indifference curves in orthant **II**, setting taxes to $\tau_t^S < \bar{\tau}^S$, according to (26), yields higher utility. This is depicted by the points $\left(\varepsilon_3^I, \varepsilon_3^{II}, \varepsilon_3^{III}, \varepsilon_3^{IV}\right)$. That $\tau_t^S$ gives higher lifetime utility for the voter, follows from the fact that lower taxes today, lead to higher savings, i.e. to a higher $a_{t+1}$, which follows from decision rule (23). The value function given by (25) shows that the well being of the voter from period $t+1$ on is increasing in $a_{t+1}$, whereas the utility derived solely by period $t$'s economic and political choices is captured by the highest-level indifference curve in orthant **II**.

## 3.3 Volatility, average taxes and average growth

Several papers, such as Ramey and Ramey (1995), Aizenman and Marion (1999), and Hnatkovska and Loayza (2003), study the link between economic volatility and long-run growth empirically. They find that there is a negative correlation between volatility and average growth. However, there is not much theoretical background addressing these empirical findings. By examining the relationship between volatility and average taxes as well as the relationship between average growth and taxes, we provide a theoretical underpinning for these empirical results.[21]

Recall that we model the distribution of the productivity shock, $A_t$, so that a change in the parameter $\sigma$ has no effect on the average shock, $E\left(A_t\right)$, whereas volatility, $Var\left(A_t\right)$, does change. The question naturally arises whether a different structural volatility, $\sigma$, of the productivity shock $A_t$, would have an effect on average taxes and as well as on average

[21]Aizenman and Marion (1999), apart from reporting a statistically significant negative link between innovation volatility and private investment GDP shares in developing countries, they suggest some theoretical explanations for their findings. Their explanations rely upon risk aversion and non-linear budget constraints.

growth in a politicoeconomic equilibrium. We show the dependence of average taxes on volatility in Proposition 2.

**Proposition 2** *If the volatility of capital productivity is higher, then the average tax rate is higher.*

**Proof.** From equation (26),

$$E(\tau_t) = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)} \left[1 + E\left(\frac{1}{A_t - \delta}\right)\right] . \tag{29}$$

We find the dependence of $E\left(\frac{1}{A_t-\delta}\right)$ on parameter $\sigma$. Since $A_t$ is lognormal, $(A_t - \delta)$ is also lognormal. Let parameters $\hat{\mu}$ and $\hat{\sigma}$ be defined as,

$$\ln(A_t - \delta) \sim N\left(\hat{\mu}\ ,\ \hat{\sigma}^2\right) .$$

Noticing that,

$$E(A_t - \delta) = e^{\mu} - \delta ,$$

$$Var(A_t - \delta) = e^{2\mu}\left(e^{\sigma^2} - 1\right) ,$$

$$E(A_t - \delta) = e^{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2} ,$$

and

$$Var(A_t - \delta) = e^{2\hat{\mu} + \hat{\sigma}^2}\left(e^{\hat{\sigma}^2} - 1\right) ,$$

we express parameters $\hat{\mu}$ and $\hat{\sigma}$ as functions of $\mu$, $\sigma$ and $\delta$. In particular,

$$e^{\hat{\sigma}^2} = \left(\frac{e^{\mu}}{e^{\mu} - \delta}\right)^2 \left(e^{\sigma^2} - 1\right) + 1 , \tag{30}$$

whereas,

$$e^{\hat{\mu}} = (e^{\mu} - \delta)\left[\left(\frac{e^{\mu}}{e^{\mu} - \delta}\right)^2 \left(e^{\sigma^2} - 1\right) + 1\right]^{-\frac{1}{2}} . \tag{31}$$

Thus,

$$-\ln(A_t - \delta) \sim N\left(-\hat{\mu}\ ,\ \hat{\sigma}^2\right)\ ,$$

and,

$$E\left(\frac{1}{A_t - \delta}\right) = e^{-\hat{\mu}+\frac{1}{2}\hat{\sigma}^2}\ ,$$

so, using (30) and (31),

$$E\left(\frac{1}{A_t - \delta}\right) = \frac{1}{e^{\mu} - \delta}\left[\left(\frac{e^{\mu}}{e^{\mu} - \delta}\right)^2\left(e^{\sigma^2} - 1\right) + 1\right]\ .$$

