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Cournot-Nash Competition in a General Equilibrium Model of International Trade *

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Abstract

We use the two-factor, two-sector, two-country model of Melvin and Warne (1973) and Markusen (1981), in which the production of one good is monopolized in each country, in order to investigate the role of the price normalization. We illustrate several puzzling effects that occur if the price normalization is changed. However, we show that Markusen's result on the direction of the trade flow between two proportional countries with constant returns to scale is robust with respect to the choice of the normalization rule. To overcome the price normalization problem in international trade we suggest to use the concept of real wealth maximization.

JEL Classification: D43, F11, F12

Keywords: International trade with imperfect competition, price normalization, real wealth maximization

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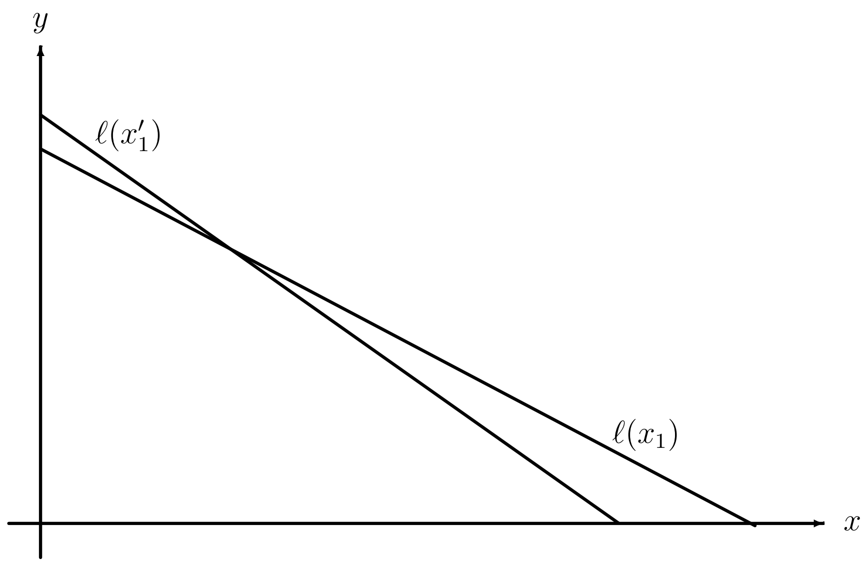
1 Introduction

We analyze Cournot competition between two countries within the framework developed by Melvin and Warne (1973) and Markusen (1981). There are two countries, $i = 1, 2$. In each country, there are two producers providing the consumption goods x and y, respectively. There are two factors, capital and labor, which are not internationally traded. The producer of y represents the perfectly competitive sector in its country. The producer of x in country i chooses its output x_i in a strategic manner. For every strategy pair (x_1, x_2), there is a relative price system in each country such that all markets clear. In the case of free trade, the relative price of x and y is the same in both countries, but other price ratios such as relative factor prices are typically country specific.

In order to determine the equilibrium production levels of (x_1, x_2) one has to specify the objectives of the strategic firms. As is common in the literature, it is assumed that each strategic firm maximizes profits. If the firms were price takers, profits were unambiguously defined. In that case, relative prices suffice to compare the values of different production plans. In the present case of imperfect competition, however, different ways to measure profits give rise to different objective functions as illustrated in the following figure.



Consider a strategy x_1 of the strategic firm in country 1. The strategy x_1 determines a budget line $\ell(x_1)$ for the firms' shareholders consisting of all commodity bundles (x, y) the shareholders can buy out of their profit income. This line depends on the relative price system and on the size of the profits generated by the strategy x_1. The line $\ell(x_1')$ denotes shareholders' budget line if the firm chooses the alternative strategy x_1'. Since the price ratio corresponding to x_1 differs from that generated by x_1' the two budget lines typically intersect. In models of perfect competition, however, all budget lines are parallel and budget sets are ordered by inclusion according to the amount of profits associated with a strategy.

Assume for the moment that profits and income are measured in terms of commodity y. That is to say, compare the two strategies x_1 and x_1' on the y-axis. Since the shareholders can buy more units of y if the firm chooses x_1', the strategy x_1' gives higher income to the shareholders than x_1. Assume now that the commodity x is used instead of y to measure profit income. On the x-axis, x_1 gives higher income than x_1'. Thus, the normalization of prices and profits matters for profit maximization.

We could have used any other consumption bundle $(\beta, 1-\beta) \gneqq 0$ instead of $(0,1)$ or $(1,0)$. Then profits are measured along the ray $y = (1-\beta)x/\beta$ and prices are normalized such that the value of the basket $b = (\beta, 1-\beta, 0, 0)$ is identically equal to 1. More generally, let $b = (\tilde{x}, \tilde{y}, \tilde{k}, \tilde{l}) \gneqq 0$ be any commodity basket. In the b-normalization, profit maximization amounts to maximizing the number of units of b that can be bought out of profits. Different baskets b give rise to different objective functions of the strategic firms. As a consequence, the associated games typically have different equilibria.

We illustrate the role of the price normalization problem by means of a particularly simple numerical example in which the basket $b = (\beta, 1-\beta, 0, 0)$ is varied. More precisely, we consider two completely identical countries $i = 1, 2$. The production of the strategic good is given by the function $x_i = k_i^{1/3} l_i^{2/3}$ and that of the competitive good is given by $y_i = k_i^{2/3} l_i^{1/3}$, where k_i and l_i denote the amount of capital and labor, respectively. Factor endowments are $\bar{K}_i = \bar{L}_i = 1$. The representative consumer in each country has the CES utility function $u(x, y) = (\sqrt{x} + \sqrt{y})^2$.

The equilibrium concept used in the present example and the bulk of the paper is familiar from the theory of international trade [see, for instance, Markusen (1981) and Wong (1995), chapt. 7]. The strategic firm in each country i chooses its supply x_i so as to maximize profits in units of some basket $b = (\beta, 1-\beta, 0, 0)$. If the firm compares the profitability of x_i with that of an alternative production plan it considers factor prices as fixed. Otherwise, general equilibrium feedbacks are taken into account. Since both countries are identical, the trade flow is zero although free trade is possible. In the case of free trade, market prices are lower than in autarky due to the competition between the strategic firms. In the example, β takes the values 0, 0.1, 0.5, and 1, respectively. To be able to compare the welfare of the shareholders separately from that of other consumers, we assume that the shareholders consume the profits and the non-shareholders consume the factor incomes.

First, we consider the traditional case of the y-normalization for reference purposes. In this case $\beta = 0$. The autarky production is $x \approx 0.188$ and the price of the strategic good x in units of y is $p_x \approx 2.12$. In the case of free trade between the two countries, the strategic firms choose $x_1 = x_2 \approx 0.34$ and $p_x \approx 1.44$.

