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Money in Consumption Economies

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MONEY IN CONSUMPTION ECONOMIES

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Abstract. Three sequential models of consumption economies are considered, where consumers' only endowment is money. The existence and unicity of temporary equilibria, the neutrality of money and the validity of quantity theory are investigated. In the …rst two models "money" is perishable; in the second one lending between consumers is possible. In the third model money is an asset and can be created through bank loans.

1. Introduction

Exchange economies are an idealization where production is so bracketed that commodities fall like manna from heaven. In this paper we attempt a di¤erent sort of idealization: production is certainly bracketed, but the essential bipolarity between consumers and producers in a modern economy is emphasized, although still centring on one of the poles. On this pole, consumers' needs are satis…ed with goods, producers' needs with money. Speci…cally, at the beginning of the week's market, consumers have money, and producers have goods, and "every man lives by exchanging", in the words of Adam Smith [9]. We do not consider here the very relevant question of why we have come to this institutional framework (v., e.g., [3], [4] and [7] for answers to this kind of question), but what is certain is that we have come to it. As in cash-in-advance models (…rst proposed by [1]), here the medium of exchange role of money is not endogenously determined, but it is imposed institutionally (v., e.g., [6] and [10] among the many contributions in this direction).

Consumers face some social entity that holds the (perishable) commodities, and supply them against the deliverance of money. Beyond that, this entity might be interpreted as the supply edge of "the productive sector". We consider three models, according to the increasing freedom that consumers have not to spend their money or spend more than they own. In the …rst model, they have none of these liberties. In the second one, they have access to them individually, but not in the aggregate: lending between consumers is possible, but money is perishable and new money cannot be created at the request of consumers. In the third model these freedoms exist also in the aggregate: money is no longer perishable (so consumers are allowed not to spend it and so to keep it for the future) and can be created through bank loans to consumers (so they are allowed to spend more than they

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have). The idea of "perishable money" appearing in the first two models may seem a contradiction in terms; the reader can always solve this problem by changing the term and calling this dismal money "pseudomoney".

Each model is defined in a sequential framework by some structural hypotheses, upon which further assumptions are introduced in the statements of the results. These assumptions are defined on aggregate demands and incorporate the effect of expectations. The stability of any monetary system supposes some restraint in the way expectations are built up. However strong the real economy is or wise the policies are, any system will collapse if the economic agents come to believe that money will be worthless tomorrow.

We consider only the short run and discuss temporary competitive equilibria ([2] and [5]). The issue of grounding microeconomically the assumptions about aggregate demands is not tackled.

Given two vectors $x, y \in \mathbb{R}^q$, $x \leq y$ means $x_i \leq y_i$ for $i = 1, \ldots, q$, and $x < y$ means $x_i < y_i$ for $i = 1, \ldots, q$; the scalar product of the two vectors is denoted by $\langle x, y \rangle$. We write $\mathbb{R}^q_+ \equiv \{x \in \mathbb{R}^q : x \geq 0\}$ and $\mathbb{R}^q_{++} \equiv \{x \in \mathbb{R}^q : x > 0\}$.

2. The simplest model

We consider a sequential exchange economy where l perishable commodities are traded and consumed by n agents in each period t, where $t = 1, 2, \ldots$

Consider the situation in period t. The consumption set for all the consumers is $\mathbb{R}^l_+$, the society has $e^t_i > 0$ units of commodity i at its disposal, $e^t \equiv (e^t_1, \ldots, e^t_l)$, and at the beginning of the period consumer j receives an allowance of $m^t_j \geq 0$ monetary units, $m^t \equiv (m^t_1, \ldots, m^t_n)$, $M^t \equiv \sum_{j=1} m^t_j$. The consumer's money consists of a credit balance in his account in the (central) bank. Money is "perishable": it is dated, and after its period is not accepted in the market. Given commodity prices $p^t = (p^t_1, \ldots, p^t_l)$, the consumer's budget set is defined by $\{x \in \mathbb{R}^l_+ : \langle p^t, x \rangle \leq m^t_j\}$. Thus consumers have in front of them a fixed stock of (different sorts of) manna, to be distributed according to a method: they have to pay for it with the money that they possess.

As in this paper almost every symbol has the superscript t referring to the current period, this superscript will be dropped after the symbol has been introduced for the first time, except for emphasis.

Consumer j has a demand function $f^t_j : \mathbb{R}^l_{++} \to \mathbb{R}^l_+$ defined for all strictly positive price[1] vectors p. The consumer's demand depends on m_j; when we want to reflect it explicitely in the notation we write $f^t_j(m_j, p)$. We also consider the aggregate demand function $\xi^t \equiv \sum_{j=1} f^t_j$. Now ξ changes with m. If we fix the proportions $\gamma^t_j \equiv m^t_j / M^t$, $j = 1, \ldots, l$, then ξ changes only with M, which we reflect in the notation when we write $\xi^t(M, p)$. For individual demands we assume:

(I.1) $f_j(p) \in \{x \in \mathbb{R}^l_+ : \langle p, x \rangle = m_j\}$, for all $p \in \mathbb{R}^l_{++}$

(I.2) $f_j(\lambda m_j, \lambda p) = f_j(m_j, p)$, for all $\lambda > 0, p \in \mathbb{R}^l_{++}$

[1] Money is here assumed to be the unit of account, i.e., the price of money p^t_0 is 1. This can be always obtained by normalizing, provided that $p_0 > 0$. At any rate, we are considering only strictly positive prices, including that of money, hoping that equilibrium should exist in this range (v. the proof of Proposition 2.2).