This last equation combined with (29) implies that,

$$\frac{\partial E(\tau_t)}{\partial \sigma} > 0\ ,$$

proving the proposition. □

Proposition 2 shows that cross-country differences in the volatility of capital productivity are linked to differences in average taxes. However, in politicoeconomic equilibrium, there is no link between volatility and average growth. To see this, combine (23) and (26), to get the growth rate of capital,

$$\gamma_t^S = \frac{\beta(1 + A_t - \delta)}{1 + (1 - \beta)(1 - \chi)} - 1\ . \tag{32}$$

Thus,

$$\frac{\partial E\left(\gamma_t^S\right)}{\partial \sigma} = \frac{\beta\frac{\partial E(A_t - \delta)}{\partial \sigma}}{1 + (1 - \beta)(1 - \chi)} = 0\ . \tag{33}$$

This result, expressed by equation (33), comes from the fact that in equilibrium, aggregate savings, private consumption and public consumption are drivem by decision rules that are linear in $A_t$, as it can be verified by (24), (24) and (27).

The empirical studies such as Ramey and Ramey (1995), Aizenman and Marion (1999), and Hnatkovska and Loayza (2003) assert that after controling for several country-specific characteristics that may capture the cross-country variation in average productivity, $E(A_t)$, then, $Var\left(\gamma_t^S\right)$ and $E\left(\gamma_t^S\right)$ should be negatively correlated. Our specification of the stochastic structure of productivity, $A_t$, mimics this claim in the regression specification of the above empirical studies.

Our theoretical findings do not support a negative correlation between $Var\left(\gamma_t^S\right)$ and $E\left(\gamma_t^S\right)$, at least through the channel of taxation for the provision of public consumption. An open question is whether assuming different intratemporal and intertemporal elasticities of substitution between private and public consumption would lead to a link between volatility and growth through the politico-economic channel. Yet, another open question is whether cross country regression analysis can control for the cross-country variation in average productivity, $E(A_t)$. If not, then it is possible that the reported negative correlation between $Var\left(\gamma_t^S\right)$ and $E\left(\gamma_t^S\right)$ comes from cross-country variations in average productivity, $E(A_t)$, or, in the language of our model, from cross-country variations in parameter $\mu$. In the last case, our model's implication about the absence of a link between volatility and growth is not necessarily erroneous.

## REFERENCES


Aizenman, Joshua and Nancy Marion (1999): "Volatility and Investment: Interpreting Evidence from Developing Countries," Economica, 66, pp. 157-179.

Amano, Robert A. and Tony S. Wirjanto (1998): "Government Expenditures and the Permanent-Income Model", Review of Economic Dynamics, v 1, n 3, Jul 1998, p 719-730.

Arreaza, Adriana, Bent E. Sørensen, and Oved Yosha (1998): "Consumption Smoothing through Fiscal Policy in OECD and EU Countries," NBER Working Paper 6372.

Barro, Robert J. (1979): "On the Determination of the Public Debt," Journal of Political Economy, 87, pp. 940-971.

Caselli, Francesco and Jaume Ventura (2000): "A Representative Consumer Theory of Distribution," American Economic Review, 90, pp. 909-926.

Chatterjee, Satyajit (1994): "Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model," Journal of Public Economics, Vol. 54, pp. 97-119.

Cohen, Daniel and Philippe Michel (1988): "How Should Control Theory Be Used to Calculate a Time-Consistent Government Policy?", Review of Economic Studies; v55, n2, April, pp. 263-274.

Denzau, Arthur T, and Robert J. Mackay (1981): "Structure-Induced Equilibria and Perfect-Foresight Expectations," American Journal of Political Science, Vol. 25(4), pp. 762-779.

Easterly, William; Rebelo, Sergio (1993): "Fiscal Policy and Economic Growth: An Empirical Investigation," Journal of Monetary Economics, v32, n3, December, pp. 417-58.

