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The Importance of Market Size and Technology Choice

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Existence, Uniqueness, and Symmetry of Free-Entry Cournot Equilibrium: The Importance of Market Size and Technology Choice*

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Abstract

This article adds technology choice to a free-entry Cournot model with linear demand and constant marginal costs. Firms can choose from a discrete set of technologies. This simple framework yields non-existence of equilibrium, existence of multiple equilibria and equilibria in which ex-ante identical firms choose different technologies as possible outcomes. I provide a full characterization of the parameter sets for which these outcomes arise. The (non-)existence problem disappears if vertical market size is large. Non-existence is largely a 'small number' phenomenon. Asymmetric equilibria emerge either because of indivisibilities or due to similarity of different technologies in terms of the average costs realized.

JEL Classification: D43; L13

Keywords: Cournot equilibrium; existence; market size; heterogeneity; integer constraint

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1 Introduction

Existence and uniqueness of Cournot equilibrium are topics of a long and ongoing debate.[1] The problem is that even for well-behaved preferences, 'examples of duopoly models in which no Cournot equilibrium (in pure strategies) exist are easily produced' (Vives, 1999, p. 94). Typically, the approaches dealing with existence start from a given number of firms. They consider the quantity setting game, taking as given technologies, i.e. the cost functions. While firms may differ, these differences are usually given exogenously (see, for instance, Novshek, 1985). From an Industrial Organization perspective it is interesting to know whether and under what conditions both market structure and technology can be endogenized without running into existence problems. Is it easily possible to add an additional stage which deals with entry and technology choice?[2]

From the above quote as well as from the well-known problem of non-existence of a pure strategy equilibrium in research tournaments without uncertainty (see Dasgupta and Stiglitz, 1980a), one might conclude that the non-existence problem becomes even more severe. I assess how important the problem is by analyzing the simple and standard case of linear demand and constant marginal costs. As regards technology choice, I assume that firms can choose from a set of two technologies, a large-scale and a small-scale technology.

This simple framework is quite rich in terms of the patterns of existence and uniqueness of (pure strategy) equilibrium it yields. Non-existence of equilibrium, existence of multiple equilibria and equilibria in which ex-ante identical firms choose different technologies are possible outcomes depending on the parameters. I provide a full characterization of the parameter sets for which these outcomes arise.

There are two main findings with respect to non-existence of equilibrium: First, the

[1] See the recent article by Long and Soubeyran (2000) for a list of contributions.

[2] Long and Soubeyran (2000) claim that an advantage of their approach is that the equilibrium is characterized in terms of marginal costs. According to these authors, this facilitates the study of a class of two-stage Cournot games (see p. 345).

existence problem disappears if vertical market size is larger than a certain threshold. Vertical market size is measured here by the vertical intercept of the demand curve. Second, non-existence is largely a 'small number' phenomenon. One may well construct examples for which an equilibrium fails to exist for arbitrarily large numbers of firms. However, the range of parameter values where non-existence may occur is large only if the market supports only a few large-scale firms. This result is quite important in the light of the wide-spread use of duopoly models. An interesting example is a recent paper by Mills and Smith (1996). In a Cournot duopoly model with technology choice, Mills and Smith characterize conditions under which ex-ante identical firms choose different technologies. My results show that the characterization is incomplete as long as entry is not explicitly accounted for. The article also derives a condition, which provides an easy check for the existence of equilibrium.

Concerning uniqueness of equilibrium I characterize the market size range in which multiple equilibria are likely to exist. It turns out that multiplicity of equilibrium requires that the different types of firms do not differ much in terms of the average costs realized in an equilibrium in which only one technology is available. Consequently the performance, i.e., prices and market output, of very different industry structures may be almost identical, a point also made by Davis (1999).

A final topic addressed in this article concerns the question under what conditions ex-ante identical potential entrants end up employing different technologies. I derive a sufficient condition for a symmetric equilibrium to exist in which all firms choose the same technology. From an empirical point of view, the most interesting parameter values seem to be those for which this sufficient condition does not hold. In the respective range a heterogeneous industry structure arises endogenously. The article shows that the results of Mills and Smith (1996) generalize even to the case of free entry. My model provides an endogenous explanation of the differences in firm size frequently found in many industries (see, e.g., Sutton, 1998) within a framework of technology choice. By explicitly allowing for heterogeneity, it differs from the main body of the literature on technology choice and R&D activities, respectively, under Cournot competition. In that literature, most authors either directly assume that a unique and symmetric equilibrium exists (see, e.g., Okuno-Fujiwara and Suzumura, 1993) or they ensure existence of such an equilibrium by making strong assumptions on the - typically continuous - set of available technologies (see, e.g., Dasgupta and Stiglitz, 1980b). Consequently, these models cannot account for heterogeneity among firms by assumption.

The remainder of the paper is organized as follows. Sections 2 and 3 present the basic model and introduce entry. Section 4 derives a sufficient condition for a unique and symmetric equilibrium to exist. Section 5 discusses non-existence of equilibrium, coexistence of different types of firms in equilibrium and non-uniqueness of equilibrium. It proves that non-existence vanishes for large values of vertical market size and presents an example on the importance of non-existence. The example also demonstrates for which parameter values both asymmetric and multiple equilibria arise. Section 6 concludes.

2 The model

Consider an industry which produces a homogeneous product. The inverse demand function is

$$p(y) = a - \frac{y}{s}, \tag{1}$$

where p and y respectively denote the price and the aggregate demand of the product. The demand function exhibits two market size parameters, a and s. The parameter a accounts for what I call vertical market size. It measures the maximum willingness to pay for that product. The parameter s is a measure of horizontal market size. One can think of it as the number of (identical) consumers. Firms may choose from a set of two technologies, a small- and a large-scale technology. The constant marginal costs associated with the small-scale technology are $c > 0$. Firms entering with the small-scale technology incur fixed costs f_S. The marginal costs for large-scale producers are zero. Their fixed costs are denoted as f_L. Of course, $f_L > f_S$. The overall number of (active) firms is denoted by n, m describes the number of small-scale (or S-)firms and $n - m$ the number of large-scale (or L-)firms.

Firms' profits depend on the technology they have chosen. The profit functions are:

$$\Pi_j(y_1, \dots, y_n) = (a - \frac{\sum_{i=1}^n y_i}{s})y_j - cy_j - f_S, \quad j \in M \tag{2}$$

$$\Pi_i(y_1, \dots, y_n) = (a - \frac{\sum_{j=1}^n y_j}{s})y_i - f_L, \quad i \in N \setminus M, \tag{3}$$

where y_k is the output of firm $k = 1, \dots, n$. M is the set of S-firms and $N := \{1, \dots, n\}$. The equilibrium quantity of an S-firm is

$$y_S(m, n) = \frac{s(a - c - (n - m)c)}{n + 1} \quad \forall m = 1, \dots, n. \tag{4}$$

The equilibrium quantity of an L-firm is

$$y_L(m, n) = \frac{s(a + mc)}{n + 1} \quad \forall m = 0, \dots, n - 1. \tag{5}$$

Substituting y_l and y_h into (2) and (3) leads to equilibrium profits of S and L firms as a function of the respective firm numbers:

$$\Pi_S(m, n) = \frac{s}{(n + 1)^2}(a - c - (n - m)c)^2 - f_S \quad \forall m = 1, \dots, n, \tag{6}$$

and

$$\Pi_L(m, n) = \frac{s}{(n + 1)^2}(a + mc)^2 - f_L \quad \forall m = 0, \dots, n - 1. \tag{7}$$

3 Entry

Suppose entry into the market is free, and a large number of identical potential entrants exists. A potential firm can enter as an S-firm or an L-firm. In a free-entry-equilibrium (m, n) the zero-profit conditions

$$\Pi_S(m + 1, n + 1) < 0 \leq \Pi_S(m, n) \tag{8}$$

and

$$\Pi_L(m, n+1) < 0 \le \Pi_L(m, n) \tag{9}$$

must hold.