The hypotheses made so far define our Model I. Some obvious consequences follow. From (I.1), Walras' law obtains:

$$\langle p, \xi(p)\rangle = M; \text{ for all } p \in \mathbb{R}^l_{++} \tag{1}$$

From (I.2), absence of money ilusion holds:

$$\xi(\lambda M; \lambda p) = \xi(M; p); \text{ for all } \lambda > 0; p \in \mathbb{R}^l_{++} \tag{2}$$

A price vector $\hat{p}^t > 0$ such that $\xi^t(\hat{p}^t) = e^t$ is called an equilibrium price vector ($f_j^t(\hat{p}^t)$, $j = 1, \dots, n$, are the corresponding equilibrium individual demands)[2].

We attempt first some unicity result. Some price vector $p^k > 0$, in period k ($k < t$), is taken as a base for price indices; given a price vector $p^t > 0$ we shall denote by P^t the corresponding (Paasche) price index, i.e., $P^t \equiv \langle p^t, \xi^t(p^t)\rangle / \langle p^k, \xi^t(p^t)\rangle$, provided $\langle p^k, \xi^t(p^t)\rangle > 0$.

Proposition 2.1. *Consider Model I, and let $t \in \mathbb{N}$. Then all equilibrium price vectors (if any) have the same price index.*

Proof. From Walras' law, we have that $\langle \hat{p}, e\rangle = M$ for any equilibrium price vector $\hat{p}$. Hence $\hat{P} = M/\langle p^k, e\rangle$. ∎

Thus, if there is some equilibrium, we can speak of the equilibrium price index. Notice that it depends on M, although it does not depend on how M is partitioned into the allowances m_j; $j = 1, \dots, n$.

In order to ensure the existence of equilibrium, we form macroeconomic hypotheses. For the aggregate demand we may assume:

(I.3) $\xi(M; p)$ is continuous as a function of $(M; p) \in \mathbb{R}^{1+l}_{++}$

(I.4) $\lim_{p \to \bar{p}} \|\xi(p)\| \to +\infty$, for all $\bar{p} \geq 0$ with $\bar{p}_i = 0$ for some i

Observe that, from (I.1) and (I.4), $M > 0$.

Proposition 2.2. *Consider Model I and assume (I.3) and (I.4). Let $t \in \mathbb{N}$. Then there is some equilibrium price vector.*

Proof. Let $z : \mathbb{R}^{1+l}_{++} \to \mathbb{R}^{1+l}$ be defined by $z_0(p_0; p) \equiv \langle p, e\rangle / p_0 - M$, $z_i(p_0; p) \equiv \xi_i(p_0 M; p) - e_i$ for $i = 1, \dots, l$; here p_0 can be interpreted as "the price of money". Thus z is continuous and homogeneous of degree zero, and $z(p_0; p) \geq (-M; -e)$ for all $(p_0; p)$. Also $\langle (p_0; p), z(p_0; p)\rangle = p_0(\langle p, e\rangle / p_0) - p_0 M + \langle p, \xi(p_0; p)\rangle - \langle p, e\rangle = 0$. Moreover, if $(\bar{p}_0; \bar{p}) \geq 0$, $(\bar{p}_0; \bar{p}) \neq 0$, and either $\bar{p}_i = 0$ for some i or $\bar{p}_0 = 0$, then

$$\lim_{(p_0; p) \to (\bar{p}_0; \bar{p})} \|z(p_0; p)\| \to +\infty \tag{3}$$

In fact, if $\bar{p}_0 > 0$, then (3) results from (I.4); on the other hand, if $\bar{p}_0 = 0$, then

$$\lim_{(p_0; p) \to (\bar{p}_0; \bar{p})} z_0(p_0; p) \to +\infty$$

since $e > 0$ and $\bar{p} \neq 0$, and (3) also follows. From these properties of z, we conclude (by a well known result; v., e.g., 17.C.1 in [8]) that there is $(p_0^*; p^*) > 0$ such that $z(p_0^*; p^*) = 0$, and thus $\xi^t(p_0^* M; p^*) = e$; the result follows from (2) for $\hat{p} \equiv p^*/p_0^*$. ∎

In the following Proposition we consider the effects of a change in the initial quantity of money M. Redistribution effects on the individual allowances can be or not excluded. Observe that (i) holds irrespective of how λM is partitioned into the allowances of the consumers.

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[2] This equilibrium does not guarantee that all consumers receive the necessities of life.

Proposition 2.3. Consider Model I and assume (I.3) and (I.4). Let $t \in \mathbb{N}$. Suppose that the quantity of money changes from M to λM (with $\lambda > 0$), within the same hypotheses. Then:

(i) The equilibrium price index for λM is that for M multiplied by λ.