Second, we assume $\beta = 0.1$ and focus on the case of autarky. Production in the autarky equilibrium becomes $x \approx 0.24$ and $p_x \approx 1.84$.[1] In comparison to the previous case $\beta = 0$, good x is cheaper and consumers' welfare is raised. A computation shows that not only the factor owners but also shareholders are better off although profits measured in units of y have decreased. Therefore, *a move from $\beta = 0$ to $\beta = 0.1$ presents a Pareto improvement.*

Third, we consider $\beta = 1/2$. We compare the autarky equilibrium in this normalization with the free trade equilibrium in the y-normalization. If $\beta = 1/2$ autarky production levels will be nearly twice as high as in the y-normalization. More precisely, $x \approx 0.37$ is produced in each country in autarky. The resulting price $p_x \approx 1.36$ is lower than the corresponding price in the free trade equilibrium based on the y-normalization. Accordingly, the welfare level reached in autarky under the $\beta = 1/2$-normalization is higher than in the case of free trade and the y-normalization. *The welfare gain achieved in the y-normalization by the introduction of free trade is surpassed by the gain obtained by replacing the y-normalization by the $\beta = 1/2$-normalization while autarky is retained.*

Finally, if $\beta = 1$ the production levels in autarky and under free trade coincide. Moreover, $p_x = p_y = 1$ in both cases. That is to say, *the monopoly equilibrium in autarky and the duopoly equilibrium in the case of free trade are Walrasian equilibria.*

In this example, market power is reduced if β is increased. In the limiting case $\beta = 1$, no market power remains. Clearly, it cannot lie in the interest of the shareholders to maximize profits in units of $b = (\beta, 1 - \beta, 0, 0)$ for β sufficiently high. However, the example also shows that a β sufficiently close to zero does not reflect the interests of the shareholders appropriately. The y-normalization that has been used traditionally represents a polar case without particular economic significance.

Following the literature on international trade, we mostly deal with equilibria in which each strategic firm takes its factor prices as fixed whenever it examines a strategy with regard to its optimality. Factor prices are expressed in units of an a priori chosen basket b. We call these equilibria fixed factor price equilibria or FFPE. Sometimes we also deal with equilibria in which factor price adjustments induced by a change of the supply x_i are fully taken into account. These equilibria are called variable factor price equilibria or VFPE.

A major goal of this paper is to study the impact of the choice of the price index b on free trade and autarky equilibria. In particular, we analyze the systematic influence of the weight of good x in the index on equilibria that is illustrated

[1] As before, p_x denotes the price of x in units of the competitive good y.

in the above example. Furthermore, we present an example in which the direction of the trade flow depends on the normalization that is used in both countries. Similarly, we show that the direction of the trade flow between two countries can depend on whether factor prices are considered as fixed or not.

If the strategic good is produced with increasing returns to scale the existence of equilibria is not guaranteed. We show that the existence of a free trade equilibrium depends on the normalization that is adopted. This is due to the fact that the profit of a strategic firm can become negative if the normalization is altered. The firm will not enter the market if it anticipates that it will make losses.

Furthermore, we consider two countries that differ only with respect to the size of their initial factor endowments $(\bar{K}_i, \bar{L}_i)$. We assume that $(\bar{K}_2, \bar{L}_2) = \lambda(\bar{K}_1, \bar{L}_1)$ where λ tends to infinity. We show that the small country does or does not specialize in the production of the strategic good x depending on the price normalization that is chosen.

A further goal is to examine the robustness of the result in Markusen (1981) according to which the small country exports good x if factor endowments are proportional and returns to scale are constant. We show that this result does neither depend on the specific linearly homogeneous utility function nor on the price normalization that are used. The result holds for FFPE as well as for VFPE. It is worth emphasizing that, in the case of FFPE, the size of the trade flow and the gains from trade tend to zero if the x-normalization is approached.

Our final goal is to discuss a way to overcome the price normalization problem. The basket b used to measure profits and to normalize prices plays the role of a consumer price index. Therefore, b ought to be related to the consumption pattern observed on the market. The concept of real wealth maximization proposed in Dierker and Grodal (1998, 1999) is independent of the arbitrary choice of a price normalization. The resulting first order condition coincides with the one for the maximization of shareholders' utility suggested by Kemp and Okawa (1995). Real wealth maximization, however, can also be used if shareholders are heterogeneous. A shareholder's demand need not even be derived from utility maximization.

The paper is organized as follows. In Section 2, we introduce the model and define fixed factor price equilibria and variable factor price equilibria. We state the first order conditions for both types of equilibria and show how they depend on the normalization of prices. The proofs of the first order conditions are relegated to the Appendix. In Section 3, we present examples illustrating the effect of the price normalization problem. In Section 4, we examine the robustness of the result in Markusen (1981) according to which the small country exports good x if factor endowments are proportional and returns to scale are constant. In Section 5, we discuss the concept of real wealth maximization.

2 Model and basic properties

The model falls into the tradition of Melvin and Warne (1973) and Markusen (1981). Chapter 7 in Wong (1995) provides a valuable reference. There are two countries, indexed $i = 1, 2$, two factors in each country, and two consumption goods, x and y, which are the only goods entering a consumer's utility function. In country i, capital k_i and labor l_i are used to produce the consumption goods. Country i has the initial factor endowment $(\bar{K}_i, \bar{L}_i) \gg 0$.

Both countries have identical technologies. In each country i, there are a strategic firm that produces x_i according to the production function F and a perfectly competitive sector that produces y_i according to the production function G. The production functions F and G are linearly homogeneous in (k_i, l_i) and satisfy the usual properties.[2] If the factor combination (k_i, l_i) is used in country i to produce the amount $x_i = F(k_i, l_i)$ of the strategic good, the output of the competitive good is $y_i = G(\bar{K}_i - k_i, \bar{L}_i - l_i)$.

Assumption (A1). *The only strategic variables are the quantities x_i. In particular, no firm chooses its inputs strategically.*

In each country i, both firms minimize costs with respect to the same relative factor prices. Therefore, production efficiency prevails within country i, that is to say, $(x_i, y_i) = (F(k_i, l_i), G(\bar{K}_i - k_i, \bar{L}_i - l_i))$ lies on the production possibility frontier of country i, which is denoted by PPF_i. We assume that PPF_i is given by a C^1 function $Y_i(x_i)$ which is strictly concave due to differences in factor intensities. The marginal rates of transformation are $MRT_i(x_i) = -Y_i'(x_i)$.

Factor prices in country i can be described as functions of the strategic variable x_i in the following way. Assign $y_i = Y_i(x_i)$ to a feasible x_i. By definition, $(x_i, y_i) \in PPF_i$. Under the usual assumptions, there is a unique input vector $(k_i(x_i), l_i(x_i))$ such that $x_i = F(k_i(x_i), l_i(x_i))$ and $y_i = G(\bar{K}_i - k_i(x_i), \bar{L}_i - l_i(x_i))$. Let $r_i(x_i) = \partial_1 G(\bar{K}_i - k_i(x_i), \bar{L}_i - l_i(x_i))$ and $w_i(x_i) = \partial_2 G(\bar{K}_i - k_i(x_i), \bar{L}_i - l_i(x_i))$, where ∂_i denotes the partial derivative with respect to the ith argument. Since marginal products and factor prices coincide due to production efficiency, $r_i(x_i)$ and $w_i(x_i)$ represent the rental rate of capital and the wage rate in country i measured in units of the competitive good y, respectively. The vector $(r_i(x_i), w_i(x_i))$ is proportional to $(\partial_1 F(k_i(x_i), l_i(x_i)), \partial_2 F(k_i(x_i), l_i(x_i)))$ because $(x_i, y_i) \in PPF_i$.