Definition 1 *A candidate equilibrium is a configuration* (m, n) *that satisfies the zero-profit conditions (8) and (9).*

A candidate equilibrium is of some interest on its own. It constitutes the equilibrium of a game with a large population of potential entrants of two different types. The types, i.e. the technology of the respective firms, are exogenous.

In our model with technology choice, an equilibrium configuration (m, n) must additionally satisfy the no-switching conditions

$$\Pi_S(m, n) \ge \Pi_L(m-1, n) \tag{10}$$

and

$$\Pi_L(m, n) \ge \Pi_S(m+1, n). \tag{11}$$

S-firms must not have an incentive to employ the L-technology, and L-firms must not have an incentive to employ the S-technology.

As the above description makes clear, I consider a two-stage game. In stage 1, firms decide on entry and technology. In the second stage, firms choose their output levels. Each equilibrium of the game with endogenous technology is, of course, an equilibrium of a game with exogenous technologies and given types. Therefore, the equilibria of the endogenous technology case also indicate possible equilibrium configurations for an environment with exogenous heterogeneity, i.e. ex-ante heterogeneity.

4 Existence of a unique and symmetric equilibrium

In this section I provide a sufficient condition for a unique equilibrium to exist in which all active firms choose the same technology. I call such an equilibrium symmetric. The derivation of the condition first proceeds in a graphical way, before it is stated and proved in more formal terms. The graphical analysis provides some intuition for the requirements of a symmetric equilibrium.

Let $T_S = c + \sqrt{f_S/s}$ and $T_L = \sqrt{f_L/s}$. T_S and T_L denote the average costs realized by an L-firm and an S-firm, respectively, in a free entry equilibrium in which only the respective technology is used.[3] $T_L = T_S$ is clearly a knife-edge case. The relation between T_S and T_L determines which of the two technologies is, roughly speaking, the efficient one.

For the graphical analysis I display the equations $\Pi_S(m, n) = 0$ and $\Pi_L(m, n) = 0$ in (n, m)-space.[4] Using equation (6), $\Pi_S(m, n) = 0$ yields $m = n(T_S/c) + (T_S - a)/c$. Using equation (7), $\Pi_L(m, n) = 0$ yields $m = n(T_L/c) + (T_L - a)/c$. The two lines intersect at $(n, m) = (-1, -a/c)$. Figure 1 depicts the two lines for $T_S > T_L$. They intersect the horizontal axis at $-1 + a/T_S$ and $-1 + a/T_L$, respectively. Note that the lines have a positive slope greater than 1.[5] Profits can be kept constant with increasing n if m, the number of small firms, increases faster than n. Profits of the respective firm types are negative in the area below the respective zero-profit line.

Figure 1 about here

Given the configuration in Figure 1 both technologies cannot coexist in equilibrium. To see this, note that the use of technology S by some firms would require a configuration (m, n) on or above the locus $\Pi_S(m, n) = 0$. Otherwise S-type firms would make losses. However, with this configuration, entry would be profitable for large-scale firms. This can be seen from the configuration at the starting point of the horizontal arrow in Figure 1. Entry of an L-firm implies that the configuration (m, n) changes in the direction of this arrow. It immediately follows that at least one firm could enter using the L-technology provided that the horizontal distance between the two lines at

[3] Average cost are calculated here neglecting the integer constraint.

[4] The following derivation ignores the integer constraint. It is taken into account in the formal derivation below.

[5] The slope of locus $\Pi_L(m, n) = 0$ is smaller than one if $c > T_L$. In this case the reduced profit function derived in equation (7) does not apply, as it would require negative output of small firms.

the respective value of m is greater than 1. The resulting configuration $(m, n+1)$ implies positive profits for the entrant. A sufficient condition for the horizontal distance between the two lines to always be greater than one is that the distance at $m = 0$ is greater than one. For future purposes I denote the respective expression as D. Thus, $D \equiv (a/T_L) - (a/T_S)$. In a sense, D gives a measure of the cost difference between the two technologies.

The above discussion shows that in equilibrium S-type firms cannot be active. To see that only L firms are active is indeed an equilibrium look at the vertical arrow in Figure 1. It starts at $(n, m) = (-1 + a/T_L, 0)$, i.e., at the free entry number of L-type firms if only this technology is used. For this configuration to be an equilibrium an L-type firm must not have an incentive to switch to the S-technology. Again, the arrow indicates in which direction the industry configuration would change in this case. If the vertical distance between the two lines for $n = -1 + a/T_L$ is greater than 1 such a move cannot be profitable. To see this, note that the resulting configuration $(1, n)$ would lie below the zero-profit line for S-technology firms. Again, the assumption $D > 1$ together with the fact that the slope of $\Pi_S(m, n) = 0$ is greater than the slope of $\Pi_L(m, n) = 0$ guarantees that the (vertical) distance is greater than 1. This establishes the existence of a symmetric equilibrium with only L-type firms. The condition on D shows what it takes in terms of a cost disadvantage in order to keep 'inefficient' firms out of the market. This is equivalent to guaranteeing the existence of a unique and symmetric equilibrium.

Uniqueness and symmetry of equilibrium requires much less in terms of the cost difference if $T_S \leq T_L$. For the respective parameter values only S-type firms can be active in equilibrium. To see this, consider the knife-edge case $T_S = T_L$. In this case the zero-profit curves coincide. Consequently, L type firms cannot be active in equilibrium. This follows from the fact that profits of all firms inclusive of the switching firm increase as soon as an L-type firm switches to the S-technology. Starting from a situation of zero profits it is always profitable for a large firm to switch to the small-scale technology.

Irrespective of what the number of L-firms is in the candidate equilibrium given n, deviation is a dominant strategy. The equilibrium is reached when only S-firms are active. Neither switching nor entry would be profitable. Switching to a large-scale technology given the total number of firms depresses profits of all firms inclusive of the switching firm. Further entry is not possible by construction of the zero-profit curves.

The graphical analysis reveals an important difference between the cases with endogenous and exogenous technology, respectively. With exogenous technology the condition $T_S \leq T_L$ is insufficient to guarantee that the 'inefficient' technology, the L-technology, is not employed in equilibrium.[6] Consider again the knife-edge case $T_S = T_L$. With exogenous technology only large firms are active is an equilibrium. The entry of neither large nor small firms is possible. Starting from a situation when only large firms are active, entry of a small firm would lead to a movement along an arrow with slope 1. As the slope of locus $\Pi_S(m,n) = 0$ is greater than 1, a small entrant cannot break even. With exogenous heterogeneity, uniqueness of equilibrium requires a sufficient distance between the two loci in the case $T_S \leq T_L$ as well. Only if the cost advantage of the 'efficient' technology, the S-technology, is sufficiently large, only large firms are active cannot be an equilibrium.

Proposition 1 states and proves the results for the endogenous technology case in more formal terms. It also takes the integer constraint into account. Therefore the value of D which is sufficient for existence and symmetry is greater than in Figure 1.

Proposition 1 *If $D \geq 2$, a unique equilibrium exists and in equilibrium all firms use the L-technology. If $T_S \leq T_L$, a unique equilibrium exists and in equilibrium all firms use the S-technology.*

Proof see Appendix.

Having derived the conditions under which a unique and symmetric equilibrium exists, I now further discuss the meaning of these conditions and how different vari-

[6] Note that only small firms are active is an equilibrium anyway as it is the equilibrium in the endogenous technology case.

ables affect them. A closely related question is how technology choice depends on the parameters in general. The graphical analysis and Proposition 1 reveal an asymmetry between the case where the S-technology and that where the L-technology is the efficient one. If the small-scale technology is efficient, the result obtained is similar to that of models with a continuum of firms (see Elberfeld and Götz, 2002). Only the efficient technology is employed in equilibrium.[7] If the L-technology is the efficient one the results differ from, for instance, a model of perfect competition with a continuum of firms. In our oligopoly model with large agents even 'inefficient' firms may be viable in the long run, if the cost difference is not too large.

