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# A Comment on "The Selection of Preferences Through Imitation"*

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**Abstract**

We observe that the imitation dynamics of Cubitt and Sugden (CS) is the same as the Replicator Dynamics for a certain class of games. Known results for such games then permit a more complete analysis of the CS imitation process, containing their results as special cases, and extending them considerably. We also offer a comment on the special role of "pure" prospects, and an *as if* interpretation of the CS process in terms of payoff-guided imitation.

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# 1 Introduction

In a recent article, Cubitt and Sugden (1998) (henceforth CS) postulate an evolutionary model where a population of agents adapt their behavior in a game against nature through a process of imitation. For binary decisions, the outcome of the process is compatible with preference relations of the type proposed by SSB utility theory and regret theory. For non-binary decision problems among lotteries, CS define a *Fishburn solution* (following Fishburn (1984)) as a population state in which the average behavior corresponds to a prospect that is weakly better in terms of SSB preferences than each of the a priori available prospects. They go on to show that such a Fishburn solution is a particular type of rest point of the process, which is Lyapunov stable but not asymptotically stable in general.

The evolution of preferences is an important subject, and CS is a pioneering paper in the sense that it provides a bridge between, first, the literature which extends the study of static preferences beyond the von Neumann-Morgenstern framework, and, second, explicitly dynamic evolutionary models. We write this note with the intention of better understanding and illustrating CS from the point of view of "classical" evolutionary game theory.

In this spirit, we first observe that the process postulated by CS is formally identical to the Replicator Dynamics (see e.g. Hofbauer and Sigmund (1998)) for a certain associated zero-sum game. A Fishburn solution is the same as a Nash equilibrium of this game. Theorems 1-3 in CS then follow immediately from standard results in evolutionary game theory. Moreover, these results permit also a more complete analysis of the dynamics studied by CS. In fact, there are two qualitatively different cases, only one of which is covered in CS. In the first case (covered by CS) we have orbits oscillating within closed invariant sets around interior rest points, and it seems difficult to interpret these orbits in terms of preferences (in fact, the interior rest points are both best and worst in terms of certain natural SSB-preferences). In the second case, we have no interior rest points, but a flow away from certain prospects in one face of the simplex towards certain other prospects in another face; the attracting prospects are SSB-best and the repelling prospects are SSB-worst, so that one might tentatively consider these SSB preferences as the "result of evolution."

We also complement the analysis in CS and Fishburn (1984) to show that the Fishburn solution $\pi^0$ induced by the set of degenerate lotteries ("simple prospects"), is the essential prediction of the process in the following sense. A Fishburn solution for a given set of lotteries can only be in the interior of the convex hull spanned by those lotteries if it coincides with $\pi^0$. If $\pi^0$ can not be spanned by the postulated lotteries, then necessarily the Fishburn solution must be in the boundary of their convex hull.

Finally, we show that the imitation process postulated by CS, which is *not* guided by any considerations of payoffs or preferences, is nevertheless *behaviourally indistinguishable* from a process generated by a certain payoff-guided imitation rule (the proportional imitation rule), for the associated zero-sum game. Thus, the agents in the CS model behave *as if* they tried to maximize some sort of expected utility, after all.[1]

Section 2 presents the model of CS and defines the associated zero-sum game. Section 3 contains a fairly complete analysis of the CS dynamics, using known results from evolutionary game theory. Section 4 presents our results on simple prospects. Section 5 proposes our alternative interpretation in terms of payoff-guided imitation.

[1] We offer this section with apologies to CS: even without discussing methodological issues, it is pretty clear that such an *as if* explanation of observed behavior is not what they have in mind.

## 2 The Cubitt-Sugden Model and the Fishburn Solution

A (large) population of individuals play a game against nature with a finite set of *consequences* $X = \{x_1, ..., x_z\}$; the set of probability distributions on $X$ is given by the $(z-1)$ - dimensional simplex $\Delta^z = \{q = (q_1, ..., q_z) \mid 0 \leq q_j \leq 1 \; \forall \; j \text{ and } \sum_{j=1}^{z} q_j = 1\}$ (this set is called $P$ in CS). Elements of $\Delta^z$ are called *prospects*. A *decision problem* is a set of $n$ different prospects $D = \{p_1, ..., p_n\}$. We denote by $P$ the stochastic $(n \times z)$-matrix whose rows are the prospects in $D$.