Epple, Dennis and Joseph, B. Kadane (1990): "Sequential Voting with Endogenous Voter Forecasts," American Political Science Review, Vol. 84(1), pp. 165-175.

Fölster, Stephan and Magnus Henrekson (2001): "Growth Effects of Government Expenditure and Taxation in Rich Countries," European Economic Review, 45, pp. 1501-1520.

Gavin, M., R. Hausmann, R. Perotti and E. Talvi (1996): "Managing Fiscal Policy in Latin America and the Caribbean: Volatility, Procyclicality and Limited Creditworthiness," Inter-American Development Bank Mimeo.

Gavin, M. and R. Perotti (1997): "Fiscal Policy in Latin America," NBER Macroeconomics Annual 12, pp. 11-70.

Hall, Robert E. and Charles I. Jones (1999): "We Do Some Countries Produce So Much More Output per Worker than Others?" Quarterly Journal of Economics, 114, pp. 83-116.

Hercowitz, Zvi and Michel Strawczynski (2002): "Cyclical Ratcheting in Government Spending: Evidence from the OECD," mimeo, Tel Aviv University.

Hnatkovska, Viktoria and Norman Loayza (2003): "Volatility and Growth," Mimeo, the World Bank.

Karp, Larry and In Ho Lee (2003): "Time-Consistent Policies," Journal of Economic Theory, 112, pp. 353-364.

Klein, Paul, Per Krusell and José-Víctor Ríos-Rull (2003): "Time-Consistent Optimal Fiscal Policy," International Economic Review, 44, 1217-1245.

Klein, Paul and José-Víctor Ríos-Rull (2003): "Time-Consistent Public Expenditures,"

Kneller, Richard, Michael F. Bleaney and Norman Gemmell (1999): "Fiscal policy and Growth: Evidence from OECD Countries," Journal of Public Economics, 74, pp. 171-190.

Koulovatianos, Christos and Leonard J. Mirman (2004): "Public Consumption and Public Infrastructure in a Dynamic Economy," mimeo, University of Virginia.

Krusell, Per, Vincenzo Quadrini and José-Víctor Ríos-Rull (1997): "Politico-Economic Equilibrium and Economic Growth", Journal of Economic Dynamics and Control, v21, n1, January, pp. 243-72.

Krusell, Per and José-Víctor Ríos-Rull (1999): "On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model," American Economic Review, Vol. 89(5), December, pp. 1186-1181. Also, a previous version in Minneapolis Fed. Research Dept. Staff Report #234, July 1997.

Krusell, Per and Anthony A. Smith Jr. (1998): "Income and Wealth Heterogeneity in the Macroeconomy," Journal of Political Economy, 106, pp. 867-896.

Lane, Philip R. (2003): "The Cyclical Behavior of Fiscal Policy: Evidence from the OECD," Journal of Public Economics, 87, pp. 2661-2675.

Lane, Philip R. and A. Tornell (1996): "Power, Growth and the Voracity Effect," Journal of Economic Growth, 1, pp. 213-241.

Lane, Philip R. and A. Tornell (1998): "Why aren't Latin American Savings Rates Procyclical?" Journal of Development Economics, 57, pp. 185-200.

Lucas, Robert E., Jr. (1990): "Supply-Side Economics: An Analytical Review," Oxford Econ. Papers, 42, pp. 293-316.

Meltzer, Alan H. and Scott F. Richard (1981): "A Rational Theory of the Size of Government," Journal of Political Economy, Vol 89(5), pp. 914-927.

Mendoza, Enrique G., Gian Maria Milesi Ferretti and Patrick Asea (1997): "On the Ineffectiveness of Tax Policy in Altering Long-Run Growth: Harberger's Superneutrality Conjecture," Journal of Public Economics, 66, pp. 99-126.

Ortiguera, Salvador (1998): "Fiscal Policy in an Endogenous Growth Model with Human Capital Accumulation," Journal of Monetary Economics, 42, pp. 323-355.