The two market size parameters a and s have quite different effects as far as the two questions are concerned. Vertical market size a does not affect technology choice as it does not enter T_S or T_L. Given that $T_S > T_L$, there is a monotonic relation between a and D. Increasing a makes it ever more likely that a symmetric equilibrium exists. Horizontal market size s has a clear impact on technology choice. Increasing s eventually makes the large-scale technology superior. It is with respect to horizontal market size that the statement holds that large markets give rise to the use of large-scale technologies. Symmetric equilibria exist for small and for large values of s but not for intermediate values. In the limit, as either a or s approach infinity, a unique and symmetric equilibrium exists. Large vertical market size does not determine which technology is used. This property of the linear demand model has also been documented by Neumann et al. (2001). They show that changes in the parameter a leave firm size and firm R&D expenditures constant. Only the number of firms changes with a.

The cost parameters affect technology choice in the way one would expect. If either marginal or fixed costs of a technology decrease it is more likely that the respective technology is used. A change in a cost parameter that increases the differences in average costs makes existence of a unique and symmetric equilibrium more likely.

[7] Note that one obtains a unique equilibrium even for the knife-edge case $T_L = T_S$. This is the only case in which equilibrium is not unique in a framework with a continuum of firms!

Before turning to the question of what happens if the sufficient condition of Proposition 1 is not satisfied, I extend Proposition 1 to the case of k different technologies. For that purpose I extend the above notation in a straightforward way. Suppose technology type t, where $t = 1, \ldots, k$, has fixed costs f_t and (constant) marginal costs c_t. Let $T_t = c_t + \sqrt{f_t/s}$. Again, T_t denotes average costs in the free-entry equilibrium with technology t. Define $T_i = min\{T_1, \ldots, T_k\}$, $T_j = min\{T_1, \ldots, T_i - 1, T_i + 1, \ldots, T_k\}$ and $D_t \equiv a/T_i - (a/T_t)$ for all $t = 1, \ldots, k$, $t \neq i$.

Proposition 2 *If $D_j \geq 2$, then an equilibrium exists and in equilibrium all firms use technology T_i.*

Proof Note that $D_j < D_t$ for all $t = 1, \ldots, k$, $t \neq i, j$ by definition. The Proposition then follows immediately from the proof of Proposition 1. By the assumption on D_j a deviation to technologies with greater marginal costs and smaller fixed costs cannot be profitable. Taking into account that $D_j \geq 2$ implies $T_i < T_t$ for all $t = 1, \ldots, k$, $t \neq i$, technologies with lower marginal but higher fixed costs cannot be profitable either. □

The condition employed in Proposition 2 is more restrictive than the respective condition of Proposition 1. The main purpose of Proposition 2 is to show that the above arguments easily extend to more general cases. Two consequences of Proposition 2 are worth mentioning. First, technology choice and therefore industry structure may change quite often as a function of horizontal market size s. Of course, this requires that the various technologies are important in the sense that they provide the minimum average costs for some values of s. Second, the range for which the sufficient conditions for existence of a unique and symmetric equilibrium does not hold increases if more technologies exist. Thus, it is even more important in the case with k technologies to examine what happens if the sufficient condition is not satisfied. I turn to this in the next section. The analysis will be constrained to the case of two technologies, which as above are labeled L- and S-technology.

5 Non-existence, non-uniqueness, and asymmetry of equilibrium

In this section I examine the outcomes in the range where the sufficient condition is not satisfied. The result that is probably the most interesting one from a theoretical and methodological view concerns the non-existence of a pure strategy equilibrium for certain parameter ranges. From an empirical point of view, the asymmetric equilibria arising in large part of the range where the sufficient condition does not hold, are well worth mentioning. They reproduce the heterogeneous structures found in many industries (see, e.g., Sutton 1998). In the relevant range ex-ante identical firms end up with different amounts of output. Thus, one obtains an endogenous explanation of firm size differences leading to an asymmetric industry structure. The final outcome I shall discuss concerns uniqueness of equilibrium. Examples show that one often obtains multiple equilibria in the range considered in this section. This is particularly true if the candidate equilibria involve several large firms. The interesting thing about these equilibria is that industry performance measured by the equilibrium price is approximately the same in the different equilibria while the implied industry configurations may be quite different.

In the following subsections, I examine the three possible outcomes in some detail. The analysis proceeds mainly by means of examples. After excluding the range for which general results are easy to derive, general results are hardly possible. There is one important exception, however, as far as non-existence is concerned. In the next subsection I show that an equilibrium always exists if vertical market size a is greater than some threshold value.

5.1 Non-existence of equilibrium

The intuition as to why non-existence arises is straightforward: As long as many rivals are active, the pivotal firm chooses the large-scale technology. Given this level of investment, (some) rivals do not have an incentive to enter in the first place. However, if there are less rivals, it is optimal for the pivotal firm to switch to the small-scale technology. Put in the terms of Figure 1, the scenario might appear as follows. Suppose that the candidate equilibrium is one with only large firms and that the respective number of large firms is $\bar{n}$. An equilibrium may not exist if the vertical distance between the two zero-profit lines is so small that a firm which switches the technology and moves along the vertical arrow, ends up making a positive profit with the S-technology.[8] At the new industry configuration entry of either S or L-type firms may be possible. If S-firms enter both the horizontal and the vertical distance between the zero-profit lines is greater than in the only large firm case. Therefore, it is well likely that either additional entry of a large firm becomes profitable or that a small firm could increase its profit by switching technologies. Both in these situations and in the case where entry with the L-technology becomes profitable once an 'incumbent' switches, the resulting configuration is one with $\bar{n}$ large firms and (at least) one small firm. Due to the assumption that only large firms are active constitutes a candidate equilibrium, we know that the small firm cannot be viable. Once it exits, we are back at the situation in which switching technologies becomes profitable for one large firm. An equilibrium does not exist.

In the following I first present an example of non-existence, which illustrates how important non-existence is in terms of the range of parameter values for which non-existence occurs. Second, I show that non-existence does not arise for 'large' values of vertical market size a. I also discuss the parameters which determine how large a must be in order to guarantee existence.

The example depicts the regions in the (a, s)-space where non-existence applies. The parameter values I use are: $f_S = 5, c = 2, f_L = 2050$. The values of the fixed costs are chosen such that the maximum number of large firms, for which non-existence can arise, equals 20. It is explained below how this number is derived. Figure 2 depicts necessary conditions for non-existence. The triangular-shaped areas are the areas where

[8]The vertical distance at $n = (a/T_L) - 1$ must be smaller than 1.

non-existence may occur. The respective candidate equilibrium starts with one large firm for values of a of about 4 (the largest 'triangle') and ends with 19 large firms for values of a around 40 (the respective area looks more like a dot in the diagram). In all 'triangles' there are (a, s) vectors for which non-existence arises.[9]

Figures 2 and 3 about here

The conditions are explained in more detail by describing Figure 3, which provides a detail of Figure 2. Figure 3 depicts the area of non-existence for the case where the candidate equilibrium is one with one large firm.

Locus s_1 gives the value of s such that the large firm (or one of the $\bar{n}$ large firms, respectively) is indifferent between the two technologies, given the firm is the sole incumbent (given there are $\bar{n}$ large incumbents). Thus, s_1 is derived from the condition $\Pi_S(1, \bar{n}) = \Pi_L(0, \bar{n})$, which yields

$$s_1 = \frac{(1+\bar{n})^2 \, (f_L - f_S)}{2\, a\, c\, \bar{n} - c^2\, \bar{n}^2}. \tag{12}$$

Below locus s_1, the firm would switch to the small-scale technology. As explained above, such a switch may cause non-existence as entry of either large or small firms might become possible.

Locus A gives the value of s, such that a large firm is viable ($\bar{n}$ large firms are viable). It solves the equation $\Pi_L(0, \bar{n}) = 0$. Locus A captures the notion of a candidate equilibrium with $\bar{n}$ large firms.