Agents are able to compare their performance with that of other, randomly sampled agents. Suppose that an agent (the reviewer) using prospect $p$ who has just obtained consequence $x_j$ samples an agent (the comparator) using prospect $q$ who has obtained consequence $x_k$. The probability that this reviewer switches to the prospect of the comparator is assumed to depend only on observed consequences, and is named $1 - M(x_j, x_k)$. We define the *index of attractiveness* of $x_j$ relative to $x_k$ by $\psi_{jk} = \psi(x_j, x_k) = M(x_j, x_k) - M(x_k, x_j)$, and note that the $(z \times z)$-matrix $\Psi := [\psi_{jk})]_{j,k=1}^{z}$ is skew-symmetric, i.e. $\psi_{jk} = -\psi_{kj}$. The bilinear extension of $\psi$ to general prospects $p, q \in \Delta^z$, is given by

$$\psi(p, q) = \sum_j \sum_k p_j q_k \psi_{jk} = p\Psi q' \tag{1}$$

The skew-symmetric bilinear (SSB) function $\psi$ can be thought of as representing (non-transitive) preferences on $\Delta^z$, namely, a prospect $p \in \Delta^z$ is (weakly) SSB-better than $q \in \Delta^z$ iff $\psi(q, p) \leq 0$.

CS postulate the following dynamics, justified on evolutionary grounds as a model of imitation giving rise to an "analogue of the replicator equation" (CS, p.768).[2] Given the decision problem $D = \{p_1, ..., p_n\}$, let $\pi_i$ be the proportion of agents in the population choosing prospect $p_i$. Agents sample other agents as above and switch or not according to $M$, in such a way that the net flow of individuals switching from $p_h$ to $p_i$ is given by $\psi(p_i, p_h)$. The (continuous-time) dynamics on the population state $\pi = (\pi_1, ..., \pi_n) \in \Delta^n$ is

$$d\pi_i/dt = \pi_i \sum_h \pi_h \cdot \psi(p_i, p_h) = \pi_i \psi(p_i, g(\pi)) \qquad (i = 1, ..., n) \tag{2}$$

where $g(\pi) = \sum_h \pi_h p_h = \pi P \in \Delta^z$ is the population average of chosen prospects.[3]

CS define a *Fishburn solution* for the decision problem $D$ as a population state $\pi^*$ such that

$$\psi(p_i, g(\pi^*)) \leq 0 \qquad \forall i = 1, \ldots n \tag{3}$$

In terms of SSB-preferences, the prospect $p^* = g(\pi^*)$ is a best element in the convex hull co$(D)$ of $D$. We may call co$(D)$ the set of *feasible prospects*[4] and shall also call a prospect $p^*$ a "Fishburn solution" for $D$ if it is an SSB-best element in co$(D)$.

It was already observed by Kreweras (1961) that an SSB-best prospect is formally equivalent to an optimal strategy in a certain symmetric zero-sum game.[5] More precisely, let

[2] The interpretation of the Replicator Dynamics as a model of imitation is well-known. See e.g. Björnerstedt and Weibull (1996) for details. See also Section 5.

[3] Given their derivation of $\Psi$, CS restrict $\psi_{jk}$ to lie in the interval $[-1, 1]$. This is not necessary if a continuous-time dynamics is built, since here the quantities $\psi_{jk}$ refer to an instantaneous flow and can be interpreted as densities. The analysis therefore holds for an arbitrary (skew-symmetric) matrix $\Psi$.

[4] In the sense that these prospects can be realized as population averages, not necessarily in the sense that such "average prospects" are directly available to the individual agents.

[5] "Le problème est formellement identique à celui du duel symétrique .." (p. 29). Note also that a symmetric zero-sum game has an antisymmetric (or skew-symmetric) payoff matrix, and that the value of such a game is always zero.

us define the *auxiliary game* $G = G(p_1, ..., p_n)$ as the zero-sum game with (pure) strategy space $D$ and payoff matrix $A := P\Psi P'$. This game is symmetric and hence has value zero because $\Psi$ (and hence $A$) is skew-symmetric. Profiles $\pi = (\pi_1, ..., \pi_n) \in \Delta^n$ correspond to mixed strategies for this game. The payoff from a pure strategy $p_i$ against strategy $p_h$ is $a_{ih} = p_i \Psi p_h' = \psi(p_i, p_h)$ and the payoff from $p_i$ against a mixed strategy $\pi \in \Delta^n$ is

$$\sum_h a_{ih}\pi_h = (A\pi')_i = p_i \Psi P' \pi' = \psi(p_i, g(\pi)) \tag{4}$$

The following observation is essentially due to Kreweras (1961) (p. 29); for completeness' sake, we also give a short proof.