Ramey, Garey and Valerie R. Ramey (1995): "Cross-Country Evidence on the Link Between Volatility and Growth," American Economic Review, 85, p. 1138-1151.

Rebelo, Sergio (1991): "Long Run Policy Analysis and Long Run Growth," Journal of Political Economy 99, pp. 500-521.

Rivas, Luis A. (2003): "Income Taxes, Spending Composition and Long-Run Growth," European Economic Review, 47, 477-503.

Romer, Paul A. (1986): "Increasing Returns and Long-Run Growth," Journal of Political Economy, 94, pp. 1002-1037.

Sørensen, Bent E., Lisa Wu, and Oved Yosha (2001): "Output Fluctuations and Fiscal Policy: U.S. State and local Governments 1978-1994," European Economic Review 45, pp. 1271-1310.

Stein, Ernesto, Ernesto Talvi and Alejandro Grisanti (1998): "Institutional Arrangements and Fiscal Performance: the Latin American Experience," NBER Working paper #6358.

Stokey, Nancy L. and Sergio Rebelo (1995): "Growth Effects of Flat-Rate Taxes," Journal of Political Economy, 103, pp. 519-550.

Talvi, Ernesto and Carlos A. Vegh (2000): "Tax Base Variability and Procyclical Fiscal Policy," NBER Working paper #7499.

Tornell, A. and Philip R. Lane (1998): "Are Windfalls a Curse? A Non-representative Agent Model of the Current Account," Journal of International Economics, 44, pp. 83-112.

Tornell, A. and Philip R. Lane (1999): "The Voracity Effect," American Economic Review, 89, pp. 22-46.

Xie, Danyang (1997): "On Time Inconsistency: A Technical Issue in Stackelberg Differential Games," Journal of economic Theory, 76, pp. 412-430.

# Appendix

**Proof of proposition 1.** The optimality condition (7) can be written as,

$$c_{t+1} = \beta \frac{\lambda_t}{\lambda_{t+1}} c_t \ , \quad t = 0, 1, ..., \tag{A1}$$

where $\lambda_t$ is the shadow price of the final good with $\lambda_0 = 1$ (the numeraire) and:

$$\frac{\lambda_{t-1}}{\lambda_t} = 1 + (1 - \tau_t)(A - \delta) \ , \quad t = 0, 1, ..., \tag{A2}$$

with $\lambda_{-1} = 1 + (1 - \tau_0)(A - \delta)$, by default. The household budget constraint combined with (A2) becomes,

$$c_t + a_{t+1} = \frac{\lambda_{t-1}}{\lambda_t} a_t \quad \text{or} \quad \lambda_t c_t = \lambda_{t-1} a_t - \lambda_t a_{t+1} \ . \tag{A3}$$

Considering equation (A3) one period ahead, using (A1), and defining,

$$z_t \equiv \lambda_{t-1} a_t \ , \quad t = 0, 1, ...,$$

yields,

$$z_{t+2} - (1 + \beta) z_{t+1} + \beta z_t = 0 \ . \tag{A4}$$

Equation (A4) is an elementary second-order linear difference equation with two obvious solutions, $z_{t+1} = \beta z_t$ and $z_{t+1} = z_t$. The second solution is ruled out since it implies that $\lambda_t a_{t+1} = \lambda_{t-1} a_t$, so from equation (A3), it must be $c_t = 0$ in all periods, which cannot be optimal. Alternatively, $z_{t+1} = z_t$ implies that $\lambda_t a_{t+1} = \lambda_{-1} a_0$, or $\frac{\lambda_t a_{t+1}}{\lambda_{-1}} = a_0$, so from equation (A2) it must be that, $0 < a_0 = \frac{\lambda_t a_{t+1}}{\lambda_{-1}} = \frac{a_{t+1}}{\prod_{s=0}^{t} [1+(A-\delta)(1-\tau_s)]} = \lim_{t \to \infty} \frac{a_{t+1}}{\prod_{s=0}^{t} [1+(A-\delta)(1-\tau_s)]}$, i.e. a violation of the transversality condition. Therefore, only the first solution is applicable, $a_{t+1} = \beta \frac{\lambda_{t-1}}{\lambda_t} a_t$, and using equation (A2), equation (8) is proved. Equation (9) comes from combining (8) and the household budget constraint. Equation (10) comes from (8) through linear aggregation. Note that (A4) would not be linear if preferences were not "log." □