Locus C captures the constraint 'only small firms are active'. It solves the condition $\Pi_S(n^*, n^*) = \Pi_L(n^* - 1, n^*)$, where n^* denotes the zero profit number of small firms in an 'only small firms are active' equilibrium. For values of s above locus C deviating from the choice of the S-technology is always profitable for a single firm.

The emergence of non-existence in the area between $s1$ and C is straightforward. Below locus $s1$ 'one large firm is active' is not an equilibrium, since the firm would have

[9]Non-existence occurs, for instance, for $(a, s) = (40.079, 510.587)$. The free entry number of firms in the case where only the L-(S-)technology is available is 19 (383) firms. To see that the equilibrium fails to exist note that $\Pi_S(1, 19) = .517 > \Pi_L(0, 19) = .423$ and that $\Pi_S(2, 20) = .004 < \Pi_L(1, 20) = .038$.

an incentive to switch to the S technology. The only remaining candidate equilibrium is one with 'only S-type firms are active'. Above C this cannot be an equilibrium as well. The result is non-existence.

Note that locus A rather than some extension of locus C limits the non-existence range for $a \leq (n-m+1)c$, i.e. for $a \leq 4$ in the case of one large firm. The reason is the following: In a situation where only small firms are active, a large firm could always profitably enter for values of s above locus A. All small firms would stop producing as prices would be below their marginal costs. Therefore, the L-firm could break even. A large firm will be active even though switching from the S-technology to the L-technology were not profitable. The latter result is due to the integer constraint which implies positive profits of the S-type incumbents in a free entry equilibrium.

The above argument for non-existence applies to the left of locus V_1 (i.e., the area indicated by the left arrow originating at NE). The (dashed) locus V_1 describes the vectors (a, s) at which a small firm becomes viable, given that one ($\bar{n}$) large firm(s) is(are) active. Thus, to the right of this line 'one large firm is active' is no longer a candidate equilibrium. Similarly, locus V_2 describes the vectors (a, s) at which two small firms become viable. Both between the loci V_1 and V_2 and to the right of V_2 non-existence arises. The respective areas are indicated by the center and the right arrow. They apply below the loci $s_{1,1}$ and $s_{1,2}$, respectively. Along $s_{1,1}$ ($s_{1,2}$) a firm is indifferent between the S and the L-technology given that one (two) small rival(s) is (are) active. The non-existence area indicated by the right arrow is so small that it is hardly visible. For parameter values where three or more small firms were viable non-existence cannot arise. Similarly, the constraint provided by locus V_1 is binding for small values of $\bar{n}$ only, namely for the case of one to four large firms. Figure 2 omits the non-existence areas to the right of the (dashed) locus V_1 for the cases in which two to four large firms are active, the respective areas would hardly be visible. For the purpose of the example it is sufficient to know that non-existence also arises in a small part of the area made up by V_1, s_1, and B. Locus B is the only part of Figure 2 which is not yet explained. I explain the construction of this locus by using Figure 4.

Figure 4 about here

Figure 4 provides another detail of Figure 2. It depicts the area of non-existence for the case where the candidate equilibrium is one with four large firms. Two things need to be explained, the array of curves drawn in grey and locus B. The grey lines are conditions which are analogous to s_1. They solve the equation $\Pi_S(m, \bar{n}+m-1) = \Pi_L(m-1, \bar{n}+m-1)$. For $m = 1$, one obtains the condition determining s_1 (see equation 12). For $m = 2$, the curve next to locus s_1 applies. Using the notation of Figure 3, this curve could be denoted as $s_{1,1}$. Between this curve and s_1 non-existence arises because the pivotal firm would switch to the S-technology if all firms use the L-technology but would switch back to the L-technology if entry occurs due to the first switch. Below the $m = 2$ locus entry of a single S-entrant would not induce a switch by the pivotal firm to the L-technology. However, the switch occurs if a second S-firm would enter. Analogous reasoning holds for the other curves ($m = 3, 4, ...$, i.e., $s_{1,2}, s_{1,3}, ...$). The number of small entrants is always incremented by one. The adjacent pairs of these curves give the non-existence area for different configurations with $\bar{n}-1$ large firms and $m-1$ small firms. These areas are bounded at the left by locus A. For values of s below locus A, $\bar{n}$ large firms are not viable. Thus, the switch to the L-technology, which causes non-existence, can no longer occur. Locus B is the lower envelope of the array of grey curves. It connects the points where the adjacent pairs of curves intersect.[10,11]

A final condition for non-existence to arise concerns the viability of potential entrants once a large incumbent switches to the small-scale technology. As mentioned above, non-existence requires that entry occurs if an incumbent switches. The importance of the viability condition becomes apparent in the case of a candidate equilibrium

[10] As shown in the appendix, non-existence arises only to the left of the intersections.

[11] The maximum number of m I consider in the construction of B is the free entry number of small firms if only the S-technology were available. This procedure yields an upper bound for the range in which an equilibrium does not exist. See also the next paragraph.

with 20 large firms. While a triangle (s_1, A, B) exists for this case, non-existence does not arise because neither a small nor a large entrant would be viable if the pivotal firm were to switch to the S-technology. This requirement for non-existence is not satisfied in part of the triangle with 19 large firms as well. For smaller numbers of large firms the 'triangles' are a good approximation for the actual non-existence area. For instance, in the case underlying Figure 4, 16 small firms would at least be viable if three large firms were active. The respective grey curves (e.g.,$s_{1,13}$) are very close to the intersection of locus A and locus B. At least in Figure 2, the difference between the approximation and the actual non-existence area would not be visible.

The above example indicates two things. First, non-existence does not seem to arise for large values of vertical market size a. Indeed, Proposition 3 below shows that, given cost parameters, a threshold value a^* exists such that non-existence cannot occur for $a > a^*$. Second, it demonstrates that the area where non-existence occurs is 'small'. This holds even in a case where, due to a large difference between f_S and f_L, non-existence may arise for candidate equilibria with as many as 19 large firms. That is, even though a^* is 'large', non-existence cannot occur in a number of intervals $[a_1, a_2[$, where $a_2 < a^*$. The area of the non-existence triangles is small in general. Exceptions are the triangles for the cases with a small number of large firms in the candidate equilibrium. If this number is five or smaller than five, such intervals no longer exist and the area of the triangles becomes 'large'. In this sense, non-existence is a small number problem.

Turning back to the general case I show now that an equilibrium of the whole game always exists in markets of a sufficiently large vertical size. What is 'sufficient' is shown below. The relevance of the parameter a should be clear from the discussion of Proposition 1. We know that a symmetric equilibrium does not exist once horizontal market size s is such that we are slightly above the knife-edge case $T_L = T_S$. It is natural to ask whether non-existence may also arise for arbitrary values of a. Proposition 3 shows that this is not the case.

Proposition 3 *Given the cost parameters of the model, an a^* exists such that for all $a > a^*$ an equilibrium of the two-stage game exists.*

Proof see Appendix.

It is instructive to look at how the proof proceeds.[12] The proof starts with the necessary conditions for a candidate equilibrium (m, n) **not** to be an equilibrium of the game (equations (30) and (31)). The conditions yield a maximum value for a denoted as $\bar{a}$ such that non-existence can arise. I then show that for sufficiently large n, n firms are not viable, given the market size vector ensuring non-existence. This is a contradiction to the assumption that the configuration is a candidate equilibrium. This result yields an intuitive explanation as well. Remember that changes in a leave the relative profitability of the two technologies unaffected. Therefore, one would expect that an increase in the number of large firms changes the condition for switching to the small technology, loosely speaking, in a proportional way. Increases in the number of firms, however, imply that the market becomes more competitive in the sense that price-cost margins decrease. Thus, a given change in the number of firms requires a more than proportional increase in market size in order for the larger number of firms to be viable.[13] As a result, the conditions for non-existence to arise eventually cease to hold as the number of (large) firms increases.