All entities are considered as functions of the quantities x_i chosen by the strategic firms. The market mechanism that gives rise to these functions can be described as follows. Assume that the strategic firms in both countries have decided to produce x_1 and x_2, respectively. Then, independently of whether free

[2] In Section 3, we exceptionally refer to a few specific examples in which increasing returns to scale prevail.

trade or autarky prevails, there is a system $(1, r_i(x_i), w_i(x_i))$ of relative prices for the competitive good y and the factors in country i such that the competitive firm maximizes profits by producing $y_i = Y_i(x_i)$, the strategic firm minimizes costs given the relative factor prices, and the factor markets clear. The optimal factor combination of the strategic firm in country i is $(k_i(x_i), l_i(x_i))$ and that of the competitive sector is $(\bar{K}_i - k_i(x_i), \bar{L}_i - l_i(x_i))$. Thus, x_i determines the system $(1, r_i(x_i), w_i(x_i))$ of relative prices for the competitive goods (y, k, l) in economy i together with the production plans. The functions $k_i(x_i), l_i(x_i), r_i(x_i)$, and $w_i(x_i)$ are assumed to be C^1. Under free trade, aggregate consumption in each country and the relative price of x and y are determined on the world market for the consumption goods. In the case of autarky, market clearing in a country determines consumption and the relative price of x and y in that country.

We describe the marginal rates of transformation $MRT_i(x_i) = -Y_i'(x_i)$ with the aid of $k_i(x_i), l_i(x_i), r_i(x_i)$, and $w_i(x_i)$. Since $Y_i(x_i) = G(\bar{K}_i - k_i, \bar{L}_i - l_i))$ the marginal rate of transformation $MRT_i(x_i)$ equals

$$\partial_1 G(\bar{K}_i - k_i(x_i), \bar{L}_i - l_i(x_i)) \cdot k_i'(x_i) + \partial_2 G(\bar{K}_i - k_i(x_i), \bar{L}_i - l_i(x_i)) \cdot l_i'(x_i).$$

Replacing each marginal product by the corresponding factor price in units of the competitive good y we obtain

$$MRT_i(x_i) = r_i(x_i) k_i'(x_i) + w_i(x_i) l_i'(x_i)\,. \tag{1}$$

Observe that only relative prices play a role in the previous part of the description of the model. This is due to the fact that the variables (x_1, x_2) are treated parametrically without regard to strategic considerations. The competitive firms maximize profits whereas the strategic firms have been only assumed to minimize costs up to now. To do so, the relative price systems $(1, r_i(x_i), w_i(x_i))$ of the competitive goods suffice. The price normalization problem enters when the objectives of the strategic firms are specified.[3]

As usual in the literature, we assume that the strategic firms also maximize profits. There are many ways in which profits can be defined. As illustrated in the introduction, the quantity supplied by a strategic firm engaged in Cournot competition depends on how the firm measures profits. Let $b = (\tilde{x}, \tilde{y}, \tilde{k}, \tilde{l}) \gneqq 0$ be any commodity basket. In principle, profits can be measured in units of b. In this case, profit maximization amounts to maximizing the number of units of the basket b that can be bought out of profit income. Therefore, one is led to ask which choice of b lies in the interest of the firm's shareholders.

[3] In contrast to the partial equilibrium literature, we avoid to speak of profits or of the price of an individual good without making the economic interpretation explicit. We distinguish between relative prices of several goods or factors and prices that are normalized with respect to a particular good or basket of goods.

The set of admissible baskets b can be narrowed down by the following argument. It is apparent that profits should not be expressed in units of a basket of the form $b = (0, 0, \tilde{k}, \tilde{l})$ because this would be justified only if the shareholders of a strategic firm were not interested in the consumption of goods x and y. Therefore, one can rule out that $\tilde{x} = \tilde{y} = 0$. On the other hand, we assume that shareholders do not own factors and, therefore, do not receive factor incomes. Since shareholders do neither consume nor supply factors we have only to consider baskets of the form $b = (\tilde{x}, \tilde{y}, 0, 0) \gneqq 0$. Observe that only the relative size of $\tilde{x}$ and $\tilde{y}$ matters for the objective of the firm. We parameterize a bundle of the type $b = (\tilde{x}, \tilde{y}, 0, 0)$ by $\beta \in [0, 1]$ and write $b = (\beta, 1 - \beta, 0, 0)$ and assume

Assumption (A2). *The value of the commodity basket* $b = (\beta, 1 - \beta, 0, 0)$ *with* $0 \leq \beta \leq 1$ *is normalized to 1.*

In a substantial part of the literature, for instance, in Markusen (1981) and in Wong (1995), profits are measured in units of the competitive good y, that is to say, prices are normalized with respect to the bundle $b = (0, 1, 0, 0)$. We think that there are no compelling reasons to do so, because the shareholders of a firm consume x as well as y. Often utility functions are used in which x and y are treated symmetrically. To illustrate the point, assume that bread and butter are complements and that butter is provided under competitive conditions whereas bread is supplied by large producers that exert market power. Why should the producer of bread measure profits in terms of butter rather than slices of bread with butter?

It has been argued that the choice of a numéraire or a basket b is of minor importance if one is only interested in qualitative features of a model such as the direction of a trade flow whose precise magnitude is of minor importance. However, specific examples presented in Section 3 show that a change of the basket b in which profits are measured can have significant consequences. Furthermore, we discuss and use a concept in Section 5, called real wealth maximization, that gives rise to an endogenous way of measuring profits such that profit maximization lies in the interest of the shareholders.

We describe the consumption side of the economy in detail. The residents in each country can be divided into two disjoint groups, shareholders of the strategic firm and factor owners. Ideally, a strategic firm is supposed to exert its market power in favor of its shareholders. Since shareholders do not receive factor incomes, they are affected by the strategy of their firm only as far as their profit income and their consumption expenditures are concerned.

The aggregate consumption in a country does not depend on how income is distributed among its inhabitants. This is achieved by assuming that all inhabitants have the same linearly homogeneous utility function.

Assumption (A3). *In each country, there is a representative consumer who generates the country's aggregate demand. Both representative consumers have the same homothetic, strictly quasiconcave utility function* $u(x, y)$ *defined on* $\mathbb{R}^2_+$. *More precisely, for* $(x, y) \gg 0$, *the marginal rate of substitution is a function* $g(x/y) = \partial_1 u(x, y)/\partial_2 u(x, y)$ *of* x/y *that has a negative derivative* g'.

There is no need to specify the utility function u further unless specific examples are considered. According to Assumption (A3), the market clearing price p_x of good x in units of y is determined as follows. Consider the strategy profile (x_1, x_2), which determines the aggregate output vector $(x, y) = (x_1 + x_2, y_1 + y_2)$ where $y_i = Y_i(x_i)$. The market clearing price $p_x(x_1, x_2)$ of the strategic good in units of y is given by $p_x(x_1, x_2) = g(x/y)$.

The literature on international trade with oligopolistic competition concentrates on equilibria in which a strategic firm maximizes its profit while disregarding the factor price changes induced by a variation of its output. According to Assumption (A1), a strategic firm does not select its factor combination in a strategic manner. Factor markets are perfectly competitive and the production in each country i is efficiently arranged since the firms in country i minimize costs with respect to the factor prices $(r_i(x_i), w_i(x_i))$ associated with x_i. It is common in the literature on international trade to stipulate that the strategic firms have the following conjectures: If the strategic firm in country i chooses the strategy x_i it anticipates $(r_i(x_i), w_i(x_i))$ correctly. However, if the firm ponders about the profitability of a potential deviation from x_i to x_i' it does not take into account that the factor prices will adjust to $(r_i(x_i'), w_i(x_i'))$. We call the equilibria based on these conjectures *fixed factor price equilibria* or FFPE.

In the literature, FFPE in the y-normalization have been analyzed, but the concept is easily extended to other normalizations. The y-normalization presents the polar case $\beta = 0$. In Section 3, we examine in which way the parameter β affects equilibria when we move to the opposite case, the x-normalization. There we show that the assumption of fixed factor prices makes the price normalization problem particularly severe.

