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A Surplus Approach.

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Objectives of an Imperfectly Competitive Firm: A Surplus Approach.

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Abstract

We consider a firm acting strategically on behalf of its shareholders. The price normalization problem arising in general equilibrium models of imperfect competition can be overcome by using the concept of real wealth maximization. This concept is based on shareholders' aggregate demand and does not involve any utility comparisons. We explore the efficiency properties of real wealth maxima for the group of shareholders. A strategy is called S-efficient (S stands for shareholders) if there is no other strategy such that shareholders' new total demand can be redistributed in a way that all shareholders will be better off. Our main result states that the set of real wealth maximizing strategies coincides with the set of S-efficient strategies provided that shareholders' social surplus is concave. Thus, even if a firm does not know the preferences of its shareholders it can achieve S-efficiency by selecting a real wealth maximizing strategy.

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1 Introduction

In this paper, we consider a simple model of a firm acting strategically on behalf of its shareholders. The firm influences relative prices in the economy either by its production decision or directly as a price setter. We assume that the firm is owned by a large number (ideally, a continuum) of small shareholders who take prices and wealth as given when choosing their consumption plans. The wealth of a consumer consists of the value of his initial endowment and his profit share. If a firm is engaged in imperfect competition, the strategy choice does not only affect the wealth of its shareholders, but also the prices shareholders face as consumers on the market. Since demand patterns and shareholdings differ across individuals, different shareholders would like their firm to pursue different objectives. A similar problem is encountered in economies with incomplete markets, where shareholders disagree because of their idiosyncratic insurance needs. Thus, in case of imperfect competition as well as incomplete markets, a social choice problem arises for which there is no obvious solution [see, in particular, Arrow (1983), p. 2].[1]

In the literature, this social choice problem is often assumed away. In the field of industrial organization, it is nearly always taken for granted that shareholders only consume and own goods whose prices do not depend on the action of their firm. Similarly, in the finance literature dealing with incomplete markets, the firm is often assumed to neglect that the choice of its production plan influences the insurance possibilities of its shareholders ("market value maximization").

In many models of imperfect competition, firms are assumed to maximize profits. However, it is well known from the literature that this objective is ill-defined unless particular, strong assumptions are made, see for example Gabszewicz and Vial (1972) and H. Dierker and Grodal (1986). Since the price level remains undetermined, profits are normalized by using one of the commodities as numéraire or, more generally, by applying some price normalization rule. But different price normalizations entail profit functions which are in general not related to each other by monotone transformations. Hence, maximization of profits in different normalizations amounts to firms pursuing different objectives.

To overcome the price normalization problem, E. Dierker and Grodal (1999) propose the concept of real wealth maximization. Given the strategies of all other firms, the strategy $\hat{\sigma}$ maximizes shareholders' real wealth if it is undominated in the following sense: There does not exist another strategy σ' such that the aggregate demand of all shareholders at $\hat{\sigma}$ is in the interior of their aggregate budget set at σ'.

Real wealth maximization is based on profits and the composition of the aggregate demand of the firm's shareholders. Moreover, it is independent of any a

[1] Headnote to Arrow (1950) in Arrow (1983). We are grateful to M. Hellwig for drawing our attention to this headnote.

priori chosen price normalization. If real wealth maximization is applied, shareholders' aggregate demand endogenously yields a yardstick to compare profits. If $\hat{\sigma}$ is a real wealth maximum and $\hat{D}$ is shareholders' total demand at $\hat{\sigma}$, then shareholders' wealth never suffices to buy more than $\hat{D}$.

In an important contribution to the theory of incomplete markets, Drèze (1974) defines the goal of a firm by using Pareto comparisons accompanied by redistribution. Given the production decisions of all other firms, a production decision of the firm under consideration leads to an allocation of goods among its shareholders. In general, these allocations will not be Pareto comparable. Therefore, Drèze proposes the following test that a production decision has to pass: It must be impossible to choose another production plan together with a redistribution scheme for the group of shareholders such that all shareholders will be better off if they keep their portfolio fixed. Clearly, since markets are incomplete, the redistribution is only allowed to involve the good available to the group of shareholders at the present date $t = 0$.

We will now formulate the analogue of the Drèze criterion for the case of imperfect competition. Consider a specific firm and assume that the strategies of all other firms are given. The strategy σ of the firm gives rise to a price system which, together with the profits $\Pi(\sigma)$, determines the budget set of each shareholder. Thus, each strategy σ induces an allocation of goods among the shareholders of the firm under consideration. As in the case of incomplete markets, these allocations will in general not be Pareto ranked. We say that strategy σ' of a firm dominates strategy σ if the aggregate demand of the shareholders of this firm at σ' can be redistributed in such a way that every shareholder will be better off. In analogy to the Drèze criterion, we propose the following test that a strategy $\bar{\sigma}$ has to pass: There is no strategy σ' that dominates $\bar{\sigma}$. An undominated strategy is called S-efficient (S stands for shareholders).

It is important to clarify the role redistribution plays in our setting. We want to emphasize that no redistribution ever takes place among the shareholders.[2] Redistribution only enters in the form of the following thought experiment. Suppose there is an omniscient hypothetical planner who can freely redistribute goods among shareholders. If the firm chooses a certain strategy σ, the hypothetical planner checks whether there exists another strategy σ' such that he can achieve a Pareto improvement for the shareholders by redistributing their aggregate demand at σ' among them. If he can, the strategy σ is discarded. If he cannot, σ can be implemented. Clearly, to implement σ the planner is not needed. Therefore, the assumption of an omniscient planner who possesses the power to perform any lump sum redistribution presents no restriction here since

[2] This point has caused a certain confusion in the literature on general equilibrium theory with incomplete markets. In our view, the usual definition of the set of feasible allocations is inappropriate since redistribution at time $t = 0$ is allowed to take place in the traditional definition of a feasible allocation.

it is used only to single out certain allocations, which can be obtained without any help of a planner.[3]

The Drèze criterion is based on Pareto comparisons and cannot be stated without reference to preferences. To make Pareto comparisons the firm is supposed to know the distribution of shareholders' preferences. By contrast, a firm maximizing shareholders' real wealth only needs to know their aggregate demand function. Obviously, these informational requirements are much less demanding, and the question arises as to how both concepts are related to each other.

In order to analyze how real wealth maxima and S-efficient allocations are related, a particular type of imperfect competition must be stipulated. We opt for a framework in which firms set prices and we study the behavior of a firm under the assumption that the prices of its competitors are given. For this purpose, it suffices to focus on an economy with two commodities and a price setting monopolist who produces good 1 using good 0, the numéraire, as input. The strategy P of the firm is the decision to offer one unit of the product in exchange for P units of the numéraire.

We will show that any S-efficient strategy maximizes shareholders' real wealth if the firm's profit function is concave. Since real wealth maximization is defined without reference to utility functions, the more interesting question is whether a real wealth maximizing strategy is S-efficient.

To address this question, we first assume that shareholders have quasilinear preferences. In this case, shareholders' preferences can be aggregated into a single preference relation, for which shareholders' social surplus is a utility representation. Therefore, S-efficiency amounts to maximizing the utility of the representative owner, that is to surplus maximization. Moreover, a surplus maximizing strategy maximizes real wealth [cf. Section 5]. Thus, if there is a unique real wealth maximizing strategy it must be S-efficient.

In order to formulate conditions ensuring a unique real wealth maximum, we introduce the notion of a shareholder's marginal willingness to pay for an increase of the strategy P. The sum of these marginal willingnesses vanishes if and only if real wealth is maximized. Thus, if the sum of the marginal willingnesses is strictly decreasing, there is only one real wealth maximum. It turns out that this monotonicity property is equivalent to the strict concavity of the surplus function. Hence, if shareholders' utilities are quasilinear, strict concavity of the surplus function implies that real wealth maximization entails a unique outcome, which is S-efficient.