It is possible to explicitly determine the maximum number n^* of L-type firms for which non-existence may arise. It can be shown that n^* is the (positive) root of the equation[14]

$$\begin{aligned}(1 + 4\,n + 12\,n^2 + 20\,n^3 + 24\,n^4 + 16\,n^5 + 4\,n^6)\ f_S \\ - (1 + 8\,n + 20\,n^2 + 16\,n^3 + 4\,n^4)\ f_L \quad &= 0. \qquad (13)\end{aligned}$$

[12] The proof follows the construction of the non-existence areas in Figures 2 and 4 closely. Unfortunately, this makes the proof quite tedious.

[13] Actually, $\bar{a}$, capturing the non-existence condition, is strictly concave in the number of large firms (see equation (34)), whereas the zero-profit condition is linear in n.

[14] The equation derives from setting $\Pi_L(0, n)$ as defined in equation (35) in the Appendix equal to 0.

Two points deriving from equation (13) are worth being mentioned. First, n^* depends only on the ratio f_L/f_S. Second, n^* is of order $\sqrt{f_L/f_S}$. n^* may well grow without bound. However, this requires quite a large difference between the two technologies in terms of their respective fixed costs. Empirically, a ratio of $f_L/f_S \approx 400$ as in the above example seems to be quite a large number. And even in this case, n^* is only 20. The reason why n^* increases with f_L/f_S seems to be the following: switching technologies seems to be more profitable when the differences between technologies are large. Note that this is not a ceteris paribus statement. The switch can only occur in a neighborhood of the knife-edge case. That is, large differences in fixed costs require either large differences in marginal costs or a large horizontal market size s.

Given n^*, we can immediately derive a^*. Substituting n^* in equation (34) and setting $m = 1$ yields

$$a^* = c\left(n^* + 1 + \frac{3 + 2\,n^*}{2\,(-1 + n^* + n^{*2})}\right). \tag{14}$$

For $n^* \geq 2$, a^* is smaller than $c(\sqrt{f_L/f_S} + 2)$. Similar to the condition for D in Proposition 1, we obtain an easy check for the question whether an equilibrium exists. The way in which a^* depends on c, the difference in marginal costs, is straightforward. If the difference is large, a must be large in order for small firms to be viable at all. The same argument applies for large values of n^*. In order for a single small firm to produce a positive amount of output in the case of n^* L-type rivals, a must be greater than cn^* (see equation (4)).

5.2 Asymmetric and multiple equilibria

In this subsection I discuss both under what conditions firms employing different technologies may co-exist in equilibrium and the circumstances which give rise to the existence of multiple equilibria. It turns out that a rather intuitive characterization is possible based on the above example. As noted above, the empirical importance of asymmetric equilibria, i.e. of equilibria in which firms of different size are active, stems from the well established empirical finding that firm sizes differ greatly within industries (see, e.g., Sutton, 1998 and Cabral and Mata, 2001). My model stresses the importance of both lumpy technology and market size in the explanation of these facts. In what follows I describe the market size vectors for which asymmetric and multiple equilibria arise, given cost parameters.

From the above discussion of Figures 2, 3, and 4 it is clear that locus V_1 will be pivotal in determining the respective range of co-existence. To the right of V_1 a small firm is viable even if the maximum feasible number of large firms is active. Even though Figure 2 draws the respective dashed lines only for the cases of one to four large firms, similar lines exist for the other non-existence areas in Figure 2 as well. Actually, in the example a small firm may be viable for as large values of a as 1900, given the number of large firms is equal to the free entry number of large firms in a situation with only large firms. The latter number is greater than 900.[15] It is now straightforward to determine areas in which asymmetric equilibria must exist. If we were to draw locus V_1 for all the different numbers of large firms considered in Figure 2, and if we extend that up to the locus where a further large firm becomes viable (extend locus A to obtain that condition), we obtain an area with an asymmetric equilibrium certainly as long as we are above locus C.[16]

The obvious question to address now is whether the respective asymmetric equilibria are unique. We know that above locus s_1 (to be exact, above locus $s_{1,1}$ in the notation used in Figure 3), an equilibrium exists with the maximum feasible number of L-type firms and one (or a 'few') small firms. However, other equilibria may exist as well as long as the market size vector (a, s) is close to s_1. To see this, consider Figure 4. Note that $s_{1,13}$ (i.e., the grey line closest to the intersection of A and B) is much flatter than s_1. Extending this line across locus V_1, we see that for the area between s_1 and the (extended) grey line the following holds. The pivotal firm would choose the L-technology if only one (or a few) small firms were active. This yields the above

[15]For $(a, s) = (1941.91, 464.129)$ an equilibrium with one small firm and 922 large firms exists.
[16]This statement does not hold, of course, for vectors from the non-existence area.

described equilibrium with the maximum number of large firms. If, however, 15 or more small firms are active, the pivotal firm chooses the S-technology. In the resulting equilibrium, the number of large firms is one short of the maximum number. This argument applies even for vectors for which a small firm would not be viable, i.e., for vectors to the left of V_1. To see this, consider the vector $(a, s) = (14.675, 491.364)$. In this case, two equilibria exist: $(m, n) = (0, 6)$ and $(m, n) = (20, 25)$. The first equilibrium implies that a small firm is not viable, given the maximum number of large firms is active. That is, we are to the left of V_1. Obviously, we are also below one of the above mentioned (extended) grey lines (below what might be called $s_{1,20}$ using the notation of Figure 3).

Given the above argument and the shape of locus B as well as the grey lines in Figure 4, it is now straightforward to provide an upper bound $\bar{s}$ with the following property. For values of horizontal market size greater than $\bar{s}$, the resulting equilibria are unique. To construct $\bar{s}$, consider Figure 4 and note that the slope of locus B close to the intersection with A is always negative, although the curve is quite flat. If the number of small firms is quite large, the locus along which the pivotal firm is indifferent between the two technologies is only slightly negatively inclined. To be sure that the pivotal firm chooses the L-technology it is sufficient to draw a horizontal line through the maximum of locus B in terms of s. Alternatively, one may join the intersections of loci A and B to obtain an upper bound for the parameters underlying Figure 2. The resulting curve is nearly a straight, horizontal line with a value of s of about 511.

Having derived an upper bound for multiple equilibria to arise, the next step consists of the derivation of a lower bound $\underline{s}$. Proposition 1 has shown that the value of s for which $T_S = T_L$ holds is the obvious candidate. In the example the respective value of s is 463.129. This value seems to be the greatest lower bound, i.e. $\underline{s} = 463.129$. It is easy to find examples for values slightly greater than $\underline{s}$ for which multiple equilibria arise.[17]

[17] I have calculated the equilibria for $\underline{s} + 1$ and a ranging from 8 to 60. In each case multiple equilibria arise. Take, for example, $(a, s) = (20, 464.129)$. The equilibrium configurations are in this case: $(m, n) = \{(30, 37), (50, 56), (71, 76), (91, 95), (111, 114), (131, 133), (152, 153), (172, 172)\}$. Multi-

As far as the existence of symmetric equilibria is concerned, note that below locus C a symmetric equilibrium exists, in which only small firms are active. Of course, the respective equilibrium is not unique as long as $s > \underline{s}$. Locus C converges to $\underline{s}$ for large a.

The above discussion shows that there are two rather different configurations under which an asymmetric equilibrium arises. First, a range in which the resulting asymmetric equilibrium is unique. Second, an area in which multiple equilibria exist. In the latter case, at least one of the equilibria is asymmetric. The economic reason as to why these different types arise are also different. The unique equilibria arise largely due to indivisibilities of technology; entry of the more 'efficient' type of firm is not possible, as it could take place only on a large scale. On the other hand, a small firm may be able to enter the market even though its average costs are higher. Small-scale entry is possible even in the case of a small residual demand.