**Proposition 2.1.** *A population profile $\pi^*$ is a Fishburn solution for the decision problem $D$ if and only if $(\pi^*, \pi^*)$ is a Nash equilibrium of the game $G$.*

*Proof.* By (4), a profile $\pi^*$ is a Nash equilibrium of the game $G$ iff

$$\psi(p_i, g(\pi^*)) \le 0 = \psi(g(\pi^*), g(\pi^*)) \le \psi(g(\pi^*), p_k) \quad \forall p_i, p_k \in D \tag{5}$$

The left inequality in (5) says that $\pi^*$ is a Fishburn solution. Conversely, if $\pi^*$ satisfies the left inequality in (5), then it also satisfies the right inequality, by skew-symmetry, i.e. it is a Nash equilibrium. ■

In view of this proposition, existence of a Fishburn solution (Theorem 1 in CS) follows immediately from the standard fixed-point argument for the existence of Nash equilibria.[6] As noted above, a Fishburn solution $p^* = g(\pi^*)$ corresponds to a best element (in terms of SSB preferences) in the set of feasible prospects. Note that existence of a *worst* element also follows from the same argument (consider the game $G^-$ with payoff matrix $-A$). Moreover, if $\pi^*$ is interior, it satisfies (5) with equality everywhere and $g(\pi^*)$ is both a best and a worst element with respect to the SSB preferences described by $\psi$.

# 3 Dynamics

The Replicator Dynamics is the most important dynamic model arising from evolutionary game theory. It is derived from a Darwinian model where strategies that fare better than average (given the population profile) thrive at the expense of others (see e.g. Hofbauer and Sigmund (1998) or Weibull (1995)). Formally, for the game $G$ defined above, the Replicator Dynamics is given by

$$d\pi_i/dt = \pi_i \left[\psi(p_i, g(\pi)) - \overline{\pi}\right] \qquad \forall i = 1, \ldots n \tag{6}$$

where $\overline{\pi} = \psi(g(\pi), g(\pi)) = \pi A \pi'$ is the average payoff in the population given profile $\pi$. Since the payoff matrix $A$ is skew-symmetric, we have $\overline{\pi} = 0$ for every profile $\pi \in \Delta^n$. Hence, the CS dynamics (2) is not only analogous but actually identical to the Replicator Dynamics (6) for the zero-sum game $G$.

The behaviour of the Replicator Dynamics for zero-sum games is well-known.[7] We summarize here some of the results and refer to Akin and Losert (1984) (henceforth AL) for

[6] Equivalently (in the present context), both Kreweras (1961) and Fishburn (1984) refer to von-Neumann's Minimax Theorem.

[7] See e.g. Hofbauer and Sigmund (1998, pp. 74 and 127) or Hofbauer and Sigmund (1988, pp. 129 and 275); the original analysis is due to Schuster, Sigmund, Hofbauer, and Wolff (1981); see also Akin and Losert (1984)).

the proofs. Consider the dynamics (2) on the simplex $\Delta^n$. The solution can be described by a smooth map $\Phi : \Delta^n \times \mathbb{R} \to \Delta^n$, called the *flow* of the system (AL p. 232). For given $\pi \in \Delta^n$, the function $t \to \Phi(\pi, t)$ describes the solution path with initial point $\pi$. In order to describe the asymptotic behavior of such paths, define the following three sets:

$$\begin{aligned} E_0 &= \{\pi \in \Delta^n \mid \psi(p_i, g(\pi)) = 0 \quad \forall i\} \\ E_+ &= \{\pi \in \Delta^n \mid \psi(p_i, g(\pi)) \geq 0 \quad \forall i, \text{ with at least one strict inequality}\} \\ E_- &= \{\pi \in \Delta^n \mid \psi(p_i, g(\pi)) \leq 0 \quad \forall i, \text{ with at least one strict inequality}\} \end{aligned}$$