(ii) If all individual allowances change also in the same scale from m_j to λm_j, then the equilibrium price vectors for λM are those for M multiplied by λ, and the corresponding equilibrium individual demands do not change.

Proof. (i) Let $\overline{P}$ be the equilibrium price index for λM and $\hat{P}$ that for M. Then $\overline{P} = \lambda M / \langle p^k, e\rangle = \lambda \hat{P}$.

(ii) From (2) and (I.2),

$$\xi(\lambda M; p) = \xi(M; p/\lambda); \text{ for all } p \in \mathbb{R}^l_{++}$$

$$f_j(\lambda m_j; p) = f_j(m_j; p/\lambda); \text{ for all } p \in \mathbb{R}^l_{++};\ j = 1; \ldots; n$$

and the result follows. ∎

Therefore, if the quantity of money changes by a factor of λ, then the price index changes by the same factor. If in addition the proportions are kept in the allowances of the consumers (the Ricardian case), then money is neutral (real magnitudes stay unchanged), and quantity theory holds (the other nominal magnitudes (i.e., prices) are also multiplied by λ).

The reader will have found the superscript indicating the period t not only cumbersome, but also useless. In Model I it is indeed useless: each period is a "world apart". But at least the patient reader has flexed his superscript muscles in preparation for the following models.

3. Trusting the neighbour

We again consider a sequential exchange economy where l perishable commodities are traded and consumed by n agents in each period t, where $t = 1; 2; \ldots$ Money is perishable, as above, but now money-denominated private loans (i.e., between consumers) allow purchasing power to be transferred between periods.

In every period t, again, $\mathbb{R}^l_+$ is the consumption set for all the consumers, the society has $e_i^t > 0$ units of commodity i at its disposal, and at the beginning of the period consumer j receives an allowance of $m_j^t \geq 0$ units of perishable money (which is also the unit of account), $M^t \equiv \sum_{j=1} m_j^t$. We suppose that $M^t > 0$. As one-period private loans are now possible, he borrows (positive sign) or lends (negative sign) b_j^t monetary units from/to the other consumers. He also pays back or cashes the loans b_j^{t-1} contracted the period before (obviously, $\sum_{j=1} b_j^{t-1} = 0$), plus the interest that was agreed at rate r^{t-1}. Thus the consumer has $a_j^t \equiv m_j^t - (1 + r^{t-1})b_j^{t-1} + b_j^t$ available units of money.

Given a rate of interest r^t and commodity prices p^t in period t, consumer j makes estimations about the future[3] and has to decide the amount he borrows (lends) and the commodity bundle in the current period. There are two markets:

[3] The past is supposed to be known, and the present is either known since the beginning of the period (so m_j^t), or represented through the parameters $r^t; p^t$.

that for loans[4] and that for commodities. Notice that in the loans market the quantity of tradeable items is not given as a datum (loans are issued by the very consumers), as it is in the commodities market. The preferences of the consumer are given by a function $h^t_j :]-\infty, +\infty[\times R^l_{++} \to R \times R^l_+$ defined for pairs $(r;p)$, where $h^t_{j1}(r;p)$ represents the desired borrowing (lending) and $h^t_{j2}(r;p)$ the desired commodity bundle when the constraint on the present budget (no longer given as a datum, like in Model I) is that determined by $(r;p)$ (v. below). We assume that $m_j - (1 + r^{t-1})b^{t-1}_j + h_{j1}(r;p) \geq 0$. Obviously, predictions affect preferences; implicit in $h^t_j(r;p)$ is the fact that, for periods $t+u$, with $u > 0$, consumer j predicts the vector of prices $\hat{p}^u_j$, the rate of interest $\hat{r}^u_j$ and his allowance $\hat{m}^u_j$.

We suppose that the desired borrowing (lending) depends on commodity prices only through the corresponding price index. Formally, some price vector $p^k > 0$, in period k ($k < t$), is fixed as a base for price indices; given a price vector $p^t > 0$ we define the corresponding (approximately Paasche) price index $P^t \equiv \langle p^t; e^t\rangle / \langle p^k; e^t\rangle$; now $h^t_{j1}(r;p) = \beta^t_j(r;P)$, where $\beta^t_j :]-\infty, +\infty[\times R_{++} \to R$. The aggregate excess demand for loans is $z^t \equiv \sum_{j=1} \beta^t_j$.