Multiple equilibria arise only if the firms are not too different in terms of average costs. The upper bound $\bar{s}$ determines this maximum difference. Below $\bar{s}$ the L-technology is not sufficiently superior in order to be always the first choice in the following sense. Suppose we are in a situation with less than the maximum feasible number of large firms and with a number of small firms which is just viable given the number of large firms. For $s > \bar{s}$ it is always profitable for a small firm to switch to the large-scale technology. Below $\bar{s}$ this does not hold. Between $\bar{s}$ and $\underline{s}$ the two technologies exhibit similar average costs. This makes it possible to produce different equilibria by replacing, for instance, one large firm by the number of small firms, which is sufficient to produce approximately the same output. The example mentioned in footnote 17 produces such a pattern. One large firm can be replaced by about 20 small firms. Note that the equilibrium output of a large firm is about 977, small firms produce about one twentieth, namely about 49 units of output. This result is very much in vein of Davis (1999) who provides conditions under which market output is uniquely

ple equilibria can also be found for $\underline{s} + .5$ even for as small a number of a as 12.

determined within the set of (multiple) equilibria.

A final point related to multiple equilibria, but also to my model of technology choice in general, concerns the relation between market concentration and market power, respectively, and performance. The multiple equilibria exhibit nearly identical performance in terms equilibrium prices. Prices typically differ by less than a tenth of a percentage point. However, concentration ratios and price-cost margins vary greatly. More generally, if an industry experiences a drastic restructuring due to increases in horizontal market size s, the new equilbrium is typically one with much higher concentration *and* with lower prices.

6 Conclusions

Once one takes entry and technology choice into account, (two-stage) Cournot games allow for a wide range of outcomes. Apart from a unique equilibrium with symmetric firms, co-existence of different types of firms and non-uniqueness of equilibrium may result for different parameter values. An outcome may also be non-existence even though the underlying game is one with linear demand and constant marginal costs. The article characterizes the parameter values for which the various outcomes result and evaluates their respective importance. The full characterization of the equilibrium outcomes in this article allows applied papers focussing on technology choice to evaluate the importance of different equilibrium configurations without performing a complete analysis on their own. An example of such an application is Elberfeld, Götz, and Stähler (2002). Technology choice enters their model in the shape of whether production should take place domestically or whether firms should choose a multinational production mode. The formal structure of the problem of vertical foreign direct investment Elberfeld, Götz, and Stähler consider is the same as in my model.

As regards the importance of the different outcomes, the article has shown that non-existence does not occur if vertical market size and the number of firms is large. Existence problems become important if the market supports only a few large firms.

The conclusion from this result for the duopoly cases often considered in the literature is the following: Entry should explicitly be taken into account in the respective models. This would ensure that the claimed equilibria can really result from underlying fully specified games.

Compared to non-existence, asymmetric equilibria are much more important in the sense that they apply for a larger set of parameter values. The interesting thing about the equilibria with co-existence of different firm types is that it provides an endogenous explanation of a stylized fact, namely that firms in an industry often differ with respect to their size. This explanation is based purely on indivisibilities in technology choice and on market size.

The combination of indivisibilities and of technology choice in my model yields two further conclusions. The first regards the integer constraint. In a case in which firms are large compared to the market size, lumpy technology may yield substantial profits due to the integer constraint (see Lambson, 1987). Once one allows for technology choice, the problem of excess profits is partly mitigated by the possibility of small-scale entry, even if the small-scale technology is lumpy as well. The second conclusion concerns the relation between market structure and performance. The discussion of multiple equilibria has shown that the various equilibrium configurations may differ greatly in market structure parameters like the number of firms and industry concentration. These differences map into differences in conduct parameters like price-cost margin in an intuitive and expected way. However, large variations in price-cost margins among industries, for instance, do not imply that these industries differ in performance. On the contrary, the article has shown that rather different industry structures are compatible with nearly identical equilibrium outcomes in terms of prices. Yet another example that policy prescriptions based on determinants of the industry structure only may be quite misleading.

7 Appendix:

Proof of Proposition 1

Case 1: $T_L < T_S$.

First, I show that a configuration in which S-firms are active, i.e., $m > 0$, cannot be an equilibrium. Entry with the L-technology would be profitable in this case. To see this note that $m > 0$ requires $\Pi_S(m, n) \geq 0$. Using (6), this inequality can be rearranged to yield

$$\frac{a - c - (n - m)c}{(n + 1)} \geq \sqrt{\frac{f_S}{s}} \tag{15}$$

Using the definition of T_S this expression can be simplified to

$$\frac{a + mc}{(n + 1)} \geq T_S. \tag{16}$$

Profitable entry of an L-firm requires $\Pi_L(m, n + 1) \geq 0$. Similar to the above inequality, using (7) this inequality can be written as

$$\frac{a + mc}{(n + 2)} \geq T_L. \tag{17}$$

Inverting the inequality and rewriting the r.h.s. yields

$$\frac{n + 1}{a + mc} + \frac{1}{a + mc} \leq \frac{1}{T_L}. \tag{18}$$

From equation (16) we know that the first term of (18) is smaller than $1/T_S$. Therefore, inequalty (18) is certainly satisfied if

$$\frac{1}{T_S} + \frac{1}{a + mc} \leq \frac{1}{T_L}. \tag{19}$$

holds. Rearranging and multiplying by a yields

$$\frac{a}{a + mc} \leq \frac{a}{T_L} - \frac{a}{T_S}. \tag{20}$$

This inequality is certainly satisfied as the l.h.s. is smaller than 1, while the r.h.s consists of the definition of D which by assumption is greater than 2. This proves that the S technology cannot be used in equilibrium. Additional entry of L-firms would occur.

The second step of the proof shows that a firm in a candidate equilibrium with only L-type firms does not have an incentive to switch to the S-technology. In formal terms a sufficient condition for such a switch not to be profitable is $\Pi_S(1, n^*) < 0$. Here, n^* is defined as the largest integer such that $\Pi_L(0, n) \geq 0$. To prove that switching is unprofitable I calculate the (real) numbers $\bar{n}$ and $\hat{n}$, respectively, for which $\Pi_S(1, n) = 0$ and $\Pi_L(0, n) = 0$, respectively. Using the definitions of T_S and T_L one obtains after some manipulations

$$\bar{n} = \frac{a + c}{T_S} - 1 \tag{21}$$

and

$$\hat{n} = \frac{a}{T_L} - 1 \tag{22}$$

respectively. To show that $\Pi_L(1, n^*) < 0$ it is sufficient that $\hat{n} > \bar{n} + 1$. Note that 1 must be added due to the integer constraint. Using the definitions of $\bar{n}$ and $\hat{n}$, the condition reads

$$\frac{a}{T_L} - 1 > \frac{a + c}{T_S}. \tag{23}$$

Rearranging yields

$$\frac{a}{T_L} - \frac{a}{T_S} > \frac{c}{T_S} + 1. \tag{24}$$

This inequality clearly holds. The l.h.s equals D and is therefore by assumption greater than 2. The expression c/T_S is smaller than 1. This proves the proposition as concerns the case $T_L < T_S$.

Case 2: $T_S \leq T_L$. First, I show that the L-technology cannot be an equilibrium choice. It is always optimal for a large firm to switch to the S-technology, irrespective of what the number of large firms is, if

$$\Pi_S(\tilde{m}(i) + 1, \tilde{m}(i) + i) - \Pi_L(\tilde{m}(i), \tilde{m}(i) + i) > 0. \tag{25}$$

Here i is an arbitrary number of large firms (of course smaller than the maximum feasible number). $\tilde{m}(i)$ is the maximum viable number of small firms, given there are

i large firms, i.e., it is determined by the condition $\Pi_S(\tilde{m}(i), \tilde{m}(i) + i) = 0$. Using the condition $T_L = T_S$ tedious calculations yield that the l.h.s. of (25) equals

$$cf_S(2a - c - 2ci)/(a - c - ci)^2. \tag{26}$$

This expression must be positive if small firms are to be viable at all. Thus, deviation from the L-technology is always profitable.