If we give up the quasilinear framework, some of these statements carry over and others are lost or need to be modified. In the quasilinear setting, the existence

[3] Drèze's characterization of the objective of a firm is intimately related to Shapley's extension of the value to NTU games. In both cases, the solution concept involves a tool that is not available to the agents. This hypothetical tool is used to formulate the following postulate: The outcome should be such that no improvement would be possible even if the tool were available.

of an S-efficient outcome is obvious. However, S-efficiency may be unobtainable in more general cases. We present an example in which there are several real wealth maxima, but all of them dominated. The nonexistence of an S-efficient outcome relies on the fact that the wealth of a shareholder is given by a fixed assignment of initial endowments and profit shares. Thus, nonexistence is due to a conflict between efficiency and distribution. The incompatibility of efficiency with a given distribution has been discovered by Guesnerie (1975) in the context of nonconvex production sets.

A natural way to ensure existence of S-efficient outcomes and the S-efficiency of real wealth maxima is to generalize the insight obtained from the quasilinear case. Let $\hat{P}$ maximize real wealth and define shareholders' compensated social surplus associated with $\hat{P}$ as the difference between the aggregate wealth they obtain at some strategy P and the expenditures needed to keep them on the utility levels they have at $\hat{P}$. If this social surplus function is concave, then $\hat{P}$ is S-efficient. To obtain uniqueness of a real wealth maximum we use strict concavity of shareholders' uncompensated surplus function.

Therefore, concavity of surplus functions plays a crucial role in the quasilinear as well as the general case. Hence, we show the invariance of this property with respect to the choice of economically meaningful linear structures on strategies and on wealth.

The paper is organized as follows: Section 2 introduces the model. In Section 3 and Section 4, real wealth maximization and S-efficiency are defined, respectively. In Section 5, we analyze the connection between social surplus maximization, real wealth maximization, and S-efficiency in the quasilinear setting. Section 6 is devoted to the invariance of the concavity of the profit and the surplus functions with respect to meaningful normalization rules. Section 7 presents the example. In Section 8, it is shown that real wealth maximization and S-efficiency are identical goals if shareholders' compensated social surplus is concave. Section 9 concludes the paper.

2 Model and Basic Notation

It suffices to consider an economy with two commodities and one price setting monopolist who produces good 1 using good 0, the numéraire, as input. The analysis will be essentially the same as that of a price setting firm in an oligopolistic market if the prices of its competitors are given. For simplicity's sake, we assume that the firm has fixed unit costs $c > 0$. The strategy P is the commitment to deliver one unit of the product in exchange for P units of the numéraire. If commodity 0 serves as numéraire, we use the subscript N. For instance, profits obtained at prices $(1, P)$ are denoted $\Pi_N(P)$.

The consumers are denoted by $I = \{1, \cdots, m\}$. Consumer $i \in I$ has shares $\vartheta^i \geq 0$ in the firm. We assume that the firm has a large set $\mathcal{I} = \{i \in I \mid \vartheta^i > 0\}$ of shareholders and that all consumers, owners as well as nonowners, take their budget sets as given. Suppose, for simplicity's sake, that the consumption set of every consumer equals $\mathbb{R}^2_+$ and that no consumer has initial endowments of the product, that is, consumer i has the initial endowment $e^i = (e^i_0, 0)$ where $e^i_0 > 0$.

Since Pareto comparisons are made, we assume that every consumer i has a C^1 demand function d^i that is generated by a strictly convex, monotone preference relation $\succ^i$, which can be represented by the C^2 utility function U^i. Moreover, whenever convenient, expenditure functions are assumed to be C^2 in prices. The demand function d^i is homogeneous of degree 0 and satisfies the budget identity $(1, P)d^i(1, P, W^i_N) = W^i_N$. The wealth of consumer i at prices $(1, P)$ is described by the function $W^i_N(P) = e^i_0 + \vartheta^i \Pi_N(P)$ and $W_N(P) = \sum_{i \in \mathcal{I}} W^i_N(P)$ denotes the aggregate wealth of the shareholders.

We assume throughout that profit expectations are correct, that is, the demand based on consumers' wealth expectations generates precisely the expected profits if the monopolist satisfies the demand for its product. That is to say, profits fulfill $\Pi_N(P) = (P - c)d_1(P)$, where $d_1(P) = \sum_{i=1}^m d^i_1(1, P, W^i_N(P))$ is the total demand of all consumers for good 1 if prices are $(1, P)$ and profits are $\Pi_N(P)$. We assume that Π_N is a continuous function. Let $D^i(P) = d^i(1, P, W^i_N(P))$ denote shareholder i's demand corresponding to strategy P. Shareholders' aggregate demand is $D(P) = \sum_{i \in \mathcal{I}} D^i(P)$. Let P_{max} be the smallest P at which Π_N attains its maximum. Clearly, no shareholder wishes the firm to charge a price above P_{max}. Therefore, we consider only strategies in $\mathcal{P} = [c, P_{max}]$. We assume that $W_N(P) > 0$ for all $P \in \mathcal{P}$. Moreover, the demand of the nonowners for the firm's product is supposed to be positive if $P = c$.

Since we only analyze commodity assignments to the group $\mathcal{I}$ of shareholders, we call such assignments allocations for short. For every strategy $P \in \mathcal{P}$, there exists exactly one allocation, namely the allocation $(D^i(P))_{i \in \mathcal{I}}$. An allocation is attainable iff it can be implemented by a strategy choice of the firm.

Definition . *The allocation $(x^i)_{i \in \mathcal{I}}$ is* attainable *iff there exists $P \in \mathcal{P}$ such that $x^i = D^i(P)$ for all $i \in \mathcal{I}$.*

Observe that no sidepayments occur in the definition of an attainable allocation. As we have already mentioned, all agents, shareholders as well as nonshareholders, buy the firm's product at market prices. Thus, consumers are treated as anonymous. Nobody knows which characteristics any other, particular consumer possesses. This fact is in accordance with the Walrasian tradition. Walras equilibria depend only on the distribution of agents' characteristics. In the present framework no additional information is assumed. Therefore, sidepayments cannot be made.

3 Real Wealth Maximization

Each strategy P defines the budget line

$$BL(P) = \{(x_0, x_1) \in \mathbb{R}^2 \mid x_0 + Px_1 = W_N(P)\} \tag{1}$$

and the corresponding budget set

$$AB(P) = \{(x_0, x_1) \in \mathbb{R}^2_+ \mid (1, P)(x_0, x_1) \leq W_N(P)\}$$

of the group of owners. Their aggregate budget set is $AB = \bigcup_{P \in \mathcal{P}} AB(P)$. Note that AB is compact since $\mathcal{P}$ is compact and W_N is continuous. Since $\mathbb{R}^2_+ \setminus AB(P)$ is convex for every P and $AB = \mathbb{R}^2_+ \setminus \bigcap_{P \in \mathcal{P}} \left(\mathbb{R}^2_+ \setminus AB(P)\right)$, the aggregate budget set is the complement of a convex set. The North-East boundary of AB is called the *aggregate budget curve* ABC. More precisely,

$$ABC = \{x \in AB \mid \nexists\, z \gg 0 \text{ such that } x + z \in AB\}.$$

We define the objective of the monopolist without making a priori assumptions on the demand behavior of the shareholders. Consider two different strategies P_1, P_2 and the corresponding aggregate budget sets $AB(P_1)$ and $AB(P_2)$. First, we look at the extreme case, in which $AB(P_1)$ is strictly contained in $AB(P_2)$. Let $x \geq 0, x \neq 0$, be any commodity bundle. Clearly, the number of units of the bundle x which the shareholders can afford if the firm chooses strategy P_2 is strictly larger than the number of units they can buy if the firm chooses strategy P_1. Whatever bundle the firm uses to evaluate the real wealth of the shareholders, their aggregate wealth is larger at P_2 than at P_1. We assume that a real wealth maximizing firm choosing between P_1 and P_2 will select P_2, although it may very well be that some shareholders, due to distributional effects, prefer strategy P_1 to P_2.

In general, the budget sets corresponding to different strategies of the firm will not be ordered by inclusion. Hence the ordering of budget sets according to the number of units of the bundle x which can be bought out of shareholders' aggregate wealth depends on the choice of the reference bundle x. However, when the firm considers a strategy P, it is assumed to know the composition $x(P) = D(P)/\|D(P)\|$ of shareholders' aggregate demand at P. In our opinion, it is natural for the firm to use $x(P)$ as the reference bundle.