Let $(p_x, 1, r, w)$ denote a price system in which p_y is normalized to 1. The corresponding price system in the b-normalization is obtained by setting the value of $b = (\beta, 1 - \beta, 0, 0)$ identically equal to 1, i.e.,

$$(p_x^b, p_y^b, r^b, w^b) = \frac{1}{\beta p_x + (1 - \beta)}(p_x, 1, r, w)\,.$$

In particular, the market clearing price of x in the b-normalization is given by

$$p_x^b(x_1, x_2) = \frac{p_x(x_1, x_2)}{\beta p_x(x_1, x_2) + (1 - \beta)}\,.$$

where $p_x(x_1, x_2)$ denotes the market clearing price of x in the y-normalization.

We define the concept of an FFPE with respect to the b-normalization in case of free trade.[4]

Definition 1. *A* fixed factor price equilibrium or FFPE *in the b-normalization consists of a strategy profile* (x_1^*, x_2^*) *with associated factor requirements* $(k_i^*, l_i^*) = (k_i(x_i^*), l_i(x_i^*))$ *and factor prices* (r_i^{*b}, w_i^{*b}) *such that, for each* $i = 1, 2$ *and all production plans* $(x_i, k_i(x_i), l_i(x_i))$,

$$p_x^b(x_i, x_{-i}^*)x_i - r_i^{*b}k_i(x_i) - w_i^{*b}l_i(x_i) \leq p_x^b(x_i^*, x_{-i}^*)x_i^* - r_i^{*b}k_i^* - w_i^{*b}l_i^*, \quad (2)$$

where x_{-i}^* *denotes the strategy of i's opponent.*

Observe that the factor prices on the left hand side of (2) are the constants r_i^{*b}, w_i^{*b} rather than the values $r_i^b(x_i), l_i^b(x_i)$ associated with x_i. In the following definition of a variable factor price equilibrium (VFPE) all prices and quantities are adjusted to their correct level and formal inconsistencies are avoided.[5] In this paper, emphasis is placed on FFPE because of their dominant role in the international trade literature. We state the first order conditions for FFPE as well as VFPE. Their proofs are relegated to the Appendix.

Proposition 1. *The first order conditions for an FFPE are*

$$MRT_i(x_i) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2)x_i - \frac{\beta \partial_i p_x(x_1, x_2)}{\beta p_x(x_1, x_2) + (1 - \beta)} p_x(x_1, x_2)x_i. \quad (3)$$

Definition 2. *A* variable factor price equilibrium or VFPE *in the b-normalization consists of a strategy profile* (x_1^*, x_2^*) *with associated factor requirements* $(k_i^*, l_i^*) = (k_i(x_i^*), l_i(x_i^*))$ *and factor prices* (r_i^{*b}, w_i^{*b}) *such that, for each* $i = 1, 2$ *and all production plans* $(x_i, k_i(x_i), l_i(x_i))$,

$$p_x^b(x_i, x_{-i}^*)x_i - r_i^b(x_i)k_i(x_i) - w_i^b(x_i)l_i(x_i) \leq p_x^b(x_i^*, x_{-i}^*)x_i^* - r_i^{*b}k_i^* - w_i^{*b}l_i^*, \quad (4)$$

where x_{-i}^* *denotes the strategy of i's opponent.*

Proposition 2. *The first order conditions for a VFPE are*

$$MRT_i(x_i) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2)\, x_i + A_i(x_i) + B_i(x_1, x_2)\,, \quad (5)$$

where

$$A_i(x_i) = -[r_i'(x_i)k_i(x_i) + w_i'(x_i)l_i(x_i)] \quad (6)$$

and

$$B_i(x_1, x_2) = \frac{-\beta \partial_i p_x(x_1, x_2)}{\beta p_x(x_1, x_2) + (1 - \beta)} \left(p_x(x_1, x_2)x_i - r_i(x_i)k_i(x_i) - w_i(x_i)l_i(x_i)\right). \quad (7)$$

[4]Adjusting the definition of an FFPE to the case of autarky is straightforward.

[5]This equilibrium concept is an adaptation of a Cournot-Walras equilibrium in the sense of Gabszewicz and Vial (1972) to the present framework.

In the y-normalization, $B_i(x_1, x_2)$ vanishes since $\beta = 0$. In this case, the difference between formulas (3) and (5) reduces to the occurrence of $A_i(x_i)$ in (5). Clearly, $A_i(x_i)$ becomes zero if the factor price changes $r_i'(x_i)$ and $w_i'(x_i)$ are assumed away. More formally, we state[6]

Remark 1. *In the y-normalization, the assumption $A_i(x_i) = 0$ is equivalent to the assumption that the derivative of the total factor income $r_i(x_i)\bar{K}_i + w_i(x_i)\bar{L}_i$ in country i vanishes.*[7]

Proof. First we show that the following equation holds for the competitive sector:

$$r_i'(x_i)(\bar{K}_i - k_i(x_i)) + w_i'(x_i)(\bar{L}_i - l_i(x_i)) = 0\,. \tag{8}$$

Since the competitive sector has constant returns to scale, its profits equal zero for every strategy x_i, i.e.

$$Y_i(x_i) = r_i(x_i)\,(\bar{K}_i - k_i(x_i)) + w_i(x_i)\,(\bar{L}_i - l_i(x_i))\,. \tag{9}$$

Differentiating (9) we get[8]

$$Y_i' = r_i'\,(\bar{K}_i - k_i) + w_i'\,(\bar{L}_i - l_i) - r_i\,k_i' - w_i\,l_i'\,. \tag{10}$$

Equation (8) follows from (1) and (10).

Equation (8) implies that $A_i(x_i) = 0$ holds if and only if the derivative of the factor income $r_i(x_i)\bar{K}_i + w_i(x_i)\bar{L}_i$ vanishes. □

3 Where does the price normalization matter?

To understand the role of the price normalization problem it is helpful to discuss specific examples. Unless stated otherwise, our examples are based on the Cobb-Douglas production functions $F(k_i, l_i) = k_i^{1/3} l_i^{2/3}$ and $G(k_i, l_i) = k_i^{2/3} l_i^{1/3}$ and the CES utility function $u(x, y) = (\sqrt{x} + \sqrt{y})^2$. This utility function gives rise to the market clearing prices

$$p_x(x_1, x_2) = \frac{\sqrt{Y_1(x_1) + Y_2(x_2)}}{\sqrt{x_1 + x_2}} \tag{11}$$

in the duopoly case and $p_x(x_i) = \sqrt{Y_i(x_i)/x_i}$ in the autarky case. These examples fit into the framework described above.

[6]A similar statement can be formulated for other normalizations.

[7]In an FFPE, a strategic firm behaves as follows: The perceived demand function is based upon the correct factor income that is generated if factor prices adjust. The perceived cost function, however, is based on the assumption that factor prices do not adjust. Therefore, the factor income that is generated differs from the factor income that is spent.

[8]For convenience, we often drop the arguments if no ambiguity arises.

A major goal of the theory of international trade is to explain the direction of trade flows. Thus, we illustrate the *sensitivity of the direction of a trade flow* with respect to the price normalization. Assume that country 1 is endowed with $K_1 = 10, L_1 = 8$ and country 2 with $K_2 = 11, L_2 = 10$. We compare the traditional y-normalization with the symmetric normalization based on the bundle $b = (1/2, 1/2, 0, 0)$. In the y-normalization, country 1 exports the strategic good x. However, if the $\beta = 1/2$-normalization is adopted, country 1 imports the strategic good. In this example, redirection of the trade flow due to a change of the normalization occurs for FFPE as well as for VFPE.