On the other hand, we suppose that the desired commodity bundle in the current period depends on the rate of interest only through the resulting planned borrowing, and also that the distribution according to sources of the available money a^t_j (as allowance, borrowing or settlement of past lending) does not affect the preferred commodity bundle. Formally, there exist functions $f^t_j(a_j;\cdot) : R^l_{++} \to R^l_+$, for $a_j \geq 0$, such that $h^t_{j2}(r;p) = f^t_j(m_j - (1 + r^{t-1})b^{t-1}_j + \beta^t_j(r;P);p)$. Given any proportions $\alpha^t_j \geq 0$ (not explicitely reflected in the notation, but they must be clear in each case), with $\sum_{j=1} \alpha^t_j = 1$, and given $A > 0$, then the function $\gamma^t(A;\cdot) : R^l_{++} \to R^l_+$ is defined by $\gamma^t(A;p) \equiv \sum_{j=1} f^t_j(\alpha^t_j A;p)$. The following assumptions will be no surprise[5]:

(II.1) $f_j(a_j;p) \in \left\{ x \in R^l_+ : \langle p;x\rangle = a_j \right\}$, for all $a_j \geq 0; p \in R^l_{++}$

(II.2) $f_j(\lambda a_j;\lambda p) = f_j(a_j;p)$, for all $a_j \geq 0$; $\lambda > 0; p \in R^l_{++}$

The hypotheses made so far in this section define our Model II. In this model, Walras' law also obtains (where $\mu^t \equiv \sum_{j=1} h^t_{j2}$):

$$\langle p;\mu(r;p)\rangle = M + z(r;P); \text{ for all } (r;p) \in]-\infty, +\infty[\times R^l_{++} \tag{4}$$

Indeed, $\langle p;\mu(r;p)\rangle = \sum_{j=1} \langle p; f_j(m_j - (1 + r^{t-1})b^{t-1}_j + \beta_j(r;P);p)\rangle$, and the result follows from (II.1).

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[4] $1/(1 + r^t)$ is the price of a unit of loan (i.e., a promise of payment in period $(t + 1)$ of 1 monetary unit). Notice that the possibility of default is excluded, and this supposes moderation and prudence on the part of borrowers. In fact, lenders are likely to help borrowers to be virtuous, and borrowers are to acknowledge this help up to the point of incorporating as second nature the lenders' solvency conditions in their preferences.

[5] The analogy between (I.1) and (II.1) (with m_j replaced by a_j) should not be overstretched: (I.1) holds, in principle, for a particular m_j, whereas (II.1) holds for any $a_j \geq 0$. Notice that Proposition 3.3 would still be satisfied if (II.1) held only for $a_j \equiv \hat{a}_j$, with $\hat{a}_j$ as defined below.

A pair $(\hat{r}^t; \hat{p}^t) \in]-\infty; +\infty[\times R^l_{++}$ is called an equilibrium pair if $\sum_{j=1} h^t_j(\hat{r}^t; \hat{p}^t) = (0; e)$.

The following result is similar to Proposition 2.1.

Lemma 3.1. Consider Model II, and let $t \in N$. Then all equilibrium pairs (if any) have the same (commodity) price index.

Proof. If $(\hat{r}; \hat{p})$ is an equilibrium pair, then $z(\hat{r}; \hat{\pi}) = 0$ and $\theta(\hat{r}; \hat{p}) = e$. Hence, from Walras' law (4), $\hat{\pi} = M/\langle p^k; e\rangle$. ∎

From the proof of Lemma 3.1, our only possible candidate for equilibrium price index is $\pi^t := M^t/\langle p^k; e^t\rangle$. We may consider assumptions on z:

(II.3) $z(r; \pi)$ is continuous and strictly decreasing as a function of $r \in]-\infty; +\infty[$

(II.4) There exist $\underline{r}^t; \overline{r}^t \in]-\infty; +\infty[$ such that $z(\underline{r}^t; \pi) \geq 0$, $z(\overline{r}^t; \pi) \leq 0$

From (II.3) and (II.4), by the intermediate value theorem, there exists[6] one, and only one, $\hat{r}^t \in]-\infty; +\infty[$ such that $z(\hat{r}^t; \pi) = 0$. The (sort of) unicity result for Model II is now immediate from Lemma 3.1.

Proposition 3.2. Consider Model II and assume (II.3) and (II.4). Let $t \in N$. Then all equilibrium pairs (if any) have the same interest rate and (commodity) price index.

As in Model I, assumptions on γ may be made. We denote $\hat{B}^t_j := \beta^t_j(\hat{r}^t; \pi)$ and $\hat{m}^t_j := m_j - (1 + r^{t-1})b^{t-1}_j + \hat{B}^t_j$. Notice that $\sum_{j=1} \hat{m}_j = M$. In (II.5) and (II.6), the proportions $\nu_j := \hat{m}_j/M$, $j = 1; \ldots; l$ are to be taken to define $\gamma(A; \cdot)$.

(II.5) $\gamma(A; p)$ is continuous as a function of $(A; p) \in R^{1+l}_{++}$

(II.6) $\lim_{p \to \bar{p}} \|\gamma(M; p)\| \to +\infty$, for all $\bar{p} \geq 0$ with $\bar{p}_i = 0$ for some i

Proposition 3.3. Consider Model II and assume (II.3)-(II.6). Let $t \in N$. Then there is some equilibrium pair.