Note that, in general, shareholders do not agree on the strategy choice of their firm. Shareholder i wants the firm to maximize $U^i(D^i(P))$. Since shareholders differ with regard to shares, endowments, and preferences, they want the firm to pursue different goals. As a consequence, there will typically be a continuum of strategies that cannot be Pareto ranked. Pareto comparisons of attainable states cannot provide us with a useful definition of the goal of the firm.

The same holds true for profit maximization unless very strong assumptions are made. The maximization of profits Π_N measured in terms of the numéraire is justified only if shareholders do neither own nor consume the firm's product. Moreover, different ways to normalize prices and measure profits lead to different profit functions and hence different maxima. If there is no clear, a priori specified connection between some commodity basket used to define profits and the shareholders' desires, the maximization of a profit function cannot be used as an objective of the firm acting on behalf of its shareholders. In E. Dierker and Grodal (1999), the following relation is used to introduce an objective of the firm:

Definition . *Shareholders' real wealth at* $P_1 \in \mathcal{P}$ *can be increased by strategy* $P_2 \in \mathcal{P}$*, in symbols* $P_1 \prec_{rw} P_2$*, iff* $(1, P_2)D(P_1) < W_N(P_2)$ *.*

The objective of the firm is to choose a strategy $\hat{P}$ such that there is no other strategy P which increases shareholders' real wealth. That is to say, there is no other strategy P such that the aggregate demand $D(\hat{P})$ lies below the budget line $BL(P)$. If such a strategy P existed, the group of shareholders could buy more units of the bundle $D(\hat{P})$ if the firm chose strategy P instead of $\hat{P}$.

Definition . *Strategy* $\hat{P} \in \mathcal{P}$ maximizes shareholders' real wealth *if there is no strategy* $P \in \mathcal{P}$ *such that* $(1, P)D(\hat{P}) < W_N(P)$*, that is to say, if* $D(\hat{P}) \in ABC$*.*

Strategies maximizing shareholders' real wealth need not exist since the relation $\prec_{rw}$ need neither be acyclic nor convex. In E. Dierker and Grodal (1998), conditions on the aggregate demand function are given which imply that $\prec_{rw}$ is acyclic. Also, they show that convexity of $\prec_{rw}$ obtains if the profit function is concave. In either case, a real wealth maximum exists.

Observe that real wealth maximization reduces to profit maximization in case of perfect competition with complete markets. Moreover, it generalizes the standard approach in industrial organization, in which it is (implicitly) assumed that shareholders only own and consume the numéraire commodity.

The first order condition for real wealth maximization states that shareholders' marginal wealth equals their aggregate demand for the product.[4] Since we assume the value of the initial endowment, which takes the form $(e_0^i, 0)$, to be independent of P, we know that marginal wealth equals marginal profits.

Remark 1. *If* Π_N *is* C^1 *and* $\hat{P}$ *maximizes shareholders' real wealth, then*

$$W_N'(\hat{P}) = \Pi_N'(\hat{P}) = D_1(\hat{P}) \ .$$

In the remainder of this section, we present two alternative characterizations of the first order condition for real wealth maximization. The first will be used to establish the uniqueness of a real wealth maximum and is based on the concept of shareholder i's *indirect utility function* $u^i : \mathcal{P} \times \mathbb{R}_+ \to \mathbb{R}$ which is defined as

$$u^i(P, \tau) = U^i(d^i(1, P, W_N^i(P) + \tau)) \, .$$

[4] For a proof, see E. Dierker and Grodal (1998).

Agent i obtains the utility $u^i(P,\tau)$ if the relative price P is chosen and if i gets τ units of the bundle $(1,0)$ as sidepayment. His marginal willingness to pay for an infinitesimal change of the relative price P of the firm's product is defined by

$$MW^i(P) = \frac{\partial_P u^i(P,0)}{\partial_\tau u^i(P,0)}\,.$$

Remark 2. *Let $\bar{P} \in \operatorname{int}\mathcal{P}$. Then $\bar{P}$ satisfies the first order condition for real wealth maximization iff $\sum_{i\in\mathcal{I}} MW^i(\bar{P}) = 0$.*

Proof. By differentiation of u^i we obtain

$$\begin{aligned} \partial_P u^i(P,0) &= \partial_P \tilde{v}^i(1,P,W_N^i(P)) \\ &\quad + \partial_W \tilde{v}^i(1,P,W_N^i(P)) \cdot \partial_P(W_N^i(P)), \end{aligned}$$

where $\tilde{v}^i$ is shareholder $i's$ ordinary indirect utility function. Roy's identity yields

$$MW^i(P) = -D_1^i(P) + \vartheta^i \Pi_N'(P)\,.$$

By summation,

$$\sum_{i\in\mathcal{I}} MW^i(P) = \Pi_N'(P) - D_1(P) = W_N'(P) - D_1(P)\,.$$

Then $\bar{P}$ satisfies the first order condition for real wealth maximization if and only if $\sum_{i\in\mathcal{I}} MW^i(\bar{P}) = 0$. □

Furthermore, it is worth noting that the first order condition for real wealth maximization characterizes the envelope of the family of aggregate budget lines. To each strategy $P \in \mathcal{P}$ there corresponds the budget line

$$L_P = \{(x_0,x_1,P) \in \mathbb{R}^2 \times \mathcal{P} \mid (1,P)\cdot(x_0,x_1) - W_N(P) = 0\}\,.$$

The difference between L_P and $BL(P)$ as defined in (1) is that L_P is embedded in $\mathbb{R}^2 \times \mathcal{P}$, whereas $BL(P) \subset \mathbb{R}^2$. The 1-parameter family of budget lines $\{L_P\}_{P\in\mathcal{P}}$ forms a smooth 2-dimensional manifold denoted $\mathcal{L}$ since the derivative of the mapping $(x_0,x_1,P) \mapsto x_0 + Px_1 - W_N(P)$ does not vanish. Now, project $\mathcal{L}$ to the commodity space $\mathbb{R}^2$. The *envelope* of the family of budget lines is defined as the set of critical values of the projection of $\mathcal{L}$ into the commodity space $\mathbb{R}^2$. It is characterized by the condition $\partial_P(x_0 + Px_1 - W_N(P)) = 0$, that is to say $W_N'(P) = x_1$, together with the budget equation. Thus, the envelope is given by

$$Env = \{(x_0,x_1) \in \mathbb{R}^2 \mid \exists P \in \mathcal{P} \text{ with } x_0 + Px_1 = W_N(P) \text{ and } W_N'(P) = x_1\}.$$

Remark 3. *Strategy $\bar{P} \in \mathcal{P}$ satisfies the first order condition $W_N'(\bar{P}) = \Pi_N'(\bar{P}) = D_1(\bar{P})$ for real wealth maximization iff $D(\bar{P})$ lies in Env.*

4 S-Efficiency

S-efficiency refers to Pareto comparisons among the shareholders of a firm. The firm wants to extract wealth from the nonowners. However, if the firm raises its price for that purpose, then the shareholders themselves also have to pay more since they must buy the firm's product at market prices.

If the firm chooses P, then the group of shareholder obtains the profit $\Pi_N(P)$ as well as the commodity bundle $D(P)$. Note that the way the profit is raised cannot be separated from the way it is spent since the choice of strategy P determines both the profit income and the consumption of every shareholder. The definition of S-efficiency must take this link into account.

The concept of S-efficiency is based on the following thought experiment: Suppose $\bar{P}$ has been chosen and is compared with the alternative P. Clearly, if P is implemented, the group of shareholders receives the bundle $D(P)$, and the profit $\Pi_N(P)$ contains the part $(P-c)D_1(P)$ derived from this bundle. In order to keep this relation intact, $D(P)$ is kept fixed in the thought experiment. Assume that the group of shareholders could, after having obtained $D(P)$, redistribute this bundle in order to compensate the losers of the move from $\bar{P}$ to P. They would then certainly not move from the original strategy $\bar{P}$ to the alternative P if they could not even obtain a Pareto improvement for themselves in this hypothetical situation.

We are now going to define S-efficiency in a more formal way.