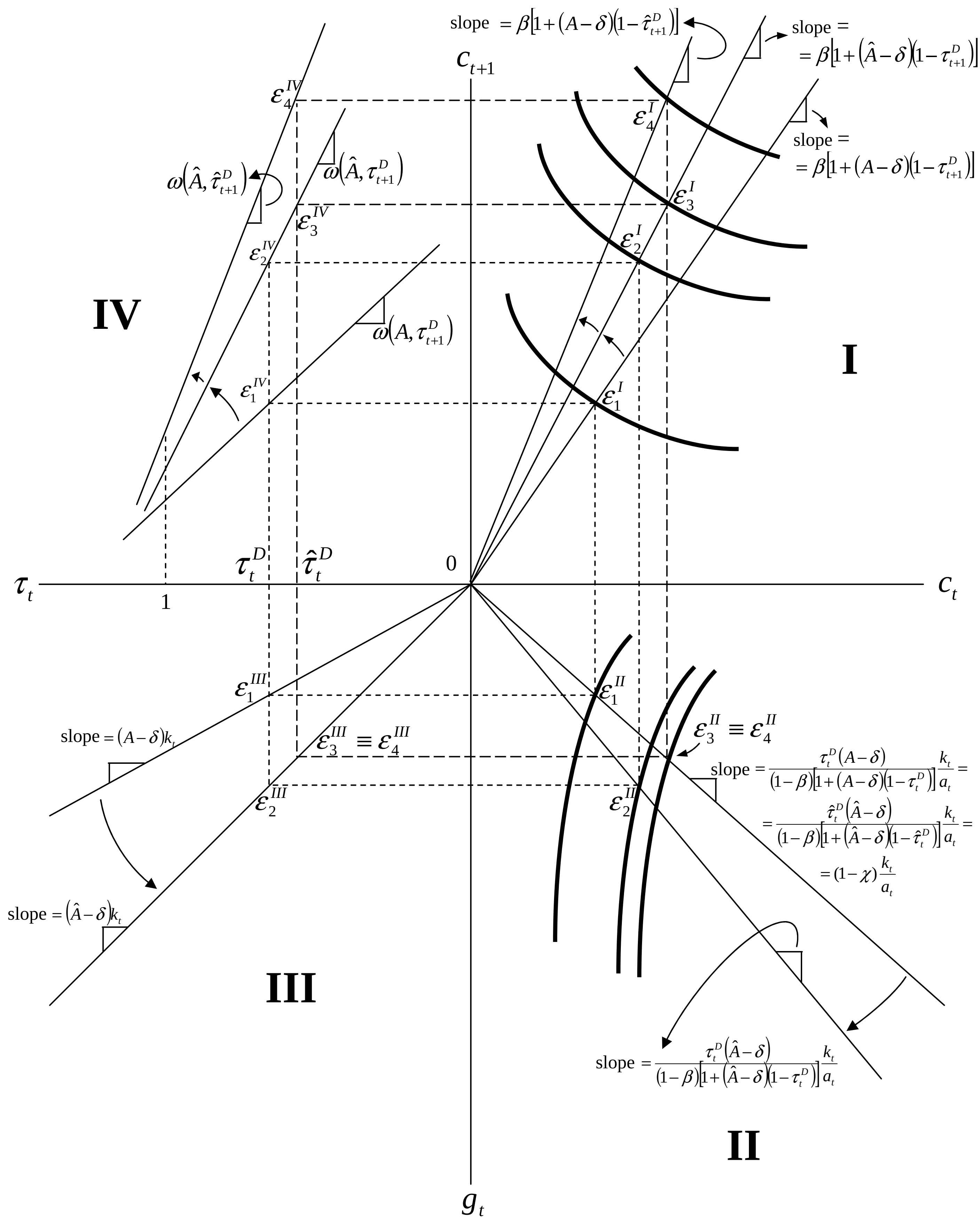


**Figure 1** Why voters choose lower taxes when productivity is higher