The equilibrium concept typically used in the trade literature with oligopolistic firms is the FFPE in which firms do not take factor price adjustments into account. One may ask whether this simplification leaves the directions of trade flows invariant. In the following example, we use the y-normalization. The trade flow changes its direction if the FFPE is replaced by the VFPE. The example is a slight modification of the previous one. Country 1 is now endowed with $K_1 = 10, L_1 = 7$ and country 2's endowment $K_2 = 11, L_2 = 10$ remains unchanged. As before, country 1 exports the strategic good x in an FFPE. In a VFPE, though, country 1 exports the competitive good y.

The literature on international trade with oligopolistic firms also deals with the case of increasing returns to scale in which the existence of free trade equilibria is not always warranted. Therefore, we address the question of whether the *existence of a free trade equilibrium* depends on the normalization. The example is based on the production functions $F(k_i, l_i) = (k_i^{1/3} l_i^{2/3})^c$ and $G(k_i, l_i) = k_i^{2/3} l_i^{1/3}$ with $c = 1.1$. It turns out that the existence of free trade FFPE with nonnegative profits for all firms depends on the normalization which is chosen. To be specific, let $K_1 = L_1 = 1$ and $K_2 = L_2 = 8$. Then, in the y-normalization there is a free trade FFPE in which both strategic firms make positive profits. However, in the $\beta = 1/2$-normalization the first order conditions for an FFPE are satisfied at a point where the profit of the strategic firm in country 1 is negative. Hence, no free trade FFPE exists in this normalization. The strategic firm in country 1 will stay out of the market in order to avoid losses if it cannot exert enough political influence to obtain a sufficient protection in form of trade barriers.

We return to the case of constant returns to scale and consider large differences in size between two countries with proportional factor endowments. More precisely, let the small country 1 have factor endowments $\bar{K}_1 = \bar{L}_1 = 1$ and let the large country 2 be endowed with $\bar{K}_2 = \bar{L}_2 = n$. The small country exports x in exchange for y and one is led to ask: *Is the small country fully specialized* in the production of x if n becomes sufficiently large? The answer to this question depends on how prices are normalized.

Observe that the choice of the equilibrium concept becomes important for large n for the following intuitive reason. Consider, for simplicity's sake, the

y-normalization. Since the small country becomes more and more negligible, the price p_x converges to the autarky equilibrium price p_x^{aut}. In addition, the influence of the small country on world market prices vanishes in the limit, that is to say, $\partial_1 p_x$ approaches zero. Therefore, the first order condition (3) for an FFPE in country 1 reduces to the degenerate formula $MRT_1(x_1) = p_x^{aut}$ if factor price changes are not taken into account. Since $\partial_1 p_x$ vanishes for n tending to infinity, the change of the factor prices becomes fundamental. For this reason, we consider VFPE in the present discussion.

If n becomes large country 2 is approximately autarkic since the trade flow per capita becomes negligible. In the case of the y-normalization, $p_x^{aut} \approx 2.21$ and the corresponding output ratio $(x_2/y_2)^{aut} \approx 0.20$. If prices are normalized with respect to $b = (0.5, 0.5, 0, 0)$ the price of x in units of y becomes $p_x^{aut} \approx 1.75$ and $(x_2/y_2)^{aut} \approx 0.26$. The output ratio (x_1/y_1) chosen in the small country for large n is less obvious. A numerical computation shows that the small country becomes fully specialized if n reaches 48 and the y-normalization is used. By contrast, if the value of the basket $b = (0.5, 0.5, 0, 0)$ is normalized to 1, specialization never takes place. More generally, one can show that country 1 becomes fully specialized in the production of x for sufficiently large n if p_x^{aut} exceeds a critical value of approximately 1.89. Such is the case for all normalizations with $\beta \geq 1/3$.

Finally, we address the question of why an increase in the parameter β raises social welfare. For that purpose, we examine in which way β affects equilibria when we move from the y-normalization to the opposite polar case, the x-normalization. The effect can most easily be explained in the case of autarky.[9]

Proposition 3. *If factor prices are kept fixed and the relative weight of good x in the basket $b = (\beta, 1-\beta, 0, 0)$ underlying the price normalization is raised, the optimal strategy of a strategic firm increases. Even in the case of autarky, the FFPE approaches the* Walrasian equilibrium *if β approaches 1. In particular, $p_x(x) = MRT(x)$ at an FFPE if $\beta = 1$.*

Proof. The first order condition (3) can be restated in the case of an autarkic country as

$$MRT(x) = p_x + \frac{dp_x}{dx} x \left(1 - \frac{\beta p_x}{\beta p_x + (1-\beta)}\right).$$

The term in parentheses is positive for $\beta < 1$ and converges monotonically to zero if β tends to 1. In a Walrasian equilibrium, $p_x = MRT(x)$. The wedge between the market price p_x and the Walrasian price decreases monotonically if β approaches 1 and vanishes in the limit. □

Remark 2. *If factor endowments are proportional and the x-normalization is approached, all potential gains from trade become negligible at an FFPE. In the limit, there is no trade between a small and a large country.*

[9]Adjusting the definition of an FFPE to the case of autarky is straightforward.

The fact that an FFPE becomes Walrasian if the x-normalization is approached can be explained on an intuitive level as follows. By considering factor prices in the b-normalization as fixed, a strategic firm links its output price p_x to its input prices r and w unless x has no weight in b. If the weight of good x in the basket b grows, this link is tightened. In the x-normalization, the strategic firm takes the relative prices between its output x and its factors k and l as fixed, that is to say, the firm becomes a perfect price taker.

4 Trade flows between countries with proportional endowments and constant returns

In this section, we show that Markusen's result on the direction of trade flows between countries with proportional factor endowments is robust with respect to the choice of the normalization as well as the choice of the equilibrium concept. That is to say, in the case of constant returns the smaller country exports the strategic good x to the larger country in an FFPE as well as in a VFPE provided that both countries measure profits in units of the same basket $b = (\beta, 1-\beta, 0, 0)$ and $\beta < 1$.

We consider countries with proportional factor endowments. More precisely, suppose $(\bar{K}_2, \bar{L}_2) = \lambda(\bar{K}_1, \bar{L}_1)$ and $x_2 = \lambda x_1$ with $\lambda > 0$. Then $(k_2(x_2), l_2(x_2)) = \lambda(k_1(x_1), l_1(x_1))$ because F and G are linearly homogeneous. Since the marginal products of G are constant along a ray through the origin we get $r_1(x_1) = r_2(x_2)$ and $w_1(x_1) = w_2(x_2)$.

Remark 3. *$A_i(x_i)$ satisfies the following homogeneity property: If $(\bar{K}_2, \bar{L}_2) = \lambda(\bar{K}_1, \bar{L}_1)$ and $x_2 = \lambda x_1$ then $A_2(x_2) = A_1(x_1)$.*

Proof. Let $x_2 = \lambda x_1$. Then

$$r_2'(\lambda x_1) = r_1'(x_1)/\lambda \text{ and } k_2(\lambda x_1) = \lambda k_1(x_1)\,.$$

Hence, $r_2'(x_2)k_2(x_2) = r_1'(x_1)k_1(x_1)$. Similarly, $w_2'(x_2)l_2(x_2) = w_1'(x_1)l_1(x_1)$. □

It is apparent from formula (11) that, in a Cournot model of international trade, the market clearing price $p_x(x_1, x_2)$ of the strategic good x is typically not a function of total output $x_1 + x_2$. Since production takes place domestically a change $\Delta x_1 = -\Delta x_2$ that leaves total output unaffected alters, in general, the world market price p_x.