Definition . *The strategy $P_1 \in \mathcal{P}$ is* dominated *by the strategy $P_2 \in \mathcal{P}$ iff there exist bundles $(x^i)_{i\in\mathcal{I}}$ such that $\sum_{i\in\mathcal{I}} x^i = D(P_2)$ and $D^i(P_1) \prec^i x^i$ for all $i \in \mathcal{I}$.*

A strategy $\bar{P}$ is undominated if there is no $P \in \mathcal{P}$ such that $D(P)$ can be distributed among the shareholders in a way which leaves them better off than at $\bar{P}$. An undominated strategy $\bar{P}$ and the corresponding allocation $(D^i(\bar{P}))_{i\in\mathcal{I}}$ are called S-efficient.

Definition . *The strategy $\bar{P}$ and the corresponding allocation $(D^i(\bar{P}))_{i\in\mathcal{I}}$ are* S-efficient *iff there does not exist a strategy $P \in \mathcal{P}$ dominating $\bar{P}$.*

We derive the first order condition for S-efficiency.

Proposition 1. *Let the profit function Π_N be C^1 and assume that the allocation $(D^i(\bar{P}))_{i\in\mathcal{I}}$ is S-efficient. Then strategy $\bar{P}$ satisfies the first order condition*

$$(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) = 0\,.$$

This condition is equivalent to the first order condition for real wealth maximization, i.e. $D_1(\bar{P}) = W_N'(\bar{P})$.

Proof. Since Π_N is C^1, shareholders' aggregate demand D is C^1. Let $\bar{P} \in \mathcal{P}$ and assume that the allocation $(D^i(\bar{P}))_{i\in\mathcal{I}}$ is S-efficient. Assume by way of contradiction that $(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) \neq 0$. Without loss of generality let $(1, \bar{P}) \cdot (D_0'(\bar{P}), D_1'(\bar{P})) > 0$. (If this expression is negative, consider strategies $P < \bar{P}$.) Then $(1, \bar{P})(D(P) - D(\bar{P})) > 0$ for $P > \bar{P}$ and $|P - \bar{P}|$ sufficiently small. Since all preferences are strictly convex and the utility functions are C^2, there exists $\varepsilon > 0$ such that for any shareholder i the following condition holds. If $u \in \mathbb{R}^2$, $\|u\| < \varepsilon$, $(1, \bar{P}) \cdot u > 0$, and $D^i(\bar{P}) + u \in \mathbb{R}^2_+$, then $D^i(\bar{P}) + u \succ^i D^i(\bar{P})$ [see, e.g., Magill and Quinzii (1996), p. 359]. Since D is continuous, there exists $\delta > 0$ such that $\|D(P) - D(\bar{P})\| < \epsilon$ for $|P - \bar{P}| < \delta$. Now let $u^i = \vartheta^i(D(P) - D(\bar{P}))$. For $|P - \bar{P}|$ sufficiently small we obtain $D^i(\bar{P}) + \vartheta^i(D(P) - D(\bar{P})) \succ^i D^i(\bar{P})$ for all shareholders i. However, as $\sum_{i\in\mathcal{I}}(D^i(\bar{P}) + \vartheta^i(D(P) - D(\bar{P}))) = D(P)$, this contradicts the fact that $(D^i(\bar{P}))_{i\in\mathcal{I}}$ is S-efficient. Hence, $(1, \bar{P})\cdot(D_0'(\bar{P}), D_1'(\bar{P})) = 0$.

To ascertain that the first order condition for S-efficiency coincides with the first order condition for real wealth maximization, we differentiate the budget equation $(1, P) \cdot D(P) = W_N(P)$ and obtain $(1, P) \cdot (D_0'(P), D_1'(P)) + D_1(P) = W_N'(P)$. Hence, $(1, P) \cdot (D_0'(P), D_1'(P)) = 0$ iff $D_1(P) = W_N'(P)$. $\square$

On the assumption that the profit function Π_N is concave, E. Dierker and Grodal (1998), Theorem 3, show that a real wealth maximum obtains whenever the first order condition for real wealth maximization is satisfied. Thus, Proposition 1 implies the following:

Proposition 2. *Let D and Π_N be C^1 and Π_N concave. Assume that the strategy $\hat{P}$ is S-efficient. Then strategy $\hat{P}$ maximizes shareholders' real wealth.*

5 S-Efficiency and Uniqueness of Real Wealth Maxima: The Quasilinear Case

To explore under which conditions the converse of Proposition 2 holds, it is instructive to investigate the case in which all shareholders have quasilinear utility functions U^i. This setting has the following major advantage: Although shareholders may differ radically in their individual assessments of the strategy of their firm, their preferences can be aggregated into a single one for the following reason. Consider any two utility profiles of the shareholders and add their individually preferred sets. Then one of the aggregate preferred sets must be contained in the other. In other words, Scitovsky curves do not intersect each other due to the absence of income effects that affect the demand for the product. Therefore, the Scitovsky curves describe the preferences of a single consumer. Let U^{Rep} denote a utility function of this consumer. Let S_1 and S_2 be two aggregate preferred sets corresponding to the utility levels U_1 and U_2, respectively. Then $U_2 > U_1$

iff $S_2 \subset \operatorname{int} S_1$.[5] Clearly, if income effects are permitted, additional phenomena enter the picture. They will be analyzed in the following sections.

For the maximization of the utility of the representative owner, there is a clear economic interpretation. Since shareholders have quasilinear utilities, their consumers' surplus is unambiguously defined and their social surplus can be written as $S_N(P) = \Pi_N(P) + \int_P^\infty D_1(p)dp$. Assume that $S_N(P_1) < S_N(P_2)$ and let U_1 and U_2 be the associated utility levels, respectively. Let E_N^{Rep} denote the expenditure function of the representative owner. Then $\Pi_N(P_1) < \Pi_N(P_2) + \int_{P_2}^{P_1} D_1(p)dp = \Pi_N(P_2) + (E_N^{Rep}(P_1, U_2) - E_N^{Rep}(P_2, U_2))$. Denoting shareholders' aggregate initial endowment by $(e_0, 0)$, we have $e_0 + \Pi_N(P_1) = E_N^{Rep}(P_1, U_1)$ and $e_0 + \Pi_N(P_2) = E_N^{Rep}(P_2, U_2)$. As a consequence, the above inequality becomes $E_N^{Rep}(P_1, U_1) < E_N^{Rep}(P_2, U_2) + (E_N^{Rep}(P_1, U_2) - E_N^{Rep}(P_2, U_2)) = E_N^{Rep}(P_1, U_2)$, and we conclude that $U_1 < U_2$. This argument shows that surplus maximization amounts to maximizing the utility of the representative owner. In particular, a surplus maximum must be undominated.

The first order condition for surplus maximization, $\Pi'_N(P) = D_1(P)$, coincides with the first order condition for real wealth maximization. Moreover, the real wealth shareholders obtain at P_1 can be increased by choice of P_2, in symbols $P_1 \prec_{rw} P_2$, iff $(1, P_2)D(P_1) < W_N(P_2)$. Therefore, $P_1 \prec_{rw} P_2$ implies $U_1 < U_2$. Note, however, that the relation $\prec_{rw}$ is not complete. It may happen that the surplus maximum is unique, but that additional real wealth maxima exist at the same time.

Remark 4. *Assume that shareholders' utility functions are quasilinear. Then surplus maximization and the maximization of the representative owner's utility coincide. Furthermore, $P_1 \prec_{rw} P_2$ implies $U_1 < U_2$. In particular, a unique real wealth maximum is a surplus maximum and, therefore, undominated.*

Thus, the question arises as to the conditions entailing the uniqueness of a real wealth maximum. Obviously, if P equals unit costs c, profits vanish. Since the demand of the nonowners is positive at $P = c$ by assumption, an infinitesimal price increase raises shareholders' wealth, and the sum of the marginal willingnesses $MW^i(c)$ is positive. Similarly, if $P = P_{max}$, no shareholder's marginal willingness MW^i will be positive, and every shareholder who consumes a positive amount of the product would benefit from a lower price. Thus, $\sum_{i\in\mathcal{I}} MW^i(P_{max})$ is negative. Naturally, a unique real wealth maximum results if one assumes that $\sum_{i\in\mathcal{I}} MW^i(P)$ is strictly decreasing in the interval $\mathcal{P} = [c, P_{max}]$.