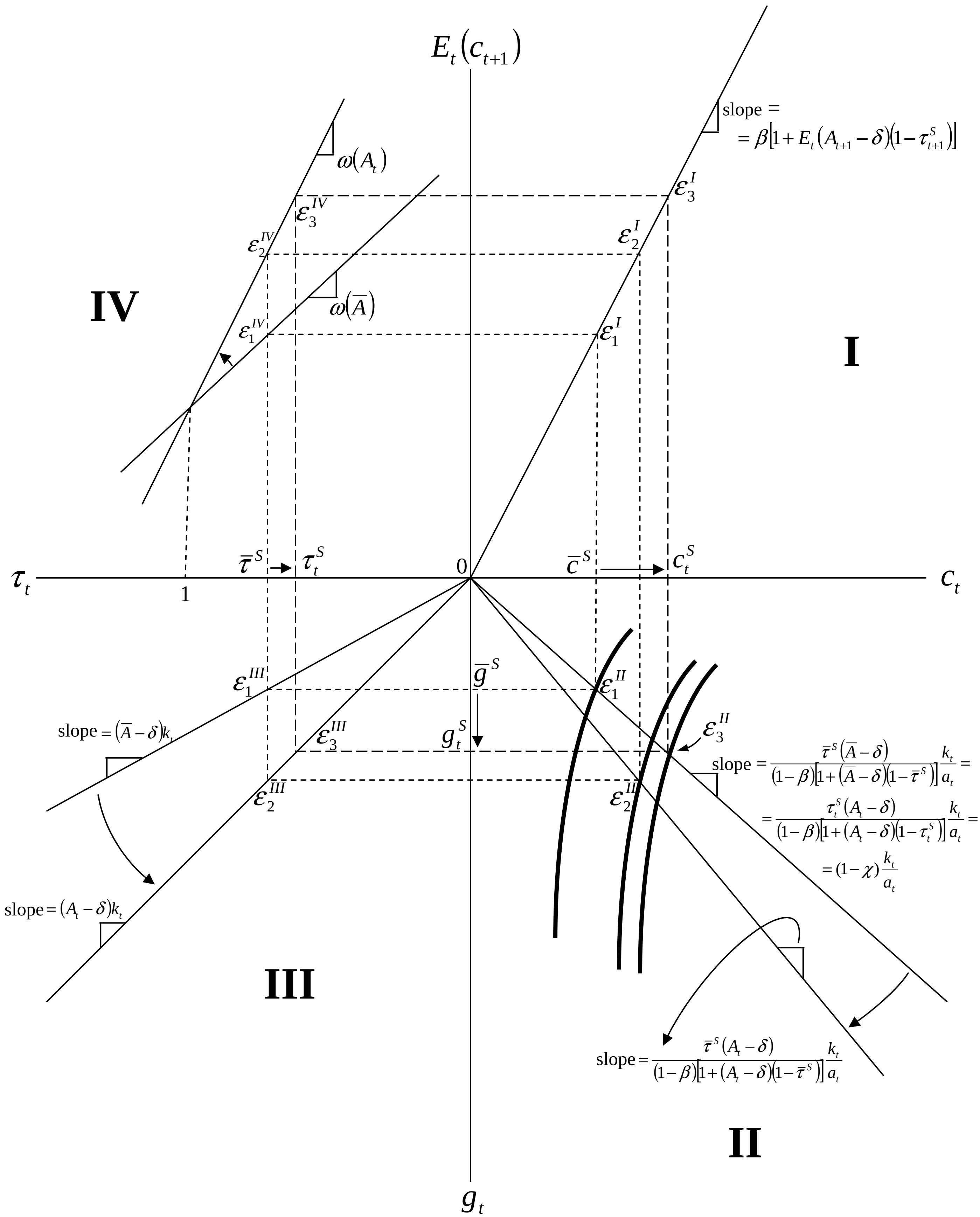


**Figure 2** Why public consumption is procyclical and taxes countercyclical