Remark 4. *Let $(\bar{K}_2, \bar{L}_2) = \lambda(\bar{K}_1, \bar{L}_1)$ and $(x_i, y_i) \gg 0$. Then $\partial_i p_x(x_1, x_2) < 0$ and*

$$\partial_1 p_x(x_1, x_2) \geq \partial_2 p_x(x_1, x_2) \text{ iff } MRT_1 \leq MRT_2. \tag{12}$$

In particular, if $(x_2, y_2) = \lambda(x_1, y_1)$ then $\partial_1 p_x(x_1, x_2) = \partial_2 p_x(x_1, x_2) < 0$.

Our formal analysis only relies on the production side of the model and on the fact that the market clearing price p_x satisfies (12). The utility considerations underlying (12) are irrelevant.

Proof. According to Assumption (A3) the market clearing price of x in units of y takes the form $p_x(x_1, x_2) = g((x_1 + x_2)/(Y_1(x_1) + Y_2(x_2))$. Since $g' < 0$ and $\partial_i(x_1 + x_2)/(Y_1(x_1) + Y_2(x_2)) > 0$ we have

$$\partial_i p_x = g' \frac{(Y_1 + Y_2) - (x_1 + x_2)Y_i'}{(Y_1 + Y_2)^2} < 0, \ i = 1, 2.$$

Moreover, $\partial_1 p_x - \partial_2 p_x = g' \cdot (x_1 + x_2)/(Y_1 + Y_2)^2 \cdot (-Y_1' + Y_2')$. Since $MRT_i = -Y_i'$ and $g' < 0$ the sign of $\partial_1 p_x - \partial_2 p_x$ equals the sign of $MRT_2 - MRT_1$. In particular, if $(x_2, y_2) = \lambda(x_1, y_1)$ then $MRT_1 = MRT_2$. Hence, $\partial_1 p_x = \partial_2 p_x < 0$. □

Observe that $\partial_1 p_x \neq \partial_2 p_x$ unless $(x_2, y_2) = \lambda(x_1, y_1)$.[10] On an intuitive level, this fact can be explained as follows. If $x_2 > \lambda x_1$ then $MRT_2 > MRT_1$. Hence, the world supply of the competitive good y decreases more if an additional unit of good x is produced in country 2 rather than in country 1. Therefore, the world market price p_x decreases more if the additional unit of good x is produced in country 2, that is to say, $\partial_2 p_x < \partial_1 p_x < 0$.

Now we derive Markusen's result for FFPE in the y-normalization.[11] In a free trade equilibrium, the consumption bundles in both countries are proportional to each other. Let $(\bar{x}_2, \bar{y}_2) = \lambda(x_1^*, y_1^*)$, where (x_1^*, y_1^*) is the equilibrium production in the small country 1. If $x_2^* < \bar{x}_2$, then $x_2^*/y_2^* < x_1^*/y_1^*$ and country 1 exports good x. Therefore, the small country exports good x if $x_2^* < \lambda x_1^*$.

Proposition 4. *Let $(\bar{K}_2, \bar{L}_2) = \lambda(\bar{K}_1, \bar{L}_1)$ with $\lambda > 1$. Let (x_1^*, x_2^*) be a free trade FFPE in the y-normalization with $(x_i^*, y_i^*) \gg 0$. Then the small country 1 exports the strategic good x to the large country 2 in exchange for the competitive good y.*

Proof. The first order condition for an FFPE in the y-normalization states

$$MRT_i(x_i^*) = p_x(x_1^*, x_2^*) + \partial_i p_x(x_1^*, x_2^*) x_i^*.$$

Assume by way of contradiction that $x_2^* \geq \lambda x_1^*$. Then $MRT_2(x_2^*) \geq MRT_1(x_1^*)$. By Remark 4, $\partial_2 p_x(x_1^*, x_2^*) \leq \partial_1 p_x(x_1^*, x_2^*) < 0$. Therefore,

$$MRT_1(x_1^*) = p_x + (\partial_1 p_x)x_1^* > p_x + (\partial_1 p_x)(\lambda x_1^*) \geq p_x + (\partial_2 p_x)x_2^* = MRT_2(x_2^*),$$

which contradicts $MRT_2(x_2^*) > MRT_1(x_1^*)$. □

[10] As a consequence, the elasticities of demand for the home and the foreign firms must take the differences between $\partial_1 p(x_1, x_2)$ and $\partial_2 p(x_1, x_2)$ into account. This point tends to be disregarded in the literature.

[11] Our line of argument corresponds to that in Markusen (1981) but makes use of Remark 4.

We extend Proposition 4 to normalizations based on any basket of the form $b = (\beta, 1-\beta, 0, 0)$ and to the case of variable factor prices. Costs in units of y are $C_i(x_i) = r_i(x_i)k_i(x_i) + w_i(x_i)l_i(x_i)$. Hence, marginal costs are $MC_i(x_i) = MRT_i(x_i) - A_i(x_i)$. In the case of VFPE, we assume that marginal costs are increasing. This corresponds to the assumption that MRT_i increases in the case of FFPE because A_i vanishes if factor prices are held fixed. Remember that, for fixed factor prices, the trade flow peters out when the basket β approaches 1 according to Proposition 3. Thus, we restrict ourselves to $0 \leq \beta < 1$ if factor prices are held fixed.

Theorem . *Assume* (A1) *to* (A3) *and let* $(\bar{K}_2, \bar{L}_2) = \lambda(\bar{K}_1, \bar{L}_1)$ *with* $\lambda > 1$. *Let the strategic firms in both countries maximize profits in units of the same bundle* $b = (\beta, 1-\beta, 0, 0)$, *where* $0 \leq \beta \leq 1$ *in the case of VFPE and* $0 \leq \beta < 1$ *in the case of FFPE. In the case of VFPE, we assume that marginal costs* $MC_i(x_i)$ *are positive and weakly increasing. Then the small country 1 exports the strategic good* x *to the large country 2 in equilibrium.*

Proof. Let (x_1^*, x_2^*) be a free trade equilibrium in the b-normalization. We consider the case of FFPE first. According to (3) in Proposition 1 we have

$$p_x + \Gamma(\partial_i p_x)x_i^* - MRT_i = 0,$$

where

$$\Gamma = 1 - \frac{\beta p_x}{\beta p_x + (1-\beta)} = \frac{1-\beta}{\beta p_x + 1 - \beta} > 0$$

because $0 \leq \beta < 1$. Assume $x_2^* \geq \lambda x_1^*$. By (12) in Remark 4,

$$p_x + \Gamma(\partial_1 p_x)x_1^* > p_x + \Gamma(\partial_1 p_x)(\lambda x_1^*) \geq p_x + \Gamma(\partial_2 p_x)x_2^*$$

holds at (x_1^*, x_2^*). Since $MRT_1(x_1^*) = MRT_2(\lambda x_1^*) \leq MRT_2(x_2^*)$ we obtain

$$\begin{aligned} 0 = p_x + \Gamma(\partial_1 p_x)x_1^* - MRT_1(x_1^*) &> p_x + \Gamma(\partial_1 p_x)\lambda x_1^* - MRT_2(\lambda x_1^*) \\ &\geq p_x + \Gamma(\partial_2 p_x)x_2^* - MRT_2(x_2^*) = 0, \end{aligned}$$

a contradiction.