The present quasilinear setting provides an ideal framework for the interpretation of this monotonicity assumption. As laid out below, the marginal willingness $MW^i(P)$ can easily be integrated with respect to P, and the result is closely connected to i's part of the social surplus.

[5]This refers to the interior relative to $\mathbb{R}^2_+$.

Given $\tilde{P}$, let $T^i : \mathcal{P} \to \mathbb{R}$ describe the compensation shareholder i needs in order to stay at the utility level $\tilde{U}^i$, that is, $T^i(P)$ is uniquely defined by the equation $u^i(P, T^i(P)) = \tilde{U}^i = u^i(\tilde{P}, 0)$. Clearly, if P is raised infinitesimally, $dT^i(P)/dP$ has to be given to i in order to keep him at the utility level $\tilde{U}^i$. That is to say, i' marginal willingness is given by $MW^i(P) = -dT^i(P)/dP$. The assumption that $\sum_{i \in \mathcal{I}} MW^i(P)$ is strictly decreasing can then be restated as follows: The aggregate compensation $T(P) = \sum_{i \in \mathcal{I}} T^i(P)$ needed to keep every shareholder i on the utility level $\tilde{U}^i$ is a strictly convex function of P. One can think of $T(P)$ as the amount of shareholders' social surplus that can be liberated if the firm moves from P to $\tilde{P}$. In this case, shareholder i's part of the social surplus increases by $T^i(P)$.

The monotonicity assumption entailing the uniqueness and S-efficiency of a real wealth maximum can be rephrased as follows. We assume that Π_N is concave. Then the domain $C^i = \{(P, \tau) \in \mathcal{P} \times \mathbb{R} \mid W_N^i(P) + \tau \geq 0\}$ of the indirect utility function u^i is convex for every $i \in \mathcal{I}$. The *aggregate preferred set* associated with the surplus maximizing strategy $\tilde{P}$, and the resulting utility profile $(\tilde{U}^i)_{i \in \mathcal{I}}$ is

$$\tilde{A} = \{(P, \sum_{i \in \mathcal{I}} \tau^i) \in \mathcal{P} \times \mathbb{R} \mid (P, \tau^i) \in C^i \text{ and } u^i(P, \tau^i) \geq \tilde{U}^i \text{ for each } i \in \mathcal{I}\}\,.$$

Note that $\tilde{A}$ is the epigraph $\{(P, \tau) \in \mathcal{P} \times \mathbb{R} \mid \tau \geq T(P)\}$ of the aggregate compensation function T and that $\tilde{A}$ is strictly convex iff T is strictly convex.[6] As argued above, this is the case iff $\sum_{i \in \mathcal{I}} MW^i(P)$ is strictly decreasing.

The results are summarized in the following proposition.

Proposition 3. *Assume that the utility functions U^i of the shareholders $i \in \mathcal{I}$ are quasilinear and assume that the profit function Π_N is concave. Let $\tilde{P}$ denote a surplus maximizing strategy and $(\tilde{U}^i)_{i \in \mathcal{I}}$ the associated utility profile of the shareholders. Then the following conditions are equivalent:*

i) *The sum $\sum_{i \in \mathcal{I}} MW^i(P)$ of the marginal willingnesses to increase P is strictly decreasing for $P \in \mathcal{P} = [c, P_{max}]$.*

ii) *The total compensation $T = \sum_{i \in \mathcal{I}} T^i : \mathcal{P} \to \mathbb{R}$ needed to keep every shareholder on the utility level $\tilde{U}^i$ is a strictly convex function.*

iii) *The aggregate preferred set $\tilde{A} \subset \mathcal{P} \times \mathbb{R}$ is strictly convex.*

Each of these conditions implies the uniqueness and S-efficiency of a real wealth maximum. Moreover, maximization of real wealth and maximization of shareholders' social surplus coincide in this case.

[6] More precisely, the lower boundary of $\tilde{A}$ is strictly convex, i.e., $\tilde{A}$ is the intersection of a strictly convex set with $\mathcal{P} \times \mathbb{R}$.

6 Linear Structures on Strategies and Wealth

In Proposition 2 the profit function Π_N is assumed to be concave. Moreover, in Proposition 3 S-efficiency and uniqueness of a real wealth maximum is derived from an assumption either requiring a certain convexity property directly [see conditions ii) and iii)], or stipulating the monotonicity of $\sum_{i \in \mathcal{I}} MW^i(P)$. However, whenever we speak about the concavity or convexity of some set or function, we need an underlying linear structure. In the present case, we need linear structures on the sets of strategies and wealth, respectively.

In order to introduce such linear structures, we have expressed prices and wealth in units of commodity 0. Instead of using a particular good as numéraire, one could have taken any commodity bundle $x = (x_0, x_1)$, $x_0 > 0$. This leads to the question of whether Propositions 2 and 3 and other statements are invariant with respect to the choice of bundle x. In this section, we show that the concavity of the profit function, the convexity of the aggregate preferred set $\tilde{A}$, and related items are invariant with respect to the choice of bundle x.[7]

Prices normalized with respect to x are denoted (π_0, π_1), that is, (π_0, π_1) satisfies $\pi_0 x_0 + \pi_1 x_1 = 1$. Thus, π_1 denotes the output price in terms of bundle x and the corresponding input price is $\pi_0 = (1 - \pi_1 x_1)/x_0$. If bundle x is used to measure wealth, then W_x denotes the maximal number of units of bundle x affordable at prices (π_0, π_1).

Consider any $A_N \subset \mathbb{R}_+ \times \mathbb{R}$. As before, the subscript N indicates that we use good 0 as numéraire. The first component of $(P, W_N) \in A_N$ corresponds to the price system $(1, P)$, the second measures the wealth in terms of the bundle $(1, 0)$. Now replace $(1, 0)$ by an alternative bundle $x = (x_0, x_1)$, $x_0 > 0$,. Then $\mathbb{R}_+ \times \mathbb{R}$, and hence A_N, are transformed as follows:

$$(P, W_N) \mapsto t_x(P, W_N) = \left(\frac{1}{x_0 + P x_1} \right) (P, W_N)\,.$$

The price system corresponding to $(1, P)$ in the x-normalization is given by $(1/(x_0 + Px_1), P/(x_0 + Px_1))$. Hence, the first coordinate of $t_x(P, W_N)$ equals $\pi_1 = P/(x_0 + Px_1)$. Moreover, the wealth W_N is the number of units of the bundle $(1, 0)$ which can be bought at the price system $(1, P)$. At the price system (π_0, π_1) the corresponding wealth is $(W_N, 0)(\pi_0, \pi_1) = \pi_0 W_N$. The number of units of the bundle x, given the wealth $\pi_0 W_N$, is $W_x = W_N \pi_0 / (\pi_0 x_0 + \pi_1 x_1) = W_N/(x_0 + Px_1)$. Thus, the second coordinate of $t_x(P, W_N)$ is $W_N/(x_0 + Px_1)$.

Proposition 4. *If prices are normalized with respect to an arbitrary consumption bundle $x \in \mathbb{R}^2_+ \setminus \{0\}$, then the set $A_x = t_x(A_N)$ is convex if and only if A_N is convex.*

[7] This follows from observations in E. Dierker and Grodal (1999). Here we shall present a shorter and more direct argument.

Proof. Consider any points (P, W_N) and (P', W'_N) in A_N. An easy calculation shows that we have, for any $\delta \in [0,1]$,

$$t_x(\delta(P, W_N) + (1-\delta)(P', W'_N)) = \lambda t_x(P, W_N) + (1-\lambda)t_x(P', W'_N)\,,$$

where $\lambda = \delta(x_0 + Px_1)/(x_0 + (\delta P + (1-\delta)P')x_1))$. Note that λ, when considered as a function of $\delta \in [0,1]$, is bijective. Hence, A_x is convex iff A_N is convex. $\square$

Proposition 4 entails that the concavity of the profit function, the expenditure function, or the surplus function does not depend on which commodity bundle is used to normalize prices and measure wealth. This is due to the following fact: If the set below the graph of one of these functions is convex if the N-normalization is used, its image under the mapping t_x is convex.

We use the profit function to illustrate the invariance. Let Π_x denote profits as function of the output price if bundle x has been used to normalize prices and measure wealth. We want to show that the profit function Π_x is concave for any bundle x iff Π_N is concave.