Proof. The assumptions of Model I are satisfied, with $\hat{m}_j$ in place of m_j. Recall that $\sum_{j=1} \hat{m}_j = M$. Applying Proposition 2.2, we have that there exists $\hat{p} \in R^l_{++}$ such that $\sum_{j=1} f_j(\hat{m}_j; \hat{p}) = e$. Further, from (1), $\langle \hat{p}; e\rangle = M$, and thus $\hat{\pi} = \pi$. We can conclude that $(\hat{r}; \hat{p})$ is an equilibrium pair:

$$\sum_{j=1} h_{j1}(\hat{r}; \hat{p}) = z(\hat{r}; \pi) = 0$$

$$\sum_{j=1} h_{j2}(\hat{r}; \hat{p}) = \sum_{j=1} f_j(\hat{m}_j; \hat{p}) = e$$

∎

As in Model I, the equilibrium price index depends on M (and does not depend on the m_j), and also in a linear way:

[6] Obviously, for the mere existence it would be sufficient to assume the condition of (II.3) for $r \in [\underline{r}; \overline{r}]$.

Proposition 3.4. Consider Model II and assume (II.3)-(II.6). Let $t \in \mathbb{N}$. Suppose that the quantity of money changes from M to λM (with $\lambda > 0$), within the same hypotheses. Then the equilibrium price index for λM is that for M multiplied by λ.

In contrast with Model I, if all individual allowances m_j (and not only the quantity of money M) change by the same factor λ, then the equilibrium (commodity) price vectors are not necessarily multiplied by λ (and the new corresponding equilibrium individual demands may be altered). In fact, the settlement of past loans term $(1 + r^{t-1})b_j^{t-1}$ does not change, and this means a redistribution of purchasing power among consumers when allowances and prices move in the same scale[7].

We conclude that quantity theory (as it is usually defined) holds: if the quantity of money is multiplied by λ, so it is the price index. On the other hand, money is not neutral, at least exactly.

4. A touch of immortality

We consider now two substantial changes in the last model (whose notation we keep in the sequel, unless otherwise stated). Firstly, money is now nonperishable: if consumer j does not spend some amount d_j^t at period t, it stays as a deposit in the bank, yielding interest at the beginning of period $(t + 1)$ at rate $\underline{\rho}^t$, the (bank) deposit rate. Secondly, the bank can grant loans; it makes so by creating new money. At period t, the consumer can contract a money-denominated credit c_j^t with the bank at rate of interest $\overline{\rho}^t$, the (bank) credit rate. Private loans (i.e., between consumers) are still possible, as in Model II, at agreed rate r^t. Thus the consumer has $a_j^t \equiv m_j^t - (1 + r^{t-1})b_j^{t-1} - (1 + \overline{\rho}^{t-1})c_j^{t-1} + (1 + \underline{\rho}^{t-1})d_j^{t-1} + b_j^t + c_j^t - d_j^t$ available units of money to spend.

In contrast to Model I and Model II, now the authorities have instruments of policy. We assume that the bank can set $\underline{\rho}^t$ and $\overline{\rho}^t$, with $\underline{\rho}^t \le \overline{\rho}^t$, and the Government can determine M^t. In both cases there are constraints: $\underline{\rho}^t \ge 0$ and $M^t \ge M_0^t$, where $M_0^t \ge 0$ is a threshold already known when period t begins[8]. The first constraint is imposed by the technology of money, the second one by the realities of governance. We denote $V^t \equiv \{x \in \mathbb{R} : x \ge M_0^t\}$ and $W \equiv \{(x_1; x_2) \in \mathbb{R}_+^2 : x_1 \le x_2\}$. The proportions m_j^t/M^t, $j = 1; \ldots; l$, depend on M^t, i.e., there are functions $g_j^t : V^t \to \mathbb{R}_+$, $j = 1; \ldots; l$, such that $m_j^t = g_j^t(M^t)$. Unless otherwise specified, $(\underline{\rho}; \overline{\rho}) \in W$ and $M \in V$ are fixed in the sequel.

The bank imposes no rationing in its deposit and credit facilities. We suppose that $r \in [\underline{\rho}; \overline{\rho}]$, i.e., consumers prefer to contract with the bank if the latter offers a better deal (rate of interest) than fellow consumers.

All in all, consumer j saves $s_j^t \equiv d_j^t - b_j^t - c_j^t$. Ex post, as for the moment we are contemplating it, $\sum_{j=1} b_j^t = 0$. Macroeconomically, if we write $D^t \equiv \sum_{j=1} d_j^t$ and $C^t \equiv \sum_{j=1} c_j^t$, we have that $S^t = D^t - C^t$. After settling the contracts of period $(t - 1)$, the aggregated amount of money held by consumers at the start of period

[7] Even if this effect could be overlooked, hardly anything has been assumed on the functions β_j, and on the way in which they incorporate expectations.

[8] Further constraints might be imposed, v.g., perhaps there is some $\varsigma \ge 0$ such that it must hold that $\overline{\rho} - \underline{\rho} \ge \varsigma$.

t is $N^t \equiv M^t + (1+\underline{\rho}^{t-1})D^{t-1} - (1+\bar{\rho}^{t-1})C^{t-1}$; within period t, C^t new money is to be created, and $\bar{D}^t$ is to be kept away from the commodity market.