Now we turn to the case in which (x_1^*, x_2^*) is a VFPE. According to (5) in Proposition 2 we have

$$p_x + \Gamma(\partial_i p_x)x_i^* - MC_i + \Delta(\partial_i p_x)C_i = 0,$$

where

$$\Delta = \frac{\beta}{\beta p_x + (1-\beta)} \geq 0.$$

Assume that $x_2^* \geq \lambda x_1^*$. At (x_1^*, x_2^*) we have

$$p_x + \Gamma(\partial_1 p_x)x_1^* > p_x + \Gamma(\partial_1 p_x)(\lambda x_1^*) \geq p_x + \Gamma(\partial_2 p_x)x_2^*.$$

Since $MC_1(x_1^*) = MC_2(\lambda x_1^*) \leq MC_2(x_2^*)$ we obtain

$$\begin{aligned} p_x + \Gamma(\partial_1 p_x)x_1^* - MC_1(x_1^*) >& p_x + \Gamma(\partial_1 p_x)\lambda x_1^* - MC_2(\lambda x_1^*) \\ \geq& p_x + \Gamma(\partial_2 p_x)x_2^* - MC_2(x_2^*). \end{aligned}$$

Furthermore, $C_2(x_2^*) > C_1(x_1^*) > 0$. Therefore,

$$\begin{aligned} 0 =& p_x + \Gamma(\partial_1 p_x)x_1^* - MC_1(x_1^*) + \Delta(\partial_1 p_x)C_1(x_1^*) \\ >& p_x + \Gamma(\partial_2 p_x)x_2^* - MC_2(x_2^*) + \Delta(\partial_2 p_x)C_2(x_2^*) = 0, \end{aligned}$$

a contradiction. □

Remark 5. *In the case of fixed as well as variable factor prices, a firm's optimal response to a strategy of its opponent increases if the normalization places more weight on the strategic good x. As a consequence, the strategic good tends to become cheaper in a free trade equilibrium if good x gains weight in the basket b.*

5 Real wealth maximization

A firm acting in the interest of its shareholders will neither choose the y- nor the x-normalization because its shareholders consume a combination of both goods. Consider, for example, the shareholders of firm 1 and let the strategy $\bar{x}_2$ of firm 2 be given. The aggregate demand of the shareholders of firm 1 for good x at the strategy profile $(x_1, \bar{x}_2)$ is denoted $D_1(x_1, \bar{x}_2)$. Suppose for the moment that firm 1 maximizes profits in units of y and denote its optimal strategy x_1^y. Since the firm neglects the expenditures of its shareholders on good x the supply x_1^y of x is too small and the price $p_x(x_1^y, \bar{x}_2)$ is too high. More precisely, firm 1's shareholders could afford to buy more units of the bundle $D_1(x_1^y, \bar{x}_2)$ if the firm increased its production beyond x_1^y and thereby lowered p_x. Similarly, if firm 1 maximizes profits with respect to a basket b that places a sufficiently high weight on good x, the price p_x becomes so low that an output reduction enables its shareholders to increase their aggregate consumption.

We say that firm 1 *maximizes the real wealth of its shareholders* given $\bar{x}_2$ if firm 1 chooses its strategy $\hat{x}_1$ in such a way that its shareholders cannot afford to buy more units of $D_1(\hat{x}_1, \bar{x}_2)$ at any other strategy x_1. In other words, firm 1 maximizes the real wealth of its shareholders if, by choice of $\hat{x}_1$, it maximizes profits in units of a basket b proportional to $D_1(\hat{x}_1, \bar{x}_2)$.

In order to maximize the real wealth of its shareholders the firm needs to know the proportion x/y of both consumption goods in their aggregate demand, but it needs no information on shareholders' utility. The concept of real wealth maximization can be applied as well if shareholders do not possess utility functions. To use this concept in a Cournot model of international trade it suffices to stipulate a linearly homogeneous demand function instead of a linearly homogeneous utility function. Observe that the concept of real wealth maximization is not built upon a representative agent. Shareholders may very well have heterogeneous tastes.

In the case of real wealth maximization, the commodity basket b used to normalize prices and to measure profits is not a priori given but determined endogenously. The additional complexity can easily be taken into account by adjusting the first order condition for an optimal response. In the present Cournot setting, marginal profits will typically be negative at the optimum, since an infinitesimal increase of x_1 reduces expenditures of the shareholders of firm 1 on their bundle $D_1(\hat{x}_1, \bar{x}_2)$. Real wealth maximization takes shareholders' expenditures explicitly into account. In particular, the first order condition for real wealth maximization states that the sum of marginal profits and marginal savings of the shareholders is zero [cf. E. Dierker and Grodal (1998)].

In contrast to profit maximization, real wealth maximization depends on relative prices only. Therefore, it can be expressed in any price normalization. Taking the y-normalization for the sake of convenience, marginal savings are $-\partial_1 p_x(x_1, \bar{x}_2) x_1^{S_1}$, where $x_1^{S_1}$ denotes the x-component of shareholders' aggregate demand $D_1(\hat{x}_1, \bar{x}_2)$. In the case of FFPE, the first order condition for firm i becomes, according to Proposition 1, $p_x + (\partial_i p_x) x_i - MRT_i(x_i) = (\partial_i p_x)\, x_i^{S_i}$, i.e.,

$$MRT_i(x_i) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2) \cdot (x_i - x_i^{S_i}). \tag{13}$$

The interpretation of the modified mark-up formula (13) is simple. Shareholders want their firm to exploit all other consumers but not themselves. Thus, their own demand $x_i^{S_i}$ has to be subtracted from the total demand x_i for the product if firm i evaluates its marginal revenues.

Using (13) to calculate the FFPE with real wealth maximization in the standard example presented in Section 1 for the autarky case we obtain $x \approx 0.24$ and $p_x \approx 1.82$. By contrast, $p_x \approx 2.12$ in the FFPE in the y-normalization, that is to say, the mark-up over the Walrasian price 1 is about 37% higher than in the case of real wealth maximization. In the duopoly with two countries identical to the one just considered, the FFPE with real wealth maximization is $x_1 = x_2 \approx 0.37$ with $p_x \approx 1.37$. For comparison, the mark-up in the FFPE based on the y-normalization is 18% higher.

We turn to the case of VFPE with real wealth maximizing firms. As above, the first order condition for i's optimal response has to be modified by replacing x_i by $x_i - x_i^{S_i}$. We continue to use the y-normalization to express the first order condition for real wealth maximization. Formula (5) in Proposition 2 is transformed into:

$$MRT_i(x_i) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2) \cdot (x_i - x_i^{S_i}) + A_i(x_i). \tag{14}$$

We compare the first order condition for real wealth maximization and the one for profit maximization with respect to a fixed basket $b_i = (\beta_i, 1-\beta_i, 0, 0)$ which, in principle, can be country specific. In particular, let $\beta_i = x_i^{S_i}/(x_i^{S_i} + y_i^{S_i})$ be the share of good x in the aggregate demand of the shareholders of the strategic firm in country i. It is easily seen that an equilibrium with real wealth maximizing firms is identical to an equilibrium in which firms maximize profits in the β_i-normalization. Both countries maximize profits with respect to the same basket b if firms maximize shareholders's real wealth because the composition of demand is identical in countries that differ only with respect to their size.