With the price system (π_0, π_1), the firm obtains the profit $\Pi_N(\pi_1/\pi_0)$ in terms of good 0, which corresponds to the value $(\pi_0, \pi_1)(\Pi_N(\pi_1/\pi_0), 0)$. This profit enables the shareholders to buy $\Pi_x(\pi_1) = \pi_0 \Pi_N(\pi_1/\pi_0)$ units of bundle x. Substitution yields

$$\Pi_x(\pi_1) = \frac{1-\pi_1 x_1}{x_0}\,\Pi_N(\frac{\pi_1}{(1-\pi_1 x_1)/x_0})\,,$$

where π_1 lies in the range of the transformed prices with $\pi_1 < 1/x_1$. Hence, we obtain $t_x(P, \Pi_N(P)) = (\pi_1, \Pi_x(\pi_1))$ for all $P \in \mathbb{R}_+$. Clearly, Π_x is concave if and only if $\{\pi_1, r) \mid r \leq \Pi_x(\pi_1)\}$ is convex. Note that, in the above argument, Π_x can be replaced by the expenditure or the surplus as a function of the output price.

Remark 5. *Assume that only normalization rules are used that have an economic interpretation. That is to say, there is a commodity bundle $x \geq 0, x \neq 0$, such that $px = 1$ for all prices under consideration. Wealth is expressed in units of the same bundle x. In that case, the concavity of the profit, expenditure or surplus function does not depend on the normalization rule used. As a consequence, the results in this paper are independent of the choice of the normalization rule.*

7 Nonexistence of S-Efficient Strategies

In the quasilinear case, there can be several real wealth maxima. They need not all be undominated. However, there is always at least one S-efficient real wealth maximum, namely the maximum of shareholders' social surplus.

Now we will consider a framework in which no representative owner exists. In our example there is no undominated attainable allocation. There are two real wealth maxima, but each of them dominates the other.

The example is constructed as follows: The aggregate demand function g of all nonowners is taken as being linear. The group of owners of the firm can "almost" be represented by one agent. In fact, there are two owners of the firm, a large one with a CES utility function and a small one with a quasilinear utility function. The weights are calibrated such that the two real wealth maxima yield approximately the same utility for the large CES owner.

In the example, the profit function Π_N of the firm is not concave. In order to show that the absence of an S-efficient allocation does not depend on the nonconcavity of the profit function, we also consider the concavification $\tilde{\Pi}_N$ of the profit function and note that the phenomenon persists.

There is one firm with constant unit costs $c = 1$. The demand function of the nonowners is given by

$$g(1, P) = 1000 - P.$$

There are two (types of) owners with initial endowments, $e^1 = (1000, 0)$ and $e^2 = (542, 0)$. They have the following CES utility function and a quasilinear utility function, respectively.

$$\begin{aligned} U^1(x_0, x_1) &= x_0^{\frac{10}{11}} + (21x_1)^{\frac{10}{11}} \\ U^2(x_0, x_1) &= x_0 + 144x_1^{\frac{1}{2}} . \end{aligned}$$

The (large) CES shareholder owns the fraction 0.999 of the firm and the (small) quasilinear shareholder the fraction 0.001.

An easy computation yields that the profit function is given by

$$\Pi_N(P) = (P-1)\frac{1000 - P + \frac{72^2}{P^2} + \frac{21^{10}\cdot 10^3}{P(21^{10}+P^{10})}}{1 - (P-1)\frac{21^{10}\cdot 0.999}{P(21^{10}+P^{10})}} .$$

The demand functions of the first and second owner are

$$\begin{aligned} D^1(P) &= \left(\frac{P^{10}(10^3 + 0.999 \cdot \Pi_N(P))}{21^{10} + P^{10}}, \frac{21^{10}(10^3 + 0.999 \cdot \Pi_N(P))}{P(21^{10} + P^{10})} \right) \\ D^2(P) &= \left(542 + 0.001 \cdot \Pi_N(P) - \frac{72^2}{P}, \frac{72^2}{P^2} \right) , \end{aligned}$$

respectively. Shareholders' total demand is $D(P) = D^1(P) + D^2(P)$.

A calculation yields that there are three strategies which satisfy the first order condition $\Pi_N'(P) = D_1(P)$ for real wealth maximization, namely

$$P_A \approx 12.94, \qquad P_B \approx 500.48, \qquad P_C \approx 26.45 .$$

However, as the profit function Π_N is not concave, the first order condition is not sufficient. A direct investigation shows that the two strategies $P_A \approx 12.94$

and $P_B \approx 500.48$ are real wealth maximizing strategies, whereas P_C is not. For instance, $D(P_C)$ lies in the interior of the budget set $AB(P_B)$ associated with P_B.

The aggregate budget curve ABC has a kink since the profit function does not coincide with its concavification $\tilde{\Pi}_N$ [cf. E. Dierker and Grodal (1999), Section 3]. However, the profit function Π_N and its concavification $\tilde{\Pi}_N$ coincide at P_A and P_B. Thus, real wealth is also maximized at P_A and P_B if $\tilde{\Pi}_N$ rather than Π_N is used.

We want to show that the strategies $P_A \approx 12.94$ and $P_B \approx 500.48$ are dominated, more precisely, that either of the real wealth maximizing strategies is dominated by the other one. First, we calculate the utility levels of the two owners when the firm chooses strategy P_B and obtain

$$U^1(D^1(P_B)) \approx 80840.74 \qquad \text{and} \qquad U^2(D^2(P_B)) \approx 801.87\,.$$

In order to show that the strategy P_A dominates strategy P_B, we calculate the aggregate demand at P_A and get $D(P_A) \approx (1496.08,\ 11828.65)$. Now we let $x^2 = (0.2, 31)$ and $x^1 = D(P_A) - x^2$ and obtain the corresponding utility levels of the owners

$$U^1(x^1) \approx 80872.80 \qquad \text{and} \qquad U^2(x^2) \approx 801.96\,.$$

Hence, we have distributed the aggregate demand at P_A such that both owners are better off, that is, P_A dominates P_B.

Similarly, the utility levels at strategy P_A are

$$U^1(D^1(P_A)) \approx 80734.10 \qquad \text{and} \qquad U^2(D^2(P_A)) \approx 1095.69$$

and the aggregate demand at strategy P_B is $D(P_B) \approx (251042.23, 0.021)$. Now we let $x^2 = (1095.70, 0)$ and $x^1 = D(P_B) - x^2$ and obtain the utility levels

$$U^1(x^1) \approx 80748.83 \qquad \text{and} \qquad U^2(x^2) = 1095.70\,.$$

We see that the strategy P_B dominates P_A. Thus, in the example, none of the real wealth maximizing strategies leads to an S-efficient allocation. Moreover, each of the two real wealth maximizing strategies dominates the other.

Remark 6. *In the example, no attainable allocation is S-efficient.*

Proof. According to Proposition 1, the first order condition $\Pi_N'(P) = D_1(P)$ for real wealth maximization holds at any S-efficient allocation. The only strategies satisfying this condition are $P_A, P_B,$ and P_C. Since P_A dominates P_B and vice versa, the only remaining candidate is P_C. However, the utility levels obtained at P_C are $U^1(D^1(P_C)) \approx 11282.17$ and $U^2(D^2(P_C)) \approx 765.40$. Both owners prefer the bundles they obtain at P_A and at P_B. Thus, the strategy P_C is dominated by P_A and by P_B. □

One could conjecture that the nonexistence of S-efficient allocations is due to the fact that the profit function is not concave. We will show that this conjecture is false.

Consider the concavification $\tilde{\Pi}_N$ of the profit function and define the demand function $\tilde{g}$ for the nonowners in such a way that $\tilde{g}$ generates the profit function $\tilde{\Pi}_N$, i.e. $\tilde{g}(P) = (\tilde{\Pi}_N - (P-1)\tilde{D}_1(P))/(P-1)$, where $\tilde{D}_1$ is the aggregate demand of the shareholders when they obtain profit $\tilde{\Pi}_N$. It turns out that $\tilde{\Pi}_N$ is obtained by replacing the graph of Π_N by a straight line in the interval given approximately by $[13.96, 397.14]$. Outside of this interval $\tilde{\Pi}_N$ coincides with Π_N.