It remains to be shown that an S-firm does not have an incentive to switch to the L-technology if the zero profit number of S-firms n^* is active. That is,

$$\Pi_S(n^*, n^*) - \Pi_L(n^* - 1, n^*) > 0. \tag{27}$$

It turns out that the resulting expression is positive both if the integer constraint is taken into account and if it is neglected. Once one employs the condition $T_L = T_S$ to substitute for f_L, one eventually obtains in the latter case

$$\frac{(2a - 3c)cf_S + (a - c)2c^2\sqrt{f_S}\sqrt{s}}{(a - c)^2}, \tag{28}$$

which is clearly positive. Condition (27) is also positive if evaluated at $n^* - 1$ thus taking the integer constraint into account. It reads

$$\frac{c\sqrt{f_S}\,\sqrt{s}\,\left(2\,f_S - 3\,c\sqrt{f_S}\,\sqrt{s} + (a - c)\,2\,c\,s\right)}{\left(\sqrt{f_S} - (a - c)\,\sqrt{s}\right)^2}. \tag{29}$$

Omitting the term $2f_S$ in the term in brackets in the numerator, the remaining expression is positive once $n^* > 1.5$. Thus one obtains that the whole term is positive.[18] Only S-type firms are active is the unique equilibrium if $T_L \leq T_S$.

Proof of Proposition 3

The proof proceeds in several steps.

First, I derive necessary conditions for the non-existence of equilibrium. Suppose that the configuration $(m-1, n)$ constitutes a candidate equilibrium. Necessary conditions for non-existence are that configurations $(m-1, n)$ and (m, n) do **not** constitute

[18] For brevity, it is omitted to show that the derivative of condition (27) with respect to n^* is monotonous, proving that the the condition must be satisfied for the actual equilibrium number which lies in the interval $[n^* - 1, n^*]$

an equilibrium. Thus, conditions

$$\Pi_S(m,n) \geq \Pi_L(m-1,n) \tag{30}$$

and

$$\max\{\Pi_S(m+1,n+1),0\} \leq \Pi_L(m,n+1), \tag{31}$$

where $m = 1,\ldots,n.$ must hold. Condition (30) implies that an L-firm would deviate to the S-technology, thus $(m-1,n)$ cannot be an equilibrium. By condition (31) the switch would induce entry of an L-firm, thus (m,n) cannot be an equilibrium. Condition (31) implies also that $(m+1,n+1)$ cannot be an equilibrium.

Second, I show that a maximum value of $\bar{a}$ exists such that the necessary conditions for non-existence, i.e., conditions (30) and (31), can be satisfied simultaneously, given the number of firms. From conditions (30) and (31) we can derive the values of s (s_1 and s_2) such that both conditions are satisfied with equality. In terms of Figure 4, s_1 and s_2 are two adjacent grey curves. In the case of (31), I drop the 0 and take $\Pi_S(m+1,n+1)$ as the term on the left hand side. This matters only if $\Pi_S(m+1,n+1) < 0$. In this case the non-existence range is a subset of the set captured by the conditions. Solving (30) and (31) for s yields

$$s_1 \equiv \frac{(1+n)^2\,(f_L - f_S)}{2\,a\,c\,n + 2c^2 n(m-1) - c^2\,n^2} \tag{32}$$

and

$$s_2 \equiv \frac{(2+n)^2\,(f_L - f_S)}{c\,(1+n)\,(2\,a - c\,(n-2m+1))}. \tag{33}$$

(30) and (31) are satisfied for values of s such that $s_2 < s < s_1$. This follows from the fact that

$$\partial\Pi_L/\partial s > \partial\Pi_S/\partial s.$$

Calculating the value of a such that $s_1 = s_2$ we obtain

$$\bar{a} \equiv 2c + c\,(n-m) + \frac{c\,(3+2\,n)}{2\,(-1+n+n^2)}. \tag{34}$$

In terms of Figure 4, the values of $\bar{a}$ for different values of m constitute locus B. It is straightforward to show that the derivative of s_1 with respect to a evaluated at $\bar{a}$ is greater in absolute terms than the respective derivative of s_2. Thus, $s_2 < s_1$ requires $a < \bar{a}$. Therefore, $\bar{a}$ is the maximum value such that conditions (30) and (31) can be satisfied simultaneously. Note two things about the relation between $\bar{a}$ and m, the number of small firms, deriving from (34). First, for given n, $\bar{a}$ assumes a maximum for $m = 1$, i.e., in a candidate equilibrium with only large firms, in which the pivotal firm may switch from the L-technology to the S-technology. Second, for a given number of large firms, i.e. $n - m$, the case without small firms again yields the maximum value of $\bar{a}$. As a consequence of these two properties it is sufficient to consider non-existence for candidate equilibria with only large firms active. Thus, in what follows the analysis assumes $m = 1$.

Third, I show that a number of large firms n^* exist such that for $n > n^*$ the necessary conditions for non-existence, i.e., (30) and (31), cannot be satisfied simultaneously. The reason is that, starting from the values of $\bar{a}$ and s_1 associated with n, an n exists such that n firms are not viable given the underlying values of a and n. To see this, calculate the profits of L-firms in the candidate equilibrium $(0, n)$ if $a = \bar{a}$ and $s = s_1(\bar{a})$. One obtains

$$\Pi_L(0, n) = \frac{(1 + 4\,n + 2\,n^2)^2\;(f_L - f_S)}{4\,n\,(1+n)^3\;(-1 + n + n^2)} - f_L \tag{35}$$

This expression is decreasing in n. Therefore, an n^* must exist such that all firms' profits are negative for all $n \geq n^*$. Thus, the candidate equilibrium $(0, n)$ with $n \geq n^*$ requires a market size vector (a, s) such that either $a > \bar{a}$ or, in the case of $a < \bar{a}$, that $s > s_1(a)$. The latter statement follows from the fact that profits are an increasing function of a if we use $s = s_1(a)$. The respective profits read

$$\Pi_L(0, n) = \frac{a^2\;(f_L - f_S)}{2\,a\,c\,n - c^2\,n^2} - f_L. \tag{36}$$

It is straightforward to show that the derivative of this expression with respect to a is positive. Therefore, it is proved that non-existence cannot occur for a number of large firms greater than n^*. Values of a and s satisfying the relevant necessary conditions for non-existence do not support the respective candidate equilibrium.

Finally, a^*, the threshold value of a which ensures existence is calculated. Substituting n^* into equation (34) and using $m = 1$ yields the respective value (see equation (14) in the main text). It follows from the above reasoning that conditions (30) and (31) cannot be satisfied simultaneously for $a > a^*$. Therefore, non-existence cannot occur for $a > a^*$. This completes the proof. □

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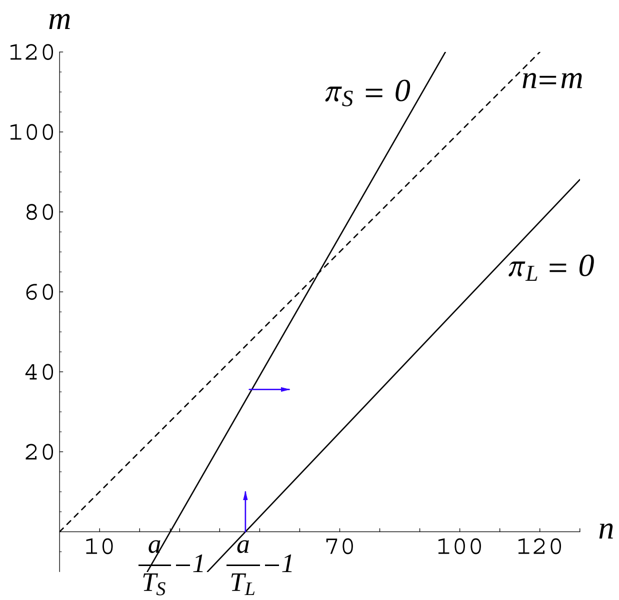

Figure 1: Condition for the existence of a unique and symmetric equilibrium ($T_S > T_L$).