In period t, given a rate of interest (for private loans) r^t and commodity prices p^t, consumer j makes estimations about the future and has to decide: how much he deposits in the bank, how much he borrows from the bank, the amount he borrows (lends) privately and the commodity bundle in the current period. There are four markets: for deposits, credits[9], private loans and commodities. The preferences of the consumer are given by a function $h^t_j : [\underline{\rho};\bar{\rho}] \times R^l_{++} \to R_+ \times R_+ \times R \times R^l_+$ defined for pairs $(r;p)$, where $h^t_{j1}(r;p)$ represents the desired bank deposits, $h^t_{j2}(r;p)$ the desired borrowing from the bank[10], $h^t_{j3}(r;p)$ the desired private borrowing (lending), and $h^t_{j4}(r;p)$ the desired commodity bundle when the constraint on the present budget is that determined by $(r;p)$. We assume that $m^t_j - (1+r^{t-1})b^{t-1}_j - (1+\bar{\rho}^{t-1})c^{t-1}_j + (1+\underline{\rho}^{t-1})d^{t-1}_j + h^t_{j3}(r;p) + h^t_{j2}(r;p) - h^t_{j1}(r;p) \geq 0$.

We suppose that h_{j1}, h_{j2} and h_{j3} depend on commodity prices only through the corresponding price index. Formally: $h^t_{j1}(r;p) = \delta^t_j(r;P)$, where $\delta^t_j : [\underline{\rho};\bar{\rho}] \times R_{++} \to R_+$; $h^t_{j2}(r;p) = \kappa^t_j(r;P)$, where $\kappa^t_j : [\underline{\rho};\bar{\rho}] \times R_{++} \to R_+$; $h^t_{j3}(r;p) = \beta^t_j(r;P)$, where $\beta^t_j : [\underline{\rho};\bar{\rho}] \times R_{++} \to R$. The aggregate demands for deposits and bank credit are $\Delta^t \equiv \sum_{j=1} \delta^t_j$ and $K^t \equiv \sum_{j=1} \kappa^t_j$, respectively, and the aggregate excess demand for private borrowing (lending) is $z^t \equiv \sum_{j=1} \beta^t_j$.

Similarly to Model II, we suppose that there exist functions $f^t_j(a_j;.) : R^l_{++} \to R^l_+$, for $a_j \geq 0$, such that $h^t_{j4}(r;p) = f^t_j(m_j - (1+r^{t-1})b^{t-1}_j - (1+\bar{\rho}^{t-1})c^{t-1}_j + (1+\underline{\rho}^{t-1})d^{t-1}_j + \beta^t_j(r;P) + \kappa^t_j(r;P) - \delta^t_j(r;P);p)$. Given $\gamma^t \in U \equiv \{x \in R^l_+ : \sum_{j=1} x_j = 1\}$, and given $A > 0$, then the function $\xi^t(\gamma^t;A;.) : R^l_{++} \to R^l_+$ is defined by $\xi^t(\gamma^t;A;p) \equiv \sum_{j=1} f^t_j(\gamma^t_j A;p)$. We assume:

(III.1) $f_j(a_j;p) \in \{x \in R^l_+ : \langle p;x\rangle = a_j\}$, for all $a_j \geq 0; p \in R^l_{++}$

(III.2) $f_j(\lambda a_j;\lambda p) = f_j(a_j;p)$, for all $a_j \geq 0; \lambda > 0; p \in R^l_{++}$

The hypotheses made so far in this section define our Model III. A kind of Walras' law already follows (where $\mu^t \equiv \sum_{j=1} h^t_{j4}$):

$$(5)\quad \langle p;\mu(r;p)\rangle = N + z(r;P) + K(r;P) - \Delta(r;P), \text{ for all } (r;p) \in [\underline{\rho};\bar{\rho}] \times R^l_{++}$$

A pair $(\hat{r}^t;\hat{p}^t) \in [\underline{\rho};\bar{\rho}] \times R^l_{++}$ is called an equilibrium pair if $\sum_{j=1} h_{j3}(\hat{r}^t;\hat{p}^t) = 0$ and $\sum_{j=1} h_{j4}(\hat{r}^t;\hat{p}^t) = e$. Observe that nothing is required here for the deposits market and the credit market, where demand is always met by supply.

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[9] Since the prices in the markets for deposits and credits (i.e., $\underline{\rho}$ and $\bar{\rho}$) are fixed, the adjustment is through the supply: we recall that in both markets the bank imposes no rationing. The consumers can deposit and borrow as much as they want. Just as for private loans, the possibility of default is excluded; cf footnote[4] .

[10] Notice the presence of r as an argument in the demand for bank credit function $h_{j2}(r;p)$ and the demand for deposits function $h_{j1}(r;p)$, as private borrowing (lending) is an alternative to them.

Two assumptions on z may be made[11], like in Model II, but now there is no such an impervious candidate for equilibrium price index as then.