First, note that P_A lies to the left of the interval and P_B lies to the right. P_A and P_B are real wealth maximizing strategies in the economy with the concavified profit function $\tilde{\Pi}_N$. As before, P_A dominates P_B and vice versa. Second, if Π_N is replaced by its concavification $\tilde{\Pi}_N$, then P_C is turned into a real wealth maximum. We know that P_C satisfies the first order condition for real wealth maximization without maximizing real wealth. Thus, according to Remark 3, $D(P_C)$ lies in Env, but not in ABC. Clearly, shareholders' aggregate demand function D also changes if the profit function is concavified, and the critical point P_C moves to the nearby point $\tilde{P}_C \approx 29.19$, which lies on the envelope after concavification. Since $\tilde{\Pi}_N$ is concave, $\tilde{P}_C$ must maximize real wealth [see the end of Section 4].

Since $\tilde{\Pi}$ is linear (i.e. barely concave) on a segment around $\tilde{P}_C$, it is not surprising that $\tilde{P}_C$ is dominated by points very close to $\tilde{P}_C$. Here we suppress these calculations and show that, more interestingly, $\tilde{P}_C$ is dominated by both other real wealth maxima, P_A and P_B. The utility levels of the two owners at $\tilde{P}_C$ are

$$U^1(\tilde{D}^1(\tilde{P}_C)) \approx 55116.89 \qquad \text{and} \qquad U^2(\tilde{D}^2(\tilde{P}_C)) \approx 882.36\,.$$

In order to show that P_A dominates $\tilde{P}_C$, we distribute the aggregate demand $\tilde{D}(P_A) = D(P_A) \approx (1496.08, 11828.65)$ as follows: We put $x^2 = (0, 40)$ and $x^1 = D(P_A) - x^2$ and obtain the utility levels

$$U^1(x^1) \approx 80817.34 \qquad \text{and} \qquad U^2(x^2) \approx 910.74\,.$$

Hence, we have distributed the aggregate demand at P_A such that both owners are better off than they were at P_A, that is, P_A dominates $\tilde{P}_C$.

Similarly, to show that P_B dominates $\tilde{P}_C$, we distribute the aggregate demand $\tilde{D}(P_B) = D(P_B) \approx (251042.23, 0.021)$ in the following way: We put $x^2 = (1000, 0)$ and $x^1 = D(P_A) - x^2$ and get utility levels

$$U^1(x^1) \approx 80776.92 \qquad \text{and} \qquad U^2(x^2) = 1000\,.$$

In the concavified economy, the profit function is, of course, not strictly concave. However, by continuity we can easily obtain the same conclusions in an economy with a strictly concave profit function.

Proposition 5. *There are robust examples with concave profit functions in which every real wealth maximum is dominated by another real wealth maximum. As a consequence, no S-efficient strategy exists.*

8 The Equivalence of Real Wealth Maximization and S-Efficiency

The analysis of the representative agent model in Section 5 provides an insight that is absent from the traditional models of general equilibrium theory and industrial organization. Clearly, the particular structure of representative consumer models also presents a risk. In his paper "On the "Law of Demand"", Werner Hildenbrand (1983), p. 998, points out: "There is a *qualitative difference* in market and individual demand functions. This observation shows that the concept of a "representative consumer," which is often used in the literature, does not really simplify the analysis; on the contrary, it might be misleading." Representative consumer models have often been misused. However, we are going to argue that, with a sufficient degree of precaution, the study of representative agent models helps improve general equilibrium theory with imperfect competition.

To illustrate this point, we come back to the discussion of Proposition 3 in terms of surplus maximization or, alternatively, in terms of the utility of the representative owner. There, shareholders' social surplus takes the form

$$S_N(P) = \Pi_N(P) + \sum_{i\in\mathfrak{I}} \int_P^\infty D_1^i(p)dp = \Pi_N(P) - E_N^{Rep}(P) + const\,, \tag{2}$$

that is, it encompasses not only the profits accruing to the representative owner but also his expenditures E_N^{Rep} (which are independent of the utility level in the present case). By contrast, in the usual G.E.- or I.O.-models, expenditures do not appear in the definition of the goal of a firm. Indeed, the models present special cases in which consumers' surplus vanishes. Clearly, if it vanishes, it can be neglected, but the price normalization problem has arisen because it has been ignored that consumers' surplus vanishes in degenerate cases only.

If the goal of the firm is based on shareholders' social surplus rather than on profits, it is natural to impose the concavity assumption on $S_N(P)$ rather than on the profit function $\Pi_N(P)$.[8] Clearly, S_N can be concave only if Π_N is concave since the expenditure function E_N^{Rep} is concave in P. Remember that i's compensation function T^i is implicitly defined by $u^i(P, T^i(P)) = \tilde{U}^i = u^i(\tilde{P}, 0)$, where the tilde indicates the surplus maximum. Thus, if i possesses $W_N^i(P) + T^i(P)$, he can just reach $\tilde{U}^i$. Therefore, the aggregate compensation $T(P) = \sum_{i\in\mathfrak{I}} T^i(P)$ is given by $e_0 + \Pi_N(P) + T(P) = E_N^{Rep}(P)$. Choosing the *const* in (2) to equal e_0, we obtain $S_N(P) = -T(P)$ and $S_N(\tilde{P}) = 0$. Hence, the aggregate compensation

[8] As pointed out in Section 5, S_N is a utility function of the representative owner and individual utility functions are ordinal concepts. However, the concavity of S_N plays an important role in the present context. The representative owner differs from an ordinary consumer in the following sense: It is often appropriate to break down his utility function into a sum of such functions. Here they consist of the profit shares and the consumer's surpluses the individual shareholders obtain [cf. (2)].

function always equals $T(P) = S_N(\tilde{P}) - S_N(P)$. Obviously, T is strictly convex iff the social surplus S_N is strictly concave. Therefore, Proposition 3 can also be interpreted as follows:

Remark 7. *In the quasilinear setting S-efficiency and uniqueness of a real wealth maximum obtain if shareholders' social surplus S_N is a strictly concave function of the strategy P.*

If one leaves the quasilinear framework, the situation becomes more complex. First, the conditions for S-efficiency and uniqueness of a real wealth maximum no longer coincide (see below). Second, according to Remark 6, S-efficient strategies may not exist. However, it will turn out that the concavity of an appropriately defined social surplus function yields S-efficiency of a real wealth maximizing strategy. Hence S-efficient strategies exist.

Now we focus on the relationship between real wealth maximization and S-efficiency. In the light of surplus theory, one is led to proceed as follows [see Luenberger (1995), chapter 6, sections 7 and 8]: Consider any strategy $\check{P}$, and let $\check{U} = (\check{U}^i)_{i\in\mathcal{I}}$ be the associated utility profile of the shareholders. Given $\check{P}$, define shareholders' compensated social surplus as

$$S_N(P, \check{U}) = W_N(P) - \sum_{i\in\mathcal{I}} E_N^i(P, \check{U}^i)\,,$$

where $E_N^i(P, \check{U}^i)$ denotes i's expenditures in terms of the input good 0. We say that $\check{P}$ maximizes social surplus if $\check{P}$ maximizes $S_N(\cdot, \check{U})$.

Let $\hat{P}$ be any real wealth maximizing strategy and $S_N(\cdot, \hat{U})$ the associated social surplus function. The argument for S-efficiency of $\hat{P}$ relies on the inequality $S_N(\cdot, \hat{U}) = W_N(\cdot) - \sum_{i\in\mathcal{I}} E_N^i(\cdot, \hat{U}^i) \leq S_N(\hat{P}, \hat{U}) = 0$, which follows from the concavity of $S_N(\cdot, \hat{U})$.