The sets $E_0$, $E_+$, and $E_-$ are convex subsets of $\Delta^n$ consisting of rest points of the dynamics. Moreover, exactly one of two cases holds. In the "Interior Equilibrium Case," $E_0$ is nonempty, and both $E_+$ and $E_-$ are empty. In the "No Interior Equilibrium Case," $E_0$ is empty and both $E_+$ and $E_-$ are nonempty (AL, Th. 2). Note that the elements of $E_0 \bigcup E_-$ are Fishburn solutions, i.e., Nash equilibria of the game $G$ with payoff matrix $A$. Symmetrically, the elements of $E_0 \bigcup E_+$ are Nash equilibria of the game $G^-$ with payoff matrix $-A$, i.e., the game where all payoffs (all indices of attractiveness) are reversed. Of course, the Replicator Dynamics for $G^-$ is also the "reverse" of the dynamic (2) (formally, its time reversal).

Next define the Lyapunov function $I^q(p) = -\sum_{i \in \text{supp}(q)} q_i \log(p_i/q_i)$ for $p, q \in \Delta^n$ with $\text{supp}(q) \subset \text{supp}(p)$. This function (known as *relative entropy*) is strictly convex in $p$ and achieves a unique minimum $I^q(p) = 0$ at $p = q$. Then, if $\Phi(\pi, t)$ is the path through any interior point $\pi \in \Delta^n$, AL (Thms. 4 and 5) show the following: $I^e[\Phi(\pi, t)]$ is constant in $t$ for $e \in E_0$, and strictly increasing (resp. strictly decreasing) in $t$ for $e \in E_+$ (resp. for $e \in E_-$). Theorems 2 and 3 in CS follow immediately from these results.[8]

In the *interior equilibrium case* (AL, Th. 4), the elements of $E_0$ are exactly the equilibria of both $G$ and $G^-$. In particular, they are rest points of both the Replicator Dynamics (2) and its reverse. Not surprisingly, for both dynamics these points are Lyapunov stable but not asymptotically stable, with the dynamics leading neither towards them nor away from them, but orbiting around in closed invariant sets which contain no equilibria (a phenomenon called "neutral stability" in evolutionary game theory).[9] It seems difficult to interpret these equilibria either evolutionarily or normatively, since they are equally compatible with both the original CS model of imitation based on the index of attractiveness $\psi$ (or, if we wish, SSB "preferences" described by $\psi$) and also with the "reverse" model based on $-\psi$. We agree with CS that in this case their model of imitation does not yield an outcome that could meaningfully be interpreted in terms of preferences.

In the *no interior equilibrium case* (typical for $n$ even), not considered in CS, the dynamics is rather different (AL, Th. 5). The elements of $E_-$ are precisely the Nash equilibria of the game $G$, i.e. the Fishburn solutions (SSB-best elements). Call a strategy $p_i$ *good* if $\psi(p_i, g(\pi)) \geq 0 \quad \forall \pi \in E_-$. Symmetrically, the elements of $E_+$ are precisely the Nash equilibria of the game $G_0^-$, i.e. the SSB-worst elements. Call a strategy (or prospect) $p_i$ *bad*

[8] For the original result (in the framework of bimatrix games), see the main Theorem of Section 8 in Schuster, Sigmund, Hofbauer, and Wolff (1981). Incidentally, this Theorem uses the same Lyapunov functions as CS (which could be traced back to Volterra), and also shows that, in the interior equilibrium case, time averages along any interior orbit converge to the equilibrium set. If orbits are closed, time averages must then be numerically equal to the equilibrium they "enclose." This latter fact is mentioned in Footnote 9 of CS, for a specific example. Actually, it turns out to be a general property of the Replicator Dynamics (see Hofbauer and Sigmund (1998, Th. 7.6.4)).