(III.3) z(r; P) is continuous and strictly decreasing as a function of r 2 [½; ½], for all P 2 R_{++}

(III.4) z(½; P) ¸ 0; z(½; P) · 0, for all P 2 R_{++}

From (III.3) and (III.4), for every P^t 2 R_{++} there exists a unique $e^t_{P^t}$ 2 [½; ½] such that z($e^t_{P^t}$; P^t) = 0. We consider the aggregate deposits and the aggregate bank credit when there is equilibrium in the private loans market, $±^t$: R_{++} ! R_+ and $·^t$: R_{++} ! R_+, de…ned by $±^t$(P) , $¢^t$(e^t_P; P) and $·^t$(P) , K^t(e^t_P; P), respectively; both are functions of the price level. Also, when there is equilibrium in the private loans market, $¹^t$, N^t + $·^t$ represents the aggregate quantity of money, $¾^t$, $±^t$ ¡ $·^t$ the aggregate saving and $Ã^t$, N^t ¡ $¾^t$ the aggregate expenditure. Notice that $Ã^t$(P) ¸ 0 for all P 2 R_{++}. For simplicity, we call ¹ the (aggregate) quantity of money function, ¾ the (aggregate) saving function and Ã the (aggregate) demand function.

With the above assumptions, the equilibrium (commodity) price indices (if there is some equilibrium) must be …xed points of a real function[12], namely $´^t$: R_{++} ! R_+ de…ned by $´^t$(P) , $Ã^t$(P))=hp^k; ei.

Proposition 4.1. Consider Model III and assume (III.3) and (III.4). Let t 2 N. If (e; p) is an equilibrium pair, then P is a …xed point of ´ and e = e_P.

Proof. If (e; p) is an equilibrium pair, then z(e; P) = 0 (thus e_P = e) and µ(e; p) = e. Hence, from Walras' law (5), hp; ei = N ¡ ¾(P), and therefore ´(P) = P. ∎

We may consider now assumptions on ³:

(III.5) ³(°; A; p) is continuous as a function of (A; p) 2 R^{1+l}_{++}, for all ° 2 U

(III.6) $\lim_{p \,!\, \overline{p}}$ k³(°; A; p)k ! +1, for all ° 2 U, A > 0 and $\overline{p}$ ¸ 0 with $\overline{p}_i$ = 0 for some i

The last two assumptions make possible to turn the statement of Proposition 4.1 into a characterization.

Lemma 4.2. Consider Model III and assume (III.3)-(III.6). Let t 2 N. Then (e; p) is an equilibrium pair if, and only if, P is a …xed point of ´ and e = e_P.

Proof. One implication is Proposition 4.1. Conversely, suppose that P is a …xed point of ´ and e = e_P. Obviously, z(e; P) = 0. We denote e_j , $¯_j$(e; P), e_j , $·_j$(e; P), e_j , $±_j$(e; P) and e_j , m_j ¡ (1+$r^{t¡1}$)$b^{t¡1}_j$ ¡ (1+$½^{t¡1}$)$c^{t¡1}_j$ + (1+$½^{t¡1}$)$d^{t¡1}_j$ + e_j + e_j ¡ e_j. Observe that $\sum_{j=1} e_j$ = N + K(e; P) ¡ ¢(e; P) = Ã(P). Also, taking r , e and (e.g.) p , Pp^k in (5), we have that Ã(P) > 0. The assumptions of Model I are again satis…ed, with e_j and Ã(P) in place of m_j and M. From Proposition 2.2,

[11] Some comment on (III.4) is in order. Private lenders (borrowers) compete with bank deposits (credits) for the favour of consumers. It seems realistic to suppose that, the rate of interest being equal, the bank tends to conquer, if only because consumers wish to save shoe leather. Hence, more often than not, $¯_j$(½; P) ¸ 0 and $¯_j$(½; P) · 0.

[12] Even without requiring (III.3) and (III.4), if (e; p) is an equilibrium pair, then (e; P) is a …xed point of a certain real function of two real variables. This is clear from the proof of Proposition 4.1.

there exists $\bar{p} \in \mathbb{R}^l_{++}$ such that $\sum_{j=1} f_j(\mathbf{e}_j;\bar{p}) = e$. Further, from (1), $\langle\bar{p}; e\rangle = \psi(\hat{P})$, and thus $\bar{P} = \phi(\hat{P}) = \hat{P}$. We can deduce that $(\mathbf{e};\bar{p})$ is an equilibrium pair:

$$\sum_{j=1} h_{j3}(\mathbf{e};\bar{p}) = z(\mathbf{e};\hat{P}) = 0$$

$$\sum_{j=1} h_{j4}(\mathbf{e};\bar{p}) = \sum_{j=1} f_j(\mathbf{e}_j;\bar{p}) = e$$

∎

In order to obtain the existence of equilibrium, it is now only natural to make further assumptions on ϕ (or, what is the same, on either σ or ψ). Notice that Model III and assumptions (III.3)-(III.6) depend on certain fixed $(\underline{\rho}^t;\bar{\rho}^t) \in W$ and $M^t \in V^t$; we are now going to hypothesize them for any $(\underline{\rho}^t;\bar{\rho}^t) \in W$ and $M^t \in V^t$. When we want to reflect explicitly that σ changes with M and $(\underline{\rho};\bar{\rho})$, we write $\sigma(M;\underline{\rho};\bar{\rho};\cdot)$; the same notation is applicable to ψ and ϕ. Also we denote $c_{M^t} \equiv \max\{0;(N^t/\langle p^k;e\rangle)\}$. The main point of the following assumption is that aggregate dissaving can be made smaller than any positive number by using the bank rates, whatever the price level is.