Theorem . *Assume that Π_N is C^1. Consider any strategy $\hat{P} \in \mathcal{P}$ and let $\hat{U}^i = U^i(D^i(\hat{P}))$ and $\hat{U} = (\hat{U}^i)_{i\in\mathcal{I}}$. Assume that shareholders' compensated social surplus $S_N(\cdot, \hat{U})$ is concave. Then $\hat{P}$ is S-efficient if and only if $\hat{P}$ maximizes shareholders' real wealth.*

Proof. Since $S_N(\cdot, \hat{U})$ is concave, Π_N is concave and Proposition 2 applies. Thus, it suffices to prove that $\hat{P}$ is S-efficient if it satisfies the first order condition for real wealth maximization. In this case, $\partial_P S_N(\hat{P}, \hat{U}) = W_N'(\hat{P}) - \sum_{i\in\mathcal{I}} \partial_P E_N^i(\hat{P}, \hat{U}^i) = D_1(\hat{P}) - \sum_{i\in\mathcal{I}} h_1^i(\hat{P}, \hat{U}^i) = 0$, where h_1^i denotes shareholder i's Hicksian demand for good 1. Hence, the concave function $S_N(\cdot, \hat{U})$ attains its maximum at $\hat{P}$. Hence, $S_N(P, \hat{U}) \leq S_N(\hat{P}, \hat{U})$ for all $P \in \mathcal{P}$.

Now let $V = \{x \in \mathbb{R}_+^2 \mid x = \sum_{i\in\mathcal{I}} x^i,\, U^i(x^i) > \hat{U}^i$ for all $i \in \mathcal{I}\}$ and consider any $x \in V$. For all $P \in \mathcal{P}$ we have $(1, P)x \geq \sum_{i\in\mathcal{I}} E_N^i(P, \hat{U}^i) \geq W_N(P)$. Indeed, $(1, P)x > W_N(P)$ for all $P \in \mathcal{P}$ since preferences are monotone and continuous.

Hence, $x \notin AB(P)$ for all $P \in \mathcal{P}$ and, consequently, $x \notin AB$. Therefore, $V \cap AB = \emptyset$. Since $D(P) \in AB$ for all $P \in \mathcal{P}$, we obtain $D(P) \notin V$ for all $P \in \mathcal{P}$. Hence, $\hat{P}$ is S-efficient. □

Remark 8.

1) *Under the assumptions of the Theorem, real wealth maximization and S-efficiency both coincide with compensated surplus maximization.*

2) *S-efficiency of a real wealth maximum obtains also under the weaker condition that $S_N(\cdot, \hat{U})$ has a unique maximizer and a positive (negative) derivative to the left (right) of the maximizer.*

The latter condition coincides with strict pseudoconcavity. However, pseudoconcavity is not invariant with respect to the choice of the bundle used to normalize prices. In the Theorem, concavity of $S_N(\cdot, \hat{U})$ is assumed in order to employ an invariant assumption yielding the full equivalence of the solution concepts. An S-efficient strategy maximizes real wealth under the weaker assumption that Π_N is concave.

Remember that strategy $\hat{P}$ is S-efficient if there is no strategy P such that shareholders' aggregate demand $D(P)$ can be redistributed in a way that all shareholders will be better off. The proof of the Theorem shows the following stronger property of the real wealth maximizing strategy: There is no other strategy such that shareholders' new wealth can be redistributed in a way that all shareholders can buy a preferred bundle on the market.

The Theorem is based on the assumption that shareholders' compensated social surplus $S_N(\cdot, \hat{U})$ is concave. As in the quasilinear case, concavity of $S_N(\cdot, \hat{U})$ is equivalent to convexity of the aggregate preferred set $\hat{A}$ corresponding to $\hat{U}$ and equivalent to convexity of the sum of the compensation functions $T^i(\cdot, \hat{U}^i): \mathcal{P} \to \mathbb{R}$ defined by the condition $u^i(P, T^i(P, \hat{U}^i)) = \hat{U}^i = u^i(\hat{P}, 0)$.

In the quasilinear case, strict concavity of shareholders' social surplus implies uniqueness and S-efficiency of a real wealth maximum. Clearly, strict concavity of $S_N(\cdot, \hat{U})$ does not imply uniqueness if income effects are permitted. Therefore, we consider shareholders' uncompensated surplus

$$S_N^{unc}(P) = \Pi_N(P) + \int_P^\infty D_1(p)dp \,.$$

Remark 9. *If S_N^{unc} is strictly concave and C^1, the real wealth maximum is uniquely determined.*

Remark 9 obtains since $dS_N^{unc}(P)/dP = \sum_{i \in \mathcal{I}} MW^i(P)$ is strictly decreasing and any real wealth maximum $\hat{P}$ has to satisfy $\sum_{i \in \mathcal{I}} MW^i(\hat{P}) = 0$ [cf. Remark 2]. Note that the conclusion also obtains under the weaker assumption of strict pseudoconcavity.

There is no clear connection between concavity of the compensated surplus function $S_N(\cdot, \hat{U})$ used in the Theorem and concavity of the uncompensated surplus function S_N^{unc}. By differentiation it is easily seen that for any P and corresponding utility profile $U_P = (U^i(D^i(P)))_{i \in \mathfrak{I}}$ we have

$$\frac{\partial^2}{\partial P^2} S_N^{unc}(P) = \frac{\partial^2}{\partial P^2} S_N(P, U_P) + \tag{3}$$

$$\sum_{i \in \mathfrak{I}} \frac{\partial}{\partial W} d_1^i(1, P, W_N^i(P)) \cdot (d_1^i(1, P, W_N^i(P)) - \vartheta^i \Pi_N'(P)) \,. \tag{4}$$

In the absence of income effects term (4) vanishes. However, in general it can have any sign.

9 Conclusion

The price normalization problem arising in general equilibrium models of imperfect competition can be overcome in two different ways. First, the objective of a firm can be described as maximization of shareholders' real wealth. This concept is based on the aggregate demand of the shareholders and does not rely on utility considerations. Second, in the spirit of Drèze's concept of the goal of a perfectly competitive firm in a setting with incomplete markets, we say that strategy P_1 of a firm is dominated by P_2 if shareholders' total demand $D(P_2)$ can be redistributed in such a way that all shareholders will be better off than at P_1. An undominated strategy is called S-efficient. In this paper, we have investigated the relationship between the two goals, real wealth maximization and S-efficiency.

First, we observe that the first order conditions for the two objectives coincide. Second, if the profit function is concave, an S-efficient strategy maximizes real wealth since the first order condition is sufficient for real wealth maximization.

S-efficiency of real wealth maxima is first explored in the setting of quasilinear preferences. In this case, shareholders' social surplus is unambiguously defined. We show that strict concavity of the social surplus function entails the coincidence of both solution concepts. Moreover, they coincide with surplus maximization. Furthermore, strict concavity of the surplus function implies uniqueness.

The quasilinear case exhibits properties that do not carry over smoothly to the general case. If utilities are quasilinear, there always exists an S-efficient strategy. By means of an example, we show that there are economies without quasilinear preferences in which each real wealth maximum is dominated by another and no S-efficient outcome exists.

In the general case, shareholders' social surplus also plays a decisive role. Since there are various nonequivalent versions of the notion of consumer's surplus we point out how we proceed. Each real wealth maximum determines a reference utility profile $\hat{U}$. Compensated social surplus $S_N(\cdot, \hat{U})$ is defined as the difference between the wealth generated and the wealth shareholders need to retain their respective utility levels. Our central result can be summarized as follows: If shareholders' compensated social surplus is concave for every real wealth maximum, both solution concepts coincide. Moreover, if $S_N(\cdot, \hat{U})$ and, hence, Π_N are concave the existence of undominated strategies can be shown.

In the quasilinear case, strict concavity of the social surplus function implies uniqueness of a real wealth maximum. Clearly, the surplus function $S_N(\cdot, \hat{U})$ is a welfare oriented concept based on compensated demand. Therefore, it is not an appropriate tool to establish the uniqueness of a real wealth maximum if income effects are permitted. However, if $S_N(\cdot, \hat{U})$ is replaced by the uncompensated surplus function $S_N^{unc}(P) = \Pi_N(P) + \int_P^\infty D_1(p)dp$, strict (pseudo)concavity of this surplus function implies that there cannot be multiple real wealth maxima.

Our arguments rely on concavity of the social surplus function. Since the price normalization problem is due to a missing invariance property of the profit function, we show that our results are independent of which commodity bundle is used to normalize prices and measure wealth.

The paper shows that surplus theory provides a useful tool to unify different concepts of the objective of a firm acting in the interest of its shareholders.

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