In the case of FFPE, real wealth maximization also amounts to maximization of profits in terms of some basket b. However, b is not given by $\beta_i = x_i^{S_i}/(x_i^{S_i}+y_i^{S_i})$. Instead, $\beta_i = x_i^{S_i}/(x_i^{S_i}+\tilde{y}_i^{S_i})$ where $\tilde{y}_i^{S_i} = p_x(x_i - x_i^{S_i})$. Observe that shareholders' demand $y_i^{S_i}$ for good y is equal to the *profit* in country i minus shareholders' expenditures on good x, whereas $\tilde{y}_i^{S_i}$ equals the *revenue* minus these expenditures. To understand why $y_i^{S_i}$ is replaced by $\tilde{y}_i^{S_i}$ compare formulas (17) and (19) in the Appendix. The last term in the first order condition (17) for VFPE contains the profit $p_x x_i - r_i k_i - w_i l_i$ as a factor. In the first order condition (19) for FFPE the corresponding factor is the revenue $p_x(x_1, x_2)x_i$.[12]

To overcome the price normalization problem in oligopolistic models with representative consumers, Kemp and Okawa (1995) suggest to maximize shareholders' utility. Assume as before that profit shares are the only source of income shareholders possess. Then a VFPE in which each firm maximizes the utility of its shareholders constitutes an equilibrium with real wealth maximization. More precisely, let D^{S_i} be the aggregate equilibrium consumption of firm i's shareholders and let b be proportional to D^{S_i}. Then firm i cannot, by unilateral deviation, enable its shareholders to buy more units of b than are contained in D^{S_i}. Discrepancies between profit and utility maximization can be overcome by an appropriate definition of profits. Observe, however, that the concept of real wealth maximization has the advantage that it can also be applied to a setting in which utility maximization is meaningless.

[12]Due to the fact that $y_i^{S_i}$ is replaced by $\tilde{y}_i^{S_i}$ in the baskets underlying real wealth maximization in the case of FFPE, proportional countries do not, in general, maximize profits with respect to the same bundle b. The problem does not arise in the case of VFPE where cost changes and factor income changes are fully taken into account.

6 Appendix

First we consider the case of a fully-fledged general equilibrium model in which factor price adjustments are taken into account. We derive the first order condition (5) for a VFPE stated in Proposition 2 for a normalization based on some bundle $b = (\beta, 1-\beta, 0, 0)$. Proposition 1 is then trivially derived by setting the derivatives of the factor prices in the b-normalization equal to zero.

6.1 Proof of Proposition 2

As shown in Section 2, the marginal rate of transformation between the two commodities x and y is given by

$$MRT_i(x_i) = r_i(x_i)k_i'(x_i) + w_i(x_i)l_i'(x_i)\,. \tag{1}$$

We use this equation to derive the first order conditions that are satisfied in a Nash equilibrium if both countries use the same basket b to normalize profits.

For simplicity, we begin with the traditional y-normalization associated with the basket $b = (0, 1, 0, 0)$. As explained in Section 2, the choice of the quantity x_i produced by the strategic firm in country i determines the factor prices $r_i(x_i), w_i(x_i)$ expressed in units of y, the factor quantities $k_i(x_i), l_i(x_i)$, and the output $Y_i(x_i)$ in country i. Furthermore, the strategy combination (x_1, x_2) determines the world market price $p_x(x_1, x_2)$ of x in units of y. Profits in the y-normalization in country i are

$$\Pi_i(x_1, x_2) = p_x(x_1, x_2)\, x_i - [r_i(x_i)k_i(x_i) + w_i(x_i)l_i(x_i)].$$

Marginal profits in units of y are

$$\partial_i\Pi_i(x_1, x_2) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2)x_i + A_i(x_i) - MRT_i(x_i),$$

where

$$A_i(x_i) = -[r_i'(x_i)k_i(x_i) + w_i'(x_i)l_i(x_i)].$$

Thus, the first order condition for an optimal response of a firm maximizing profits in units of y is

$$MRT_i(x_i) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2)x_i + A_i(x_i). \tag{15}$$

We turn to the case in which the firm maximizes profits measured in units of $b = (\beta, 1-\beta, 0, 0) \geq 0$. Absolute prices $(p_x^b, p_y^b, r_i^b, w_i^b)$ are such that

$$(p_x^b, p_y^b, r_i^b, w_i^b) \cdot (\beta, 1-\beta, 0, 0) = 1.$$

Consider a strategy profile (x_1, x_2) that gives rise to the relative price system $\pi_i(x_1, x_2)$ associated with $(p_x(x_1, x_2), 1, r_i(x_i), w_i(x_i))$ in each country i. The b-normalization assigns the following absolute price system to $\pi_i(x_1, x_2)$:

$$\pi_i(x_1, x_2) \mapsto \pi_i^b(x_1, x_2) = \alpha^b(x_1, x_2)\,(p_x(x_1, x_2), 1, r_i(x_i), w_i(x_i)).$$

where the normalization factor equals $\alpha^b(x_1, x_2) = 1/(\beta p_x + (1-\beta))$.

We derive the first order condition for an optimal response of a firm that maximizes profits in the b-normalization. The profit $\Pi_i^b(x_1, x_2)$ measured in units of b is

$$\alpha^b(x_1, x_2)\,p_x(x_1, x_2)x_i - [\alpha^b(x_1, x_2)r_i(x_i)k_i(x_i) + \alpha^b(x_1, x_2)w_i(x_i)l_i(x_i)].$$

Marginal profits in the b-normalization are

$$\partial_i\Pi_i^b(x_1, x_2) = \alpha^b p_x + \partial_i(\alpha^b p_x)x_i - [\alpha^b r_i k_i' + \alpha^b w_i l_i' + \partial_i(\alpha^b r_i)k_i + \partial_i(\alpha^b w_i)l_i]. \quad (16)$$

We put $\partial_i\Pi_i^b(x_1, x_2) = 0$, divide by α^b and use (1) to obtain

$$MRT_i(x_i) = r_i k_i' + w_i l_i' = p_x + \frac{\partial_i(\alpha^b p_x)x_i - [\partial_i(\alpha^b r_i)k_i + \partial_i(\alpha^b w_i)l_i]}{\alpha^b}.$$

Inserting the definition of α^b we obtain the following first order condition for a firm maximizing profits in units of b

$$MRT_i(x_i) = p_x + (\partial_i p_x)x_i + A_i(x_i) - \frac{\beta(\partial_i p_x)(p_x x_i - r_i k_i - w_i l_i)}{\beta p_x + (1-\beta)}. \quad (17)$$

Therefore, (5) holds and Proposition 2 is shown.

6.2 Proof of Proposition 1

We turn to the case of fixed factor price equilibria. Equation (16) above reduces to

$$\partial_i\Pi_i^b(x_1, x_2) = \alpha^b p_x + (\partial_i \alpha^b p_x)x_i - [\alpha^b r_i k_i' + \alpha^b w_i l_i'] \quad (18)$$

if the derivatives $\partial_i(\alpha^b r_i)$ and $\partial_i(\alpha^b w_i)$ are set equal to zero. If $\partial_i\Pi_i^b(x_1, x_2) = 0$ we obtain

$$MRT_i(x_i) = p_x(x_1, x_2) + \frac{\partial_i(\alpha^b(x_1, x_2)p_x(x_1, x_2))}{\alpha^b(x_1, x_2)}x_i.$$

Using the definition of α^b we obtain the following first order condition for an FFPE in the b-normalization

$$MRT_i(x_i) = p_x(x_1, x_2) + \partial_i p_x(x_1, x_2)x_i - \frac{\beta\partial_i p_x(x_1, x_2)}{\beta p_x(x_1, x_2) + (1-\beta)}p_x(x_1, x_2)x_i. \quad (19)$$

Thus, Proposition 1 is shown.

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