(III.7) There exists $M^t_1 \in V^t$ satisfying: for every $P > c_{M^t_1}$ and $\varepsilon > 0$, there exists $(\underline{\rho};\bar{\rho}) \in W$ such that $\sigma(M^t_1;\underline{\rho};\bar{\rho};P) \geq -\varepsilon$

The possibility of using the bank rates in an expansional policy has a limit: they must be nonnegative. However, if (III.8) below is assumed (where M_1 is as defined in (III.7)), an aggregate demand as large as required is obtained by levering M.

(III.8) For every $P > c_{M_1}$ and $\lambda > 0$, there exist $M \in V$; $(\underline{\rho};\bar{\rho}) \in W$ such that $\psi(M;\underline{\rho};\bar{\rho};P) \geq \lambda$

A continuity assumption suggests itself in the way to reaching equilibrium (again M_1 is as defined in (III.7)).

(III.9) $\psi(M;\underline{\rho};\bar{\rho};P)$ is continuous as a function of $(M;(\underline{\rho};\bar{\rho})) \in V \times W$, for all $P > c_{M_1}$

In the next result, any price index larger than c_{M_1} is compatible with the existence of equilibrium.

Proposition 4.3. Let Model III and (III.3)-(III.6) hold for any $(\underline{\rho};\bar{\rho}) \in W$ and $M \in V$, and assume (III.7)-(III.9). Let $t \in \mathbb{N}$. Then, for all $\bar{P} > c_{M_1}$, there are $\hat{M} \in V$ and $(\hat{\underline{\rho}};\hat{\bar{\rho}}) \in W$ for which there is some equilibrium pair with (commodity) price index $\bar{P}$.

Proof. Let $\bar{P} > c_{M_1}$. From (III.7),

$$\exists(\underline{\rho}_1;\bar{\rho}_1) \in W \text{ such that } \sigma(M_1;\underline{\rho}_1;\bar{\rho}_1;\bar{P}) \geq N_1 - \langle p^k;e\rangle\bar{P}$$

where clearly $N_1 \equiv M_1 + (1+\underline{\rho}^{t-1})D^{t-1} - (1+\bar{\rho}^{t-1})C^{t-1}$. By (III.8),

$$\exists(M_2;(\underline{\rho}_2;\bar{\rho}_2)) \in V \times W \text{ such that } \psi(M_2;\underline{\rho}_2;\bar{\rho}_2;\bar{P}) \geq \langle p^k;e\rangle\bar{P}$$

Hence $\phi(M_1;\underline{\rho}_1;\bar{\rho}_1;\bar{P}) \leq \bar{P}$ and $\phi(M_2;\underline{\rho}_2;\bar{\rho}_2;\bar{P}) \geq \bar{P}$. Since $V \times W$ is connected, it follows from (III.9) that there exists $(\hat{M};(\hat{\underline{\rho}};\hat{\bar{\rho}})) \in V \times W$ such that $\bar{P}$ is a fixed point of $\phi(\hat{M};\hat{\underline{\rho}};\hat{\bar{\rho}};\cdot)$. Applying Lemma 4.2, $(\mathbf{e};\mathbf{p})$ is an equilibrium pair, where $\mathbf{e} \equiv \mathbf{e}_{\bar{P}}$ and $\mathbf{p} \equiv \bar{P}p^k$. ∎

A contentious issue is whether the descent of prices can be able to overcome a contractionary shock on demand. In order to obtain the existence of equilibrium (this time not necessarily compatible with every price index larger than c_M), an alternative to (III.7)-(III.9) is to keep M …xed and replace (III.8) by

(III.8') For every À > 0 there exist (½; ⅟₂) 2 W; P > 0 such that Ã(½; ⅟₂; P)=P ¸ À

Then (III.7) and (III.9) should also be altered in the obvious way:

(III.7') For every " > 0 there exist (½; ⅟₂) 2 W; P > c_M such that ¾(½; ⅟₂; P) ¸ ¡"

(III.9') Ã(½; ⅟₂; P) is continuous as a function of ((½; ⅟₂); P) 2 W £ $\bar{R}_{++}$

The proof of Proposition 4.4 is similar to that of Proposition 4.3. Recall that now M is again …xed.

Proposition 4.4. Let Model III and (III.3)-(III.6) hold for any (½; ⅟₂) 2 W, and assume (III.7')-(III.9'). Let t 2 N. Then there is (½; ⅟₂) 2 W for which there is some equilibrium pair.

In Model III (assuming also (III.3) and (III.4), the aggregate quantity of money and the aggregate demand are endogenously determined. From Proposition 4.1, any equilibrium price index (if it exists) is proportional to the corresponding aggregate demand (the proportionality factor is $1{=}\mathrm{h}p^k; e\mathrm{i}$). In order that the quantity theory may hold, the permanence of deposits as a constant fraction of the quantity of money (under the change of the latter) has to be hypothesized.

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