[9] Although the typical text-book example is that of closed orbits, in general the actual shape of the orbits can be extremely complex. Sato, Akiyama, and Farmer (2002) show that, in a $3 \times 3$ zero-sum game, the two-population Replicator Dynamics presents chaotic behavior. Such situations could also arise in the one-population case for higher dimensions.

if $\psi(p_i, g(\pi)) \leq 0 \quad \forall \pi \in E_+$. Denote by $\Delta_-$ (resp. $\Delta_+$) the face of the simplex $\Delta^n$ spanned by all good (resp. all bad) strategies.[10] Take any interior point $\pi \in \Delta^n$, and consider the path $\Phi(\pi, t)$ of the dynamics passing through $\pi$ ($\pi$ is not a rest point because $E_0$ is empty). Then, the $\omega$-limits of this path are contained in $\Delta_-$ and the $\alpha$-limits are contained in $\Delta_+$. In other words, all interior paths lead away from the bad prospects in $\Delta_+$ and towards the good prospects in $\Delta_-$. Moreover, the dynamics within the "attracting face" $\Delta_-$ can be analyzed considering the game restricted to the pure strategies which span the face. Eventually, we will end either in an interior equilibrium situation restricted to a certain face of the simplex, or in a corner of the original simplex.

# 4 Simple Fishburn Solutions

Let us now consider the decision problem $D_0 = \{x_1, ..., x_z\}$ where $x_k$ stands for the prospect which gives consequence $x_k$ with certainty. Such prospects will be called *simple* prospects. Obviously, in this case, $P = I$ and hence $g(\pi) = \pi P = \pi$. A Fishburn solution $p^0 = g(\pi^0) = \pi^0 \in \Delta^z$ for $D_0$ is called a *simple* Fishburn solution. We want to show that, for a (nonsimple) decision problem $D = \{p_1, ..., p_n\}$ in the sense of CS, there are, qualitatively, only two relevant situations. Either $p_1, ..., p_n$ can span a simple solution $p^0$, and then $p^0$ is a Fishburn solution for $D$ too, or they can't, and then there can not be any interior Fishburn solution: the Fishburn solution for $D$ must prescribe not to use some of the prospects.

We interpret this result as follows. Suppose, for simplicity, that $p^0$ is interior and unique. Either it is also a Fishburn solution of the decision problem $D$, or the latter solution is in the boundary of $\text{co}(D)$, pointing to an evolutionary pressure to introduce new prospects, replacing old ones. Whenever (maybe through mutation or experimentation) a new prospect is introduced in $D$ which allows to span $p^0 = \pi^0$, this will become the Fishburn solution of $D$. Hence, in a sense, there is not much generality gained by considering a decision problem $D$ restricted to certain prospects $p_1, ..., p_n$ instead of the unrestricted problem $D_0$ containing all pure strategies $x_1, ..., x_z$.[11]

We call a decision problem $D = \{p_1, ..., p_n\}$ *nondegenerate* if the matrix $P$ has full rank (equal to $z$).

**Proposition 4.1.** *Consider a decision problem* $D = \{p_1, ..., p_n\}$.

*(a) Suppose a simple Fishburn Solution* $p^0$ *belongs to the convex hull* $\text{co}(D)$ *of* $p_1, ..., p_n$. *Then,* $p^0$ *is also a Fishburn solution for* $D$.

*(b) Suppose that the decision problem* $D$ *is non-degenerate and admits an interior Fishburn solution* $p^* = g(\pi^*)$ *with* $\pi^* >> 0$. *Then,* $p^*$ *is a simple Fishburn Solution.*

*Proof.* (a) By assumption, $p^0 \in \text{co}(D)$, and since it is an SSB-best element in the whole simplex $\Delta^z$, it is *a fortiori* an SSB-best element in the subset $\text{co}(D)$.

(b) Let $p = g(\pi)$ be a Fishburn solution for $D$. This means that $\psi(p_i, g(\pi)) \leq 0 \; \forall \; i$. In fact, if $\pi$ is interior (i.e. $\pi_i > 0$ for all $i$), we must have

$$\psi(p_i, g(\pi)) = 0 \; \forall \; i$$

because the average payoff is zero, $\sum_i \pi_i \cdot \psi(p_i, g(\pi)) = 0$. By (4), the equations above can be written in matrix form as

$$P \cdot \Psi \cdot P' \cdot \pi' = 0$$

[10] Of course, by the definition of $E_-$ resp. $E_+$ we must actually have equality in the equations defining good resp. bad prospects. In degenerate cases, $\Delta_+$ and $\Delta_-$ may not be disjoint.