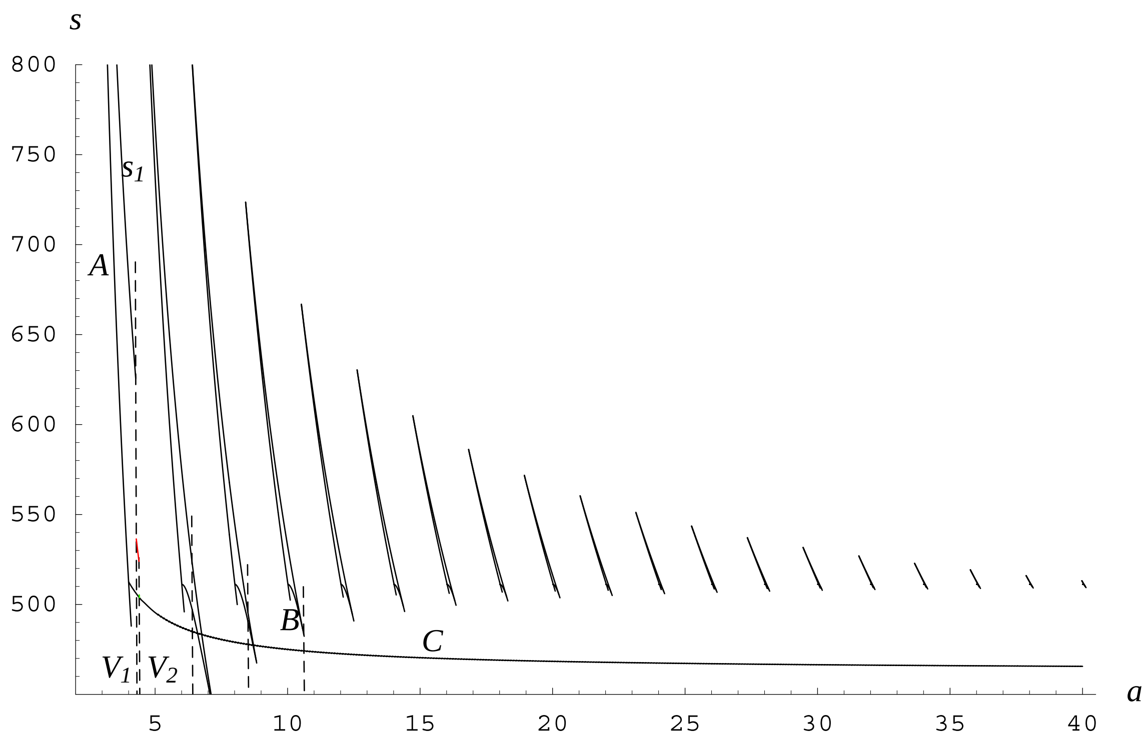

Figure 2: Regions of non-existence of an equilibrium

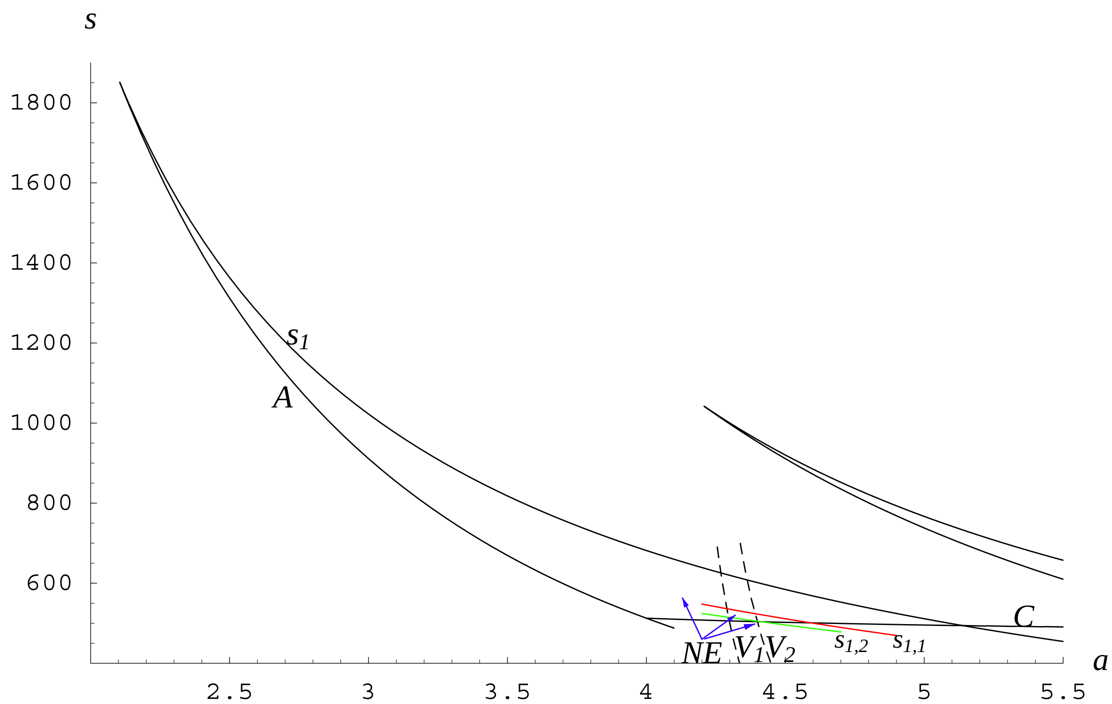

Figure 3: Regions of non-existence of an equilibrium (detail): Candidate equilibria with one or two large firms

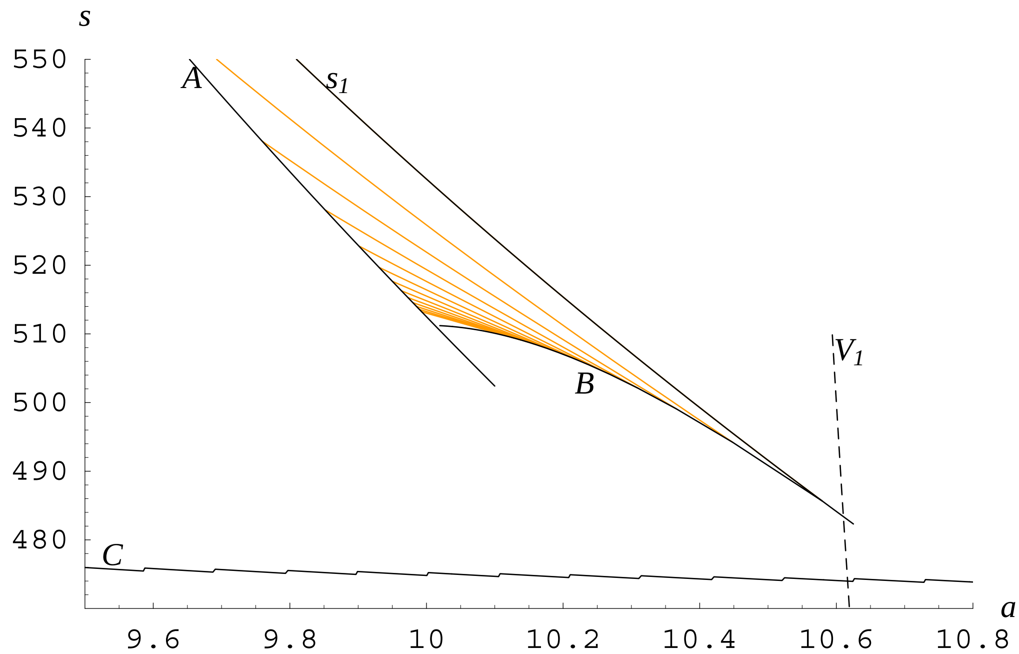

Figure 4: Non-existence of an equilibrium (detail): Candidate equilibrium with four large firms