[11] This, of course, is a different view of the model than the one given by CS. Under this view, it is arguable whether the model describes "evolution of preferences" or merely adds to the literature on the evolutionary foundations of Nash equilibria, in the spirit of Björnerstedt and Weibull (1996).

Since $P$ has full rank, it follows that $\Psi \cdot P' \cdot \pi' = 0$. That is, $p = g(\pi) = \pi \cdot P$ is a Fishburn solution for $D_0 = \{x_1, ..., x_z\}$. ■

Proposition 4.1 relates the Fishburn solution $p^0$ of the "unrestricted" problem $D_0$ to the Fishburn solution of the "restricted" problem $D$, with given prospects $\{p_1, \ldots p_n\}$, and says, roughly, that these solutions are the same provided $p^0 \in \text{co}(D)$. There is an analogous result on evolutionarily stable strategies (ESS) in evolutionary game theory. Here, the $x_k$ are "pure strategies," and the $p_i$ are "phenotypes." If some $p^*$ is an ESS in the unrestricted game with all pure strategies, then it is also stable (w.r.t. the replicator dynamics) in the game restricted to the phenotypes $\{p_1, \ldots p_n\}$, provided, again, $p^* \in \text{co}(\{p_1, \ldots p_n\})$ (see Hofbauer and Sigmund (1998, p. 73)).

# 5 An Alternative Interpretation

CS emphasize that in their model of social evolution they take the concept of *imitation* as primitive; "there is no independent concept of preference or utility" (p. 763). The agents' behaviour is governed by "indices of attractiveness" $\psi_{ij}$ which are not derived from any idea of payoff associated with the various prospects. On the contrary, the objective is to "investigate whether this process tends to select behaviour which maximizes something which we may interpret as preference satisfaction."

The conclusion of CS in this regard is rather skeptical; we wish to point out here that it is indeed possible to give such an interpretation, in the following sense: for any (skew-symmetric) "imitation matrix" $\Psi$ and any decision problem $D = \{p_1, \ldots p_n\}$ the associated CS dynamic (2) on the population state $\pi \in \Delta^n$ is behaviourally indistinguishable from the dynamics generated by a conventional (i.e. payoff-guided) imitation rule in our auxiliary game $G$, viz. the *proportional rule*[12] "imitate prospects that perform better, with a probability proportional to the expected payoff gain."

Thus, even if the agents imitate each other blindly, so to speak, according to arbitrary indices $\psi_{ij}$, they behave *as if* they tried to maximize "payoff", by following a certain boundedly rational imitation rule in some suitably defined zero-sum game. This follows from our observation in Section 2 that the CS dynamic (2) is the same as the replicator dynamic (6) for the auxiliary game $G$, and from the known fact that the proportional imitation rule gives rise to the replicator dynamic (Hofbauer and Sigmund (1998, p. 87)).

More precisely, let $A = P\Psi P'$ be the payoff matrix of $G$, and $\pi = (\pi_1, \ldots \pi_n) \in \Delta^n$ be a population state (strategy). Define the (state-dependent) "utility" $u_\pi(p_i)$ of a prospect $p_i$ as the expected "payoff" from this prospect in state $\pi$: $u_\pi(p_i) = (A\pi')_i = \psi(p_i, g(\pi))$. Then, under proportional imitation, the net switching rate from $p_h$ to $p_i$ is $u_\pi(p_i) - u_\pi(p_h)$ and we obtain the dynamic

$$d\pi_i/dt = \pi_i \sum_h \pi_h [u_\pi(p_i) - u_\pi(p_h)] \tag{7}$$

or (remember that the average payoff $\sum_h \pi_h u_\pi(p_h) = \bar{\pi} = 0$),

$$d\pi_i/dt = \pi_i u_\pi(p_i) = \pi_i \psi(p_i, g(\pi)) \tag{8}$$

which is the same as (2).

The "preferences" represented by the "utility function" $u_\pi$ give a complete transitive ordering over all prospects (even satisfying the expected utility principle of von Neumann -

[12]Schlag (1998) studies this rule in games against multi-armed bandits.

Morgenstern) for every population state $\pi$, but as $\pi = \pi(t)$ changes over time, so do these preferences. However, if the process does converge to a rest point $\pi^*$ a positive answer can be given to the question quoted at the beginning of this section: in this case, the CS imitation process selects a stable transitive preference order represented by the vNM-utility function $u_{\pi^*}$.

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